Computers and Chemical Engineering 96 (2017) 169–182

Contents lists available at ScienceDirect

Computers and Chemical Engineering

journal homepage: www.elsevier.com/locate/compchemeng

Review

Novel method for looped pipeline network resolution

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ARTICLE INFO

Article history:
Received 19 July 2016
Received in revised form 1 October 2016
Accepted 3 October 2016
Available online 5 October 2016

Keywords:
Looped pipeline network
Simultaneous-modular procedure
Hardy-cross
Kirchhoff's laws
Nonlinear system of equations
Process simulation

ABSTRACT

It is proposed a novel method to solve looped pipeline network problems that seeks to deal with limitations of the available methods The problem is modeled as a nonlinear system of equations formed by equations that cannot be solved sequentially, characterizing the resolution as a simultaneous-modular procedure. The equations of the system are the differences between the final pressure of the pipes that end at the same network nodes and the difference between the specified and calculated design variables. At the solution both Kirchhoff's laws are met, being the method main advantages the no need of independent loops selection and the formulation of a reduced system of equation. Case studs with a small and a big looped water pipeline network, and an industrial installation with looped pipeline configuration, are solved. The latter shows the method applicability for design process, highlighting its advantages in comparison with the traditional simulation procedures.

Contents

1. Introduction

The pipeline network problem is formulated when the flowrate and pressure in all pipes and nodes of a network are needed. The problem formulation requires the physical characteristics of the pipes and the specification of some network pressure and flowrate variables. Many networks problems are easily solved by a sequential procedure. However, when different pipes flow to a same node, or the network presents loops, different procedures must be employed.

In order to solve the looped pipeline network problem, it is very common the employment of the two Kirchhoff's laws. The first and the second laws, originally developed to solve electrical circuits problems, are equivalent, respectively, to the continuity and the energy conservation equation (Martinez and Puigjaner, 1988), and can be written as follow:

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http://dx.doi.org/10.1016/j.compchemeng.2016.10.001
0098-1354/

1ª: The algebraic sum of currents at a node on a network of conductors is zero.

2ª: The directed sum of the electrical potential differences (voltage) around any closed circuit is zero.

Appling the laws on the looped pipeline network problem, the current at the first law is understood as the mass flow and the electrical potential at the second law is understood as the pressure drop. When solving the looped pipeline network problem employing the Kirchhoff's laws, one of the laws is satisfied in the problem modeling and the other one is satisfied solving a nonlinear system of equations (Cross, 1936). Based on the linearization of such system of equations, Cross (1936) proposed the most relevant methods to solve the problem, which are essentially a relaxation method suitable to solve the problem by hand (Gay and Middleton, 1970). Depending on which law is firstly satisfied in the modeling, different methods are obtained, named: *Method of Balanced Flow*, which firstly satisfies the first Kirchhoff's law; and *Method of Balanced Pressure*, which firstly satisfies the second Kirchhoff's law (Cross, 1936).

In order to improve the problem resolution, methods based on the partitioning of the problem representative matrix were developed (Sargent, 1978; Shacham, 1984), but the application on large problems was still not satisfactory (Martinez and Puigjaner, 1988). Furthermore, other linearization methods were developed (Krope et al., 2011) and, more recently, the Newton-Raphson method was applied to solve the looped pipeline network problem for both Hardy-Cross methods (Altman and Boulos, 1995; Brkic, 2011). As any Newton-Raphson application, the characteristics of the equations and the initial guesses of the dependent variables define the problem convergence property.

Analyzing the Hard-Cross procedures, the *Method of Balanced Flow* is appropriate to solve problems with known pressures at any point of the network. Firstly, the network model must be built satisfying the second Kirchhoff's law, assuming pressures at all network nodes and calculating the flowrates in all pipes. As the pressure nodes and, consequently, the calculated flowrates in pipes are not the problem solution, corrections at the node pressure must be made until the mass balance, first Kirchhoff's law, be satisfied. In principle, for the problem resolution, it is needed the employment of a pressure drop equation with explicit flowrate variable. Aiming the improvement of the problem resolution, for any pressure drop equation, new procedures were proposed for the iterative correction of the nodes pressure (Rao, 1987) or by using non-deterministic optimization techniques, such as *Simulated Annealing* (Yeh and Lin, 2008; Tospornsampam et al., 2007).

The *Method of Balanced Pressure* is suitable to solve problems when the inlet and outlet flowrates of the looped network are known. By satisfying, in the network modeling, the mass balance in the nodes, the nonlinear system of equation is formed by the energy conservation equations of the selected network pipeline loops. The main drawback of this method lies on the selection of such representative loops (Gay and Middleton, 1970; Rao, 1987), since the number of the network loops is commonly greater than the number of necessary equations to solve the problem, and not every loops group forms a solvable problem. In order to overcome this drawback, Gay and Middleton (1970) proposed a procedure to perform the initial choice of the representative network loops and to compute the pressure drop with the Darcy-Weisbach equation, and Martinez and Puigjaner (1988) proposed a procedure to choose the loops in large networks. Currently, this choice can be done by graph procedures and it is already known that the representative network loops must be verified for the independency among each other (Jha, 2007).

Aiming the use of the *Method of Balanced Pressure* on problems with specified pressure at the inlet or outlet network nodes, a procedure to obtain pseudo-loops equations, as found in Streeter and Wylie (1984) and Sârbu and Valea (2011), was developed. With this procedure, the method can be applied on networks with either pressure or flowrate as specified variables and, because of that, nowadays, the method is widely employed to solve any looped network problem.

Another method, called hybrid method, has no need to firstly satisfy any Kirchhoff's law, which leads to a looped pipeline network problem characterized by both node mass balance and energy conservation equations (Hamam and Brameller 1971; Todini and Pilati, 1987; Osiadacz, 1987; EPANET, 2015). The method has as main drawback the need of simultaneous resolution of large set of equations, making even more difficult the problem convergence.

Despite the greater attention on water pipeline network distribution, the introduced methods are applied on looped pipeline network flowing any kind of fluid, being applicable on wide industrial and urban pipeline installations. In industrial plants, the looped pipeline arrangements are found on by-pass of equipment, utilities distribution systems and firefight systems, for instance; and in urban installations, on heat gas, fuel gas and water distribution networks. The calculation of looped pipeline network of gas distribution systems has gained some attention given the higher difficulty on compute pressure drop (Krope et al., 2011; Woldeyohannes and Majid, 2011). Furthermore, the calculations of pumps, valves and pipe accidents pressure drops were incorporated into the looped pipeline problem, making possible the resolution of more realistic problems (Krope and Goricanec, 1991).

Currently, different methodologies can be employed to solve looped pipeline network problems. The identification and analysis of the pipes, devices and equipment's characteristics, the problem specifications and the flowing fluid must be done previously to make possible the choice of the best resolution method (Brkic, 2011).

In this work, a novel method for looped pipeline network resolution is proposed. The new method consists in attending the mass balance in the nodes (first Kirchhoff's law), group all equations that can be solved sequentially, identifying the ones that need simultaneous convergence. Such equations formulate the nonlinear system of equations of the problem being characterized by the differences between the final pressures of the pipes that ends on the same node, which replace the loop equations (second Kirchhoff's law), and the differences between specified and calculated design variables, which replace the pseudo-loop equations. With the proposed method, there is no need of identifying and selecting the independent loop equations, being more suitable to incorporate in a process simulator.

This paper is structured as follows: in Section 2, the proposed method is presented, highlighting the steps of modeling and the numerical resolution of the whole network problem. In Section 3, the developed methodology is demonstrated by the resolution of three looped pipeline network problems. The first and the second problems are a small and a big looped water pipeline network, respectively, with both of them introduced by Yeh and Lin (2008). The third one shows the method applied to an industrial looped pipeline, where pumps and pipe accidents compose the problem, showing that the method can be used in any process simulation problem. The conclusions of the work are presented in Section 4.

2. Proposed method

It is understood that the Hardy-Cross *Method of Balanced Pressure* is the most suitable procedure to solve the looped pipeline network problems, since it requires the simplest network modeling. The method allows the natural attendance of the mass balance in any network node, pipe or equipment; and the employment of pressure drop equations with explicit pressure drop.

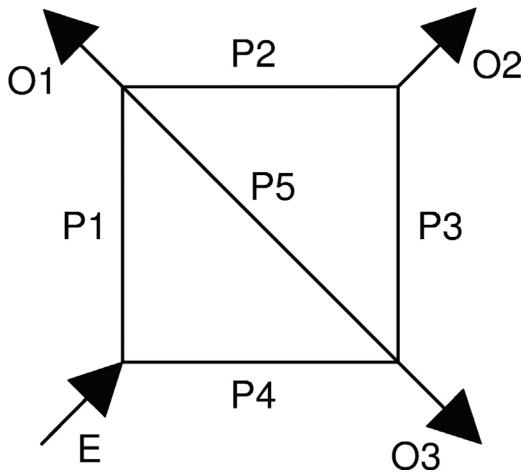

Fig. 1. Simple looped pipeline network.

Connecting generic and independent models for the pipe, node and equipment, which hold their representative equations and satisfy the mass balance, it is possible to build the pipeline network simulation that resembles a real network. Aiming the resolution of the built model, the specifications of some independent variables, pressures or flowrates for instance, to seek a solvable system of equation with zero degrees of freedom are necessary.

For a non-looped pipeline network with X inlets and Y outlets, it is necessary the specification of X + Y independent variables to obtain a solvable problem. Whereas to solve a looped pipeline network with the same number of pipes, which has less inlets and/or outlets, the number of network inlet and outlet (X + Y) specifications is not enough to remove all degrees of freedom. Therefore, in order to obtain the number of degrees of freedom of a looped pipeline network problem, with no more inlet and outlet independent variables to be specified, the equation found at Martinez and Puigjaner (1988), Eq. (1), can be employed.

$$M = T - N + 1 \tag{1}$$

where T is the number of pipes, M is the number of degrees of freedom to be covered by the loop equations and N is the number of nodes.

Using Eq. (1), M loop equations must be identified, selected and joined to the problem to remove the remaining degrees of freedom. As mentioned, the main drawback of the *Method of Balanced Pressure* lies in defining the group of network loops to be joined to the problem, since the number of the network loops is commonly greater than the number of remaining degrees of freedom and not every group of loops forms a solvable problem.

In this work, we propose new equations, defined as the difference between the final pressures of pipes that flow to the same node, to handle this degree of freedom drawback. The inclusion of the pressure difference equations to solve the problem removes the difficulty of identifying and selecting the independent network loops, since the number of the pressure difference equations is always equal to the remaining degrees of freedom. As shown below, these new equations (pressure difference equations) are equivalent to the loop equations, which mean that the resolutions of the problem by both kinds of equations are equivalent.

2.1. Introductory example

The looped pipeline network presented by Streeter and Wylie (1984), shown in Fig. 1, was used to introduce the proposed method.

According to Eq. (1), the network has two degrees of freedom to be covered by the loop equations. In order to formulate the pressure difference equations it is necessary to establish flow directions in the network pipes, which could be understood as the way that the pipe models were used to build the whole network problem, as illustrated in Table 1. It must be emphasized that wrong choice of flow directions are acceptable, because it is handled by the proposed procedure and the signal of the obtained flowrates will indicate the true direction of the flow. However, coherence with the built network helps the problem formulation and resolution due to its nonlinear nature and, consequently, the possible dependence on the initial guess by the chosen numerical algorithm.

Table 1
Chosen flow directions in pipes of Fig. 1.

P1	P2	P3	P4	P5
↑	→	↑	→	↓

The initial guess of the flow directions can be interpreted as how the pipes are connected among each other, with specified inlet and outlet connections, in order to form the whole pipeline network. This guess does influence the formulation of the pressure difference equations, since these equations are formed according to the identification of the pipes that needs its final pressure converged or to a specified value, just as: (i) when some pipe ends at an outlet network node that has a specified pressure (pressure difference equations for pseudo loop); (ii) or when different pipes end on a same node, needing to have the same final pressure (pressure difference equations for loop). Thus, if the same problem is modeled with different flow directions, different pipes need to converge to their final pressures. In this sense, a coherent pipe connection (e.g, node with at least one inlet and one outlet connection) helps the problem formulation and resolution.

However, independently of the initial guess of the flow direction, the problem resolution converges to its unique solution, where all node pressure and mass balance are coherent with the model equations. In this sense, the result for the pressure values at the nodes and the absolute value of the pipes flowrates does not change. But the signal of the flowrate in pipes does change in accordance with the chosen initial pipe flow direction: (i) if the flowrate is positive, then it is in accordance with the initial chosen flow direction, and (ii) if the flowrate is negative, then the correct flow direction is contrary to the initial chosen flow direction.

Knowing the flow direction and identifying which pipes flow to each node, it is possible to formulate two pressure difference equations, Eqs. (2) and (3), and realize that the number of formulated equations is equal to the problem degree of freedom.

$$P_{4f} - P_{5f} = 0 \tag{2}$$

$$P_{2f} - P_{3f} = 0 \tag{3}$$

where P_{if} is the outlet pressure of pipe i.

For comparison purpose, when solving the same problem with the traditional loop equations, it is necessary to identify the network loops. Analyzing the network, it is possible to identify three pipes arrangements that form loops (P1, P5 and P4; P2, P3 and P5; and P1, P2, P3 and P4), obtaining one surplus loop equation than the required equations to solve the problem.

2.2. Method procedure

Process simulators may roughly be classified into two groups: sequential modular and equation-oriented (Boston et al., 1993). The sequential modular procedure is the one that, given the inlet problem specification, the whole problem is solved sequentially from the first module (unit or equipment) until the last one; while the equation-oriented technique groups all problem equations to solve simultaneously as a system of equations.

In the proposed method, a sequential modular procedure is applied to solve some of the equations of the looped pipeline network problem. Since the characteristics of the problem make its entire resolution not possible by the sequential modular procedure without to resort to iterative approaches with tear variables, a reduced set of nonlinear equations are solved simultaneously, just like the equation-oriented procedure, characterizing a simultaneous-modular approach (Chen and Stadtherr, 1985).

Explaining the approach, when the sequential modular procedure reaches a node with K outlets, to satisfy the mass balance, there are K – 1 flowrate variables that need to be guessed; and when Z different pipes flow to a single node, Z – 1 pressure difference equations need to be formulated to obtain a unique node pressure. These approaches are represented in Figs. 2 and 3, respectively, where K – 1 variables and Z – 1 equations are the ones that cannot be solved sequentially, and form the nonlinear system of equations that need simultaneous convergence.

Furthermore, following the modular procedure, the idea of guessing non-specified inlet model variables, which need to feed a model to solve its internal equations just as the guessing of flowrate F_{p2} to solve the $Pipe_2$ model illustrated in Fig. 2; and the formulation of difference equation to seek the convergence of outlet models variables, which are the outlet information of a model just as the final pressure P_{1f} of the $Pipe_1$ illustrated in Fig. 3, could be widespread applied for guessing and converging any variable of the network to be solved. These difference equations and outlet flowrate variables need to be incorporated into the nonlinear system of equations in order to solve the whole network problem.

An example of difference equation that can be formulated using the network of Fig. 1 is: if the pressure of the outlet O1 is specified, the difference equation shown in Eq. (4) can be formulated. This procedure also replaces the pseudo-loop equations of traditional approaches (Streeter and Wylie, 1984; Sârbu and Valea, 2011).

$$P_{1f} - P_{O1} = 0 \tag{4}$$

As the flow paths in a looped pipeline network can be split in the nodes, a straightforward modular resolution cannot solve the entire network problem. Using the illustration showed in Fig. 2 as example, when the modular procedure reaches the node, the sequential resolution should continue solving one of the downstream pipes ($Pipe_2$ or $Pipe_3$). Assuming that the resolution is continued solving $Pipe_2$, when there is no further downstream model to be solved by the modular procedure, the problem resolution must return to the referred node in order to solve the pipeline path of the $Pipe_3$.

Generalizing the term “outlet flowrate variable” to any dependent variable that needs to be guessed during the problem resolution and the term “difference equation” to any equation that needs to be formulated to seek the convergence of an outlet model variable, it is possible to propose the pipeline network problem resolution. On the proposed methodology, the problem formulation has to identify dependent variables and the difference equations, for further mathematical resolution of the network models by the simultaneous-modular procedure. This procedure is presented in the flowchart of Fig. 4 for the problem formulation, and in the flowchart of Fig. 5 for the simultaneous-modular resolution.

Following the procedure, attention must be taken in order to formulate a system of equations with zero degree of freedom. If the problem is formulated with only node pressure difference equations and there is no independent variable to be specified, it is verified that the number of obtained dependent variables is always equal to the number of formulated difference equations. However, if an outlet model variable is specified, then a difference equation and a non- specified inlet model variable are necessary to formulate a problem with zero degree of freedom.

The obtained set of nonlinear equations can be solved by numerical procedures, such as the Newton-Raphson method, and its main advantage is grouping some equations with a modular procedure to make possible the simultaneous resolution of a reduced system of equations.

Table 2
Pressure difference equations of the looped pipeline network of Fig. 1.

Eq. (2′)
$(P_E - \Delta P_4) - (P_E - \Delta P_1 - \Delta P_5) = 0$
$-\Delta P_4 + \Delta P_1 + \Delta P_5 = 0$
Eq. (3′)
$(P_E - \Delta P_1 - \Delta P_2) - (P_E - \Delta P_4 - \Delta P_3) = 0$
$-\Delta P_1 - \Delta P_2 + \Delta P_4 + \Delta P_3 = 0$
Eq. (4′)
$P_E - \Delta P_1 - P_{O2} = 0$

2.3. Pressure difference equations analysis

The pressure difference equations (Eqs. (2)–(4)) of the looped pipeline network of Fig. 1, can be rewritten as shown in Table 2, where P_E is the pressure at the inlet point “E” and the ΔP_i is the pressure drop of pipe *i*.

It is possible to note that Eqs. (2′) and (3′) are network loop equations, and Eq. (4′) is a network pseudo-loop equation. These results show that the pressure difference equations are actually equal to the traditional loop and pseudo-loop equations.

2.4. Process modeling

Aiming the resolution of a pipeline network, it is possible to combine independent models of pipes and nodes to build the whole network. Since the all network models must have their inlets and outlets specified or connected with each other, the formulation of an inconsistent network, where some model presents no inlet or outlet connection, is prevented. Such models characteristics make possible the construction of a network with consistent paths for the sequential modular resolution. Given such paths, the sequential models are searched until the identification of a not solvable path, which must form one equation of the nonlinear system of equations for further simultaneous resolution.

Since these connections represent the flow directions in the network and, during the problem formulation, these directions could not be in accordance with the problem results, the pipe and node models used to build the network also need to handle negative flowrates, which leads to negative pressure drop on pipes. As shown further, these negatives flowrates and pressure drops are not an issue for the proposed method.

2.5. Numerical resolution

In order to start the iterative procedure of the Newton-Raphson method, which is an effective method to solve looped pipeline network problem, good initial guess of the dependent variables are needed to guarantee the problem convergence (Martinez and Puigjaner, 1988; Yeh and Lin, 2008).

Taking this convergence difficulty into account to solve the looped pipeline network problem, it is proposed a procedure to manage its nonlinear system of equations to be solved as an optimization problem. The objective function is the sum of the square of all difference equations of the original nonlinear system of equations. This reformulation makes also possible the employment of non-deterministic optimization techniques, such as Simulated Annealing (Yeh and Lin, 2008; Tospornsampam et al., 2007), Genetic Algorithm, and Particle Swarm (Kennedy and Eberhart, 1995), which have no need of initial guess.

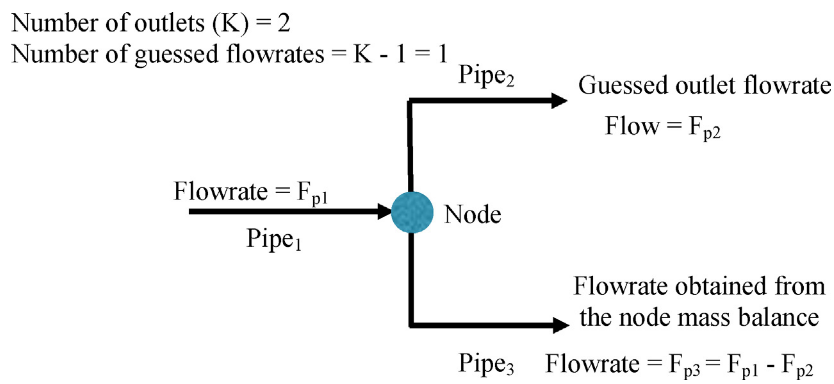

Fig. 2. Illustration of guessed outlet flowrate variables.

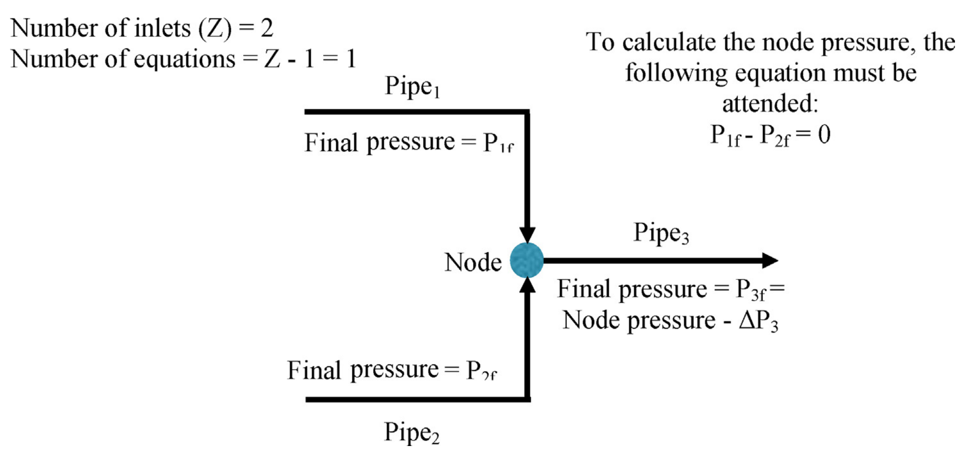

Fig. 3. Illustration of pressure difference equation formulation.

The objective function of the network problem formed by Eqs. (2)–(4) is written in Eq. (5), where the dependent variables are the same variables from the original system of equations.

$$OF = (P_{4f} - P_{5f})^2 + (P_{2f} - P_{3f})^2 + (P_{1f} - P_{O1})^2 \quad (5)$$

Despite the attenuation of good initial guess, the optimization resolution has as drawback the high computational cost. Thus, its employment is recommended to obtain an approximate problem solution, finding better values for the dependent variables, to be used as initial guess for the Newton-Raphson procedure, but only if the straight Newton-Raphson employment does not converge.

2.6. Further comments of the proposals

One of the major advantages of the proposed procedure relays on the no need of loop identification in order to formulate the loop equations and solve the problem. Following the simultaneous-modular formulation problem procedure showed in Fig. 4, the pressure difference equations, which are, according to Section 2.3, equivalent to the loop equation, are automatically identified. Thus, differently of the existing methods, this proposal allows to formulate the whole problem with a systematic procedure that can be computationally implemented. This characteristic also makes possible the integration of the loop network problem with a process simulation problem, just as the third case study shown in Section 3.3 bellow.

After the problem formulation, the simultaneous-modular resolution procedure, showed in Fig. 5, groups all identified pressure difference equations and dependent variables to form the representative nonlinear system of equations to be solved. Differently of the existing methods that solve the problem as a unique system of equations, the proposed procedure solves, in each iteration, all sequential parts of the network problem, in order to compute the pressure difference equations values, and also the Jacobian matrix, in order to consider the influence of all dependent variables of the problem. Furthermore, the proposed procedure allows any network variable to be set as dependent variable, without changing the procedure characteristic. This feature enables pressure of network inlet nodes and other pipes flowrate to be set together as dependent variables, just as shown in the further big looped pipeline network case study.

3. Case studies

The proposed procedure was applied to solve three different problems. The first two problems, representing a small and a big looped water pipeline network, were obtained from Yeh and Lin (2008); and the third problem was built to show the methodology applyed to solve a looped pipeline network commonly found in industrial instalations with large pumps flowing oil.

3.1. Small looped pipeline network

The small looped pipeline network to be solved is shown in Fig. 6, where the pipes are identified by "Pi", the nodes by "Ni", the inlet by "E" and outlets by "Oi". The physical characteristics of the pipes, required for the problem resolution, are shown in Table 3 and the flow direction in pipes, required to obtain the pressure diference equations, are sketched in Table 4.

In order to solve the problem, it is necessary to specify X + Y independents variables, where X = 1 and Y = 7 are equal to the number of the network inlets and outlets, respectively, that could be both

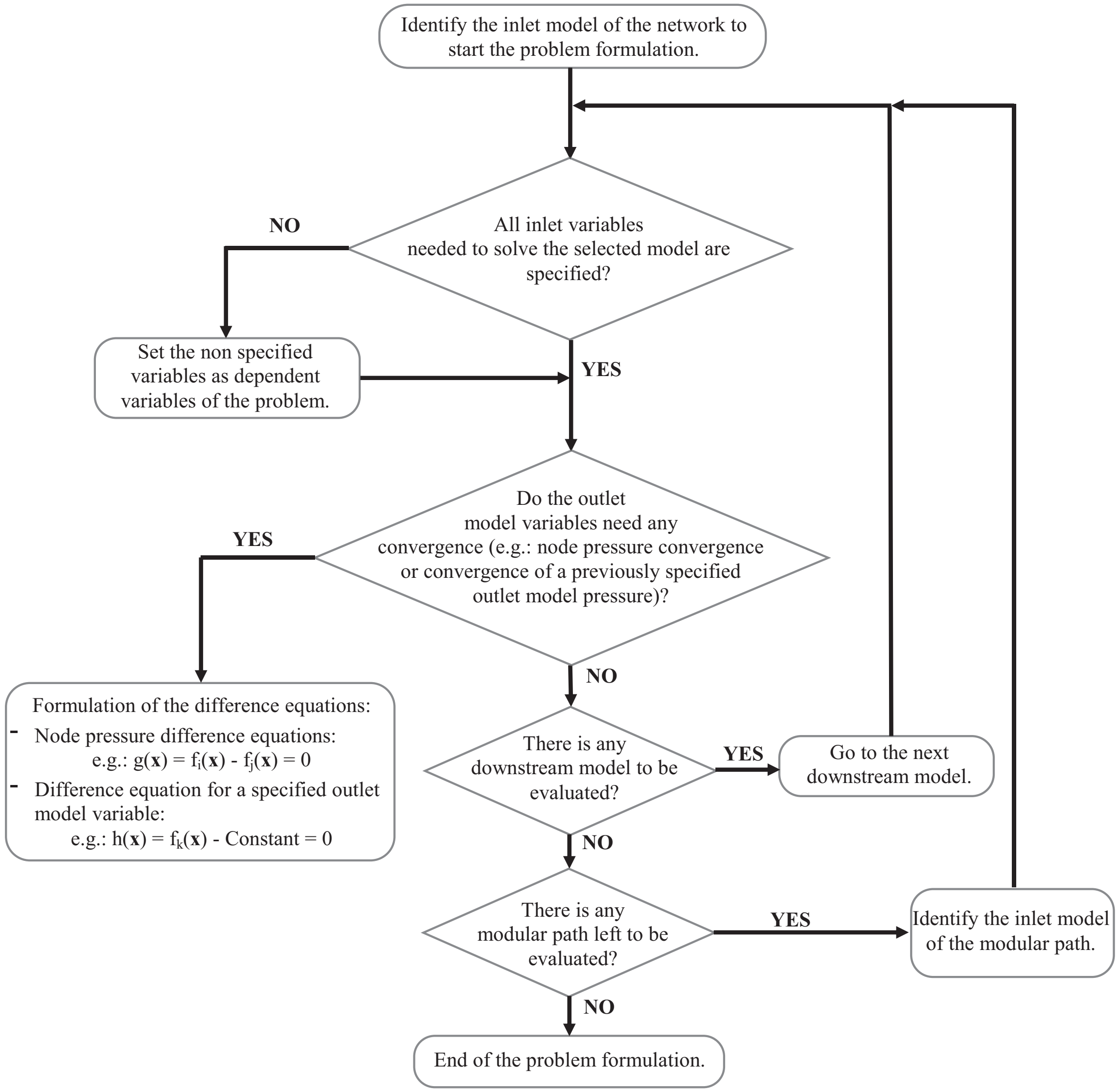

Fig. 4. Flowchart of the simultaneous-modular problem formulation procedure.

Table 3
Physical characteristics of the pipes in Fig. 6 (small network).

Pipe	Length (m)	Diameter (m)
P1	1000	0.305
P2	1000	0.305
P3	1100	0.250
P4	1250	0.405
P5	500	0.200
P6	400	0.400
P7	500	0.200
P8	400	0.355
P9	600	0.355
P10	1100	0.305
P11	1250	0.305

Table 4
Flow directions in pipes of Fig. 6 (small network).

P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11
←	↓	→	↓	→	↑	→	↓	↑	→	↓

Table 5
Specified variables for the small network of Fig. 6.

P_{N1} = 980.7 kPa	P_{5f} = 871.2 kPa
P_{1f} = 897.6 kPa	P_{7f} = 870.9 kPa
P_{2f} = 850.8 kPa	P_{10f} = 843.0 kPa
P_{4f} = 890.4 kPa	P_{11f} = 778.9 kPa

pressure or flowrate values. At the present problem, as shown in Table 5, the inlets and outlets pressures obtained from Yeh and Lin (2008) were chosen, with P_{if} meaning the final pressure of pipe i. Since the final pressure of the pipe is an outlet model result, each specified outlet pressure formulates one pseudo-loop pressure difference equation. The specification of all outlet pressure leads to a problem with the highest number of pressure difference equations.

Starting the problem resolution at the N1 node (network inlet), it is verified that the inlet flowrate ("E" flowrate), not specified, is split into two outlets. Following the modular procedure, to start the problem resolution, the inlet flowrate and one of its outlet flowrates must be guessed, being dependent variables of the problem (E and P1 flowrates), while the other outlet N1 node flowrate (P4 flowrate) needs to be obtained from the node mass balance to allow the pressure drop calculation of pipes P1 and P4. In order to emphasize that no restriction is imposed to the equations to be used in the problem formulation, differently from Yeh and Lin (2008) that used the Hazen-Williams, the pressure drop calculation was carried out

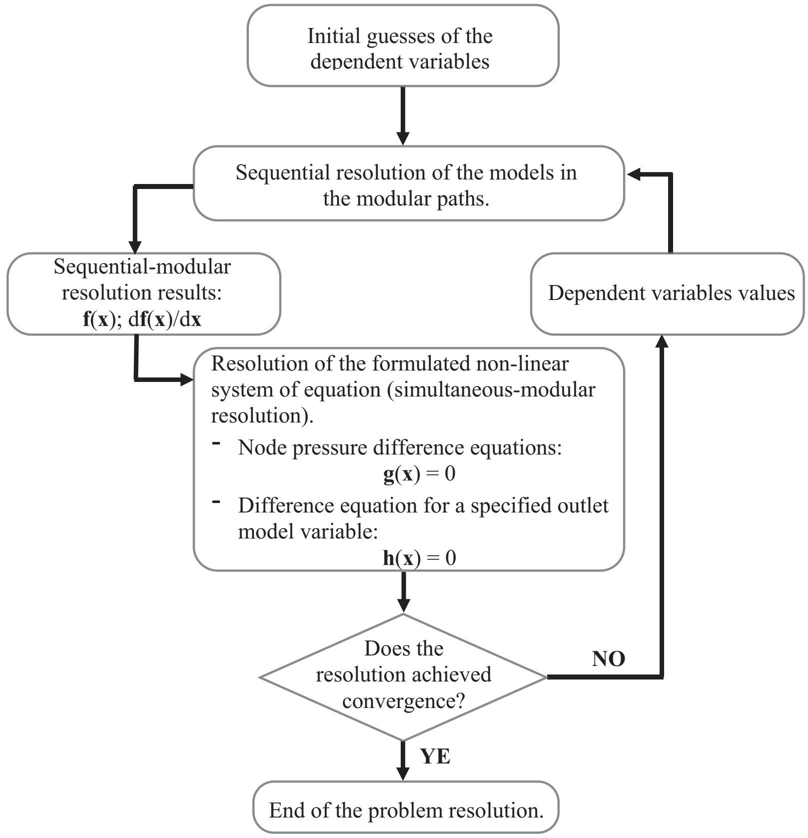

Fig. 5. Flowchart of the simultaneous-modular problem resolution procedure.

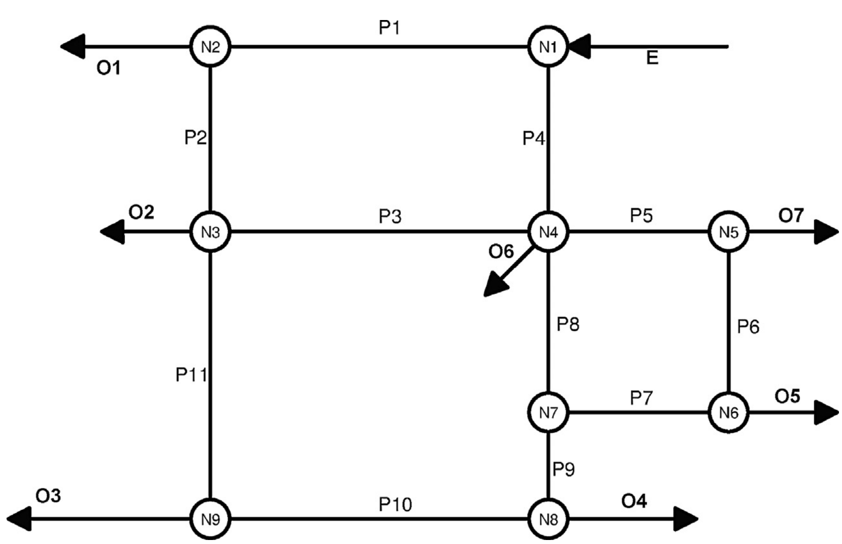

Fig. 6. Small looped pipeline network.

employing the Darcy-Weisbach Eq. (6), adapted to handle negative flowrate values.

$$\Delta P_i = \frac{f\rho L_i v_i |v_i|}{2D_i} \quad (6)$$

where ΔP_i is the pressure drop (Pa) of pipe i, f is the Darcy factor (dimensionless and calculated according to the Churchill (1977) equation), L_i is the pipe length (m), D is the pipe inner diameter (m), $v = 4F/(\pi\, D^2)$ is the fluid velocity (m/s), F is the flowrate (m^3/s) and ρ is the fluid specific mass (kg/m^3).

Knowing the entrance pressure (P_{N1}) and the P1 pressure drop, it is possible to obtain the P1 final pressure, which needs to be converged to the specified value (P_{1f} presented in Table 5) employing the respective pressure difference equation for the pipeline pseudo-loop. The P2 final pressure follows the same procedure already described, where one of the O1 or P2 flowrates must be guessed while the other is obtained by the N2 node mass balance. Following the procedure, when two or more pipes end on a single node, all final pressures must have the same value, which is guaranteed by the employment of the pressure difference equations for the pipeline loops.

Table 6
Difference equations and dependent variables of the small network.

Dependent variables	F_E, F_{P1}, F_{P3}, F_{P5}, F_{O1}, F_{O2}, F_{O3}, F_{O4}, F_{O5} and F_{O6}
Eq. (1)	$P_{3f} - P_{4f} = 0$
Eq. (2)	$P_{5f} - P_{6f} = 0$
Eq. (3)	$P_{8f} - P_{9f} = 0$
Eq. (4)	P_{1f} (calculated) – P_{1f} (given) = 0
Eq. (5)	P_{2f} (calculated) – P_{2f} (given) = 0
Eq. (6)	P_{11f} (calculated) – P_{11f}(given) = 0
Eq. (7)	P_{10f} (calculated) – P_{10f} (given) = 0
Eq. (8)	P_{4f} (calculated) – P_{4f} (given) = 0
Eq. (9)	P_{5f} (calculated) – P_{5f} (given) = 0
Eq. (10)	P_{7f} (calculated) – P_{7f} (given) = 0

Table 7
Sequential equations of the problem for the small network.

Mass Balance N1	$F_E = F_{P1} + F_{P4}$
Mass Balance N2	$F_{P1} = F_{O1} + F_{P2}$
Mass Balance N3	$F_{P2} = F_{O2} + F_{P3} + F_{P11}$
Mass Balance N4	$F_{P4} + F_{P3} = F_{O6} + F_{P5} + F_{P8}$
Mass Balance N5	$F_{P5} + F_{P6} = F_{O7}$
Mass Balance N6	$F_{P7} = F_{P5} + F_{O5}$
Mass Balance N7	$F_{P8} + F_{P9} = F_{P7}$
Mass Balance N8	$F_{P10} = F_{P9} + F_{O4}$
Mass Balance N9	$F_{P11} = F_{P10} + F_{O3}$
Final Pressure P1	$P_{1f} = P_{N1} - \Delta P_1$
Final Pressure P2	$P_{2f} = P_{1f} - \Delta P_2$
Final Pressure P3	$P_{3f} = P_{2f} - \Delta P_3$
Final Pressure P4	$P_{4f} = P_{N1} - \Delta P_4$
Final Pressure P5	$P_{4f} = P_{4f} - \Delta P_5$
Final Pressure P6	$P_{7f} = P_{7f} - \Delta P_6$
Final Pressure P7	$P_{7f} = P_{8f} - \Delta P_7$
Final Pressure P8	$P_{8f} = P_{4f} - \Delta P_8$
Final Pressure P9	$P_9 f = P_{10f} - \Delta P_9$
Final Pressure P10	$P_{10f} = P_{11f} - \Delta P_{10}$
Final Pressure P11	$P_{11f} = P_{2f} - \Delta P_{11}$

At the end of the described procedure, it was identified 10 dependent variables, three pressure difference equations for the pipeline loops and six pressure difference equations for the pipeline pseudo-loops. The identified equations and dependent variables, shown in Table 6, formulate the nonlinear system of equations that must be solved simultaneously.

It is important to note that the equations presented in Table 6 group all sequential equations, representing the whole problem. The sequential equations of the problem are shown in Table 7, where the pressure drops are evaluated by Eq. (6). Since it is a water pipeline problem, the fluid properties were considered constants (specific mass equal to 1000 kg/m^3 and viscosity of 0.89 cP).

The procedure was implemented in MATLAB software and the *fsolve* function was used to solve the nonlinear system of equations. A value of 1 m^3/h was guessed for all dependent variables and the convergence criterion was set to 10^{-8}. In order to improve the convergence properties, the residuals of all pressure difference equations were squared. The results are shown in Table 8.

It is observed that some obtained flowrate variables have negative value, indicating that the flow direction at the problem solution is contrary to the initial choice (Table 4). As expected, the obtained results are in accordance with the ones presented by Yeh and Lin (2008), despite the use of the Darcy-Weisbach equation. The objective function value (OF) also indicates that the obtained results are the right problem solution, having also good accuracy.

3.2. Big looped pipeline network

The looped water pipeline network of this case study is shown in Fig. 7. The network was solved using the Hazen-Williams pressure drop Eq. (7), adapted to handle negative flowrates, where the pipes physical characteristics are shown in Table 9, the guesses of the flow directions in pipes are shown in Table 10, and the specified independent variables are shown in Table 11.

Table 8
Results of the small network problem.

Variables	Results
P_{1f}	897.6 kPa
P_{2f}	850.8 kPa
P_{3f}	890.4 kPa
P_{4f}	890.4 kPa
P_{5f}	871.2 kPa
P_{6f}	871.1 kPa
P_{7f}	870.8 kPa
P_{8f}	871.1 kPa
P_{9f}	871.1 kPa
P_{10f}	843.1 kPa
P_{11f}	778.9 kPa
F_{P1}	0.1409 m^3/s
F_{P2}	0.1042 m^3/s
F_{P3}	−0.0537 m^3/s
F_{P4}	0.2758 m^3/s
F_{P5}	0.0308 m^3/s
F_{P6}	−0.0223 m^3/s
F_{P7}	0.0026 m^3/s
F_{P8}	0.1580 m^3/s
F_{P9}	−0.1553 m^3/s
F_{P10}	−0.1171 m^3/s
F_{P11}	0.1163 m^3/s
OF Value	0.4121×10^{-6}
N° of iterations	27

$$\frac{h_{f,i}}{L_i} = \frac{10.67 Q_i |Q_i|^{0.85}}{{C_i}^{1.85} {d_i}^{4.87}} \tag{7}$$

where $h_{f,i}$ is the head loss (meters of water), L_i the length of pipe (m), Q_i the volumetric flowrate (m^3/s), C_i the pipe roughness coefficient (dimensionless), and d_i is the inner pipe diameter (m) of the pipe *i*.

The network degrees of freedom, to be covered by the pressure difference equations, are given by Eq. (1). The identification, by Fig. 7, that node N2 is a reservoir precludes the direct use of a node mass balance equation, as done on other network nodes. To model the reservoir, it is needed to consider the N2 node as three independent nodes, as it has three inlets or outlet connections, with pressure equal to the specified N2 pressure (897 kPa). Thus, it is assumed a network with 50 nodes and 75 pipes, obtaining a problem with 25 degrees of freedom.

Following the modular procedure and analyzing the specified variables and the flow directions in pipes, it is verified that three final pressures variables were specified (P_{3f}, P_{4f} and P_{10f}, which is equal to P_{N2}), leading to three pressure difference equations for the pseudo-loops. Just as in the previously examples, the identification of one pseudo-loop lead to one more pressure difference equation and one more dependent variable. It is also verified that two network inlet pressures (P_{N31} that is equal to the inlet pressure of P_{31i} and P_{N9} that is equal to the inlet pressure of P_{43i}) were not specified. In this case, following the steps to formulate the network problem, presented in Fig. 4, these inlet pressures must be set as dependent variables and, to formulate a solvable problem, two more difference pressure equations must be added to the problem.

Since the problem has 25 degrees of freedoms due to its loops, 3 variables that must be converged due to the pseudo-loops, and 2 variables that must be converged due to the problem specifications, it is expected 30 pressure difference equations and 30 dependent variables for the problem. Following the problem formulation of the proposed procedure, Fig. 4, it is obtained exactly the right number of pressure difference equations and dependent variables, as shown in Table 12.

Table 9
Physical characteristics of the pipes of Fig. 7 (big network).

Pipe	P1	P2	P3	P4	P5	P6
Length	240 m	60 m	1830 m	3550 m	1220 m	640 m
Diameter	0.95 m	0.90 m	1.45 m	1.15 m	1.45 m	1.45 m
HW Coef.	120	110	130	135	130	130
Pipe	P7	P8	P9	P10	P11	P12
Length	640 m	60 m	50 m	3660 m	60 m	60 m
Diameter	0.90 m	0.9 m	1 m	0.9 m	0.9 m	1 m
HW Coef.	110	110	110	115	110	110
Pipe	P13	P14	P15	P16	P17	P18
Length	800 m	3140 m	3140 m	3140 m	60 m	60 m
Diameter	0.9 m	1.45 m	1.15 m	1.65 m	0.9 m	1 m
HW Coef.	115	130	130	135	110	110
Pipe	P19	P20	P21	P22	P23	P24
Length	2300 m	60 m	4040 m	60 m	4050 m	4050 m
Diameter	0.8 m	0.9 m	1.15 m	0.9 m	0.8 m	1.15 m
HW Coef.	115	110	130	110	115	130
Pipe	P25	P26	P27	P28	P29	P30
Length	60 m	60 m	2150 m	180 m	2980 m	2980 m
Diameter	0.9 m	0.9 m	0.8 m	0.8 m	1.45 m	1.45 m
HW Coef.	110	110	110	110	135	135
Pipe	P31	P32	P33	P34	P35	P36
Length	12000 m	670 m	60 m	13400 m	80 m	4290 m
Diameter	1.65 m	0.95 m	1 m	1.65 m	0.90 m	0.95 m
HW Coef.	135	110	110	135	110	120
Pipe	P37	P38	P39	P40	P41	P42
Length	4290 m	60 m	2590 m	60 m	2960 m	2960 m
Diameter	0.9 m	0.05 m	0.95 m	0.05 m	0.9 m	1.15 m
HW Coef.	115	110	120	110	115	135
Pipe	P43	P44	P45	P46	P47	P48
Length	2280 m	370 m	90 m	60 m	1610 m	60 m
Diameter	1.15 m	0.95 m	1 m	0.05 m	0.9 m	0.05 m
HW Coef.	130	120	130	110	115	110
Pipe	P49	P50	P51	P52	P53	P54
Length	1350 m	2960 m	6530 m	60 m	230 m	7200 m
Diameter	0.95 m	0.05 m	0.95 m	0.9 m	0.95 m	0.95 m
HW Coef.	115	120	120	110	120	120
Pipe	P55	P56	P57	P58	P59	P60
Length	60 m	3200 m	4300 m	3200 m	80 m	90 m
Diameter	1 m	1.15 m	1.45 m	1.15 m	0.8 m	0.75 m
HW Coef.	110	135	135	135	115	130
Pipe	P61	P62	P63	P64	P65	P66
Length	2050 m	2380 m	3050 m	670 m	60 m	60 m
Diameter	0.95 m	0.8 m	1.15 m	0.05 m	0.05 m	0.05 m
HW Coef.	120	115	135	115	110	110
Pipe	P67	P68	P69	P70	P71	P72
Length	1830 m	60 m	1950 m	3780 m	60 m	60 m
Diameter	0.8 m	0.9 m	0.8 m	0.95 m	0.05 m	0.9 m
HW Coef.	115	110	115	120	110	120
Pipe	P73	P74				
Length	4290 m	60 m				
Diameter	1.15 m	0.05 m				
HW Coef.	135	110				

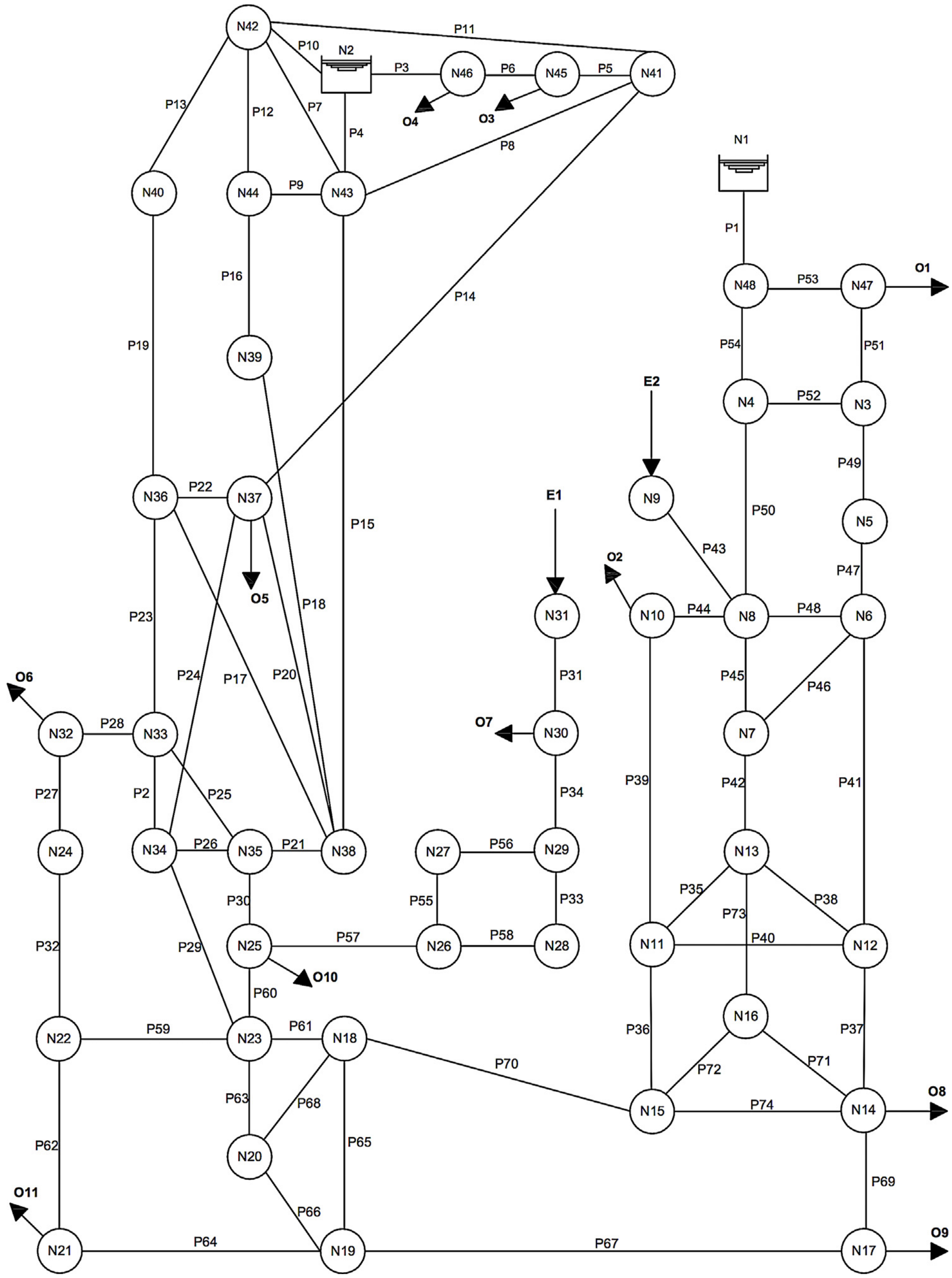

Fig. 7. Big looped pipeline network.

The procedure was implemented in MATLAB software and the *fsolve* function was used to solve the nonlinear system of equations. A value of 1 m^3/h was guessed for all dependent variables and the convergence criterion was set to 10^{-8}. In order to improve the convergence properties, the residuals of all pressure difference equations were squared. The results are shown in Table 13.

The obtained results are in accordance with the ones presented by Yeh and Lin (2008), where the objective function value highlights the accuracy. The big network problem was also solved with the PSO non-deterministic optimization algorithm (Kennedy and Eberhart, 1995), where the results were used as initial guesses for the nonlinear system of equations resolution that converged with lower number of Newton-Raphson iterations to the same results presented in Table 13.

3.3. *Industrial installation with lopped pipeline network*

A looped pipeline network commonly found in industrial installation with high-capacity pumps (large flowrate and high discharge pressure) used for oil transfer through ducts is solved. The system, shown in Fig. 8, is composed by pipe arrangement for the oil recirculation done by two groups of pumps, connected in series, suctioning from an oil tank. The series connection of booster pumps on the suction of the main pumps is to attend the main pumps high-required NPSH (Net Positive Suction Head), and the pipes arrangement forms the loop configuration to connect the pumps discharges to their suctions allowing the oil recirculation.

For both groups of pumps, the recirculation procedure is needed on the pumps starts to minimize their required potency and avoid possible electric damages, which could cause fire and/or others fur-

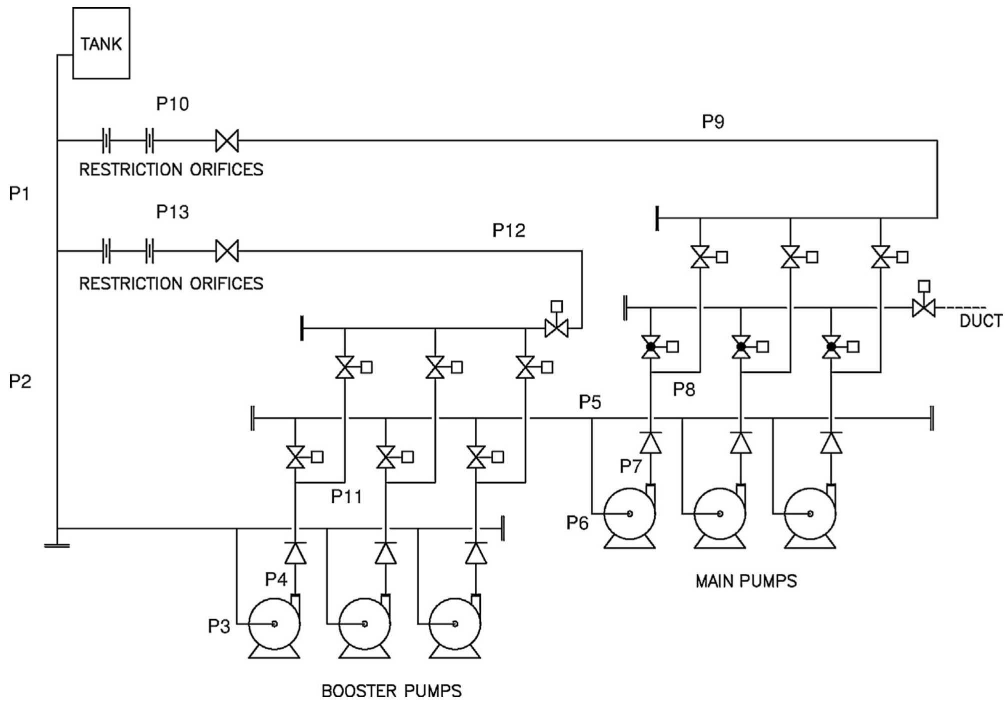

Fig. 8. Industrial lopped pipeline network.

Table 10
Flow directions in pipes of Fig. 7 (big network).

Pipe	Direction
P1	↓
P2	↑
P3	←
P4	↑
P5	←
P6	←
P7	↓
P8	↑
P9	→
P10	↓
P11	→
P12	↑
P13	↑
P14	↑
P15	↑
P16	↑
P17	↓
P18	↑
P19	↑
P20	↓
P21	→
P22	→
P23	↑
P24	↑
P25	↑
P26	←
P27	↑
P28	→
P29	↑
P30	↑
P31	↓
P32	↑
P33	↓
P34	↓
P35	↑
P36	↓
P37	↓
P38	↑
P39	↓
P40	←
P41	↓
P42	↓
P43	↓
P44	←
P45	↓
P46	↓
P47	↓
P48	←
P49	↓
P50	↓
P51	↓
P52	→
P53	→
P54	↓
P55	↓
P56	←
P57	←
P58	←
P59	←
P60	↓
P61	←
P62	↓
P63	↑
P64	←
P65	↑
P66	↑
P67	←
P68	↓
P69	↓
P70	←
P71	↑
P72	↓
P73	↓
P74	←

Table 11
Specified variables for the big network.

P_{N1}	1363 kPa
P_{N2}	897 kPa
F_{E1}	1.620370 m^3/s
F_{E2}	1.620370 m^3/s
F_{O1}	0.016203 m^3/s
F_{O2}	0.023148 m^3/s
F_{O3}	0.138888 m^3/s
F_{O4}	0.254629 m^3/s
F_{O5}	0.092592 m^3/s
F_{O6}	0.012731 m^3/s
F_{O7}	0.104166 m^3/s
F_{O8}	0.017361 m^3/s
F_{O9}	0.162037 m^3/s
F_{O10}	0.104166 m^3/s
F_{O11}	0.074074 m^3/s

Table 12
Difference equations and dependent variables of the big network.

Num.	Equations	Variables
1	$P_{51f} - P_{52f} = 0$	F_{P1}
2	$P_{50f} - P_{43f} = 0$	F_{P53}
3	$P_{50f} - P_{48f} = 0$	F_{P52}
4	$P_{46f} - P_{45f} = 0$	F_{P48}
5	$P3_{8f} - P_{42f} = 0$	F_{P46}
6	$P_{35f} - P_{42f} = 0$	F_{P44}
7	$P_{40f} - P_{39f} = 0$	F_{P38}
8	$P_{36f} - P_{72f} = 0$	F_{P40}
9	$P_{74f} - P_{36f} = 0$	F_{P35}
10	$P_{71f} - P_{73f} = 0$	F_{P71}
11	$P_{14f} - P_{8f} = 0$	F_{P69}
12	$P_{68f} - P_{66f} = 0$	F_{P65}
13	$P_{63f} - P_{61f} = 0$	F_{P66}
14	$P_{60f} - P_{61f} = 0$	F_{P68}
15	$P_{64f} - P_{62f} = 0$	F_{P56}
16	$P_{55f} - P_{58f} = 0$	F_{P60}
17	$P_{28f} - P_{2f} = 0$	F_{P59}
18	$P_{28f} - P_{25f} = 0$	F_{P26}
19	$P_{26f} - P_{29f} = 0$	F_{P25}
20	$P_{27f} - P_{21f} = 0$	F_{P2}
21	$P_{20f} - P_{21f} = 0$	F_{P17}
22	$P_{24f} - P_{22f} = 0$	F_{P19}
23	$P_{13f} - P_{12f} = 0$	F_{P20}
24	$P_{15f} - P_{9f} = 0$	F_{P18}
25	$P_{7f} - P_{9f} = 0$	F_{P9}
26	$P_{8f} - P_{11f} = 0$	F_{P7}
27	$P_{3f} - P_{4f} = 0$	F_{P10}
28	$P_{10f} - P_{3f} = 0$	F_{P8}
29	$P_{3f} - P_{N2} = 0$	P_{N9}
30	$P_{70f} - P_{65f} = 0$	P_{N31}

Table 13
Results of the big network problem.

Pipe	Results	
	P (kPa)	F (m^3/s)
P1	1362.2588	0.250626
P2	958.4615	0.134666
P3	896.6340	1.216537
P4	896.6340	0.820946
P5	904.6488	1.667946
P6	902.1178	1.413296
P7	911.4383	0.023051
P8	911.2166	0.336781
P9	911.4383	0.615998
P10	896.6340	0.361295
P11	911.2166	0.350073
P12	911.4657	0.543399
P13	911.4657	0.191019
P14	911.2166	0.981071
P15	911.4383	0.518678
P16	911.7924	1.159398
P17	917.4479	0.202430
P18	916.0696	1.159398
P19	912.4507	0.191019
P20	917.4479	0.196712
P21	917.4479	1.278934
P22	917.5234	0.040881
P23	917.5381	0.434330
P24	917.5234	1.275790
P25	958.4615	0.134667
P26	958.5135	0.000281
P27	958.7902	0.257589
P28	958.4615	0.164997
P29	958.5135	1.410175
P30	958.5135	1.413882
P31	1036.6060	1.620370
P32	967.7565	0.257589
P33	1002.4810	0.803819
P34	1003.1657	1.607639
P35	1195.3534	0.000816
P36	1165.9008	0.577825
P37	1343.5913	0.211710
P38	1195.3534	0.005646
P39	1195.3534	0.572996
P40	1195.3534	0.005646
P41	1350.0581	0.223001
P42	1195.3534	1.035648
P43	1215.5453	1.620370
P44	1212.8554	0.596144
P45	1214.3250	1.030286
P46	1214.3250	0.005362
P47	1354.9621	0.233700
P48	1215.5453	0.005337
P49	1357.8619	0.233700
P50	1215.5453	0.000722
P51	1359.7376	0.119317
P52	1359.7376	0.114383
P53	1362.1509	0.135520
P54	1359.7690	0.115105
P55	989.6367	0.803820
P56	990.3313	0.803820
P57	969.5037	1.607639
P58	989.6367	0.803819
P59	968.9661	0.329089
P60	969.4595	0.089591
P61	969.4595	0.678943
P62	968.1244	0.071500
P63	969.4595	0.970730
P64	968.1244	0.002574
P65	988.4164	0.008789
P66	986.7987	0.008811
P67	1339.5938	0.020174
P68	986.7987	0.961919
P69	1339.6467	0.182211
P70	988.4164	1.632073
P71	1167.5254	0.006154
P72	1165.9008	1.048163
P73	1167.5254	1.042109
P74	1165.8999	0.006084
OF value	2.3641×10^{-13}	
Nº of Iterations	46	

thers undesirable consequences. To stabilize the pumps discharge and suction pressure, a pipe accident (the restriction orifice) is used to obtain the required pressure drop. The restriction orifice equations are found in standards as EN ISO-5167. Both groups of pumps (main and booster pumps) have their own recirculation systems, designed to operate with only one pump at time, and using the orifice arrangement with two restriction orifices in series.

Analyzing the system, it is easy to identify two pipes that end at a same node, the pipes 13 and 1, and a pipe with final pressure equal to the oil column in the tank, the pipe 10. The difference equations and the dependent variables of this problem are shown in Table 14, and the problem specifications are shown in Table 15.

The procedure was implemented in MATLAB software and the *fsolve* function was used to solve the nonlinear system of equations. A value of 1 m^3/h was guessed for all dependent variables and the convergence criterion was set to 10^{-8}. In order to improve the convergence properties, the residuals of all pressure difference equations were squared. The results are shown in Table 16.

Table 14
Difference equations and dependent variables of the industrial network.

Dependent variables	F_{P1} e F_{P11}
Eq. (1)	$P_{10f} - P_{tank} = 0$
Eq. (2)	$P_{13f} - P_{1f} = 0$

Table 15
Specification of the simulation for the industrial network.

P_0 (petroleum column)	110,2 kPa
Pressure drop equation	Darcy-Weisbach (Eq. (6))
Pipe roughness	4752×10^{-5} m
Pipe length (for the 13 pipes)	[20 30 10 10 70 16.5 6 15.9 80 10 10 20 10] m
Pipe inner diameter (for the 13 pipes)	[1.40 1.40 0.74 0.58 1.04 0.58 0.48 0.23 0.58 0.23 0.18 0.38 0.18] m
Oil specific mass	937 kg/m^3
Oil viscosity	206.14 cP
Main pumps ΔP equation (F[=] m^3/h)	$(-1.8 \times 10^{-8}$ $F^3 + 1.8 \times 10^{-4}$ $F^2 - 0.28$ $F + 8456.53)$ kPa
Booster pumps ΔP equation (F[=] m^3/h)	$(2.8 \times 10^{-8}$ $F^3 - 8.3 \times 10^{-5}$ $F^2 + 0.051$ $F + 1303.1)$ kPa
Diameters of the main pumps restriction orifices	0.0660 m
Diameters of the booster pumps restriction orifices	0.0381 m

Table 16
Problem results of the industrial network.

Variables	Value	Variables	Value
P_{1f}	110.2 kPa	F_{P1}	0.210 m^3/s
P_{2f}	110.2 kPa	F_{P2}	0.237 m^3/s
P_{3f}	110.0 kPa	F_{P3}	0.237 m^3/s
P_{4f}	1408.6 kPa	F_{P4}	0.237 m^3/s
P_{5f}	1408.2 kPa	F_{P5}	0.210 m^3/s
P_{6f}	1407.5 kPa	F_{P6}	0.210 m^3/s
P_{7f}	9664.6 kPa	F_{P7}	0.210 m^3/s
P_{8f}	9597.7 kPa	F_{P8}	0.210 m^3/s
P_{9f}	9594.2 kPa	F_{P9}	0.210 m^3/s
P_{10f}	110.3 kPa	F_{P10}	0.210 m^3/s
P_{11f}	1403.7 kPa	F_{P11}	0.027 m^3/s
P_{12f}	1403.1 kPa	F_{P12}	0.027 m^3/s
P_{13f}	110.2 kPa	F_{P13}	0.027 m^3/s
$\Delta P_{Booster.orifice}$	1308.7 kPa	$\Delta P_{Booster.Pump}$	1299.2 kPa
$\Delta P_{Main.orifice}$	9441.9 kPa	$\Delta P_{MainPump}$	8257.9 kPa
OF Value	1.6367×10^{-12}	N° of iterations	24

The low objective function value shows that the proposed method is also applied to looped pipeline network problem with more devices than just pipes and nodes.

Furthermore, the proposed method was applied to the same system, but on the system design perspective. The design calculation aims to obtain the diameters of the restriction orifices. Thus, the problem was remodeled specifying, according to the results in Table 16, all flowrate variables and leaving the diameters of the main and booster pumps restriction orifices as dependent variables. The obtained diameters results were, as expected, the same values showed in Table 15, showing that the proposed method can be employed for any kind of process simulation problem.

4. Conclusions

On the looped pipeline network problem, a simultaneous resolution is needed to solve some of its equations, making not possible the direct employment of a modular procedure. The impossibility on use the sequential resolution lies on the required attendance of both Kirchhoff's laws since, initially satisfying the node mass balance (first Kirchhoff's law), it is needed a simultaneous convergence of the loop equations obtained from the second Kirchhoff's law.

The proposed method uses modular procedure to identify the problem pressure difference equations, grouping the equations that can be solved sequentially. The identified pressure difference equations replace the traditional loop and pseudo-loop equations and form the nonlinear system to be solved simultaneously. Since the proposed method uses features from both modular and equation-oriented procedures, it was characterized as simultaneous-modular. Furthermore, was proposed the employment of the Newton-Raphson method to solve the problem nonlinear system of equations and, to obtain better initial guesses for the dependent variables, it was proposed a procedure to formulate an objective function for the initial employment of non-deterministic optimization methods.

The presented case studies showed that the pressure difference equations are easily identified and, different of the loop equations, are always applicable to the problem. It was also shown that the pressure difference equations are equal to the loop equations formulated by the traditional employment of the second Kirchhoff's Law, making no difference between problems solved by both procedures.

The particularity of the lopped pipeline network problem leaded to the development of simulators to solve this specific problem, as the EPANET (2015) software. However, the development of simulators capable to solve problems composed by both looped pipeline and others process equipment's are not a simple task. Using the proposed method to identify the pressure difference equations, there is no more need to identifying and choosing the independent loop equations, and was also verified that the equations can be used on problems that aims the obtainment of any design variable, as showed at the case study *Industrial installation with lopped pipeline network* (item 3.3).

It is understood the described simultaneous-modular procedure makes possible the development of a process simulator capable to solve any process problem, with or without looped pipeline. The main advantage of the procedure is groups all sequential equation to formulate the smallest possible problem system of equations, turn easier and less computationally costly a simultaneous convergence by numerical procedures. The big looped pipeline problem resolution (item 3.2) exemplifies the conclusion, where the whole problem was represented by a system of equation composed by only 30 equations.

Acknowledgments

The present work was accomplished with the support of CNPq, Conselho Nacional de Desenvolvimento Científico e Tecnológico – Brasil and CAPES, Coordenação de Aperfeiçoamento de Pessoal de Nível Superior – Brasil.

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