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An improved semi-circular bend specimen for investigating mixed mode brittle fracture

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ABSTRACT

An edge cracked semi-circular specimen subjected to asymmetric three-point bend loading was suggested for investigating mixed mode fracture in brittle materials. Using finite element analysis, the crack parameters were obtained for various crack lengths and different locations of loading points. It was shown that by selecting appropriate positions for the loading points, full mode mixities from pure mode I to pure mode II could be achieved. Then, a series of fracture tests were conducted on PMMA using the proposed specimen. Very good agreement was found between the experimental results and those predicted from the generalized maximum tangential stress criterion.

1. Introduction

The presence of flaws and cracks are very often inevitable in engineering structures and components. The cracks can be generated during the manufacturing processes or due to cyclic loading or environmental causes, etc. Pure mode I and pure mode II are two modes of deformation that take place for a cracked component subjected to in-plane loading. In practical situations, the cracked structures sometimes experience mixed mode loading, i.e. a combination of mode I and II. Mixed mode brittle fracture is one of the common types of mechanical failure in cracked components made of brittle or quasi-brittle materials. Therefore, it is important to investigate the structural integrity of cracked components under mixed mode loading.

Several theoretical and experimental methods have been suggested by researchers for exploring mixed mode brittle fracture. While the experimental fracture studies on real components are often expensive and difficult, researchers prefer to conduct their experiments on laboratory specimens. However, appropriate fracture criteria are also required to correlate the experimental results obtained from the simple laboratory specimens to the fracture event in cracked structures under their complex service loading conditions. In order to validate a fracture criterion, researchers have to conduct a series of experiments on appropriate test materials by using suitable test specimens. PMMA (polymethylmethacrylate or Perspex) has been recognized as a favorite model material for conducting brittle fracture experiments. The brittle type of fracture at room temperature, the convenience of machining and introducing a sharp crack and the optical transparency (which allows direct observation of fracture path) are among the advantages of PMMA in brittle fracture experiments.

In addition to the choice of test material, a valid fracture test requires an appropriate test configuration. For mixed mode fracture experiments, a suitable test configuration should have simple geometry and loading condition, inexpensive

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Nomenclature

a	crack length
a/R	crack length ratio
ASCB	asymmetric semi-circular bend specimen
GMTS	generalized maximum tangential stress
K_I	mode I stress intensity factor
K_{II}	mode II stress intensity factor
K_{Ic}	mode I fracture toughness
K_{eff}	effective mixed mode stress intensity factor
M^e	mixity parameter
MTS	maximum tangential stress
P	applied load
PMMA	polymethylmethacrylate
R	radius of CCCD and SCB specimens
r, θ	crack tip co-ordinates
r_c	critical distance from crack tip
SCB	semi-circular bend specimen
$2S$	loading span in the SCB test
$S1$, $S2$	bottom-support distances from the crack line
S/R	loading point span ratio in the SCB specimen
t	specimen thickness
T	T-stress
T^*	normalized T-stress
Y_I	mode I geometry factor
Y_{II}	mode II geometry factor

Greek symbols

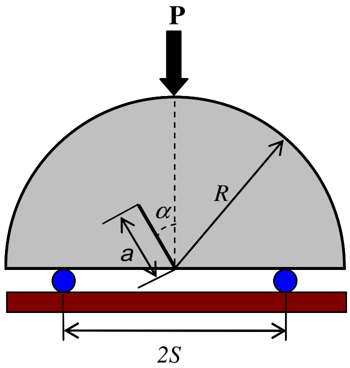
α	crack inclination angle
θ_0	crack initiation direction
$\sigma_{\theta\theta}$	tangential stress component
σ_t	tensile strength
$\sigma_{\theta\theta c}$	critical tangential stress component

preparation procedure, convenience of testing set up and also the ability of introducing complete combinations of mode I and mode II. Some of the test configurations proposed in literature for investigating mixed mode I/II fracture are briefly described here. Erdogan and Sih [1], Williams and Ewing [2] and Theocaris [3] used a rectangular plate containing an inclined center crack and subjected to a uniform far field tension in their mixed mode fracture studies. The asymmetrically loaded three or four-point bend specimens were also employed by researchers for investigating mixed mode brittle fracture [4–11]. Disc type specimens including the centrally cracked Brazilian disk (BD) specimen and the semi-circular bend (SCB) specimen have been frequently employed for determining the mixed mode fracture resistance of various engineering materials such as rocks and PMMA [12–20]. The compact tension-shear specimen [21–23] is another configuration used for mixed mode fracture experiments. Ewing et al. [24] also made use of the inclined edge-crack plates subjected to far field tension and bending to study mixed mode fracture in PMMA. More recently Ayatollahi and Aliha [25] proposed a diagonally loaded square plate containing an inclined center crack for investigating mixed mode fracture behavior. In the above-mentioned studies, brittle fracture experiments have been conducted either on PMMA or other brittle or quasi-brittle materials like ceramics and rocks. However, some of these specimens have certain shortcomings. For example, some of the mixed mode test configurations are able to provide only limited mode mixities or require complicated loading fixtures.

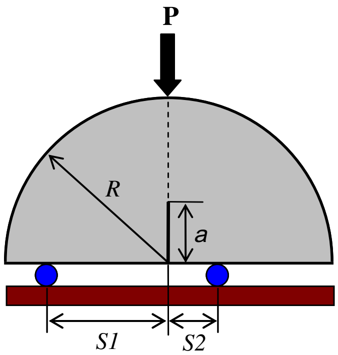
The specimens of circular or semi-circular shape are very suitable for fracture testing on rock or asphalt materials because they can be easily cut from cylindrical cores which are traditionally prepared from such materials. However, as elaborated in the next section, the classical SCB specimen has certain shortcomings for mixed mode fracture experiments. Therefore, a modified SCB test specimen is proposed in this paper to overcome the previous weaknesses. In the forthcoming sections, the suggested specimen is described and then its capabilities and advantages are investigated by means of finite element analysis and also through some fracture tests conducted on PMMA.

2. New test configuration

The classical semi-circular bend (SCB) specimen shown in Fig. 1a has been used by several researchers in the past for investigating mixed mode fracture in brittle materials, e.g. [16–20]. The SCB specimen that contains an angled crack is subjected to three-point bending. The two bottom supports in this specimen are always of the same distance from the direction



(a) Classical SCB specimen (angled crack – symmetric loading)



(b) The ASCB configuration (vertical crack – asymmetric loading)

Fig. 1. Semi-circular specimen under three-point bend loading.

of top load. In order to control the relative combination of mode I and mode II in the classical SCB specimen, one has to produce specimens with different crack angles. One major drawback for the classical SCB specimen is related to the practical difficulties in producing an angled crack in the specimen, particularly for mode II dominant loading conditions in which the crack angle α is relatively large (about 50°). For example, Lim et al. [26] faced with an unwanted fracture while preparation mode II dominant SCB specimens having large angles α. Therefore, an improved SCB configuration was suggested in this research to prevent the disadvantage described above.

Fig. 1b shows the geometry and loading conditions for the improved test configuration called the asymmetric semi-circular bend (ASCB) specimen. In this test configuration, a semi-circular specimen of radius R that contains an edge crack of length a emanating normal to the flat edge of the specimen is loaded asymmetrically by a three-point bend fixture. While the specimen is easily manufactured, it does not need complicated loading fixtures for using in the conventional testing machines. Moreover, the crack is always along the symmetry line of the semi-circle. The state of mode mixity in the ASCB specimen can be easily altered by changing the locations of two bottom supports ($S1$ and $S2$). When the bottom loads are applied symmetric to the crack line (i.e. when $S1 = S2$) the specimen is subjected to pure mode I. But for asymmetric loading (i.e. $S1 \neq S2$), mode II appears in the crack deformation in addition to mode I. The mode I and mode II contributions can be controlled simply by choosing appropriate values for $S1$ and $S2$. Hence different mode mixities can be obtained in the proposed specimen. The specimen has been frequently used in the past but only for the simple case of symmetric loading conditions in order to obtain pure mode I fracture toughness for several engineering materials including, rocks, concrete, asphalt, polymers, etc. [18–20,27–29].

In order to study mixed mode fracture in brittle materials using this configuration, it is necessary to define the state of mode mixity by calculating the mode I and mode II stress intensity factors (K_I and K_{II}) for different loading positions and crack lengths. The finite element method was employed in this research to determine K_I and K_{II} in the ASCB specimen. More details of calculations will be given in the next section.

3. Numerical analysis

The stress intensity factors K_I and K_{II} for the ASCB specimen are functions of the crack length (a) and the locations of loading supports defined by $S1$ and $S2$ and can be written as:

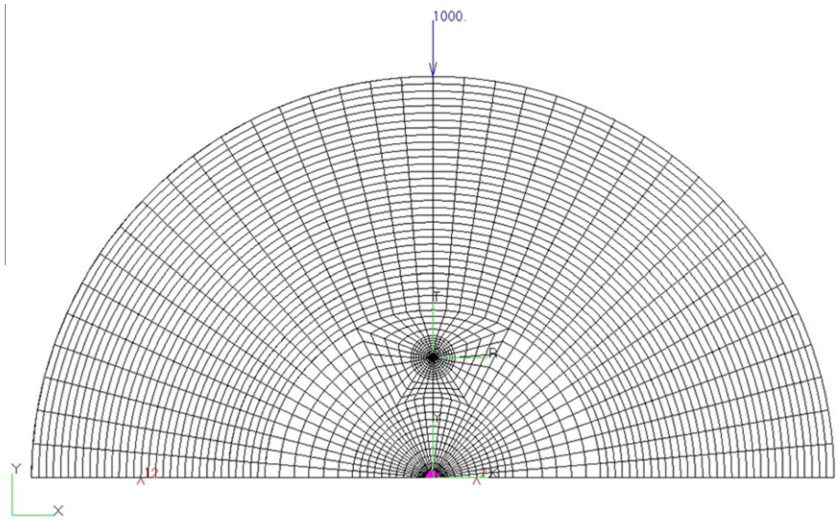


Fig. 2. A typical finite element mesh used for simulating ASCB specimen.

$$K_{\mathrm{I}} = \frac{P}{2Rt}\sqrt{\pi a}Y_{\mathrm{I}}(a/R, S1/R, S2/R) \tag{1}$$

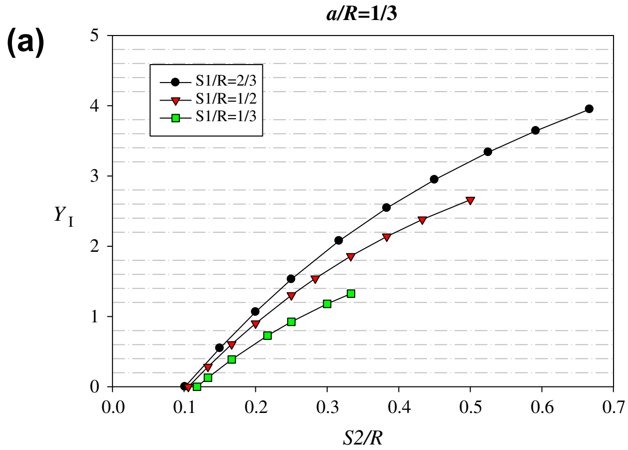
$$K_{\mathrm{II}} = \frac{P}{2Rt}\sqrt{\pi a}Y_{\mathrm{II}}(a/R, S1/R, S2/R) \tag{2}$$

where t is the specimen thickness and Y_{I} and Y_{II} are the geometry factors corresponding to mode I and mode II, respectively. For calculating Y_{I} and Y_{II}, different finite element models of the ASCB specimen were analyzed using the finite element code ABAQUS. Fig. 2 shows a typical mesh pattern generated for simulating the ASCB specimen. In the models, the following geometry and loading conditions were considered: R = 60 mm, t = 6 mm, P = 1000 N and different values for crack lengths. $S1$ was set at a fixed value of 40 mm and $S2$ varied from zero to 40 mm to change the state of mode mixity. The elastic material properties of PMMA as E = 2970 MPa and ν = 0.35 were also considered in the finite element models. A total number of 2162 eight-noded plane stress elements were used for each model. The singular elements were considered in the first ring of elements surrounding the crack tip for producing the square root singularity of stress/strain field. A J-integral based method built in ABAQUS was used for obtaining the stress intensity factors directly from software. Figs. 3 and 4 show the values of Y_{I} and Y_{II} calculated from several finite element analyses performed for different loading conditions in the ASCB specimen.

It is seen from these figures that for the symmetric loading conditions (i.e. $S1 = S2$), Y_{II} equals zero and thus the specimen is subjected to pure mode I loading. By changing the location of the second loading support $S2$, the mode II component also appears in the ASCB specimen. It is seen from Figs. 3 and 4 that by moving $S2$ towards the crack plane, the mode I geometry factor Y_{I} decreases and the mode II geometry factor Y_{II} increases. Fig. 5 shows the von Mises stress contour plot when $a/R = 1/3$, $S1/R = 2/3$, R = 60 mm and for two different mode mixities: pure mode I ($K_{\mathrm{II}} = 0$) and mixed mode loading with $K_{\mathrm{I}} = K_{\mathrm{II}}$. In pure mode I, the stress contour is symmetric relative to the crack plane. But in mixed mode conditions, the contour plot is asymmetric with respect to the crack plane. According to Figs. 3 and 4, the mode I geometry factor increases by increasing a/R, $S1/R$ and $S2/R$. But Y_{II} decreases when $S2/R$ becomes greater. For each value of crack length ratio (a/R), there is a specific value for $S2$ where Y_{I} becomes zero while Y_{II} is non-zero. This loading situation corresponds to pure mode II conditions. Fig. 6 shows the loading and geometry conditions that correspond to pure mode II deformation in the ASCB specimen for some combinations of a/R, $S1/R$ and $S2/R$.

4. Mixed mode fracture tests

In order to investigate the practical applicability of the ASCB specimen, a series of mixed mode fracture tests were conducted on PMMA. A total number of 30 ASCB specimens were manufactured from a PMMA sheet of 6 mm thickness. The dimensions of the produced specimens were chosen as: R = 60 mm, a = 20 mm and t = 6 mm. Thus the crack length ratio a/R was equal to $\frac{1}{3}$ in the test samples. For creating the cracks, first a very thin fret saw blade of thickness 0.4 mm was used to generate a notch with the initial depth of slightly less than 20 mm. Then, a sharp pre-crack was introduced by pressing a razor blade carefully to make the final length of each crack 20 mm. The distance $S1$ was set equal to 40 mm for all the experiments and then in order to cover the full range of mixed mode I/II cases, the following $S2$ values in mm were considered for experiments: $S2$ = {40 (pure mode I), 23, 15, 12, 9, 7.5 and 6.07 (pure mode II)}. For each mode mixity, at least four ASCB specimens were prepared. Then each specimen was located inside a three-point bend loading fixture with desired values



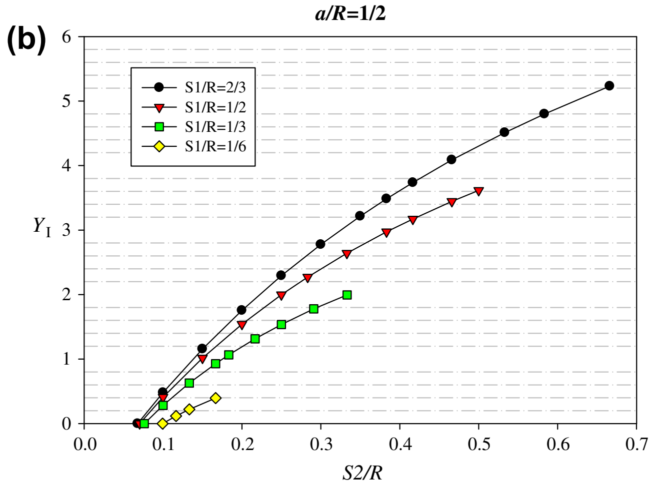


Fig. 3. Variations of mode I geometry factor (Y_I) with $S2/R$ in the ASCB specimen; (a) $a/R = \frac{1}{3}$, (b) $a/R = \frac{1}{2}$.

of $S1$ and $S2$, and loaded at a constant rate of 0.5 mm/min up to its final fracture. The tests were conducted by means of a screw-driven tensile test machine having a capacity of 150 kN. The load–displacement data were recorded during the tests. All the test samples fractured suddenly from the crack tip and with negligible non-linear deformation showing the brittle fracture behavior of the tested PMMA samples. Fig. 7 shows the loading set up for one of the ASCB specimens and a sample fractured specimen. Using the fracture load obtained from each specimen, the critical stress intensity factors of the tested ASCB specimens were calculated from Eqs. (1) and (2). Details of each test including, the fracture loads and the corresponding stress intensity factors are listed in Table 1.

5. Results and discussions

5.1. Experimental results

The results for mixed mode fracture resistance of brittle materials are usually presented in a normalized form as K_{II}/K_{Ic} versus K_I/K_{Ic} where K_{Ic} is a material constant called the pure mode I fracture toughness. The average value of mode I fracture toughness K_{Ic} obtained from the symmetrically loaded ASCB specimens (i.e. $S1 = S2 = 40$ mm) was 1.51 MPa $\sqrt{m}$ (see the results presented in Table 1). This figure is in the range of 1–2 MPa $\sqrt{m}$ reported in previous papers for fracture toughness of PMMA [9,18,30]. The mixed mode test results obtained in this research for various combinations of mode I and mode II are shown in Fig. 8 in a K_{II}/K_{Ic}–K_I/K_{Ic} diagram. Also shown in this figure is the theoretical prediction of a well known fracture criterion called the maximum tangential stress (MTS) criterion [1]. However, as shown in Fig. 8, the results obtained from mixed mode fracture experiments using the ASCB specimen are not consistent with the predictions of the MTS criterion and the mixed mode test results are higher than the MTS curve. The discrepancy between the experimental results and the MTS criterion becomes more pronounced by moving towards pure mode II conditions. For example, the average value for the mode II fracture toughness ratio K_{IIc}/K_{Ic} for the tested ASCB specimen is 1.13 (see the vertical axis in Fig. 8) that is

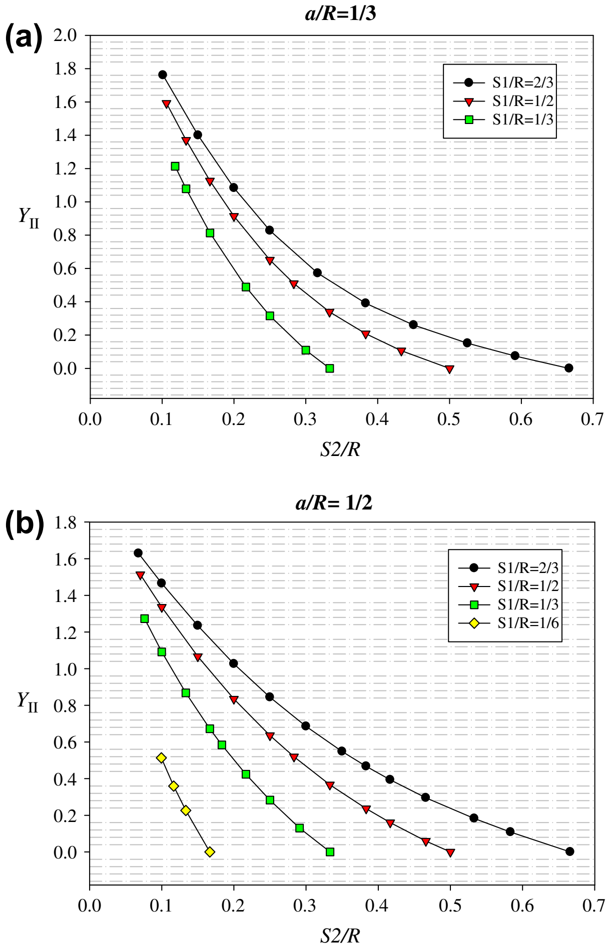


Fig. 4. Variations of mode II geometry factor (Y_{II}) with $S2/R$ in the ASCB specimen; (a) $a/R = \frac{1}{3}$, (b) $a/R = \frac{1}{2}$.

about 30% higher than the figure 0.87 predicted by the MTS criterion. Although not shown here for the sake of brevity, other criteria like minimum strain energy density criterion [31] and maximum energy release rate criterion [32] also fail to provide acceptable predictions for the experimental results. Therefore, in this research a generalized form of the MTS criterion (called the GMTS criterion) was employed to improve the theoretical predictions for the experimental results obtained from the ASCB specimen.

5.2. Fracture criterion

Based on the GMTS criterion [33], mixed mode brittle fracture takes place along the direction of maximum tangential stress around the crack tip (θ_0). It also commences when the value of tangential stress along θ_0 and at a critical distance (r_c) from the crack tip reaches a critical value. The critical distance r_c can be considered as a parameter that describes the length of damage zone developing around the crack tip. This damage zone is generated because of high stress/strain concentration in the vicinity of the crack tip.

The tangential stress $\sigma_{\theta\theta}$ around the crack tip is written as an infinite series expansion [34]:

$$\sigma_{\theta\theta} = \frac{1}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left[K_I \cos^2\frac{\theta}{2} - \frac{3}{2}K_{II}\sin\theta\right] + T\sin^2\theta + O(r^{1/2}) \tag{3}$$

where r, θ are the conventional crack tip polar co-ordinates. While the first term in Eq. (3) is singular and depends on the stress intensity factors (K_I and K_{II}), the second term (often called the T-stress) is a non-singular stress term that is independent of distance r from the crack tip. $O(r^{1/2})$ stands for the remaining terms of the tangential stress which are negligible near the crack tip. According to the GMTS criterion the angle of fracture initiation θ_0 is determined from:

$$\left.\frac{\partial\sigma_{\theta\theta}}{\partial\theta}\right|_{\theta=\theta_0} = 0 \quad \Rightarrow \quad [K_I \sin\theta_0 + K_{II}(3\cos\theta_0 - 1)] - \frac{16T}{3}\sqrt{2\pi r_c}\cos\theta_0 \sin\frac{\theta_0}{2} = 0 \tag{4}$$

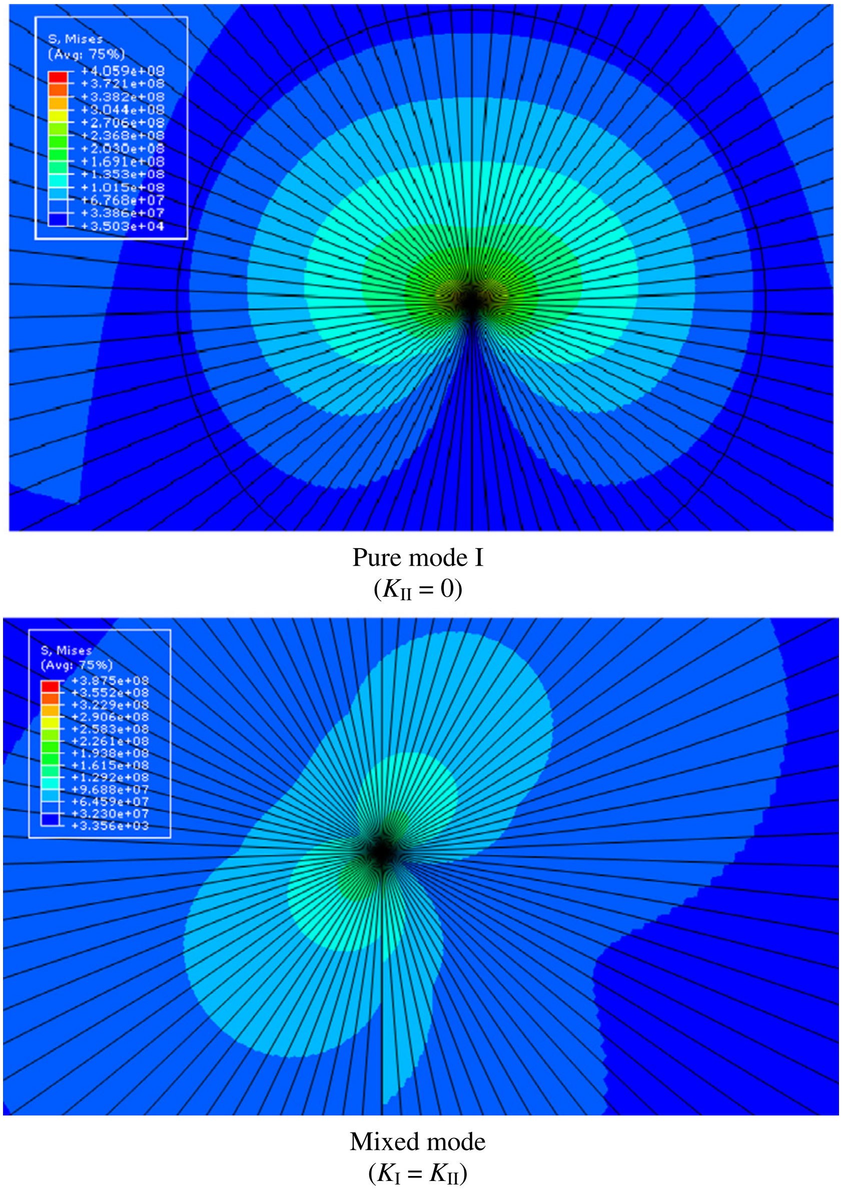


Fig. 5. Plots of von Mises stress contours in front of the crack tip in the ASCB specimen ($a/R = 1/3$, $S1/R = 2/3$, $R = 60$ mm).

Eq. (4) indicates that the angle of mixed mode fracture initiation (θ_0) for any mode mixities depends on four parameters of K_I, K_{II}, T and r_c. The angle θ_0 determined from Eq. (4) can then be used for predicting the onset of mixed mode fracture based on the GMTS criterion as:

$$\sigma_{\theta\theta}(r_c, \theta_0) = \sigma_{\theta\theta c} \tag{5}$$

By replacing the angle θ_0 from Eq. (4) into Eq. (5), brittle fracture is anticipated to initiate at critical conditions when:

$$\sqrt{2\pi r_c}\sigma_{\theta\theta c} = \cos\frac{\theta_0}{2}\left[K_I \cos^2\frac{\theta_0}{2} - \frac{3}{2}K_{II}\sin\theta_0\right] + \sqrt{2\pi r_c}T\sin^2\theta_0 \tag{6}$$

For pure mode I fracture (i.e. when $K_I = K_{Ic}$, $K_{II} = 0$ and $\theta_0 = 0$), Eq. (6) simplifies to:

$$\sqrt{2\pi r_c}\sigma_{\theta\theta c} = K_{Ic} \tag{7}$$

By replacing Eq. (7) into Eq. (6), the conditions required for mixed mode brittle fracture can be rewritten in terms of mode I fracture toughness K_{Ic} as:

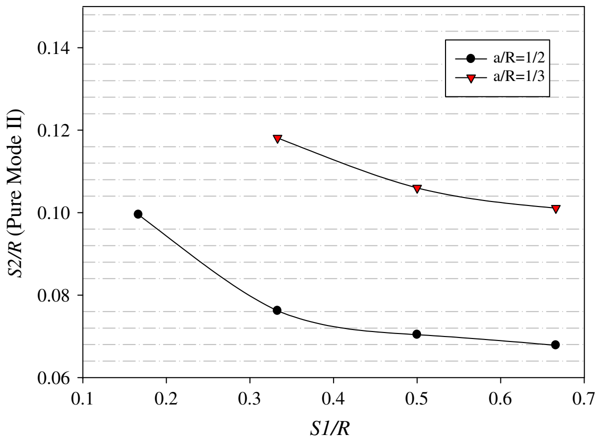


Fig. 6. Positions of bottom supports for pure mode II conditions in the ASCB specimen.

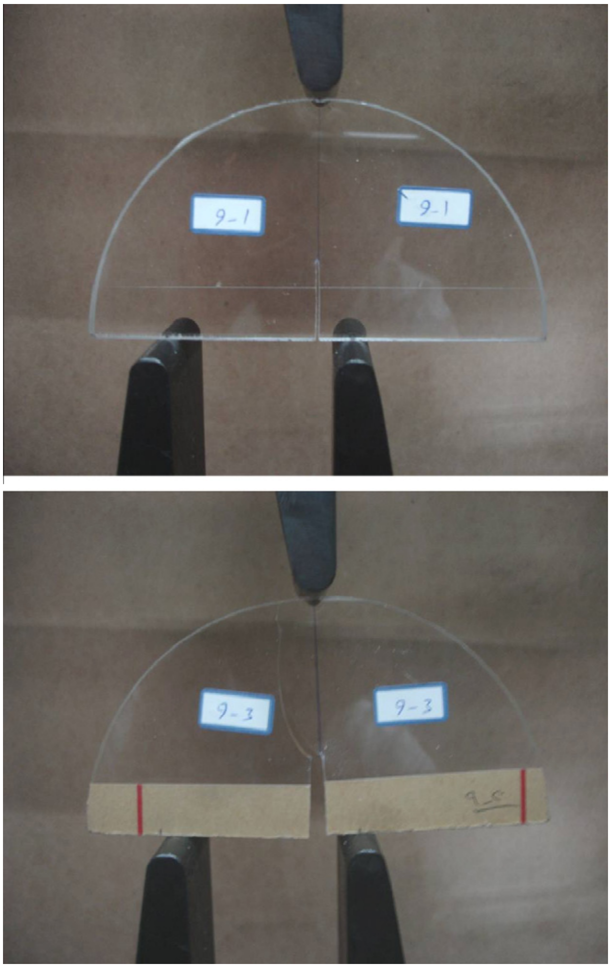


Fig. 7. Loading set up and fracture pattern in a typical ASCB specimen.

$$K_{Ic} = \cos\frac{\theta_0}{2}\left[K_I\cos^2\frac{\theta_0}{2} - \frac{3}{2}K_{II}\sin\theta_0\right] + \sqrt{2\pi r_c}T\sin^2\theta_0 \tag{8}$$

Eq. (8) shows the GMTS criterion for mixed mode fracture in terms of three crack parameters K_I, K_{II} and T. By ignoring the effect of T-stress in Eq. (8), the GMTS criterion will be identical to the conventional MTS criterion. Based on the GMTS

Table 1
Summary of the results obtained from mixed mode I/II fracture tests conducted on ASCB specimens made of PMMA. (Specimen code *X*–*Y*–*Z*: *X* = *S*1 (mm), *Y* = *S*2 (mm), *Z* = test number).

Specimen code	P_{cr} (kN)	K_I (MPa$\sqrt{m}$)	K_{II} (MPa$\sqrt{m}$)
40–6.07–1 (pure mode II)	2.413	0	1.479
40–6.07–2 (pure mode II)	2.44	0	1.496
40–6.07–3 (pure mode II)	2.86	0	1.753
40–6.07–4 (pure mode II)	2.458	0	1.507
40–6.07–5 (pure mode II)	2.713	0	1.663
40–6.07–6 (pure mode II)	3	0	1.839
40–7.5–1	2.603	0.247	1.431
40–7.5–2	3.17	0.301	1.743
40–7.5–3	2.851	0.271	1.567
40–7.5–4	2.87	0.272	1.578
40–9–1	2.915	0.556	1.421
40–9–2	2.494	0.476	1.215
40–9–3	2.64	0.504	1.286
40–9–4	2.58	0.492	1.257
40–12–1	2.361	0.874	0.891
40–12–2	2.426	0.898	0.915
40–12–3	2.343	0.867	0.883
40–12–4	2.324	0.860	0.876
40–15–1	2.297	1.223	0.662
40–15–2	2.086	1.110	0.601
40–15–3	2.034	1.083	0.586
40–15–4	2.095	1.115	0.604
40–23–1	1.559	1.380	0.212
40–23–2	1.556	1.378	0.211
40–23–3	1.659	1.469	0.225
40–23–4	1.554	1.376	0.209
40–40–1 (pure mode I)	1.072	1.474	0
40–40–2 (pure mode I)	1.159	1.593	0
40–40–3 (pure mode I)	1.033	1.420	0
40–40–4 (pure mode I)	1.182	1.625	0
40–40–5 (pure mode I)	1.075	1.478	0

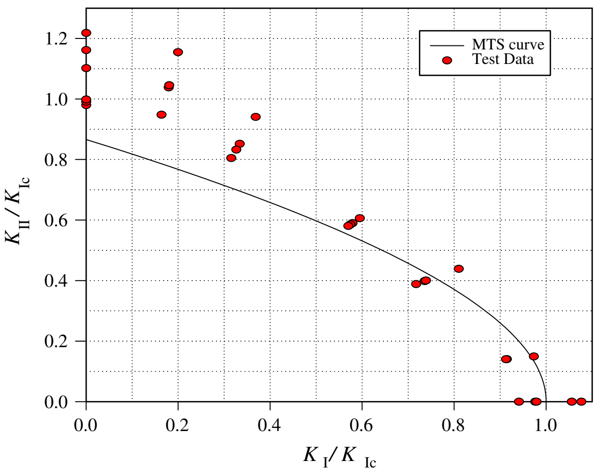


Fig. 8. Experimental results obtained from mixed mode I/II fracture tests on PMMA using the ASCB specimen.

criterion, the sign and magnitude of *T*-stress which depends on the geometry and loading condition of the cracked specimen can influence significantly the mixed mode fracture toughness. In general, the fracture resistance of cracked specimens under mixed mode loading increases by a negative *T*-stress and decreases by a positive *T*-stress [33].

5.3. Mixed mode fracture study for the ASCB specimen

According to the GMTS criterion, five parameters K_I, K_{II}, T, K_{Ic} and r_c are required for estimating the mixed mode I/II fracture resistance. Three parameters (K_I, K_{II} and T) depend on the geometry and loading conditions in the test specimen whereas the critical distance r_c and K_{Ic} are considered to be material properties. The T-stress in the ASCB specimen is a function of crack length (a) and the location of bottom supports ($S1$ and $S2$) and can be written as:

$$T = \frac{P}{2Rt} T^*(a/R, S1/R, S2/R) \tag{9}$$

where T^* is the normalized form of T-stress. The T-stress was also determined for the ASCB specimen directly from ABAQUS. Fig. 9 shows the variations of T^* in terms of $S1/R$ and $S2/R$ and for $a/R = 1/3$ and $a/R = 1/2$. According to this figure, T^* is always negative but its value increases when the normalized support distance $S2/R$ becomes larger.

Now, by knowing the fracture parameters in the tested ASCB specimens, the mixed mode fracture resistance can be predicted theoretically in terms of the normalized fracture parameters (i.e. Y_I, Y_{II} and T^*) of ASCB specimen. The procedure for a theoretical prediction of mixed mode fracture resistance in the ASCB specimen is outlined here. If Eqs. (1), (2), and (9) are replaced into Eq. (4), the angle of fracture initiation (θ_0) in the ASCB specimen can be determined from:

$$[Y_I \sin\theta_0 + Y_{II}(3\cos\theta_0 - 1)] - \frac{16T^*}{3}\sqrt{\frac{2r_c}{a}}\cos\theta_0 \sin\frac{\theta_0}{2} = 0 \tag{10}$$

Meanwhile, if both sides of Eq. (8) are divided by K_I, one gets:

$$\frac{K_{Ic}}{K_I} = \cos\frac{\theta_0}{2}\left[\cos^2\frac{\theta_0}{2} - \frac{3}{2}\frac{K_{II}}{K_I}\sin\theta_0\right] + \sqrt{2\pi r_c}\frac{T}{K_I}\sin^2\theta_0 \tag{11}$$

Now, the ratio K_I/K_{Ic} in the ASCB specimen is rewritten in terms of Y_I, Y_{II} and T^* as:

$$\frac{K_I}{K_{Ic}} = \left[\cos\frac{\theta_0}{2}\left[\cos^2\frac{\theta_0}{2} - \frac{3}{2}\frac{Y_{II}}{Y_I}\sin\theta_0\right] + \sqrt{\frac{2r_c}{a}}\frac{T^*}{Y_I}\sin^2\theta_0\right]^{-1} \tag{12}$$

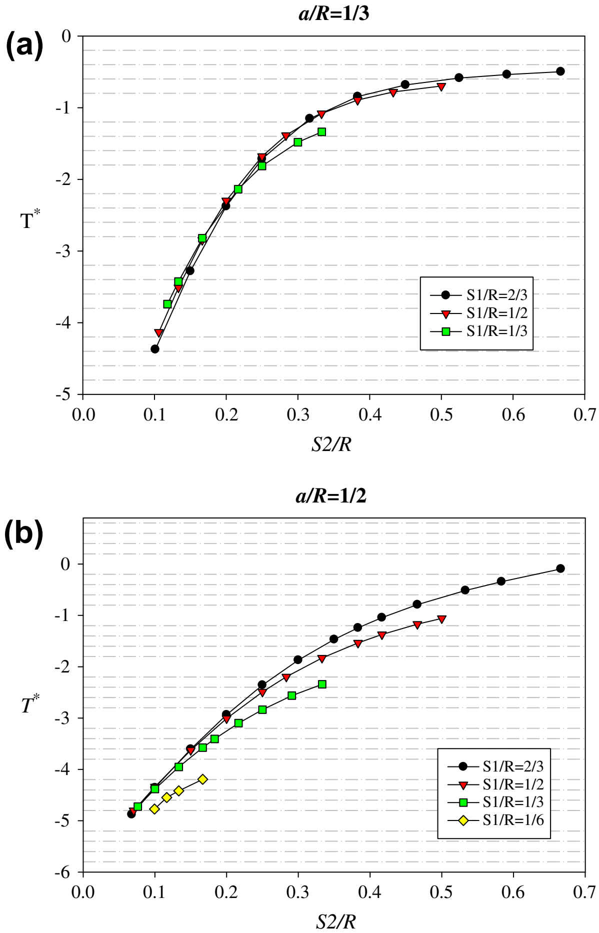


Fig. 9. Variations of normalized T-stress (T^*) with $S2/R$ in the ASCB specimen; (a) $a/R = \frac{1}{3}$, (b) $a/R = \frac{1}{2}$.

Using a similar procedure, the ratio K_{II}/K_{Ic} is also derived as:

$$\frac{K_{II}}{K_{Ic}} = \left[\cos\frac{\theta_0}{2}\left[\frac{Y_I}{Y_{II}}\cos^2\frac{\theta_0}{2} - \frac{3}{2}\sin\theta_0\right] + \sqrt{\frac{2r_c}{a}}\frac{T^*}{Y_{II}}\sin^2\theta_0\right]^{-1} \tag{13}$$

In order to predict the experimental results, the numerical values of Y_I, Y_{II} and T^* extracted from Figs. 3, 4 and 9 for an assumed value of $S2$ are substituted into Eq. (10) to calculate the fracture initiation angle θ_0. Then, by replacing the calculated angle θ_0 and the related values of Y_I, Y_{II} and T^* into Eqs. (12) and (13), the corresponding ratios of K_I/K_{Ic} and K_{II}/K_{Ic} are determined for the given mode mixity. The same procedure can be repeated for other mode mixities (by changing $S2$) from pure mode I to pure mode II to obtain a fracture curve based on the GMTS criterion for the tested ASCB specimens made of PMMA. However, using Eqs. (10), (12), and (13) requires a reasonable value for r_c. Several values were examined for r_c and it was found that r_c = 0.14 mm can provide very good estimate for the average experimental results. Fig. 10 shows the mixed mode fracture curves of PMMA predicted by the GMTS criterion for the ASCB specimen by using r_c = 0.14 mm. It is noteworthy that based in Fig. 9, the T-stress in the ASCB specimen is negative for all combinations of mode I and mode II. Therefore, as proposed by the GMTS criterion the enhanced fracture resistance of the tested PMMA specimens can be attributed to the influence of negative T-stress that exists in the ASCB specimen.

Indeed, the conventional MTS criterion takes into account only the effect of singular stress term characterized by the stress intensity factors K_I and K_{II}. Therefore, it is not able to provide acceptable predictions for the experimental results obtained from the ASCB specimen because the effect of non-singular T term on the onset of fracture is noticeable in this specimen. However, by using the GMTS criterion which considers a more accurate description for the crack tip stress field, the test data could be predicted very well.

It is worth mentioning that there are many other brittle or quasi-brittle materials like rocks which have a larger critical distance r_c compared to PMMA. For such materials, one would expect to obtain higher values of mixed mode fracture toughness from the ASCB experiments; because the GMTS suggests that the effect of T-stress in mixed mode brittle fracture is more pronounced for materials having larger values of r_c [34].

According to the experimental results obtained for pure mode I and pure mode II fracture tests, the average values of K_{Ic} and K_{IIc} are 1.52 and 1.63 MPa $\sqrt{m}$, respectively. Therefore, the average ratio of K_{IIc}/K_{Ic} for the tested PMMA specimens is about 1.07. This finding do not comply with the predictions of the available conventional criteria e.g. [1,31,32] which often estimate a value of K_{IIc} less than K_{Ic}. These fracture criteria are based only on the singular stress terms and do not consider the effect of T-stress. However, when the effect of T-stress is also taken into account using the GMTS criterion, the enhanced mode II fracture toughness of the tested PMMA can be predicted quantitatively. The noticeable negative T-stress that exists in the ASCB specimen under pure mode II loading increases the value of K_{IIc} such that its magnitude becomes even greater than K_{Ic}.

Fig. 10 indicates that the fracture curve predicted from the GMTS criterion using r_c = 0.14 mm is in very good agreement with the experimentally obtained mixed mode fracture data for the PMMA material tested by using the ASCB specimen. As mentioned earlier for obtaining the GMTS curve for cracked specimens, an appropriate value should be taken for r_c. For brittle polymers like PMMA, the size of r_c is often taken as the length of the localized damage zone (comparable to the craze zone) in front of the crack tip. As a simple model for estimating the size of r_c, the following equation has been suggested by Taylor et al. [35]:

$$r_c = \frac{1}{2\pi}\left(\frac{K_{Ic}}{\sigma_t}\right)^2 \tag{14}$$

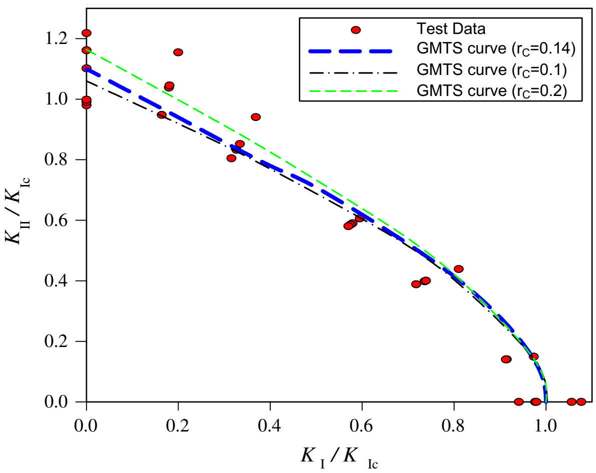


Fig. 10. GMTS fracture curves for PMMA specimens tested using ASCB configuration (with different values of r_c).

where σ_t is the material tensile strength which is determined from a standard tensile test. Eq. (14) can also be derived simply from Eq. (7) if $\sigma_{\theta\theta c}$ in Eq. (7) is replaced by σ_t. Indeed, as a material property $\sigma_{\theta\theta c}$ is commonly considered to be the ultimate tensile strength σ_t for brittle and quasi-brittle materials. This is because the final fracture for a specimen subjected to tensile loading will occur only when the molecular bonds of the material are broken. This condition can be assumed to be valid for both cracked and crack-free samples. The only difference between a cracked specimen and a flawless one is that, for the cracked sample, the stress gradient in the vicinity of the crack tip is very high and hence the local stress reaches σ_t under lower values of load.

For the PMMA material tested in this research, the average values of K_{Ic} and σ_t were about 1.51 MPa $\sqrt{m}$ and 51 MPa, respectively. Thus, by using Eq. (14) the numerical value of r_c is again obtained for the tested PMMA as 0.14 mm. This figure is in the typical range reported by other papers for the size of r_c in PMMA [30,36–38]. Also shown in Fig. 10 are the GMTS curves related to r_c = 0.1 mm and r_c = 0.2 mm. It is seen that the GMTS curves are not significantly sensitive to r_c, implying that the simple model suggested by Eq. (14) is adequate for estimating the critical distance, at least for the PMMA specimens studies in this research. It should be added that a fixed value of r_c was used in this study for predicting the onset of mixed mode fracture based on the GMTS criterion. As shown in Fig. 10, the calculated value of r_c = 0.14 mm is able to provide good estimates for the experimental results obtained from ASCB specimens under different combinations of mode I and mode II. Since the calculated value of r_c was suitable for the investigated specimens made of PMMA, it can be suggested that r_c is independent of the mode mixity and can be considered as a constant material property.

Fig. 11 shows some of the ASCB specimens fractured under different mode mixities. It is seen from this figure that except for pure mode I loading in which the crack growth takes place along the original crack, mixed mode I/II cracks extend in a non-coplanar manner and kink from the original crack line. The fracture initiation angles for the tested ASCB specimens can also be studied by means of the GMTS criterion. Fig. 12 shows the fracture initiation angles θ_0 measured for the tested ASCB specimens. In this figure, M^e is a mode mixity parameter defined as:

$$M^e = \frac{2}{\pi} \arctan\left(\frac{K_I}{K_{II}}\right) \tag{15}$$

These angles were measured from the broken ASCB specimens. In order to measure the angle θ_0 for each specimen, first a photo was taken from the crack front area of the broken specimen using an optical microscope. Then a tangent line was drawn from the tip of crack along the direction of fracture initiation angle. Afterwards, θ_0 was determined by measuring the angle between the line of original crack and the tangent line. Also shown in Fig. 12 are the theoretical predictions of the MTS and GMTS criteria for fracture initiation angle. For plotting the fracture angle curve related to the GMTS criterion, similar to the fracture resistance calculations described earlier, the same fracture parameters (Y_I, Y_{II}, T^* and r_c = 0.14 mm) were again used in Eq. (10) for different mode mixities. While there is a discrepancy between the measured angles and the MTS curve, a good agreement is seen between the experimental results and the curve of GMTS criterion. Therefore, it can be concluded that the GMTS criterion is able to provide good prediction for mixed mode fracture initiation angle in the ASCB specimens as well.

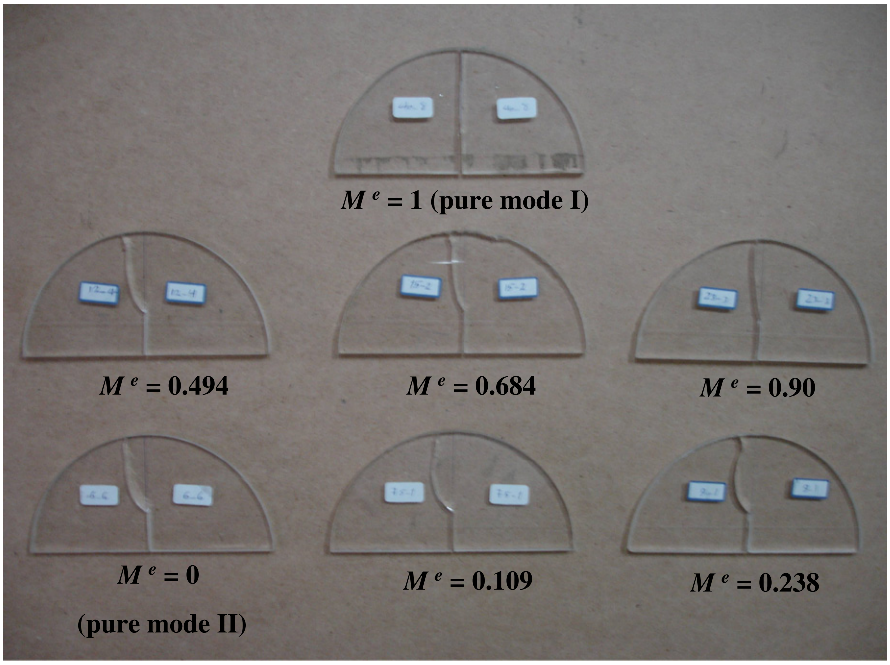


Fig. 11. Fracture patterns in the PMMA specimens tested using the ASCB configuration under different combinations of mode I and II.

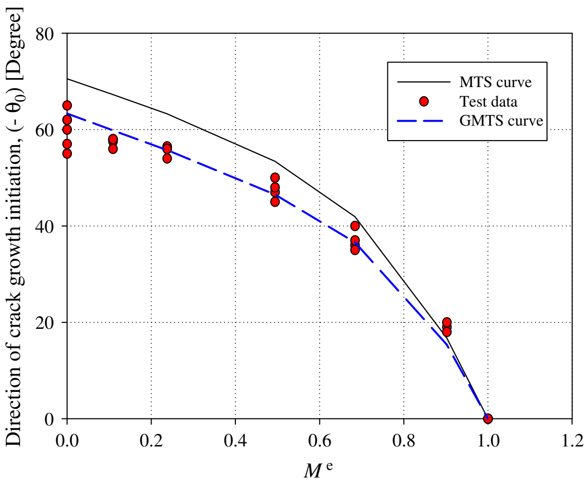


Fig. 12. Direction of fracture initiation angle for the tested ASCB specimens and comparison with the conventional MTS criterion.

A review of the test specimens suggested in the past for conducting mixed mode fracture experiments shows that some of these specimens require a complicated test set up. For example, the compact tension-shear (CTS) specimen suggested by Richard and Benitz [22] and Arcan et al. [23] consists of a cracked specimen loaded through a complicated loading fixture. The auxiliary fixture not only makes the experiments more expensive but also may sometimes be a source of error in the test results due to possible manufacturing inaccuracies. The ASCB specimen suggested in this paper does not need any additional loading fixture since it can directly be tested by an ordinary three-point bend fixture which is normally available in standard fracture testing machines. Furthermore, the application of compressive loads in the ASCB experiments makes it more suitable for conducting fracture tests on brittle materials like rocks, ceramics or concretes which are weak against tensile loads. Although almost all of the mixed mode test specimens can be used for pure mode I and mixed mode fracture tests, some of them are not able to provide pure mode II. For instance, the classical center cracked plate [1–3] or the edge cracked rectangular beam specimen under asymmetric three-point bend loading [4,5] can produce only limited combinations of mode I and mode II. In particular, they cannot be used for pure mode II tests. Another advantage for the ASCB specimen is its ability for providing complete combinations of mode I and mode II including pure mode I, pure mode II and various intermediate mode mixities. It is worth mentioning that the cracked test specimens which contain only one crack tip (like the ASCB specimen), are often preferred to centrally cracked specimens that contain two crack tips (e.g. the centrally cracked square plate, or the centrally cracked Brazilian disk specimen). This is because the crack extension does not necessarily initiate from the two crack tips simultaneously. The delay between the fracture initiations at the two crack tips is not controllable and can sometimes be a likely source of error in the test results.

The classical semi-circular bend (SCB) specimen containing an inclined edge crack but loaded with equal bottom-support distances from the crack (i.e. $S1 = S2$) has also been suggested in the past for mixed mode fracture studies [16–20]. However, there are some complications for accurately introducing the inclined crack in the SCB specimen. Any minor inaccuracy in setting the crack line relative to the loading direction can influence the state of mode mixity. Moreover, since pure mode II is provided in the classical SCB specimen at large crack inclination angles (typically more than 50° relative to the vertical direction), as stated by Lim et al. [26], the SCB specimen can be vulnerable to damage while generating a mode II initial crack. Because of the advantages elaborated above, the ASCB specimen can be recommended as a favorite test sample for conducting mixed mode fracture experiments on brittle materials.

It is finally worth mentioning that if the size of damage zone around the crack tip (represented here by the critical distance) becomes large relative to the dimensions of ASCB specimen, the experimental and theoretical results are expected to be size dependent and the GMTS criterion would need some modifications. This point might be practically of little concern for PMMA since it has a very small critical distance (typically between 0.1 and 0.2 mm). However, for some other brittle or quasi-brittle materials like rocks which sometimes exhibit a critical distance in the order of several millimeters, the size dependency of the experimental results obtained from the ASCB specimen should also be investigated.

6. Conclusions

1. A new test configuration called the asymmetric semi-circular bend (ASCB) specimen was suggested for mixed mode I/II fracture experiments on brittle materials.
2. The simple geometry and loading set up, the ease of generating a crack in the specimen, and the ability of introducing full combinations of mode I and mode II are the main advantages of the ASCB specimen.

3. The experimental results obtained from mixed mode fracture tests on PMMA using the ASCB specimens were in very good agreement with the theoretical predictions of the GMTS criterion.
4. The numerical, experimental and theoretical studies of the ASCB specimen showed that the suggested test configuration is very suitable for mixed mode fracture investigations on brittle and quasi-brittle materials.

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