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QoS provisioning for multiple Femtocells via game theory☆

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ABSTRACT

We consider a system with multiple Femtocells (FCs) operating in a Macrocell. The transmissions in one Femtocell interfere with its neighboring Femtocells. Each Femtocell has multiple users, each requiring a minimum transmission rate. There is also peak transmit power constraint in each channel to control interference to the BS and the users in the Macrocell. We formulate the problem of channel allocation and power control in each Femtocell as a noncooperative Game. We develop efficient decentralized algorithms to obtain a Nash equilibrium (NE) that satisfies the Quality of Service (QoS) of each user, in an efficient way. We also obtain efficient decentralized algorithms to obtain fair NE when it may not be feasible to satisfy the QoS of all the users in FCs. Finally, we extend our algorithms to the case where there may be voice and data users in the system.

1. Introduction

The channel gain of cellular wireless channels indoors attenuates from the walls, causing significant degradation of voice quality. On the other hand, it has also been found that more than 60% of the cellular traffic generates indoors [55]. Thus, to improve the quality of service, the Base Station (BS) will have to significantly increase the transmit power which may not be allowed by the telecom regulators due to health concerns. Hence, Femtocells [6] are being considered as an option.

Femtocell access points (FAP) are small, inexpensive, low powered base stations designed for indoor deployment. When mobile stations (MSs) are indoor and encounter outage from the (Macro) Base Station outside these can be handed over to the FAP in their building. A FAP is connected to the wireline infrastructure. Thus, it can receive data from a MS in its building and transfer to the backhaul network through the wireline connection. This improves the quality of service (QoS) to the indoor applications and also offloads a significant fraction of cellular traffic to wireline network.

Although, deployment of FAPs in a macrocell (MC) environment improves the performance of the users inside the FC, it causes interference to the MSs in the MC and adjacent FCs especially when these are densely deployed in a MC. Therefore, to reap the benefits of a FC one needs careful interference management [10,30,34]. This requires FCs to control the transmit power in each of the channels. Furthermore, the FCs have to schedule channels within its area to ensure QoS to each user. The power control and channel allocation in each FC can be done centrally by the macro BS but it will require the BS to have the information about the channel gains and QoS requirements of all the users in the network (including those in the FCs). This may be manageable for very small networks only. Thus, generally decentralized control, where the FAPs decide about the transmit powers and channel allocations for their users using local information is recommended [46,53]. We consider this problem in this paper. First we provide the related literature survey in the next subsection and also state our contribution.

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1.1. Literature survey and our contribution

In future, to meet the ever increasing traffic in wireless networks [32], Femtocells, microcells, picocells and WiFi networks within a Macrocell will possibly be working in the same geographical area. These heterogeneous networks (Hetnets) bring additional challenges such as spectrum sharing between different networks, admission control, hand over control and interference management from transmissions in different networks. Considering the importance of these problems, several survey papers [2,11,29,32,36] have appeared in recent years to address these issues. In the following we specifically mention the papers which are directly related to our problem of interference management in Hetnets and in particular for Femtocells.

A common technique used for interference management is successive and parallel interference cancellation [53]. However, it requires significant computational resources and information about the signaling from other FCs and is not practical in the present context. Therefore, interference avoidance, [29,53] mainly via power control on individual channels is the most commonly used technique in Hetnets. In [41] interference avoidance is achieved via partitioning the bandwidth. Also, centralized control being not feasible in a practical network, self organization and tuning by the FAPs is required [2,29,53]. This is feasible specifically in the Cognitive Femtocells [29] where a FAP can sense the channels and avoid using channels with high interference.

Interference avoidance schemes for CDMA networks are surveyed in [53] and use time hopping by the users, multi sector antennas by FAPs and power control. OFDMA systems (WiMAX, LTE, LTE-A) offer more flexibility in designing interference avoidance schemes. One solution to reduce interference is to use different channels for the Macrocell users and Femtocells. However this is not favored by the operators because the spectrum is very expensive.

A survey on energy efficient resource management in multicell scenario is provided in [42]. Distributed power control algorithms used by FCs have been presented in [3,8,24]. Joint subchannel allocation and power control has been addressed in [1,12,27,38,54]. Due to mixed integer programming, the joint power control and channel allocation problem is computationally more demanding.

Game theory has been used to obtain distributed algorithms in several studies. Non-cooperative game theory has been used in [4,5,7,22,25,26,51] for power control. Channel allocation has been provided in [33,46] and [52]. Channel allocation via correlated equilibrium is provided in [23] while [15] uses cooperative game theory. [16] and [31] provide power control and channel allocation algorithms. Certain minimal rates to FCs are ensured via distributed auctions in [12].

In this paper we consider the problem of providing satisfactory QoS in the uplink and downlink for each MS in a multi FC dense environment via non-cooperative game theory. We consider subchannel and power allocation jointly. Interference between different FCs is considered. Our problem has some similarity to that in [54]. However [54] does not consider game theoretic framework and also limits itself to the case when the minimum rate requirements of all the users can be satisfied. We develop novel low complexity distributed algorithms to compute Nash Equilibria (NE) which are energy efficient and provide QoS to each user in each FC. For this we exploit the fact that the game is a potential game [37]. When it is not possible to satisfy the QoS of each user in each FC, we develop efficient distributed NE which are *fair*. Now the game is no longer a potential game. Thus, we discretize the power levels and use regret matching algorithm [18] to obtain a correlated NE. Since, computation of fair NE using regret matching procedure is complex, we modify the problem into a submodular game and obtain a much less complex algorithm to compute a fair NE. Later, we extend this model to the case where voice and data users coexist and obtain a fair NE. Although [16] and [31] also address joint power control and channel allocation via game theory, [16] does not consider the problem of QoS guarantees to multiple users in a FC and [31] uses game theory only for power allocation.

This paper is organized as follows. Section 2 describes the system model and formulates the problem. Section 3 provides a game theoretic framework and also provides efficient algorithms to obtain a NE. Section 4 considers a case when the QoS of all users may not be satisfied. Section 5 extends the algorithms to a systems with voice and data users. Section 6 shows the efficacy of the algorithms via a few examples. Section 7 concludes the paper.

2. System model and problem formulation

We consider a two tier cellular system in which within a MC there may be many FCs. The cellular system has multiple subchannels, perhaps using OFDMA (e.g., LTE or LTE-A) and the subchannels are shared by the FCs and the outdoor users in the MC. The transmission between a FAP and its MSs may be in the uplink or downlink or in both directions. The allocation of channels and powers to different users will be done by the FAP using the same algorithms. Thus, for simplicity we will consider downlink only. Also, we consider closed door mode of operation: FCs do not allow outside users to use their resources.

The MSs using a FC are also the MSs for the Macro BS. Thus, these MSs can get the information from the Macro BS about which subchannels are being used by the users in the MC (e.g., from PDCCH/PUCCH in LTE/LTE-A sent by the MBS in each frame). These channels may not be used by the FC at that time. The indoor MSs can also sense the SINR in each subchannel and can further be directed by the MBS on the maximum power they can use in transmitting in different available subchannels. This information can be sent to the FAP by the MSs within its domain or by the MBS directly (see e.g., [56]). FAP uses this information to decide on subchannel allocation and power control within its FC to provide QoS to its users while using minimum power within the limits prescribed by the MBSs. Minimizing power reduces interference to other FCs and to MSs outside and also reduces the overall energy consumed in the cellular network. We assume a control channel via which the different FCs [2,29] can coordinate with each other and self-configure.

Motivated by the above setup, in the following we formulate our problem. Let there be K FCs deployed in a neighborhood sharing N channels. FC k has M_k users. We consider allocation of channels to different users in a slot although our algorithms can be used for multiple slots also. Let $\bar{P}_i^k$ be the maximum power that can be used in channel i in FC k. Let $\bar{R}_j^k$ be the rate requirement of user j in FC k. Let $G_{i,j}^k$ be the channel gain of channel i for user j in FC k. This includes the effect of fading and shadowing. We assume that it will stay same during the slot. Since FCs cater to indoor users, this is a reasonable assumption. We make the same assumption for other channel gains we consider in the paper. We denote by $G_{i,j}^{l,k}$ the channel gain for user j in channel i in FC k from FC l. Also, $I_{i,j}^k$ denotes the interference (caused by outdoor users and MBS) experienced by user j in channel i in FC k. Let σ^2 denote the receiver noise at a receiver, assumed same for simplicity, for all receivers. The FAP k is supposed to have all this information. Such assumptions are also made in [27]. Based on this information each FAP k has to decide the subchannel allocation to different users and the power P_i^k to be used in subchannel i such that the rate $\bar{R_j}^k$ is received by each user j if possible. Also we desire an energy efficient solution. The decisions of one FAP will affect the decisions of others due to the interference caused to MSs in their FCs.

In the following we formulate this problem mathematically. Superscript k will refer to FC k. Let

$$A_{i,j}^k = \begin{cases} 1, & \text{if subchannel } i \text{ is assigned to user } j, \\ 0, & \text{otherwise.} \end{cases}$$

Also let $C_{i,j}^k = \log_2(1 + \frac{P_i^k G_{i,j}^k}{\Gamma(\sigma^2 + I_{i,j}^k + \sum_{l \neq k} G_{i,j}^{l,k} P_i^l)})$, where Γ is the SNR gap included for practical rates achievable depending on the modulation and coding scheme [9].

We consider the following problem: Find $P_i^k, i = 1, 2, .., N$ and $A_{i,j}^k, i = 1, 2, \ldots, N, j = 1, 2, \ldots, M_k, k = 1, 2, \ldots, K$ to

$$min \quad \sum_{i=1}^{N} P_i^k \tag{1}$$

such that

$$\sum_{i=1}^{N} C_{i,j}^k A_{i,j}^k \geq \bar{R_j}^k, \quad \forall j = 1, 2, \ldots, M_k, \tag{2}$$

$$0 \leq P_i^k \leq \bar{P_i}^k, \quad \forall i = 1, 2, \ldots, N, \tag{3}$$

$$\sum_{j=1}^{M_k} A_{i,j}^k \leq 1, \quad \forall i = 1, 2, \ldots, N. \tag{4}$$

Eq. (2) specifies QoS requirements and (3) specifies power constraints. The constraint (4) ensures that any subchannel is allocated to only one user within a FC. While trying to provide QoS to each of its users, each FC also wants to minimize the total power (1) it needs to use to reduce the costs involved as well as to address the green communication related issues [20]. If one central controller has all the global information of the system then this problem could be solved by it. This is a mixed constrained integer, nonlinear programming problem and is NP hard. However, presently we are assuming that each FAP k knows only its own $\bar{R}_j^k, G_{i,j}^k, G_{i,j}^{l,k}, I_{i,j}^k$ and not of other FCs. Also each FAP does scheduling of channels and power control of its own users only. We provide a decentralized solution for this problem using non-cooperative game theory assuming that each FC behaves selfishly to satisfy the QoS of its own users.

3. Game theoretic solution

We first formulate our problem as a noncooperative game and then provide computationally inexpensive decentralized solutions.

A non-cooperative game [40] $\mathscr{G} = \langle \mathscr{I}, \mathscr{X}, (\Phi_k)_{k \in \mathscr{I}} \rangle$, consists of set $\mathscr{I}$ of K players, with $\mathscr{X} \subseteq R^{mK}$ and the set of pure strategies $\mathbf{x} \triangleq [\mathbf{x}_1^T, \ldots, \mathbf{x}_K^T]$. The m-length vector $\mathbf{x}_k$ represents a strategy of the kth player and the function $\Phi_k : \mathscr{X} \to R$ is the utility function of the k^{th} player, which depends on the strategies $\mathbf{x}$ for all players. The following definitions and results will be used in the sequel.

Definition 1 [45]**.** A strategy $\mathbf{x} \in \mathscr{X}$ is a *Nash Equilibrium* (NE) for the game if $\forall k \in \mathscr{I}, \Phi_k(\mathbf{x}) \geq \Phi_k(\mathbf{x}_k^*, \mathbf{x}_{-k}), \forall \mathbf{x}_k^* \in \mathscr{X}_k$, where $\mathbf{x}_{-k}$ denotes the strategies of all players in $\mathscr{I} - \{k\}$.

Definition 2. A strategic game $\mathscr{G} = \{\mathscr{I}, \mathscr{X}, (\Phi_k)_{k \in \mathscr{I}}\}$ is called an *exact potential game* if there exists a function $f : \mathscr{X} \to R$ such that $\forall k \in \mathscr{I}$ and $(\mathbf{x}_k, \mathbf{x}_{-k}), \mathbf{y}_k \in \mathscr{X}_k$,

$$\Phi_k(\mathbf{x}_k, \mathbf{x}_{-k}) - \Phi_k(\mathbf{y}_k, \mathbf{x}_{-k}) = f(\mathbf{x}_k, \mathbf{x}_{-k}) - f(\mathbf{y}_k, \mathbf{x}_{-k}). \tag{5}$$

Theorem 1 [17]**.** *Let $\mathscr{G} = \{\mathscr{I}, \mathscr{X}, (\Phi_k)_{k \in \mathscr{I}}\}$ be a potential game, with a potential function f, and let $\mathscr{P}_{max}$ denote the set of global maxima of f (assumed non empty). Then each point $x \in \mathscr{P}_{max}$ is a NE of $\mathscr{G}$.*

We consider decentralized algorithms which converge to a NE. First we define ϵ-*Better Response*. Let, for an $\epsilon > 0$ and $\mathbf{x} = (\mathbf{x}_k, \mathbf{x}_{-k})$,

$$\mathscr{D}_k(\mathbf{x}) = \{\mathbf{x}_k^* \in \mathscr{X}_k(\mathbf{x}_{-k}) : \Phi_k(\mathbf{x}_k^*, \mathbf{x}_{-k}) \geq \Phi_k(\mathbf{x}_k, \mathbf{x}_{-k}) + \epsilon\}. \tag{6}$$

Then ϵ-Better Response provides a point in $\mathscr{D}_k(\mathbf{x})$.

In the following we develop iterative algorithms for our system using these abstract algorithms and will also show that these algorithms converge to a NE (or a correlated equilibrium, defined below).

We formulate the game for our system as $\mathscr{G} = \langle \mathscr{I}, \mathscr{X}, (\Phi_k(x))_{k \in \mathscr{I}} \rangle$, where $\mathscr{I} = \{1, 2, \ldots, K\}$, the set of FCs. Let $\underline{P}^k = \{P_{i,j}^k, \quad i = 1, 2, \ldots, N, j = 1, 2, \ldots, M_k\}$. We define strategy set $\mathscr{X} = \{(\underline{P}^k \in R_+^N, A_{i,j}^k \in \{0, 1\}, k = 1, 2, \ldots, K) : \sum_{i=1}^N C_{i,j}^k A_{i,j}^k \geq \bar{R_j}^k, j = 1, 2, \ldots, M_k, \sum_{j=1}^{M_k} A_{i,j}^k \leq 1, P_i^k \leq \bar{P_i}^k, i = 1, 2, \ldots, N\}$. We are interested in finding a decentralized energy efficient solution which provides QoS to each user in the system (if at all possible). If it is not possible we provide a *fair* NE (to be defined later).

Define the utility function for FC k by

$$\Phi_k(\mathbf{x}) = -\sum_i P_i^k, \forall \mathbf{x} \in \mathscr{X}. \tag{7}$$

Maximizing $\Phi_k(\mathbf{x})$ will minimize the total power used by FC k. It is easy to verify that the game $\mathscr{G}$ defined above is an exact potential game with potential function

$$f(\mathbf{x}) = -\sum_k \sum_i P_i^k. \tag{8}$$

We observe that $\mathscr{X}$ is a compact set. Also the potential function f is continuous. Therefore, it has a global maximizer and hence from Theorem 1, has a NE. Furthermore, now since the potential function is bounded and in the ϵ-better response algorithm, the potential function increases by atleast ϵ, it converges in a finite number of steps to an ϵ-Nash Point. We point out, that ours is a constrained optimization problem. Hence the best response [13] algorithm may not converge to a (generalized) NE.

In the following we develop ϵ-better response algorithms for our game. At each FC k we can use these algorithms iteratively to reach a ϵ-NE.

The ϵ-better response algorithm obtains, at iteration $n+1$, at each FC k,

$$\mathbf{x}_k^{n+1} \in \mathscr{D}_k(\mathbf{x}_1^{n+1}, \mathbf{x}_2^{n+1}, .., \mathbf{x}_{k-1}^{n+1}, \mathbf{x}_{k+1}^{n}, .., \mathbf{x}_K^{n}), \tag{9}$$

and then passes on the ϵ-better response to FC $k+1$. It can find $\mathbf{x}_k^{n+1}$ via solving the optimization problem (1)–(4) to obtain $A_{i,j}^{k,n}$ and $P_i^{k,n}$. However this algorithm has a high complexity because of mixed integer and real variables.

For computationally simple algorithms, we consider random ϵ-better response [35] which converges for the above potential game [39]. These algorithms have lower complexity per iteration. But for a continuous strategy space one may need a large number of iterations. Thus, we consider a variation of these algorithms which converges much faster. We take a random sample over $A_{i,j}^k$. Once a random sample of $A_{i,j}^k$ is taken, we pick P_i^k that optimizes (1)–(4). This is a convex optimization problem. The KKT solution [48], which provides the global optimum for this, is

$$P_i^k = \left(\frac{\lambda_{k,j} A_{i,j}^k}{A_{i,j}^k + \mu_{k,i}} - \frac{1}{d_{i,j}^k} \right)^+, i = 1, 2, \ldots, N, \tag{10}$$

where $d_{i,j}^k = \frac{G_{i,j}^k}{\Gamma(\sigma^2 + I_{i,j}^k + \sum_{l \neq k} G_{i,j}^{l,k} P_i^l)}$, $(x)^+ = max(x, 0)$ and $\lambda_{k,j}$, $\mu_{k,i}, i = 1, 2, \ldots, N, j = 1, 2, \ldots, M_k, k = 1, 2, \ldots, K$ are Lagrange multipliers such that (2) and (3) is satisfied with equality. This way we eliminate a large number of dominated strategies. Third, even for $A_{i,j}^k$ we do not sample from the whole sample space, but only a neighborhood of $A_{i,j}^k$ (by changing only two rows of A^k) which has a higher probability of providing an improved strategy.

The random strategy so obtained in iteration $n+1$ at FC k may not provide an ϵ-improvement over its policy in iteration n (or may not even be able to satisfy the QoS of all users). Then, FC k can again take a random sample of $A_{i,j}^k$ and repeat the whole process. We allow this to happen up to $N_1 \geq 1$ times in each iteration. If in N_1 trials also FC k does not get an ϵ-improved policy, then it keeps the policy of the iteration n itself.

This procedure provides ϵ-improvement with a low computational complexity with a high probability. By increasing N_1 we can increase the probability of success. If in N_1 random samplings we do not get ϵ improvement then either the algorithm has converged to an ϵ-Nash point or it failed to find ϵ-improvement even though we have not reached the ϵ-Nash point. This algorithm is summarized in Algorithm 1 below.

Algorithm 1 Randomized Algorithm (RA).

Generate initial channel allocation from Algorithm 2 and compute power allocation for this matrix using Eq. (10).
(1) $n = 0$, $\mathbf{x}_k(0) = (\underline{P_0}^k, A_0^k)$, $\Phi_k^0 = \Phi_k(\mathbf{x}_k(0))$
(2) $\forall n > 0$
if (Rate requirements of all users satisfied simultaneously) **then**
  if $(\Phi_k^n \geq \Phi_k^{n-1} + \epsilon)$ **then**
    Pick $\mathbf{x}_k(n)$
  else
    Pick randomly A^k via Algorithm 3 and test for above condition for a specified number of times ($\triangleq N_1$) till satisfies elsetake $\mathbf{x}_k(n) = \mathbf{x}_k(n-1)$
  end if
else
  Stop and go for fair allocation algorithm in Section 4.
end if
(3) Do until convergence

We use Algorithm 2 for a good initial channel allocation. It also checks whether the rate requirements of all the users can be satisfied or not. A good initial point can help obtain a good NE in perhaps fewer number of steps. Algorithm 2 tries to allocate channels to users with good channel gains while satisfying their QoS. For this we define $\bar{C}_{i,j}^k = C_{i,j}^k|_{P_i^k = \bar{P}_i^k}, \forall i$.

Remark 1. The fact that $\mathscr{G}$ is a potential game, and our potential function (8) is the sum of the utility functions (7) implies that the NE we obtain by our algorithms provided above when each user is trying to maximize (improve) its utility, should provide an NE which performs well from the global perspective [21,39].

4. Fair allocation

If the QoS of all the users in all the FCs cannot be satisfied then we obtain "fair" NE from the algorithms developed in this section. Of course, we need to first know if QoS of all the users can be satisfied in each FC or not. We can check this fairly quickly by using maximum possible power in each channel in each FC and obtaining good channel allocations for each FC separately via Algorithm 3 as explained below.

Now we can allocate channels to each FC independently of others. For each FC, we randomly allocate channels via Algorithm 3 till QoS of all users are satisfied. If it does not happen for some time for a FC, then we will use the algorithms for the present section. This will generally provide

Algorithm 2 Initial Subchannel Allocation Algorithm (SA).

Input = $N, M, \bar{C}^k_{i,j}, \bar{R}^k_j, i = 1,2,\ldots N, j = 1,2,\ldots M_k$.
Take $A^k_{i,j} = 0, i = 1,2,\ldots N, j = 1,2,\ldots M_k$, $R^k_{remain}(j) = \bar{R}^k_j, \forall j$ and $R^k_{alloted}(j) = 0, j = 1,2,\ldots M_k$
for $i = 1$ to $i = N$ **do**
  $C^k_1(j) = \min(\bar{C}^k_{i,j}, R^k_{remain}(j)), \forall j$
  $j^* = \arg\max_j C^k_1(j)$
  if $(\bar{C}^k_{i,j^*} \neq 0)$ **then**
    $A^k_{i,j^*} = 1, R^k_{alloted}(j^*) = R^k_{alloted}(j^*) + \bar{C}^k_{i,j^*}$
    $R^k_{remain}(j^*) = \bar{R}^k_{j^*} - R^k_{alloted}(j^*)$
  end if
  if $((\bar{R}^k_{j^*} - R^k_{alloted}(j^*) < 0)$ **then**
    $\bar{C}^k_{i,j^*} = 0, i = 1,2,\ldots N$
  end if
  if $((\bar{C}^k_{i,j} \forall i = 1,2,\ldots N, j = 1,2,\ldots M_k) = 0)$ **then**
    Exit from the for loop
  end if
end for
if (Rate requirements of all users are not satisfied simultaneously) **then**
  Pick randomly A^k via Algorithm 3 and test for above condition for a specified number of times ($\triangleq N_1$) till satisfies, return it as initial channel allocation and proceed to Algorithm 1.
else
  Proceed to Algorithm 1 directly.
end if

Algorithm 3 Generation of RSAM.

(1) Randomly pick a channel i_1.
(2) Let this be allocated to user j_1.
 • Take away this allocation
(3) Pick at random a user j'_1 and allocated i_1 to j'_1.
(4) Take at random a channel i'_1 of j'_1 and allocate to user j_1,
 • exit.
(5) If the channel i_1 is not allocated to any user, then we do not take away a channel from corresponding j'_1.
 • exit.

a good test. If all QoS can be satisfied then we get the NE from Section 3. If not, we proceed as follows. If by mistake we decided by above test that all the QoS cannot be met, our Algorithm 4 or 5 in this section will provide the right NE.

If the QoS of all the users in the system cannot be satisfied simultaneously, then we try to satisfy the largest fraction of QoS of all the users in each FC. In particular, for each FC k we obtain power and subchannel allocation P^k and A^k that

$$\max \quad \alpha_k \tag{11}$$

such that

$$\frac{1}{\bar{R}^k_j}\sum_{i=1}^{N} C^k_{i,j}A^k_{i,j} \geq \alpha_k, \quad \forall j = 1,2,\ldots,M_k, \tag{12}$$

Algorithm 4 Regret-Matching Learning Algorithm.

Initialize arbitrarily probability for taking action of player k, $p^1_k(\mathbf{x}_k), \forall k \in \mathscr{I}$
for $n = 1,2,3,\ldots$
1.Find $D^k_n(\mathbf{x}_k, \mathbf{x}'_k)$ as in (15).
2. Find average regret $R^k_n(\mathbf{x}_k, \mathbf{x}'_k)$ as in (16).
3. Let $\mathbf{x}_k \in \mathscr{X}_k$ be the strategy choosen by user k at time n, i.e. $\mathbf{x}^n_k = \mathbf{x}_k$. Then probability distribution p^k_{n+1} for the action for next period is defined as,
$p^k_{n+1}(\mathbf{x}'_k) = \frac{1}{\mu}R^k_n(\mathbf{x}_k, \mathbf{x}'_k)\forall \mathbf{x}'_k \neq \mathbf{x}_k$,
$p^k_{n+1}(\mathbf{x}_k) = 1 - \sum_{\mathbf{x}'_k \neq \mathbf{x}_k} p^k_{n+1}(\mathbf{x}'_k)$.
where μ is a sufficiently large constant.

and

$$\sum_{j=1}^{M_k} A^k_{i,j} \leq 1, \quad \forall i = 1,2,\ldots,N. \tag{13}$$

where $A^k_{i,j} \in \{0,1\}$ and

$$C^k_{i,j} = \log_2\left(1 + \frac{P^k_i G^k_{i,j}}{\Gamma(\sigma^2 + I^k_{i,j} + \sum_{q\neq k} G^q_{i,j}P^q_i)}\right).$$

We use α_k as the utility function for FC k. This max-min fairness criterion within a FC has been used before [28,47]. We define strategy space for FC k as $\mathscr{X}_k = \{(\underline{P}^k \in R^N_+, A^k_{i,j} \in \{0,1\} : 0 \leq P^k_i \leq \bar{P}^k_i, \sum_{j=1}^{M_k} A^k_{i,j}, \leq 1 \quad i = 1,2,\ldots,N\}$.

It is observed that $\mathscr{X}_k$ is compact. Also the utility function $\Phi^k(\mathbf{x}_k, \mathbf{x}_{-k})$ for FC k is continuous. Thus the game has a (mixed) NE [14]. However there is no known algorithm to compute a NE in this general setup. In the following we provide algorithms to compute (approximately) the NE.

The first algorithm actually computes an approximate Correlated Equilibrium (CE) [19], defined below, which also exists.

Definition 3. A probability distribution $p = (p(\mathbf{x}))_{\mathbf{x}\in\mathscr{X}}$ is a CE of game $\mathscr{G}$ if $\forall k \in \mathscr{I}, \mathbf{x}_k \in \mathscr{X}_k$, $\mathbf{x}_{-k} \in \mathscr{X}_{-k}$ and $\forall \mathbf{x}'_k \in \mathscr{X}_k$

$$\sum_{\mathbf{x}_{-k}\in\mathscr{X}_{-k}} p(\mathbf{x}_k, \mathbf{x}_{-k})\{\Phi_k(\mathbf{x}_k, \mathbf{x}_{-k}) - \Phi_k(\mathbf{x}'_k, \mathbf{x}_{-k})\} \geq 0. \tag{14}$$

To get the CE point, we first discretize the strategy space (by discretizing the power). For the corresponding finite game we compute a CE (which exists) via the regret matching algorithm [18] provided below. We can show that as we reduce the discretization step of the strategy space, the CE obtained for the finite game converges to the CE of the original game [43,49].

A stationary solution of a regret algorithm exhibits no regrets. In this algorithm the probability of a playing action is proportional to the “regret” for not having played this action. We use the following notation to describe this algorithm. The average utility of player k at time n is

$$D^k_n(\mathbf{x}_k, \mathbf{x}'_k) = \frac{1}{n}\sum_{\tau\leq n}\{\Phi_k(\mathbf{x}'_k, \mathbf{x}^\tau_{-k}) - \Phi_k(\mathbf{x}_k, \mathbf{x}^\tau_{-k})\}. \tag{15}$$

Average regret of player k for not playing action $\mathbf{x}'_k$ is:

$$R^k_n(\mathbf{x}_k, \mathbf{x}'_k) = max(D^k_n(\mathbf{x}_k, \mathbf{x}'_k), 0). \tag{16}$$

Algorithm 4 describes the procedure for calculating CE for our game $\mathscr{G}$.

Let the relative frequency of actions $\mathbf{x}$ played by all the players in first n periods be $Z_n(\mathbf{x}) = \frac{1}{n}\sum_{l=1}^{n} 1_{\{\mathbf{x}_l=\mathbf{x}\}}$. The following theorem guarantees the convergence of Algorithm 4 to a CE.

Theorem 2 [18]**.** *If every player plays according to Algorithm 4, then the empirical distribution Z_n of the play converges almost surely, as $n \to \infty$, to the set of correlated equilibria of the game $\mathscr{G}$.*

The regret algorithm provided above can converge slowly. Also, computing regret in each iteration is quite complex. Thus, we also obtain a faster algorithm to obtain a NE approximately. For this we rewrite Eq. (12) as

$$\frac{1}{M_k}\sum_{j=1}^{M_k}\min\left\{\frac{1}{\bar{R}_j^k}\sum_{i=1}^{N}C_{i,j}^k A_{i,j}^k, \alpha_k\right\} \geq \alpha_k, \tag{17}$$

for each k.

We again discretize the strategy space. For the finite game so obtained, a (mixed) NE exists. Below we describe our proposed algorithm to compute a NE for this game. We will compare it with Algorithm 4 in the next section on some examples.

First we fix α_k and we use the LHS of (17) as the utility Φ_k of player k. We show in Theorem 3 below that now the game becomes a submodular game, although the game with utilities α_k is not. Thus, our game has a weak finite improvement property [39] and a random better response algorithm converges to a NE. We provide below (Algorithm 5) one such algorithm. As long as it is possible

Algorithm 5 Computation of NE.

(1) Initialization: $\alpha_k = \alpha_k(0)$, Fix $\Delta > 0$, a small constant.
(2) Use Algorithm 6 to obtain NE for $\alpha(m) = (\alpha_k(m), k = 1, 2, \ldots, K)$.
(3) $\alpha_k(m+1) = \alpha_k(m) + \Delta$
(4) Repeat (2) and (3) till a NE is obtained.
(5) if $\alpha_k(m) > 1$ for all k, then the rate requirement of all users are satisfiedand hence use Algorithm 1 to compute NE.

to achieve QoS upto α_k fraction for each k, we will obtain a NE. Once a NE is achieved we increase each α_k by a small amount Δ and again run our random better response algorithm to obtain a NE corresponding to the new α_ks. We keep increasing each α_k till eventually atleast one of the FCs has an empty strategy space. Then for that FC we decrease α by $\frac{\Delta}{2}$. We keep doing this till the strategy space of all FCs become empty. Then we keep the NE of the previous α's. We write this overall algorithm as Algorithm 5. The α_ks in the m^{th} cycle are denoted by $\alpha_k(m)$, the corresponding utility functions are $\Phi_k(m)$.

Now we explain Algorithm 6 to obtain a NE for fixed α_ks. As before, for each α_k we use randomization at each step of obtaining the NE. This algorithm computes NE in two loops. In the outer loop for iteration n, each FC k uses Algorithm 7 to change its channel allocation from A_{n-1}^k to a new allocation A_n^k. In the inner loop at each iteration n, FC k picks $\underline{P}_n^k$ randomly. Then it computes $\Phi_k(A_n^1, \underline{P}_n^1, \ldots, A_n^{k-1}, \underline{P}_n^{k-1}, A_n^k, \underline{P}_n^k, A_{n-1}^{k+1}, \underline{P}_{n-1}^{k+1}, \ldots, A_{n-1}^K, \underline{P}_{n-1}^K)$. If $\Phi_k(A_n^1, \underline{P}_n^1, \ldots, A_n^k, \underline{P}_n^k, A_{n-1}^{k+1}, \underline{P}_{n-1}^{k+1}, \ldots, A_{n-1}^K, \underline{P}_{n-1}^K) > \Phi_k(A_n^1, \underline{P}_n^1, \ldots, A_n^{k-1}, \underline{P}_n^{k-1}, A_{n-1}^k, \underline{P}_{n-1}^k, A_{n-1}^{k+1}, \underline{P}_{n-1}^{k+1}, \ldots, A_{n-1}^K, \underline{P}_{n-1}^K)$, then it retains the power and channel allocation. Otherwise, it again randomly picks a new power and checks for improvement of Φ_k. If in N_1 trials it does not improve then it keeps $(A_{n-1}^k, \underline{P}_{n-1}^k)$. Otherwise it changes to the new strategy. It then forwards the new $\underline{P}_n^k, A_n^k$ to another FC to change its strategies. This algorithm is summarized as Algorithm 6.

Algorithm 6 Randomized Fair Allocation Algorithm (RFA) for NE.

Requirement: For each time instant generate Random Subchannel Allocation Matrix (RSAM) and randomly pick power allocation vector.
(1) Initialization: $n = 0$, $\mathbf{x}_k(0) = (A_0^k, \underline{P}_0^k)$, $\Phi_k^0 = \Phi_k(x_k(0))$
repeat
  (2) $\forall n > 0$
  Pick randomly $A^k, \underline{P}^k$,
  if $(\Phi_k(A^k, \underline{P}_k) > \Phi_k^{n-1})$ **then**
    $\mathbf{x}_k(n) = (A^k, \underline{P}_k)$
  else
    Pick randomly power allocation $\underline{P}_k$ and test for above condition for a specified number of times ($\triangleq N_1$) till satisfies elsetake $\mathbf{x}_k(n) = \mathbf{x}_k(n-1)$ and $\Phi_k^n = \Phi_k^{n-1}$
  end if
  (3) $n = n + 1$
until convergence

Algorithm 7 Generation of FRSA.

Requirement: Randomly pick (i, j)th element of subchannel allocation matrix A_{n-1}^k
if $(A_{n-1}^k(i, j) = 0)$ **then**
- $A_{n-1}^k(i, j) = 1$
- make another random element of ith row which is 1, as 0 (if there is noother element of ith row 1 then no change is required).

else
- $A_{n-1}^k(i, j) = 0$
- make another random element of ith row as 1.

end if
- return new modified matrix as A_n^k

This algorithm provides a NE much faster than the regret matching based Algorithm 4. Also, in this algorithm in each iteration, the different FCs need to know only the total interference due to other FCs in each channel but the regret matching algorithm requires the power allocation of other FCs in each channel.

In the following we show that for each feasible α_k our present game is a submodular game [14].

Theorem 3. *The game $\mathscr{G}_{FA}(\alpha_k) = \langle \mathscr{I}, (\mathscr{X}_k)_{k\in\mathscr{I}}, (\Phi_k)_{k\in\mathscr{I}}\rangle$ with utility $\Phi_k(x_k, x_{-k}, \alpha_k)$ is submodular.*

Proof. Please see Appendix. □

As the discretization step is reduced to zero, the corresponding sequence of NE obtained from Algorithm 5 converges to that of the original game [14].

Since the overall aim is to satisfy the QoS of all the users and minimize the powers consumed by each FC, at an equilibrium point obtained in Algorithm 5 some users in an FC may get more rate than required. For all such users at their channels allocated we can minimize power using KKT conditions while ensuring their rates.

5. Coexistence of voice and data users

There are situations where both data users and voice users coexist in a FC. It is easier to satisfy rate requirements of voice users since their rate requirements are low (e.g., 32 Kbps) compared to data users. Also, for data users if we cannot satisfy their minimum requirements but a large fraction of it (large α_k) they may still be acceptable. But, if a voice user does not get its minimum rate then its voice quality at the receiver may actually become quite unacceptable. This motivates the following setup. In [44], we developed distributed algorithms for this situation for sparsely deployed FC's case. Below we consider the case of densely deployed FC's in a MC. We extend our set up developed in Section 4 for this scenario.

Let D be the set of data users and V be the set of voice users. Then at each FC k,

$$\max \quad \alpha_k \tag{18}$$

such that

$$\sum_{i=1}^{N} C_{i,j}^k A_{i,j}^k \geq \alpha_k \bar{R}_j^k, \quad \forall j \in D, \tag{19}$$

$$\sum_{i=1}^{N} C_{i,j}^k A_{i,j}^k \geq \bar{R}_j^k, \quad \forall j \in V, \tag{20}$$

and

$$\sum_{j=1}^{M_k} A_{i,j}^k \leq 1, \quad \forall i = 1, 2, \ldots, N. \tag{21}$$

Eq. (19) specifies that the largest fraction of QoS of data users are satisfied and Eq. (20) specifies that QoS of voice users are fully satisfied.

We can use regret matching Algorithm 4 to obtain a CNE. We obtain a more efficient algorithm via Algorithm 5 in the following.

Eqs. (18)–(21) can be rewritten as

$$\max \quad \alpha_k \tag{22}$$

such that,

$$\frac{1}{|V|} \sum_{j \in V} \min\{\frac{1}{\bar{R}_j^k} \sum_{i=1}^{N} C_{i,j}^k A_{i,j}^k, 1\}$$

$$+ \frac{1}{|D|} \sum_{j \in D} \min\{\frac{1}{\bar{R}_j^k} \sum_{i=1}^{N} C_{i,j}^k A_{i,j}^k, \alpha_k\} \geq 1 + \alpha_k,$$

$$\sum_{j=1}^{M_k} A_{i,j}^k \leq 1, \quad \forall i = 1, 2, \ldots, N. \tag{24}$$

We obtain NE for this problem using Algorithm 5. For a fixed α_k, we take LHS of (23) as the utility of player k. Then, again from Theorem 3 we obtain a submodular game. Thus Algorithm 6 converges to a NE of the game for each α_k (if possible). Then as in Section 4 we can increase α_k's and repeat. For all voice users (and data users whose QoS is satisfied) at their channels allocated we can minimize power using KKT conditions while ensuring their rate requirements are satisfied.

6. Examples

In this section we first consider a system with 2 FCs each with 4 users and 10 subchannels deployed in a MC. We consider Time Division Duplex (TDD) scenario where all parameters, such as rate requirements, interference matrix and subchannel gain matrices do not change significantly within a given TDD duration. We use the following parameters.

Subchannel bandwidth B = 180 kHz, SNR Gap $\Gamma = 1$, Noise Power $N_0 = 1 * 10^{-9}$ W, Noise variance $\sigma^2 = N_0 B = 1.8 * 10^{-4}$ Ws, $\epsilon = 0.003$.

Maximum Power constraint on each subchannel (mw): $\bar{P}_i^1 = \bar{P}_i^2 = 8, \quad \forall i = 1, 2, \ldots, 10.$

Rate requirement of the users in kbps:

$$\bar{R}^1 = \begin{bmatrix} 260, & 300, & 380, & 430 \end{bmatrix},$$

$$\bar{R}^2 = \begin{bmatrix} 295, & 395, & 385, & 445 \end{bmatrix}.$$

Subchannel gain matrices; G^1, G^2, $G^{1,2}$, $G^{2,1}$ were obtained by sampling from Gaussian distributions $\mathcal{N}(1,1), \mathcal{N}(0.5,1), \mathcal{N}(0,1), \mathcal{N}(0,1)$ and taking square.

Interference matrices (mw):

$$I^1 = \begin{pmatrix} 11 & 12 & 9 & 3 \\ 1 & 11 & 8 & 6 \\ 13 & 9 & 3 & 4 \\ 11 & 5 & 4 & 13 \\ 7 & 12 & 7 & 6 \\ 6 & 7 & 3 & 2 \\ 6 & 5 & 12 & 13 \\ 4 & 14 & 2 & 14 \\ 7 & 13 & 3 & 6 \\ 7 & 8 & 2 & 1 \end{pmatrix}, I^2 = \begin{pmatrix} 10 & 1 & 3 & 8 \\ 11 & 6 & 12 & 2 \\ 6 & 1 & 6 & 12 \\ 1 & 14 & 13 & 9 \\ 3 & 0 & 2 & 5 \\ 13 & 11 & 3 & 7 \\ 2 & 12 & 2 & 6 \\ 12 & 13 & 2 & 1 \\ 8 & 1 & 13 & 3 \\ 14 & 5 & 8 & 1 \end{pmatrix}.$$

For this example, there were enough resources to satisfy each user's requirements in each FC. The solution obtained for the downlink via Algorithm 1 is as follows.

Subchannel allocations at NE: A_{RA} obtained by RA is given by ($CA^k(i) \triangleq j$ if $A_{i,j}^k = 1$)

$$CA_{RA}^1 = \begin{bmatrix} 2 & 1 & 4 & 2 & 3 & 3 & 2 & 2 & 1 & 4 \end{bmatrix},$$

$$CA_{RA}^2 = \begin{bmatrix} 2 & 1 & 3 & 3 & 4 & 4 & 3 & 4 & 1 & 1 \end{bmatrix},$$

Minimal powers needed for transmission for the RA at the NE are (in mw) given as

$$P_{RA}^1 = [7.8, \quad 8.0, \quad 0.0, \quad 8.0, \quad 5.2, \quad 8.0, \quad 0.0, \quad 3.3, \quad 5.8, \quad 6.5],$$

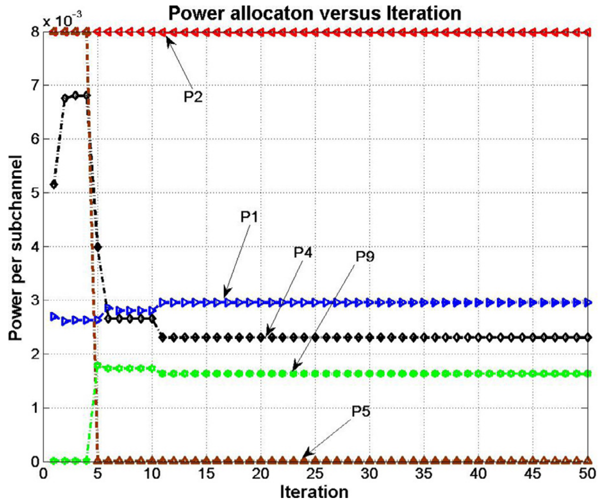

Fig. 1. FC2 using Algorithm 1.

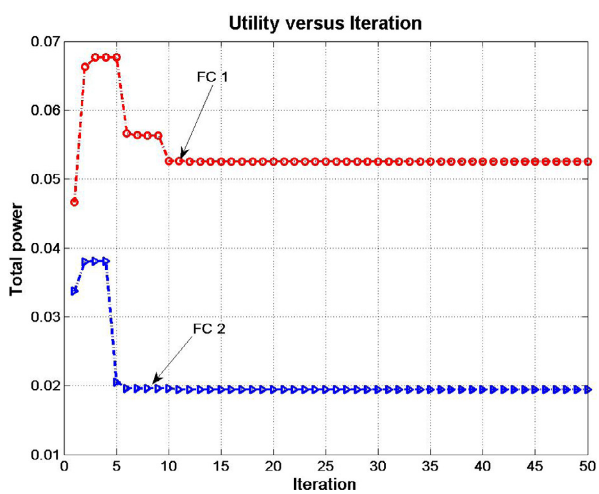

Fig. 2. Utility using Algorithm 1.

$$P_{RA}^2 = [3.0, \quad 8.0, \quad 2.3, \quad 2.3, \quad 0.0, \quad 0.0, \quad 1.2, \quad 1.1, \quad 1.6, \quad 0.0],$$

with corresponding utilities (sum of powers in mw) as 52.6 and 19.5. Power in each subchannel for FC 2 for Algorithm 1 is plotted in Fig. 1 (for readability, plotted for only 5 subchannels). Convergence of the utility (sum of powers) in each FC for the RA is plotted in Fig. 2.

Next we provide an example to obtain NE via the regret matching Algorithm 4. We use a system with 2 FCs each with 2 users and 4 subchannels for calculation of NE. We generated the subchannel gain matrices G^1, G^2, $G^{1,2}$, $G^{2,1}$ as before. The interference matrices (mw) are

$$I^1 = \begin{pmatrix} 4 & 3 \\ 11 & 4 \\ 8 & 9 \\ 9 & 13 \end{pmatrix}, I^2 = \begin{pmatrix} 10 & 1 \\ 11 & 6 \\ 6 & 1 \\ 1 & 14 \end{pmatrix}$$

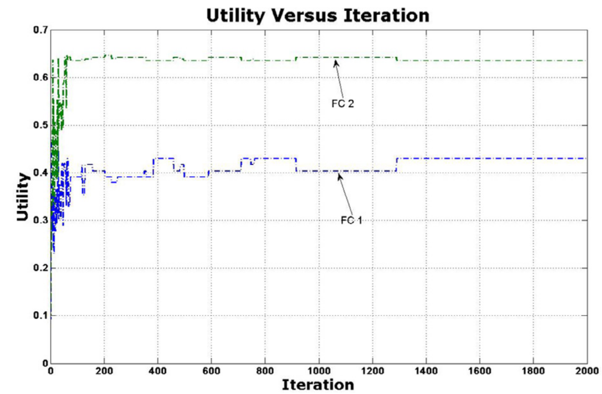

Fig. 3. Correlated Equilibrium using Algorithm 4.

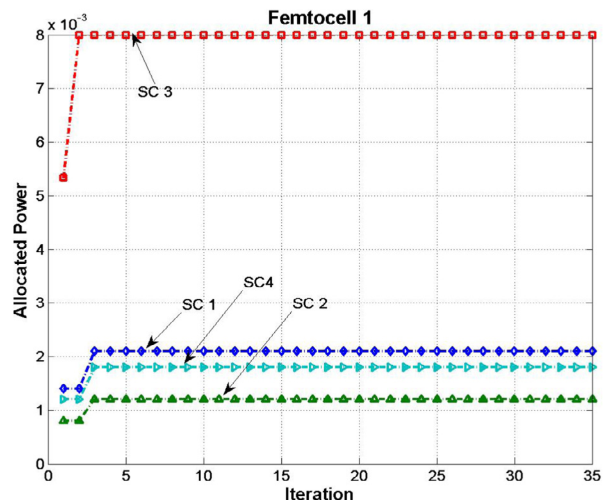

Fig. 4. Power allocation with variation of α_k using Algorithm 5.

Maximum powers (mw) allocated to the subchannels are,

$$\bar{P}^1 = \left[2.1, \quad 1.2, \quad 8.0, \quad 1.8\right],$$

$$\bar{P}^2 = \left[1.1, \quad 1.5, \quad 7.5, \quad 1.9\right].$$

Rate requirement of the users, in kbps, is,

$$\bar{R}^1 = \left[200, \quad 220\right],$$

$$\bar{R}^2 = \left[160, \quad 180\right].$$

We have verified that the rate requirement of all the users cannot be satisfied for this case. Thus, we obtain a fair NE via Algorithms 4 and 5. There are multiple NE. Depending on the initial conditions, the algorithms converge to different NE. We show the convergence of our algorithms in Figs. 3–5. Convergence of power allocation is shown in Figs. 4 and 5, convergence of utility is in Fig. 6. We observe that the regret algorithm converges to the same NE in these Figs. even though we started with different initial conditions. For this NE, the subchannel allocation for the two FC's converges to $CA^1 = [2, 1, 1, 1], CA^2 = [2, 2, 1, 2]$ and the power allocations are $P^1(\text{in mw}) = [2.1, 1.2, 8.0, 1.8], P^2(\text{in mw}) = [1.1, 1.5, 7.5, 1.9]$ with the

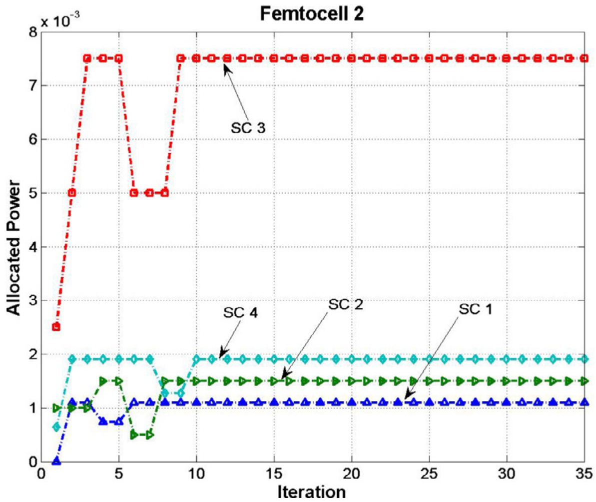

Fig. 5. Power allocation with variation of α_k using Algorithm 5.

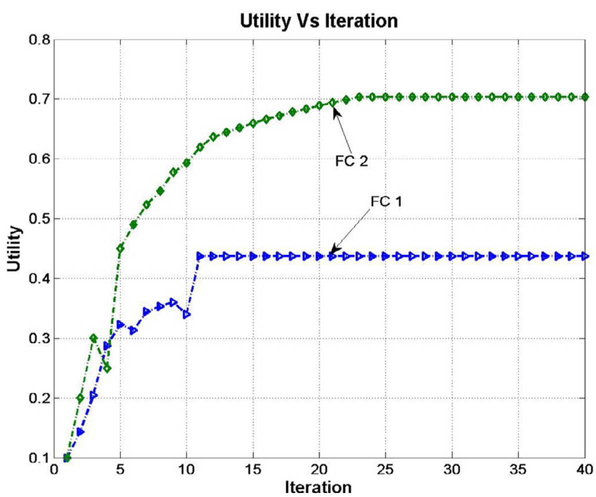

Fig. 6. Variation of Utility with α_k using Algorithm 5.

utilities 0.4294, 0.6348 and 0.4374, 0.7036 with the allocated rates (in kbps) [85.8, 97.9], [123.5, 114.2].

One of the NE for this example is such that maximum power is allocated to all the subchannels. This corresponds to the optimal solution when the FCs do not play any game together (i.e., maximize their utility ignoring the strategy of the other player). Now the subchannel allocation is given by $CA^1 = [1, 2, 2, 2], CA^2 = [2, 2, 1, 2]$ with utilities 0.2290, 0.6348 and with rate allocations (in kbps) [78.8, 50.3], [123.5, 114.2]. Comparing this to the NE provided above, we see that the first FC suffers substantially in the present solution while the second does not gain anything.

Voice and data users: Next we provide an example to obtain a NE for a system with 2 FCs each with 2 voice users and 1 data user (for FC 1, {2, 3} are voice users and {1} is data user, for FC 2, {1, 3} are voice users and {2} is data user) and 4 subchannels. We generated the subchannel gain matrices G^1, G^2, $G^{1,2}$, $G^{2,1}$ as before. The interfer-

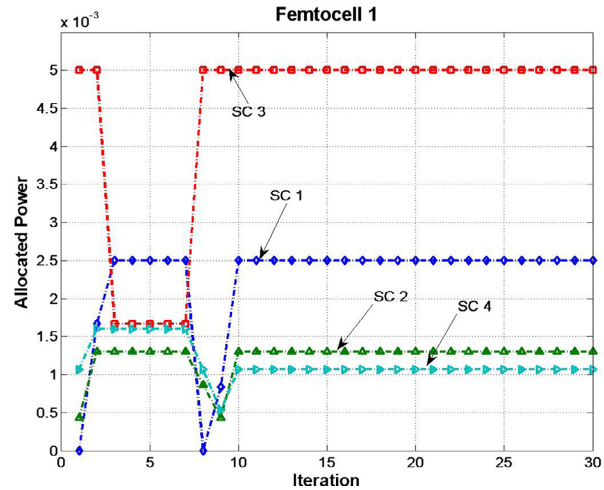

Fig. 7. Power allocation with variation of α_k for voice and data users using Algorithm 5.

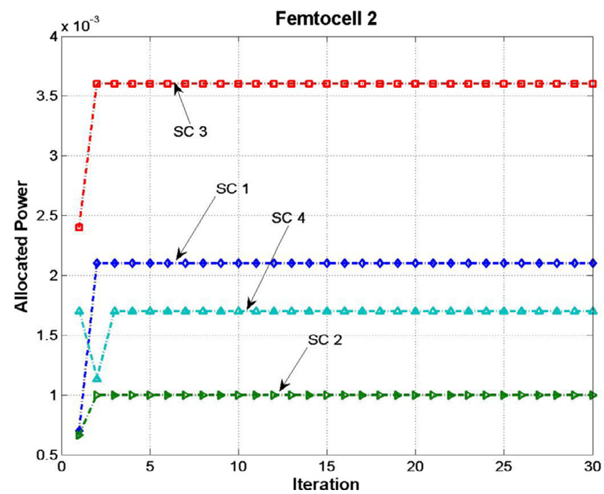

Fig. 8. Power allocation with variation of α_k for voice and data users using Algorithm 5.

ence matrices (in mw) are

$$I^1 = \begin{pmatrix} 10 & 6 & 6 \\ 3 & 9 & 12 \\ 1 & 11 & 12 \\ 9 & 5 & 3 \end{pmatrix}, I^2 = \begin{pmatrix} 10 & 7 & 5 \\ 1 & 5 & 2 \\ 7 & 10 & 8 \\ 7 & 11 & 3 \end{pmatrix}$$

Maximum powers (mw) allocated to the subchannels are,

$$\bar{P}^1 = \begin{bmatrix} 2.5, & 1.3, & 7.0, & 1.6 \end{bmatrix},$$

$$\bar{P}^2 = \begin{bmatrix} 2.1, & 1, & 6.6, & 1.7 \end{bmatrix}.$$

Rate requirement of the users, in kbps, is,

$$\bar{R}^1 = \begin{bmatrix} 220, & 25, & 17 \end{bmatrix},$$

$$\bar{R}^2 = \begin{bmatrix} 15, & 180, & 30 \end{bmatrix}.$$

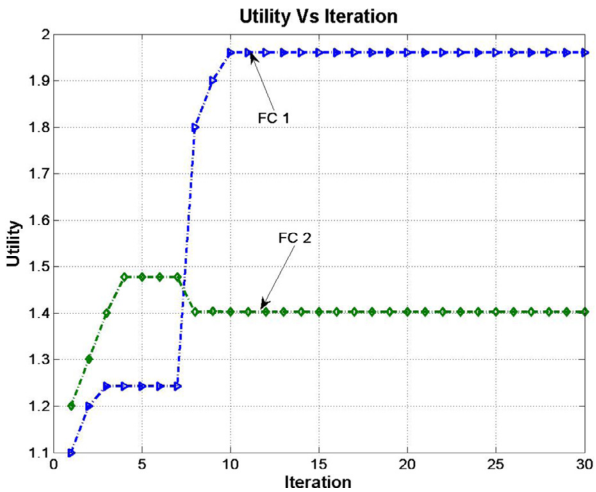

Fig. 9. Variation of Utility with α_k for voice and data users using Algorithm 5.

We show the convergence of power allocations for Algorithm 5 in Figs. 7 and 8 and convergence of utilities in Fig. 9. For this NE, the subchannel allocation for the two FC's converges to $CA^1 = [1, 2, 1, 3], CA^2 = [1, 3, 2, 2]$ and the power allocations are P^1(in mw)= [2.5, 0.3, 5.0, 0.2], P^2(in mw)= [0.4, 0.4, 3.6, 1.7] with the utilities 1.9601, 1.4027 and the allocated rates (in kbps) [211.27, 25.06, 17.03], [15.01, 72.66, 39.97].

7. Conclusions

We have considered a channel and power allocation problem in a multiple FC environment when there are multiple users in each FC requiring different minimum rates. The channels need to be shared by the different FCs such that they do not cause much interference to each other and to the MC users. We have formulated the problem in a non-cooperative game theory setup and provided low complexity distributed algorithms to obtain Nash equilibria and correlated equilibria. We have extended our study to include the case when both the voice and data users are present in the system.

Appendix

Let $C_j^k = \frac{1}{R_j^k}\sum_{i=1}^N C_{i,j}^k A_{i,j}^k$ and $\Phi_k(x_k, x_{-k}, \alpha_k) = \frac{1}{M_k}\sum_{j=1}^{M_k} \min\{C_j^k, \alpha_k\}$.

We will use the following notations for vectors $a = (a_1, a_2, \ldots, a_N)$ and $b = (b_1, b_2, \ldots, b_N)$: $a \vee b = max(a, b)$ component wise and $a \wedge b = min(a, b)$ component wise. We will also use the following definitions.

Definition 4 [14,50]**.** $\Phi_k(\mathbf{x}_k, \mathbf{x}_{-k})$ has decreasing differences in $(\mathbf{x}_k, \mathbf{x}_{-k})$ if, for all $\mathbf{x}_k, \widetilde{\mathbf{x}}_k \in \mathscr{X}_k$ and $\mathbf{x}_{-k}, \widetilde{\mathbf{x}}_{-k} \in \mathscr{X}_{-k}$, $\mathbf{x}_k \geq \widetilde{\mathbf{x}}_k$ and $\mathbf{x}_{-k} \geq \widetilde{\mathbf{x}}_{-k} \Longrightarrow \Phi_k(\mathbf{x}_k, \mathbf{x}_{-k}) - \Phi_k(\widetilde{\mathbf{x}}_k, \mathbf{x}_{-k}) \leq \Phi_k(\mathbf{x}_k, \widetilde{\mathbf{x}}_{-k}) - \Phi_k(\widetilde{\mathbf{x}}_k, \widetilde{\mathbf{x}}_{-k})$.

Definition 5. $\Phi_k(\mathbf{x}_k, \mathbf{x}_{-k})$ is submodular in $\mathbf{x}_k$ if for each $\mathbf{x}_{-k}$, $\Phi_k(\mathbf{x}_k \vee \widetilde{\mathbf{x}}_k, \mathbf{x}_{-k}) + \Phi_k(\mathbf{x}_k \wedge \widetilde{\mathbf{x}}_k, \mathbf{x}_{-k}) \leq \Phi_k(\mathbf{x}_k, \mathbf{x}_{-k}) + \Phi_k(\widetilde{\mathbf{x}}_k, \mathbf{x}_{-k})$.

Definition 6. A submodular Game in $(\mathbf{x}_k, \mathbf{x}_{-k})$ is such that $\forall k$, $\mathscr{X}_k$ is a Lattice, Φ_k has decreasing difference in $(\mathbf{x}_k, \mathbf{x}_{-k})$ and Φ_k is submodular in $\mathbf{x}_k$.

Lemma 1. *Strategy space of* $\{A_{i,j}^k \in \{0, 1\} : \sum_{j=1}^{M_k} A_{i,j}^k \leq 1, \forall i = 1, 2, \ldots, N\}$ *Subchannel allocation matrix is sublattice.*

Proof. The strategy space for channel allocations is, for each i, $\mathcal{A}_i^k = \{A_{i,j}^k \in \{0, 1\} : \sum_{j=1}^{M_k} A_{i,j}^k \leq 1\}$. This is not a lattice. However, take integers $B_{i,\,j}$,

$$B_{i,1} = A_{i,1}, \quad B_{i,2} = A_{i,1} + A_{i,2},$$

$$B_{i,j} = \sum_{k=1}^{j} A_{i,k}, \; j = 1, 2, \ldots, M_k.$$

Then, $\mathcal{B}_i^k = \{(B_{i,1}, B_{i,2}, \ldots, B_{i,M_k}) \in \{0, 1\}^{M_k} : B_{i,1} \leq B_{i,2} \leq \ldots \leq B_{i,M_k} \leq 1\}$ is a sublattice in $\{0, 1\}^{M_k}$. Also, there exists one to one relationship between elements of $\mathcal{B}_i^k$ and $\mathcal{A}_i^k$. We form matrix $\mathcal{B}^k = [\mathcal{B}_1^k, \mathcal{B}_2^k, \ldots, \mathcal{B}_{M_k}^k]^T$, is also a sublattice. Hence, $\mathcal{A}^k = [\mathcal{A}_1^k, \mathcal{A}_2^k, \ldots, \mathcal{A}_{M_k}^k]^T$ is also sublattice. □

Similarly strategy space of $\{\underline{P}_k \in \mathbb{R}_+^n : 0 \leq P_i^k \leq \bar{P}_i^k, \forall i = 1, 2, \ldots, N\}$ power allocation is also a sublattice. Thus the overall strategy space of each player k is a sublattice of $\mathbb{R}^{NM_k}$.

Proof of Theorem 3. We first show that $C_{i,j}^k$ has decreasing differences property. Let $P_i^k, \widetilde{P}_i^k$ and $P_i^{-k}, \widetilde{P}_i^{-k}$ be such that $P_i^k \geq \widetilde{P}_i^k$ and $P_i^{-k} \geq \widetilde{P}_i^{-k}$. We show

$$C_{i,j}^k(P_i^k, P_i^{-k}) - C_{i,j}^k(\widetilde{P}_i^k, P_i^{-k})$$

$$\leq C_{i,j}^k(P_i^k, \widetilde{P}_i^{-k}) - C_{i,j}^k(\widetilde{P}_i^k, \widetilde{P}_i^{-k}).$$

The LHS

$$= \log_2\left(\frac{\Gamma(\sigma^2 + I_{i,j}^k + \sum_{q\neq k} G_{i,j}^q P_i^q) + P_i^k G_{i,j}^k}{\Gamma(\sigma^2 + I_{i,j}^k + \sum_{q\neq k} G_{i,j}^q P_i^q) + \widetilde{P}_i^k G_{i,j}^k}\right)$$

$$= \log_2\left(1 + \frac{G_{i,j}^k(P_i^k - \widetilde{P}_i^k)}{\Gamma(\sigma^2 + I_{i,j}^k + \sum_{q\neq k} G_{i,j}^q P_i^q) + \widetilde{P}_i^k G_{i,j}^k}\right).$$

Similarly, RHS

$$= \log_2\left(1 + \frac{G_{i,j}^k(P_i^k - \widetilde{P}_i^k)}{\Gamma(\sigma^2 + I_{i,j}^k + \sum_{q\neq k} G_{i,j}^q \widetilde{P}_i^q) + \widetilde{P}_i^k G_{i,j}^k}\right).$$

Since, $P_i^k \geq \widetilde{P}_i^k$, the denominator term for RHS is less than that of LHS. Therefore $C_{i,j}^k$ has decreasing property.

We also have

$$C_{i,j}^k(P_i^k \vee \widetilde{P}_i^k, P_i^{-k}) + C_{i,j}^k(P_i^k \wedge \widetilde{P}_i^k, P_i^{-k})$$

$$= C_{i,j}^k(P_i^k, P_i^{-k}) + C_{i,j}^k(P_i^k, P_i^{-k})$$

$$\leq C_{i,j}^k(P_i^k, P_i^{-k}) + C_{i,j}^k(P_i^k, P_i^{-k}).$$

Therefore $C_{i,j}^k$ is submodular. Hence, $\sum_{i=1}^N C_{i,j}^k(P_i^k, P_i^{-k})$ is also submodular.

Now we show that $C_j^k(\mathbf{x}_k,\mathbf{x}_{-k}) = \frac{1}{\bar{R}_j^k}\sum_{i=1}^{N} C_{i,j}^k(P_i^k, P_i^{-k})A_{i,j}^k$ is Submodular in $(\bar{P}^k,\bar{P}^{-k})$ for fixed (A^k,A^{-k}). Let $\mathbf{x}_k = (P^k,A^k)$ and $\mathbf{x}_{-k} = (P^{-k},A^{-k})$. We show the decreasing property,

$$C_j^k(\mathbf{x}_k,\mathbf{x}_{-k}) - C_j^k(\widetilde{\mathbf{x}}_k,\mathbf{x}_{-k}) \le C_j^k(\mathbf{x}_k,\widetilde{\mathbf{x}}_{-k}) - C_j^k(\widetilde{\mathbf{x}}_k,\widetilde{\mathbf{x}}_{-k}),$$

when $\mathbf{x}_k \ge \widetilde{\mathbf{x}}_k$ and $\mathbf{x}_{-k} \ge \widetilde{\mathbf{x}}_{-k}$. The LHS

$$= \frac{1}{\bar{R}_j^k}\sum_{i=1}^{N} C_{i,j}^k(P_i^k,P_i^{-k})A_{i,j}^k - \frac{1}{\bar{R}_j^k}\sum_{i=1}^{N} C_{i,j}^k(\widetilde{P}_i^k,P_i^{-k})A_{i,j}^k$$

$$= \frac{1}{\bar{R}_j^k}\sum_{i\in N_j^k}\{C_{i,j}^k(P_i^k,P_i^{-k}) - C_{i,j}^k(\widetilde{P}_i^k,P_i^{-k})\},$$

where N_j^k is set of subchannels i allocated to user j in A^k, i.e.,

$$A_{i,j}^k = \begin{cases} 1 & \text{if subchannel } i \in N_j^k, \\ 0 & \text{otherwise.} \end{cases}$$

Thus, LHS

$$= \frac{1}{\bar{R}_j^k}\sum_{i\in N_j^k}\log_2\left(1 + \frac{G_{i,j}^k(P_i^k - \widetilde{P}_i^k)}{\Gamma(\sigma^2 + I_{i,j}^k + \sum_{q\ne k} G_{i,j}^q P_i^q) + \widetilde{P}_i^k G_{i,j}^k}\right)$$

Similarly, RHS

$$= \frac{1}{\bar{R}_j^k}\sum_{i\in N_j^k}\log_2\left(1 + \frac{G_{i,j}^k(P_i^k - \widetilde{P}_i^k)}{\Gamma(\sigma^2 + I_{i,j}^k + \sum_{q\ne k} G_{i,j}^q \widetilde{P}_i^q) + \widetilde{P}_i^k G_{i,j}^k}\right)$$

Since denominator term of RHS is less than that of LHS, LHS $\le$ RHS. Furthermore,

$$LHS = C_j^k(\mathbf{x}_k \vee \widetilde{\mathbf{x}}_k,\mathbf{x}_{-k}) + C_j^k(\mathbf{x}_k \wedge \widetilde{\mathbf{x}}_k,\mathbf{x}_{-k})$$

$$= \frac{1}{\bar{R}_j^k}\sum_{i=1}^{N} C_{i,j}^k(P_i^k \vee \widetilde{P}_i^k,P_i^{-k})A_{i,j}^k$$
$$+\frac{1}{\bar{R}_j^k}\sum_{i=1}^{N} C_{i,j}^k(P_i^k \wedge \widetilde{P}_i^k,P_i^{-k})A_{i,j}^k$$

$$= \frac{1}{\bar{R}_j^k}\left[\sum_{i=1}^{N} C_{i,j}^k(P_i^k,P_i^{-k})A_{i,j}^k + \sum_{i=1}^{N} C_{i,j}^k(\widetilde{P}_i^k,P_i^{-k})A_{i,j}^k\right]$$

$$= C_j^k(\mathbf{x}_k,\mathbf{x}_{-k}) + C_j^k(\widetilde{\mathbf{x}}_k,\mathbf{x}_{-k})$$

$$\le C_j^k(\mathbf{x}_k,\mathbf{x}_{-k}) + C_j^k(\widetilde{\mathbf{x}}_k,\mathbf{x}_{-k}).$$

Therefore, $C_j^k(\mathbf{x}_k,\mathbf{x}_{-k})$ is submodular in (P^k,P^{-k}) for fixed (A^k,A^{-k}). Then, for any given α_k, $\min\{C_j^k,\alpha_k\}$ is also submodular. Hence $\sum_{j=1}^{M_k}\min\{C_j^k,\alpha_k\}$ is submodular. Therefore, $\Phi_k(\mathbf{x}_k,\mathbf{x}_{-k},\alpha_k)$ is submodular in (P^k,P^{-k}) for fixed (A^k,A^{-k}).

Now we show that $C_j^k(\mathbf{x}_k,\mathbf{x}_{-k})$ is Submodular in (A^k,A^{-k}) for fixed $(\bar{P}^k,\bar{P}^{-k})$. We consider (B^k,B^{-k}) instead of (A^k,A^{-k}) for proving submodularity since B^k is sublattice and also has one to one correspondence with A^k. So, let $\mathbf{x}_k = (P^k,B^k)$. We show the decreasing property,

$$C_j^k(\mathbf{x}_k,\mathbf{x}_{-k}) - C_j^k(\widetilde{\mathbf{x}}_k,\mathbf{x}_{-k}) \le C_j^k(\mathbf{x}_k,\widetilde{\mathbf{x}}_{-k}) - C_j^k(\widetilde{\mathbf{x}}_k,\widetilde{\mathbf{x}}_{-k}),$$

The, LHS

$$= \frac{1}{\bar{R}_j^k}\sum_{i=1}^{N} C_{i,j}^k(P_i^k,P_i^{-k})B_{i,j}^k - \frac{1}{\bar{R}_j^k}\sum_{i=1}^{N} C_{i,j}^k(P_i^k,P_i^{-k})\widetilde{B}_{i,j}^k$$

similarly, RHS

$$= \frac{1}{\bar{R}_j^k}\sum_{i=1}^{N} C_{i,j}^k(P_i^k,P_i^{-k})B_{i,j}^k - \frac{1}{\bar{R}_j^k}\sum_{i=1}^{N} C_{i,j}^k(P_i^k,P_i^{-k})\widetilde{B}_{i,j}^k$$

Hence,

$$LHS = RHS \le RHS.,$$

Moreover,

$$LHS = C_j^k(\mathbf{x}_k \vee \widetilde{\mathbf{x}}_k,\mathbf{x}_{-k}) + C_j^k(\mathbf{x}_k \wedge \widetilde{\mathbf{x}}_k,\mathbf{x}_{-k})$$

$$= \frac{1}{\bar{R}_j^k}\sum_{i=1}^{N} C_{i,j}^k(P_i^k,P_i^{-k})(B_{i,j}^k \vee \widetilde{B}_{i,j}^k)$$

$$+\frac{1}{\bar{R}_j^k}\sum_{i=1}^{N} C_{i,j}^k(P_i^k,P_i^{-k})(B_{i,j}^k \wedge \widetilde{B}_{i,j}^k)$$

$$= \frac{1}{\bar{R}_j^k}\left[\sum_{i=1}^{N} C_{i,j}^k(P_i^k,P_i^{-k})B_{i,j}^k + \sum_{i=1}^{N} C_{i,j}^k(P_i^k,P_i^{-k})\widetilde{B}_{i,j}^k\right]$$

$$= C_j^k(\mathbf{x}_k,\mathbf{x}_{-k}) + C_j^k(\widetilde{\mathbf{x}}_k,\mathbf{x}_{-k})$$

$$\le C_j^k(\mathbf{x}_k,\mathbf{x}_{-k}) + C_j^k(\widetilde{\mathbf{x}}_k,\mathbf{x}_{-k}).$$

Therefore, $C_j^k(\mathbf{x}_k,\mathbf{x}_{-k})$ is submodular in (A^k,A^{-k}) for fixed (P^k,P^{-k}). Then, for any given α_k, $\min\{C_j^k,\alpha_k\}$ is also submodular. Hence $\sum_{j=1}^{M_k}\min\{C_j^k,\alpha_k\}$ is submodular. Therefore, $\Phi_k(\mathbf{x}_k,\mathbf{x}_{-k},\alpha_k)$ is submodular in (A^k,A^{-k}) for fixed (P^k,P^{-k}).

For the general case, let $(P^k,A^k) \ge (\widetilde{P}^k,\widetilde{A}^k)$, $(P^{-k},A^{-k}) \ge (\widetilde{P}^{-k},\widetilde{A}^{-k})$. Then we can consider $(P^k,A^k) \ge (\widetilde{P}^k,A^k) \ge (\widetilde{P}^k,\widetilde{A}^k)$ and $(P^{-k},A^{-k}) \ge (\widetilde{P}^{-k},A^{-k}) \ge (\widetilde{P}^{-k},\widetilde{A}^{-k})$ and use the above to show the submodularity of the game. □

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