

Energy-efficient joint BS and RS sleep scheduling in relay-assisted cellular networks



Hongbin Chen*, Qiong Zhang, Feng Zhao

School of Information and Communication, Guilin University of Electronic Technology, Guilin 541004, China

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ABSTRACT

The deployment of relay stations (RSs) offers a promising and viable approach to satisfy the increasing need of high data rate in cellular networks. With the expected increase of the number of RSs, the energy efficiency (EE) becomes a crucial system design parameter. One of the most effective energy saving methods is to switch off some stations. In this paper, joint base station (BS) and RS sleep scheduling algorithms are investigated in relay-assisted cellular networks. We aim to maximize the EE of the relay-assisted cellular networks under the spectral efficiency (SE) constraint. First, we establish a mathematical model which is a mixed integer nonlinear fractional programming problem. To solve it with globally optimal solution, a branch and bound (BnB) algorithm based on the denominator interval values of the objective function is designed and its convergence is proved theoretically. Then, two kinds of sub-optimal algorithms are proposed to compare with the optimal algorithm: one is a greedy algorithm and the other is based on the Cellular Automata (CA) theory. Simulation results are presented to demonstrate the effectiveness of our algorithms.

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1. Introduction

Nowadays, the mobile data traffic is increasing exponentially with the popularity of smart mobile devices. Therefore, a great deal of technologies have been considered, one of which is the deployment of relay stations (RSs) in cellular networks to improve network capacity and quality of service (QoS) [1,2]. However, the deployment of RSs also increases energy consumption of cellular networks. Moreover, it has been revealed that up to 80% of energy consumption in a cellular network is attributed to the base stations (BSs) [3]. A large number of techniques for energy saving in cellular networks have been proposed. In order to minimize the total energy cost, a joint BS switching-on/off and user association algorithm

was proposed [4]. The maximum energy saving that could be achieved in a cellular network under a given performance constraint was characterized through optimizing the density of BSs [5]. However, these techniques face great challenges to maintain reliable coverage and QoS while saving energy simultaneously. Therefore, the design of energy-efficient cellular network becomes a crucial goal. To improve the energy efficiency (EE), a lot of methods have been proposed in almost all aspects of cellular networks [6]. Among these approaches, switching off BSs is an effective way to reduce the total energy consumption and improve the EE [7,8]. A survey showed the importance of green mobile networking and emphasized the concept of “sleep mode” in BSs [9]. In recent years, many BS sleep scheduling schemes have been put forward in some studies. To save energy, in [10], the authors proposed the concept of cell zooming which adaptively adjusts cell size according to the traffic load. In [11], a scheme of switching off BSs dynamically was proposed by switching light-load BSs into the sleep mode. In [12], the authors investigated

* Corresponding author. Tel.: +86 15078366203.

E-mail addresses: chbscut@guet.edu.cn (H. Chen), 1033806866@qq.com (Q. Zhang), zhaofeng@guet.edu.cn (F. Zhao).

the network sleep schemes by applying the Markov Decision Processes to reduce energy consumption. Sleeping control and power matching for energy-delay trade-offs in one cell of a cellular network with burst traffic was studied [13]. Deploying RSs in cellular networks can increase coverage and capacity. However, energy consumption also increases. Obviously, it is imperative to reduce energy consumption and improve EE in relay-assisted cellular networks. An energy-efficiency cell breathing and offloading scheme was studied while the BS switching-off aggressiveness was considered [14]. In [15], the authors showed that the cooperation among the relay nodes reduced energy consumption in the transmission toward the end users, and offset the increase of energy consumption of BSs. In [16], the authors formulated a network energy consumption minimization framework, where the constraints of coverage performance and jointly determination of the optimal BS density and BS transmission power were taken into account. Taking coverage probability and wake-up times into consideration, [17] studied the maximization of EE under different sleeping policies by switching off the small cell BSs. A novel EE sleep scheduling protocol was proposed for maximizing the sleep time lengths of each node while satisfying the delay constraints in wireless local area networks [18]. In our early work [19], we proposed a distributed BS sleep scheduling algorithm according to the traffic load considering the fluctuations in both temporal and spatial domains in relay-assisted cellular networks, and optimized the transmission powers of the active BSs via game theory. However, we did not consider that the deployment of RSs increases the total energy consumption in relay-assisted cellular networks. In [20], the authors demonstrated that about 50% small cells could be turned off for saving energy in hyper-cellular networks. A traffic-aware RS sleep control scheme was proposed to put some RSs into sleep when the traffic in their serving areas was lower than a threshold [21]. To the best of our knowledge, researches on RS sleeping are relatively scarce in relay-assisted cellular networks. Accordingly, we design joint BS and RS sleep scheduling algorithms in relay-assisted cellular networks. Due to the formulation of the mixed integer nonlinear fractional programming optimization problem and to obtain the globally optimal solution, we propose a branch and bound (BnB) algorithm which requires much lower computational complexity compared with the exhaustive search method. The proposed BnB algorithm is based on the denominator interval values of the objective function to obtain the globally optimal solution with proper branching and bounding rules. Then, two heuristic algorithms, i.e., a greedy algorithm and an algorithm based on the Cellular Automata (CA) theory are proposed to compare with the BnB algorithm. In general, the CA theory has been investigated in wireless sensor networks to save energy of nodes and prolong the network lifetime. Some attempts have also been made by applying the CA theory in cellular networks. The authors developed a self-organizing channel assignment scheme by applying the CA theory in wireless cellular systems [22]. In [23], a fractional frequency reuse scheme was proposed based on the CA theory in cellular networks. There is little work on sleep scheduling of BSs

and RSs in cellular networks based on CA theory. Our work differs from the above-mentioned works as follows.

Firstly, we construct a mixed integer nonlinear fractional programming mathematical model to study the sleep scheduling of BSs and RSs according to the system model. Secondly, the BnB algorithm based on the denominator interval values of the objective function is proposed to obtain the globally optimal EE under the spectral efficiency (SE) constraint in the relay-assisted cellular network. Finally, a greedy algorithm and an algorithm based on the CA theory are presented to compare with the BnB algorithm and the convergence of the algorithms is demonstrated.

The remainder of this paper is structured as follows. In Section 2, we describe the relay-assisted cellular network model and formulate the optimization problem. In Section 3, the joint BS and RS sleep scheduling algorithms are proposed, and the BnB algorithm based on the denominator interval values of the goal function is employed to obtain the globally optimal EE. Furthermore, the greedy and CA algorithms are proposed to compare with the BnB algorithm. In Section 4, performance of the proposed algorithms is demonstrated by some numerical results. Finally, concluding remarks are made in Section 5.

2. System model and problem formulation

2.1. System model

We consider a downlink, two-hop, and hexagonal multi-cell relay-assisted cellular network, where the BSs are located at the center of each cell and the RSs are deployed in the edge of the cells uniformly to service the cell-edge Mobile Users (MUs). Moreover, we do not consider the coordinated multi-point transmission. We consider that there are H cells in the relay-assisted cellular network. The sets of MUs and RSs are denoted by $\mathcal{N} = \{N_1, N_2, \dots, N_H\}$ and $\mathcal{K} = \{K_1, K_2, \dots, K_H\}$ respectively. There are K_h RSs and N_h MUs staying within the h th cell, for $h \in \{1, 2, \dots, H\}$, which follow the Poisson distribution with parameter λ_K and λ_N , respectively. Without loss of generality, we assume that the number of MUs is greater than or equal to the number of RSs, i.e., $N_h \geq K_h$. Each RS is allowed to associate with only one MU but the BS is allowed to associate with many MUs. The model of the h th cell is shown in Fig. 1.

The decode-and-forward relaying protocol is adopted and it is assumed that the decoding is always successful. Moreover, the interference among cells is not considered. All the stations are assumed to be half-duplex and are equipped with a single antenna. Each transmission frame is divided into two stages, and the direct links between BS and MUs that are associated with the RSs are ignored, e.g. due to shadowing. In the first stage, the BS transmits a signal to each associated MU or RS, and the received signals at the RSs and the MUs in the h th cell can be represented by

$$y_{k,h}^{(1)} = \sqrt{P_b} g_{k,h}^{(1)} x + n_s^{(1)}, k = 1, 2, \dots, K_h \quad (1)$$

$$y_{n,h}^{(1)} = \sqrt{P_b} g_{n,h}^{(1)} x + n_s^{(1)}, n = K_h + 1, \dots, N_h \quad (2)$$

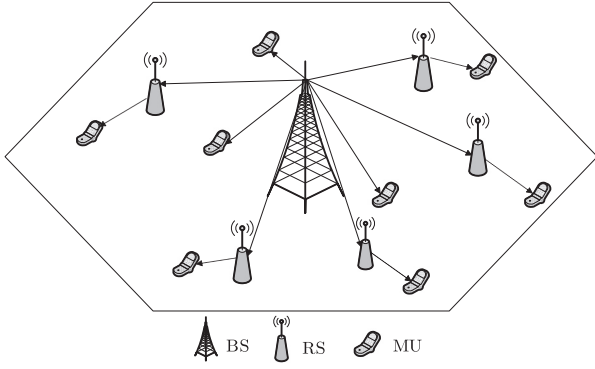


Fig. 1. The model of h th cell in the relay-assisted cellular network.

respectively, where P_b is the transmission power of BS, x is the signal transmitted by the BS with zero mean and unit variance, $g_{k,h}^{(1)}$ and $g_{n,h}^{(1)}$ are channel coefficients from the h th BS to the k th RS and the n th MU, respectively (including path loss and shadow fading), $n_s^{(1)}$ is an additive white Gaussian noise (AWGN) with zeros mean and variance σ^2 .

In the second stage, the RSs forward their decoded signals to the associated MUs. The received signals at the corresponding MUs in the h th cell can be represented by

$$y_{k,h}^{(2)} = \sqrt{P_r} g_{k,h}^{(2)} x + n_s^{(2)} \quad (3)$$

where P_r is the transmission power of RS, $g_{k,h}^{(2)}$ is the channel coefficient from the k th RS to the associated MU (including path loss and shadow fading), $n_s^{(2)}$ is an AWGN with zero mean and variance σ^2 . Accordingly, the signal-to-noise ratios (SNRs) of the MUs associated with the BSs directly and the k th RS in the h th cell can be respectively calculated as

$$SNR_{n,h} = \frac{P_b |g_{n,h}^{(1)}|^2}{\sigma^2}, n = K_h + 1, \dots, N_h \quad (4)$$

$$SNR_{k,h} = \frac{P_r |g_{k,h}^{(2)}|^2}{\sigma^2}, k = 1, 2, \dots, K_h \quad (5)$$

We consider that the BS and RS have an active mode and a sleep mode, and the power consumption of the modes are as follows [24]:

$$P_c = \begin{cases} P_{i0} + \Delta_{pi} P_i, & \text{active mode} \\ P_{i_sleep}, & \text{sleep mode} \end{cases}, i = b, r \quad (6)$$

where b and r represent the BS and RS, respectively, P_{i0} and P_{i_sleep} are the static power consumption of the stations in the active mode and sleep mode, respectively, Δ_{pi} accounts for power consumption that scales with the average radiated power, and P_i is the transmission power of the stations. We set the power consumption parameters of the BS and RS as in Table 1 [25]:

We assume that all MUs have the same bandwidth requirement w . The information rate of each MU that is associated with the BSs directly and the k th RS in the h th cell can be respectively expressed as

$$R_{n,h} = \frac{1}{2} \cdot w \cdot \log_2(1 + SNR_{n,h}), n = K_h + 1, \dots, N_h \quad (7)$$

Table 1
Power consumption parameters.

Type	P_{i0}	Δ_{pi}	P_{i_sleep}	P_i
BS	130.0 W	4.7	75 W	20 W
RS	56 W	2.6	39 W	2 W

$$R_{k,h} = \frac{1}{2} \cdot w \cdot \log_2(1 + SNR_{k,h}), k = 1, 2, \dots, K_h \quad (8)$$

Consequently, the h th cell throughput can be written as $R_h = \sum_{k=1}^{K_h} R_{k,h} + \sum_{n=K_h+1}^{N_h} R_{n,h}$. The EE in the h th cell is $\eta_{EE,h} = \frac{R_h}{P_h}$ and the total SE and EE of the system can be respectively expressed as

$$\eta_{SE} = \frac{\sum_{h=1}^H R_h}{B}, \eta_{EE} = \frac{\sum_{h=1}^H R_h}{\sum_{h=1}^H P_h} \quad (9)$$

where P_h is the energy consumption in the h th cell, and $B = \sum_{h=1}^H w \cdot N_h$ is the system bandwidth.

2.2. Problem formulation

A set of RSs in the h th cell and the BSs of the system can be denoted as $\{RS_1, RS_2, \dots, RS_{K_h}\}$ and $\{BS_1, BS_2, \dots, BS_H\}$, respectively. The state of the BSs and RSs of the h th cell are denoted as $\mathcal{A} = (a_1, a_2, \dots, a_H)$ and $\mathcal{B} = (b_1, b_2, \dots, b_k, \dots, b_{K_h})$, respectively, where $a_h = \begin{cases} 1 & \text{active} \\ 0 & \text{sleep} \end{cases}$ and $b_k = \begin{cases} 1 & \text{active} \\ 0 & \text{sleep} \end{cases}$ are the state of the BS and the RS in the h th cell in active/sleep mode, respectively.

Therefore, considering the sleep scheduling of the BS and RS, the throughput and the energy consumption in the h th cell can be respectively expressed as

$$R_h = \sum_{k=1}^{K_h} b_k \cdot \frac{1}{2} \cdot w \cdot \log_2(1 + SNR_{k,h}) + \sum_{n=K_h+1}^{N_h} \frac{1}{2} \cdot w \cdot \log_2(1 + SNR_{n,h}) \quad (10)$$

$$P_h = \left(P_{b0} + \Delta_{pb} P_b + \sum_{k=1}^{K_h} [(P_{r0} + \Delta_{pr} P_r) \cdot b_k + P_{r_sleep} \cdot (1 - b_k)] \right) \cdot a_h + P_{b_sleep} \cdot (1 - a_h) \quad (11)$$

Therefore, the total throughput and energy consumption of the system based on formulas (10) and (11) can be respectively written as

$$R_{tot} = \sum_{h=1}^H R_h = \sum_{h=1}^H a_h \cdot \left[\sum_{k=1}^{K_h} b_k \cdot \frac{1}{2} \cdot w \cdot \log_2(1 + SNR_{k,h}) + \sum_{n=K_h+1}^{N_h} \frac{1}{2} \cdot w \cdot \log_2(1 + SNR_{n,h}) \right] \quad (12)$$

$$P_{tot} = \sum_{h=1}^H P_h = \sum_{h=1}^H \left[\left(P_{b0} + \Delta_{pb}P_b + \sum_{k=1}^{K_h} [(P_{r0} + \Delta_{pr}P_r) \cdot b_k + P_{r_sleep} \cdot (1 - b_k)] \right) \cdot a_h + P_{b_sleep} \cdot (1 - a_h) \right] \quad (13)$$

Based on formula (12) and (13), the SE and EE of the system can be respectively calculated as

$$\eta_{SE} = \frac{R_{tot}}{B} = \frac{\sum_{h=1}^H a_h \cdot \left[\sum_{k=1}^{K_h} b_k \cdot \frac{1}{2} \cdot w \cdot \log_2(1 + SNR_{k,h}) + \sum_{n=K_h+1}^{N_h} \frac{1}{2} \cdot w \cdot \log_2(1 + SNR_{n,h}) \right]}{\sum_{h=1}^H w \cdot N_h} \quad (14)$$

$$\eta_{EE} = \frac{R_{tot}}{P_{tot}} = \frac{\sum_{h=1}^H a_h \cdot \left[\sum_{k=1}^{K_h} b_k \cdot \frac{1}{2} \cdot w \cdot \log_2(1 + SNR_{k,h}) + \sum_{n=K_h+1}^{N_h} \frac{1}{2} \cdot w \cdot \log_2(1 + SNR_{n,h}) \right]}{\sum_{h=1}^H \left[(P_{b0} + \Delta_{pb}P_b + \sum_{k=1}^{K_h} [(P_{r0} + \Delta_{pr}P_r) \cdot b_k + P_{r_sleep} \cdot (1 - b_k)]) \cdot a_h + P_{b_sleep} \cdot (1 - a_h) \right]} \quad (15)$$

In this work, we aim to maximize the EE under the SE constraint by considering the BS and RS sleep scheduling in the relay-assisted cellular network. The optimization problem can be formulated as

$$\begin{aligned} \max_{\mathcal{A}, \mathcal{B}} \quad & \eta_{EE} \\ \text{s.t.} \quad & \eta_{SE} \geq \bar{\eta}_{SE} \\ & \sum_{h=1}^H a_h \geq 1 \\ & a_h = \{0, 1\}, h \in \{1, \dots, H\} \\ & b_k = \{0, 1\}, b \in \{1, \dots, K_h\} \end{aligned} \quad (16)$$

where $\bar{\eta}_{SE}$ is the minimum required SE. The first constraint meets the need of the SE of the system. The second constraint dictates not all the BSs get into sleep to ensure the QoS of MUs, and the last two constraints dictate the state of BS and RS can only take the value 0 or 1 for sleep mode or active mode.

Proposition. *Since the elements of \mathcal{A}, \mathcal{B} are only 0 or 1, we relax them to continuous variables in the closed interval [0,1]. The system spectral efficiency η_{SE} is not a convex function with respect to \mathcal{A}, \mathcal{B} . Hence, the optimization problem (16) is not a convex optimization problem.*

Proof. Refer to [Appendix](#).

Since the optimization problem (16) is not a convex problem which is difficult to be transformed into a relaxed problem, it will be hardly solved compared with a general optimization problem. Besides, solving a nonconvex optimization problem requires a lot of computation. The optimization problem (16) is a mixed integer nonlinear fractional programming for which finding the optimal solution is extreme challenging in general. A simple approach to solve it is exhaustive search, but it has very high complexity. The specific form of the problem motivates us to construct two sub-problems for simple analysis and to propose the BnB algorithm to obtain the globally optimal solution. The BnB algorithm may still have high complexity, thus two heuristic algorithms based on greedy principle and CA theory are proposed.

3. Sleep scheduling algorithm

The above objective function includes multiple variables while it is difficult to solve the optimization problem directly. What is more, because the BS and RS belong to different network architecture level, the BS and RS sleep scheduling should be discussed hierarchically. Therefore, the above optimization problem is divided into two sub-problems.

Firstly, the BS sleep scheduling algorithm is designed while all the RSs are active, that is $b_k = 1$. Therefore, the above optimization problem can be simply made as

$$\begin{aligned} \max \quad & \eta_{EE1} = \frac{\alpha_1 a_1 + \alpha_2 a_2 + \dots + \alpha_H a_H}{\beta_1 a_1 + \beta_2 a_2 + \dots + \beta_H a_H + \mu} \\ \text{s.t.} \quad & \alpha_1 a_1 + \alpha_2 a_2 + \dots + \alpha_H a_H \geq \bar{\eta}_{SE} \\ & a_1 + a_2 + \dots + a_H \geq 1 \\ & a_h = \{0, 1\}, h \in \{1, \dots, H\} \end{aligned} \quad (17)$$

where $\alpha_h = \sum_{k=1}^{K_h} \frac{1}{2} \cdot w \cdot \log_2(1 + SNR_{k,h}) + \sum_{n=K_h+1}^{N_h} \frac{1}{2} \cdot w \cdot \log_2(1 + SNR_{n,h})$, $\mu = H \cdot P_{b_sleep}$ and $\beta_h = P_{b0} + \Delta_{pb}P_b + (P_{r0} + \Delta_{pr}P_r) \cdot K_h - P_{r_sleep}$.

Secondly, we consider the RSs sleep scheduling in one cell, that is $H = 1$ and $a_h = 1$. Then, the above optimization problem can be simplified as

$$\begin{aligned} \max \quad & \eta_{EE2} = \frac{\varepsilon_1 b_1 + \varepsilon_2 b_2 + \dots + \varepsilon_K b_K + \rho}{\delta(b_1 + b_2 + \dots + b_K) + \zeta} \\ \text{s.t.} \quad & \varepsilon_1 b_1 + \varepsilon_2 b_2 + \dots + \varepsilon_K b_K + \rho \geq \bar{\eta}_{SE} \\ & b_k = \{0, 1\}, b \in \{1, \dots, K\} \end{aligned} \quad (18)$$

where $\varepsilon_k = \frac{1}{2} \cdot w \cdot \log_2(1 + SNR_{k,h})$, $\rho = \sum_{n=K+1}^N \frac{1}{2} \cdot w \cdot \log_2(1 + SNR_{n,h})$,

$\delta = P_{r0} + \Delta_{pr}P_r - P_{r_sleep}$, and $\zeta = P_{b0} + \Delta_{pb}P_b + P_{r_sleep} \cdot K$. Besides, N and K are the number of MUs and RSs in one cell respectively.

Since the EE of the above two objective functions in (17) and (18) are determined by the variables in numerator and denominator. Moreover, the numerator, denominator, and constraints are linear. Both optimization problems (17) and (18) are integer linear fractional programming problems [26]. An exhaustive search algorithm can be adopted to obtain the optimal solution when the variables are few. However, it has a high computational complexity $O(2^H)$ and $O(2^K)$, respectively, when there are a lot of variables.

Accordingly, to avoid the exponential growth of computational complexity and obtain the globally optimal solution, the BnB algorithm is proposed.

3.1. Branch and bound algorithm

In the worst case, the BnB algorithm ends up with examining for all possibilities, and thus the resulting complexity is equal to that of exhaustive search. However, with proper branching and bounding rules, a large number of subsets can be eliminated in the BnB algorithm which potentially results in significant complexity reduction compare with exhaustive search.

The basic idea is constructing the initial upper and lower bound functions of the objective function. Then, the denominator interval values are binary. Finally, we solve the linear programming sub-problems which are determined by the sub-interval through making the upper bound decrease and the lower bound increase. The upper bound is close to the optimal value of the objective function ultimately.

Since $\eta_{EE1} > 0$, maximizing this objective function in (17) is equivalent to

Minimize

$$f = 1/\eta_{EE1} \quad (19)$$

Accordingly, we need to solve the following optimization problem (19.1) to obtain an optimal value:

$$\begin{aligned} \min f &= \frac{\beta_1 a_1 + \beta_2 a_2 + \dots + \beta_H a_H + \mu}{\alpha_1 a_1 + \alpha_2 a_2 + \dots + \alpha_H a_H} \\ \text{s.t. } &\alpha_1 a_1 + \alpha_2 a_2 + \dots + \alpha_H a_H \geq \bar{\eta}_{SE} \\ &a_1 + a_2 + \dots + a_H \geq 1 \\ &a_h = \{0, 1\}, h \in \{1, \dots, H\} \end{aligned} \quad (19.1)$$

First of all, the upper and lower bound functions of the objective function in (19.1) are calculated. For describing easily, we define $\mathcal{X}(a) = \beta_1 a_1 + \beta_2 a_2 + \dots + \beta_H a_H + \mu$, $\mathcal{Y}(a) = \alpha_1 a_1 + \alpha_2 a_2 + \dots + \alpha_H a_H$, and the constraint is $\mathbf{A}a \geq \mathbf{b}$, $a = (a_1, \dots, a_h, \dots, a_H)^T$, where $\mathbf{A} = [a_1, a_1, \dots, a_H; 1, 1, \dots, 1]$ and $\mathbf{b} = [\bar{\eta}_{SE}, 1]^T$, for $h \in \{1, \dots, H\}$. It is easy to solve the two problems:

$$\begin{cases} \min \mathcal{Y}(a) \\ \text{s.t. } \mathbf{A}a \geq \mathbf{b} \end{cases} \quad \text{and} \quad \begin{cases} \max \mathcal{Y}(a) \\ \text{s.t. } \mathbf{A}a \geq \mathbf{b} \end{cases} \quad (19.2)$$

We assume that the optimal solutions are a_1 and a_2 , and the optimal values are L and U , respectively. Therefore, the upper and lower bound functions of the objective function are

$$\frac{\mathcal{X}(a)}{U} \leq \frac{\mathcal{X}(a)}{\mathcal{Y}(a)} \leq \frac{\mathcal{X}(a)}{L} \quad (19.3)$$

The problem (19.2) turns to solving the following linear programming problems:

$$\begin{cases} \min \frac{\mathcal{X}(a)}{U} \\ \text{s.t. } \mathbf{A}a \geq \mathbf{b} \end{cases} \quad \text{and} \quad \begin{cases} \min \frac{\mathcal{X}(a)}{L} \\ \text{s.t. } \mathbf{A}a \geq \mathbf{b} \end{cases} \quad (19.4)$$

Assume that the optimal solutions to (19.4) are a_0^1 and a_0^2 , and the optimal values are l_0 and u_0 , respectively. For the optimal value in (19.1) f^* , l_0 and u_0 , we have the following conclusion:

Theorem 3.1. Assume $l_0 = \min \frac{\mathcal{X}(a)}{U}$, $u_0 = \min \frac{\mathcal{X}(a)}{L}$, $f^* = \min \frac{\mathcal{X}(a)}{\mathcal{Y}(a)}$, then $l_0 \leq f^* \leq u_0$.

Algorithm 1 The BnB Algorithm.

1. Initialization: make the initial interval $S_1 = [L_1, U_1] = [L, U]$, and the current lower bound and upper bound are $lb^{(0)} = l_0$ and $ub^{(0)} = \min\{f(a_0^1), f(a_0^2)\}$, respectively, initialize $\mathcal{P} = \{S_1\}$, convergence accuracy ν , and iteration $i = 1$.
2. Stop criterion: if $ub^{(i)} - lb^{(i)} \leq \nu$, go to Step 3; else go to Step 4.
3. Output: the optimal solution and the optimal value are a and $ub^{(i)}$, respectively.
4. Branch: select the maximum length range $S_j = [L_j, U_j]$ from \mathcal{P} , make $V_j = \frac{1}{2}(L_j + U_j)$, $S' = [L_j, V_j]$, and $S'' = [V_j, U_j]$, then make $\mathcal{P} = (\mathcal{P} \setminus \{S_j\})$ and $\mathcal{R} = \{S', S''\}$.
5. Determine bounds and cut down branch: select $S \in \mathcal{R}$ and solve the problem (19.5). Assume the optimal solution and optimal value are a_S^* and l_S , respectively.
If $l_S \geq ub^{(i-1)}$, $\mathcal{R} = \mathcal{R} \setminus \{S_j\}$. If $\mathcal{R} = \emptyset$, go to Step 4, else if there has an interval in \mathcal{R} , solve the problem (19.5) and obtain the optimal solution which is a_S^* and the optimal value which is l' , the upper bound and the lower bound are $lb^{(i)} = l'$ and $ub^{(i)} = \min\{ub^{(i-1)}, f(a_S^*)\}$, respectively, else if there are two intervals in \mathcal{R} , solve the problem (19.5) and obtain the optimal solutions which are a_S^1 and a_S^2 , respectively, and the optimal values are l' and l'' , respectively, the upper bound and lower bound are $lb^{(i)} = \min\{l', l''\}$ and $ub^{(i)} = \min\{ub^{(i-1)}, f(a_S^1), f(a_S^2)\}$, respectively. Then, make $\mathcal{P} = \mathcal{P} \cup \mathcal{R}$ and $i = i + 1$, and go to Step 2.

Proof. Assume $f^* = \min \frac{\mathcal{X}(a^*)}{\mathcal{Y}(a^*)}$, through the definition of U , we know $\frac{\mathcal{X}(a^*)}{U} \leq \frac{\mathcal{X}(a^*)}{\mathcal{Y}(a^*)}$, but $l_0 = \min \frac{\mathcal{X}(a)}{U} \leq \frac{\mathcal{X}(a^*)}{U}$, therefore $l_0 \leq f^*$. The same way can be used to prove $f^* \leq u_0$. \square

lb and ub are used to represent the lower bound and upper bound of the objective function, respectively. Hence, the initial lower bound and upper bound are $lb^{(0)} = l_0$ and $ub^{(0)} = \min\{f(a_0^1), f(a_0^2)\}$, respectively. Initialize $S_1 = [L_1, U_1] = [L, U]$, a set of \mathcal{P} includes the sub-interval of the $[L, U]$, and make $\mathcal{P} = S_1$, select the maximum length interval $S_j = [L_j, U_j] \subset [L, U]$ ($j \in I(\mathcal{P})$, $I(\mathcal{P})$ is an index set) from \mathcal{P} , make $V_j = \frac{1}{2}(L_j + U_j)$, $S' = [L_j, V_j]$ and $S'' = [V_j, U_j]$, then make $\mathcal{P} = (\mathcal{P} \setminus \{S_j\}) \cup \{S', S''\}$. For each sub-interval S_j in \mathcal{P} , it is easy to obtain the linear programming problem

$$\begin{aligned} \min \frac{\mathcal{X}(a)}{U_j} \\ \text{s.t. } \mathbf{A}a \geq \mathbf{b} \\ L_j \leq \mathcal{Y}(a) \leq U_j \end{aligned} \quad (19.5)$$

The BnB algorithm based on the denominator interval values of the objective function is described as follows **Algorithm 1**.

If the BnB algorithm is terminated in a finite step i , it is concluded that the optimal solution and the optimal value of the objective function can be obtained and the algorithm converges. Otherwise, if the BnB algorithm is terminated in an infinite step, the convergence of the algorithm is proved by the following theorem.

Theorem 3.2. (Convergence behavior): If the proposed algorithm is terminated in an infinite step, then lb and ub have the following relation with the increase of the iteration i : $\lim_{i \rightarrow \infty} (ub - lb) = 0$.

Proof. In the step i , set \mathcal{P}_i to represent the sub-interval sets, which is obtained by the interval S_{i-1} in \mathcal{P}_{i-1} based on the strategy of the algorithm. By the definition of S_j

Algorithm 2 Greedy BS switch-off algorithm.

1. Initialization: set $a_h = 1, \forall h \in \mathcal{H}, \mathcal{H} = \{1, 2, \dots, H\}$, make $op = \eta_{EE}^*(\mathcal{A})$ and $iter = 1$.
2. While $\mathcal{A} \neq \emptyset$ do
3. Let $h^* = \arg \min_{h \in \mathcal{H}} \eta_{EE,h}^*$, set $a_{h^*} = 0$, and calculate $\eta_{EE}^*(\mathcal{A})$ by solving (17).
4. If (17) is feasible and $\eta_{EE}^*(\mathcal{A}) > op$, then assign $op := \eta_{EE}^*(\mathcal{A})$, else set $a_{h^*} = 1$.
end if
5. Update $iter := iter + 1$.
6. End while

and the infinite termination of the algorithm, the interval length of S_i is $d(S_i) = U_{ji} - L_{ji} \rightarrow 0$. Thus, there has $\lim_{i \rightarrow \infty} (ub^{(si)} - lb) = 0$ ($ub^{(si)}$ is the upper bound of the objective function in S_i). By the definition of ub , it is obvious that there has $ub^{(si)} \geq ub$ in the step i . Therefore, $\lim_{i \rightarrow \infty} (ub - lb) = 0$. \square

Then, the reciprocal of the optimal value in (19.1) is the optimal EE of the optimization problem (17). For the optimization problem (18), the procedure is similar to the above algorithm. Since the complexity of the BnB algorithm is related to the branch rule and the bound, we can know that the complexity of the BnB algorithm is lower than that of exhaustive search due to cutting down the branch.

By analyzing the BnB algorithm, it is equivalent to a binary tree search with depth l , where l is the times of convergence. In the worst case, each branch is feasible, and the complexity of the BnB algorithm is $O(2^l)$. However, the use of pruning will make the times of comparison be far less than $O(2^l)$ in the actual operation. The proposed BnB algorithm terminates when the convergence accuracy is satisfied or when \mathcal{P} is empty. However, in reality, finding the globally optimal solution by using the BnB algorithm generally brings high complexity. Therefore, two heuristic algorithms are proposed in the following.

3.2. Two sub-optimal algorithms

Despite the fact that the proposed BnB algorithm requires lower complexity compared with exhaustive search, the complexity of the proposed BnB algorithm is still high for large-scale networks. In this subsection, we propose two sub-optimal algorithms to compare with the BnB algorithm.

3.2.1. Greedy switch-off algorithm

We find a cell with the minimum EE and make it sleep. Then, we calculate the total EE and set up it as the optimal value. Specifically, we switch off the BSs by setting $a_h = 0$ in each step and compute the EE. If the result is feasible and leads to better EE, the set of BSs are switched off and the iterative procedure continues.

Algorithm 2 terminates after H steps, i.e., the complexity of the greedy is $O(H)$. The RS sleep scheduling is similar to the greedy BS sleep scheduling algorithm.

3.2.2. Cellular automata algorithm

By applying the self-organizing characteristics of the cellular network, a sleep scheduling algorithm based on CA theory is proposed. CA is a decentralized, discrete space-time system that can be used to model physical systems [27]. It consists of a large number of small cells. The state of an individual simple cell changes synchronously and is stimulated by state updating in neighboring cells. A 2-D CA can be formally defined as a five tuple (C, V, Q, ψ, t) .

- C represents a series of cells in the CA, $C = \{C_{nb}, nb = 1, 2, \dots, N\}$, where N is the size of the neighbor. V is a set of neighboring cells, which is a finite subset of C , i.e., $V \subset C$.
- Q represents a finite set of configuration state of each cell, $Q_m^{t+1} = f(Q_{m-N}^t \dots Q_{m-1}^t, Q_m^t, Q_{m+1}^t \dots Q_{m+N}^t)$, where Q_m^t is the state of the m th cell at time t .
- ψ is the localized transmission rule, and the local rule is a function $f: Q^N \rightarrow Q$.
- t is the transmission time of a cell moving from its current state to its final state.

Each cell updates its individual state based on its current state and that of the adjacent cells. The most common types of neighbors are the Von Neumann, Moore, and Hexagonal [28]. In this paper, when considering the BS sleep scheduling, a Hexagonal neighbor model is adopted because it is analogous to our system model where a hexagonal cellular cell shape is considered. While considering the RS sleep scheduling, a Moore neighbor model is adopted for simple analysis. Based on the Life Game, which considers the growth condition in the nature to guarantee the whole system not too desolate or crowded, the CA local rules can be defined. In judging the state of a current cell based on the local rule of the CA theory, the current cell is called the reference cell.

Local rule 1 for the BS sleep scheduling: given each cell have six neighboring cells in our system model, if there are two neighboring cells which are active, the state of the cell will not be changed. If more than two neighboring cells are active, the reference cell will go into sleep to save energy. If there is only one active neighboring cell, the reference cell must be active.

Similarly, the local rule 2 for the RS sleep scheduling can be defined. Taking a Moore neighbor model into account, each cell has eight neighboring cells. If three neighboring cells are active, the reference cell must be active. If there are two active neighboring cells, the state of the cell will remain unchanged. In other cases, the reference cell must be sleep considering the Life Game, which considers the growth condition in the nature to guarantee the whole system not too desolate or crowded. Then, the sleep scheduling algorithm based on CA is described as follows [Algorithm 3](#).

The number of active BS and RS gets to convergence gradually depending on the CA. The analysis of a steady state is indicated theoretically in [29]. The steady state means the existence of a steady state distribution. The state of the cell can reach convergence by evolution gradually based on the CA theory. In cellular networks, the system converges to a steady state after sufficient negotiations and coordination of neighboring cells using the local rules.

Algorithm 3 Sleep scheduling of BS and RS based on CA.

1. Initiation: a set of state of the BS and RS are $A = (a_1, a_2, \dots, a_h, \dots, a_H)$ and $B = (b_1, b_2, \dots, b_k, \dots, b_{K_h})$ respectively.
 2. Based on the local rule 1 and 2 of the CA, the state of BS (a_1, a_2, \dots, a_H) and the state of RS $(b_1, b_2, \dots, b_{K_h})$ is obtained respectively in time $t + 1$, where $t = 0, 1, \dots$
 3. If the values of (a_1, a_2, \dots, a_H) and $(b_1, b_2, \dots, b_{K_h})$ satisfy the constraints in (17) and (18), Calculate the EE η_{EE1} and η_{EE2} , $t = t + 1$. Else return 2, until the number of active BS and RS get to convergence.
- End

Table 2
Simulation parameters.

Parameter	Value
Total system bandwidth	10 MHz
Inter-site distance d_{ISD} (BS-BS)	1000 m
Path loss (dB)	$128.1 + 37.6 \log_{10}(d)$ (d in km)
Noise power	-174 dBm
Minimum value of SE	0.5 bit/s/Hz
Convergence accuracy ν	$1.0e-10$
Shadow fading	$\exp(1)$

The relevant work [30] has indicated that the system converges to a steady state and shows the final steady state by simulation. We also show the convergence of the CA algorithm in the simulation.

Remark: In our system model, each RS is allowed to associate with one MU only, in order to simplify the expressions of the SNRs of the MUs. Each RS can associate with multiple MUs. In this case, the expressions of the SNRs, the system spectral efficiency, and the energy efficiency will change, but they will not affect the implementation of the proposed algorithms.

4. Simulation results

In this section, we carry out numerical simulations to evaluate the performance of the proposed algorithms. Some simulation parameters are specified in Table 2 [31].

The side length of the hexagonal cell can be calculated as $r_{cell} = \frac{d_{ISD}}{\sqrt{3}}$. The distance between BS and RS is set to $d_{RS} = 0.7r_{cell}$ in one cell. Here, in the h th cell, the distance from the BS to the n th MU $d_{n,h}$ and from the k -th RS to the MU $d_{k,h}$ follows uniform distribution $d_{n,h} \sim U[0, 0.7r_{cell}]$ and $d_{k,h} \sim U[0, 0.3r_{cell}]$, respectively. We set $H = 9$. The computer program is repeated for 1000 times.

Fig. 2 shows the upper and lower bound evolution of the objective function in (19.1) with $\lambda_K = 5, \lambda_N = 50$. It is seen that the upper bound decreases while the lower bound increases until it satisfies the convergence accuracy and the convergence behavior of Algorithm 2 is proved.

By taking the reciprocal of the value of the upper bound, we can obtain the optimal EE value with the BS sleep scheduling of the system. In order to prove the optimality of our proposed algorithm, the globally optimal

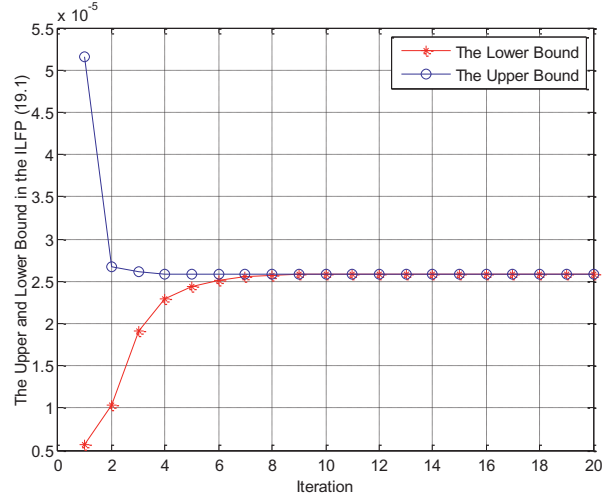


Fig. 2. Upper and lower bound evolution of the objective function in (19.1) with $\lambda_K = 5, \lambda_N = 50$.

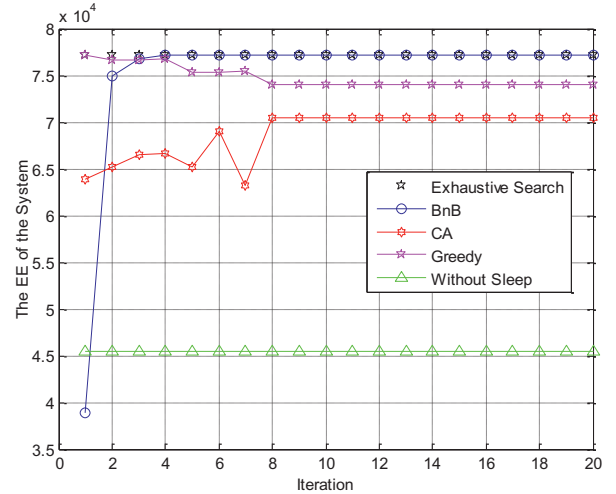


Fig. 3. The EE of the system with $\lambda_K = 5, \lambda_N = 50$.

solution is also plotted by exhaustive search. In Fig. 3 we compare the EE of the system with exhaustive search, BnB, greedy, and CA algorithms.

It is noticed that the BnB algorithm achieves the globally optimal EE of the system and coincides with exhaustive search which has a high computational complexity, and the EE of the BnB algorithm is better than that of the greedy and the CA algorithms. The EE of the greedy algorithm is superior to the CA algorithm which does not need central control. Moreover, the EE of the proposed algorithms is better than that of scheme without sleep scheduling. The convergence behaviors of the proposed algorithms are demonstrated.

In Fig. 4, under different λ_K and λ_N , we compute the energy consumption when the proposed algorithms get to convergence. As shown in Fig. 4, the BnB algorithm has the minimum energy consumption while the algorithm

Table 3
Algorithm comparisons.

	Performance of solution	Operation manner	Required information
BnB	Globally optimum	Decentralized	Local information
Greedy	Locally optimum	Centralized	Global information
CA	Locally optimum	Decentralized	Local information

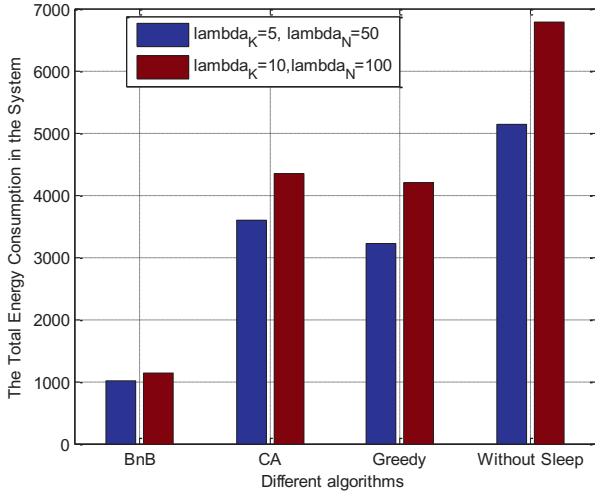


Fig. 4. Energy consumption of the system after reaching convergence with $\lambda_K = 5$, $\lambda_N = 50$ and $\lambda_K = 10$, $\lambda_N = 100$.

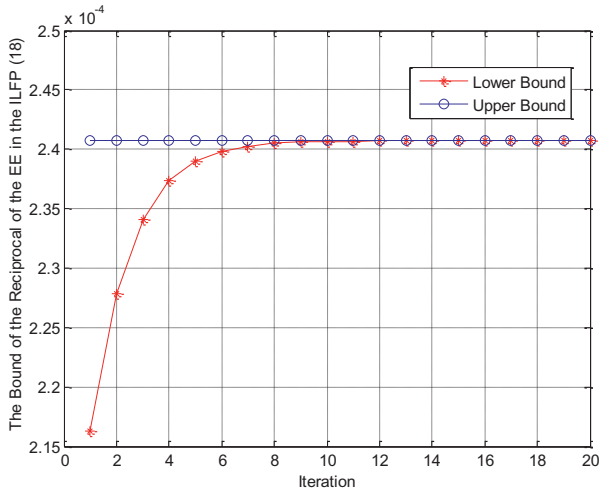


Fig. 5. The upper and lower bound evolution of the reciprocal of the objective function in (18).

without sleep scheduling has the maximum energy consumption. It can be seen that the energy consumption of the BnB algorithm decreases dramatically, followed by the energy consumption of the greedy and CA algorithms. Besides, the larger λ_K and λ_N , the more obvious of the effectiveness of energy saving.

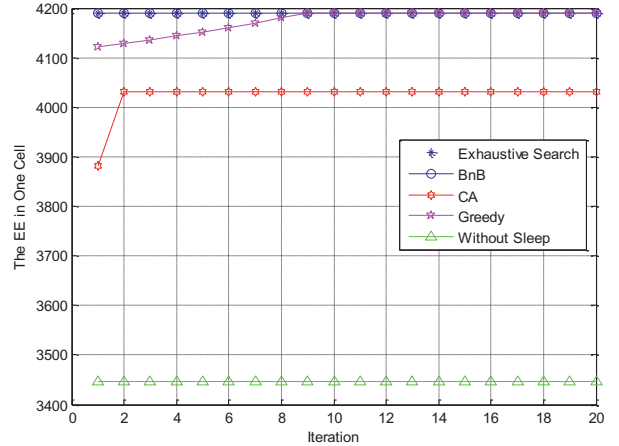


Fig. 6. The EE with RS sleep scheduling in one cell.

Next, we simulate the sleep scheduling algorithm of the RSs in one cell. Fig. 5 shows the upper and lower bound evolution of the reciprocal of the objective function in (18) with $K = 9$, $N = 100$. It is shown that the obtained value of the upper bound remains unchanged in each iteration while the lower bound increases gradually until it satisfies the convergence accuracy.

After the bounds get to convergence, by taking the reciprocal of the value of the upper bound, Fig. 6 plots the optimal EE with the RS sleeping in one cell. It is noticed the BnB algorithm achieves the globally optimal EE. Recall that the upper bound remains unchanged while the lower bound increases gradually until convergence in Fig. 5 where the BnB algorithm converges in 10 iterations. Therefore, the EE of the BnB algorithm by taking the reciprocal of the value of the upper bound always equal to the optimal solution which coincide with the exhaustive search. When the greedy algorithm does not converge, the EE of the BnB algorithm is superior to the greedy algorithm, however, when the greedy algorithm gets to convergence, the EE of the greedy algorithm is same with the BnB algorithm. The reason is that the greedy algorithm achieves the optimal EE. Specially, the reason for the fast convergence of the CA algorithm is that the number of RSs is few. As a whole, the EE of the BnB algorithm is superior to the greedy and CA algorithms. Moreover, the EE of the proposed algorithms is better than that of the scheme without sleep scheduling. In addition, Table 3 shows more comprehensive comparisons for the three algorithms.

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Hongbin Chen was born in Hunan Province, China in 1981. He received the BEng degree in electronic and information engineering from Nanjing University of Posts and Telecommunications, China, in 2004; and the PhD degree in circuits and systems from South China University of Technology, China, in 2009. He is presently a Professor in the School of Information and Communication, Guilin University of Electronic Technology, China. From October 2006 to May 2008, he was a Research Assistant in the Department of Electronic and Information Engineering, Hong Kong Polytechnic University, China. From March to April 2014, he was a Research Associate in the same department. His research interests lie in energy-efficient wireless communications. He has published more than 30 papers in renowned international journals including *IEEE Transactions*. He is serving as an Editor for *IET Wireless Sensor Systems*.



Qiong Zhang received a BEng degree in communications engineering from North China University of Water Resources and Electric Power, China in June 2013. She is working towards her MEng degree in Guilin University of Electronic Technology from September 2013. Her research focuses on energy-efficient cellular networks.



Feng Zhao received a PhD degree in communications and information systems from Shandong University, China in 2007. Now he is a Professor in the School of Information and Communication, Guilin University of Electronic Technology, China. His research interests include wireless communications, signal processing, and information security.