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Multicast routing algorithms for sparse splitting optical networks

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This work proposes novel routing approaches for transparent optical networks where only a fraction of the network nodes are multicast capable (MC) and can split the optical power from a single input to multiple output ports. The remaining, multicast-incapable (MI) nodes, can have either Drop-or-Continue (DoC) or Drop-and-Continue (DaC) capabilities. For the case of DaCs, if a MI node is a destination of the multicast group, it can drop a fraction of the incoming signal locally and transmit the rest to the next node. The current paper presents an Integer Linear Programming (ILP) formulation as well as novel heuristic multicast routing algorithms under the sparse-splitting constraint, for networks with or without DaC nodes. Performance results show that the proposed algorithms achieve an important decrease of the average cost of the derived solutions, compared to existing relevant techniques, and attain results very close to the lower bound provided by the ILP.

1. Introduction

Most connections carried over an optical mesh network have been and are still currently unicast connections, such as high-bandwidth point-to-point connections for enterprise customers. However, new traffic requirements are driving the evolution of the telecommunications service providers' network architectures. For example, the service providers receive their different programs from content producers and aggregate them at a few specific locations before distributing the information to their end-customers. Multicasting is an obvious choice to carry this aggregated content to different local distribution points. In addition, for the use of applications such as telepresence that has grown in the past few years, video training, e-learning, and on-line teaching (with start-ups such as Udacity and Coursera), as well as telemedicine and remote medical diagnosis, multicast connectivity appears to be the best solution to transport such applications [1,2]. For example, deployment of holographic technologies, such as the use of telepresence is seen as a way to eliminating costly, time and energy consuming travel in the near future. For some situations, multicast connectivity will potentially be the most efficient way to transmit high-definition video, voice, and data signals between multiple telepresence locations.

Optical multicasting is based on the calculation of light-trees, utilizing optical splitters in the network nodes [3]. An optical splitter is a passive device that splits the input signal into multiple identical output signals [4]. The nodes that have splitting capability are called *Multicast-Capable (MC)* nodes. If not, they are called *Multicast Incapable (MI)*. To limit the impact of optical splitters on the cost of the network, they can be placed at only some of the network nodes (MC nodes), resulting in a *sparse-splitting* network [3,4]. The remaining MI nodes of the network may be *Drop-and-Continue* (DaC) or *Drop-or-Continue* (DoC) nodes [5,6]. A DaC node can transmit the optical signal to the following node *and* can also drop it locally as well, while a DoC node can either transmit the optical signal to the following node *or* drop it locally. This work deals with both types of networks as both DoC and DaC nodes are viable architectures currently under consideration (a preliminary work for DoC networks has been presented in [7]).

The problem of multicast routing in sparse-splitting networks is NP-hard, since the NP-hard Steiner problem in graphs [8] is a special case of it. Therefore, polynomial-time heuristics that give approximate solutions are used in practice. In this work, a novel ILP formulation is developed for multicast routing in sparse splitting networks, that accounts for networks with MI nodes that can be either DoC or DaC, in order to have a benchmark against which to evaluate the heuristics that constitute the state of the art, as well as the heuristics proposed in the current paper. Performance results via simulations on the USNET and NSFNET as well as on several randomly created networks, have shown that the proposed heuristics achieve an important decrease of the average cost of the derived solutions, compared

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to existing relevant techniques. For the USNET and NSFNET networks, in particular, the performance of the proposed heuristic algorithms is very close to the lower bound provided by the ILP. In fact, for these networks, the proposed heuristic algorithms achieve to derive the optimal solution for the majority of the investigated cases.

The remaining of the paper is organized as follows. The definitions, notation and assumptions as well as the applicable network architectures for the proposed work, are given in Section 2. Section 3 presents a literature review on the state of the art, as well as a detailed description of some of the most efficient existing sparse-splitting multicast routing heuristic algorithms, while the proposed heuristic algorithms are presented in Section 4. Section 5 gives a novel Integer Linear Programming (ILP) formulation for the derivation of the optimal solution, and the evaluation, through simulations, of the existing and proposed heuristic algorithms, is presented in Section 6. Finally, in Section 7, the conclusions of the paper are presented, as well as ongoing future work.

2. Preliminaries

2.1. Definitions and notation

Throughout the paper, the following definitions and notation are utilized.

- The network is modeled as a directed graph $G = (V_G, A_G)$, where V_G ($|V_G| = n$) and A_G ($|A_G| = m$) are the sets consisting of the network nodes and arcs respectively.
- A cost c_{ij} is assigned to each arc $[i, j]$.
- The network directed graph is considered to be symmetric, i.e., for every arc $[i, j]$ in A_G, the corresponding reversed arc $[j, i]$ also belongs to A_G, with $c_{ij} = c_{ji}$ (as each network link consists of two fibers with opposite orientation).
- Each fiber carries W wavelengths, denoted by $\lambda_1, ...\lambda_W$.
- The notation $P_{uv} = (V_{P_{uv}}, A_{P_{uv}})$ (or P_{uv} for simplicity) stands for the directed shortest path originating from node u and ending at node v. The sets of its corresponding nodes and arcs are $V_{P_{uv}}$ and $A_{P_{uv}}$, respectively.
- The *cost* of a path is defined as the sum of the costs of its arcs and is denoted by $c_{P_{uv}}$.
- Let the corresponding undirected path derived from P_{uv} denoted by $\mathcal{P}_{uv}$. Since G is a symmetric directed graph, $\mathcal{P}_{uv} = \mathcal{P}_{vu}$ for every u, v in V_G. Therefore, P_{vu} can be derived from P_{uv} simply by reversing the direction of the arcs of the latter.
- The union of two subgraphs $SG = (V_{SG}, A_{SG})$ and $SG' = (V_{SG'}, A_{SG'})$ is equal to $SG \cup SG' = (V_{SG} \cup V_{SG'}, A_{SG} \cup A_{SG'})$.
- The set consisting of the MC nodes of the network is denoted by MC_{set}, and $|MC_{set}| = z$.
- The multicast session is denoted by $S = \{s, d_1, d_2, \ldots, d_k\} = \{s, D\}$, where s is the source node and $D = \{d_1, d_2, \ldots, d_k\}$ is the destination set consisting of k destinations (i.e., $|D| = k$).

The reader should note that the cost metric for each link in this work is left as a general cost (as is done in a large body of work in the literature) and is not tied to a specific physical layer metric, as in this work physical layer impairments are not taken into consideration. However, the general cost metric utilized in this work could represent a number of entities, such as monetary cost, actual distance in kilometers, etc.

2.2. Problem definition

The problem under investigation in this paper considers a fiber-optic backbone telecommunications network modeled as a graph where the vertices of the graph represent optical switching nodes and the arcs of the graph represent fiber-optic links. Wavelength division multiplexing (WDM) is employed in these networks, with W wavelengths being available for every fiber-optic link. For the networks considered, some of the network nodes provide multicasting capability (utilizing optical splitters), while the rest do not. Thus, these types of networks can provide the capability for the provisioning of multicast connections (point-to-multipoint connectivity from a source to multiple destinations). In order to provision each multicast request, the optical fibers (arcs) that will be used in order to establish this request must be initially found. The set of these optical fibers constitutes the *Routing SubGraph (RSG)* that is used for establishing the requested multicast session. The objective of the problem at hand is to find the *RSG* with the minimum cost for each multicast connection request established in these types of networks.

The problem under investigation is thus defined as follows:

- *Input*:
  - Network graph: $G = (V_G, A_G)$.
  - Number of wavelengths on each network fiber (arc): W.
  - Set of MC nodes on network graph: MC_{set}.
  - Multicast session consisting of a source and k destinations: S.
- *Output*: Routing SubGraph $RSG = (V_{RSG}, A_{RSG})$ with the minimum possible cost.

This *Routing SubGraph* $RSG = (V_{RSG}, A_{RSG})$, which is the output of the problem, is the subgraph of the network graph $G = (V_G, A_G)$ that is used for establishing the requested multicast session (with nodes on the *RSG* with out-degree equal to zero called *leaf nodes*). The *subgraph* notation is used for the derived topology instead of the *Tree* notation, since cycles may exist on it. The reason is that, because of the sparse-splitting constraint, *RSG* may use both arcs of a link in the opposite direction. One or more wavelengths are also utilized on each arc of the *RSG* (from a total number of W wavelengths on each network arc). The possible existence of cycles on the *RSG* as well as the possible utilization of more than one wavelengths on each of its arcs are explained in more detail in Section 2.3 (an example is shown in Fig. 4). Also, note that the *RSG* is derived under the constraint that only a fraction of the network nodes are MC (MC_{set} is the set consisting of these nodes).

As stated previously, the cost metric for each link in this work is left as a general cost. Examples of optical networks with a cost assigned to each arc are the well known USNET ([9], illustrated in Fig. 10) and NSFNET ([10], illustrated in Fig. 11). Since it is desirable to derive an *RSG* with cost as low as possible, the efficiency of an algorithm for the problem under investigation is evaluated according the cost of the *RSG*s it derives. The cost c_H^S of the RSG_H^S derived by heuristic H for multicast session S is defined as:

$$c_H^S = \sum_{[i,j] \in A_{RSG_H^S}} w_{ij}^{H,S} c_{ij} \tag{1}$$

In more detail, since multiple wavelengths may be used on each arc, the number of wavelengths used on arc $[i, j]$, for RSG_H^S, is denoted by $w_{ij}^{H,S}$. The cost c_{ij} of each arc $[i, j]$ that is part of the RSG_H^S is multiplied by the number of the wavelengths $w_{ij}^{H,S}$ that are utilized by the RSG_H^S on $[i, j]$, and the products $w_{ij}^{H,S} \cdot c_{ij}$, for every arc $[i, j]$ that is part of the RSG_H^S, are summed in order to derive the cost c_H^S of the RSG_H^S. If the target is the derivation of an *RSG* that utilizes the lowest possible number of wavelengths, then cost equal to 1 is assigned to each network arc.

If a set of multicast sessions $\mathcal{S} = \{S_1, S_2, \ldots, S_l\}$ is routed using heuristic H, the average cost $\bar{c}_H$ is defined as:

$$\bar{c}_H = \frac{1}{l} \sum_i c_H^{S_i} \quad i = 1, \ldots, l \tag{2}$$

In more detail, the average cost for heuristic H over several multicast sessions is equal to the summation of the cost for each one of

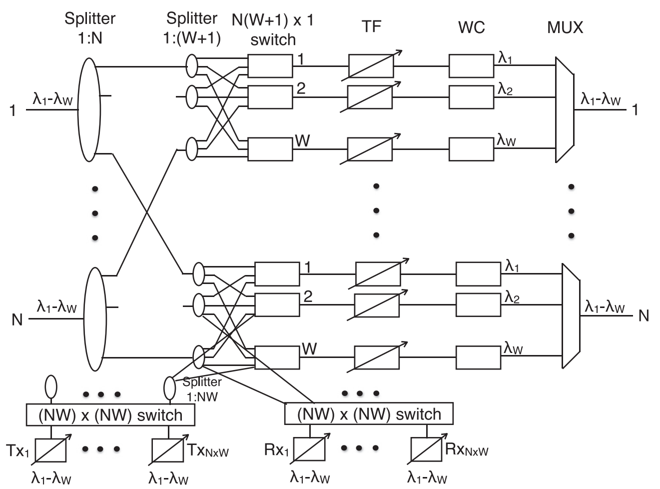

Fig. 1. MC node architecture utilized in the current work (adopted from [5]) (WC: wavelength converters, TF: tunable filters, *W* wavelengths per link, node degree *N*).

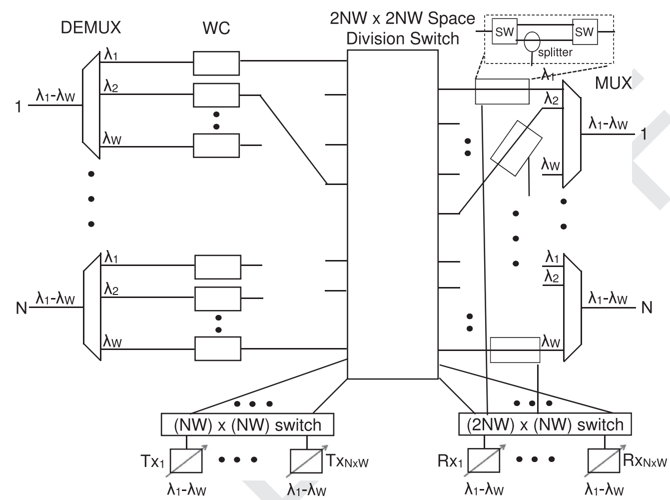

Fig. 3. MI (DaC) node architecture utilized in the current work (*WC*: wavelength converters, *SW*: switches, *W* wavelengths per link, node degree *N*).

them (as defined by Eq. (1)), divided by the number of the multicast sessions.

The utilization of *RSG*s that may include cycles for cost efficient routing was first proposed in [11]. In the aforementioned paper, *RSG* is called *Light-Hierarchy* and it is shown theoretically and via simulations (using ILP formulation) that the light-hierarchy structure gives optimal results whereas the light-tree structure is sub-optimal. It is important to note here that the work in [11] is concentrated on the optimal (non-polynomial) solution (ILP formulation - and only for the case of MI nodes that are DaC) and does not give any polynomial-time heuristics as is the work presented in this paper.

2.3. Network architectures and assumptions

In this work, the following network assumptions are valid.

1. All network nodes are equipped with wavelength converters, therefore the information can be transmitted through the derived *RSG* using multiple wavelengths. Examples of the architectures for MC and MI (DoC) nodes with degree *N* (i.e., the maximum possible number of copies of the incoming signal) utilized in the current paper can be found in [5] and are shown in Figs. 1 and 2. A similar architecture to that illustrated in Fig. 2 is adopted for the multicast-incapable node that is now drop-and-continue (DaC). The only difference between the two architectures is that in the DaC case, at each output fiber of the 2*NW* × 2*NW* space division switch, the architecture provides the capability for each signal to pass through and continue to the next node, get dropped at the receiver, or continue to the next node *and* get dropped at the receiver. This functionality can be achieved utilizing a number of different approaches. A simple architecture to achieve the drop, continue, and drop-and-continue functionalities combines a 1 × 2 switch, a 1:2 splitter, and a 2 × 1 switch (in that order) at each output fiber of the 2*NW* × 2*NW* space division switch (as shown in Fig. 3).
2. Each node is equipped with a bank of tunable transmitters and receivers allowing the source of the multicast session, even in the case of an MI node, to transmit the information through multiple fibers, and (if needed) to utilize more than one wavelength at each one of these fibers.

These aforementioned assumptions are illustrated with an example for a simple network graph shown in Fig. 4. In this case, a multicast connection has been established with *s* being the source node

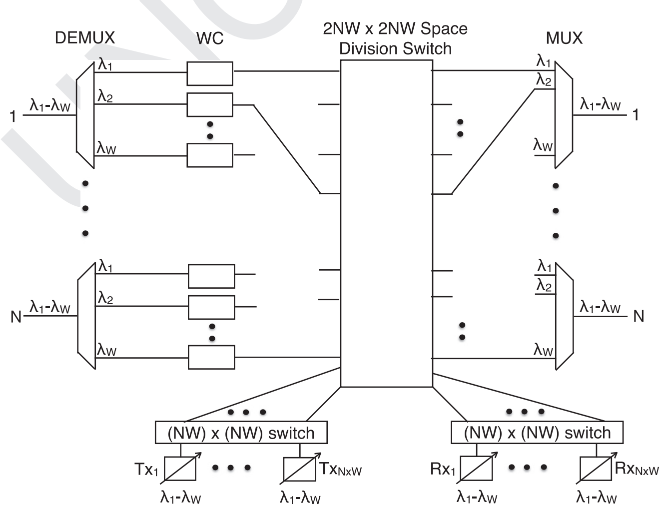

Fig. 2. MI (DoC) node architecture utilized in the current work (adopted from [5]) (WC: wavelength converters, *W* wavelengths per link, node degree *N*).

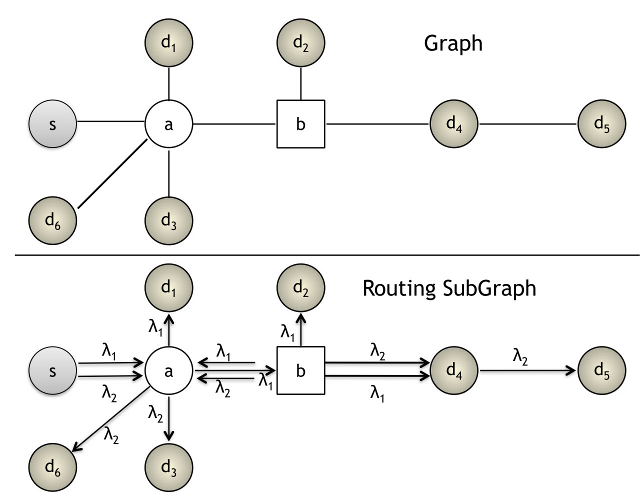

Fig. 4. Network example illustrating the problem assumptions (MC nodes are represented by squares and MI nodes by circles).

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and d_1–d_6 the destination nodes and where node b is MC while the rest of the nodes are MI (DoC). The top figure gives the network graph, where each connection refers to a pair of fibers with opposite orientation. The lower figure illustrates the resulting *RSG*. The signal is transmitted from the source to the MC node b through path $P_0 = s - a - b$ using wavelength λ_1, it is split into five copies, and from node b it is transmitted to destinations $d_1 - d_5$ through paths $P_1 - P_5$, respectively. The signal arrives to d_6 through P_6. In more detail:

- P_0: $s - a - b$, using wavelength λ_1
- P_1: $b - a - d_1$, using wavelength λ_1
- P_2: $b - d_2$, using wavelength λ_1
- P_3: $b - a - d_3$, using wavelength λ_2
- P_4: $b - d_4$, using wavelength λ_1
- P_5: $b - d_4 - d_5$, using wavelength λ_2
- P_6: $s - a - d_6$, using wavelength λ_2

From this simple example, it can be seen that the MC node b can transmit the multiple copies of the incoming signal through different wavelengths, either through the same fiber or through different fibers. It can also be seen that several wavelengths can be utilized on the same fiber, that both fibers of the same link may be utilized in the same light-tree, and that the source can transmit the signal multiple times, using different wavelengths.

It is important for the reader to note that other node architectures are also applicable for the work described in this paper. For example, reconfigurable optical add/drop multiplexers (ROADMs) [12–15] that are the key elements for building the next-generation optical networks, can also be considered. A ROADM takes in signals at multiple wavelengths and selectively drops some of these wavelengths locally, while letting others pass through, switching them to the appropriate output ports. The choice of ROADM architecture and underlying technology depends on how effectively current and future traffic can be addressed. ROADM architecture and technology influences cost, power consumption, optical performance, and configuration flexibility. Wavelength selective switch (WSS) technology [16,17] is currently being used for the implementation of ROADMs and for the deployment of cost-effective dynamic wavelength switched networks. The WSSs are complex multiplexers/demultiplexers that select the corresponding outputs to forward the data carried by each wavelength.

ROADMs based on broadcast-and-select (BS) or route-and-select (RS) are current choices in deployed optical networks and can remotely configure all transit traffic. BS nodes (Fig. 5) include a splitter first-stage that implicitly provides multicast towards the outputs, whereas RS nodes (Fig. 6) have a WSS first-stage that provides on-demand multicast. Both implementations have a WSS second-stage that provides the selection of the wavelengths at the outgoing links (allowing full flexibility on which wavelength to pass through from the incoming links or which wavelength to add from the add/drop terminals).

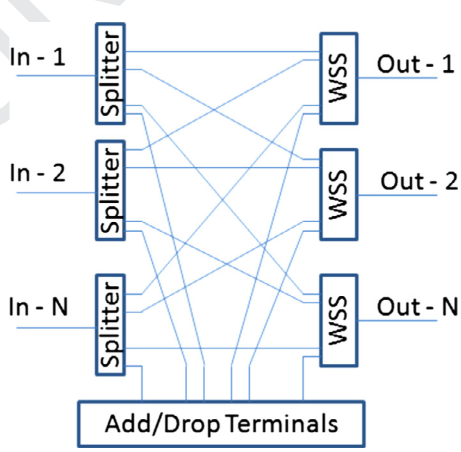

Fig. 5. BS architecture.

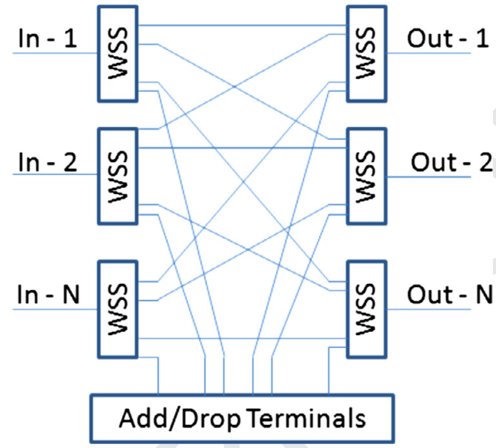

Fig. 6. RS architecture.

Further, at the add/drop terminals, tunable transponders and WSSs are utilized, ensuring the tunability and the re-configurability of the architecture. Thus, a ROADM architecture offers full flexibility of add/drop ports, meaning that traffic can be added/dropped to/from an arbitrary transmission fiber originating from or terminating at the node and on any wavelength. ROADMs have the ability to support dynamic traffic evolution in a flexible and economic manner and are very cost-efficient architectures from the operator's perspective, since components can be added on a node that needs to be upgraded, without affecting existing transit traffic. Such nodes that are remotely configurable and utilize colorless and directionless add/drop ports are also called optical crossconnects (OXCs).

Clearly, these architectures can also be used in order to provide multicast capabilities to the network. Thus, these architectures can also be considered as MC nodes. However, in current deployment networks where optical nodes are already installed, the upgrade of all the nodes to ROADMs will have a big impact on the network cost. For this reason, it is envisioned that a fraction of the network nodes will be upgraded in order to provide cost-reducing solutions without compromising optical performance and flexibility. Depending on the network traffic, it will most likely be preferable to keep the legacy network nodes as well due to the high cost of the node upgrades (high cost of the required WSSs). Thus, it is envisioned that a fraction of the (upgraded) network nodes will be multicast-capable (utilizing the BS or RS architecture) and the legacy nodes (that are multicast-incapable) will also remain in the network. Therefore, the proposed algorithms can be used by considering that the ROADMs nodes are the MC nodes, and the legacy nodes are the nodes with no multicasting capability (MI nodes that are either DoC or DaC).

A literature review on multicast routing in sparse splitting networks, as well as a more detailed description of some of the most important existing heuristic algorithms for multicast routing in sparse-splitting networks are presented in the section that follows and will be used for comparison purposes against the proposed techniques.

3. Existing heuristic algorithms

There are a number of approaches in the literature on the problem of multicast routing in sparse-splitting optical networks. In [18], a sparse splitting multicasting algorithm is proposed, aiming at the derivation of low diameter and average delay multicast trees. The work in [19] focuses on deriving multicast trees with a good trade-off among minimizing the link stress, total cost, and end-to-end delay. The problem of provisioning multiple multicast requests with the lowest possible number of wavelengths is investigated in [20].

Other related work is included in [5,21,22] where efficient solutions are identified for the problem of multicast routing in sparse splitting networks where the multicast incapable nodes are either drop-or-continue or drop-and-continue.

In [23], authors propose an algorithm that minimizes the wavelength usage by constructing a light-forest for a given multicast session so that the multicast data can be delivered to all the members of the session and the delay of communication never exceeds the permissible quality of service value assigned with the session request (delay limit) without violating the sparse-splitting constraints. Moreover, in [24] authors investigate the problem of provisioning a set of multicast requests simultaneously with the objective of minimizing the blocking probability. In particular, authors propose an integer linear programming (ILP) formulation and adaptive heuristic algorithms to compute approximated solutions. Instead of using light-trees, both ILP and heuristics use light-hierarchy under sparse-splitting configurations.

Furthermore, in [25] the sparse-splitting quality-of-service-driven multicast routing problem is investigated, with the objective of minimizing the total cost of wavelength channels that are used by the multicasting tree. The case of repetitive multicast demands whose source and destinations are known a priori is investigated in [26]. In that work, the network infrastructure is adapted according to the recurrent transmissions, by setting available branching routers in the selected nodes of a predefined tree. In [27], a virtual-node-based multicast routing algorithm is proposed to satisfy the requirements of interactive real-time multicasting as well as the constraints from underlying optical networks. The problem of low-cost multiple-session multicasting in mixed-line-rate optical networks is investigated in [28], while in [29], heuristics are proposed for low-cost multicasting under power constraints.

Additional work in [30] investigates the problem of sub-wavelength traffic grooming in WDM optical networks, where the advantages of MC nodes are exploited in grooming the sub-wavelength traffic. Furthermore, the problem of multicast routing in relation to the protection/restoration of multicast connections was also investigated. For example, the work in [31] and in [32] investigates the problem of protecting multicast sessions in optical networks, taking into account physical layer impairments. In [33], a restoration method that provides relatively fast restoration of multicast demands is proposed, while in [34] the problem of sub-wavelength level protection for dynamic multicast traffic grooming is investigated, and a new technique is proposed that aims at minimizing the network resources allocated for the protection of the traffic requests.

Recently, optimization algorithms were also proposed for multicast routing in elastic optical networks (EONs)[35–38]. Specifically, in [35] authors optimize the spectrum efficiency of multicast requests in EONs based on ILP and genetic algorithms. Further, in [36], authors propose ILP algorithms and a heuristic algorithm that implement distance-adaptive transmission for multicasting in EONs. These algorithms are based on candidate tree modeling of multicasting. In [37], authors propose algorithms for generation of candidate trees for EON multicasting, since according to [36], one of the most efficient approaches to model and optimize multicasting in EONs is the candidate tree concept. Finally, in [38], authors study the multicast-capable routing, modulation, and spectrum assignment schemes that consider the physical impairments due to both the transmission and light splitting in EONs.

As it is not possible to compare the proposed solution with all the heuristics in the literature, the heuristics that produce the most efficient solutions amongst the existing works were identified and are the ones that are compared with the proposed heuristic algorithms. These are the Member-Only (MO) heuristic [5], the On-Tree MC Node First (OTMCF) and Nearest MC Node First (NMCF) heuristics [21], and the Cost-Effective Multicasting Using Splitters (MUS) heuristic [22]. These heuristics are described below.

3.1. Member-only (MO) heuristic

The Member-Only (MO) heuristic algorithm is presented in [5] and it is the most efficient among the heuristics presented in the aforementioned paper. It was created for DaC networks and in this approach, the existence of cycles is not permitted in the derived *RSG*. Therefore, for this heuristic, the derived topology for the establishment of the multicast request is called a *Routing Tree (RT)*, which is the union of created Routing SubTrees (*RST*s). Specifically, for the creation of Routing SubTree RST_i, initially two sets are defined, set X containing the source node ($X = \{s\}$) and set Y that is initially empty ($Y = \emptyset$). Each path connecting a still unconnected destination is added in RST_i under the constraints that it originates from a node in X and it does not include any nodes of set Y. After the addition of a path in RST_i, sets X and Y are updated such that now X consists of only the MC nodes and leaf nodes of RST_i and set Y consists of the rest of the nodes of RST_i. The destinations are added in RST_i in a non-decreasing order, according to the cost of their corresponding paths. If no more destinations can be connected to the current RST_i, the procedure is repeated for a new *RST*, with X and Y again initialized to $X = \{s\}$ and $Y = \emptyset$.

The MO technique has the following drawbacks: (i) it forbids the existence of cycles, leading to high-cost solutions. (ii) If a destination can be connected through a path originating from a node in set X, this path is preferred to a path originating from the source, even in the case where the cost of the former is larger than that of the latter. (iii) The constraint that the path to be added cannot include any nodes of set Y leads to high-cost solutions.

The aforementioned drawbacks of MO constituted the motivation for the creation of the proposed MPH* heuristic (Section 4.1), that was designed in order to be capable of: (i) permitting the existence of cycles. (ii) Selecting a path originating from a node in set X, if it has lower cost compared to a path originating from the source. (iii) Permitting the path to be added to include nodes of set Y. These characteristics of MPH* lead to lower-cost solutions compared to MO, as shown in Section 6.

3.2. On-Tree MC Node First (OTMCF) and Nearest MC Node First (NMCF) heuristics

The On-Tree MC Node First (OTMCF) and Nearest MC Node First (NMCF) heuristic algorithms were initially presented in [21]. These techniques were created for DoC networks and in these approaches the existence of cycles is permitted in the derived *RSG*s.

Specifically, in OTMCF, initially each MI destination is connected with the closest MC node in G. Let the union of these paths constitute $PU = (V_{PU}, A_{PU})$ and let the MC nodes where the MI destinations are connected, constitute set X. Subsequently, a minimum-cost RSG' is generated, that connects the source with the MC destinations as well as the MC nodes of set X. The final *RSG* is derived by the union $RSG' \cup PU$.

On the other hand, NMCF works reversely compared to OTMCF. Initially, a minimum-cost RSG' is generated, that connects the source with the MC destinations. Subsequently, each MI destination is connected with the closest MC node in RSG'. Let the union of these paths constitute $PU = (V_{PU}, A_{PU})$. The final *RSG* is derived by the union $RSG' \cup PU$.

3.3. Cost-Effective Multicasting Using Splitters (MUS) heuristic

This heuristic algorithm is presented in [22] and was also created for DoC networks. It works similarly to NMCF, with two improvements that make it more efficient:

- In NMCF, the MI destinations are connected with the closest MC node in RSG' in random order, while in MUS they are connected in non-decreasing order according to the cost of the shortest path between them and the closest MC node in RSG'.

- In NMCF, all MI destinations are connected only with MC nodes in *RSG′*. In MUS, however, an MI destination can be connected either with an MC node in *RSG′* or with an MC node belonging to a path of a previously connected MI destination. The minimum-cost solution between the two is subsequently selected.

All the presented existing heuristics lack the capability of locating the MC nodes that can lead to lower-cost solutions if inserted in the destination set. This was exactly the motivation for the creation of the proposed SSMRH (base) heuristic (Section 4.2). The latter was designed in order to be capable of deriving low-cost solutions (as shown in Section 6) by the insertion of the appropriate MC nodes in the destination set.

4. Proposed heuristic algorithms

The proposed heuristic algorithms for multicast routing in sparse-splitting networks are presented in this section. Specifically, two heuristics are proposed, namely the MPH* and the Sparse-Splitting Multicast Routing Heuristic (SSMRH (base)) heuristics. Both of them are described in detail below.

4.1. MPH* heuristic algorithm

A heuristic algorithm that is used extensively in the literature for multicast routing is the *Minimum Path Heuristic (MPH)* [39]. It consists of the following Steps:

1. $RSG = (V_{RSG}, A_{RSG}) = (\{s\}, \emptyset)$, $Y = D$
2. While $(Y \neq \emptyset)$
   (a) Find P_{uv}, $u \in V_{RSG}$, $v \in Y$ such that:
   $c_{P_{uv}} \leq c_{P_{u^*v^*}}$, $\forall u^* \in V_{RSG}$, $v^* \in Y$
   (b) $RSG \leftarrow RSG \cup P_{uv}$
   (c) $Y \leftarrow Y/\{v\}$

In simple words, initially the *RSG* consists only of the source. At each iteration of Step 2, from the unconnected destinations, the one that is closest to the current *RSG* is added to it. The algorithm terminates when all destinations are connected to the tree.

The MPH heuristic can be applied only in networks with full-splitting capability, since the paths of Step 2a can originate from any network node. In order to be applicable to sparse-splitting networks without DaC nodes, this path must originate either from the source or from an MC node that belongs to the current tree. In the case that the MI nodes are DaC, the path can also originate from destinations that are leaves on the current tree. This modified heuristic is called *MPH** and it consists of the following Steps ($DaC = 1$ for DaC networks, $DaC \neq 1$ for DoC networks, $c^S_{MPH^*}$ and $w^{MPH^*,S}_{ij}$ are written as c and w_{ij}, respectively, for simplicity):

4.1.1. Formulation of the MPH* heuristic

1. $RSG = (V_{RSG}, A_{RSG}) = (\{s\}, \emptyset)$, $X = \{s\}$, $Y = D$, $c = 0$
   $\forall i, j \in V_G$: $w_{ij} = 0$
2. $\forall v' \in D$, $v'' \in V_G$: Calculate $P_{v'v''}$ → Derive $P_{v''v'}$ by reversing the arcs of $P_{v'v''}$
3. While $(Y \neq \emptyset)$
   (a) Find P_{uv}, $u \in X$, $v \in Y$: $c_{P_{uv}} \leq c_{P_{u^*v^*}}$, $\forall u^* \in X$, $v^* \in Y$
   (b) $\forall v' \in V_{P_{uv}}$: If $(v' \in MC_{set})$ $X \leftarrow X \cup \{v'\}$
   (c) If $(DaC = 1)$
      (1) $X \leftarrow X \cup \{v\}$
      (2) If $(u \notin MC_{set})$ $X \leftarrow X/\{u\}$
   (d) $RSG \leftarrow RSG \cup P_{uv}$
   (e) $c \leftarrow c + c_{P_{uv}}$
   (f) $\forall [v', v''] \in A_{P_{uv}}$: $w_{ij} \leftarrow w_{ij} + 1$
   (g) $Y \leftarrow Y/\{v\}$

MPH* works for both DoC and DaC networks. At each iteration, from the unconnected destinations, the one that is closest to the current *RSG* is added to it. It terminates when all the destinations are connected to *RSG*. Step 2 calculates all paths that may be used during the construction of *RSG*. Since every path added in *RSG* ends at a node in *D*, it is sufficient to calculate every path $P_{v'v''}$ such that v' is in *D* and v'' is in V_G and derive $P_{v''v'}$ from it (by reversing the arcs of $P_{v'v''}$). This can be realized using a shortest-path algorithm, such as Dijkstra's algorithm [40]. Steps 3b and 3c update *X*. Step 3b ensures that the path to be added in Step 3a in the next iteration of the *while* loop, must originate from an MC node of the current *RSG*. If the network is DaC, the path can also originate from a node that is a leaf on *RSG*. This is settled in Step 3c: After the addition of path P_{uv}, node v is added in *X* (Step 3c(*i*)), since it is now a leaf node on *RSG*. If node u is MI, it is excluded from *X* (Step 3c(ii)), since after the addition of P_{uv} in *RSG*, u is now not a leaf node in the latter. Steps 3d–3f update *RSG*, c, and w_{ij} respectively. Finally, in Step 3g the just added destination is removed from *Y*.

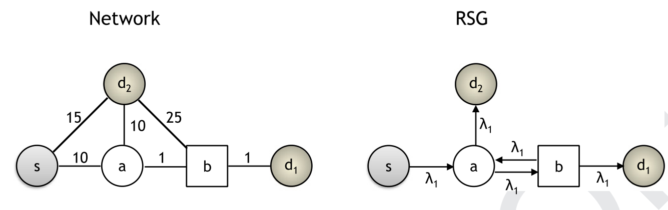

Fig. 7. Simple example of MPH* heuristic.

Contrary to MO, in the case of MPH*, if there is a choice between a path originating from the source and one originating from any other appropriate node of the current *RSG*, the least-cost one is selected. This leads to decreased cost compared to the MO heuristic. Consider for example the case where MPH* is applied to the DaC network of Fig. 7. It can be easily verified that the derived *RSG* is the one given in the figure on the right, with a cost equal to 23. On the other hand, if MO is applied, first d_1 will be connected with the source, with the same path as in the MPH* case. Then, for the connection of d_2, under the constraint of MO that the MI non-leaf nodes of the current *RSG* cannot be part of any newly added path, paths $\{[b, a], [a, d_2]\}$ $\{[s, a], [a, d_2]\}$ cannot be used. Furthermore, under the constraint that a path originating from an MC node or a leaf node of the current *RSG* is preferable compared to a path originating from the source, the path $\{[b, d_2]\}$ with cost equal to 25 would be selected for the connection of d_2 (instead of $\{[s, d_2]\}$ with cost equal to 15), leading to an *RSG* of total cost equal to 37.

4.1.2. Computational complexity of MPH*

In Step 2, Dijkstra's algorithm (with complexity $O(m + n \log n)$ if implemented with Fibonacci heaps [41]) must be executed $|D| = k$ times. Therefore, the complexity of this step is $O(km + kn \log n)$. The part of Step 3 with the highest order complexity is 3*a*, where at most kn paths are compared in terms of their cost. Since Step 3 is repeated at most k times, it has complexity $O(k^2 n)$.

Thus, the result of the above is that the complexity of MPH* is $O(km + kn \log n + k^2 n)$.

4.2. SSMRH (base) heuristic algorithm

The study of various examples has shown that MPH* as well as the existing algorithms have improved performance *if specific MC nodes are added in the destination set*. An example where this happens is shown in Fig. 8, for a multicast session with source node *s* and destination set $D = \{d_1, d_2\}$. If MPH* is applied either to the DaC network of Fig. 8(i) or to the DoC network of Fig. 8(ii), the derived *RSG* will be the one given in Fig. 8(iii), with cost equal to 40. However, if MC node *b* is added into the destination set, the derived *RSG* will be the one given in Fig. 8(iv), with cost equal to 33 for the DaC case and 22 for

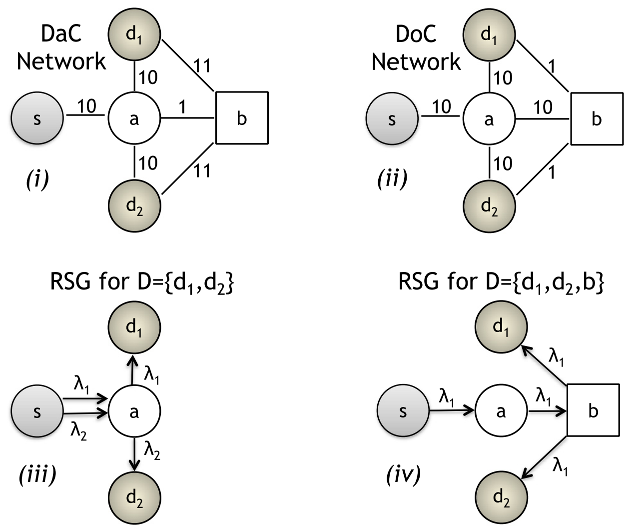

Fig. 8. Improving the performance of MPH* by adding an MC node in the destination set.

the DoC case. After the addition of node b in D, the optimal solution is derived by MPH* for both DaC and DoC cases.

The observation that the derived solution may be improved by adding MC nodes in the destination set, has led to a development of a *general solution-enhancing technique* called *Sparse-Splitting Multicast Routing Heuristic (SSMRH (base))*. SSMRH starts with the solution derived by a *base* heuristic and, subsequently, it sequentially adds MC nodes in the destination set to improve this solution. All the existing heuristics as well as the proposed MPH* can be set as the *base* for SSMRH. The latter consists of the following Steps (again, c^{S}_{SSMRH} and $w_{ij}^{SSMRH,S}$ are written as c and w_{ij} for simplicity):

4.2.1. Formulation of the SSMRH (base) heuristic

1. $X = D$
2. Apply *base* heuristic for destination set X: $\rightarrow RSG, c, w_{ij}\ \forall i, j \in V_G$
3. $\forall u \in MC_{set}$ and $u \notin V_{RSG}$:
   (a) $X \leftarrow X \cup \{u\}$
   (b) Apply *base* heuristic for destination set X: $\rightarrow RSG\{u\}, c\{u\}, w_{ij}\{u\}\ \forall i, j \in V_G$
   (c) $X \leftarrow X / \{u\}$
4.
   (a) Find $v \in MC_{set}$ and $v \notin V_{RSG}$: $c\{v\} \le c\{v^*\}$, $\forall v^* \in MC_{set}$ and $v^* \notin V_{RSG}$
   (b) If $c\{v\} \ge c$, Stop. Else Continue
5. $RSG = RSG\{v\}$, $c = c\{v\}$, $w_{ij} = w_{ij}\{v\}\ \forall i, j \in V_G$
6. $X \leftarrow X \cup \{v\}$
7. Return to Step 3

The SSMRH algorithm works as follows: initially, the base heuristic is used for the derivation of RSG, c, and $w_{ij}\ \forall i, j \in V_G$, for destination set $X = D$ (Step 2). One of the MC nodes (u) that is not part of the RSG derived by the *base* heuristic, is added *temporarily* in the destination set X (Step 3a), the corresponding $RSG\{u\}$, $c\{u\}$ and $w_{ij}\{u\}\ \forall i, j \in V_G$ are calculated using *base* (Step 3b) and u is removed from X (Step 3c). This procedure is repeated for every MC node not in RSG. Subsequently, the MC node (v) that, if added in X, gives the $RSG\{v\}$ with the least cost is found (Step 4a). If the cost of $RSG\{v\}$ is greater or equal to the cost of RSG, further decrease of the cost of RSG cannot be obtained and SSMRH stops; otherwise, it continues (Step 4b). In Step 5 RSG, c and $w_{ij}\ \forall i, j \in V_G$ are updated to $RSG\{v\}$, $c\{v\}$ and $w_{ij}\{v\}\ \forall i, j \in V_G$ respectively. Node v is added *permanently* into X (Step 6) and the algorithm returns to Step 3. It terminates either in Step 4b (as described previously) or in Step 3, if no node u exists such that u is in MC_{set} and not in V_{RSG}.

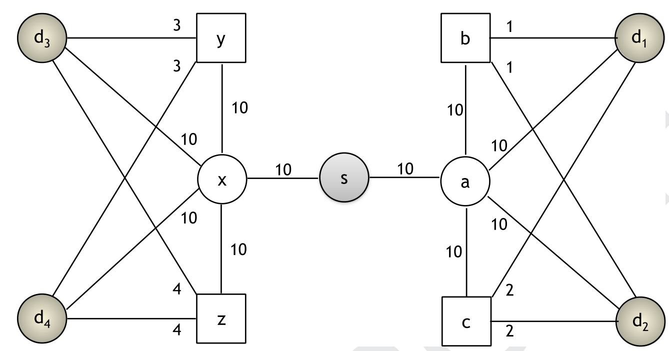

Fig. 9. Example for explanation of the SSMRH algorithm (DoC network).

Since SSMRH is initialized with the solution derived by the *base* heuristic, the solution of the former is always at least as good as the one derived from the latter.

The reader should note that even though Step 3 can find all MC nodes that give a RSG with less cost if added into the destination set X, this step is not executed only once resulting in the permanent addition in X of all MC nodes identified. The reason for not doing this is explained using the example in Fig. 9. In this example (where the network is considered to be DoC) if MPH* is used, the resulting RSG will consist of paths $[s, a]$, $[a, d_1]$, $[s, a]$, $[a, d_2]$, $[s, x]$, $[x, d_3]$ and $[s, x]$, $[x, d_4]$, with total cost equal to 80. After the execution of Step 3, it is found that the addition of each one of the MC nodes b, c, y, z in X will give a RSG with less cost, with $RSG\{b\}$ having the least cost among $RSG\{i\}$, $i = b, c, y, z$. If MC node b is added in X, the addition of MC node c as well in X will not give any improvement (the opposite will actually occur), since this node "serves" the same destinations as MC node b. Therefore, it is not prudent to add in X *all* MC nodes that can give a more efficient RSG in Step 3. After the addition of MC node b, Step 3 must be repeated, to find the next MC node to be added in D, node y in this case, and in the third iteration the procedure stops, since further improvement cannot be achieved.

4.2.2. Computational complexity of SSMRH

Assume that the *base* heuristic of SSMRH has time complexity equal to $O(b)$, and z MC nodes exist in the network. Therefore, the complexity of Step 2 is $O(b)$. Since Step 3 has complexity $O(zb)$ and it is repeated at most z times, SSMRH has complexity $O(z^2b)$.

SSMRH was evaluated using MPH* as *base* (resulting to SSMRH (*MPH**)). All the existing heuristics described in Section 3 were used as *base* as well. However, only SSMRH (*MUS*) is presented in Section 6, since it gave the best results among SSMRH (*base*), *base*=MO, OTMCF, NMCF, MUS. According to the aforementioned complexity of SSMRH, SSMRH (*MPH**) has complexity $O(z^2km + z^2kn\log n + z^2k^2n)$. In addition, it can be easily derived that MUS has the same complexity as MPH*. Consequently, SSMRH (*MUS*) has the same complexity as SSMRH (*MPH**).

5. Optimal solution obtained by integer linear programming (ILP)

In this section, a novel Integer Linear Programming (ILP) formulation for the sparse-splitting multicast routing and wavelength assignment problem is presented in networks where the MI nodes can be either DaC or DoC. The proposed formulation aims to minimize the cost of the derived RSG in terms of the cost of the utilized arcs (as defined in Eq. (1)), in sparse-splitting optical networks where all nodes have wavelength conversion capabilities. Specifically, the ILP is given as input a specific multicast instance; that is, a network topology, the set of wavelengths that can be used, a placement of optical splitters, the characteristics of destination nodes (DaC or DoC) and a multicast session. It returns the optimal combination of used wavelengths and arcs in order to establish the requested multicast session and minimize the cost of the derived *RSG*. The ILP computes lightpath pairs from source to every destination with minimum aggregate cost. Different lightpaths (belonging to the same multicast session) can share common arcs and wavelengths which is the main characteristic of multicast routing.

The following parameters and variables are used to formulate the aforementioned problem.

Parameters:

- Set of MI nodes: $MI_{set} \subseteq V_G$
- Cost of arc $[i, j]$: c_{ij}
- Multicast session: $S = \{s, d_1, \ldots, d_k\}$, where s is the source node and d_j, $j = 1, \ldots, k$ are the destination nodes of the multicast session
- Set $\mathcal{W} = \{1, 2, \ldots, W\}$ of W distinct wavelengths
- A large constant B

Variables:

- $Q_{ij}^{d,w}$: Boolean variable representing the path from source node s to destination node d, equal to one if the path of the multicast session from source node s to destination node d occupies the arc $[i, j]$ between nodes i, j and wavelength w; zero otherwise.
- R_{ij}^{w}: Boolean variable equal to one if the session uses arc $[i, j]$ and wavelength w; zero otherwise.

Objective:

$$Minimize : \sum_{ij} \sum_{w} R_{ij}^{w} \cdot c_{ij}$$

Subject to the following constraints:

- Source node has one outgoing flow unit for each path and zero incoming flow

$$\sum_{i} \sum_{w} Q_{si}^{d,w} - \sum_{i} \sum_{w} Q_{is}^{d,w} = 1, \forall d \in S \quad (3)$$

- Destination node has one incoming flow unit for each path of the session and zero outgoing flow

$$\sum_{i} \sum_{w} Q_{id}^{d,w} - \sum_{i} \sum_{w} Q_{di}^{d,w} = 1, \forall d \in S \quad (4)$$

- Every intermediate node has the same incoming and outgoing flow for each path

$$\sum_{i} \sum_{w} Q_{ij}^{d,w} = \sum_{i} \sum_{w} Q_{ji}^{d,w}, \forall d \in S, \forall j \neq s, d \quad (5)$$

- Every intermediate MI node cannot split the incoming signal

$$\sum_{i} \sum_{w} R_{ij}^{w} \geq \sum_{i} \sum_{w} R_{ji}^{w}, \forall j \in MI_{set}, j \neq s, d \quad (6)$$

- Number of used wavelengths per fiber

$$\sum_{d} Q_{ij}^{d,w} \leq B \cdot R_{ji}^{w}, \forall w \in \mathcal{W}, \forall i, j \in V_G \quad (7)$$

- Used arcs-wavelengths should be utilized by at least one lightpath

$$R_{ji}^{w} \leq \sum_{d} Q_{ji}^{d,w}, \forall w \in \mathcal{W}, \forall i, j \in V_G \quad (8)$$

- DoC nodes can only tap or forward the incoming signal

$$\sum_{i} \sum_{w} R_{id}^{w} - \sum_{i} \sum_{w} R_{di}^{w} \geq 1, \forall d \in S \cap MI_{set} \quad (9)$$

- DaC nodes can both tap and forward the incoming signal

$$\sum_{i} \sum_{w} R_{id}^{w} - \sum_{i} \sum_{w} R_{di}^{w} \geq 0, \forall d \in S \cap MI_{set} \quad (10)$$

The objective function accounts for the cost of the arcs used multiplied by the wavelengths utilized on these arcs. Constraints (1)–(3) correspond to the flow conservation constraints. Specifically, constraint (1) ensures that the incoming traffic is satisfied and the source node has one flow unit for each destination. Constraint (2) ensures that each destination node has one incoming flow unit, while constraint (3) ensures flow conservation for intermediate nodes with wavelength conversion capability. Constraint (4) prohibits the splitting in a node with no splitting capabilities. This means that the outgoing traffic in an intermediate MI node should not be greater than the incoming traffic in order to prevent the splitting of the incoming signal. Signal splitting is only possible in MC nodes. Constraints (5) and (6) are used to define the connection between variables $Q_{ij}^{d,w}$ and R_{ij}^{w}. Specifically, inequality (5) is a wavelength usage constraint that is used to count the wavelengths used on each arc. The large constant B is used to count only once the usage of a specific arc and wavelength by different lightpaths. Constraint (6) ensures that if wavelength w is used on arc $[i, j]$, then at least one session should occupy the wavelength w on arc $[i, j]$. Constraints (7) and (8) are used for destination nodes that are MI. Specifically, constraint (7) ensures that when the destination is a DoC node then the incoming signal can only be dropped or forwarded in that node and constraint (8) ensures that when the destination is a DaC node then the incoming signal can be both dropped and forwarded at that node.

The proposed ILP is used for comparison purposes (providing the lower bound) with the existing and proposed heuristic algorithms.

6. Performance evaluation

The existing and proposed heuristics were evaluated under the assumptions of the current paper, as given in Section 4. The cost of the derived *RSG*s as defined in Eqs. (1) and (2) was used as the performance criterion. The evaluation was performed using simulations on the well known USNET [9] and NSFNET [10] networks illustrated in Figs. 10 and 11 respectively, as well as on several larger, randomly created networks. The USNET and NSFNET are used mainly for comparison with the optimal solution, since they are small enough for the ILP formulation to be solved in a reasonable time. On the other hand, the randomly created networks are large enough so as to stress the improved performance of the proposed heuristics, compared to the existing ones.

The reader should note that since the MO heuristic was created only for DaC networks, for comparison purposes, it was modified so that it can be simulated for DoC networks as well. Similarly, MUS, that was created for DoC networks, was also modified so as to work efficiently for DaC networks as well. The rest of the existing heuristics (NMCF, OTMCF), also created for DoC networks, were simulated without any changes for DaC networks, since it is not possible to be adapted for this category of networks.

6.1. Evaluation on the USNET and NSFNET graphs

The USNET graph consists of 24 nodes and 43 connections, where each connection consists of two opposite arcs with equal cost. The simulation was repeated for $k = 3, 6, 9, 12$, where k is the number of destinations. Five hundred multicast requests were generated for each k, with the source and destination nodes randomly selected for each multicast request. The *RSG* for each multicast request was derived utilizing each one of the existing and proposed heuristics. The optimal *RSG* was derived as well, using the proposed ILP formulation. For each k simulated, the corresponding average cost $\bar{c}_H$ of the 500 derived *RSG*s using heuristic H, was calculated using Eq. (2). For the

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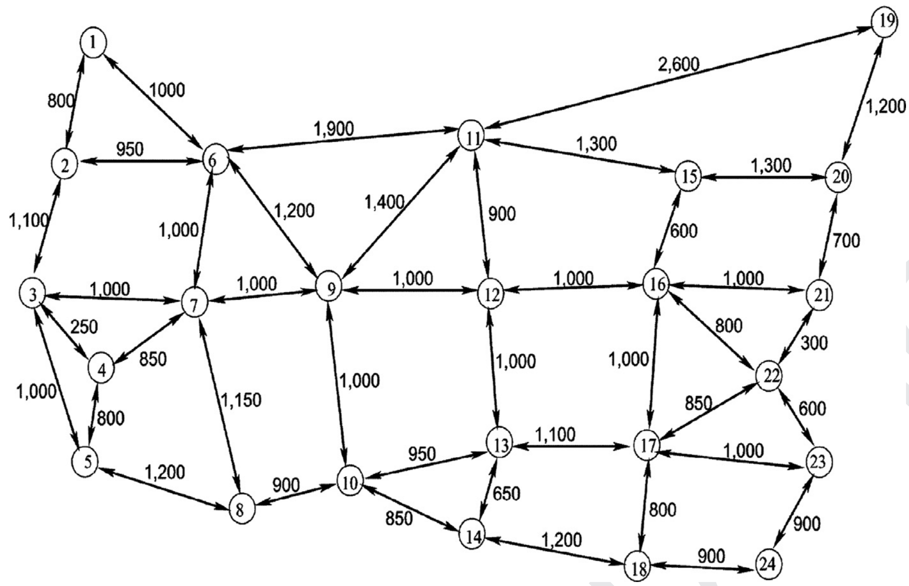

Fig. 10. USNET graph.

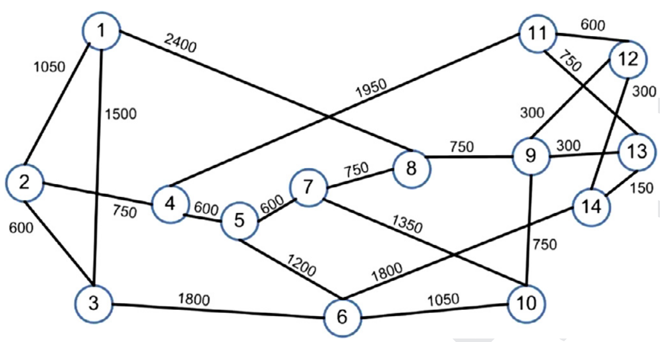

Fig. 11. NSFNET graph.

optimal solution, this cost is written as $\bar{c}_{opt}$. The following were calculated as well, for a detailed evaluation of the simulation results:

$$I_H = 100 \times \frac{\bar{c}_H - \bar{c}_{opt}}{\bar{c}_{opt}} \quad (11)$$

$$SO_H = 100 \times \frac{\text{number of suboptimal } RSG\text{ s}}{500} \quad (12)$$

where I_H gives the extra average cost of heuristic H, compared to the optimal solution, while SO_H gives the percentage of the cases where heuristic H fails to give the optimal solution.

The aforementioned procedure was repeated for $z = 4, 8, 12$, where z is the number of MC nodes. As the problem of placement of the MC nodes is beyond the scope of this work, the MC nodes were allocated in the network utilizing the *kmaxD* method as described in [42] (i.e., the MC nodes were placed at the nodes that have the largest degree). All the above are repeated for both cases where the MI nodes have the DoC or the DaC ability.

The same procedure was repeated for the NSFNET graph, consisting of 14 nodes and 22 connections, where each connection consists of two opposite arcs with equal cost. Here, the simulation was repeated for $k = 2, 4, 6, 8$, and $z = 3, 6$.

The results of the simulation on the USNET are given in Table 1. Amongst the existing heuristics MUS gave the best results for every simulated case (for both DoC and DaC networks); its evaluation parameters are presented in Table 1.

The results of the simulation on the NSFNET are given in Table 2. The best result among the existing heuristics was selected to be presented for each case (given in the parenthesis next to each result).

The proposed SSMRH technique, whose results are presented in the same table, utilizes MUS as the *base* algorithm for the DoC case and MPH* as the *base* algorithm for the DaC case.

From the results of Tables 1 and 2, it can be seen that:

- SSMRH outperforms the best existing heuristic, for every simulated case.
- In terms of the cost of the derived *RSGs*, SSMRH gives results very close to the optimal ones. For the DoC case of the USNET, it gives on average 0.09% extra cost compared to the optimal solution, and 0.17% for the worst case, while for the DaC case of the USNET, it gives 0.7% and 2.63% extra cost compared to the optimal solution for the average and worst case respectively. For the DoC case of the NSFNET, it gives on average 0.01% extra cost compared to the optimal solution, and 0.04% for the worst case, while for the DaC case of the NSFNET, it gives 0.35% and 1.03% extra cost compared to the optimal solution for the average and worst case, respectively.
- SSMRH outperforms the best existing heuristic in terms of the percentage of the derived optimal solutions as well. For the DoC case of the USNET, SSMRH succeeds in obtaining the optimal solution for the majority of the simulated cases. It fails to give the optimal solution only in 4.98% and 10.8% of the simulated cases for the average and worst case respectively, whereas MUS (i.e., the best existing for this network) fails in 52.87% and 70.6% of the simulated cases for the average and worst case respectively. For the DaC case of the USNET, the relevant results are 20.1% and 63.6% for the average and worst case of SSMRH, and 66.7% and 95% for the average and worst case of MUS. For the DoC case of the NSFNET, SSMRH fails to give the optimal solution only in 0.30% and 1.00% of the simulated cases for the average and worst case respectively, whereas the best existing heuristic for each case fails in 17.04% and 38.60% of the simulated cases for the average and worst case respectively. For the DaC case of the NSFNET, the relevant results are 7.33% and 20.60% for the average and worst case of SSMRH, and 49.90% and 73.20% for the average and worst case of the best existing heuristic.

Summarizing the performance analysis for the USNET and NSFNET, it is clear that SSMRH gives results almost identical with the optimal ones, leaving little space for improvement for any possible subsequent heuristics, especially for the DoC case.

Table 1
Evaluation on the USNET graph.

	z	k	$\bar{c}_{opt}$	$\bar{c}_{MUS}$	$\bar{c}_{SSMRH}$	I_{MUS}	I_{SSMRH}	SO_{MUS}	SO_{SSMRH}
	4	3	7204,6	7452,3	7205,3	3,44	0,01	26,4	0,4
		6	12454,2	12902,9	12462,0	3,60	0,06	44,4	3,2
		9	16972,7	17466,2	16985,6	2,91	0,08	51,4	6,0
		12	21408,8	21881	21422	2,20	0,06	56,4	3,6
	8	3	6477,1	6679,2	6483,4	3,12	0,10	35,6	2,0
		6	10227,2	10614,0	10244,6	3,78	0,17	53,2	6,2
DoC		9	13178,0	13607,5	13197,5	3,26	0,15	62,2	10,8
		12	15908,6	16360	15924	2,83	0,09	70,6	9,2
	12	3	6182,0	6378,5	6184,0	3,18	0,03	37,8	1,6
		6	9295,3	9682,3	9310,7	4,16	0,17	58,4	5,8
		9	11680,2	12206,4	11692,1	4,51	0,10	70,2	5,8
		12	13773,5	14334	13784	4,07	0,07	67,8	5,2
		Avg	**12063,5**	**12463,7**	**12074,6**	**3,42**	**0,09**	**52,87**	**4,98**
	4	3	6205,5	6340,2	6211,6	2,17	0,10	23,4	2,2
		6	9303,9	9804,6	9383,4	5,38	0,85	57,4	18,8
		9	11484,2	12286,0	11686,9	6,98	1,77	79,0	45,4
		12	13397,2	14505	13749	8,27	2,63	90,0	63,6
	8	3	6127,7	6317,7	6129,4	3,10	0,03	35,6	1,4
		6	9113,5	9622,5	9144,3	5,59	0,34	67,6	10,6
DaC		9	11183,2	12002,4	11273,4	7,33	0,81	85,6	29,6
		12	13011,2	14168	13161	8,89	1,15	95,0	46,6
	12	3	6043,7	6214,7	6044,2	2,83	0,01	37,2	0,6
		6	8921,1	9276,1	8927,9	3,98	0,08	66,8	3,2
		9	10927,3	11441,6	10940,9	4,71	0,12	79,2	6,8
		12	12681,8	13271	12700	4,65	0,14	83,4	12,2
		Avg	**9866,7**	**10437,5**	**9946,0**	**5,3**	**0,7**	**66,7**	**20,1**

Table 2
Evaluation on the NSFNET graph.

	z	k	$\bar{c}_{opt}$	$\bar{c}_{existing}$	$\bar{c}_{SSMRH}$	$I_{existing}$	I_{SSMRH}	$SO_{existing}$	SO_{SSMRH}
	3	2	3499,2	3595,8 (MUS)	3499,2	2,76	0,00	11,60	0,00
		4	5949,9	6388,2 (MUS)	5949,9	7,37	0,00	34,20	0,00
		6	8044,8	8674,5 (MUS)	8046,0	7,83	0,01	38,60	0,40
		8	9837,0	10516,5 (MUS)	9838,2	6,91	0,01	31,20	0,20
	6	2	3335,4	3408,6 (MUS)	3335,4	2,19	0,00	11,60	0,00
		4	5386,2	5655,3 (OTMCF)	5388,0	5,00	0,03	37,20	0,40
DoC		6	6884,7	7044,6 (OTMCF)	6887,4	2,32	0,04	34,40	1,00
		8	8094,6	8237,4 (OTMCF)	8096,4	1,76	0,02	33,40	0,40
		Avg	**6379,0**	**6690,1**	**6380,1**	**4,52**	**0,01**	**17,04**	**0,30**
	3	2	3202,5	3286,5 (MUS)	3202,5	2,62	0.00	14,60	0,00
		4	4776,9	5123,7 (MUS)	4789,8	7,26	0,27	42,80	4,40
		6	5904,0	6508,8 (MO)	5937,3	10,24	0,56	63,60	11,20
		8	6767,1	7584,6 (MO)	6837,0	12,08	1,03	73,20	20,60
	6	2	3185,4	3285,3 (MUS)	3185,4	3,14	0.00	19,00	0,00
		4	4732,2	5115,3 (MUS)	4737,6	8,10	0,11	53,40	2,00
DaC		6	5828,7	6266,7 (MO)	5841,0	7,51	0,21	62,00	5,00
		8	6636,9	7174,2 (MO)	6675,6	8,10	0,58	70,60	15,40
		Avg	**5129,2**	**5543,1**	**5150,8**	**7,38**	**0,35**	**49,90**	**7,33**

6.2. Evaluation on randomly created networks

The improved performance of the proposed heuristics can be illustrated more clearly on larger networks. Six network configurations were randomly created, consisting of $n = 40$, 60, and 80 nodes, where a pair of configurations was created for each n; one consisting of $m = 2n$ connections and one consisting of $m = 3n$ connections. Three networks were randomly created for each configuration, i.e., a total number of 18 random networks were created.

Each connection consisted of two opposite arcs with equal cost, and 64 wavelengths are assumed to be available on each arc. A random (integer) cost, varying from 1 to 1000, was assigned to each connection. Let the *nominal distance* d_{nom}^{ij} between two nodes i and j be defined as $d_{nom}^{ij} = |i - j|$. The constraint that every network connection had to connect nodes that satisfy $d_{nom}^{ij} \leq \frac{n}{5}\ \forall i, j$, was used for every randomly created network. The reason is that the created network graphs should simulate a real telecommunications network, where the nodes that are connected belong to the same "neighborhood".

The performance of the existing and proposed heuristics was evaluated for $k = \frac{n}{10}, \frac{2n}{10}, \frac{3n}{10}$ destinations, where 500 multicast sessions were randomly created for each k. For the evaluation on the random networks, the simulation was set up in a way analogous to the one of the USNET network (i.e., the proposed SSMRH utilizes MUS and MPH* as the *base* algorithm for the DoC and DaC cases, respectively).

The existing heuristics (as well as the proposed MPH* heuristic) were compared to SSMRH, using the following equations (where H stands for the heuristic that is compared to SSMRH and j refers to the random network being used, i.e., $j = 1, \ldots, 18$):

$$\mathcal{I}_H^j = \frac{1}{3} \sum_i \left(100 \times \frac{\bar{c}_H^j[i] - \bar{c}_{SSMRH}^j[i]}{\bar{c}_{SSMRH}^j[i]} \right), \quad i = \frac{n}{10}, \frac{2n}{10}, \frac{3n}{10} \qquad (13)$$

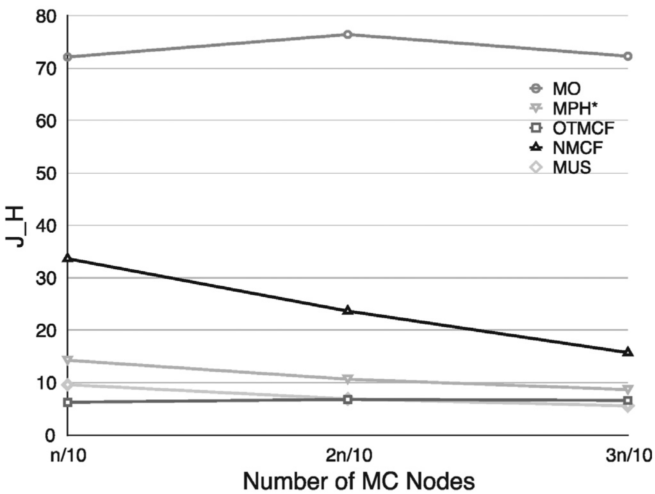

Fig. 12. Evaluation on the randomly created networks (DoC networks, actual cost).

$$\mathcal{J}_H = \frac{1}{18}\sum_j \mathcal{I}_H^j, \quad j = 1, \ldots, 18 \tag{14}$$

In Eq. (13), $\bar{c}_H^j[i]$ stands for the average cost $\bar{c}_H$ of heuristic H, for number of destinations $k = i$ (i.e., the average cost over the 500 multicast requests for this k), for the case of jth random network (from a total number of 18 randomly created networks). Simply, $\mathcal{I}_H$ gives, for heuristic H and network j, the *average value* of the *% relative increase of the average cost* compared to SSMRH, over all k investigated. In Eq. (14), $\mathcal{J}_H$ gives the average value of $\mathcal{I}_H^j$ over all randomly created networks.

The comparison of the existing and proposed algorithms was performed using Eqs. (13) and (14) defined above (results presented in Figs. 12–15 and in Table 3), since we considered that the relative (increased) cost of an existing heuristic compared to the proposed SSMRH technique can more clearly illustrate the improved performance of the latter, rather than presenting the actual cost of each of the existing and proposed heuristics.

This procedure is repeated for different numbers of MC nodes in the random networks, z ($z = \frac{n}{10}, \frac{2n}{10}, \frac{3n}{10}$), for both the DoC and DaC cases. The entire simulation is executed twice, once having the cost of each connection as described, so as to evaluate the existing and proposed heuristics in terms of the cost of the derived *RSGs* ("actual

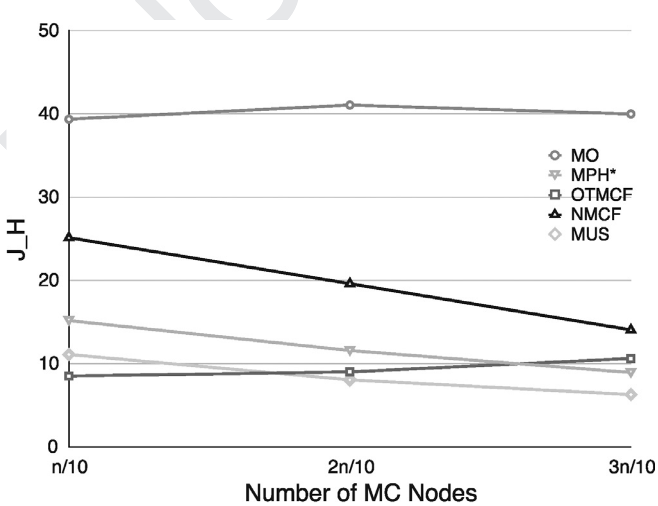

Fig. 13. Evaluation on the randomly created networks (DoC networks, wavelength usage).

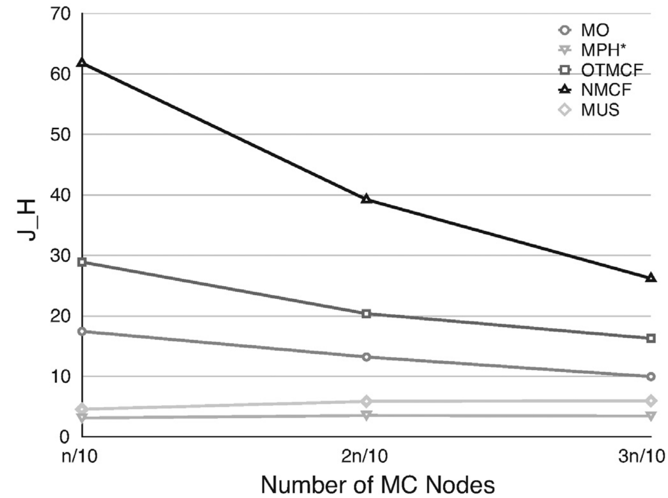

Fig. 14. Evaluation on the randomly created networks (DaC networks, actual cost).

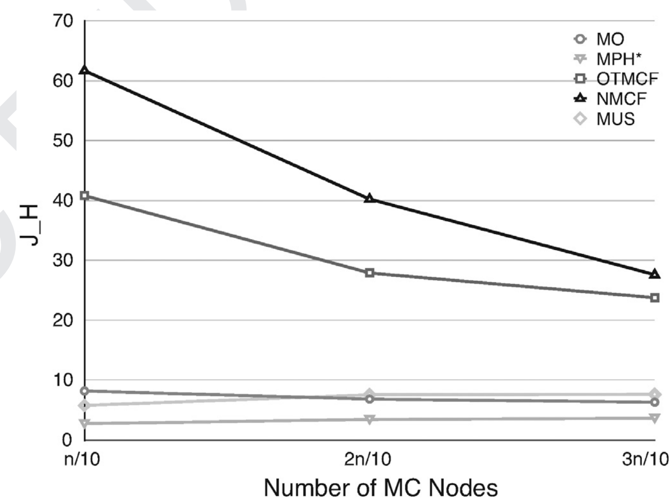

Fig. 15. Evaluation on the randomly created networks (DaC networks, wavelength usage).

Table 3
Evaluation on the randomly created networks.

		z	$\mathcal{J}_{MO}$	$\mathcal{J}_{MPH^*}$	$\mathcal{J}_{OTMCF}$	$\mathcal{J}_{NMCF}$	$\mathcal{J}_{MUS}$
DoC	**Act. cost**	$\frac{n}{10}$	72,12	14,25	6,26	33,64	9,60
		$\frac{2n}{10}$	76,41	10,63	6,77	23,64	6,87
		$\frac{3n}{10}$	72,27	8,66	6,58	15,71	5,56
		Avg	**73,60**	**11,18**	**6,54**	**24,33**	**7,34**
	W. usage	$\frac{n}{10}$	39,33	15,14	8,47	25,10	11,06
		$\frac{2n}{10}$	41,03	11,54	8,97	19,57	8,02
		$\frac{3n}{10}$	39,96	8,89	10,59	14,03	6,25
		Avg	**40,11**	**11,86**	**9,34**	**19,57**	**8,44**
DaC	**Act. cost**	$\frac{n}{10}$	17,45	3,11	28,91	61,81	4,56
		$\frac{2n}{10}$	13,22	3,54	20,37	39,24	5,87
		$\frac{3n}{10}$	9,97	3,46	16,29	26,20	5,97
		Avg	**13,55**	**3,37**	**21,86**	**42,42**	**5,47**
	W. usage	$\frac{n}{10}$	8,16	2,70	40,80	61,67	5,75
		$\frac{2n}{10}$	6,81	3,41	27,88	40,22	7,52
		$\frac{3n}{10}$	6,29	3,62	23,75	27,59	7,59
		Avg	**7,09**	**3,24**	**30,81**	**43,16**	**6,96**

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cost"), and once having the cost of each connection equal to one, so as to evaluate the existing and proposed heuristics in terms of the wavelength usage of the derived *RSG*s ("wavelength usage").

The total number of the scenarios investigated during the simulation for the case of randomly created networks was 648 (18 (number of networks) × 2 (DaC or DoC) × 2 (actual cost or wavelength usage) × 3 ($z = \frac{n}{10}, \frac{2n}{10}, \frac{3n}{10}$) × 3 ($k = \frac{n}{10}, \frac{2n}{10}, \frac{3n}{10}$)), with 500 multicast requests for each scenario.

For better visualization, the results for the randomly created networks are presented both in Figs. 12–15 as well as in Table 3. From these results it can be seen that:

- SSMRH again outperforms all the existing heuristics, as well as the proposed heuristic MPH*, for every simulated case. This is due to the ability of it to locate and utilize the MC nodes that lead to a lower cost *RSG*, if added in the destination set.
- MUS outperforms NMCF for every case, as expected, since the former is an improvement of the latter.
- For the DoC case, OTMCF and MUS seem to have the best performance. Both these heuristics though, give average results with at least 6.54% and 8.44% extra *actual cost* and *wavelength usage* respectively, compared to the proposed SSMRH heuristic.
- For the DaC case, the proposed MPH* gives results closer to the ones obtained by SSMRH (compared to the existing heuristics), with both outperforming MO and MUS that give the best results amongst the existing heuristics. Both these heuristics, give average results with at least 5.47% and 6.96% extra *actual cost* and *wavelength usage* respectively, compared to the proposed SSMRH heuristic.
- The reader should note that the positive slope of the curve of OTMCF in Figs. 12 and 13 and the (approximately) zero slope of several heuristics in Figs. 14 and 15 does not mean that the average cost is increased or remains constant with the increase of the number of MC nodes, since the graphs of Figs. 12–15 give the % relative increase of the average cost compared to SSMRH rather than the actual average cost.
- Analyzing the way MUS, OTMCF and MPH* function, it can be seen that three different policies are applied. MUS and OTMCF split the destination set into MC and MI destinations. In MUS, all the MC destinations are added in the derived *RSG* prior to the MI ones, whereas in OTMCF the opposite happens. In MPH*, the destination set is not split. For the DaC case it can be seen that the policy of MPH* outperforms the other two. For the DoC case, the policies of MUS and OTMCF are more efficient. If the percentage of the MC nodes is small, OTMCF is superior. As this percentage increases, MUS becomes more efficient. One possible explanation for the decreased performance of OTMCF (*compared to SSMRH as well as MUS*) is the following: OTMCF first connects each MI destination with the closest MC node. Then, the source is connected with the MC destinations as well as with the MC nodes that are connected with the MI destinations. For the case of many MC nodes in the network, it is quite possible that most of the MI destinations are connected to a distinct MC node, and then all these distinct MC nodes must be connected with the source, thus leading to excessive use of network resources for the establishment of the *RSG* (i.e., to high cost). If the same MC node was used for the connection of two (or more) MI destinations, then only this MC node should be connected with the source. From Figs. 12 and 13 it is concluded that, on average, the cost decrease incurred in the case where each MI destination is connected with the closest MC node is less compared to the cost increase incurred for connecting all these MC nodes (that are connected to MI destinations) with the source. This does not happen to DaC networks (Figs. 14 and 15), leading to the conclusion that the existence of DaC MI nodes cancels this cost increase.

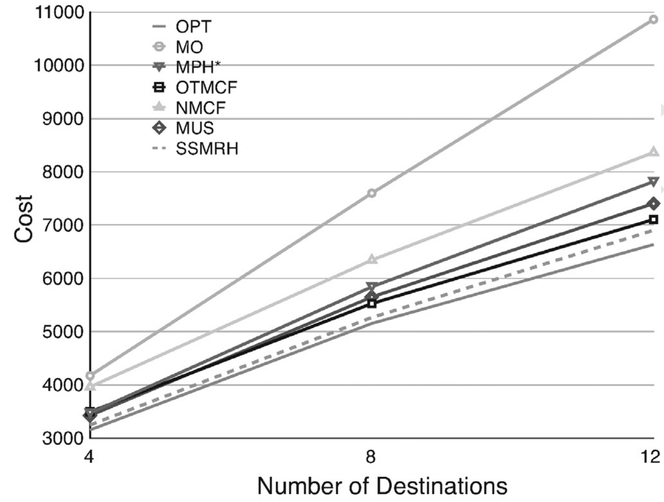

Fig. 16. Analytical results for an indicative network (DoC case).

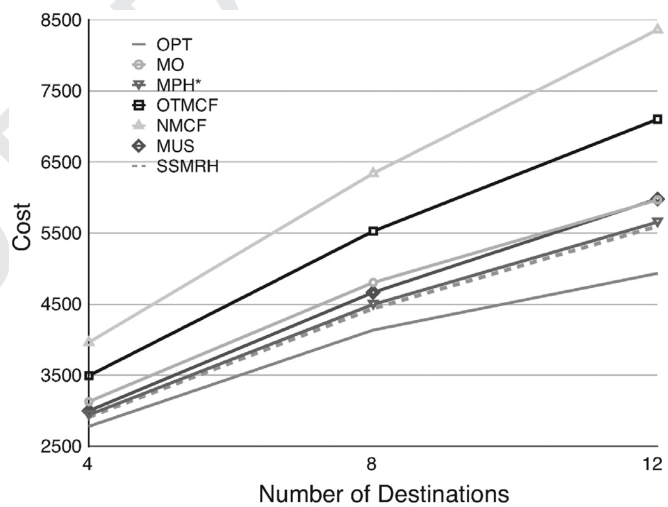

Fig. 17. Analytical results for an indicative network (DaC case).

- MO seems to have poor performance for the DoC case and much better performance for the DaC case. The reason is that the policy that MO applies (i.e., the same as MPH* where the destination set is not split), as stated above, is efficient for DaC networks but not for DoC. Compared to MPH*, MO has poorer results since, as described in Section 3.1, MO has some drawbacks compared to MPH* that lead to higher cost solutions.
- NMCF seems to have average performance for the DoC case and poor performance for the DaC case. The reason is that the policy that NMCF applies (i.e., all the MC destinations are connected with the source prior to the MI ones), as stated above, is efficient for DoC networks but not for DaC. Compared to MUS, NMCF has poorer results since, as described in Section 3.3, NMCF has some drawbacks compared to MUS, that lead to higher cost solutions.

The large number of different simulation scenarios has led to the necessity of averaging the results (Eqs. (13) and (14)). The analytical results (i.e., separately for each k) are presented for an indicative network consisting of 40 nodes and 80 links (with a random (integer) cost, varying from 1 to 1000, assigned to each link), for the case of 8 MC nodes. The results are presented in Figs. 16 (DoC case) and 17 (DaC case), where the average actual cost is presented for every investigated heuristic, for $k = 4, 8, 12$ (i.e., averaged over 500 multicast requests for each k). Figs. 16 and 17 also include the optimal results

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Table 4
Running time (in milliseconds) of investigated heuristics (per multicast request).

$\frac{m}{n}$ → Heuristic ↓	2	3	4	5
MO	0.51	0.53	0.55	0.54
MPH*	0.058	0.051	0.054	0.050
OTMCF	0.063	0.067	0.067	0.063
NMCF	0.051	0.049	0.052	0.047
MUS	0.087	0.081	0.083	0.072
SSMRH	1.101	1.124	1.223	1.102

obtained by the proposed ILP formulation. The optimal results were not derived for all of the investigated networks due to the large running time of the ILP.

6.2.1. Running time of investigated heuristics

The simulations were performed on a computer with a 2.6 GHz Intel Core i5 processor and 8 GB of RAM. Simulations on 4 randomly created networks, that consisted of $n = 40$ nodes and $m = 80, 120, 160, 200$ links (with each link having a random integer cost varying from 1 to 1000), for the case of $z = 8$ MC nodes and $k = 8$ destinations, gave the average running times per multicast request (derived from 10,000 multicast requests) for each investigated heuristic. These are presented in Table 4 (DoC case was considered).

From the results of Table 4 it can be seen that all investigated heuristics are fast enough for practical applications. It can also be seen that the average node degree (i.e., $\frac{m}{n}$) does not have any impact on the running time. The reader should note that the slight decrease of the running time with the increase of $\frac{m}{n}$ is due to the fact that a different network is created for each m (rather than adding links to the previous (sparser) network).

The heuristics that deviate from the running times of the rest are MO and SSMRH. MO is slower than the others due to the need to re-compute all-pair shortest paths in each iteration [5]. The running time of SSMRH is further investigated below.

6.2.2. Running time of SSMRH

As analyzed in Section 4.2, the computational complexity of SSMRH is of order $O(z^2b)$, where $O(b)$ is the computational complexity of its *base* heuristic. If either MUS or MPH* is used as base, this complexity is of order $O(z^2km + z^2kn\log n + z^2k^2n)$. In practice, though, for all simulated cases, a very small number of MC nodes were located by SSMRH to be utilized in order to derive an RSG with reduced cost. According to the simulation results, in practice, the running time of SSMRH was found to be of order $O(zb)$, i.e., much faster compared to the (theoretical) worst-case complexity. This is illustrated in Table 5. Here, the average number of repetitions (*rep*) of the base algorithm of SSMRH over the number of MC nodes (z) is presented for each investigated scenario, separately for each n.

6.3. Comparison between DoC-based and DaC-based architectures

Fig. 18 presents a comparison between DoC- and DaC-based architectures for the USNET network topology. The average cost that is presented is the one derived by the best heuristic for each case: SSMRH(*MUS*) for the DoC and SSMRH(*MPH**) for the DaC case. The

Table 5
Running time of SSMRH.

n	$\frac{rep}{z}$ (avg)
40	1,33
60	1,55
80	1,73

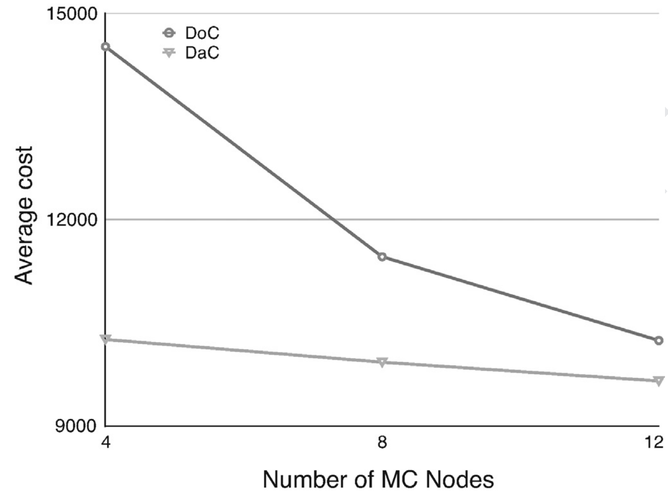

Fig. 18. Comparison between DaC and DoC for the USNET.

first observation is that, as expected, the average cost of the derived *RSGs* for the DaC case is smaller compared to the DoC-based architecture. The second observation is that as the percentage of MC nodes increases, the performance of DoC networks approaches that of DaC networks. More precisely, the increase of the average cost if the network is DoC compared to DaC is 41.54%, 15.47%, and 6.11% for $z = 4, 8,$ and 12, respectively (with an average value of 21.04%). Therefore, for cost-efficient routing in sparse-splitting networks, either the MI nodes must have the DaC ability, or the percentage of the MC nodes must be appropriately large.

7. Conclusions

In the current paper, the problem of multicast routing for networks with sparse-splitting capabilities was investigated, for networks where the MI nodes are either DoC or DaC. A novel Integer Linear Programming formulation was presented for both types of networks, as well as novel multicast routing heuristic algorithms under the sparse-splitting constraint. Simulations on the USNET, NSFNET, as well as on several randomly created networks, have shown that the proposed algorithms achieve an important decrease of the average cost of the derived solutions, compared to existing relevant techniques. For the USNET and NSFNET networks the performance of SSMRH is very close to the lower bound provided by the ILP, leaving very little room for any further improvement. Specifically, for these networks, the proposed algorithms obtain the optimal solution for the majority of the investigated cases.

Future work focuses on the development of a novel Integer Linear Programming formulation as well as heuristic algorithms that can provide survivable multicast routing in sparse-splitting optical networks.

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