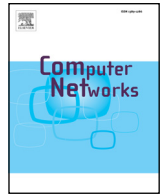




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Optimal resource allocation in wireless-powered OFDM relay networks[☆]

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ABSTRACT

This paper studies resource allocation in wireless-powered orthogonal-frequency-division multiplexing (OFDM) amplify-and-forward (AF) or decode-and-forward (DF) relay networks with time-switching (TS) based relaying. Our objective is to maximize end-to-end achievable rates by optimizing TS ratios of energy transfer (ET) and information transmission (IT), power allocation (PA) over all subcarriers for ET and IT as well as subcarrier pairing (SP) for IT. The formulated resource allocation problem is a mixed integer programming (MIP) problem, which is prohibitive and fundamentally difficult to solve. To simplify the MIP problem, we firstly provide an optimal ET policy and an optimal SP scheme, and then obtain a nonlinear programming problem to optimize TS ratios and PA for IT. Nevertheless, the obtained nonlinear programming problem is non-convex and still hard to tackle directly. To make it tractable, we transform the non-convex problem into a fractional programming problem, which is further converted into an equivalent optimization problem in subtractive form. By deriving the optimal solution to the equivalent optimization problem, we propose a globally optimal resource allocation scheme which bears much lower complexity as compared to the suboptimal resource allocation in the literature. Finally, our simulation results verify the optimality of our proposed resource allocation scheme and show that it outperforms the existing scheme in literature.

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1. Introduction

Energy transfer via radio frequency (RF)¹ has recently emerged as a new potential energy harvesting (EH) technique in wireless communications [2]. In RF energy transfer, RF signals radiated from a dedicated source is captured by receivers' antennas and then converted to a direct current voltage through appropriate circuits (rectennas) [3,4]. Due to electromagnetic wave propagation with large-scale path loss, a RF EH receiver may harvest only a small fraction of energy transferred by the source. However, by efficiently utilizing available energy, RF energy transfer may be energy-efficient in energy-constrained wireless networks. Specially, conventional battery-powered wireless networks suffer from short lifetime and require periodic replacement or recharging in order to maintain network connectivity, which may result in high operation

cost. RF energy transfer offers an efficient solution to this problem, since it can recharge energy-constrained wireless nodes at little cost to extend the wireless nodes' lifetime. In consequence, RF energy transfer has been a hot research area in wireless communications over several years, often under the umbrella of the green radio/communications [5–7].

Since RF signals can carry both energy and information, energy-constrained wireless nodes can scavenge energy and receive information from the RF signals in wireless communications, which results a new wireless communication technique named as simultaneous wireless information and power transfer (SWIPT) [8] or wireless-powered communications [9]. In wireless-powered communications, how to design energy transfer and information transmission strategies is very important to improve the performance of wireless networks because a non-trivial tradeoff usually exists for information transfer versus energy transfer [8]. Therefore, wireless-powered communication becomes more and more popular in wireless communications and has been investigated in various wireless communication networks [8–14].

An important application of wireless-powered communications is the cooperative relay networks with EH at relay nodes. This is because energy-constrained relays are often employed in wireless

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¹ For the reader's convenience, the abbreviation notations in this paper are summarized in Table 1 in the next page.

Table 1
Abbreviation notations.

Notation	Representation	Notation	Representation
RF	Radio frequency	EH	Energy harvesting
WSN	Wireless sensor network	WBAN	Wireless body area network
ET	Energy transfer	IT	Information transmission
AF	Amplify-and-forward	DF	Decode-and-forward
TS	Time splitting	PS	Power splitting
TSR	Time-splitting relaying	PSR	Power-splitting relaying
PA	Power allocation	SP	Subcarrier pairing
SWIPT	Wireless information and power transfer	MIP	Mixed-integer programming
SR	Source-to-relay	RD	Relay-to-destination
SNR	Signal noise ratio	ES	Exhaustive search

communications. In wireless cellular networks, due to random positions and mobility of users, energy-constrained relays are employed when relays need to be opportunistically deployed where most needed. Because running power cables to supply energy may be impractical or cumbersome in these scenarios, relays are usually supplied by a pre-charged battery and thus energy-constrained. Other more applications of energy-constrained relay nodes can also be found in wireless sensor networks (WSNs) [15] or wireless body area networks (WBANs) [16], where relays are usually wireless sensor nodes powered by the batteries with limited capacity, since power grid connections are usually not available in WSNs and it is impossible to connect relay nodes in WBANs to the power grid. Thus, how to solve the energy scarcity problem for relay nodes is important. Otherwise, periodic replacement or recharging may result in high operation cost, especially when relay nodes frequently assist in communications or are installed in an environment where manual operations may be inconvenient, dangerous (e.g., in a toxic environment) or even impossible (e.g., for sensors implanted in human bodies). Another approach is to harvest energy at relay by scavenging energy from natural sources. However, this type of energy may be unreliable (i.e., weather-dependent) and hence insufficient for a relay node that frequently supports communication activities. Thus, a more promising approach is to employ wireless-powered communications in which a wireless-powered relay first harvests energy (carried by the received RF signals) radiated from the source and then uses the harvested energy to forward the source information to the destination. Recent papers have discussed many application scenarios, e.g., emerging ultra-dense small scale cell deployments, wireless multicell networks, sensor networks and extremely dense wireless networks, where a combination of wireless-powered communication and relaying can be useful and practical [17–25].

In a wireless-powered relay network, the relay can harvest energy as well as implement information transmission (IT) in either time-switching relaying (TSR) or power-splitting relaying (PSR) manners [18]. If TSR is employed, the relay spends some time on EH and the remaining time on IT. If PSR is employed, the relay splits a portion of received power for EH and the remaining power for IT. In [18], the performance of TSR or PSR is studied respectively in a *narrow-band* single-carrier amplify-and-forward (AF) relay network. In [19], by allowing the relay to harvest sufficient energy to transmit information (sent by the source) at a fixed power level, an improved wireless-powered communication scheme is proposed for the narrow-band single-carrier network operating with TSR. Another interesting study in narrow-band relay networks is the power allocation for PSR-based EH relays with multiple source-destination pairs [21,22]. More recent studies on wireless-powered communications focused on broadband relay networks as more and more wireless communication networks operate in broadband channels in order to achieve large transmission capacities [26,27]. In [24], the resource allocation scheme in a PSR-based broadband orthogonal-frequency-division multiplexing (OFDM) network with

AF relay has been investigated. In [25], both PSR-based and TSR-based resource allocations in a multi-antenna OFDM network with AF relay have been studied. However, these studies are either sub-optimal resource allocations [19,21,22] or have high computational complexity in order to obtain the optimal TS or PS ratios by exhaustive search (ES) [18,24,25], where TS ratio of EH (IT) is referred to as the ratio of the time allocated for EH (IT) to the total time of EH and IT, and the PS ratio of EH (IT) is referred to as the ratio of the power allocated for EH (IT) to the total power used for EH and IT.

In this paper, we focus on TSR-based wireless-powered OFDM relay networks. The reason that we study a TSR-based OFDM network (instead of a PSR-based OFDM network) is because TSR reduces the complexity of a receiver in applying the wireless-powered communication technology at its relay as current commercial circuits are usually designed to decode information and harvest energy separately [8]. The approach through time switching between wireless power and information transfer was proposed for point-to-point networks in [8] and then employed in wireless-powered relay networks in [18], which is "practically appealing since state-of-the-art wireless information and energy receivers are typically designed to operate separately with very different power sensitivities" [8]. Thus, though employing PS-based receiver at relay may improve the system performance since no extra time is spent on EH, it is reasonable to consider a TS-based receiver at relay with separate EH and information processing circuits, as in [18,23,25].

For a TSR-based wireless-powered OFDM relay network, we study resource allocations when the relay employs AF or decode-and-forward (DF) protocols respectively. To the best of our knowledge, there are few research on TSR-based wireless-powered OFDM relay networks. Unlike these few work that focuses on obtaining a suboptimal resource allocation scheme with high computational complexity [25], in this paper, we allocate resources for wireless-powered OFDM AF or DF relay networks by a global optimal and efficient approach. The goal is to maximize end-to-end achievable rates, where the resource allocation includes TS ratios of EH and IT, PA over subcarriers for ET and IT as well as subcarrier pairing (SP) for IT. The formulated problem is a mixed integer programming (MIP) problem which is NP-hard and difficult to solve. By firstly providing an optimal PA for ET and an optimal SP scheme for IT, we simplify the MIP problem into a nonlinear programming problem to jointly optimize PA for IT and TS ratios. Nevertheless, the simplified nonlinear programming problem is non-convex because its EH constraint and rate objective function are both non-convex, which makes it still hard to solve directly. To tackle the non-convex problem, we transform it into a fractional programming problem, which is then converted into an equivalent optimization problem in subtractive form with a tractable solution. By solving the equivalent optimization problem, we propose an efficient low-complexity algorithm to achieve global optimal resource allocations. Finally, by computer simulations, we verify the optimality of our proposed

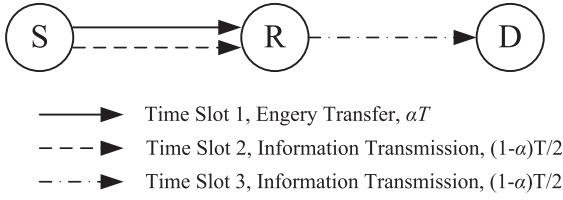


Fig. 1. The system model of two-hop wireless-powered OFDM relay networks.

algorithm. Also, by computer simulations, we show that our proposed resource allocation scheme outperforms the suboptimal resource allocation scheme in [25] in wireless-powered AF OFDM relay networks.

This paper is organized as follows. Section 2 proposes the system model. Section 3 presents our detailed studies on the optimal resource allocation for wireless-powered AF or DF OFDM relay networks respectively. Section 4 demonstrates our simulation results which verify the optimality of our proposed resource allocation scheme. Section 5 concludes the paper.

2. System model and problem formulation

Consider a wireless-powered OFDM relay network with a source node (S), a destination node (D), and an EH relay node (R) as illustrated in Fig. 1. We assume that there is no direct link between the source and the destination, e.g., due to physical obstacles, which is a valid assumption in many real-world communication scenarios [18,19,22–25,28–30]. Thus, the communications between the source and the destination are assisted by the relay, which operates with AF or DF relaying protocol. The source has fixed energy supply while the relay is energy-constrained which needs to harvest energy from RF signals transmitted by the source and operates with the TSR protocol.

It is assumed that the total frequency band is divided into N subcarriers. IT assisting by the relay is implemented on a SP basis, where the information transmitted by the source on one subcarrier at the SR (source-to-relay) link is forwarded by the relay to the destination on one designated subcarrier at the RD (relay-to-destination) link [28–30]. The subcarrier pair set is denoted as $\mathcal{N} = \{1, 2, \dots, N\}$,² where N is the subcarrier number in the considered network.

The transmission from the source to the destination is on a time-frame basis. Each time frame with equal duration, denoted as T , is divided into three time slots. The first time slot allocated for ET from the source to the relay has a TS ratio of α . Both the second and third time slots, allocating for IT from the source to the relay and from the relay to the destination respectively, have a TS ratio equal to $(1 - \alpha)/2$. That is, like studies in [18,21,22,25], the two time slots in our scheme have the same transmission periods. For AF relaying, it is necessary to assume the symmetric IT because an AF relay amplifies information signals received from an incoming subcarrier and directly forwards them to the next node via an outgoing subcarrier. For DF relaying, the assumption of symmetric IT simplifies its implementation because the relay can re-encode information signals with the same codebook as the one used by the source. It is also worth pointing out that the dynamic allocation of the two IT time slots may further improve the system performance [31,32] but potentially at the cost of the increased complexity of relay networks. We will study such a scenario in our future work.

The channel is assumed to be block fading, i.e., the channel gains are constant within the duration of one frame, but vary in-

Table 2
Symbol notations.

Notation	Representation
\mathcal{N}	Subcarrier pair set
T	Length of one time frame
α	TS ratio of ET
τ	Energy conversion efficiency of the EH receiver at relay
h_n^{SR}	Channel response of SR link over subcarrier n
h_n^{RD}	Channel response of RD link over subcarrier n
σ_R^2	Variance of the received noise at the relay
σ_D^2	Variance of the received noise at the destination
γ_n^{SR}	Normalized channel gain of SR link over subcarrier n
γ_n^{RD}	Normalized channel gain of RD link over subcarrier n
$p_m^{S,E}$	Transmit power at source over m th subcarrier for ET
$p_n^{S,I}$	Transmit power at source over n th subcarrier pair for IT
p_n^R	Transmit power at relay over n th subcarrier pair for IT
\mathcal{P}_S	Maximum allowable transmit power at source
R_{AF}	End-to-end achievable rate of wireless-powered OFDM AF relay network
R_{DF}	End-to-end achievable rate of wireless-powered OFDM DF relay network
d_{SR}	Source-to-relay distance
d_{RD}	Relay-to-destination distance
κ	Ratio of d_{RD} to d_{SR}

dependently from one frame to another. Denote h_n^{SR} and h_n^{RD} as the channel responses of SR and RD links over subcarrier n , respectively. The variances of the received noises at the relay and the destination are denoted as σ_R^2 and σ_D^2 , which is uniformly distributed over all subcarriers. Then $\gamma_n^{SR} = \frac{|h_n^{SR}|^2}{\sigma_R^2/N}$ and $\gamma_n^{RD} = \frac{|h_n^{RD}|^2}{\sigma_D^2/N}$ are the normalized channel gains.

Let $\mathbf{p}_E = \{p_m^{S,E}, m \in \mathcal{N}\}$ be the PA policy for ET, where $p_m^{S,E}$ denotes the source's transmit power allocated to the m th ($m \in \mathcal{N}$) subcarrier for the purpose of ET. Then, the energy harvested at the relay can be expressed as [10]

$$E = \alpha T \sum_{m=1}^N \tau p_m^{S,E} |h_m^{SR}|^2, \quad (1)$$

where $0 < \tau < 1$ is the energy conversion efficiency which depends on the rectification process and the EH circuitry. Moreover, let $\mathbf{p}_I = \{p_n^{S,I}, p_n^R, n \in \mathcal{N}\}$ be the PA policy for IT, where $p_n^{S,I}$ and p_n^R denote the transmission power allocated on the subcarriers belonging to subcarrier pair $n \in \mathcal{N}$ at the source and the relay respectively for IT purposes. It is assumed that the harvested energy at the relay is used for the relay's IT, and the harvested energy should be larger than the energy consumed in IT at the relay [18,21,22,24,25]. Thus, we have

$$E \geq \frac{(1 - \alpha)T}{2} \sum_{n=1}^N p_n^R. \quad (2)$$

For the considered wireless-powered OFDM AF relay network, the end-to-end achievable rate can be expressed as [28]

$$R_{AF} = \frac{(1 - \alpha)}{2N} \sum_{n=1}^N \log \left(1 + \frac{p_n^{S,I} \gamma_n^{SR} p_n^R \gamma_n^{RD}}{p_n^{S,I} \gamma_n^{SR} + p_n^R \gamma_n^{RD} + 1} \right). \quad (3)$$

Meanwhile, for the considered wireless-powered OFDM DF relay network, the end-to-end achievable rate can be expressed as [30]

$$R_{DF} = \frac{(1 - \alpha)}{2N} \sum_{n=1}^N \min \{ \log (1 + p_n^{S,I} \gamma_n^{SR}), \log (1 + p_n^R \gamma_n^{RD}) \}. \quad (4)$$

Therefore, we can formulate the resource allocation problem to maximize the end-to-end achievable rate for the wireless-powered OFDM AF or DF relay network such as follows:

$$\max_{\alpha \in [0,1], \mathbf{p}_E, \mathbf{p}_I, \mathcal{N}} R, \quad (5a)$$

² For the reader's convenience, the symbol notations in this paper are summarized in Table 2.

$$\text{s.t. } \frac{(1-\alpha)}{2} \sum_{n=1}^N p_n^R \leq \alpha \sum_{m=1}^N \tau p_m^{S,E} |h_m^{SR}|^2, \quad (5b)$$

$$\sum_{m=1}^N p_m^{S,E} \leq \mathcal{P}_S, \quad p_m^{S,E} \geq 0, \quad m \in \mathcal{N}, \quad (5c)$$

$$\sum_{n=1}^N p_n^{S,I} \leq \mathcal{P}_S, \quad p_n^{S,I} \geq 0, \quad p_n^R \geq 0, \quad n \in \mathcal{N}, \quad (5d)$$

where (5b) is the EH constraint derived by (2), \mathcal{P}_S is the maximum allowable transmit power at the source and R refers to (3) or (4) for AF or DF relay networks respectively.

3. Optimal power allocation for energy transfer and subcarrier pairing

Problem (5) is an MIP problem since the combinatorial optimization of \mathcal{N} is involved, which has been proven to be NP-hard and is fundamentally difficult to solve [33]. To simplify problem (5), we firstly investigate the PA for ET in wireless-powered OFDM relay networks. According to (1) and (5c), we obtain

$$E = \alpha T \sum_{m=1}^N \tau p_m^{S,E} |h_m^{SR}|^2 \leq \alpha \tau T \mathcal{P}_S \max_m \left(|h_m^{SR}|^2 \right). \quad (6)$$

The inequality (6) indicates that to maximize the harvested energy at the relay, the source should allocate all the available power over the subcarrier which has the maximum channel gain. Thus, we obtain the optimal PA policy for ET such as illustrated in the following proposition.

Proposition 1. For wireless-powered OFDM relay networks, the optimal PA for ET to problem (5) is

$$\bar{p}_m^{S,E} = \begin{cases} \mathcal{P}_S, & m = \arg \max_m \left\{ |h_m^{SR}|^2 \right\} \\ 0, & \text{otherwise} \end{cases} \quad (7)$$

On the basis of Proposition 1, the harvested energy at the relay in (1) can be rewritten as $E = \alpha GT$, where $G \triangleq \tau \mathcal{P}_S \max_m \left(|h_m^{SR}|^2 \right)$ is a constant. Then the EH constraint at the relay in (5b) can be expressed as

$$\frac{(1-\alpha)}{2} \sum_{n=1}^N p_n^R \leq \alpha G. \quad (8)$$

Thus, we can simplify problem (5) as

$$\max_{\alpha \in [0,1], \mathbf{p}_1, \mathcal{N}} R, \quad (9a)$$

$$\text{s.t. } \frac{(1-\alpha)}{2} \sum_{n=1}^N p_n^R \leq \alpha G, \quad (9b)$$

$$\sum_{n=1}^N p_n^{S,I} \leq \mathcal{P}_S, \quad p_n^{S,I} \geq 0, \quad p_n^R \geq 0, \quad n \in \mathcal{N}. \quad (9c)$$

Note that as α is given, problem (9) is equivalent to the resource allocation problem for a traditional OFDM AF [28,29] or DF [28,30] relay network with separate power constraints at the source and the relay. For the traditional OFDM relay networks, it has been proved that the ordered-SNR (signal noise ratio) SP, i.e. the SR subcarrier with the strongest channel gain is paired with the RD subcarrier with the strongest channel gain, the SR subcarrier with the second strongest channel gain is paired with the RD subcarrier with the second strongest channel gain, and so forth, is optimal to maximize the end-to-end achievable rate [28–30]. Thus,

the optimization of \mathcal{N} is independent of that of α and \mathbf{p}_1 . Then, we have the following proposition.

Proposition 2. The optimal subcarrier pair set $\bar{\mathcal{N}}$ for problem (5) can be obtained with the ordered-SNR SP.

In this paper, without loss of generality, it is assumed that the subcarriers at SR and RD links are sorted by decreasing orders and then paired with the ordered-SNR SP to obtain the optimal $\bar{\mathcal{N}}$.

4. Joint optimization of TS ratios and power allocation for information transmission

As the optimal SP is determined, problem (9) is reduced to optimize TS ratio α and PA for IT \mathbf{p}_1 only. Note that given α , problem (9) is equivalent to the PA problem in traditional OFDM AF [28,29] or DF [28,30] relay networks. Thus, we can solve problem (9) with two alternative steps. The first step is to solve problem (9) as α is given, which is just as computing the optimal PA in a traditional AF or DF relay network. Then, in the second step, α is optimized by ES. Such alternative optimization technique has been employed to solve the resource allocation problem for AF OFDM relay networks with multiple antennas [25]. However, it is suboptimal because high SNR approximation is adopted to simplify the solution of the equivalent PA problem. Moreover, it has high computational complexity and hence may not be practical because both achieving optimal PA in a traditional relay network and obtaining optimal α by ES are non-trivial, which will be analyzed in detail later. Thus, it is novel and important to solve problem (9) with an efficient approach that can achieve global optimization, which motivates us to propose the following approach.

According to (5b), we obtain

$$\alpha \geq \frac{\sum_{n=1}^N p_n^R}{\sum_{n=1}^N p_n^R + 2G}. \quad (10)$$

Meanwhile, we observe that the objective function in problem (9) is a non-increasing function of α . Thus, under optimal resource allocation for problem (9), α must satisfy

$$\alpha = \frac{\sum_{n=1}^N p_n^R}{\sum_{n=1}^N p_n^R + 2G}. \quad (11)$$

Submitting (11) into (3), we have the end-to-end achievable rate for AF relay such as

$$R'_{AF} = \frac{G}{N \left(\sum_{n=1}^N p_n^R + 2G \right)} \sum_{n=1}^N \log \left(1 + \frac{p_n^{S,I} \gamma_n^{SR} p_n^R \gamma_n^{RD}}{p_n^{S,I} \gamma_n^{SR} + p_n^R \gamma_n^{RD} + 1} \right). \quad (12)$$

Submitting (11) into (4), we have the end-to-end achievable rate for DF relay such as

$$R'_{DF} = \frac{G}{N \left(\sum_{n=1}^N p_n^R + 2G \right)} \times \sum_{n=1}^N \min \left\{ \log \left(1 + p_n^{S,I} \gamma_n^{SR} \right), \log \left(1 + p_n^R \gamma_n^{RD} \right) \right\}. \quad (13)$$

Then, we can equivalently rewrite problem (9) as

$$\max_{\mathbf{p}_1} R', \quad (14a)$$

$$\text{s.t. } \sum_{n=1}^N p_n^{S,I} \leq \mathcal{P}_S, \quad p_n^{S,I} \geq 0, \quad p_n^R \geq 0, \quad n \in \bar{\mathcal{N}}. \quad (14b)$$

where R' refers to (12) or (13) for AF or DF relay networks respectively.

It is observed that both the rate functions in (12) and (13) have fractional structures. Thus, problem (14) is a nonlinear fractional

programming problem [34]. In the follows, we solve these fractional programming problems for AF or DF relay networks respectively.

4.1. AF relaying

To solve problem (14) for AF relay networks, we have the following proposition.

Proposition 3. Define a function $F_{AF}(\mathbf{p}_1, \mu)$ as

$$F_{AF}(\mathbf{p}_1, \mu) = \mu N \left(\sum_{n=1}^N p_n^R + 2G \right) - G \sum_{n=1}^N \log \left(1 + \frac{p_n^{S,I} \gamma_n^{SR} p_n^R \gamma_n^{RD}}{p_n^{S,I} \gamma_n^{SR} + p_n^R \gamma_n^{RD} + 1} \right), \quad (15)$$

and define

$$T_{AF}(\mu) = \min_{\mathbf{p}_1} F_{AF}(\mathbf{p}_1, \mu), \quad (16a)$$

$$\text{s.t. } \sum_{n=1}^N p_n^{S,I} \leq \mathcal{P}_S, \quad (16b)$$

$$p_n^{S,I} \geq 0, p_n^R \geq 0, n \in \bar{N}. \quad (16c)$$

Then if and only if $T_{AF}(\bar{\mu}) = 0$, the optimal value of problem (14) for AF relay networks is $\bar{\mu}$ and the optimal solution to problem (14) for AF relay networks is the optimal solution to the following problem

$$\min_{\mathbf{p}_1} F_{AF}(\mathbf{p}_1, \bar{\mu}), \quad (17a)$$

$$\text{s.t. } \sum_{n=1}^N p_n^{S,I} \leq \mathcal{P}_S, p_n^{S,I} \geq 0, p_n^R \geq 0, n \in \bar{N}. \quad (17b)$$

Proof. See Appendix A. \square

On the basis of Proposition 3, we firstly solve problem (16) to obtain $T_{AF}(\mu)$ for a given μ . In problem (16), we rewrite the objective function in (16a) such as follows

$$F_{AF}(\mathbf{p}_1, \mu) = \mu N \left(\sum_{n=1}^N p_n^R + 2G \right) - G \left[\sum_{n=1}^N \log \left(1 + p_n^{S,I} \gamma_n^{SR} \right) + \sum_{n=1}^N \log \left(1 + p_n^R \gamma_n^{RD} \right) - \sum_{n=1}^N \log \left(1 + p_n^{S,I} \gamma_n^{SR} + p_n^R \gamma_n^{RD} \right) \right]. \quad (18)$$

The function in (18) is not a concave function, thus the Karush–Kuhn–Tucker (KKT) conditions are not sufficient for calculating the optimal solution to problem (16) [35]. Nevertheless, the KKT conditions are still the necessary conditions for all possible candidates of the optimal solution. Thus, construct the Lagrangian such as follows

$$L(\mathbf{p}_1, \mu, \lambda) = F_{AF}(p_n^{S,I}, p_n^R, \mu) + \lambda \left(\sum_{n=1}^N p_n^{S,I} - \mathcal{P}_S \right),$$

where $\lambda \geq 0$ is a Lagrange multiplier. Then, the necessary conditions for optimality can be expressed as follows [36]

$$\begin{cases} \frac{\partial L(\bar{\mathbf{p}}_1, \mu, \lambda)}{\partial \mathbf{p}_1} \geq 0 \\ \frac{\partial L(\bar{\mathbf{p}}_1, \mu, \lambda)}{\partial \mathbf{p}_1} \bar{\mathbf{p}}_1 = 0 \end{cases} \quad (19a)$$

where $\bar{\mathbf{p}}_1$ is the optimal solution to problem (16). According to (19), we obtain that the optimal $\bar{p}_n^{S,I}$ and \bar{p}_n^R must satisfy either the equations such as

$$\frac{\partial L(\bar{\mathbf{p}}_1, \mu, \lambda)}{\partial p_n^{S,I}} = \frac{\partial L(\bar{\mathbf{p}}_1, \mu, \lambda)}{\partial p_n^R} = 0 \quad (20)$$

or the equations such as

$$\bar{p}_n^{S,I} = \bar{p}_n^R = 0 \quad (21)$$

for each $n \in \bar{N}$.

According to (20), we have

$$\begin{cases} \frac{G \gamma_n^{SR}}{1 + p_n^{S,I} \gamma_n^{SR}} - \frac{G \gamma_n^{SR}}{1 + p_n^{S,I} \gamma_n^{SR} + p_n^R \gamma_n^{RD}} = \lambda \\ \frac{G \gamma_n^{RD}}{1 + p_n^R \gamma_n^{RD}} - \frac{G \gamma_n^{RD}}{1 + p_n^{S,I} \gamma_n^{SR} + p_n^R \gamma_n^{RD}} = \mu N \end{cases} \quad (22a)$$

Furthermore, according to (22), we can construct a cubic equation such as follows:

$$u^3 - \left(1 + \frac{2\lambda}{G \gamma_n^{SR}} - \frac{\mu}{G \gamma_n^{RD}} \right) u^2 + \frac{\lambda}{G \gamma_n^{SR}} \left(2 + \frac{\lambda}{G \gamma_n^{SR}} - \frac{\mu}{G \gamma_n^{RD}} \right) u - \frac{\lambda}{G^2 \gamma_n^{SR}} \left(\frac{\lambda}{\gamma_n^{SR}} - \frac{\mu}{\gamma_n^{RD}} \right) = 0. \quad (23)$$

Then the solutions to the equations in (22) can be solved through the cubic equation in (23) by

$$\begin{cases} \bar{p}_n^{S,I} = \frac{1}{\gamma_n^{SR}} \left[\frac{1}{u} - 1 \right]^+ & \text{(a)} \\ \bar{p}_n^R = \frac{1}{\gamma_n^{RD}} \left[\frac{1}{u - \frac{\lambda}{G \gamma_n^{SR}} + \frac{\mu}{G \gamma_n^{RD}}} - 1 \right]^+ & \text{(b)} \end{cases} \quad (24)$$

where $n \in \bar{N}$ and $[x]^+ \triangleq \max(x, 0)$.

Note that for certain n , the number of positive solutions to Eq. (23) may be zero, one or more than one. If there is no positive solutions for Eq. (23), it indicates that the necessary condition (20) can not be satisfied and thus the power allocation should be set as the necessary condition in (21), i.e. $\bar{p}_n^{S,I} = \bar{p}_n^R = 0$. If there is multiple positive solutions for Eq. (23), in order to obtain the global optimal solutions, we just need to compare the resulting values of the corresponding term in $L(\bar{\mathbf{p}}_1, \mu, \lambda)$, and keep the solution that leads to the maximum [35,36].

In addition, to obtain the solutions by (24), the Lagrange multiplier λ should be determined. Note that the objective function in problem (16a) is monotone increasing with $p_n^{S,I}$, thus we have the lemma such as follows.

Lemma 1. The power constraint in (16b) must be satisfied with equality for the optimality of problem (16).

Remark 1. On the basis of Lemma 1, we can calculate the optimal Lagrange multiplier λ with the bisection method. Moreover, Lemma 1 indicates that under optimal power allocation, the source uses all the available power to transmit information in the wireless-powered OFDM AF relay network.

We have derived the power allocation policy for problem (16) based on the necessary condition (20). However, such power allocation policy is not sufficient for optimality of problem (16), since (21) is also a necessary condition for optimality of problem (16). Thus, there are $(2^N - 1)$ possible ways to allocate power over the subcarriers based on (21) and (24). Enumerating all those candidate solutions is much overwhelming in complexity. Fortunately, we can obtain that the optimal power allocation solution has the following truncated structure.

Lemma 2. Under optimal power allocation, there is an integer $K \in \bar{N}$ such that

$$p_n^{S,I} > 0, p_n^R > 0, \forall n \leq K, \quad (25)$$

Algorithm 1 Optimization Algorithm for Problem (16)

```

1: Initialization: Set  $k = 1$  and  $F_{\text{opt}} = -\text{Inf}$ ;
2: Repeat:
   Set  $p_n^{S,1} = p_n^R = 0$  for all  $n > k$ ;
   Compute  $p_n^{S,1}$  and  $p_n^R$  by (24) for all  $n \leq k$ ;
   Compute  $F_{\text{AF}}(\mathbf{p}_1, \mu)$  by (18);
   If  $F_{\text{AF}}(\mathbf{p}_1, \mu) > F_{\text{opt}}$ ,
      $F_{\text{opt}} = F_{\text{AF}}(\mathbf{p}_1, \mu)$ ,  $\bar{p}_n^{S,1} = p_n^{S,1}$ ,  $\bar{p}_n^R = p_n^R$ ;
   End
    $k = k + 1$ ;
3: Until:  $k = N$ .

```

and $p_n^{S,1} = p_n^R = 0$, otherwise. (26)

Proof. See Appendix B. \square

On the basis of Lemma 2, we can adopt a linear search procedure to seek the optimal solution for problem (16), which is illustrated such as in Algorithm 1.

As the optimal solution for problem (16) is obtained, we can solve the equation $T_{\text{AF}}(\bar{\mu}) = 0$ to obtain optimal $\bar{\mu}$ by using a standard bisection procedure. Thus, we have the optimization algorithm to solve problem (14) as summarized in Algorithm 2, where ϵ denotes the predefined accuracy of bisection search over μ and U is the upper bound of the optimal value of problem (14),

$$U = N \log \left(1 + P_S \max_n \gamma_n^{\text{SR}} \right), \quad (27)$$

which is obtained by considering the maximum achievable rate from source to relay.

4.2. DF relaying

For DF relay networks, problem (14) has a complex objective function, and thus is difficult to solve directly. To simplify this objective function, we provide the proposition such as follows:

Proposition 4. For wireless-powered OFDM DF relay networks, the optimal power allocation policy $p_n^{S,1}$ and p_n^R must satisfy

$$p_n^R \gamma_n^{\text{RD}} \leq p_n^{S,1} \gamma_n^{\text{SR}}, \quad \forall n.$$

Algorithm 2 Joint TS Ratio and PA for IT Optimization Algorithm for Wireless-powered OFDM AF Relay Networks

```

1: Initialization:  $\mu_l = 0$ ,  $\mu_u = U$  and  $t = 0$ ;
2: Repeat:
    $\mu(t) = \frac{1}{2}(\mu_l + \mu_u)$ ;
   Obtain  $\bar{p}_n^{S,1}$  and  $\bar{p}_n^R$ ,  $n \in \bar{N}$ , by Algorithm 1;
    $p_n^{S,1}(t) = \bar{p}_n^{S,1}$ ,  $p_n^R(t) = \bar{p}_n^R$ ;
   If  $F_{\text{AF}}(\mathbf{p}_1, \mu) \geq 0$ ,
      $\mu_u = \mu(t)$ ;
   Else
      $\mu_l = \mu(t)$ ;
   End
    $t = t + 1$ ;
3: Until:  $\mu_u - \mu_l < \epsilon$ ;
4: Obtain the optimal TS ratio and PA for IT:
    $\bar{p}_n^{S,1} = p_n^{S,1}(t)$ ,  $\bar{p}_n^R = p_n^R(t)$ ;
   Calculate the optimal TS ratio  $\bar{\alpha}$  by (11).

```

Proof. We prove Proposition 4 by contradiction. Assume that there is a PA policy $\hat{\mathbf{p}}_1$ of which the power allocation over some subcarrier pair k satisfies $\hat{p}_k^R \gamma_k^{\text{RD}} > \hat{p}_k^{S,1} \gamma_k^{\text{SR}}$. The corresponding end-to-end achievable rate is denoted as \hat{R} . Then as \hat{p}_k^R is reduced to $\beta \hat{p}_k^R$, where $0 < \beta < 1$, such that $\beta \hat{p}_k^R \gamma_k^{\text{RD}} = \hat{p}_k^{S,1} \gamma_k^{\text{SR}}$, the end-to-end achievable rate \hat{R} will not decrease. Meanwhile, according to (2), as \hat{p}_k^R decreases, α can also decrease. Note that according to (4), as α decreases, a higher end-to-end transmission rate can be achieved. This is contrary to the assumption that $\hat{\mathbf{p}}_1$ is optimal. \square

Remark 2. Proposition 4 indicates that the source may reserve some power while transmitting information to the relay. Moreover, Proposition 4 indicates that under optimal PA, the rate over subcarrier pair n is just determined by the $p_n^R \gamma_n^{\text{RD}}$. Thus, we can let $p_n^R \gamma_n^{\text{RD}} = p_n^{S,1} \gamma_n^{\text{SR}}$ without decreasing the rate over subcarrier pair n . Then we have

$$p_n^{S,1} = \frac{\gamma_n^{\text{RD}}}{\gamma_n^{\text{SR}}} p_n^R, \quad \forall n, \quad (28)$$

and the end-to-end rate in (13) can be expressed as

$$R'_{\text{DF}} = \frac{G}{N(\sum_{n=1}^N p_n^R + 2G)} \sum_{n=1}^N \log(1 + p_n^R \gamma_n^{\text{RD}}). \quad (29)$$

Then, we can solve problem (14) for DF relay networks by the following proposition.

Proposition 5. Define a function $F_{\text{DF}}(\mathbf{p}_1^R, \mu)$ as

$$F_{\text{DF}}(\mathbf{p}_1^R, \mu) = \mu N \left(\sum_{n=1}^N p_n^R + 2G \right) - G \sum_{n=1}^N \log(1 + p_n^R \gamma_n^{\text{RD}}), \quad (30)$$

where $\mathbf{p}_1^R = \{p_n^R, n \in \bar{N}\}$, and define

$$T_{\text{DF}}(\mu) = \min_{\mathbf{p}_1^R} F_{\text{DF}}(\mathbf{p}_1^R, \mu), \quad (31a)$$

$$\text{s.t.} \quad \sum_{n=1}^N \frac{\gamma_n^{\text{RD}}}{\gamma_n^{\text{SR}}} p_n^R \leq P_S, \quad (31b)$$

$$p_n^R \geq 0, n \in \bar{N}. \quad (31c)$$

Then if and only if $T_{\text{DF}}(\bar{\mu}) = 0$, the optimal value of problem (14) for DF relay networks is $\bar{\mu}$ and the optimal solution to problem (14) for DF relay networks is the optimal solution to the following problem

$$\min_{\mathbf{p}_1^R} F_{\text{DF}}(\mathbf{p}_1^R, \bar{\mu}), \quad (32a)$$

$$\text{s.t.} \quad \sum_{n=1}^N \frac{\gamma_n^{\text{RD}}}{\gamma_n^{\text{SR}}} p_n^R \leq P_S, \quad (32b)$$

$$p_n^R \geq 0, n \in \bar{N}. \quad (32c)$$

Proof. See Appendix C. \square

Problem (31) is a convex problem, which has a globally unique optimal solution. Using the Karush–Kuhn–Tucker (KKT) conditions [35], we can obtain the optimal solution to problem (31) as

$$\bar{p}_n^R = \left[\frac{G \gamma_n^{\text{SR}}}{\mu N \gamma_n^{\text{SR}} + \nu \gamma_n^{\text{RD}}} - \frac{1}{\gamma_n^{\text{RD}}} \right]^+, \quad \forall n, \quad (33)$$

where ν is the Lagrange multiplier determined by the power constraint in (31b).

Algorithm 3 Joint TS Ratio and PA for IT Optimization Algorithm for wireless-powered OFDM DF Relay Networks

```

1: Initialization:  $\mu_l = 0$ ,  $\mu_u = U$  and  $t = 0$ ;
2: Repeat:
    $\mu(t) = \frac{1}{2}(\mu_l + \mu_u)$ ;
   Calculate  $\bar{p}_n^R$  by (33) and let  $p_n^R(t) = \bar{p}_n^R$ ;
   If  $F_{DF}(\mathbf{p}_1^R, \mu) \geq 0$ ,
      $\mu_u = \mu(t)$ ;
   Else
      $\mu_l = \mu(t)$ ;
   End
    $t = t + 1$ ;
3: Until:  $\mu_u - \mu_l < \epsilon$ ;
4: Obtain the optimal TS ratio and PA for IT:
    $\bar{p}_n^R = p_n^R(t)$  and calculate  $\bar{p}_n^{S,1}$  by (28);
   Calculate the optimal TS ratio  $\bar{\alpha}$  by (11).
  
```

As the optimal solution for problem (31) is obtained, we can solve the equation $T_{DF}(\bar{\mu}) = 0$ to obtain optimal $\bar{\mu}$ by using a standard bisection procedure. Thus, we have the optimization algorithm to solve problem (14) for DF relay networks as summarized in Algorithm 3, where ϵ denotes the predefined accuracy of bisection search over μ and U is the upper bound of the optimal value of problem (14), which is also determined by (27).

4.3. Complexity analysis

As mentioned at the beginning of section 4, for TSR-based wireless-powered AF OFDM relay networks, a suboptimal resource allocation scheme has been proposed in [25], where an alternative optimization algorithm is required to obtain the optimal α by ES and the suboptimal \mathbf{p}_1 by calculating the PA in a traditional AF OFDM relay network with high SNR approximation, but it has much high complexity and may be impractical. On the other hand, our proposed optimization algorithm for the AF OFDM relay network, i.e. Algorithm 2, has much lower computational complexity. In the follows, we give the details on the complexity analysis of the above-mentioned algorithms, including Algorithm 3 for the DF relay network.

Firstly, for our proposed resource allocation scheme, the computational complexity of the bisection search used in Algorithms 2 and 3 is $\mathcal{O}(\log_2(\epsilon^{-1}))$. To calculate the optimal PA, one optimal Lagrange multiplier is to be determined (refer to (24) for the AF relay network and (33) for the DF relay network). Let δ be the predefined accuracy of searching for the optimal Lagrange multiplier. Then, for the AF relay network, since k ($k \in \{2, 3, \dots, N\}$) cubic equations need to be solved for the k -th alteration in Algorithm 1, the computational complexity is like $\mathcal{O}(N^2 \log_2(\delta^{-1}))$ as the bisection search method is employed to calculate the optimal Lagrange multiplier. For the DF relay network, the computational complexity is $\mathcal{O}(1/\delta)$ as the gradient method is employed to calculate the optimal Lagrange multiplier [37]. Thus, the general computational complexity of our proposed resource allocation scheme is $\mathcal{O}(N^2 \log_2(\epsilon^{-1}) \log_2(\delta^{-1}))$ for the AF relay network and $\mathcal{O}(\log_2(\epsilon^{-1})\delta^{-1})$ for the DF relay network.

Secondly, for the suboptimal scheme in [25], the computational complexity of the ES can be evaluated as $\mathcal{O}(\frac{1}{\Delta\alpha})$, where $\Delta\alpha$ is an update step for the exhaustive search on a continuous interval $[0, 1]$. Furthermore, the calculation of suboptimal PA in a traditional AF OFDM relay network [28] with high SNR approximation requires calculating two optimal Lagrange multipliers, whose computational complexity is $\mathcal{O}(1/\delta^2)$ as the gradient method is employed to calculate two optimal Lagrange multipliers. Thus, the

general computational complexity of the suboptimal scheme in [25] is $\mathcal{O}(\frac{1}{\Delta\alpha} 1/\delta^2)$.

Without loss of generality, let $\epsilon = \delta = \Delta\alpha = \zeta = 0.0001$ and $N = 2048$. Then for AF relay networks, the general computational complexity is about 7×10^8 for our proposed resource allocation scheme and 10^{12} for the suboptimal scheme in [25]. That is, the computational complexity of our scheme is about 10^4 times lower than that of the suboptimal scheme in [25]. Meanwhile, for DF relay networks, the general computational complexity is about 10^5 for our proposed resource allocation scheme, which also has low computational complexity.

5. Simulation results

This section evaluates the performance of our proposed resource allocation scheme by implementing Monte-Carlo simulations. In the simulations, the total frequency band is set as 5 MHz and the total number of subcarriers is set as $N = 32$. Over each subcarrier, the channel responses of SR and RD links are independent and identically distributed (i.i.d.) complex Gaussian random variables with zero mean. The large-scale path loss is modeled as $d^{-2.5}$, where d is the distance between two nodes. We denote the SR distance as d_{SR} , which is a reference distance set to be 10m. The maximum allowable transmit power at the source is $\mathcal{P}_S = 10$ dBm. The variance of the receiving noise at the relay and destination is set as the same, i.e. $\sigma_R^2 = \sigma_D^2 = \sigma^2$. The SNR is defined as $\text{SNR} = \mathcal{P}_S/\sigma^2$. The energy conversion efficiency is $\tau = 0.9$. The simulation results are obtained by averaging over 1000 channel realizations. All configuration parameters mentioned above will not change in the following simulations unless specified otherwise.

In Fig. 2, we compare the rates of the AF relay network achieved by the proposed optimal scheme (denoted as "Optimal AF rate" in the legend) and fixed-TS schemes (denoted as "Fixed-TS AF Rate") as the RD distance varies, where $\kappa = \frac{d_{RD}}{d_{SR}}$, for which d_{RD} denote the RD distance. SNR is set as 20dB. The fixed-TS schemes are obtained by solving problem (9) as α is fixed. Also, in Fig. 2, we illustrate the optimal EH TS ratio α obtained by our proposed resource allocation scheme in the AF relay network (denoted as "Optimal AF EH TS ratio α "). From Fig. 2, it is observed that the proposed optimal resource allocation scheme for the AF relay network outperforms the fixed-TS schemes. It is interesting to observe that the rate achieved by the fixed-TS scheme with $\alpha = 0.5$ approaches that of the optimal scheme as $\kappa = 1$. This is because the optimal α is approximately equal to 0.5 when $\kappa = 1$.

In Fig. 3, we compare the rates of the AF relay network achieved by the proposed optimal scheme and the suboptimal scheme proposed in [25] (denoted as "Suboptimal AF Rate proposed in [25]") as κ varies, where SNRs are set as 0 dB, 20 dB and 40 dB, respectively. From Fig. 3, it is observed that the proposed optimal resource allocation scheme outperforms the suboptimal scheme proposed in [25], especially when SNR is low, e.g. SNR = 0 dB. Meanwhile, note that our proposed optimal resource allocation scheme is much more efficient than the suboptimal scheme proposed in [25] because the ES method is employed in the suboptimal scheme proposed in [25], for which the complexity comparison has been provided in section 4.

In Fig. 4, we compare the rates of the DF relay network achieved by the proposed optimal scheme (denoted as "Optimal DF rate") and fixed-TS schemes (denoted as "Fixed-TS DF Rate") as κ varies, where SNR=20dB. Also, in Fig. 4, we illustrate the optimal EH TS ratio α in the DF relay network (denoted as "Optimal DF α "). From Fig. 4, it is observed that the proposed optimal scheme outperforms the fixed-TS schemes. Moreover, it is observed that the rate achieved by the fixed-TS scheme with $\alpha = 0.5$ approaches that of the optimal scheme when $\kappa = 0.5$. This is because the optimal α is approximately equal to 0.5 when $\kappa = 0.5$.

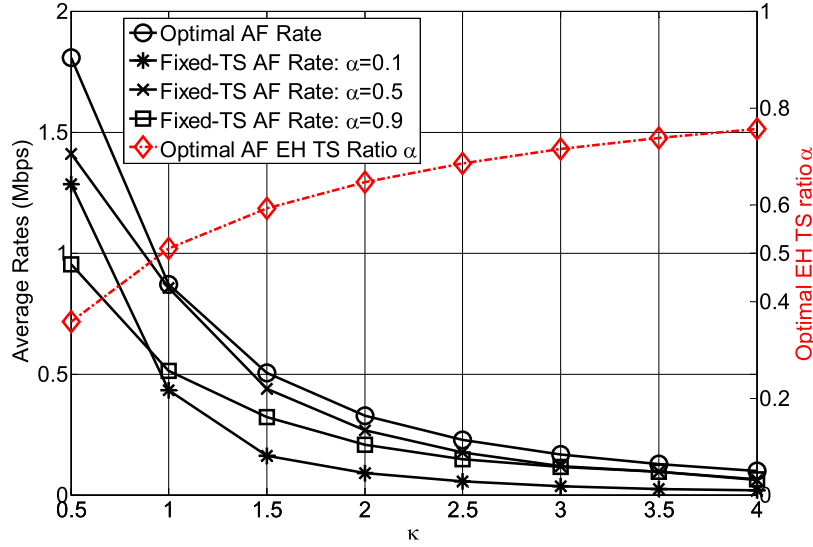


Fig. 2. Average rates versus κ in wireless-powered OFDM AF relay networks; performance comparison of the proposed optimal resource allocation scheme and fixed-TS schemes, where SNR = 20 dB.

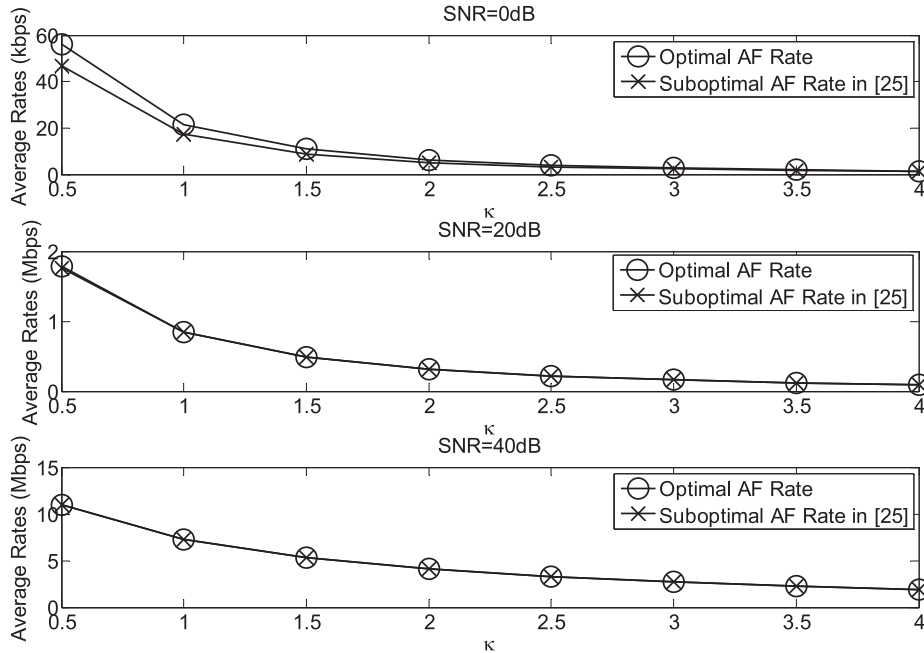


Fig. 3. Average rates versus κ in wireless-powered OFDM AF relay networks; performance comparison of the proposed optimal resource allocation scheme and the suboptimal scheme proposed in [25], where SNR = 0 dB, 20 dB and 40 dB.

In Fig. 5, we compare the optimal rates of the DF and AF relay networks achieved by our proposed optimal schemes as κ varies, where SNRs are set as 0 dB, 20 dB and 40 dB, respectively. From Fig. 5, it is observed that DF relaying outperforms AF relaying, especially when κ is small (e.g. $\kappa \leq 1$) or SNR is low (e.g. SNR = 0 dB). This is because noises are suppressed by DF relays but amplified by AF relays. However, when κ is large (e.g. $\kappa \geq 3$) and SNR is high (e.g. SNR = 40 dB), the rates achieved in the DF relay network are similar to those achieved in the AF relay network. The reasons are illustrated as follows. First, at high SNR region, we have $\log(1 + \frac{p_n^{S,1,SR} p_n^{R,SR} p_n^{R,RD}}{p_n^{S,1,SR} \gamma_n^{SR} + p_n^{R,SR} \gamma_n^{RD} + 1}) \approx \log(1 + \frac{p_n^{S,1,SR} p_n^{R,SR} p_n^{R,RD}}{p_n^{S,1,SR} \gamma_n^{SR} + p_n^{R,SR} \gamma_n^{RD}})$. Second, we have $\gamma_n^{RD} \ll \gamma_n^{SR}$ when $\kappa \geq 3$, since $d_{RD} \geq 3d_{SR}$. Third, by Fig. 6, where the optimal transmit powers at source and relay used for IT in the AF and DF relay networks are illustrated when SNR = 40 dB, the AF relay power is around 2 dBm and the AF source power is

10 dBm when $\kappa \geq 3$. This indicates that for the AF relay network, the optimal p_n^R is much less than the optimal $p_n^{S,1}$. Thus, for the AF relay network with a large κ , we have $p_n^R \gamma_n^{RD} \ll p_n^{S,1} \gamma_n^{SR}$, which results in that $\log(1 + \frac{p_n^{S,1,SR} p_n^{R,SR} p_n^{R,RD}}{p_n^{S,1,SR} \gamma_n^{SR} + p_n^{R,SR} \gamma_n^{RD}}) \approx \log(1 + p_n^{R,SR} \gamma_n^{RD})$. Note that for the DF relay network, the end-to-end rate over subcarrier n is also determined by $\log(1 + p_n^{R,SR} \gamma_n^{RD})$, which has been demonstrated in Proposition 4. Moreover, from Fig. 6, it is observed that the DF relay power is almost the same as the AF relay power when $\kappa \geq 3$. Therefore, the optimal rates in the AF relay network are similar to those in the DF relay network when κ is large. Nevertheless, from Fig. 6, it is observed that the source in the DF relay network reserves some power while the source in the AF relay network always uses up its available power when κ is large. Therefore, the total consumed power in DF relay networks is less than that in AF

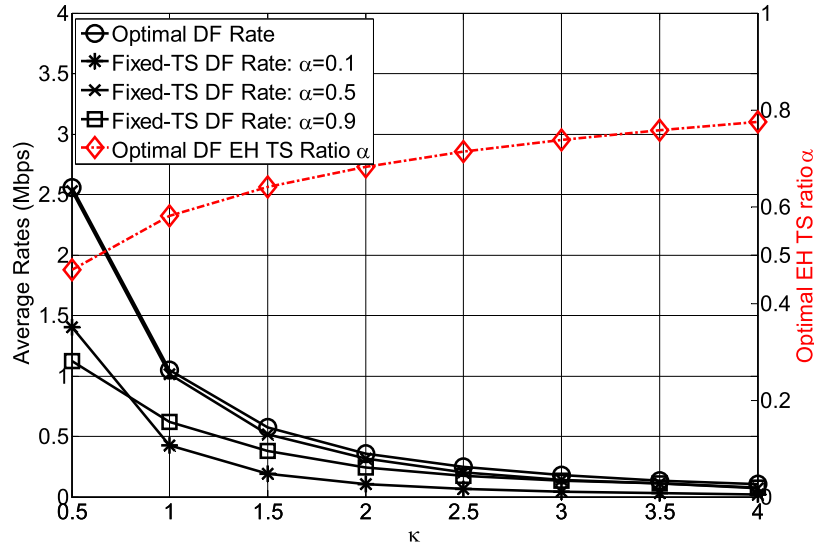


Fig. 4. Average rates versus κ in wireless-powered OFDM DF relay networks; performance comparison of the proposed optimal resource allocation scheme and fixed-TS schemes, where SNR = 20 dB.

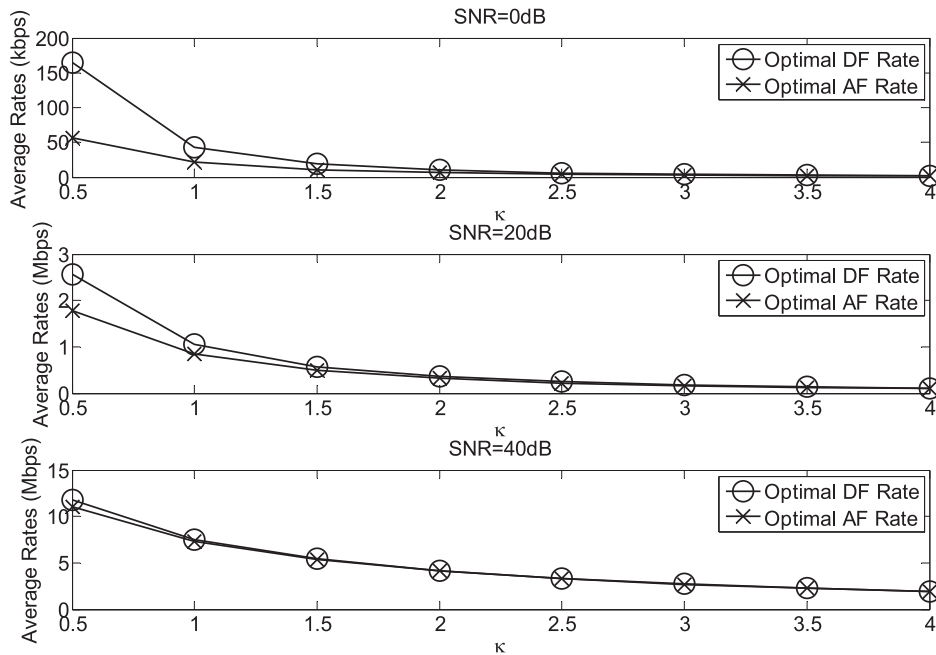


Fig. 5. Average rates versus κ in wireless-powered OFDM DF and AF relay networks achieved by the proposed optimal scheme, where SNR = 0 dB, 20 dB and 40 dB.

relay networks when κ is large. This indicates that DF relaying is more energy-efficient than AF relaying.

In Fig. 7, we compare the optimal EH TS ratios, α , in the DF and AF relay networks as κ varies, where SNRs are set as 0 dB, 20 dB and 40 dB, respectively. From Fig. 7, it is observed that the optimal EH TS ratios in the DF relay network are usually larger than those in the AF relay network. When κ is small (e.g., $\kappa \leq 1$), the difference between the optimal EH TS ratio in the DF relay network and that in the AF relay network is large. However, as κ becomes large, the difference becomes small and even disappears when $\kappa \geq 3$ and SNR = 40 dB. This can be verified by (11) and Fig. 6. (11) shows that the larger the relay power is, the larger the EH TS ratio α is. Meanwhile, Fig. 6 illustrates that when κ is small, the DF relay power is larger than the AF relay power, which results in that the optimal EH TS ratios in the DF relay network are larger than those in the AF relay network. On the other hand, when κ is

large, it is observed from Fig. 6 that the DF relay power is almost the same as the AF relay power, which results in that the optimal EH TS ratios in the DF relay network are similar to those in the AF relay network.

In Figs. 8 and 9, we further illustrate the rates achieved by our proposed optimal scheme and fixed-TS schemes in the AF or DF relay network as \mathcal{P}_S varies, where $\sigma^2 = 0$ dBm and $\kappa = 1$. For illustrations, we also illustrate the optimal α in Figs. 8 and 9. Obviously, our proposed optimal scheme outperforms fixed-TS schemes. Moreover, as illustrated in Fig. 8 for the AF relay network, the rate achieved by the fixed-TS scheme with $\alpha = 0.5$ approaches that achieved by the proposed optimal scheme when \mathcal{P}_S is 20 dBm. Meanwhile, as illustrated in Fig. 9 for the DF relay network, the rate achieved by the fixed-TS scheme with $\alpha = 0.5$ approaches that achieved by the proposed optimal scheme when \mathcal{P}_S is 25 dBm. This is because when \mathcal{P}_S is 20 dBm in the AF relay network or 25 dBm

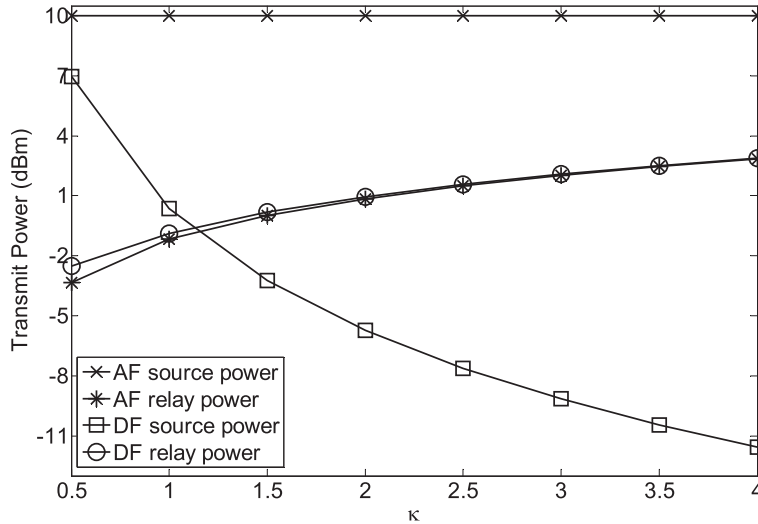


Fig. 6. Optimal transmit power at source and relay versus κ in wireless-powered AF and DF OFDM relay networks, where SNR = 40 dB.

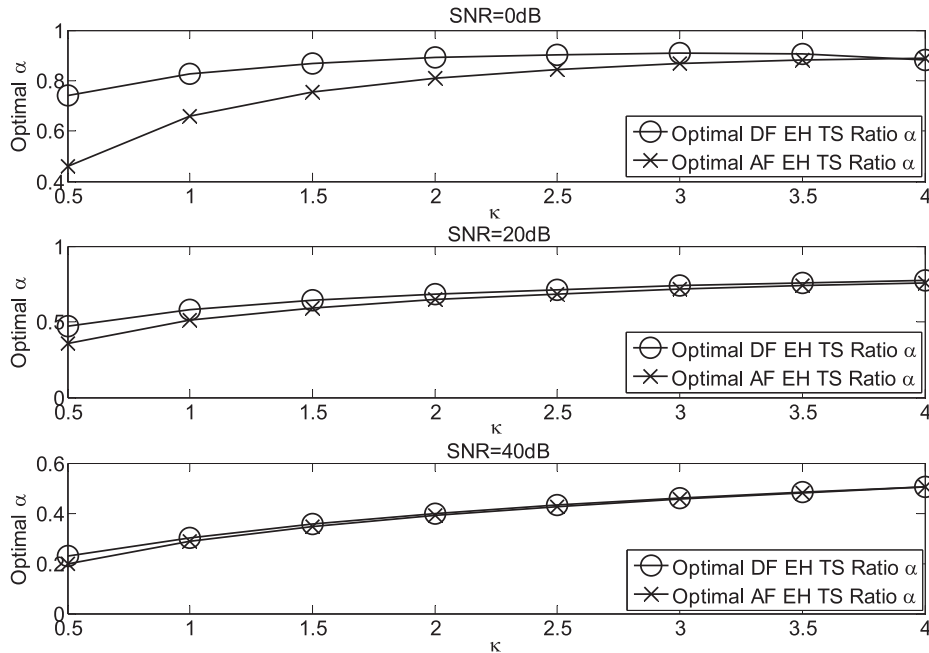


Fig. 7. Optimal TS ratio α versus κ in wireless-powered OFDM DF and AF relay networks, where SNR = 0 dB, 20 dB and 40 dB.

in the DF relay network, the value of optimal α is approximately equal to 0.5.

In Fig. 10, we compare the rates achieved by our proposed optimal scheme and the suboptimal scheme in [25] for the AF relay network as \mathcal{P}_S varies, where $\sigma^2 = 0$ dBm and $\kappa = 1$. For observation convenience, we just illustrate the results in low and medium SNR regions, i.e. \mathcal{P}_S varies from 0 dBm to 20 dBm. From Fig. 10, it is observed that our proposed optimal scheme outperforms the suboptimal scheme in [25] when SNR is low or medium. This is because high SNR approximation was adopted in [25] to achieve the suboptimal scheme.

Finally, in Fig. 11, we compare the optimal rates and α in the AF relay network with those in the DF relay network as \mathcal{P}_S varies, where $\sigma^2 = 0$ dBm and $\kappa = 1$. From Fig. 11, it is observed that DF relaying outperforms AF relaying under variant values of \mathcal{P}_S . This is consistent with the results illustrated in Fig. 5, which shows the same conclusion when $\kappa = 1$. Also, from Fig. 11, it is observed that the optimal EH TS ratios α in the DF relay network are larger than

those in the AF relay network, but the difference becomes small as SNR increases. This is consistent with the results illustrated in Fig. 7, which shows the same conclusion when $\kappa = 1$.

6. Conclusion

In this paper, we have investigated the resource allocation in wireless-powered OFDM AF or DF relay networks to maximize end-to-end achievable rates. We firstly studied the optimal energy transfer policy and SP scheme in both AF and DF relay networks. Then, we investigated the optimization of TS ratios and PA for IT in AF or DF relay networks respectively. The optimization problem was formulated as an MIP problem. By providing the optimal energy transfer policy and SP scheme, we simplified the MIP problem as a nonlinear programming problem which is non-convex. By transforming the non-convex problem into a fractional programming problem, we convert it into an equivalent optimization in subtractive form which has a tractable solution. By solving the

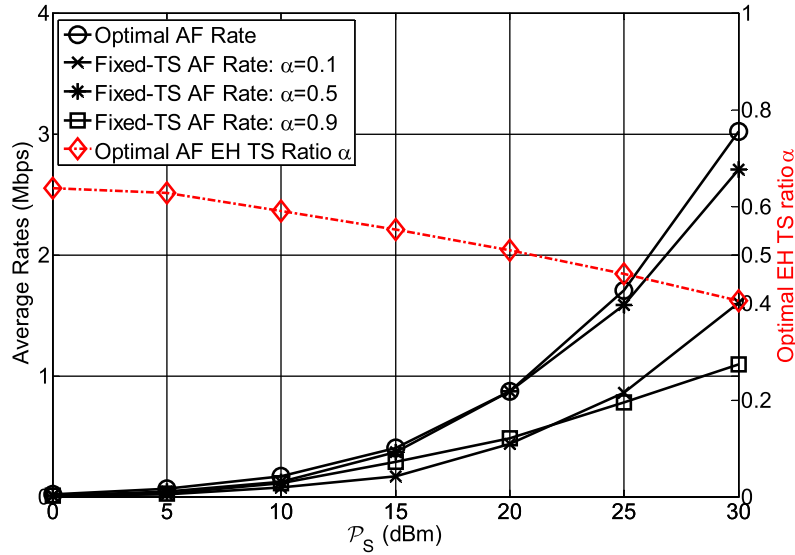


Fig. 8. Average rates versus \mathcal{P}_S in wireless-powered OFDM AF relay networks; performance comparison of the optimal resource allocation scheme with fixed-TS schemes, where $\kappa = 1$ and $\sigma^2 = 0$ dBm.

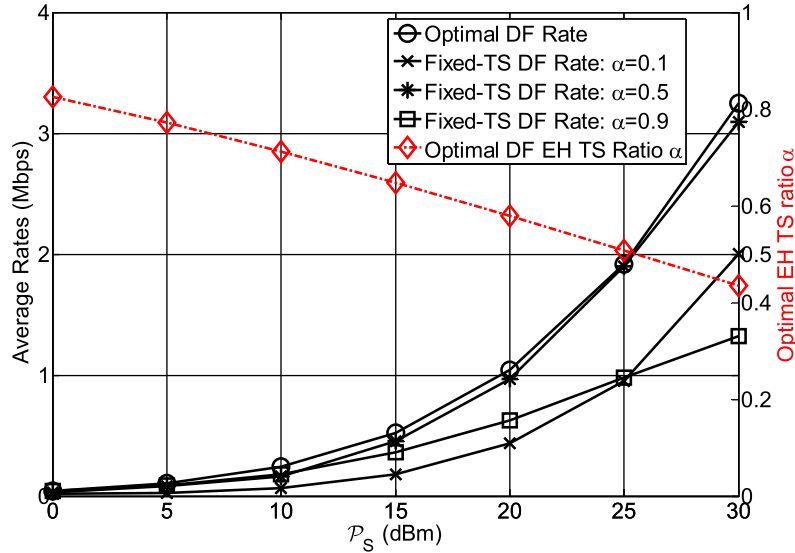


Fig. 9. Average rates versus \mathcal{P}_S in wireless-powered OFDM DF relay networks; performance comparison of the optimal resource allocation scheme with fixed-TS schemes, where $\kappa = 1$ and $\sigma^2 = 0$ dBm.

equivalent optimization problem, we proposed an efficient low-complexity algorithm to achieve global optimal resource allocation. Finally, the simulation results demonstrated the optimality of our proposed resource allocation scheme.

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Appendix A. Proof of Proposition 3

Firstly, for AF relay networks, problem (14) can be equivalently rewritten as

min : μ ,
s.t.

$$\left[\begin{array}{l} \max_{\mathbf{p}_i} \frac{G}{N(\sum_{n=1}^N p_n^R + 2G)} \sum_{n=1}^N \log \left(1 + \frac{p_n^{S,1} \gamma_n^{SR} p_n^{R,RD}}{p_n^{S,1} \gamma_n^{SR} + p_n^{R,RD} + 1} \right) \leq \mu, \\ \text{s.t. } \sum_{n=1}^N p_n^{S,1} \leq \mathcal{P}_S, p_n^{S,1} \geq 0, p_n^R \geq 0, n \in \bar{N} \end{array} \right] \quad (34)$$

by introducing a variable μ as an upper bound on problem (14). This step holds since minimizing μ is the same as finding the least upper bound of the objective function in problem (14). This is equal to the maximum value of the objective function in problem (14), which exists, as seen by straightforward continuity argument [38].

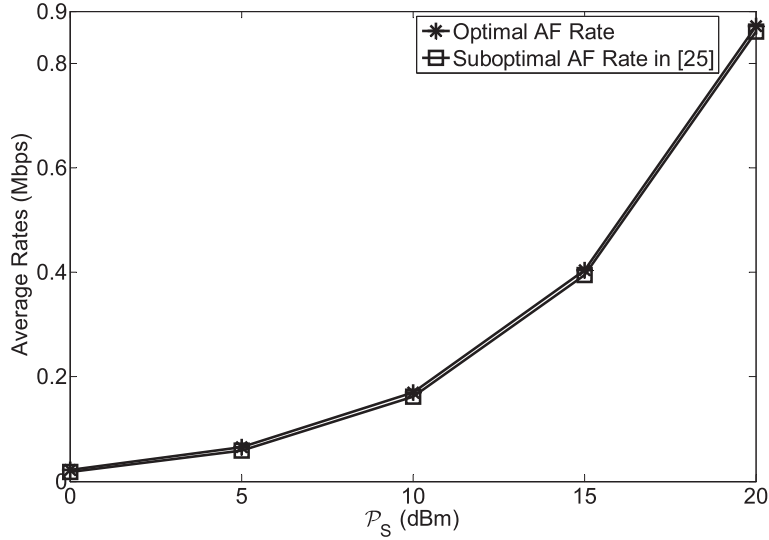


Fig. 10. Average rates versus \mathcal{P}_S in wireless-powered OFDM AF relay networks; performance comparison of the optimal resource allocation scheme with the suboptimal scheme in [25], where $\kappa = 1$ and $\sigma^2 = 0$ dBm.

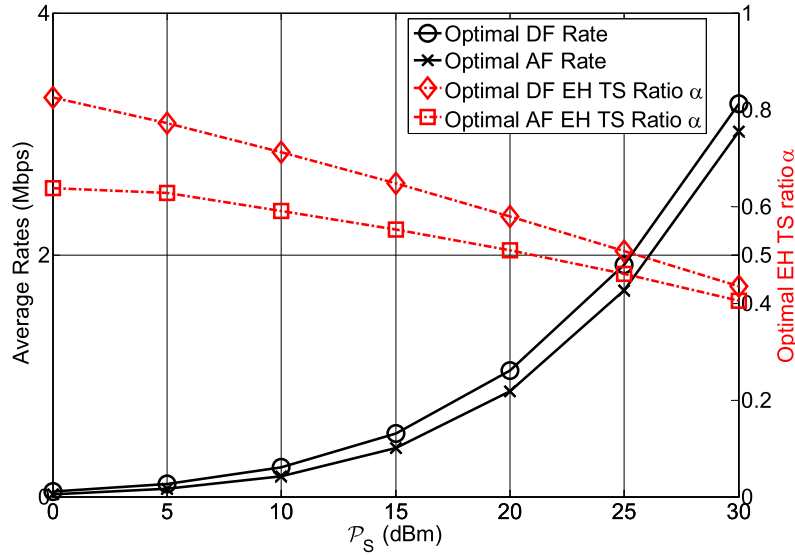


Fig. 11. Average optimal rates and α versus \mathcal{P}_S in wireless-powered OFDM relay networks; Performance comparisons of DF and AF relaying, where $\kappa = 1$ and $\sigma^2 = 0$ dBm.

Furthermore, problem (34) can be equivalently rewritten as follows:

$$\begin{aligned} \min &: \mu, \\ \text{s.t.} & \end{aligned}$$

$$\left[\begin{array}{l} \min_{\mathbf{p}_1} F_{\text{AF}}(\mathbf{p}_1, \mu) \geq 0, \\ \text{s.t. } \sum_{n=1}^N p_n^{S,1} \leq \mathcal{P}_S, p_n^{S,1} \geq 0, p_n^R \geq 0, n \in \bar{\mathcal{N}} \end{array} \right] \quad (35)$$

where $F_{\text{AF}}(\mathbf{p}_1, \mu)$ is defined as in (15).

Then, μ is a true upper bound of problem (14) if the problem

$$\min_{\mathbf{p}_1} F_{\text{AF}}(\mathbf{p}_1, \mu), \text{ s.t. } \sum_{n=1}^N p_n^{S,1} \leq \mathcal{P}_S, p_n^{S,1} \geq 0, p_n^R \geq 0, n \in \bar{\mathcal{N}} \quad (36)$$

has a non-negative optimal value. Specially, if and only if $\mu = \bar{\mu}$, where $\bar{\mu}$ is the solution of the equation $T_{\text{AF}}(\bar{\mu}) = 0$, in which

$T_{\text{AF}}(\mu)$ is defined in (16), the optimal value of problem (14) for AF relay networks is $\bar{\mu}$, and $\bar{\mathbf{p}}_1 = \arg \max_{\mathbf{p}_1} F_{\text{AF}}(\mathbf{p}_1, \bar{\mu})$ is the optimal solution to problem (14) for AF relay networks. The proof is completed.

Appendix B. Proof of Lemma 1

We prove Lemma 1 by contradiction. Assume that at optimality of power allocation for problem (16), there exist n_1 and n_2 satisfying $p_{n_1}^{S,1} = p_{n_1}^R = 0$ and $p_{n_2}^{S,1}, p_{n_2}^R > 0$, where $n_1, n_2 \in \bar{\mathcal{N}}$ and $n_1 < n_2$. Recall that $\bar{\mathcal{N}}$ is the subcarrier pair set where the channel gains are sorted with decreasing orders, thus we have $\gamma_{n_1}^{\text{SR}} > \gamma_{n_2}^{\text{SR}}$ and $\gamma_{n_1}^{\text{RD}} > \gamma_{n_2}^{\text{RD}}$.

Denote the objective function of problem (16) in (15) such as follows

$$F_{\text{AF}}(\mathbf{p}_1, \mu) = F_1(\mathbf{p}_1) - F_2(\mathbf{p}_1, \mu), \quad (37)$$

where

$$F_2(\mathbf{p}_1, \mu) \triangleq \mu N \left(\sum_{n=1}^N p_n^R + 2G \right) \quad (38)$$

and

$$F_2(\mathbf{p}_1) \triangleq GB \left[\sum_{n=1}^N \log(1 + p_n^{S,1} \gamma_n^{SR}) + \sum_{n=1}^N \log(1 + p_n^R \gamma_n^{RD}) - \sum_{n=1}^N \log(1 + p_n^{S,1} \gamma_n^{SR} + p_n^R \gamma_n^{RD}) \right]. \quad (39)$$

Then by swapping the values of $\{p_{n_2}^{S,1}, p_{n_2}^R\}$ and $\{p_{n_1}^{S,1}, p_{n_1}^R\}$, the value of $F_2(\mathbf{p}_1)$ will be increased, since it can be verified that $F_2(\mathbf{p}_1)$ is a monotonically increasing function on $p_n^{S,1}$ and p_n^R . Moreover, such swapping does not change the value of $F_1(\mathbf{p}_1)$, thus the value of $F_{AF}(\mathbf{p}_1, \mu)$ will be increased. This is contradictory with the assumption that the initial power allocation is optimal to minimize the value of $F_{AF}(\mathbf{p}_1, \mu)$. Therefore, the statement of Lemma 1 is proved.

Appendix C. Proof of Proposition 5

For DF relay networks, we can equivalently rewrite problem (14) as

min : μ ,

s.t.

$$\left[\begin{array}{l} \max_{\mathbf{p}_1, \mathbf{R}} \frac{G}{N(\sum_{n=1}^N p_n^R + 2G)} \sum_{n=1}^N \log(1 + p_n^R \gamma_n^{RD}) \leq \mu, \\ \text{s.t. } \sum_{n=1}^N \frac{\gamma_n^{RD}}{\gamma_n^{SR}} p_n^R \leq \mathcal{P}_S, p_n^R \geq 0, n \in \bar{N} \end{array} \right]$$

Furthermore, problem (40) can be equivalently rewritten as

min : μ ,

s.t.

$$\left[\begin{array}{l} \min_{\mathbf{p}_1} F_{DF}(\mathbf{p}_1, \mu) \geq 0, \\ \text{s.t. } \sum_{n=1}^N p_n^{S,1} \leq \mathcal{P}_S, p_n^R \geq 0, n \in \bar{N} \end{array} \right] \quad (41)$$

where $F_{DF}(\mathbf{p}_1, \mu)$ is defined as in (30).

Then, μ is a true upper bound if the problem

$$\min_{\mathbf{p}_1} F_{DF}(\mathbf{p}_1, \mu), \quad \text{s.t. } \sum_{n=1}^N p_n^{S,1} \leq \mathcal{P}_S, p_n^R \geq 0, n \in \bar{N} \quad (42)$$

has a non-negative optimal value. Specially, if and only if $\mu = \bar{\mu}$, where $\bar{\mu}$ is the solution of the equation $T_{DF}(\bar{\mu}) = 0$, in which $T_{DF}(\mu)$ is defined in (31), the optimal value of problem (14) for DF relay networks is $\bar{\mu}$, and $\bar{\mathbf{p}}_1 = \arg \max_{\mathbf{p}_1} F_{DF}(\mathbf{p}_1, \bar{\mu})$ is the optimal solution to problem (14) for DF relay networks. The proof is completed.

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