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# Fault-tolerant interference-aware topology control in multi-radio multi-channel wireless mesh networks



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# ABSTRACT

Wireless Mesh Networks offer an effective technology to establish broadband wireless services. Due to this application, robustness against failures and throughput improvement are the two main objectives in designing such networks. In this paper, we use the "topology control" as a set of tools to reach these objectives. Topology control in wireless mesh networks is an NP-hard problem; therefore, we propose a heuristic method known as Fault-Tolerant Interference-Aware Topology Control (FITC) to solve it in a distributed fashion. In this method, we first guarantee that the network is K-Connected using the graph modification, routing and channel assignment. Then, power control, rate adaptation, channel selection and scheduling is applied to enhance the network throughput. Due to the static channel assignment and the fixed location of the nodes in the network, the first part of the algorithm is conducted using a centralized method, while a distributed cross layer method is applied for the second part. Simulation results validate the efficiency of the proposed method in achieving the main objectives.

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# 1. Introduction

Wireless Mesh Networks (WMNs) are a class of multi-hop networks, which offer a wireless backbone for broadband wireless services such as public access to internet, neighborhood networks, supervisory-security applications, organizational networks and building automation [1]. This network consists of wireless mesh routers, which provides the infrastructure required for traffic transmission between mesh clients and to/from clients out of the mesh network [2]. There is no limitation on power consumption neither is there any mobility challenges in wireless mesh networks unlike wireless sensor networks and mobile Ad-hoc networks where power consumption and nodes mobility are the major challenges. As such, we cannot use the algorithms and proposed methods for sensor and mobile ad hoc networks in this type of network. Robustness against failures and maximizing throughput are the two major challenges in wireless mesh networks. In order to increase the network throughput, multi-radio multi-channel technology is introduced in wireless mesh networks [3]. In this

http://dx.doi.org/10.1016/j.comnet.2016.08.026 1389-1286/© 2016 Elsevier B.V. All rights reserved. technology, each mesh router is equipped with some radio interfaces and there are some non-overlapping frequency channels which can be assigned to the radios. These networks are known as Multi-Radio Multi-Channel WMNs (MR-MC WMNs). The ability to use multiple radios on different channels allow the routers to send/receive packets to/from several neighboring nodes, simultaneously. This feature brings about the efficient use of spectrum and increases the available bandwidth. The effective assignment of channels to radio interfaces can affect the network throughput considerably. In this situation, some nodes have more than one channel to communicate and therefore, selection of proper channel has effects on load balancing and network performance. In this paper, we refer to this problem as *channel selection*, which is different from *channel assignment*.

Due to the limited number of radio interfaces and nonoverlapping channels, some links interfere with others and therefore, there is no possibility to concurrent transmission on these links. More nodes are able to transmit simultaneously with more data rates when the transmission power and rate is controlled. Moreover, by using a contention-free MAC protocol based on scheduling and by assigning different time slots to transmissions with co-channel interference, prevention of collision is possible. Therefore, it is essential to use an efficient strategy for resource assignment in order to improve the network throughput. This strat-

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egy includes using the channel assignment, power control, rate adaptation, channel selection, scheduling and path selection.

From the network robustness point of view, failure of one or more nodes can interrupt the connection between some routers, if the requirements are not considered. According to definition, a network is said to be robust against failures if it still remains connected despite the occurrence of disruption in arbitrary *K* nodes. This property is called *K*-*Connectivity*. In this case, the loss and delay resulting from nodes failure can be minimized using the proper recovery algorithms.

Unlike in wired networks where topology changes only in the case of node failure, in WMNs there are also other factors which change the topology such as channel assignment and variations in power/rate. Therefore, the tools for controlling power, rate adaptation, channel assignment and channel selection have significant impact on network robustness against failures as well as on its throughput. Although a large amount of study has been conducted on maximizing throughput in wireless mesh networks, most of them assume single radio, single channel networks and a vast majority of them do not consider the robustness of the network against failures. In addition, many researches do not use the power control, rate adaptation, channel assignment/selection, scheduling and routing tools concurrently.

In this paper, we use all of the topology control tools to improve the throughput and load balancing in multi-radio multichannel wireless mesh networks, while preserving the network robustness against failures. In this manner, we propose a heuristic method known as Fault Tolerant Interference-Aware Topology Control (FITC) for topology control. The proposed algorithm employs the set of available tools, which are power control, rate adaptation, scheduling, routing and channel assignment/selection. In our proposed strategy, the solution is divided into two parts. In the first part, an algorithm called Network Formation and Modification Algorithm (NFMA) is proposed in which its main objective is building a K-Connected network topology, while trying to reduce the potential interference in the network. In the second part, by using the resulted topology from the first part, the objective is maximizing the network throughput along preserving the K-Connectivity feature of the network. The algorithm proposed for the second part is named Throughput Maximization along with preserving K-Connectivity (TMKC).

The rest of the paper is organized as follows. Section 2 reviews the previous works. The details of network model, definitions, assumptions and the problem description is described in section 3. In the fourth section, the proposed method for topology control is presented. Section 5 is dedicated to the simulation results and their analysis. The paper is concluded in section 6.

# 2. Related works

In recent years, extensive research has been carried out to improve the throughput in wireless mesh networks. In [4], we introduce an iterative algorithm for finding the best multicast tree based on the minimum number of hops between source and destinations by using the rate adaption and channel selection. The proposed strategy enhances the throughput and reduces the co-channel interferences. In this paper, the protocol interference model is employed for simplicity and the transmission power is assumed to be fixed. Conversely, the network robustness against failures and channel assignment is not considered.

In [5], Capone et al. investigated the problem of cross-layer optimization of scheduling along with power control and rate adaptation. They assumed a single radio – single channel network with the specified traffic on the links. In other words, they did not consider the routing and channel assignment problem. As a result of the computational complexity of the proposed problem, the authors presented an alternative formulation in which the decision variables indicated the compatible sets of links with the capability of activation in each time slot. Furthermore, the column generation approach was used to calculate the lower band. The extended problem of resource allocation in [6] propose a comprehensive cross-layer optimization model. Here, in addition to the scheduling along with power control and rate adaptation, the routing and channel assignment are also investigated.

In 2010, Luo et. al. [7] developed two computational tools for joint optimization of rate control, power control, scheduling and routing. Moreover, they studied the relation between frequency reuse and network performance as well as multi-hop benefits against single-hop. The tools developed by Luo et al. are based on column generation and provide a suboptimal solution in acceptable time. In [8], the authors proposed a Mixed-Integer Linear Programming(MILP) model and formulate the problem of gateway nodes selection along with the power control, routing and time slot assignment in order to maximize the service level of nodes. They proposed a heuristic method which comprised two steps in order to solve the problem in a serial manner. In the first step, only the selection of gateways is considered. In the second step, given the fixed number of gateways, the solution of the primary problem is studied. The issues of rate adaptation, channel assignment/selection and network robustness against failures is not considered in this reference.

In [9], Hedayati et. al. first formulate the throughput maximization as an optimization problem. To this end, they exploit the power control and rate adaptation. Since the problem is NP-hard, they suggest a centralized heuristic method. Afterward, they introduced a distributed method in [10]. In these papers, the network is supposed to be a single-radio single-channel and therefore, the issues of channel assignment and its impact on throughput is not considered. Conversely, the traffic on the links is assumed to be known; therefore, routing impact on throughput is not studied. Moreover, the network robustness against failures is not considered.

In [11], the authors proposed algorithms to select a set of nodes in order to establish an infrastructure in ad hoc networks, which are efficient in load distribution and robustness against failures. The authors in [12] investigated the critical node density required to ensure that an arbitrary node in a large-scale wireless multi-hop network is connected (via multi-hop path) to infinitely many other nodes with an arbitrary probability. Power consumption reduction and robustness against failures are the two significant challenges in high scale WSNs discussed in [13]. In this paper, the network architecture is assumed to be cluster-based, where the cluster head receives the information and sends them to the base station. Therefore, failure of the cluster heads poses serious problems. In this manner, the authors proposed a distributed fault- tolerant energyaware routing method. In [14], different methods of fault tolerant topology management in WSNs are studied. In this paper, due to failure and energy discharge, some applications of WSNs in harsh environments are pointed where there is a possibility of node movement. Therefore, the topology management is supposed to be an essential factor. Moreover, the related works in this area are briefly studied by dividing the methods into two groups of proactive and reactive. The algorithms and methods presented for WSNs and MANETs are not applicable in WMNs, because the main challenges of these networks are the power consumption and nodes movement, which are insignificant in backbone architectures such as WMNs.

In [15], a Multi-Radio Multi-Channel WMN is considered. Given that the primary network is *K*-Connected; the authors presented an algorithm for channel assignment which reduces the potential interference while preserving the *K*-Connectivity feature. However, other topology control tools and their impacts on connectivity are

not considered. Moreover, the protocol interference model which is an unrealistic model is employed.

In [16], a topology control method is proposed for multi-hop networks. The authors recommended a distributed method for assigning the minimum required power to all nodes so that the network topology remains *K*-Connected. In this algorithm, each node collects the information about the location of its adjacent nodes and calculates the power of adjacent nodes so that they can access this node from *K* disjoint paths. At first, the network is considered to be *K*-Connected with given maximum transmission power. Then the nodes power is adjusted so that *K*-Connectivity is preserved. Nevertheless, this assumption may not necessarily be established. In this paper, the network is assumed to be single-radio, single-channel and single rate. Moreover, similar to the previous work, the protocol interference model is exploited.

The research on robustness against failure is divided into software and hardware subgroups in [17]. The software subgroup includes routing techniques, network coding etc., while the hardware subgroup includes compact placement of nodes and backup stations. In this paper, the software level is investigated and a network coding based on the robust routing mechanism against failures for instantaneous retrieving of packets is proposed.

In [18], given a *K*-Connected graph, the authors proposed a channel assignment method that maximizes the throughput while preserving the *K*-Connectivity feature. In this paper, the use of other tools such as power control, rate adaptation and channel selection is not considered and the protocol interference model which cannot properly model the interference in practice is utilized.

In [19], a topology control method for wireless client mesh networks is presented. In client mesh networks, despite the backbone mesh networks, the nodes are often equipped with a single radio interface with limited features and so their transmission power and rate is fixed. The authors try to reduce the overload by using activation/deactivation of routing capability in some nodes, since the overload resulting from routing management packets are so high in social compensated networks with handheld devices.

The different methods of resource allocation in wireless mesh networks presented in recent years can be divided into two main groups which are the optimization-based methods and heuristic methods. The optimization-based methods are computationally complex, centralized and not scalable; therefore, these methods are not applicable in practice. On the contrary, most heuristic methods presented so far do not use the complete set of topology control tools including power control, rate adaptation, channel assignment/selection, scheduling and routing. In addition, the objectives of most previous works are not as comprehensive as what we consider in our proposed algorithm. They do not consider the complete set of objectives like throughput maximization, balancing and robustness against failures and many of them are for single radio or single channel networks.

In the heuristic method proposed in this paper, we employ all of the topology control tools to improve the throughput and load balancing in multi-radio multi-channel wireless mesh networks, while preserving the network robustness against failures. In summary, the contributions of our work are the following:

- All the available tools for topology control are used, namely power control, rate adaptation, scheduling, routing and channel assignment/selection.
- A new hybrid interference model that combines the simplicity of protocol interference model and accuracy of physical interference model is introduced.
- The solution to the topology control problem is divided into two parts. The main objective of the first part is building a *K*-Connected network topology, whereas that of the second

part is maximizing the throughput while preserving the *K*-Connectivity feature of the network. In previous related works, only one of these parts is considered as topology control.

- A new combinational metric is defined for routing which helps to select the best *K* paths between nodes in the first part of the proposed algorithm.
- For channel assignment, an Interference-aware method is introduced whose performance is much better than the random channel assignment previously used in several works.
- The main part of the proposed solution is distributed and this avoids the overhead and complexity of using centralized solutions proposed in some previous recognized researches. The proposed method also considers the *K*-Connectivity, which is not investigated in any of the previous similar works.
- In addition to maximizing the throughput and considering the network robustness, load balancing is also included in the proposed method. This aspect is often neglected in previous works.

## 3. Preliminaries and problem definition

In this section, we define the preliminaries including the network model, assumptions and definitions. In addition, we describe the problem in details. The variables and symbols are also introduced. Table 1 summarizes the most used notations in the rest of the paper.

# 3.1. Network modeling and assumptions

Consider a MR-MC WMN with *n* static routers. Each node *i* is equipped with  $In_i$  half-duplex radio transceivers (interface cards) equipped with omnidirectional antennas. One of the non-overlapping channels  $\Omega = \{\omega_1, \omega_2, ..., \omega_{|\Omega|}\}$  is assigned to each radio. In practice  $In_i \prec |\Omega|$ ; therefore, we need a channel assignment strategy to assign channels to the interfaces. Each interface card can adjust its transmission power in the range of  $[0, P^{\max}]$ , and can transmit/receive with any of the available rates  $\mathbf{R} = \{\rho_1, \rho_2, ..., \rho_M | \rho_1 \prec \rho_2 \prec ..., \prec \rho_M \}$ .

Here, we assume the traffic flows as the end-to-end unicast sessions expressed with **TD**, that is a  $n \times n$  matrix, in which,  $TD_{uv}$  indicates the volume of traffic flow from node u to node v. The traffic between the end nodes must be guaranteed in WMNs at the worst case. The network is modeled as a directed network graph G = (V, E) where the set of vertices  $V = \{v_1, v_2, v_3, ..., v_n\}$  indicate the n mesh routers and E represents the set of possible directed communication links. The interference model must be introduced prior to the definition of the possible communication links between nodes.

# 3.2. Interference modeling

According to the suggested classification in [20], two interference models known as protocol interference and physical interference model are defined. The protocol interference model is identified with two parameters: transmission range of node j ( $r_t^j$ ) and its interference range ( $r_i^j$ ). The transmission range of a transmitting node is the radius where all the nodes within this radius can receive the message correctly. According to this model, if we represent the set of assigned channels to node i with**c** $h_i$ , the directed link  $e_{ij}^{\omega}$  from node i to node j over the frequency channel  $\omega$  is defined as follows:

$$e_{ij}^{\omega} \in E \Leftrightarrow d_{ij} \le r_t^i , \quad \exists \omega \in \left\{ \boldsymbol{ch}_i \cap \boldsymbol{ch}_j \right\} \quad ; \forall i, j \in \boldsymbol{V}$$

$$\tag{1}$$

in which  $d_{ij}$  is the geometrical distance between *i* and *j*. Conversely, there is an interference from link  $e_{ij}^{\omega}$  to link  $e_{pq}^{\omega}$  if  $d_{iq} \leq r_i^i$ . Fig. 1 indicates that the node *j* is in the transmission range of node *i*,

Table 1		
List of the	most used	notations

Notation	Definition	Notation	Definition
n In <sub>i</sub>	Number of mesh routers in the network Number of interface cards in node <i>i</i>	$I^{\omega}_{ij,mn}$ $P^{min}$	Interference of link $e_{mn}^{\omega}$ on $e_{ij}^{\omega}$ in the common frequency channel $\omega$ Minimum power required for the network to be K-Connected with high probability
Ω	Set of non-overlapping frequency channels	Path <sub>1111</sub>	Set of vertices in the $l$ th path between nodes $u$ and $v$
R	Set of available rates	NoP <sub>uv</sub>	Number of paths between the nodes $u$ and $v$
G	Network graph	$I_{ii,mn}^{\omega,\max}$	Maximum value of $I_{ij,mn}^{\omega}$ for maximum transmission power
V	Set of vertices in the network graph	$H_{\mu\nu I}$	Hop counts along the <i>l</i> th path between the nodes $u$ and $v$
Ε	Set of directed communication links	$P_{uv,l}^{max}$	Maximum transmission power from node $u$ to node $v$ on the <i>l</i> th path
$r_t^j$	Transmission range of node <i>j</i>	$P^{sum}_{uv,l}$	Total power consumption along the $l$ th path between the nodes $u$ and $v$
$r_i^j$	Interference range of node j	$\hat{P}_{uv,l}$	Transmission power metric for path $l$ between nodes $\boldsymbol{u}$ and $\boldsymbol{v}$
P <sup>max</sup>	Maximum transmission power	$B_{uv,l}^{n_x}$	Number of using node $n_x$ on path <b>Path</b> <sub>uvl</sub> in other paths
<b>ch</b> <sub>i</sub>	Set of assigned channels to node <i>i</i>	$B_{uv,l}^{sum}$	Total usage of all nodes on the path $l$ between the nodes $u$ and $v$
$e_{ii}^{\omega}$	Directed link from node $i$ to $j$ over the frequency channel $\omega$	$\hat{B}_{\mu\nu}$	Load balancing metric for path $l$ between the nodes $u$ and $v$
$p_{ij}^{\omega}$	Transmission power from node $i$ to node $j$ over the channel $\omega$	RCF <sub>uv,l</sub>	Routing Cost Function(RCF) of the <i>l</i> th path between the nodes $u$ and $v$
G <sub>ii</sub>	Propagation gain resulting from the effective power loss	$\Gamma^{FTL}$	Set of necessary links for preserving K-Connectivity
ε	Path loss exponent	I <sup>sumMax</sup>	Total amount of potential interference on each connection of $\Gamma^{FTL}$
d <sub>ij</sub>	Geometrical distance between nodes <i>i</i> and <i>j</i> .	$I_i^{\omega}$	Potential Interference on the receiver of $e_{ii}^{\omega}$
$SINR_{ij}^{\omega}$	Signal to Interference and Noise Ratio (SINR) in receiver $j$ on transmission from node $i$ to node $j$ over the channel $\omega$	$\dot{f_{ij}}$	Amount of traffic on each connection between nodes $i$ and $j$
$\gamma(\rho)$	SINR threshold corresponding to the rate $\rho$	$\Gamma_{uv}^{BR}$	Set of connections on the best path selected between nodes $u$ and



Fig. 1. Transmission and interference range in protocol interference model.

and node q is in the transmission range of node p. If a common frequency channel exists between each pair; then the transmission from i to j and from p to q is possible. On the other hand, the transmission from i to j interferes on the receiver q if both links work on the same channel  $\omega$ . According to [13], we can set the interference range roughly as twice the transmission range.

The physical interference model is based on the Signal to Noise Ratio (SNR). Assuming no other link is active on the network simultaneously, the necessary condition for the existence of direct link  $e_{ii}^{ii}$  is

$$e_{ij}^{\omega} \in E \Leftrightarrow \exists P_{ij}^{\omega} \in [0, P_{\max}] \mid P_{ij}^{\omega} G_{ij} / \eta_j \ge \gamma (\rho_1) \\ ; \omega \in \{ ch_i \cap ch_j \}, \forall i, j \in V , i \neq j$$

$$(2)$$

in which  $\eta_j$  is the noise power in the *j*th receiver often set as the constant value  $N_0$ .  $P_{ij}^{\omega}$  indicates the transmission power from node *i* to node *j* over the channel  $\omega$  and  $G_{ij} = (1/d_{ij})^{\varepsilon}$  is the propagation gain resulting from the effective power loss[13]. In addition,  $\varepsilon$  is a parameter dependent on the shadow and fading phenomena and  $d_{ij}$  is the geometrical distance between nodes *i* and *j*. In the above relation,  $\gamma(\rho_1)$  is the threshold corresponding to the minimum rate. This threshold is dependent on the transmission rate and service quality requirement such that  $\rho_x \prec \rho_y \rightarrow \gamma(\rho_x) \prec \gamma(\rho_y)$ .

In general, to have a successful reception at the link  $e_{ij}^{\omega}$  with rate $\rho$ , the relation  $SINR_{ij}^{\omega} \ge \gamma(\rho)$  must be held in which  $SINR_{ij}^{\omega}$  indicates the Signal to Interference and Noise Ratio (SINR) in the *j*th receiver, which is

$$SINR_{ij}^{\omega} = \frac{p_{ij}^{\omega}G_{ij}}{N_0 + \sum\limits_{\forall e_{mn}^{\omega} \in \mathbb{N} \setminus e_{ij}^{\omega}} X_{ij,mn}^{\omega} p_{mn}^{\omega} G_{mj}}$$
(3)

The second term of the denominator represents the interference resulting from the simultaneous transmission of other links with the transmission from *i* to *j* over the frequency channel  $\omega$ . In this term,  $X_{ii,mn}^{\omega}$  is defined as

$$X_{ij,mn}^{\omega} = \begin{cases} 1 & ; if \ e_{ij}^{\omega} \ and \ e_{mn}^{\omega} \ simultaneously \ transmit\\ 0 & ; otherwise \end{cases}$$
(4)

The protocol interference model is simple, but it cannot model the real conditions of the network. Conversely, (3) indicates that for physical interference model, we must calculate the interference resulting from all links with the similar frequency channel which is time consuming for large-scale networks. In this paper, we propose a hybrid interference model based on the two described models. In the proposed model, we exploit the physical interference model within the interference range, whereas no interference is assumed in the out. In this manner, we define the interference of link  $e_{mn}^{\omega}$  on  $e_{ii}^{\omega}$  in the common frequency channel  $\omega$  as

$$I_{ij,mn}^{\omega} = \begin{cases} P_{mn}^{\omega} G_{mj} & ; if \ d_{mj} \le r_i^m, X_{ij,mn}^{\omega} = 1\\ 0 & ; otherwise \end{cases}$$
(5)

Now, we modify the definition of SINR presented in (3) as

$$SINR_{ij}^{\omega} = \frac{p_{ij}^{\omega}G_{ij}}{N_0 + \sum\limits_{\forall e_{mn}^{\omega} \in E \setminus e_{ij}^{\omega}} I_{ij,mn}^{\omega}}$$
(6)

# 3.3. Problem description and motivation

In this paper, we consider a multi-radio multi-channel WMN, in which an available frequency channel must be assigned to each radio interface. This problem is called Channel Assignment. Here, we consider the static channel assignment in which no channel switching is allowed. We assume different non-overlapping channels are assigned to the radio interfaces of each node in order to reach the maximum efficiency. Therefore, we have

$$\boldsymbol{ch}_{i} = \left\{ \boldsymbol{\omega}_{i}^{1}, \, \boldsymbol{\omega}_{i}^{2}, \, ..., \, \boldsymbol{\omega}_{i}^{ln_{i}} \middle| \begin{array}{l} \boldsymbol{\omega}_{i}^{\chi} \in \boldsymbol{\Omega}; \, \forall \boldsymbol{x}, \\ \boldsymbol{\omega}_{i}^{\kappa} \neq \boldsymbol{\omega}_{i}^{h}; \, \forall \kappa \neq h \end{array} \right\} \quad ; \forall i \in \boldsymbol{V}$$
(7)

in which  $\omega_i^x$  is the frequency channel assigned to the *x*th interface of ith node. With this assumption, each node can send/receive packets on all of its network interfaces, simultaneously. In a MR-MC WMN, there may be more than one common channel between each two adjacent nodes after the assignment of the frequency channels to the radio interfaces. In this case, the frequency channel selection is used as a tool to select a proper channel from all the available channels between two nodes considering some criteria. After channel assignment and channel selection, the network graph G = (V, E) can be specified using (2).

Although using several non-overlapping channels reduces the interference between links considerably as compared to the singlechannel ones, but the interference still exists between links as a result of limited number of non-overlapping channels. In wireless backbone networks, a contention-free Media Access Control (MAC) protocol must be used to prevent the collisions due to the interference between links. In this paper, inspired from [5,6,9,10], we use a media access control protocol based on time scheduling, in which each time frame is divided into  $T_{max}$  time slots and the interfering links operate on different time slots. In this way, scheduling is a tool that assigns a proper time slot to each active link.

As previously mentioned, the robustness against failures is one of the major requirements for WMNs. In order to satisfy this requirement, the network graph G = (V, E) must be K-Connected [16,18]. In other words, the connection number of the network graph must be K. Menger theory [21] states that the connection number of a graph is equal to K, if and only if there exist K nodedisjoint paths between each two nodes. The necessary condition for a graph to be K-Connected is that the minimum degree of the graph must be *K* [22]. To define the minimum degree of a graph, first we must define the degree of a node. The degree of node *i* in graph *G* represented by *deg*<sub>i</sub> is the number of links connected to *i*. In other words, the degree of a node is the number of its neighbors. If we define  $X_{ii}$  as

$$X_{ij} = \begin{cases} 1 & ; if \ e_{ij}^{\omega} \in E \ \forall \omega \in \left\{ \boldsymbol{ch}_i \cap \boldsymbol{ch}_j \right\} \\ 0 & ; otherwise \end{cases}$$
(8)

then, the degree of node *i* is defined as

$$deg_i = \sum_{j \in V \setminus i} X_{ij} \quad ; \forall i \in \mathbf{V}$$
(9)

Using (9), the minimum degree of the graph is defined by

$$deg_G^{min} = Min_{\forall i \in V} (deg_i) \tag{10}$$

In [20], Penrose proved a probabilistic relation between the connection number and the minimum degree of a geometricrandom graph. According to the theory presented by Penrose, if the minimum degree of a geometric-random graph is K, the network is K-Connected with high probability.

$$P(G \text{ is } K - Connected) \cong P(deg_G^{min} \ge K)$$

$$(11)$$

By definition, G is a geometric graph if the vertices are independent and are uniformly distributed in the area. Conversely, G is random if the probability of existence of a link between two nodes is determined by the distance between them. Considering the proposed hybrid interference model, the network graph of the wireless mesh network, is a geometric-random graph. Therefore, Eq. (11) is applicable in our scenario.

According to the above definitions, the fault-tolerant interference-aware topology control can be defined as an optimization problem in which the objectives of mitigating interference, maximizing throughput and preserving the K-connectivity must be achieved using power control, rate adaptation, routing, scheduling, channel assignment and selection. This optimization problem is NP-hard. Therefore, we introduce a heuristic practical solution. Simulation results will demonstrate the efficiency of the proposed algorithm in comparison to the other state of the art solutions.

# 4. The proposed fault-tolerant interference-aware topology control (FITC) algorithm

The proposed algorithm offers a practical solution for network topology control and it comprises two parts. In the first part, the network graph is formed and it is modified to achieve the K-Connectivity feature, if necessary. Since this part is performed only once in the network development phase, the proposed algorithm is implemented in a centralized manner to achieve the best performance. In the second part, we focus on the throughput maximization by minimizing the interference between nodes during the data transmission. We perform this procedure in a distributed manner to decrease the control packets overhead.

# 4.1. First part – network formation and modification algorithm (NFMA)

This part consists of three steps. In the first two steps, we do not consider the multi-channel property of the network. In these steps, we assume a common single channel is assigned to all radio interfaces. In the last step, we modify the network graph by considering the multi-channel scenario and by assigning the channels to the radio interfaces.

At first, the primary network graph is formed using minimum power adjustment. Then, by increasing the minimum power and adding nodes, the network graph is modified to become K-Connected. In the second step, the paths are prioritized using an efficient algorithm and the K best paths between each two nodes is selected. Finally, in the third step, the channel assignment is performed such that the potential interference is reduced while the K-Connectivity feature is preserved.

#### 4.1.1. First step – construction of K-Connected network topology

In this step, we try to have a network graph with minimum degree of K in accordance to Penrose theory. Here, we assume that a single common channel is assigned to all radio interfaces. The following equation can be used for determining the required minimum power for node *i*:

$$P_i^{\min} = \min\left(P \in [0, P^{\max}] \mid \deg_i^{worst} \ge K\right) \; ; i = 1, ..., n \tag{12}$$

In this equation,  $deg_i^{worst}$  is the degree of node *i* in the worst case. This is because we need to preserve the minimum degree K in the worst case. If this condition is not satisfied, we consider the value of  $P_i^{\min}$  equal to  $P^{\max}$ . In this way, we define  $X_{ii}^{worst}$  as

$$X_{ij}^{worst} = \begin{cases} 1 & ; if \quad P_{ij}^{\omega}G_{ij}/N_0 \ge \gamma\left(\rho_M\right) \\ 0 & ; otherwise \end{cases}$$
(13)

In (13), we used $\gamma(\rho_M)$ , that is the threshold corresponding to the maximum rate. This is because we need to preserve the K-Connectivity in the worst case. The amount of  $deg_i^{worst}$  can be obtained by the replacement of  $X_{ij}^{worst}$  instead of  $X_{ij}$  in (9). According to the values obtained from (12), the minimum power required for the network to be K-Connected is *P*<sup>min</sup>

$$= Max_{\forall i \in V} \left( P_i^{\min} \right) \tag{14}$$

6:

Pseudo code #1

1: Inputs- network graph G (based on the minimum required
power obtained from (14))
2: Outputs- all disjoint path between all pairs
3: <b>for</b> <i>u</i> in G
4: for $v$ in G except $u$
5: $G_t = (V_t, E_t)$ ; $G_t = G$
6: $l = 0;$
7: while $u$ can reach $v$ do
8: find shortest path from $u$ to $v$ with BFS algorithm;
9: $l = l + 1;$
10: save all vertices on the path from $u$ to $v$ in $Path_{w, i}$ ;
11: $V_t = V_t \setminus Path_{w_t};$
12. $E_t = E_t$ \ all links that are connected to vertices of
$Path_{w,l}$
13: end;
14 $NoP_{uv} = l;$
15: <b>end;</b>
16: <b>end;</b>
17: if $\min_{\forall u, v \in V} (NoP_{uv}) \ge K$ then
18: Graph is K-Connected;
19: end;

Fig. 2. Pseudo code for checking the *K*-Connectivity of the network graph.

By assigning a common channel to all interfaces and by assigning the minimum required transmission powerPmin to all nodes, the primary network graph is obtained. According to Penrose theory, this graph is K-Connected with high probability. As stated earlier, for a K-Connected graph, there must be at least K nodedisjoint paths between each two nodes. We examine this requirement by finding all of the node-disjoint paths between each node pair in the network graph. For this purpose, the pseudo code presented in Fig. 2 that is based on Breadth First Search (BFS) is suggested. In this pseudo code,  $Path_{uv,l}$  is the set of vertices in the *l*th path between nodes u and v. Likewise,  $NoP_{uv}$  is the number of paths between nodes *u* and *v*.

There are three sets of node pairs in the output of the algorithm named E<sub>EK</sub>, E<sub>GK</sub> and E<sub>LK</sub>, which are set of node pairs with Equal, Greater and Less than K disjoint paths between them. The number of components in  $E_{LK}$  is  $2 \times {\binom{n}{2}} - |E_{EK}| - |E_{GK}|$ . According to the Penrose theory, a few number of node pairs are in this set. If the  $E_{LK}$  is not null, we have to modify the network graph to obtain a K-Connected graph. One strategy to solve this problem is to increase the minimum transmission power obtained in first step. Since the transmission power has a maximum value Pmax, this strategy may not be applicable. Furthermore, increasing the transmission power of all nodes decreases the potential throughput of the network due to the increasing interference. The second strategy is adding new disjoint paths between the pairs of  $E_{LK}$  by increasing the corresponding transmission power or by placement of new nodes in appropriate locations. This problem is solved in a centralized manner in which the decision maker node recognizes the position information of all nodes. The detail of this strategy is described below.

The algorithm of adding new disjoint paths between the node pairs of  $E_{LK}$  starts with the node pair with the highest number of disjoint paths. We denote this node pair by u and v and the number of disjoint path between them byK'. It is evident that K-K'disjoint paths must be constructed between u and v. For constructing the first path, all the nodes and their connected links that lies on the available K'disjoint paths must be removed from the network graph. Now, we select a node that is nearest to one of the nodes u or v and has at least a disjoint path to other node v or

Pse	eudo code #2
1: I	<b>nputs</b> - all node pairs in $E_{LK}$
2: <b>v</b>	<b>vhile</b> ( $E_{LK}$ is not null )
3:	Select the node pair $u$ and $v$ with the maximum number of disjoint paths;
4:	Denote the number of path between the given nodes with $K'$ ;
5:	Remove all nodes and links that lie on available $K'$ paths from the graph;
6:	while $(K' < K)$
7:	Select the nearest neighbors to nodes $u$ and $v$ with at least a

- v with at least a 7: disjoint path to other node v or u and represent them with xand y, respectively.
- 8: Denote the distance of these nodes to u and v with d', d''respectively;

$$9: \quad \text{if} (d' < d'') (u_1 = u, v_1 = v, x_1 = x, d_1 = d')$$

else  $(u_1 = v, v_1 = u, x_1 = v, d_1 = d'')$ 10:

- 11: end:
- Increase transmission power of  $u_1$  up to  $P^{\max}$ ; 12:
- 13: if (the connection between  $u_1$  and  $x_1$  is established)
- 14: no operation;
- 15: else
- Add  $N^* = \left[ d_1 / d^* \right]$  new equal-distance nodes between 16:  $u_1$  and  $x_1$  where  $d^*$  denotes the maximum transmission range obtained from (15);
- 17: end
- 18: K' = K' + 1;
- 19: Remove all nodes and links that lie on the new path from the graph;
- 20: end
- 21: Eliminate the node pair (u,v) from  $E_{LK}$ ;
- 22: Update the number of paths between node pairs of  $E_{LK}$ ; 23: end

Fig. 3. Pseudo code for topology modification in order to achieve the K-Connected graph.

*u*. Without loss of generality, we assume that the selected node *x* is close to *u*, its distance to *u* is *d*'and has at least a disjoint path to *v*. We must establish a physical connection between *u* and *x*. If this can be done by increasing the transmission power of *u*, then there is no need for the addition of any other node; otherwise, we must add  $N^* = \lfloor d'/d^* \rfloor$  new equal-distance nodes between *u* and *x* where  $d^*$  represents the maximum transmission range that is achievable when the transmission power is maximum and the transmission rate is maximum. This parameter can be obtained from the following equation;

$$\frac{P^{\max}G}{N_0} \ge \gamma\left(\rho_M\right) \xrightarrow{G = \left(\frac{1}{d}\right)^{\epsilon}} \left(\frac{1}{d}\right)^{\epsilon} \ge \frac{\gamma\left(\rho_M\right)N_0}{P^{\max}}$$
$$\xrightarrow{d^* = \max\left(d\right)} d^* = \sqrt[\epsilon]{\frac{P^{\max}}{\gamma\left(\rho_M\right)N_0}}$$
(15)

In the above equation, the interference from other transmissions is ignored for simplicity; therefore, in practice,  $d^*/2$  can be used instead of *d*\*as a rule of thumb in order to have the safety margin.

The above procedure is repeated K-K'times in order to establish the required disjoint paths between u and v. Now, this node pair is removed from  $E_{LK}$  and the number of disjoint paths between other node pairs of  $E_{LK}$  must be updated. Then, the above procedure is repeated in order to modify the number of disjoint paths between other node pairs of  $E_{LK}$  until this set is empty. Fig. 3 represents the pseudo code of the algorithm.

# 4.1.2. Second step – selection and ordering of K best paths

In this step, we select K best available paths between node pairs, and prioritize them based on some criteria defined later. We use the best path for traffic routing in the second part of the proposed algorithm. Due to the *K*-Connectivity feature of the network, it has the capability to recover failure in the case of fault occurrence. Several different recovery algorithms that are beyond the scope of this paper have been proposed in literature. In order to select and prioritize the paths, we propose an efficient cost function to minimize the interference and to improve the load balancing. To this end, three metrics based on Min-Max and Min-Sum techniques are proposed and the linear combination of their normalized values is used as a cost function for path selection. The three proposed metrics are: hop count number, transmission power, load balancing.

# A. Hop count number

Paths with fewer hop counts, cause less delay and less resources consumption and therefore result in greater throughput. The number of hop counts along the *l*th path between the nodes u and v is represented as

$$H_{uv,l} = | \boldsymbol{Path}_{uv,l} | -1 \qquad ; \forall u, v \in \boldsymbol{V}, l = 1, ..., NoP_{uv}$$
(16)

in which  $NoP_{uv}$  is equal or greater than *K*. Moreover,  $|Path_{uv,l}|$  is the number of nodes in the *l*th path between nodes *u* and *v*. For each pair *u* and *v*, this metric can be normalized as

$$\hat{H}_{u\nu,l} = \frac{H_{u\nu,l}}{Max_{\forall l' \in \{1,2,...,NoP_{u\nu}\}} (H_{u\nu,l'})} \quad ; \forall u, \nu \in \mathbf{V}, l = 1, ..., NoP_{u\nu}$$
(17)

## **B.** Transmission power

High transmission power causes a considerable amount of interference on adjacent nodes. Maximum transmission power from node u to node v on the *l*th path is defined as

$$P_{u\nu,l}^{max} = Max(P_{ij}^{\omega}) ; \forall u, \nu \in \mathbf{V}, \forall i, j \in \mathbf{Path}_{u\nu,l}, l = 1, ..., NoP_{u\nu}$$
(18)

in which  $P_{ij}^{\omega}$  that is the minimum required transmission power from *i* to *j* along the *l*th path corresponding to the maximum rate, can be calculated by

$$P_{ij}^{\omega} = \gamma(\rho_M) N_0 d_{ij}^{\varepsilon} \qquad ; \forall i, j \in \boldsymbol{Path}_{uv,l}$$
(19)

This metric can be normalized as follows:

$$\hat{P}_{uv,l}^{max} = \frac{P_{uv,l}^{max}}{Max_{\forall l' \in \{1,2,...,NoP_{uv}\}}(P_{uv,l'}^{max})} ; \forall u, v \in \mathbf{V}, l = 1, ..., NoP_{uv}$$
(20)

Conversely, the reduction of total power consumption along the path reduces the interference and prolongs the lifetime of the network (in battery-powered devices). This metric is defined and normalized using the following equations, respectively.

$$P_{uv,l}^{sum} = \sum_{\forall e_{ij}^{\omega} \in \textbf{Path}_{uv}} P_{ij}^{\omega} \qquad ; \forall u, v \in \textbf{V}, l = 1, ..., NoP_{uv}$$
(21)

$$\hat{P}_{u\nu,l}^{sum} = \frac{P_{u\nu,l}^{sum}}{Max_{\forall l' \in \{1,2,...,NoP_{u\nu}\}} \left(P_{u\nu,l'}^{sum}\right)} \qquad ; \forall u, \nu \in \mathbf{V}, l = 1, ..., NoP_{u\nu}$$
(22)

By combining the parameters defined in (20) and (22), the transmission power metric is defined as

$$\hat{P}_{uv,l} = \frac{\hat{P}_{uv,l}^{max}}{2} + \frac{\hat{P}_{uv,l}^{sum}}{2} \qquad ; \forall u, v \in \mathbf{V}, l = 1, ..., NoP_{uv}$$
(23)

# C. Load balancing

In order to consider the load balancing in the procedure of path selection, we choose the paths that their vertices are less used on other paths. For node  $n_x$  on path **Path**<sub>uv,l</sub>, if we represent the number of its usage on other paths by  $B_{uv,l}^{n_x}$ , then we have</sub>

$$B_{uv,l}^{n_x} = \sum_{\forall i,j \in \mathbf{V} \setminus u, \nu} \sum_{h=1}^{NoP_{ij}} X_{ij,h}^{n_x} \quad ; \forall u, \nu \in \mathbf{V}, n_x \in \mathbf{Path}_{uv,l}$$
(24)

in which the variable  $X_{ij,h}^{n_x}$  is equal to 1 if the node  $n_x$  is on the *h*th path between nodes *i* and *j*. By this definition, the maximum value of  $B_{uv,l}^{n_x}$  for all nodes on the *l*th path between nodes *u* and *v* must be minimized. Therefore, the normalized value is

$$\hat{B}_{uv,l}^{max} = \frac{Max_{\forall n_x \in \boldsymbol{path}_{uv,l}}(B_{uv,l}^{n_x})}{Max_{\forall l' \in \{1,2,\dots,NoP_{uv}\}}\left(Max_{\forall n_x \in \boldsymbol{path}_{uv,l'}}(B_{uv,l'}^{n_x})\right)}; \forall u, v \in \boldsymbol{V}, l = 1, \dots, NoP_{uv}$$
(25)

Conversely, the total usage of all nodes on one path should be minimized. The corresponding parameter and its normalized value are defined by the following equations, respectively,

$$B_{uv,l}^{sum} = \sum_{\forall n_x \in path_{uv,l}} B_{uv,l}^{n_x} \qquad ; \forall u, v \in V, l = 1, ..., NoP_{uv}$$
(26)

$$\hat{B}_{uv,l}^{sum} = \frac{B_{uv,l}^{sum}}{Max_{\forall l' \in \{1,2,...,NoP_{uv}\}} (B_{uv,l'}^{sum})} \qquad ; \forall u, v \in \mathbf{V}, l = 1, ..., NoP_{uv}$$
(27)

By combining the parameters defined in (25) and (27), the load balancing metric is defined as

$$\hat{B}_{uv,l} = \frac{\hat{B}_{uv,l}^{max}}{2} + \frac{\hat{B}_{uv,l}^{sum}}{2} \qquad ; \forall u, v \in \mathbf{V}, l = 1, ..., NoP_{uv}$$
(28)

In this paper, we use the linear combination of all defined metrics as the Routing Cost Function (RCF) for path selection:

$$RCF_{uv,l} = \alpha_1 \hat{H}_{uv,l} + \alpha_2 \hat{P}_{uv,l} + \alpha_3 \hat{B}_{uv,l} , \quad \alpha_1 + \alpha_2 + \alpha_3 = 1 ; \forall u, v \in \mathbf{V}, l = 1, 2, ..., NoP_{uv}$$
(29)

In the above equation, the effect of each metric can be controlled by changing the values of the coefficients  $\alpha_i$ ; i = 1, ..., 3. In this step, using the Routing Cost Function defined in (29), *K* paths with the lower cost are selected between each two nodes. Fig. 4 reveals the path selection pseudo code which is based on the cost function*RCF*<sub>*uv*,*l*</sub>. In the pseudo code presented in Fig. 4,  $\Gamma_{uv}^{FTL}$  is the set of all links of the *K* selected paths between two nodes *u* and *v*. By having  $\Gamma_{uv}^{FTL}$  for all node pairs of the network,  $\Gamma^{FTL}$  is defined as follow:

$$\boldsymbol{\Gamma}^{FTL} = \left\{ \boldsymbol{\Gamma}_{uv}^{FTL} | \forall u, v \in \boldsymbol{V} \right\}$$
(30)

In the selection of *K* best disjoint paths between any pair of nodes, all available disjoint paths must be sorted using objective function (29), and then the best *K* paths must be selected. The maximum number of disjoint paths between each node pair *u* and *v* denoted by*NOP*<sub>*uv*</sub> is equal to the minimum degree of these two nodes. We show the maximum number of paths between all node pairs with *K*'. In other words,  $K' = \max_{\forall (u, v)} (NOP_{uv})$ . If we use the Merg-sort algorithm [23] for sorting paths between each node pair, the computational and memory complexities are equal to  $O(K'\log K')$ andO(K'), respectively [23]. Since the algorithm is repeated for all node pairs of the graph, the total computational and memory complexities are  $O((K'\log K')|V|(|V|-1))$  and O(K'|V|(|V|-1)), respectively; where |V| is the number of nodes and |V|(|V|-1) is the number of node pairs. The maximum amount of

Pseudo code #3

1: Inputs- all disjoint paths between all nodes in graph:
$Path_{uv,l}$ ; $\forall u, v \in V, l = 1, 2,, NoP_{uv}$
2: <b>Outputs-</b> <i>K</i> best path between any pair of nodes
3: for u in V
4: for v in V except u
5: <b>for</b> l=1 to $NoP_{uv}$
6: Calculate $\hat{H}_{uv,l}$ based on (17);
7: Calculate $\hat{P}_{uv,l}$ based on (23);
8: Calculate $\hat{B}_{uv,l}$ based on (28);
9: Calculate cost function $RCF_{uv,l}$ ;
10: end;
11: end;
12: end;
13: Sort all paths between all pairs of the nodes $u, v \in V$ .
14: Select <i>K</i> best paths with minimum cost for all node
15: Save all connections $(i, j)$ belong to all paths between u and
$v  ext{ in } \mathbf{\Gamma}_{uv}^{FTL}$ .



*K* is obtained when the network is full mesh connected. In this case,  $|\mathbf{E}| = |\mathbf{V}|(|\mathbf{V}| - 1)$  and  $K = |\mathbf{V}| - 1$ ; therefore, the computational and memory complexities can be written as  $O(|\mathbf{V}|^3 \log |\mathbf{V}|)$  and  $O(|\mathbf{V}|^3)$ , respectively.

Now we analyze the computational complexity of the algorithm for finding all disjoint paths between nodes presented in Fig. 2. This algorithm is based on the Breadth First Search (BFS) method for finding the shortest path. The computational complexity of finding the shortest path between the two nodes *u* and *v* using BFS algorithm is O(|V| + |E|) in which |V| and |E| represent the dimension of Vand *E*, respectively [23]. Consequently, the computational complexity of the given problem is O(K'(|V| + |E|)), where *K* is the maximum number of paths between all node pairs as earlier defined. As previously stated, the maximum of |E| is |V|(|V| - 1) and the maximum of *K* is |V| - 1; therefore, the computational complexity of finding all disjoint paths between two nodes is equal to  $O(|V|^3)$ . The number of node pairs in the network is equal to |V|(|V| - 1), therefore the total complexity of finding all disjoint paths between all node pairs is  $O(|V|^5)$ .

# 4.1.3. Third step - channel assignment

In the previous two steps, assuming a single common channel is assigned to nodes, we modified the network topology using available tools in order to be *K*-Connected and then the *K* best paths is selected for each node pair. All of the connections in  $\Gamma^{FTL}$  must be preserved during the process of channel assignment in order to maintain the *K*-Connectivity feature of the network. In this section, to utilize the available non-overlapping channels, two different channel assignment methods known as Random Common Channel Assignment (RC-CA) and Interference Aware Channel Assignment (IA-CA) methods are proposed.

# 4.1.3.1. Random-common channel assignment (RC-CA) method

We assign a common frequency channel to one of the radio interfaces of each node in order to meet the *K*-Connectivity feature in the RC-CA method. For the remaining radio interfaces, we assign channels randomly such that different radios on each node have different channels. Therefore, for each node  $i \in \mathbf{V}$  and interface  $\kappa$  of this node, the assigned channels  $\omega_i^{\kappa} \in c\mathbf{h}_i$  is obtained as follows:

$$\omega_{i}^{\kappa} = \begin{cases} \omega_{1} & ; if \ \kappa = 1 \\ Srand(\mathbf{\Omega} \setminus \omega_{1}) & ; otherwise \end{cases} ; \forall i \in \mathbf{V}$$
(31)

# Pseudo code #4

1: Inputs- all links in  $\Gamma^{FTL}$ 2: **Outputs-**  $\omega_i^{\kappa}$ ;  $\forall i \in V, \forall \kappa \in \{1, 2, ..., In_i\}$ 3: Step 1. Prioritize links in  $\Gamma^{FTL}$  based on potential interference for h = 1 to  $|\mathbf{\Gamma}^{FTL}|_{SumMax}$ Calculate  $I_{ij}^{SumMax}$  for  $\forall (i, j) \in \mathbf{\Gamma}^{FTL}$ ; 4: 5: 6: end; Sort links based on descending order of  $I_{ij}^{SumMax}$  in  $\Gamma^{LLA}$  set 7: 8: Step 2. Channel assignment to all connections in for h = 1 to  $|\Gamma^{LLA}|$ 9: if  $|ch_i| \prec In_i, |ch_i| \prec In_i, ch_i \cap ch_i = \varphi$ 10: A channel with minimum interference to be allocated 11: to Each end node based on (36); else if  $|ch_i| \prec In_i, |ch_i| = In_i, ch_i \cap ch_i = \varphi$ 12: The channel with minimum interference in  $ch_i$ 13: assigned to node i based on (39); 14: else if  $|ch_i| = In_i$ ,  $|ch_j| = In_j$ ,  $ch_i \cap ch_j = \varphi$ 15: Replacing the worst channel allocated to the node i(or j)with the best node channel from the node j( or i). 16: Call Modify\_Channel\_Assignment() for each node  $m \in V$  that  $\omega_j^{maxInt} \in ch_m$  (or  $\omega_j^{maxInt} \in ch_m$ );

- 17: end:
- 18: end;
- 19: Call Remain Channel Assignment();

in which  $k_1$  is the common channel assigned to the first radio interface of each node. Also, the *Srand* function assigns channels randomly to other interfaces of the nodes (except interface 1) so that a different channel is assigned to each interface.

# 4.1.3.2. The interference-aware channel assignment (IA-CA) method

RC-CA channel assignment method is simple, but its performance is unacceptable from the interference point of view. The main objective of IA-CA method is minimizing the potential interference. Although the complexity of IA-CA is much more than RC-CA, but it is an insignificant factor, since the channel assignment is static and is performed only once in the network development stage. IA-CA method is an iterative algorithm, in which, at each iteration, one link is selected and the channel will be assigned to it. In this way, we must prioritize the links and select the link with the most priority at each iteration. Here, the link with highest potential interference has the highest priority for channel assignment. Furthermore, at each iteration, we consider the interference range and the channels with minimum usage at the range assigned to this link. Therefore, we consider the balancing in using the frequency channels. Fig. 5 represents the pseudo code for IA-CA algorithm. The algorithm is composed of two phases: prioritization and channel assignment.

# A. Prioritization

The prioritization is done such that the link with higher potential interference has higher priority for channel assignment. The hybrid interference model introduced in Sections 3 and 2 is used for the measurement of the potential interference, such that the potential interference is calculated inside interference range according to the physical interference model (Eqs. (4) and (5)), while in the outside, it is assumed zero. In order to consider the worst case, during the calculation, we assume maximum transmission power; therefore, the maximum potential interference on link (i,

Fig. 5. Pseudo code for interference-aware channel assignment.

*j*) resulting from (*m*, *n*) is:

$$I_{ij,mn}^{\max} = \begin{cases} P^{\max}G_{mj} & ; if \ d_{mj} \le r_i^m \\ 0 & ; otherwise \end{cases}$$
(32)

Also, the total amount of potential interference on each link (i, j) of  $\mathbf{\Gamma}^{FTL}$  set is:

$$I_{ij}^{sumMax} = \sum_{\forall (m,n) \in \mathbf{\Gamma}^{FTL} \setminus (i,j)} I_{ij,mn}^{\max} \quad ; \forall (i,j) \in \mathbf{\Gamma}^{FTL}$$
(33)

Now, we define the set  $\Gamma^{LIA}$  in which its members are links sorted in descending order with regard to the  $I_{ij}^{SumMax}$ . In the second phase, the channels are assigned to the nodes corresponding to the links in order of preference.

# **B.** Channel assignment

In channel assignment to interfaces of  $\forall (i, j) \in \Gamma^{LIA}$  in order of preference, we consider the following reasonable assumptions:

- Two radio interfaces at the end of each link must have similar channel.
- In each node, different channels must be assigned to different radio interfaces.

The proposed algorithm selects the channel in which the amount of potential interference (resulting from the interfering transmissions based on the hybrid interference model) is minimal. To this end, we define  $I_j^{\omega}$ , which represents the potential interference on the receiver of link (*i*, *j*) assuming that the channel  $\omega$  is assigned to this link,

$$I_{j}^{\omega} = \sum_{\forall (m,n) \in \Gamma^{\sqcup A} \setminus (i,j)} I_{ij,mn}^{\omega,\max} \; ; \forall \omega \in \mathbf{\Omega}$$
(34)

Similar to (33), we again consider the worst case. Here  $I_{ij,mn}^{\omega,max}$  is defined as

$$I_{ij,mn}^{\omega,\max} = \begin{cases} P^{\max}G_{mj} & ; if \ d_{mj} \le r_i^m \\ 0 & ; otherwise \end{cases}$$
(35)

In channel assignment to radio interfaces of a node, any of the three following cases may occur.

*Case* 1- if  $|ch_i| < ln_i$ ,  $|ch_j| < ln_j$ ,  $ch_i \cap ch_j = \phi$  which means the end nodes of the selected link do not have any common channel and empty radio interfaces (without channel assignment) are available in these two nodes. In this situation, the channel assignment to the  $\kappa$ *th*interface of node *i* and *h*th interface of node *j* is performed according to

$$\omega_{i}^{\kappa} = \omega_{j}^{h} = \arg \min_{\omega' \in \Omega - \{ch_{i} \cup ch_{j}\}} \left( I_{i}^{\omega'} + I_{j}^{\omega'} \right)$$
(36)

According to the above equation, the channel which minimizes the total potential interference on both nodes i and j is selected for assignment to the interfaces. This strategy implies that the channel with minimum potential interference is assigned to the nodes i and j.

*Case* 2- if  $|ch_i| < ln_i$ ,  $|ch_j| = ln_j$ ,  $ch_i \cap ch_j = \phi$  (or similarly  $|ch_i| = ln_i$ ,  $|ch_j| < ln_j$ ,  $|ch_j| < ln_j$ ,  $ch_i \cap ch_j = \phi$ ) which implies that the two end nodes do not have any common channel and empty radio interfaces (without channel assignment) is available only in one of them. In this situation, we assign the best channel with the minimum interference on node *j* (or *i*) to an empty radio interface of node *i* (or *j*):

$$\omega_{i}^{\kappa} = \arg \min_{\omega' \in ch_{j}} \left( I_{j}^{\omega'} \right) \left\langle or \ \omega_{j}^{h} = \arg \min_{\omega' \in ch_{i}} \left( I_{i}^{\omega'} \right) \right\rangle$$
(37)

*Case* 3- if  $|ch_i| = ln_i$ ,  $|ch_j| = ln_j$ ,  $ch_i \cap ch_j = \phi$ . In this condition, for each node we must determine the channel with the maximum potential interference among the channels assigned to its radio interfaces, which can be written as follow

$$\omega^{MaxInt}{}_{i} = \arg Max \left( I_{i}^{\omega'} \right) \tag{38}$$

$$\omega^{\text{MaxInt}}{}_{i} = \arg_{\omega' \in ch_{i}}^{} Max \left( I_{j}^{\omega'} \right)$$
(38)

Without loss of generality, we assume  $I_j^{\omega_j^{maxInt}} \prec I_i^{\omega_i^{maxInt}}$ . The assignment is performed by replacing the worst channel ( $\omega_i^{maxInt}$ ) with the best channel assigned to node *j*. This can cause the disconnection of some links, which connect to node *i* through $\omega_i^{maxInt}$ . In this situation, if a common channel does not exist between these two nodes, the assigned channel to these links must be replaced with the new one. This phase is performed repeatedly. The algorithm is stopped if at least one common channel existed between the end nodes of each links of  $\Gamma^{LIA}$ set. Finally, if any node  $n \in \mathbf{V}$  with empty radio interface remained, the channel assignment process to these interfaces is performed according to

$$\omega_n^{rem} = \arg_{\omega' \in \mathbf{\Omega} \setminus \mathbf{ch}_n} Min \left( I_n^{\omega'} \right) \tag{39}$$

**Theorem 1.** The IA-CA algorithm preserves the *K*-Connectivity feature of the graph.

**Proof.** The input to the channel assignment algorithm is the *K*-Connected graph resulting from the second step. According to the IA-CA algorithm described earlier, the links of the input *K*-connected graph will remain connected after channel assignment. This implies that the network remains *K*-Connected after channel assignment.  $\Box$ 

After channel assignment, the first part of the proposed algorithm is completed and the network graph G is obtained in which K disjoint paths existed between each two nodes. This graph may be multi-graph, in which more than one frequency channel may exist between two nodes. An efficient method for throughput maximization is proposed in the second part of the algorithm.

# 4.2. Second part – throughput maximization along with preserving K-Connectivity (TMKC)

After creating the network graph and selecting the best path and channel assignment, we focus on throughput maximization. To this end, we propose a distributed method using the power control, rate adaptation, scheduling and channel selection. This method is known as Throughput Maximization along with preserving *K*-Connectivity (TMKC). Table 2 summarizes some notations that will be used in TMKC.

As mentioned in Section 3.1, we consider the traffic flows as end-to-end unicast sessions in this paper. The traffic demands are represented by matrix **TD** in which  $TD_{uv}$  represents the traffic volume between nodes *u* and *v*. The amount of traffic on the link between nodes *i* and *j* is

$$f_{ij} = \sum_{u \in \mathbf{V}} \sum_{\nu \in \mathbf{V} \setminus u} TD_{u\nu} X_{ij}^{u\nu}$$
(40)

in which  $X_{ii}^{uv}$  is defined as

$$X_{ij}^{uv} = \begin{cases} 1 & ; if(i,j) \in \Gamma_{uv}^{BR} \\ 0 & ; otherwise \end{cases}$$
(41)

where  $\Gamma_{uv}^{BR}$ ;  $\forall u, v \in V$  is the set of links on the best route selected between nodes u and v. After determining the amount of traffic on all links, the set  $\Gamma^{Act}$  is defined as

$$\boldsymbol{\Gamma}^{Act} = \left\{ e_{ij}^{\omega} \in \boldsymbol{E} \, \middle| \, f_{ij} \neq 0, \, \forall i, \, j \in \boldsymbol{V}, \, \forall \omega \in \boldsymbol{\Omega} \right\}$$
(42)

Table 2				
List of some	notations	used	in	TMKC.

Notation	Definition	Notation	Definition
$e_z^{\omega}$	Members of $\Gamma_{t,z}^{\omega,Act}$	$I_z^{\omega,mar}$	Interference margin for successful transmission in the $\Gamma^{\omega,Act}_{r,x}$ members.
$\Gamma^{Act}$	Set of links transferring the traffic load	$P_z^{\omega,\max}$	Maximum transmission power for transmitter of the added link in the <i>z</i> th iteration
$\Gamma_{t,z}^{\omega,Act}$	Set of all transmissions having the potential of activation in the given time slot $t$ at iteration $z$ and frequency channel $\omega$	$P_z^{\omega,\min}$	Minimum transmission power of new added link in the zth iteration
$\Gamma_t^{\omega,Act}$	Set of all transmissions having the potential of activation in the given time slot $t$ and frequency channel $\omega$	$P_z^{\omega}$	Transmission power for each added members to $\Gamma_{t,z}^{\omega,Act}$
$\Gamma^{\omega,Act}$	Set of links transmitting on frequency channel $\omega$	$NI_{ns}^{\omega,sum}$	Total potential interference in the receivers of non-scheduled neighbor transmissions on channel $\omega$
Gz	Propagation gain for each added members to $\Gamma^{\omega,Act}_{t,z}$	$I_{ns,q}^{\omega}$	Interference of added member in the <i>q</i> th iteration on non-scheduled neighbor transmissions on channel $\omega$
$G_{z,q}$	Propagation gain between transmitter of the $\Gamma_{t,z}^{\omega,Act}$ member and the receiver of the $\Gamma_{t,z}^{\omega,Act}$ members	$SINR_{ns}^{\omega}$	Amount of potential SINR in each transmission $e_{ns}^{\omega}$
$I^{\omega}_{z,q}$	Interference of the set $\Gamma_{t,q}^{\omega,Act}$ members on the members of $\Gamma_{t,z}^{\omega,Act}$ in the frequency channel $\omega$ .	$\sigma_c^2$	Variance of frequency channel utilization
$ ho_z^{\omega,\max}$	Maximum rate for each added members to $\Gamma_{t,z}^{\omega,Act}$ on frequency channel $\omega$	$U_c^{\omega}$	Amount of utilization for channel $\omega$
$\gamma(\rho_z^{\omega,\max})$	SINR threshold for each added members to $\Gamma_{l,z}^{\omega,Act}$ on frequency channel $\omega$	$\bar{U}_c$	Average utilization of all frequency channel
$P_z^{\omega,\max}(e_q^\omega)$	Power limitation for added members in the <i>z</i> th iteration so that the transmissions of the previously scheduled members in $\Gamma_{t,q}^{\omega,Act}$ don't disturb	$f_{ij,t}^{\omega}$	Amount of traffic on link $e_{ij}^{\omega}$ at time slot $t$

which represents the set of active links carrying the traffic load.

The transmissions on the two links with non-overlapping frequency channels have no interference. In order to carry out successful transmissions on similar frequency channels, the interference must be controlled using power control and rate adaptation. Different time slots must be used for transmissions if the simultaneously successful transmission on two or more links is not possible for any power and rate. Based on the hybrid interference model, TMKC selects the possible simultaneously transmissions using a heuristic method. Since the network topology after the channel assignment may be multi-graph, more than one frequency channel may be available for connecting two nodes. In this section, two different channel selection methods named Random Channel Selection(R-CS) and Interference Aware Channel Assignment (IA-CS) methods are proposed. TMKC with R-CS algorithm is described in Section 4.2.1. Section 4.2.2 enumerates the required modifications in order to use TMKC with IA-CS. TMKC algorithm runs repeatedly in each time slot. We represent the set of all transmissions having the potential of activation in the given time slot tand frequency channel  $\omega$  with  $\Gamma_t^{\omega,Act}$  which can be obtained by:

$$\Gamma_t^{\omega,Act} = \bigcup_h \Gamma_{t,h}^{\omega,Act} \quad ; \forall \omega \in \Omega$$
(43)

in which  $\bigcup$  represents the union of sets. In each execution round, a number of transmissions are added to the  $\Gamma_{t,h}^{\omega,Act}$  set, where *h* is the iteration index. In this manner, some power and rate constraints must be satisfied in order to preserve the *K*-Connectivity.

# 4.2.1. TMKC with R-CS

In *R*-*CS*, if more than one channel exist for connection between each nodes *i* and *j*, a frequency channel is randomly selected. In this case, the set  $\Gamma^{\omega,Act}$  is defined as:

$$\boldsymbol{\Gamma}^{\boldsymbol{\omega},\boldsymbol{Act}} = \left\{ \boldsymbol{e}_{ij}^{\boldsymbol{\omega}} \middle| \boldsymbol{e}_{ij}^{\boldsymbol{\omega}} \in \boldsymbol{\Gamma}^{\boldsymbol{Act}} \right\} \quad ; \forall \boldsymbol{\omega} \in \Omega \tag{44}$$

which shows the set of links performing transmission on frequency channel  $\omega$ . TMKC runs simultaneously for all  $\Gamma^{\omega,Act}$ ;  $\forall \omega \in \Omega$ . The compatible links must be scheduled at each iteration. Without loss of generality, it is assumed that only one link is added to  $\Gamma_{t,h}^{\omega,Act}$  on frequency channel  $\omega$  in each iteration. This is done solely for the simplification of the notations. The following constraints must be fulfilled in order to add a new member in the (h+1)th iteration to  $\Gamma_{t,h}^{\omega,Act}$ .

# A. Maximum power of added link

The maximum power of the added link must fulfill the SINR limitation determined in (45) so that the transmissions on the previously scheduled links in *h*th iteration of current time slot *t* are not disturb.

$$\frac{P_{z}^{\omega}G_{z}}{N_{0} + \sum_{q=1,2,...,h\backslash z} I_{z,q}^{\omega} + P_{h+1}^{\omega,\max}(e_{z}^{\omega})G_{h+1,z}} \geq \gamma\left(\rho_{z}^{\omega,\max}\right) \rightarrow P_{h+1}^{\omega,\max}(e_{z}^{\omega}) \leq \frac{P_{z}^{\omega}G_{z} - N_{0}\left(\gamma\left(\rho_{z}^{\max}\right) + \sum_{q=1,2,...,h\backslash z} I_{z,q}^{\omega}\right)}{G_{h+1,z} \times \gamma\left(\rho_{z}^{\omega,\max}\right)} P_{h+1}^{\omega,\max}(e_{z}^{\omega}) \leq \frac{I_{z}^{\omega,\max}}{G_{h+1,z}}; \forall z \in 1, 2, ..., h \qquad (45)$$

In this equation,  $I_z^{\omega,mar}$  represents the tolerable interference margin in the receiver of  $\Gamma_{t,z}^{\omega,Act}$ ; z = 1, ..., h members and  $e_z^{\omega}$ shows the added link in *z*th iteration. Moreover,  $P_z^{\omega}$ ,  $\gamma(\rho_z^{\omega,max})$ and  $G_z$  represent the power, SINR threshold (corresponding to the transmission with the maximum possible rate) and propagation gain between the receiver and transmitter node on the member added to  $\Gamma_{t,z}^{\omega,Act}$ ; z = 1, ..., h, respectively.

added to  $\Gamma_{t,z}^{\omega,Act}$ ; z = 1, ..., h, respectively. The propagation gain  $G_{h+1,z} = 1/(d_{h+1,z})^{\varepsilon}$  and the geometric distance  $d_{h+1,z}$  are calculated between the transmitter of new member added in (h+1)th iteration with the receivers of  $\Gamma_{t,z}^{\omega,Act}$ ; z = 1, ..., h members. In (45),  $I_{z,q}^{\omega}$  shows the interference imposed by  $\Gamma_{t,z}^{\omega,Act}$  members on the members of  $\Gamma_{t,z}^{\omega,Act}$  in the frequency channel  $\omega$ . The constraint (45) is applied to the links of  $\Gamma_{t,z}^{\omega,Act}$  in which their receivers lie in the interference range of the transmitter of added link, i.e.:  $d_{h+1,z} \prec r_i^{h+1}$ . Therefore, the maximum transmission power for the transmitter of the added link is calculated by

$$P_{h+1}^{\omega,\max} = Min_{z=1,\,2,\,\dots,\,h} \Big( P_{h+1}^{\omega,\max}(e_z^{\omega}) \Big) \tag{46}$$

# B. Maximum rate of added link

According to the maximum power resulted from (46), we can calculate the maximum transmission rate of the added members to  $\Gamma_{t,h+1}^{\omega,Act}$  by  $\rho_{h+1}^{\omega,\max} = \underset{\forall \rho \in \mathbb{R}}{\operatorname{arg}Max} \left( \frac{P_{h+1}^{\omega,\max}G_{h+1}}{N_0} \ge \gamma(\rho) \right)$ (47)

# C. Minimum power of added link

In order to preserve the *K*-Connectivity feature in the topology graph and for faster recovery in the event of node failure, each mesh router calculates the minimum power  $P^{\min}$  from (14). On the contrary, in normal conditions, for correct receiving at the added link to  $\Gamma_{t,h+1}^{\omega,Act}$ , the following relation must be held:

$$\frac{P_{h+1}^{\omega,\min} \times G_{h+1}}{N_0 + \sum\limits_{q=1,2,\dots,h} I_{h+1,q}^{\omega,\max}} \geq \gamma\left(\rho_{h+1}^{\omega,\max}\right) \rightarrow P_{h+1}^{\omega,\min} \geq \frac{\gamma\left(\rho_{h+1}^{\omega,\max}\right) \times \left(N_0 + \sum\limits_{q=1,2,\dots,h} I_{h+1,q}^{\omega,\max}\right)}{G_{h+1}}$$
(48)

Therefore, the minimum power for the transmitter of the added link is

$$P_{h+1}^{\omega,\min} = Max \left\{ P_{h+1}^{\omega,\min}, P^{\min} \right\}$$
(49)

From (46) and (49), the transmission power of added member to  $\Gamma_{t,h+1}^{\omega,Act}$  must be  $P_{h+1}^{\omega,\min} \prec P_{h+1}^{\omega} \prec P_{h+1}^{\omega,\max}$ . The left hand side of this equation is essential for preserving the *K*-Connectivity feature and the correct receiving at the added link. Conversely, the right hand side of this equation prevents creating destructive interferences on other active links of the set  $\Gamma_{t,z}^{\omega,Act}$ ; z = 1, ..., h. By selecting the transmission power as the average of  $P_{h+1}^{\omega,\min}$  and  $P_{h+1}^{\omega,\max}$ , the power is far enough from the limit values. In the first iteration of each time slot, it is assumed that the transmissions are performed with the maximum power $P^{\max}$ .

During the execution of the algorithm, a set of transmissions is determined which can be simultaneously activated so that the required SINR threshold is established in their receivers. Assuming that the algorithm is in the *h*th iteration at a given time slot *t*; therefore, the links of  $\Gamma_{t,h}^{\omega,Act}$ ;  $\forall \omega \in \Omega$  are specified. In order to add the new link in the (h+1)th iteration, first, the transmitter of each member of  $\Gamma_{t,h}^{\omega,Act}$  sends a packet with the power  $P_h^{\omega}$  including the following content to all of its neighboring links.

$$I_{h}^{\omega,mar} = \frac{P_{h}^{\omega}G_{h} - N_{0}\left(\gamma\left(\rho_{h}^{\omega,\max}\right) + \sum_{q \in 1, 2, \dots, h-1} I_{h,q}^{\omega}\right)}{\gamma\left(\rho_{h}^{\omega,\max}\right)}$$
(50)

where this amount is the tolerable interference margin in the receiver of added link in the *h*th iteration. By receiving this packet to the receivers of the already scheduled links, the total interference of each link is updated. Then a control packet with power  $P_z^{\omega}$  will be broadcasted to all non-scheduled neighbors including  $I_z^{\omega,mar}$ ;  $\forall z \in 1, 2, ..., h - 1$ . After receiving the control packets, each receiver node of the non-scheduled neighboring links updates the amount of its total potential interference and sends it to the corresponding transmitter node. The amount of interference will be obtained as follows

$$NI_{ns}^{\omega,sum} = N_0 + \sum_{q \in 1,2,\dots,h} I_{ns,q}^{\omega}$$
(51)

in which  $I_{ns,q}^{\omega}$  shows the interference resulting from the scheduled transmissions up to the *h*th iteration on the unscheduled neighboring transmission in the current time slot. Conversely, the transmitter corresponding to these receivers can calculate their maximum allowed transmission power by substituting  $I_z^{\omega,mar}$ ;  $\forall z \in$ 

1, 2, ..., h-1 from previously scheduled transmission in (46). Therefore, the amount of potential SINR in non-scheduled link is

$$SINR_{ns}^{\omega} = \frac{P_{ns}^{\omega, \max}G_{ns}}{NI_{ns}^{\omega, sum}}$$
(52)

Each transmitter node sends this value to its neighbors and saves the set of received SINRs from its neighbors in addition to the SINR of its corresponding transmission. Now, a link can add itself to the scheduling transmissions of slot t (and (h+1)th iteration) according to the following equation

$$\Gamma_{t,h+1}^{\omega,Act} = \left\{ \Gamma_{t,h}^{\omega,Act} \cup e_{h+1}^{\omega} \middle| e_{h+1}^{\omega} = \underset{\forall e_{ns}^{\omega}}{\arg \max} \left( SINR_{ns}^{\omega} \right) \right\}$$
(53)  
;  $\forall \omega \in \mathbf{\Omega}$ 

Eq. (53) indicates that the transmission with the maximum potential SINR between the unscheduled neighbors of the (h+1)th iteration is added to the set of scheduled nodes. Moreover, the maximum rate and minimum transmission power of the newly added member is calculated using (47) and (49), respectively. After adding a link and determining its transmission power, the procedure of link addition in current time slot *t* is repeated until the limit value from (46) becomes less than (49) on each frequency channel. This implies that there is no other links in the neighborhood of the previously scheduled transmission that can satisfy the limits at the same time. At this time, the next time slot t + 1 will be considered. The procedure of adding transmissions is repeated until all the members of  $\Gamma^{Act}$  are added to  $\Gamma_t^{\omega,Act}$ ;  $\forall \omega \in \Omega$ ,  $\forall t$ .

# 4.2.2. TMKC with IA-CS

In Section 4.2.1, a channel is randomly selected prior to the execution of TMKC for the multi-graph topology. In this section, the selection of the frequency channel during algorithm execution is performed such that the interference attains its minimum value. The algorithm pseudo code is represented in Fig. 6. Assuming the algorithm is in the *h*th iteration from the time slot *t*, the limitations and the method of adding new links are listed below.

#### A. Power and rate constraints

In order to determine the maximum power, the transmitters of the unscheduled links calculate Eq. (46) for each channel between the receiver and transmitter. After adding a new link to the current time slot in the (h+1)th iteration, the corresponding maximum rate and minimum power on selected frequency channel is calculated using (47) and (49), respectively.

# B. Selecting new transmission for addition

When receivers of non-scheduled neighboring transmissions receive the amount of interference resulting from the last added transmission to  $\Gamma_{t,z}^{\omega,Act}$ ;  $\forall z = 1, 2, ..., h$ , each transmitter node calculates the SINR value for all common frequency channels with the corresponding receiver using (52). Then each transmitter node saves the set of SINRs that resulted from neighboring transmissions and its corresponding transmission in the common frequency channels. Finally, the best transmission with corresponding frequency channel (with highest SINR) is added to  $\Gamma_{t,h+1}^{\omega,Act}$  using (53).

# 4.3. FITC and load balancing problem

According to [4], the load balancing can be discussed from two different points of view: spatial load balancing and channel load balancing. From the viewpoint of a specific channel, when a part of the network experiences congestion, the new traffic flows should

# Pseudo code #5

1: **Inputs** - all members in  $\Gamma^{Act}$ .

- 2: Outputs  $\Gamma_t^{\omega,Act}$ ;  $\forall \omega \in \Omega$ ,  $\forall t$ , power and rate in each member of these sets.
- 3: **Initialization-** t = 1, h = 1,

Set the power of each  $\Gamma^{Act}$  members to  $P^{\max}$ .

- 4: While(1)
- 5: **if** (all  $\Gamma^{Act}$  members added to  $\Gamma_t^{\omega,Act}$ ;  $\forall \omega \in \Omega, \forall t$ )
- 6: break;
- 7: else
- 8: Calculate the amount of the receiver interference of each non-scheduled transmissions on all of the radio interfaces.
- 9: Transmit calculated interference to corresponding transmitter on common frequency channels.
- 10: Calculate the maximum allowed power on any common frequency channels in each transmitter as follows:
- 11: **if** (h=1)
- 12:  $P_{ns}^{\omega,\max} = P^{\max};$
- 13: else
- 14: Calculate based on (46).
- 15: end;
- 16: Determine  $SINR_{w}^{\alpha}$ ;  $\forall \omega$  based on (52) and broadcast them to neighboring transmissions.
- 17: Candidate the links with maximum SINR by corresponding transmitters.
- 18: Determine the maximum rate and minimum power from (47) , (49).

19: **if**  $(P_{ns}^{\omega,\min} \succ P_{ns}^{\omega,\max})$  for all non-scheduled links

- 20: h = 0, t = t + 1;
- 21: else
- 22: Add the candidate links to  $\Gamma_{t,h}^{\omega,Act}$ ;  $\forall \omega$ .
- 23: Calculate the power of new added link according to:  $P_{h}^{\omega} = \left(P_{h}^{\omega, \max} + P_{h}^{\omega, \max}\right) / 2$ 24: Send control packet with power  $P_{h}^{\omega}$  containing the amount
- of tolerable interference of this link.25: Update the interference margin of all previously schedule
- 25: Update the interference margin of all previously scheduled links and broadcast them.
- 26: end; 27:  $\Gamma_{t}^{\omega,Act} = \Gamma_{t}^{\omega,Act} \cup \Gamma_{t,h}^{\omega,Act}; \forall \omega$ 28: h = h + 1;29: end:
- 30: end;

Fig. 6. Pseudo code for algorithm TMKC with IA-CS.

not be routed through that part. Furthermore, from the perspective of a specific location, the traffic load must be balanced over all available channels in the network. In FITC, load-balancing is considered in three levels, which are balancing in the channel assignment stage using the channels with lower usage among the neighboring nodes, balancing in the TMKC stage using power control, rate adaptation and the frequency channels selection and balancing in the routing. All of the three proposed levels have positive effects on the spatial load balancing. Moreover, the first two levels enhance the channel load balancing.

To evaluate FITC algorithm from the viewpoint of channel load balancing, we can define the variance of frequency channel utilization as follows:

$$\sigma_c^2 = \frac{1}{|\Omega|} \sum_{\omega \in \Omega} \left( U_c^{\omega} - \bar{U}_c \right)^2 \tag{55}$$

in which,  $U_c^{\omega}$  and  $\bar{U}_c$  are the amount of utilization for channel  $\omega$  and the average utilization of all frequency channels among the active

Table 3

SINR threshold required for supported rates in IEEE802.lla std.

Coding Rate	Modulation	SINR Threshold(dB)	Rate(Mbps)
3/4	64-QAM	24.56	54
2/3	64-QAM	24.05	48
3/4	16-QAM	18.80	36
1/2	16-QAM	17.04	24
3/4	QPSK	10.79	18
1/2	QPSK	9.03	12
3/4	BPSK	7.78	9
1/2	BPSK	6.02	6

time slots, respectively. These two parameters are defined as follows

$$U_{c}^{\omega} = \sum_{\forall t} \sum_{\forall e_{ij}^{\omega} \in \Gamma_{t}^{Act}} f_{ij,t}^{\omega} \quad ; \forall \omega \in \mathbf{\Omega}$$
(56)

$$\bar{U}_c = \frac{1}{|\Omega|} \sum_{\forall k \in \Omega} U_c^{\ k} \tag{57}$$

In (56),  $f_{ij,t}^{\omega}$  shows the amount of traffic on link  $e_{ij}^{\omega}$  at time slot *t*. We can define the variance of node utilization in order to investigate the performance of the proposed algorithm from the spatial load balancing aspect. This measure is in fact the variance of the total traffic on the nodes of the network. The traffic load of each node is the sum of all input and output traffics to/from the node. The smaller amount of this measure represents better load balancing in the network.

# 5. Simulation and results analysis

In this section, we carried out a complete assessment of the performance of the proposed algorithm FITC. Here, the proposed hybrid model is employed for modeling interference. In simulations, we consider up to 100 wireless mesh routers randomly distributed in an area of  $1000 \times 1000 \text{ m}^2$ , unless otherwise specified. According to IEEE 802.lla, the maximum number of nonoverlapping frequency channels is set to 12. Moreover, as a result of the hardware limitations, each node has a maximum of three radio interfaces. The channels will be assigned to these radios by using one of the two methods presented in 4.1.3. For each radio transmitter, the power can be adjusted in the range  $[0, P^{max}]$ , where *P*<sup>max</sup> is equal to 20 dbm. Furthermore, the thermal noise power in the receiver is set  $toN_0 = -90 \, dBm$ . On the other hand, radios can transmit with different rates 6, 9, 12, 18, 24, 36, 48, 54 Mbps. To this end, the required SINR threshold must be provided in the receiver according to the IEEE 802.lla. Table 3 expresses the SINR threshold, modulation type and coding rate for each of the above rates [24].

According to [13], the path loss exponent  $\varepsilon$  is set to 2.5 and the interference range is set to 350 m unless otherwise specified. In addition, the coefficients of the (29) are set to 1/3 and the time slot duration is set to 576.8 $\mu$ s unless otherwise specified.

We simulate and analyze different scenarios in order to perform a comprehensive evaluation on different aspects of the proposed algorithm. For each scenario, each data point is the average obtained from 150 simulation runs. In each run, the nodes are randomly distributed in the network, and 10 pairs of the nodes are selected randomly as source and destination. The traffic flow is assumed to be CBR and its volume is set randomly between 15 to 30 Mbyte.

**First scenario** - In the first scenario, we evaluate the operation of NFMA in network topology formation and modification. Here, 20 wireless mesh routers is assumed to be distributed in a network area of  $500 \times 500 \text{ m}^2$  where each node is equipped to three



Fig. 7. The network topology considering maximum power and minimum data rate.



Fig. 8. 3-Connected network graph obtained by NFMA.

radio interfaces. Moreover, all 12 frequency channels is assumed to be available. Fig. 7 denotes the resulted graph assuming maximum power transmission and minimum data rate. Here, the hybrid model proposed in Section 3.2 is used for link establishment.

Now, it is assume that the connection number (K) is set to 3. In accordance to the first step of NFMA, the minimum power of each node must be equal to -16.19 dBm for the achievement of a 3-Connected network with high probability. After determining the connections, all paths between different node pairs of the network is prioritized using the measure defined in (29). Then, three paths with the highest priority are selected between any node pairs. Fig. 8 represents the network graph after selecting the paths with higher priority.

In the next step, we investigate the effect of parameter K on network throughput using RC-CA and IA-CA channel assignment methods. In RC-CA, considering the 3-Connectivity limitation, we assign one common channel to one of the radio interfaces of each node. The remaining channels are assigned randomly to other radio interfaces. In IA-CA, the channel assignment is performed recursively for each of the links that resulted from the third step of the NFMA. The diversity of the assigned channels is reduced by increasing K as a result of the limited number of radio interfaces in each node and the limitation resulting from the 3-connection feature.

Fig. 9 illustrates the variation of the network throughput vs. parameter K in both channel assignment methods. Here, the network throughput is defined as the average traffic rate on the active links in each time slot [8,9]. By increasing the amount of K, the mini-



Fig. 9. Throughput variation vs. K using IA-CA and RC-CA.

mum power from Eq. (12) is also increased, which can lead to the increment of nodes transmission power and throughput reduction in TMKC algorithm. The throughput reduction in the case of IA-CA is more than that of RA-CA. As mentioned earlier, in IA-CA, the diversity of the assigned channels is reduced with the increase in *K*, therefore the throughput is reduced by increasing the value of *K*. In RC-CA, the channel assignment is independent of *K*, therefore the assigned channels are fixed and the throughput does not change significantly by varying *K*.

Second scenario - In this scenario, we investigate the effect of the number of frequency channels and radio interfaces on the performance of the proposed algorithm. Likewise, the performance of the two channel assignment methods is evaluated. Here, we assume K=2,  $|\Omega|=5$ , n=100. Fig. 10a shows a comparison between the performances of the two channel assignment methods assuming the number of radio interfaces is varied from 2 to 8. It is evident that any increase in the number of radio interfaces with separated channels results in more concurrent transmissions for each node. Since the purpose in the IA-CA channel assignment is the reduction of potential interference, the network throughput is more than random assignment in RC-CA. Moreover when the number of channels is limited to 5, the amount of throughput approximately remained unchanged when the number of interfaces is more than 5, because repetitive channels are assigned to additional interfaces of a node. Therefore, these interfaces cannot perform concurrent transmission.

In the second section of this scenario, the number of radio interfaces is set to 3. The results are shown in Fig. 10b. If less than three frequency channels was available and 2-Connectivity criterion was held, all the channels is assigned to all the radio interfaces of the nodes, using each of the two channel assignment methods. Therefore, the throughput is similar in both methods. According to Fig. 10b, in RC-CA, the throughput is increased by increasing the number of channels. Since the channel assignment is performed randomly, more increase in the number of channels, results in the less number of common channels between neighboring nodes, which results in the gradual saturation of throughput.

However, in IA-CA, the throughput is saturated because the recursive procedure of channel assignment in the IA-CA leads to the reduction of diversity of the used frequency channels. Therefore, increasing the number of channels above a threshold does not have any significant effect on the throughput. Of course, it is evident that the IA-CA results in more throughput than RC-CA.

**Third scenario** - In this scenario, we first investigate the effect of power control, rate adaptation and channel selection on the network throughput. In addition, the performance of the routing based on (29) is evaluated. At this point, the number of nodes is



Fig. 10. (a) Throughput variation vs. the number of radio interfaces (b) Throughput variation vs. the number of channels.

varied from 10 to 100. However, the number of traffic requests is left unchanged. Other assumptions are  $|\Omega| = 8$ ,  $In_i = 3$ , K = 2.

Power control and rate adaptation effectively improved the throughput as illustrated in Fig. 11a. The length of the links is reduced by increasing the number of nodes. Therefore, the amount of interference using fixed power  $P^{\max}$  is increased and the throughput is reduced compared to the TMKC algorithm with power and rate control. The other observations are:

- In the TMKC algorithm, by increasing the number of nodes (along with the power control, rate adaptation and channel selection ), the throughput is slightly increased at first, because increasing the number of nodes results in the reduction of links length and the interference is reduced by using power control. Subsequently, with further increase in the number of node, the throughput slope is slightly reduced due to the increase in the number of interfering nodes.
- TMKC with IA-CS compared to the R-CS, improves the throughput significantly.
- Fig. 11b demonstrates the results of changing the coefficients in (29). Points of interest in this figure are as follows:
- When the routing measure is minimizing the number of hops  $(\alpha_1 = 1, \alpha_i = 0 \quad i = 2, 3)$ , throughput is improved by the increase in the number of nodes, because the average length of the links is reduced in different paths. By the continuous increase in the number of nodes, throughput is reduced with a slight slope, because the possibility of concurrent transmissions on different links is reduced.



Fig. 11. (a) The effect of power control, rate adaptation and channel selection on throughput variation (b) The effect of routing measure coefficients on throughput.

- In the case of using the measure of (29) with α<sub>1</sub>=0.5, α<sub>2</sub>=0.5, α<sub>3</sub>=0, throughput is significantly increased when compared to the measure of minimum number of hops(α<sub>1</sub>=1, α<sub>i</sub>=0 i=2, 3). By increasing the number of nodes, first, throughput is increased because the average length of links in a path and the required power for transmission on the links are reduced. With further increase in the number of nodes, it is impossible to use paths with lower node utilization. Therefore, the throughput is reduced slightly.
- Any increment in the coefficient  $\alpha_2$  results in the selection of paths with less required power. Therefore, the interference is decreased while the throughput is increased. By increasing the number of nodes, both maximum and average power of the paths is decreased; hence, the effect of these parameters on throughput improvement is decreased. Therefore, the throughput slope is reduced.
- Increasing the value of  $\alpha_3$  when the node density is high has a significant effect on balancing. Increasing the weight of this factor on the routing measure leads to the selection of paths with minimum use of intermediate nodes.

**Fourth scenario** - In this scenario, the performance of FITC is compared to MSITD [18], DRRL [10] and proposed method in [16]. It is assumed that the number of nodes is varied from 10 to 100. The other assumptions are:  $|\Omega| = 12$ ,  $In_i = 3$ , K = 2. The results of this simulation are shown in Fig. 12. It is evident that the throughput of our proposed method is significantly higher than MSITD. It is due to the fact that the MSITD method only exploits interference aware channel assignment, whereas our proposed method uses dif-



Fig. 12. . Comparison between FITC with other methods DPRL [10], MSITD[17] and proposed method in [16].



Fig. 13. The effect of interference range on the performance of FITC algorithm.

ferent tools for topology control, which are power and rate control, interference aware channel assignment/selection based on the hybrid interference model and multi-criteria routing to decrease theinterference and to improve the balancing in the network.

In the proposed method of [16], the authors have only considered the robustness of the network against failures, while in our proposed method, a complete set of resource allocation tools and several objective are investigated. Therefore, the throughput in FITC is significantly higher when compared to the method of [16].

In DPRL, *K*-Connectivity is not considered. Moreover, the interference aware channel assignment/selection is not used. In this paper, only the throughput maximization is considered by using the power control and rate adaptation tools. It is evident from Fig. 12 that in a fixed amount of *K*, throughput of our proposed method is higher than DPRL. Conversely, in FITC, the amount of throughput improvement is slightly reduced by increasing *K*. However, FITC throughput is significantly higher than DPRL.

**Fifth scenario** - In this scenario, the effect of interference range in the proposed hybrid interference model is investigated. As stated in Section 4, the interference range is an effective parameter in IA-CA. On the other hand, in TMKC, a proper interference model should be employed for power control, rate adaptation and channel selection. The simulation parameters for this scenario are: n = 100,  $|\Omega| = 12$ ,  $ln_i = 3$ , K = 2. Moreover, it is assumed that the nodes are distributed in a 1000\*1000 m<sup>2</sup> area and the interference range is varied from 100 m to 1000 m.

The results of the simulation are represented in Fig. 13. According to (5), for higher values of interference range, the model



**Fig. 14.** (a) The variance of frequency channels utilization vs. the number of channels (b) The variance of nodes utilization vs. the parameter $\alpha_3$ .

is similar to the physical interference model, while for lower values, it is similar to the protocol model. Moreover, it is clear from (2) that increasing the distance between two nodes results in the exponential reduction of the interference between them. Therefore, further increase in the interference range reduces the impact of other far nodes on a given node. In Fig. 13, by increasing the interference range, first throughput is reduced. However, the reduction is insignificant for interference range above 450 m. The model is equal to the physical interference model for interference range of 1000 m. According to Fig. 13, for interference range of 350–450 m, the results of using hybrid interface model are nearly similar to the physical interference model.

**Sixth scenario** - In this scenario, the load balancing capabilities of the proposed method is evaluated. The simulation parameters of this scenario are n = 100,  $In_i = 3$ , K = 2. Here, the number of frequency channels is varied from 3 to 12. Fig. 14a indicates the variance of frequency channels utilization in the network which is a good measure in evaluating the load balancing between different channels. The use of FITC method (along with the IA-CA and IA-CS), significantly reduces the variance of using frequency channels compared to the random channel assignment /selection. Conversely, with slight increase in the number of frequency channels, the distribution of traffic between different channels improves; therefore, the variance of utilization of the channels decreases.

In the second section of this scenario, the number of channels is set to 7. We first use FITC method along with IA-CA and IA-CS, and investigate the effect of  $\alpha_3$  on the spatial load balancing. According to Fig. 14b, any increase in  $\alpha_3$  decreases the variance of



Fig. 15. The comparison of FITC performance with optimum method [25] (a) throughput graph vs. the number of nodes. (b) the node utilization vs. the number of nodes (c) the channel utilization variance vs. the number of channels.

nodes utilization. This is because increasing this parameter results in the selection of the paths with less utilized nodes. The other observations are:

- By the continuous increase in  $\alpha_3$ , the slope of the variance is decreased, because the effects of other factors (the number of hops and power consumption) decrease in the routing cost function. In this situation, paths with longer links may be selected between nodes. Therefore, during the implementation of TMKC, some links require higher power for successful transmission which causes the rate of transmission to be higher. This in turn leads to the reduction of transmission rate for neighboring nodes.
- In lower values of  $\alpha_3$ , the selection of the routes is based on the reduction of power consumption and hop numbers, which can increase the number of similar nodes in different paths. Conversely, slight increase in the number of nodes improves the load balancing. Since the network dimensions are fixed, the algorithm selects the paths with less utilized nodes in the process of best path selection. Therefore, more freedom is achieved in distributing the traffic over the network.
- Moreover, in the case of using the power control, IA-CA and IA-CS, the node utilization variance is decreased. As stated earlier, a set of links with maximum rate in each time slot is activated in TMKC. On the contrary, the maximum number of possible links in each time slot is selected for transmission according to

the power control implementation. Therefore, the variance of node utilization is improved.

**Seventh scenario** – In this scenario, we compared the performance of the FITC with the optimization-based method presented in [25]. In the optimization problem modeled in [25], the objective function is a normalized sum of three metrics including the variance of node and channel utilizations and throughput maximization. The constraints considered in this optimization problem are the constraints of the first three layers of the network and cross-layer constraints including the SINR constraint, half-duplex constraint of radio interfaces, maximum/minimum limits of the transmission power, the input and output flows constraints, routing constraints and *K*-Connectivity constraint. In this scenario, the number of nodes is varied from 10 to 25. These nodes are randomly distributed in an area of  $750 \times 750 \text{ m}^2$ . Moreover, the number of radio interfaces and non-overlapping channels are set to 3 and 12, respectively, unless otherwise specified.

Fig. 15(a) shows that throughput is slightly reduced by increasing the robustness parameter K from 1 to 2 in both FITC and in the optimized method. Moreover, the throughput of FITC in average is reduced by approximately 13.5% and 17.9% compared to the optimized method for K = 1 and K = 2, respectively. This amount of reduction in throughput is an acceptable cost for the benefit of huge reduction in computational complexity of FITC when compared to the optimization-based method.

In Fig. 15(b) and(c), the variances of frequency channel utilization and node utilization are depicted, respectively. Here, the parameter  $\alpha_3$  of FITC method is set to 0.4. In both the proposed optimization model in [25] and FITC, we consider the balancing of node and frequency channel utilization. The variances of node and frequency channel utilizations for the heuristic method FITC are increased compared to the optimized model that shows a little better balancing in optimization-based method. Averagely, the variance of node utilization is increased by approximately 31. 6% and 36.9% for K = 1 and K = 2, respectively and the variance of channel utilization is increased by approximately 23.9% and 29.3% for K = 1and K = 2, respectively. Moreover, the amount of node and channel utilization are decreased with slight increase in the number of nodes and frequency channels, respectively.

# 6. Conclusion

Exploiting advanced radio interfaces in Multi-Radio Multi-Channel WMNs (MR-MC WMNs) offers the potential of throughput improvement by topology control. In these types of networks, the main objective is maximizing the throughput and providing load balancing. Moreover, in these networks, the fault tolerance requirements must be considered in network development stage in order to enable the fast recovery of the routes in the case of faults. In several recent researches, the problem of topology control in MR-MC WMNs using tools such as power control, rate adaptation and channel assignment is considered without the consideration of fault tolerance requirements. In comparison to the previous works, in this paper, we proposed a cross-layer topology control method that aims at improving the throughput while preserving the *K*-Connectivity feature.

In the first part of the proposed method, the *K*-Connectivity feature is obtained by using the minimum transmission power at each transmitter and by adding new nodesif necessary. Then, *K* best paths between each two nodes are selected using a cross layer measure. This measure is based on minimizing the potential interference and maximizing the spatial balancing in the network. Finally, an interference aware channel assignment method is proposed where its aim is choosing appropriate channel for radio interfaces to minimize the potential interference in the network. Preservation of *K*-Connectivity of the network is the major limitation of the channel assignment method.

In the second part of the algorithm, a cross layer distributed method is used to select the best set of links for transmission in every time slot. This set of links is selected such that the maximum number of link for simultaneous transmission on each time slot is provided while at the same time, the transmission rate was maximized. In this part, the power control and rate adaptation tools along with channel selection and scheduling is used for improving the throughput and providing frequency channel balancing.

In summary, we developed a cross layer distributed method, which exploits all the required tools in order to improve the throughput and provide balancing in the network. Conversely, the *K*-Connectivity feature is also provided for network robustness against failures. Many simulations in the form of different scenarios are performed in order to evaluate the performance of the pro-

posed algorithm. Analyzing the results revealed that the proposed algorithm achieves the main objectives when compared with the previous methods.

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