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Simplified and improved multiple attributes alternate ranking method for vertical handover decision in heterogeneous wireless networks

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ABSTRACT

Multiple Attribute Decision Making (MADM) is one of the best candidate network selection methods used for Vertical Handover Decision (VHD) in heterogeneous wireless networks (4G). Selection of the network in MADM is predominantly decided by two steps, i.e., attribute normalization and weight calculation. This dependency in MADM results in an unreliable network selection for handover, and in a rank reversal (abnormality) problem during the removal and insertion of the network in the network selection list. Hence, this paper proposes a Simplified and Improved Multiple Attributes Alternate Ranking method referred to as SI-MAAR to eliminate the attribute normalization and weight calculation methods, thereby solving the rank reversal problem. Further, the MATLAB simulation results demonstrate that the proposed SI-MAAR method outperforms MADM methods such as TOPSIS, SAW, MEW and GRA with respect to the network selection reliability and rank reversal problems.

1. Introduction

The characteristics of heterogeneous wireless networks, Quality of Service (QoS) stipulations of applications [1,2], users' anticipation in terms of perceived QoS [3], monetary cost and battery power [4,5], and service providers' obligations introduced many challenges in 4*th* Generation (4G) heterogeneous wireless networks [6–9]. Integration of heterogeneous wireless networks like non-IEEE: GPRS, UMTS and IEEE: WiMAX, WiFi etc., is essential in 4G networks in order to provide *Always Best Connected* (ABC) anywhere at anytime [10]. Designing an optimized and efficient mobility management scheme for seamless communication [11] in heterogeneous wireless networks is a key challenge because of the diverse properties of non-IEEE and IEEE wireless access systems [9,10,12], user and application stipulations, multiple interface mobile device capabilities [13] and service providers' obligations.

Mobility management in heterogeneous wireless networks is defined as the process of Heterogeneous Information Gathering (HIG), Vertical Handover Decision (VHD) and Vertical Handover Execution (VHE) for the seamless communication of mobile devices between disparate radio access networks [14,15]. VHD is one of the prominent steps of mobility management in heterogeneous wireless networks for selecting the next suitable network for a seamless handover of mobile devices between the '*n*' available heterogeneous wireless networks [16]. Many VHD strategies [17–20] have been proposed in the literature, such as Function-based, User-Centric, Markov [21], Fuzzy Logic [22], Multiple Attribute Decision Making (MADM) [23] and Game Theory [24] with differing complexity, flexibility and reliability. In heterogeneous wireless networks, the handover may not always be due to weak received signal strength (Horizontal handover) [25], but could also be due to an improvement or degradation in the Quality of Service (QoS) attributes of the networks or variations in the expectations of the user, mobile device or applications. Multiple interface Mobile Nodes (MN) have limited resources of battery lifetime [26], computational capabilities and memory; hence it is essential to have an optimized and simple multiple attributes VHD method for seamless migration of MN in heterogeneous wireless networks. Among the existing VHD schemes, MADM is one of the strategies that considers multiple attributes with medium complexity [19,27]. On the other hand, the complexity of other VHD strategies increases with an increase in the number of attributes. Hence, in our proposed work we focus only on classical MADM methods. Table 1 shows a comparison of the salient features of different existing VHD strategies for heterogeneous wireless networks.

Many classical MADM methods [18,29] such as Simple Additive Weight (SAW), Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) [30,31], Multiplicative Exponent Weighting

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Table 1
Comparisons of existing VHD strategies [17,19,27,28].

VHD strategies	User consideration	Multi-attribute	Complexity	Flexibility	Reliability	Multi-service
Function-based	Medium	Yes	Low	High	Medium	No
	Merits: minimum degradation in high load and congestion situations.					
	Demerits: time consuming if services and/or available access points increase.					
User-centric	Strong	Yes	Low	High	Medium	No
	Merits: maximizes users' utility and low implementation complexity.					
	Demerits: non-real-time support, simple rate prediction method and medium precision.					
MADM	Medium	Yes	Medium	High	Medium	No
	Merits: multiple criteria consideration, easy to implement, scalable and accurate results.					
	Demerits: medium implementation complexity, selection of suitable method and normalization.					
Markov	Low	Yes	Medium	Medium	High	No
	Merits: adaptive and applicable to a wide range of conditions.					
	Demerits: implementation complexity.					
Fuzzy Logic	Medium	Yes	High	Low	High	No
	Merits: makes decisions in an autonomous way, considers multiple criteria.					
	Demerits: complexity increases if additional input parameters are considered.					
Game Theory	Strong	Yes	Medium	Medium	High	No
	Merits: efficient resource management.					
	Demerits: additional decision parameters are required in practice to ensure better quality of service.					
Reputation	Medium	Yes	Medium	Medium	Medium	No
	Merits: faster VHD decision-making.					
	Demerits: reputation sustainability needs to be addressed in greater depth.					

(MEW) [29], ELimination Et Choice Translating REality (ELECTRE) [29], Grey Relational Analysis (GRA) [29,32] and Preference Ranking Organization METHods for Enrichment Evaluations (PROMETHEE) are extensively discussed in the literature [20]. These classical MADM methods are not only limited to the field of networking, but are also popularly used in areas like Logistics and Supply Chain Management, Design, Engineering and Manufacturing Systems, Business and Marketing Management, Health, Safety and Environmental Management, Human Resources Management, Energy Management, Chemical Engineering, and Water Resources Management [29,30].

Table 2 shows a comparison between the different classical MADM methods with respect to procedure- and application- based merits and demerits.

The major problem with classical MADM methods is their dependency on the attribute normalization and weight calculation methods. Hence, these dependencies not only provoke unreliable selection of the network for handover [27], but also give rise to a rank reversal (abnormality) problem in the case of the removal and insertion of the network in the network selection list during network ranking [34]. The rank reversal problem with classical MADM methods leads to the reversal of the relative ranking of the networks if an alternative network is removed from or inserted into the candidate network selection list. For example, consider the ranks of three different networks N1, N2 and N3 as $Rank_{N1} > Rank_{N2} > Rank_{N3}$. The removal of network N1 should result in the rank of the other two networks becoming $Rank_{N2} > Rank_{N3}$. However, as in classical MADM methods, computation of network rank and score depends on the remaining other networks' attribute values, the removal of network N1 may result in the rank of the other two networks becoming $Rank_{N3} > Rank_{N2}$, resulting in a rank reversal problem. The same may be observed when a new network is inserted into the network selection list. Moreover, the presence of a rank reversal problem with classical MADM methods raises a reliability issue with respect to the selected network for handover.

The unreliability in network ranking and the rank reversal problem of classical MADM methods are the key challenges for the seamless handover of MNs in heterogeneous wireless networks. This is as wrong selection of a network during the VHD in heterogeneous wireless networks results in poor Quality of Service (QoS) of the applications in terms of high packet delay, packet loss, unnecessary handover (ping-pong effect) and handover failure (rejecting the MN handover request by the selected network due to insufficient resources). Further, this may lead to user dissatisfaction and high consumption of mobile device resources such as memory and battery life. Hence, there is a need for improving the reliability of the network selection of classical MADM methods, thereby eliminating dependence on the attribute normalization and weight calculation methods.

The main challenge in improving the reliability of network selection and eliminating the rank reversal problem is replacing the existing VHD's heterogeneous attribute normalization by its equivalent, as well as removing the VHD's heterogeneous attribute weight (which signifies its importance) during the computation of the network rank. In classical MADM methods, executing attribute normalization and weight computation are the two specific steps to proceed with the ranking of the networks: this motivated us to design a novel and simplified MADM method to overcome the unreliable network selection and the rank reversal problem of classical MADM methods. The main idea of our proposed simplified and improved MADM method is to overcome the key limitations of existing classical MADM methods used for VHD in heterogeneous wireless networks. To overcome unreliable network selection and the rank reversal problem of classical MADM methods, we propose a mobile device controlled Simplified and Improved Multiple Attributes Alternate Ranking method referred to as SI-MAAR. This can be achieved by replacing attribute normalization and weight calculation methods by a simple closeness index (utility) matrix which is computed by the networks' attributes and the expectations of the same. Further, to overcome the rank reversal problem, we propose new positive and negative ideal solutions based on the benefit and cost attributes.

Our key research contributions in this paper are, to the best of our knowledge:

- The first paper on mobile device controlled Simplified and Improved Multiple Attributes Alternate Ranking method referred to as SI-MAAR which is independent of the attribute normalization and weight calculation methods for achieving 100% network selection reliability.
- The first approach for completely avoiding the rank reversal problem of classical MADM methods.

The rest of the paper is organized as follows: Section 2 addresses the background and related work in the area of VHD; Section 3 presents a performance analysis of the classical MADM methods: TOPSIS, SAW, MEW and GRA; Section 4 presents the proposed method for solving the network selection unreliability and rank reversal problems; finally, the conclusion and future work are given in Section 5.

Table 2
Comparison of classical MADM methods [24,28].

MADM	Merits		Demerits	
Method	Procedure-based	Application-based [33]	Procedure-based	Application-based [33]
SAW	Easy to understand, easy to implement and support multiple attributes.	Good performance for interactive applications.	Poor-valued attributes can be outweighed by a very good value of another attribute [24], unreliable ranking and rank reversal problem [34].	Poor performance for data applications.
TOPSIS	Simple and comprehensive, scalable, high efficiency, high flexibility, and accurate results.	Good performance for interactive and background applications [18].	Imprecise data cannot be handled, unreliable ranking and rank reversal problems.	Poor performance in bandwidth and delay applications [18].
MEW	Medium implementation complexity and the least sensitive method.	Good performance for interactive applications [35].	Penalizes alternatives with poor-valued attributes more heavily, unreliable ranking and rank reversal problem.	Poor performance for data applications.
ELECTRE	Integrates subjective assessments with numerical data.	Satisfactory performance for data applications.	Complicated, uses pairwise comparisons, unreliable ranking and rank reversal problem.	Poor performance for voice applications [36].
AHP-GRA	Integrates subjective appraisals with numerical statistics, supports multiple attributes with precise solutions.	Good performance for bandwidth and delay applications.	Complicated, uses pairwise comparisons, the length of the process increases with the number of levels, unreliable ranking and rank reversal problem.	Poor performance for interactive and background applications [18].

2. Related work

This section of the paper presents an overview of recent work in the areas of heterogeneous wireless network mobility management and Vertical Handover Decision methods.

Several research papers on mobility management and Vertical Handover Decision methods are available in the literature presenting several issues such as key features of the heterogeneous wireless networks; future heterogeneous wireless technologies; different types of handover in wireless overlay networks; differences between traditional and next generation handover strategies; handover metrics; mobile device requirements for future heterogeneous wireless networks; and research challenges for mobility management in future heterogeneous wireless networks [9,11]. In [17], [19] and [27], authors presented vertical handover processes (system discovery, handover decision and handover execution) along with a comparison of different vertical handover decision strategies.

Xenakis et al. [37] presented a survey of mobility management for femtocells in LTE-A with a comparative list of different handover decision algorithms based on different handover decision parameters. Further, the authors also presented a comparison of different handover decision algorithms with respect to handover metrics. The different handover decision algorithms listed by the authors with respect to the LTE-A femtocells are also used in mobility management for heterogeneous wireless networks.

Mrquez-Barja et al. [14] presented the use of Media Independent Handover (MIH) for system discovery along with additional open research issues such as Quality of Service (QoS), Quality of Expectation (QoE), security, and service provider management and evaluation.

Yan and Narayanan [20] highlighted four different VHD algorithms based on RSS, Bandwidth, Cost and Hybrid with their comparative features. Further, the authors concluded that none of these VHD methods comprehensively consider various network parameters.

Wang and Kuo [28] presented an analytical framework used for modelling network selection in the classical MADM methods: TOPSIS, SAW, GRA and ELECTRE. The authors listed the key attributes (benefit and cost) used for the network selection problem along with the different subjective and objective attribute normalization and weight computation methods.

Stevens-Navarro and Wong [33] discussed handover metrics (bandwidth, latency, reliability, power, price, security and availability) and the different traffic classes (conversational, streaming, interactive and background). Further, the authors simulated the classical MADM methods: TOPSIS, SAW, MEW and GRA, but did not address the unreliability in network ranking due to existing attribute normalization and weight computation methods.

Wanga and Luoc [34] demonstrated the rank reversal problem of the Analytical Hierarchy Process (AHP) along with the other classical MADM methods: TOPSIS and SAW. The main reason identified by the authors for the rank reversal in MADM is its dependency on the attribute normalization method. Further, the authors concluded that normalization cannot prevent the rank reversal problem.

Chamodrakas and Martakos [22] developed an energy efficient utility function-based fuzzy TOPSIS for network selection, thereby addressing the rank reversal problem. Their method is based on the trade-off between performance and energy consumption. The major drawback of this method is its dependency on attribute weight calculation using linguistic values.

Yang and Tseng [31] proposed a novel approach for attribute rating referred to as Weighted Rating of Multiple Attributes (WRMA), which relies on TOPSIS for network ranking. Similar to AHP, the major drawbacks of WRMA are the decision maker competency in identifying the application level priority, and its dependency on TOPSIS.

Wang and Binet [38] proposed a new subjective attribute weighting method referred to as TRigger-based aUtomatic Subjective weighTing (TRUST). The major drawback of their proposed approach is that the important attributes usually obtain a larger weight and the unimportant attributes obtain a weight of 0, resulting in neglect of all unimportant attributes.

Zhang [39] developed handover decision using the fuzzy MADM method. The input data is fuzzy in nature, but the TOPSIS, SAW, AHP and DEA methods are finally used for ranking the alternates; hence, their approach did not address the rank reversal problem.

He et al. [40] proposed a VHD method based on the classification of mobile nodes (resource-poor and resource-rich) and dynamic new call blocking probability. The major drawbacks of this method are the classification of mobile nodes and the weighting of decision parameters. The final ranking of the networks depends on the attribute normalization and weight computation methods; hence, their method did not address the network rank unreliability and the rank reversal problem.

Ahuja et al. [41] proposed an algorithm for network selection in heterogeneous wireless networks using received signal strength, distance and outage probability. The proposed algorithm consists of two stages: during the first stage, a distance parameter is used to select the best network for handover, and in the second stage, received signal strength and outage probability are used for the same. The major problem with this algorithm is its complexity increase with an increase in the number of decision-making parameters and available networks during the handover decision. Further, this leads to an unreliable selection of the network.

Wen and Hung [42] proposed three algorithms for selecting the most energy efficient networks in heterogeneous wireless networks based on application characteristics and transmission loads. However, since the main constraints of mobile devices are memory and computational capabilities apart from energy consumption, these algorithms are too complex for mobile devices.

Khloussy et al. [43] proposed a Markov Decision Process (MDP)-based distribution of overlay heterogeneous wireless networks among MNs with the objective of maximizing the operators' revenue. This method did not consider the time taken to complete handover decision and handover execution. Further, the authors did not address the computation of the weight associated with a request to handover to a particular network.

Table 3 summarizes the literature survey conducted related to the mobility management, VHD and classical MADM methods.

3. Performance analysis of the classical MADM methods: TOPSIS, SAW, MEW and GRA

This section numerically substantiates the unreliability in network ranking and the rank reversal problem of the classical MADM methods: TOPSIS, SAW, MEW and GRA. Fig. 1 illustrates the sequential steps of the network (alternate) ranking of the aforementioned four classical MADM methods [28,32,33,35,36]. As shown in Fig. 1, TOPSIS, SAW, MEW and GRA are primarily dependent on the attribute normalization [44] and weight calculation methods [28] during network ranking. Some of the popularly used attribute normalization methods in classical MADM methods are: (i) Vector normalization (ii) Sum normalization and (iii) Max-Min normalization. Similarly, weights of the attributes can be computed by (i) Entropy [45] and (ii) Variance [28] for objective attributes and (iii) Simplified and Improved AHP (SI-AHP) for subjective attributes [38].

As shown in Fig. 1, attribute normalization is the essential step of classical MADM methods to present the attributes: bandwidth, delay, energy consumption, etc., of different dimensions: bits per sec (bps), millisecond (msec), mWatt (mWatt), etc., as dimensionless for common comparison of the attributes. The attribute weight calculation methods compute the importance (common to all alternates) of the attributes, either by using all available network attributes values (Objective method: Entropy and Variance) or by using inputs from the decision maker (Subjective method: Analytical Hierarchy Process (AHP)) [27,28,46]. Moreover, AHP uses the reciprocal matrix input from the decision maker in computing attribute weights, after which the computed attribute weights are accepted into the network ranking if the Consistency Ratio (CR) is less than 0.1. The major problem with AHP is the competency of the decision maker in producing the reciprocal matrix, which may lead to unreliable attribute weights. The Simplified and Improved AHP (SI-AHP) method proposed in our previous work [47] reduces the involvement of the decision maker in producing the reciprocal matrix, leading to 100% reliable attribute weights. The remaining steps of network ranking methods shown in Fig. 1 are unique to the respective classical MADM methods as listed below:

- TOPSIS [28,33,35,36]:
  - Weight cast matrix: computes the product of the normalized network attributes and the respective attribute weights.
  - Ideal solution: computes the positive and negative ideal values of the attributes using the weight cost matrix.
  - Euclidean distance: computes the separations from the positive (best) and negative (worst) values of the attributes among networks.
  - Relative closeness: computes the closeness to the positive value of the attributes among networks.
- SAW [28,33,35,36]:
  - Weight cast matrix: computes the product of the normalized network attributes and the respective attribute weights.
  - Network score: computes the score of individual networks by adding the respective network weight cost matrix entries.
- MEW [28,33,35,36]:
  - Network score: computes the score of the individual networks by the weighted product of the respective network normalized attributes.
  - Value ratio: ratio of the individual network score to the weighted product of the positive ideal value of the attributes.
- GRA [28,32,33]:
  - Reference sequence: computes the positive ideal values of the attributes using normalized network attributes.
  - Grey relational coefficient: describes the similarities between networks and the reference sequence.
  - Grey relational grade: computes the correlation between reference sequence and comparability sequence.

The major drawbacks with classical MADM methods (Fig. 1) are:

- Dependency of classical MADM methods on attribute normalization and weight calculation methods substantially influences the ranking of the networks. Further, this results in an uncertainty in selecting the appropriate attribute normalization and weight calculation method with respect to a specific classical MADM method.
- The way the attributes are normalized or the weights are calculated depends on the relation between one network's attributes and those of other networks existing in the network selection list. The outcome of this dependency is the rank reversal problem with classical MADM methods during the removal and insertion of the network in the network selection list [34].

An extensive MATLAB simulation is carried out to numerically justify the network rank unreliability and the rank reversal problem of the classical MADM methods: TOPSIS, SAW, MEW and GRA. The remaining part of this section is detailed as follows:

- Section 3.1: Effect of the different attribute normalization and weight calculation methods on the network rank and score in TOPSIS, SAW, MEW and GRA, for common networks and the corresponding attribute values (a_{ij}).
- Section 3.2: The Rank Reversal Problem (RRP) in TOPSIS, SAW, MEW and GRA with different pairs of attribute normalization and weight calculation methods for 1000 different combinations of a_{ij}.

In Sections 3.1 and 3.2 to demonstrate the unreliability in network rank and the RRP (reversal of the relative ranking of the networks if an alternative network is removed from or inserted into the candidate network selection list during the selection procedure) of the classical

Table 3
A summary of literature survey.

Heuristic	Contribution	Remarks
Siddiqui and Zeadally [9]	Survey paper	Mobility management, heterogeneous wireless networks key features, its challenges, different type of handovers and their metrics.
Nasser et al. [11]		
Kassar et al. [17]		
Zekri et al. [19]		
Charilas and Panagopoulous [27]		
Xenakis et al. [37]		Mobility management for femtocells in LTE-A.
Mrquez-Barja et al. [14]		MIH in system discovery, along with VHD open research issues.
Yan and Narayanan [20]		Categories of VHD algorithms.
Wang and Kuo [28]	Survey and simulation	Theory behind the classical MADM methods. Detailed list of subjective and objective attributes.
Stevens-Navarro and Wong [33]		Handover metrics and simulation of classical MADM methods.
Wanga and Luoc [34]	Verification	Demonstrated the rank reversal problem in SAW, TOPSIS, MEW and DEA.
Chamodrakas and Martakos [22]	New TOPSIS	Energy efficient fuzzy TOPSIS.
Yang et al. [31]	Weight computation method of the attributes.	Five steps of attribute weight computation method referred to as Weighted Rating of Multiple Attributes (WRMA).
Wang Binet [38]		TRigger-based aUtomatic Subjective weighTing (TRUST) method.
Zhang [39]	New MADM	Fuzzy MADM.
He et al. [40]	New VHD method	VHD method based on MN classification.
Kiran Ahuja et al. [41]		Received signal strength, distance and outage probability-based VHD method.
Wen and Hung [42]		Energy efficient VHD method.
Khloussy et al. [43]		Markov Decision Process (MDP)-based VHD method.

MADM methods, a MATLAB simulation is carried out with the following three cases:

- Case 1 - removal of network N4 from the network selection list {N1, N2, N3, N4}, resulting in {N1, N2, N3}
- Case 2 (Normal) - with four networks {N1, N2, N3, N4}
- Case 3 - inserting a new network N5 into the network selection list {N1, N2, N3, N4}, resulting in {N1, N2, N3, N4, N5}

Further, the attributes in the TOPSIS, SAW, MEW and GRA methods are normalized with the Vector, Sum and Max-Min normalization methods. Similarly, attribute weight is computed with the Entropy, SI-AHP and Variance methods.

3.1. *The effect of different attribute normalization and weight calculation methods on the classical MADM methods*

To demonstrate numerically the network selection unreliability in the classical MADM methods: TOPSIS, SAW, MEW and GRA, a MATLAB simulation is carried out with five heterogeneous wireless networks (N1, N2, N3, N4 and N5) each with six attributes (four benefit attributes and two cost attributes) in three different cases: Case 1, Case 2 and Case 3. The attributes in MADM methods can be benefit attributes (i.e., the higher the attribute value, the better is the network rank and score) or cost attributes (i.e., the lower the attribute value, the better is the network rank and score). The benefit attributes in VHD's network selection can be bandwidth, SNR, throughput, etc., and the cost attributes can be packet delay, packet loss, monetary cost, energy consumption, etc. [28]. The heterogeneity of the five networks is accomplished in the MATLAB simulation with the different range values for the six attributes with respect to the individual networks.

Table 4 enumerates the MATLAB simulation parameters for one combination of networks and the corresponding attribute values (a_{ij}), the expected attribute's value (e_j) used in the SI-MAAR and the linguistic crisp value of the six attributes (Very Low (1), Low (3), Medium (5), High (7) and Very High (9)) [22] used in the SI-AHP method.

Table 5 shows the attribute weights computed with the Entropy, SI-AHP and Variance methods after normalizing the attributes with the Vector, Sum and Max-Min normalization methods with respect to a_{ij} and e_j as shown in Table 4 with the condition $\sum_{j=1}^{m} w_j = 1$ and $w_j \geq 0$. As illustrated in Fig. 1, attribute weight computation in the Entropy and Variance methods depends on the normalized values of the attributes, resulting in variation in attribute weight with changes in attribute normalization method. As shown in Table 5, Sl. nos.: (1, 2) and (8, 9, 10) indicate the variation in attribute weights computed using the Entropy method with the Vector and Sum normalization methods respectively. However, Entropy with the Vector (Sl. no.: 3) and Max-Min (Sl. no.: (15, 16, 17)) normalization methods results in an invalid attribute weight ($w_j = \infty$ or $w_j < 0$). In the Entropy attribute weighting method, a few of the attributes with vector normalization (Sl. no.: 3) result in entropy $E_j > 1$, which leads to $w_j < 0$. Also, in Max-Min normalization (Sl. nos.: (15, 16, 17)), the Entropy weighting method computes $w_j = \infty$, since $x_{ij} = 0$ for some attributes, leading to $\ln(x_{ij}) = \infty$. Further, as shown in Table 5, Sl. nos.: (5, 12, 19), (6, 13, 20) and (7, 14, 21) indicate the variation in attribute weight computed using the Variance method with Vector and Sum normalization. However, in SI-AHP (Sl. nos.: 4, 11, 18), the computed weight of the attributes remains the same, since SI-AHP is independent of attribute normalization methods.

Tables 6–9 illustrate the variation in network rank and score in the TOPSIS, SAW, MEW and GRA methods respectively for different pairs of attribute normalization and weight calculation methods in three

Table 4
MATLAB simulation parameters.

Simulation case			Network (Alternate)	Attribute (a_{ij})					
Case 1	Case 2	Case 3		Benefit				Cost	
				1	2	3	4	5	6
N1	N1	N1	Network 1 (N1)	30	31	47	30	9.75	33
N2	N2	N2	Network 2 (N2)	60	57	10	118	3.75	37
N3	N3	N3	Network 3 (N3)	22600	91	50	106	8.25	16
-	N4	N4	Network 4 (N4)	45000	1	50	94	3.75	5
-	-	N5	Network 5 (N5)	55000	161	128	104	3	10
e_j				10030	61	77	148	3.75	73
SI-AHP (Attributes linguistic value)				M-5	L-3	VL-1	M-5	H-7	L-3

Table 5
Attribute weights in different attribute normalization and weight methods.

Sl. no.	Normalization	Weight method	Case	Attributes weight					
				Benefit				Cost	
				1	2	3	4	5	6
1	Vector	Entropy	1	0.461429	0.09974	0.137427	0.12054	0.093301	0.08756
2			2	0.436522	0.22816	0.085489	0.06464	0.059212	0.12598
3			3	-	-	-	-	-	-
4		SI-AHP	-	0.170455	0.05682	0.034091	0.17045	0.511364	0.05682
5		Variance	1	0.406652	0.1191	0.147529	0.13312	0.101723	0.09187
6			2	0.301955	0.20182	0.118236	0.10707	0.115139	0.15578
7			3	0.238596	0.20928	0.175534	0.08932	0.125051	0.16221
8	Sum	Entropy	1	0.683413	0.05532	0.10231	0.08011	0.043524	0.03532
9			2	0.469046	0.21932	0.076826	0.05879	0.057001	0.11901
10			3	0.363999	0.23759	0.144743	0.04775	0.072814	0.13311
11		SI-AHP	-	0.170455	0.05682	0.034091	0.17045	0.511364	0.05682
12		Variance	1	0.406652	0.1191	0.147529	0.13312	0.101723	0.09187
13			2	0.301955	0.20182	0.118236	0.10707	0.115139	0.15578
14			3	0.238596	0.20928	0.175534	0.08932	0.125051	0.16221
15	Max-Min	Entropy	1	-	-	-	-	-	-
16			2	-	-	-	-	-	-
17			3	-	-	-	-	-	-
18		SI-AHP	-	0.170455	0.05682	0.034091	0.17045	0.511364	0.05682
19		Variance	1	0.243252	0.1477	0.122146	0.12284	0.175762	0.18829
20			2	0.233195	0.15913	0.122312	0.12598	0.167662	0.19172
21			3	0.204371	0.18171	0.182122	0.11437	0.150567	0.16686

Table 6
TOPSIS - variations in network rank and score with respect to different attribute normalization and weight methods for three different cases.

Sl. no.	Normalization	Weight method	Case	Rank 1		Rank 2		Rank 3		Rank 4		Rank 5	
				Network	Score	Network	Score	Network	Score	Network	Score	Network	Score
1	Vector	Entropy	1	3	0.93549	2	0.14781	1	0.13511				
2			2	4	0.68547	3	0.58474	2	0.23008	1	0.14786		
3			3	–	–	–	–	–	–	–	–	–	–
4		SI-AHP	1	2	0.58919	3	0.5375	1	0.05808				
5			2	4	0.84568	2	0.60097	3	0.41233	1	0.07084		
6			3	5	0.9587	4	0,79522	2	0.62209	3	0.33257	1	0.04222
7		Variance	1	3	0.92255	2	0.17762	1	0.15802				
8			2	4	0.64613	3	0.61784	2	0.29342	1	0.19548		
9			3	5	0.94831	4	0.49166	3	0.47081	2	0.24837	1	0.16952
10	Sum	Entropy	1	3	0.98587	1	0.04945	2	0.044				
11			2	4	0.74193	3	0.554477	2	0.18298	1	0.11301		
12			3	5	0.96889	4	0.53852	3	0.45771	2	0.18211	1	0.12758
13		SI-AHP	1	3	0.63237	2	0.47289	1	0.05003				
14			2	4	0.84653	2	0.52713	3	0.43157	1	0.06834		
15			3	5	0.96158	4	0.78351	2	0.57765	3	0.33806	1	0.04495
16		Variance	1	3	0.94948	2	0.12175	1	0.11089				
17			2	4	0.67772	3	0.59436	2	0.25629	1	0.16714		
18			3	5	0.95274	4	0.49365	3	0.46885	2	0.23363	1	0.16368
19	Max-Min	Entropy	1	–	–	–	–	–	–	–	–	–	–
20			2	–	–	–	–	–	–	–	–	–	–
21			3	–	–	–	–	–	–	–	–	–	–
22		SI-AHP	1	3	0.76024	1	0.67479	2	0.25106				
23			2	3	0.72708	1	0.67844	4	0.29119	2	0.25306		
24			3	3	0.72778	1	0.67573	5	0.31135	4	0.29564	2	0.27969
25		Variance	1	3	0.64518	1	0.45311	2	0.41142				
26			2	3	0.6234	1	0.48327	4	0.47981	2	0.44017		
27			3	5	0.62394	3	0.49841	1	0.41948	2	0.39432	4	0.38963

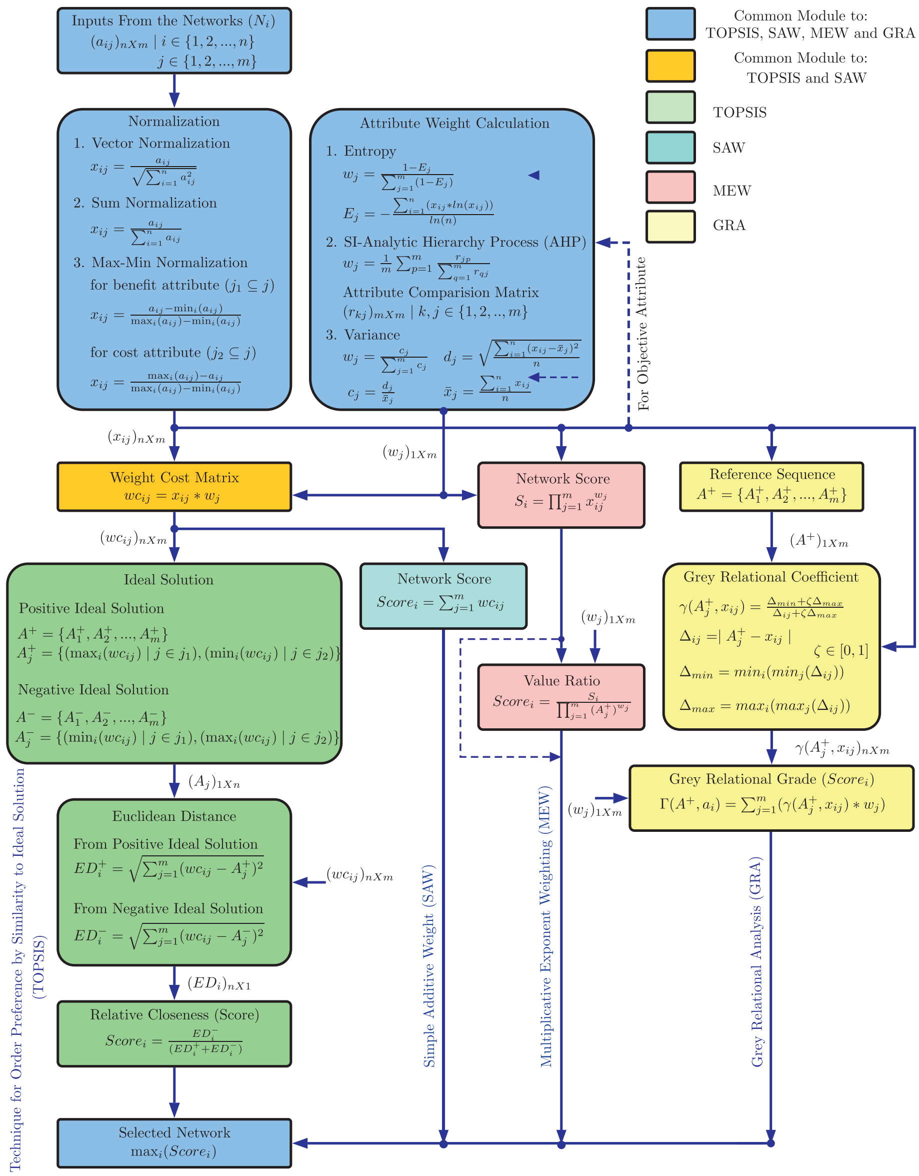

Fig. 1. The classical MADM methods: TOPSIS, SAW, MEW and GRA.

different cases (Case 1, Case 2 and Case 3) with respect to a_{ij} and e_j as shown in Table 4.

The following are the pairs of different attribute normalization and weight calculation methods considered for MATLAB simulation results, as shown in Tables 6–9:

- Attributes Vector normalization method with the Entropy (Sl. nos.: 1–3), SI-AHP (Sl. nos.: 4–6) and Variance (Sl. nos.: 7–9) methods
- Attributes Sum normalization method with the Entropy (Sl. nos.: 10–12), SI-AHP (Sl. nos.: 13–15) and Variance (Sl. nos.: 16–18) methods
- Attributes Max-Min normalization method with the Entropy (Sl. nos.: 19–21), SI-AHP (Sl. nos.: 22–24) and Variance (Sl. nos.: 25–27) methods

Following are the pairwise entries in Table 6 which illustrate the network rank unreliability with the TOPSIS method:

- Vector normalization with the Entropy, SI-AHP and Variance methods: Sl. nos. - (1, 4) and (4, 7) in Case 1; Sl. nos. - (2, 5) and (5, 8) in Case 2; and Sl. no. - (6, 9) in Case 3.
- Sum normalization with the Entropy, SI-AHP and Variance methods: Sl. nos. - (10, 13) and (10, 16) in Case 1; Sl. nos. - (11, 14) and (14, 17) in Case 2; and Sl. nos. - (12, 15) and (15, 18) in Case 3.
- Max-Min normalization with the entropy, SI-AHP and Variance methods: Sl. no. - (24, 27) in Case 3.

Inference:

- In the TOPSIS method, the Entropy weighting method is suitable for the Sum normalization method.
- The dependency of TOPSIS on attribute normalization methods results in an unreliable network rank and score, despite SI-AHP being independent of the normalized attributes.
- The TOPSIS method with the Vector and Max-Min normalization methods is suitable for both SI-AHP and Variance methods, but not the Entropy method.

Table 7 illustrates the following unreliable network rank pairwise entries with the SAW method:

- Vector normalization with the Entropy, SI-AHP and Variance methods: Sl. nos. - (2, 5) and (5, 8) in Case 2; and Sl. no. - (6, 9) in Case 3.

Table 7
SAW - variations in network rank and score with respect to different attribute normalization and weight methods for three different cases.

Sl. no.	Normalization	Weight method	Case	Rank 1		Rank 2		Rank 3		Rank 4		Rank 5	
				Network	Score	Network	Score	Network	Score	Network	Score	Network	Score
1	Vector	Entropy	1	3	0.80559	1	0.26762	2	0.24849				
2			2	3	0.54219	4	0.50275	2	0.27283	1	0.2421		
3			3	–	–	–	–	–	–	–	–	–	–
4		SI-AHP	1	3	0.6876	1	0.48132	2	0.34336				
5			2	3	0.56185	1	0.45833	4	0.40263	2	0.31967		
6			3	3	0.48881	1	0.4309	5	0.40273	4	0.33001	2	0.28792
7		Variance	1	3	0.78867	1	0.29101	2	0.27431				
8			2	3	0.54606	4	0.44077	2	0.3261	1	0.31775		
9			3	5	0.59427	3	0.3924	4	0.2897	1	0.28595	2	0.26754
10	Sum	Entropy	1	3	0.81315	1	0.09795	2	0.0889				
11			2	4	0.36831	3	0.34922	2	0.15147	1	0.131		
12			3	5	0.37219	3	0.20921	4	0.1857	1	0.11713	2	0.11578
13		SI-AHP	1	3	0.49026	1	0.29621	2	0.21353				
14			2	3	0.32384	1	0.25089	4	0.24885	2	0.17642		
15			3	3	0.24954	5	0.21722	1	0.21565	4	0.17421	2	0.14338
16		Variance	1	3	0.64576	1	0.18254	2	0.1717				
17			2	3	0.33776	4	0.29393	2	0.18828	1	0.18003		
18			3	5	0.33438	3	0.21344	4	0.16198	1	0.14974	2	0.14046
19	Max-Min	Entropy	1	–	–	–	–	–	–	–	–	–	–
20			2	–	–	–	–	–	–	–	–	–	–
21			3	–	–	–	–	–	–	–	–	–	–
22		SI-AHP	1	2	0.70667	3	0.59323	1	0.04236				
23			2	4	0.89669	2	0.71729	3	0.4888	1	0.05758		
24			3	5	0.964	4	0.78633	2	0.64498	3	0.41164	1	0.02845
25		Variance	1	3	0.85143	2	0.36293	1	0.14885				
26			2	4	0.80652	3	0.67501	2	0.39281	1	0.19015		
27			3	5	0.95573	4	0.61281	3	0.4896	2	0.31192	1	0.11203

Table 8
MEW - variations in network rank and score with respect to different attribute normalization and weight methods for three different cases.

Sl. no.	Normalization	Weight method	Case	Rank 1		Rank 2		Rank 3		Rank 4		Rank 5	
				Network	Score	Network	Score	Network	Score	Network	Score	Network	Score
1	Vector	Entropy	1	3	0.76785	2	0.03854	1	0.0299				
2			2	3	0.51737	4	0.20422	2	0.0325	1	0.02277		
3			3	–	–	–	–	–	–	–	–	–	–
4		SI-AHP	1	3	0.66615	1	0.18502	2	0.15946				
5			2	3	0.55178	4	0.29427	1	0.15325	2	0.13208		
6			3	3	0.47271	5	0.33944	4	0.2521	1	0.13129	2	0.11315
7		Variance	1	3	0.75138	2	0.0508	1	0.0407				
8			2	3	0.52122	4	0.1942	2	0.06884	1	0.05615		
9			3	5	0.5087	3	0.37982	4	0.12928	2	0.06609	1	0.06334
10	Sum	Entropy	1	3	0.74857	2	0.01078	1	0.00708				
11			2	3	0.33517	4	0.14229	2	0.01759	1	0.01204		
12			3	5	0.32365	3	0.20468	4	0.07241	2	0.1777	1	0.01478
13		SI-AHP	1	3	0.44696	1	0.12414	2	0.10699				
14			2	3	0.3188	4	0.17002	1	0.08854	2	0.07631		
15			3	3	0.24449	5	0.17556	4	0.13039	1	0.06791	2	0.05852
16		Variance	1	3	0.569	2	0.03847	1	0.03082				
17			2	3	0.32233	4	0.1201	2	0.04257	1	0.03473		
18			3	5	0.2789	3	0.20824	4	0.07088	2	0.03624	1	0.03472
19	Max-Min	Entropy	1	–	–	–	–	–	–	–	–	–	–
20			2	–	–	–	–	–	–	–	–	–	–
21			3	–	–	–	–	–	–	–	–	–	–
22		SI-AHP	1	–	–	–	–	–	–	–	–	–	–
23			2	–	–	–	–	–	–	–	–	–	–
24			3	–	–	–	–	–	–	–	–	–	–
25		Variance	1	–	–	–	–	–	–	–	–	–	–
26			2	–	–	–	–	–	–	–	–	–	–
27			3	–	–	–	–	–	–	–	–	–	–

- Sum normalization with the Entropy, SI-AHP and Variance methods: Sl. no. - (11, 14, 17) in Case 2; and Sl. nos. - (12, 15) and (15, 18) in Case 3.
- Max-Min normalization with the Entropy, SI-AHP and Variance methods: Sl. no. - (22, 25) in Case 1; Sl. no. - (23, 26) in Case 2; and Sl. no. - (24, 27) in Case 3.

Inference:

- In the SAW method, the Entropy weighting method is suitable for the Sum normalization method.
- The dependency of SAW on attribute normalization methods results in an unreliable network rank and score, despite SI-AHP being independent of the normalized attributes.

Table 9
GRA - variations in network rank and score with respect to different attribute normalization and weight methods for three different Cases.

Sl. No.	Normalization	Weight method	Case	Rank 1		Rank 2		Rank 3		Rank 4		Rank 5	
				Network	Score	Network	Score	Network	Score	Network	Score	Network	Score
1	Vector	Entropy	1	3	0.94671	2	0.54198	1	0.48796				
2			2	4	0.83878	3	0.70874	2	0.50004	1	0.44717		
3			3	–	–	–	–	–	–	–	–	–	–
4		SI-AHP	1	2	0.82125	3	0.77147	1	0.50018				
5			2	4	0.92536	2	0.81308	3	0.65975	1	0.48238		
6			3	5	0.96581	4	0.80555	2	0.74336	3	0.58354	1	0.4428
7		Variance	1	3	0.94168	2	0.56382	1	0.50138				
8			2	4	0.84625	3	0.73745	2	0.56643	1	0.48328		
9			3	5	0.95733	4	0.67062	3	0.55727	2	0.50268	1	0.41449
10	Sum	Entropy	1	3	0.98026	2	0.47377	1	0.45819				
11			2	4	0.85811	3	0.70886	2	0.51332	1	0.467		
12			3	5	0.97124	4	0.65366	3	0.53935	2	0.45064	1	0.4044
13		SI-AHP	1	2	0.83737	3	0.83498	1	0.59308				
14			2	4	0.93656	2	0.82178	3	0.70633	1	0.54302		
15			3	5	0.97022	4	0.81678	2	0.75005	3	0.0646	1	0.4693
16		Variance	1	3	0.95857	2	0.60255	1	0.55599				
17			2	4	0.86033	3	0.75773	2	0.59415	1	0.52225		
18			3	5	0.96132	4	0.67535	3	0.56596	2	0.50728	1	0.4265
19	Max-Min	Entropy	1	–	–	–	–	–	–	–	–	–	–
20			2	–	–	–	–	–	–	–	–	–	–
21			3	–	–	–	–	–	–	–	–	–	–
22		SI-AHP	1	3	0.75514	1	0.71473	2	0.49259				
23			2	1	0.72445	3	0.67571	4	0.52317	2	0.4983		
24			3	1	0.70647	3	0.63574	5	0.58227	2	0.50427	4	0.47193
25		Variance	1	3	0.78956	1	0.58959	2	0.56083				
26			2	3	0.69192	1	0.61532	4	0.60986	2	0.58277		
27			3	5	0.76724	2	0.5433	1	0.53628	3	0.5354	4	0.47268

- The SAW method with the Vector and Max-Min normalization methods is suitable for both SI-AHP and Variance methods, but not Entropy.

Table 8 illustrates the following unreliable network rank pairwise entries with the MEW method:

- Vector normalization with the Entropy, SI-AHP and Variance methods: Sl. nos. - (1, 4) and (4, 7) in Case 1; Sl. nos. - (2, 5) and (5, 8) in Case 2; and Sl. no. - (6, 9) in Case 3.
- Sum normalization with the Entropy, SI-AHP and Variance methods: Sl. nos. - (10, 13) and (13, 16) in Case 1; Sl. nos. - (11, 14) and (14, 17) in Case 2; and Sl. nos. - (12, 15) and (15, 18) in Case 3.
- Max-Min normalization with the Entropy, SI-AHP and Variance methods: This is not applicable with respect to network ranking.

Inference:

- Sl. nos.: (19–27) indicate that the MEW method is not suitable for Max-Min normalization irrespective of attribute weight calculation methods.
- In MEW, irrespective of the attribute normalization method, the rank of networks remains the same for the Entropy and Variance methods.
- In MEW, Entropy attribute weight method is suitable for the Sum normalization method.
- The dependency of MEW on attribute normalization methods results in an unreliable network rank and score, despite SI-AHP method being independent of the normalized attributes.

Table 9 illustrates the following unreliable network rank pairwise entries with the GRA method:

- Vector normalization with the Entropy, SI-AHP and Variance methods: Sl. nos. - (1, 4) and (4, 7) in Case 1; Sl. nos. - (2, 5) and (5, 8) in Case 2; and Sl. no. - (6, 9) in Case 3.
- Sum normalization with the Entropy, SI-AHP and Variance methods: Sl. nos. - (10, 13) and (13, 16) in Case 1; Sl. nos. - (11, 14) and (14, 17) in Case 2; and Sl. nos. - (12, 15) and (15, 18) in Case 3.
- Max-Min normalization with the Entropy, SI-AHP and Variance methods: Sl. no. - (23, 26) in Case 2; and Sl. no. - (24, 27) in Case 3.

Inference:

- The GRA with the Entropy weighting method is suitable for the Sum normalization method.
- The dependency of GRA on attribute normalization methods results in an unreliable network rank and score, despite SI-AHP method being independent of the normalized attributes.
- The GRA with the Vector and Max-Min normalization methods is suitable for both SI-AHP and Variance methods, but not the Entropy method.

Similarly, as shown in Tables 6–9, the variation in the network score is observed in all pairs of attribute normalization and weight calculation methods with the TOPSIS, SAW, MEW and GRA methods respectively.

Ultimately, this section numerically demonstrated (Tables 6–9) the network rank unreliability of the classical MADM methods: TOPSIS, SAW, MEW and GRA. Subsequently, the following subsection will highlight the second major limitation of the classical MADM methods i.e. rank reversal problem.

3.2. Rank reversal problem of the classical MADM methods

The second major challenging issue of the classical MADM methods is the rank reversal problem. As explained in Section 1, the rank reversal problem with classical MADM methods leads to the reversal of the relative ranking of the networks if an alternative network is removed from or inserted into the candidate networks selection list.

Further, we use the same Tables 6, 7 and 9 to illustrate the rank reversal problem of the classical MADM methods: TOPSIS, SAW and GRA respectively (Note: no rank reversal problem entries of MEW - Table 8 for a_{ij} shown in Table 4.)

Table 6 illustrates the following entries with the rank reversal problem in the TOPSIS method:

Table 10
Rank reversible problem for 1000 different a_{ij} in TOPSIS, SAW, MEW and GRA.

Sl. no.	Normalization	Weight method	Total no. of a_{ij}	Classical MADM methods							
				No. of invalid a_{ij}				No. of a_{ij} with RRP			
				TOPSIS	SAW	MEW	GRA	TOPSIS	SAW	MEW	GRA
1	**Vector**	Entropy	1000	981	981	981	981	9	13	6	16
2		SI-AHP		0	0	0	0	578	587	0	564
3		Variance		0	0	0	0	496	706	261	594
4	**Sum**	Entropy		0	0	0	0	456	676	280	570
5		SI-AHP		0	0	0	0	671	645	0	622
6		Variance		0	0	0	0	487	700	261	583
7	**Max-Min**	Entropy		1000	1000	1000	1000	-	-	-	-
8		SI-AHP		15	15	717	15	261	350	33	503
9		Variance		15	15	717	15	654	489	32	748

- Sl. nos.: 10 (Case 1) and 11 (Case 2) present the removal of network N4 in Case 2 that results in the ranking of the other three networks (Case 1) as $Rank_{N3} > Rank_{N1} > Rank_{N2}$ instead of $Rank_{N3} > Rank_{N2} > Rank_{N1}$
- Sl. nos.: 13 (Case 1) and 14 (Case 2) present the removal of network N4 in Case 2 that results in the ranking of the other three networks (Case 1) as $Rank_{N3} > Rank_{N2} > Rank_{N1}$ instead of $Rank_{N2} > Rank_{N3} > Rank_{N1}$
- Sl. nos.: 26 (Case 2) and 27 (Case 3) present the insertion of a new network N5 in Case 2 with better network attributes that results in the ranking of the networks (Case 3) as $Rank_{N5} > Rank_{N3} > Rank_{N1} > Rank_{N2} > Rank_{N4}$ instead of $Rank_{N5} > Rank_{N3} > Rank_{N1} > Rank_{N4} > Rank_{N2}$

Table 7 illustrates the following entries with the rank reversal problem in the SAW method:

- Sl. nos.: 1 (Case 1) and 2 (Case 2) present the removal of network N4 in Case 2 that results in the ranking of the other three networks (Case 1) as $Rank_{N3} > Rank_{N1} > Rank_{N2}$ instead of $Rank_{N3} > Rank_{N2} > Rank_{N1}$
- Sl. nos.: 7 (Case 1) and 8 (Case 2) present the removal of network N4 in Case 2 that results in the ranking of the other three networks (Case 1) as $Rank_{N3} > Rank_{N1} > Rank_{N2}$ instead of $Rank_{N3} > Rank_{N2} > Rank_{N1}$
- Sl. nos.: 8 (Case 2) and 9 (Case 3) present the insertion of a new network N5 in Case 2 with better network attributes that results in the ranking of the networks (Case 3) as $Rank_{N5} > Rank_{N3} > Rank_{N4} > Rank_{N1} > Rank_{N2}$ instead of $Rank_{N5} > Rank_{N3} > Rank_{N4} > Rank_{N2} > Rank_{N1}$
- Sl. nos.: 10 (Case 1) and 11 (Case 2) present the removal of network N4 in Case 2 that results in the ranking of the other three networks (Case 1) as $Rank_{N3} > Rank_{N1} > Rank_{N2}$ instead of $Rank_{N3} > Rank_{N2} > Rank_{N1}$
- Sl. nos.: 11 (Case 2) and 12 (Case 3) present the insertion of a new network N5 in Case 2 with better network attributes that results in the ranking of the networks (Case 3) as $Rank_{N5} > Rank_{N3} > Rank_{N4} > Rank_{N1} > Rank_{N2}$ instead of $Rank_{N5} > Rank_{N4} > Rank_{N3} > Rank_{N2} > Rank_{N1}$
- Sl. nos.: 16 (Case 1) and 17 (Case 2) present the removal of network N4 in Case 2 that results in the ranking of the other three networks (Case 1) as $Rank_{N3} > Rank_{N1} > Rank_{N2}$ instead of $Rank_{N3} > Rank_{N2} > Rank_{N1}$
- Sl. nos.: 17 (Case 2) and 18 (Case 3) present the insertion of a new network N5 in Case 2 with better network attributes that results in the ranking of the networks (Case 3) as $Rank_{N5} > Rank_{N3} > Rank_{N4} > Rank_{N1} > Rank_{N2}$ instead of $Rank_{N5} > Rank_{N3} > Rank_{N4} > Rank_{N2} > Rank_{N1}$

Table 9 illustrates the following entries with the rank reversal problem in the GRA method:

- Sl. nos.: 22 (Case 1) and 23 (Case 2) present the removal of network N4 in Case 2 that results in the ranking of the other three networks (Case 1) as $Rank_{N3} > Rank_{N1} > Rank_{N2}$ instead of $Rank_{N1} > Rank_{N3} > Rank_{N2}$
- Sl. nos.: 23 (Case 2) and 24 (Case 3) present the insertion of a new network N5 in Case 2 with better network attributes that results in the ranking of the networks (Case 3) as $Rank_{N1} > Rank_{N3} > Rank_{N5} > Rank_{N2} > Rank_{N4}$ instead of $Rank_{N1} > Rank_{N3} > Rank_{N5} > Rank_{N4} > Rank_{N2}$.
- Sl. nos.: 26 (Case 2) and 27 (Case 3) present the insertion of a new network N5 in Case 2 with better network attributes that results in the ranking of the networks (Case 3) as $Rank_{N5} > Rank_{N2} > Rank_{N1} > Rank_{N3} > Rank_{N4}$ instead of $Rank_{N5} > Rank_{N3} > Rank_{N1} > Rank_{N4} > Rank_{N2}$

Further, to study the rank reversal problem's detrimental effect on the classical MADM methods: TOPSIS, SAW, MEW and GRA, an extensive MATLAB simulation is carried out on the randomly generated 1000 different combinations of a_{ij}. Table 10 shows the number of invalid a_{ij} and the number of a_{ij} with the rank reversal problem.

Fig. 2 illustrates the flowchart to study the rank reversal problem's detrimental effect on the classical MADM methods as shown in Table 10. As shown in Fig. 2, an invalid a_{ij} represents the a_{ij} with $x_{ij} = \infty$ (Max-Min normalization) or $w_j < 0$ (Entropy) or $Score_i = \infty$ (MEW ranking). In Table 10, the number of combinations of a_{ij} with the rank reversal problem is computed for the valid a_{ij}, if the rank reversal exists in Case 2 and Case 1 or Case 2 and Case 3.

As shown in Table 10, rank reversal problem is observed in all the classical MADM methods with the different pairs of attribute normalization and weight calculation methods. With the help of the results shown in Table 10, it is difficult to compare the classical MADM methods: TOPSIS, SAW, MEW and GRA, with respect to the rank reversal problem.

Inference: Dependency of the classical MADM methods on attribute normalization and weight calculation methods and the interdependence between one network attribute and the other network attributes will cause an unreliable network ranking and the rank reversal problem.

Hence, we propose a novel, simplified and improved solution to address the two key limitations of classical MADM methods, i.e., the network rank unreliability and the rank reversal problem as demonstrated in Sections 3.1 and 3.2 respectively.

4. Proposed Simplified and Improved Multiple Attributes Alternate Ranking (SI-MAAR) method

This section of the paper presents the proposed Simplified and Improved Multiple Attributes Alternate Ranking (SI-MAAR) method. The main objectives of the SI-MAAR method are:

- To eliminate the dependency of attribute normalization and weight calculation methods, thereby improving the reliability of network selection.

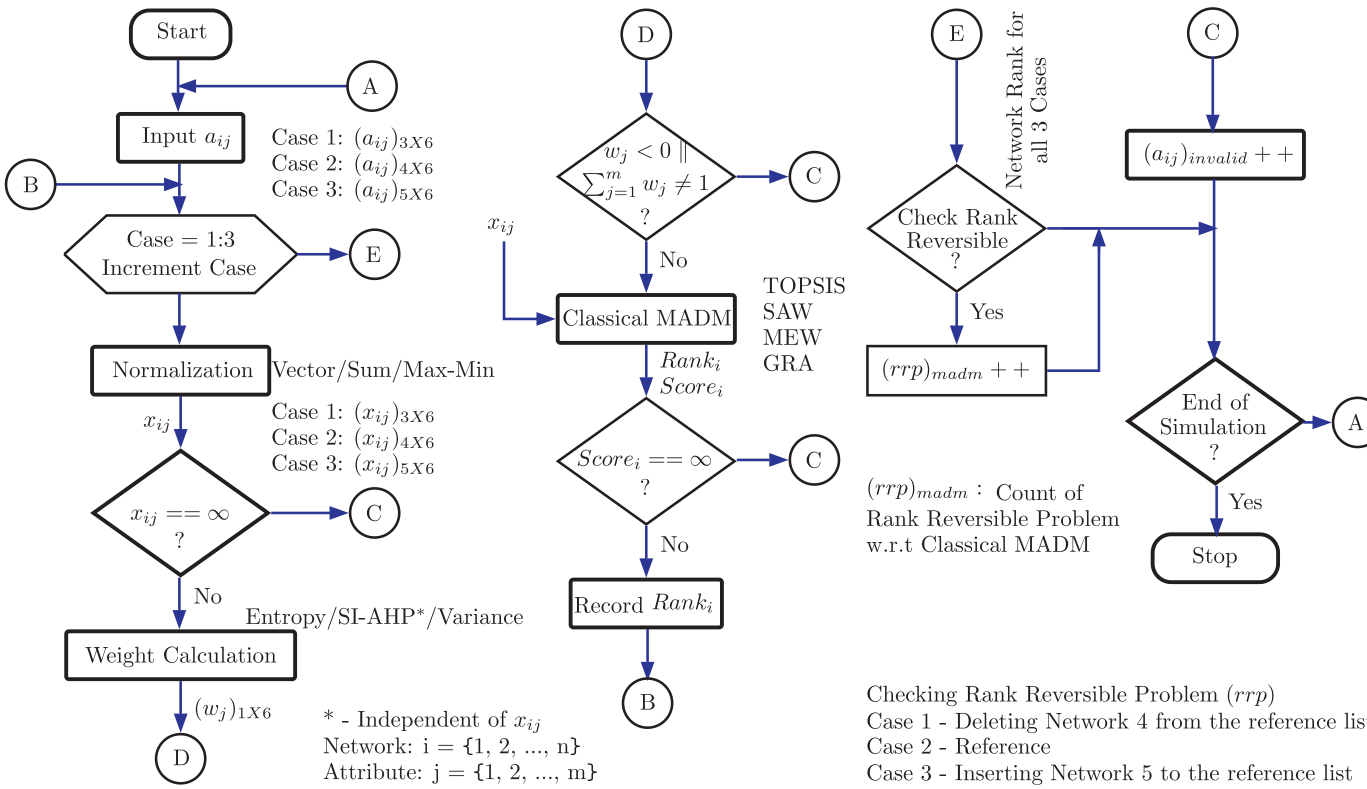

Fig. 2. Flowchart for counting the rank reversible problem of the classical MADM methods for 1000 different a_{ij}.

- To compute the network rank and score, which are independent of other network attributes, thereby avoiding the rank reversal problem.

Fig. 3 shows the proposed SI-MAAR method of ranking the available heterogeneous wireless networks during the handover. The SI-MAAR accepts the expectation of the attributes (e_j) instead of weight of the attributes. The e_j is depicted based on the network, user, mobile device and the application (k) [1,2]. Further, e_j is used along with the networks and the corresponding attribute values (a_{ij}) in computing the Closeness Index matrix or Utility matrix. This step of the SI-MAAR method eliminates the dependency from the attribute normalization and weight calculation methods.

Closeness Index (CI) or Utility matrix: It represents the closeness or utility (satisfaction) between the a_{ij} and e_j. The higher the difference between a_{ij} and e_j, the better is the CI of the ith network with respect to the jth benefit attribute, however the same is not true if the jth attribute is the cost attribute. The CI between the a_{ij} and e_j is computed using the following equation:

$$CI_{ij} = \frac{a_{ij}}{(a_{ij} + e_j)}, \text{i} = 1, 2, 3, \ldots, \text{n and j} = 1, 2, \ldots, \text{m}. \quad (1)$$

where a_{ij} is ith network jth attribute value, e_j is the jth attribute expectation from the network, user, mobile device or applications.

Observation 1: The higher the CI of a network with respect to the benefit attributes, the better is the network with respect to the same attributes, however the same is not true for the cost attributes.

Ideal solution: It represents an ideal (best/worst) value of the attributes. To eliminate the dependency (rank reversal problem) among the network attributes on the network ranking procedure, the best (positive) and the worst (negative) ideal value considered for the attributes are:

Positive Ideal Solution $(A^+) = \{A_1^+, A_2^+, \ldots, A_m^+\}$
Negative Ideal Solution $(A^-) = \{A_1^-, A_2^-, \ldots, A_m^-\}$

where

$$A_j^+ = \begin{cases} 1, & \text{for all benefit attributes} \\ 0, & \text{for all cost attributes} \end{cases} \quad (2)$$

$$A_j^- = \begin{cases} 0, & \text{for all benefit attributes} \\ 1, & \text{for all cost attributes} \end{cases} \quad (3)$$

Euclidean distance: Euclidean distance is the distance measuring approach between two real valued vectors [48]. Minkowsky class of distance measure between two vectors x and $y \in D^m$ is given by

$$d_m(\mathbf{x}, \mathbf{y}) = \left(\sum_{j=1}^{m} (x_j - y_j)^p \right)^{1/p} \quad (4)$$

Euclidean Distance (ED) between two real valued vectors which satisfies the properties: Permutation Invariance, Extension Invariance, One dimensional value-sensitivity, Sub-vector Consistency and Monotonic Order Sensitivity with p = 2 in Eq. 4 is given by

$$d_m(\mathbf{x}, \mathbf{y}) = \sqrt{\sum_{j=1}^{m} (x_j - y_j)^2} \quad (5)$$

Euclidean distance with respect to the positive ideal solution (A^+) is:

$$ED_i^+ = \sqrt{\sum_{j=1}^{m} (CI_{ij} - A_j^+)^2} \quad (6)$$

Euclidean distance with respect to the negative ideal solution (A^-) is:

$$ED_i^- = \sqrt{\sum_{j=1}^{m} (CI_{ij} - A_j^-)^2} \quad (7)$$

where ED_i^+ and ED_i^- represents the Euclidean distance of the ith network for the positive ideal (A^+) and the negative ideal (A^-) solution respectively.

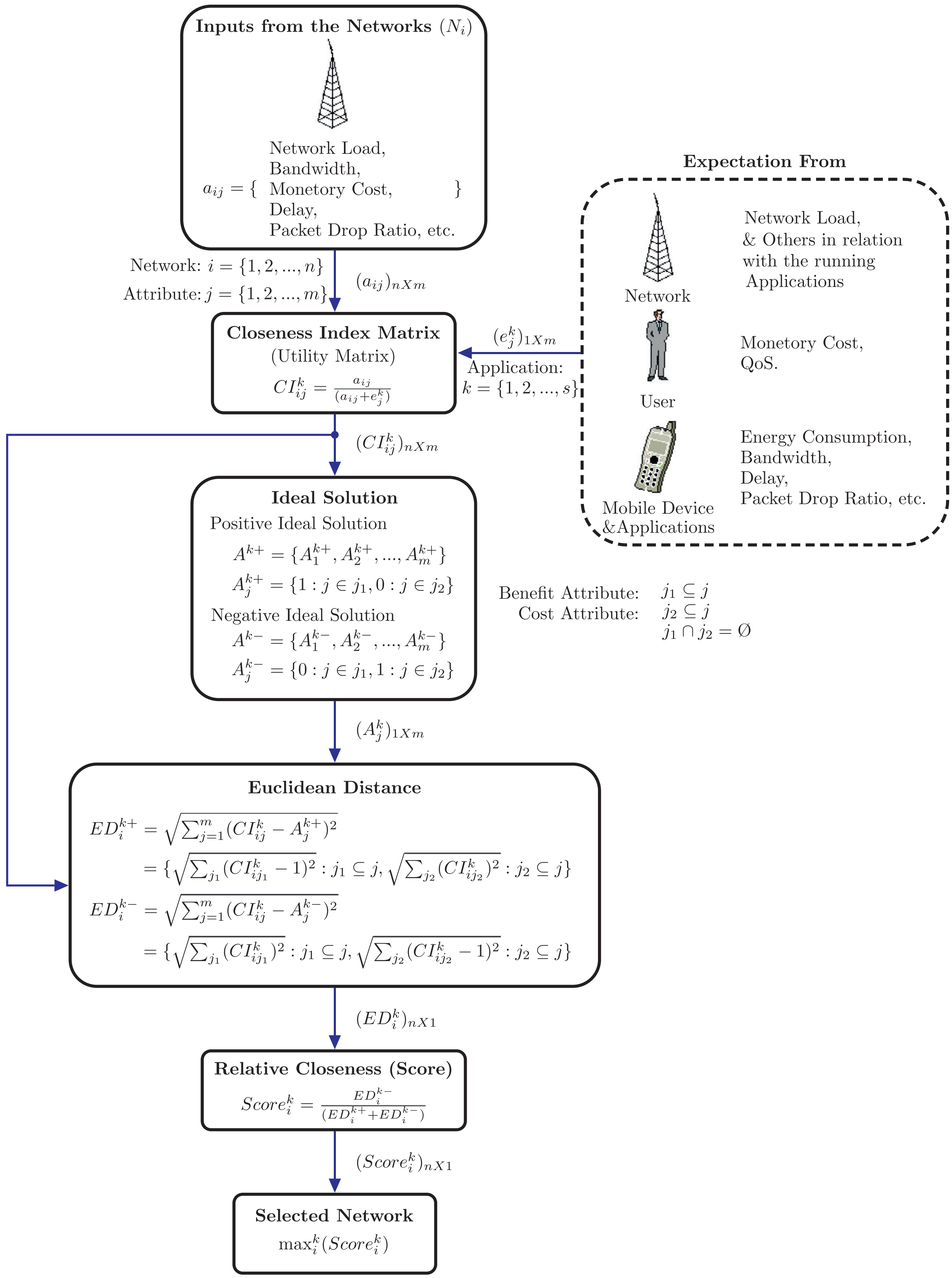

Fig. 3. The SI-MAAR method.

Observation 2: Positive and the negative ideal solutions shown in Eqs. (2) and (3) results into $ED^+_{p \in n}$ independent of $ED^+_{q \in n}$ and $ED^-_{p \in n}$ independent of $ED^-_{q \in n}$. This property of ED^+ and ED^- eliminates the rank reversal problem because of its independent nature.

Relative Closeness (RC): It represents the closeness of the *i*th network to the positive and the negative Euclidean distance. The *i*th network's, minimum distance with the positive and maximum distance with the negative Euclidean distance results into maximum score.

$$Score_i = \frac{ED_i^-}{(ED_i^+ + ED_i^-)} \tag{8}$$

Final selection of the network for handover: Finally, the networks are ranked based on the relative closeness (score) and further, a

Table 11
Network ranking results for parameters shown in Table 4.

Case	Rank 1		Rank 2		Rank 3		Rank 4		Rank 5	
	Network	Score	Network	Score	Network	Score	Network	Score	Network	Score
1	3	0.5347	2	0.3927	1	0.3409	-	-	-	-
2	3	0.5347	4	0.5063	2	0.3927	1	0.3409	-	-
3	5	0.6554	3	0.5347	4	0.5063	2	0.3927	1	0.3409

Table 12
Variation in the rank count of the individual network for 1000 different combinations of a_{ij} in Case 1, Case 2 and Case 3.

Network	Network (i) count in a particular Rank (r) for the particular Case (c) ($(Count_i)_r^c$)											
	Case 1			Case 2				Case 3				
	Rank 1	Rank 2	Rank 3	Rank 1	Rank 2	Rank 3	Rank 4	Rank 1	Rank 2	Rank 3	Rank 4	Rank 5
N1	177	483	340	16	175	471	338	2	19	175	467	337
N2	88	310	602	4	91	305	600	0	8	92	300	600
N3	735	207	58	174	568	200	58	36	176	533	197	58
N4	-	-	-	806	166	24	4	280	544	148	24	4
N5	-	-	-	-	-	-	-	682	253	52	12	1
No. of a_{ij} with rank reversal = 0												

Table 13
Variation in the rank count of the individual network for 1000 different combinations of e_j and the common a_{ij} (Table 4)in Case 1, Case 2 and Case 3.

Network	Network (i) count in a particular Rank (r) for the particular Case (c) ($(Count_i)_r^c$)											
	Case 1			Case 2				Case 3				
	Rank 1	Rank 2	Rank 3	Rank 1	Rank 2	Rank 3	Rank 4	Rank 1	Rank 2	Rank 3	Rank 4	Rank 5
N1	0	1	999	0	0	1	999	0	0	0	9	991
N2	0	999	1	0	8	991	1	0	8	237	754	1
N3	1000	0	0	559	441	0	0	559	441	0	0	0
N4	-	-	-	441	551	8	0	441	551	8	0	0
N5	-	-	-	-	-	-	-	0	0	755	237	8
No. of a rank reversal problem for 1000 different combinations of e_j = 0												

network with the maximum score is selected by the MN for the handover in heterogeneous wireless networks.

$$\text{Selected network for handover} = \max_i (Score_i) \tag{9}$$

4.1. MATLAB simulation results and analysis

Objective: To numerically justify the proposed SI-MAAR, a MATLAB simulation with respect to Table 4 is carried out for computing the network rank and score, and also for checking the rank reversal problem of the SI-MAAR method.

Table 11 shows the network rank and score with respect to Table 4 for three different cases: Case 1, Case 2 and Case 3. As shown in Table 11, even after the removal of network N4 in Case 1 or the insertion of network N5 in Case 3 with respect to Case 2, there is no change in the network rank and score. The removal and insertion of the network in the network selection list in the proposed SI-MAAR method does not make any change in the rank and score of the networks. As shown in the Table 11, the score of network N3 (0.5347) in Case 2 remains unchanged even after the removal of network N4 in Case 1 or the insertion of network N5 in Case 3 with respect to Case 2, and similarly with other networks.

Further, to check the existence of the rank reversal problem in SI-MAAR, an extensive MATLAB simulation is carried out on the similar 1000 combinations of a_{ij} (used for Table 10) with the fixed e_j shown in Table 4. Table 12 shows the variation in the network (N1, N2, N3, N4 and N5) count in a particular rank for 3 different cases (Case 1, Case 2 and Case 3), where the number of invalid a_{ij} and the number of 'a_{ij}' with the rank reversal problem is zero. Table 12 also shows the sensitivity of the SI-MAAR method for the variation of a_{ij}.

In Table 12, $(Count_i)_r^c$ represents the ith network count in rth rank for the cth case, where

Case 1 (c = 1, n = 3): i = r = {1, 2, 3},
Case 2 (c = 2, n = 4): i = r = {1, 2, 3, 4} and
Case 3 (c = 3, n = 5): i = r = {1, 2, 3, 4, 5}.

As shown in Table 12, $(Count_1)_r^1$ is 177, 483 and 340 representing the network N1 count in rank 1, rank 2 and rank 3 respectively for Case 1, and so on for the other networks with the following conditions:

- $\sum_{r=1}^{n} (Count_i)_r^c$ = No. of combinations of a_{ij}
- $\sum_{i=1}^{n} (Count_i)_r^c$ = No. of combinations of a_{ij}

With respect to Table 12 following are some of the observations:

- For Case x, after removing the nth network, the remaining $(n-1)$ networks rank count in a particular rank for Case $x-1$ can be computed as,

$$(Count_i)_{r\neq 1}^c = [(Count_i)_{r+1}^{c+1} - (Count_i)_{r+1,r\neq n}^{c+1*}] + (Count_i)_r^{c+1*} \tag{10}$$

$$(Count_i)_{r=1}^c = [(Count_i)_r^{c+1} + (Count_i)_{r+1}^{c+1}] - (Count_i)_{r+1}^{c+1*} \tag{11}$$

where $\sum_{i=1}^{n} (Count_i)_{r,r\neq 1}^{c+1*} = \sum_{r=r}^{n} (Count_{n+1})_{r+1}^{c+1} \sum_{i=1}^{n} (Count_i)_1^{c+1*} = \sum_{r=2}^{n} (Count_{n+1})_{r+1}^{c+1}$, represents the rank r count of the network i of Case x which remains in the same rank r in Case $x-1$.

- For no rank reversal problem,

$$(Count_i)_r^{c+1*} \geq 0 \tag{12}$$

Similarly, SI-MAAR is also verified for different e_j to study the rank reversal problem for the fixed a_{ij} (Table 4). Table 13 shows the variation in the rank count of the networks for 1000 different random

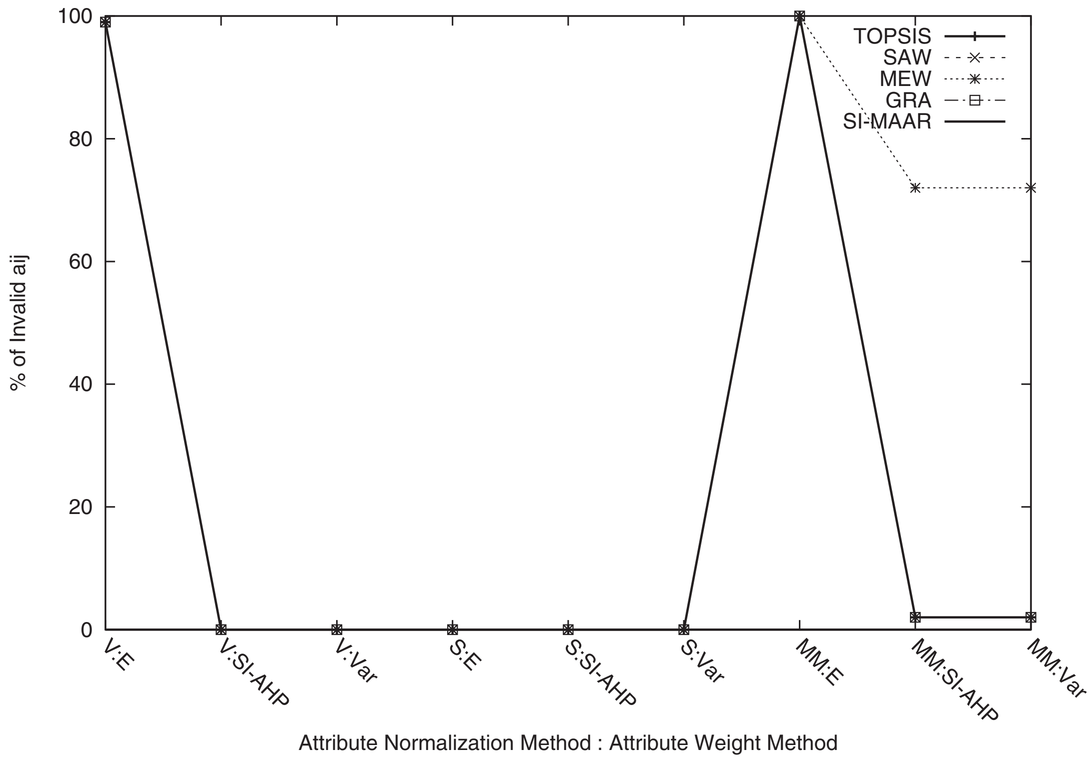

Fig. 4. The percentage of Invalid a_{ij} of the classical MADM methods and SI-MAAR for 10,000 Combinations of a_{ij} (**Note -** S: Sum, V: Vector, MM: Max-Min normalization and E: Entropy, SI-AHP, Var: Variance weight methods).

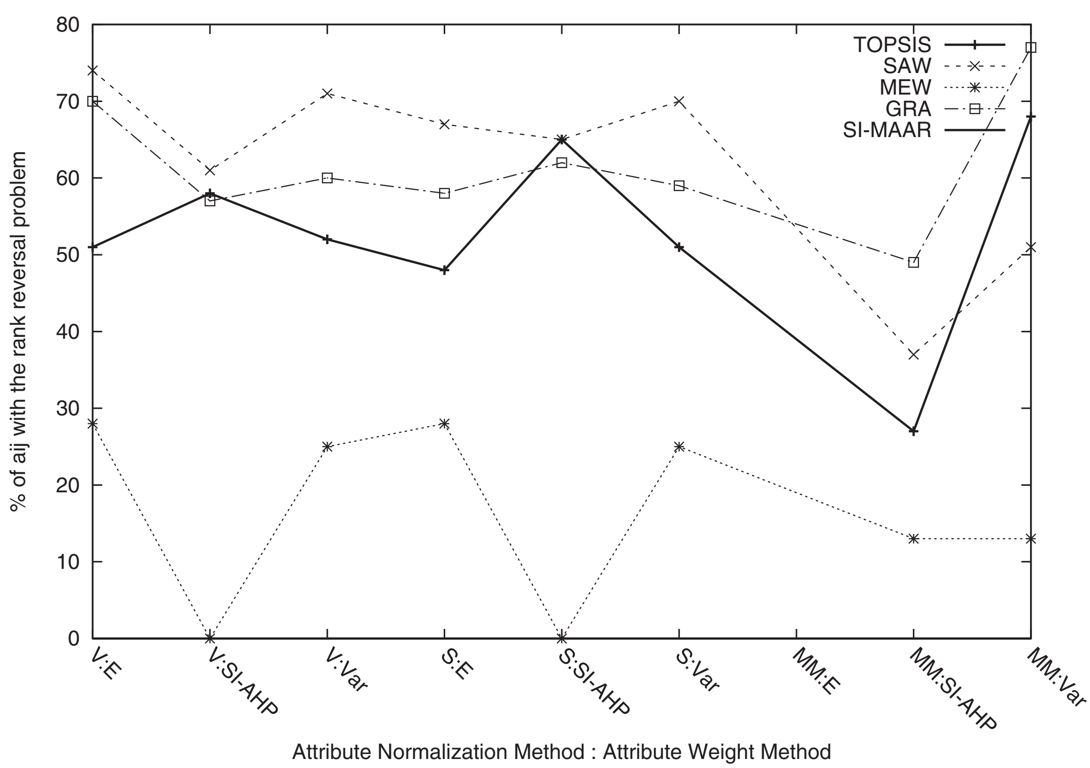

Fig. 5. The percentage of the rank reversal problem of the classical MADM methods and SI-MAAR for 10,000 combinations of a_{ij} (**Note -** S: Sum, V: Vector, MM: Max-Min normalization and E: Entropy, SI-AHP, Var: Variance weight methods).

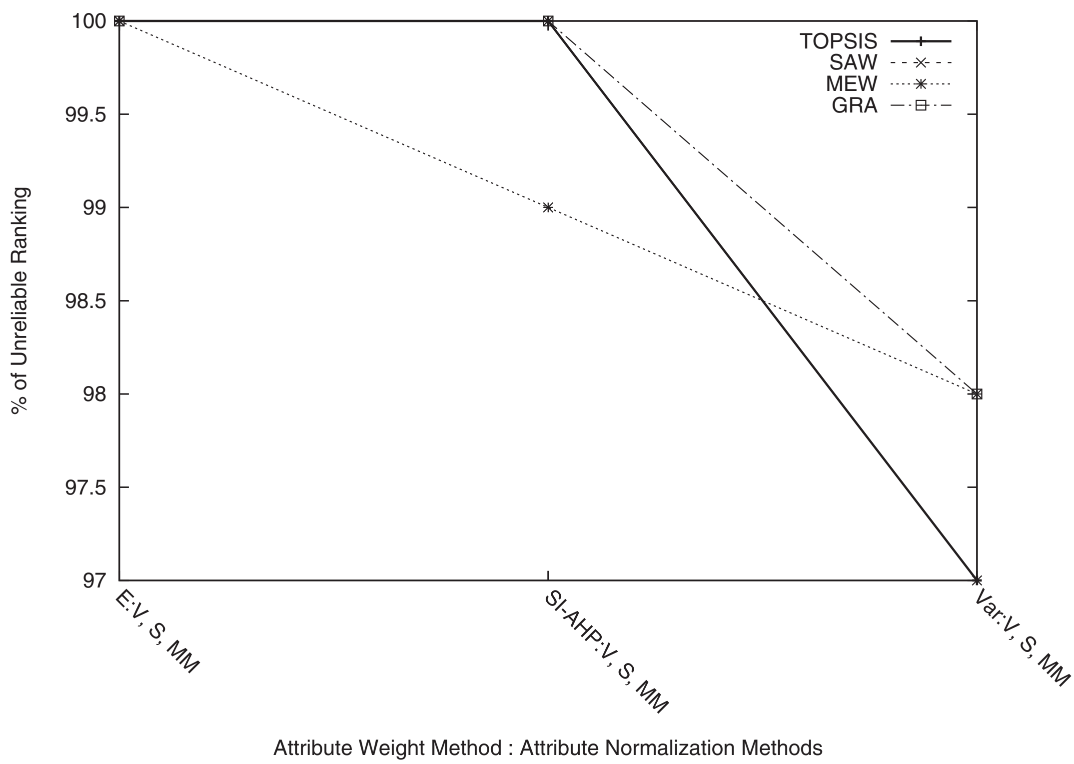

Fig. 6. The percentage of the unreliable ranking of the classical MADM methods for 10,000 combinations of a_{ij} (**Note -** S: Sum, V: Vector, MM: Max-Min normalization and E: Entropy, SI-AHP, Var: Variance weight methods).

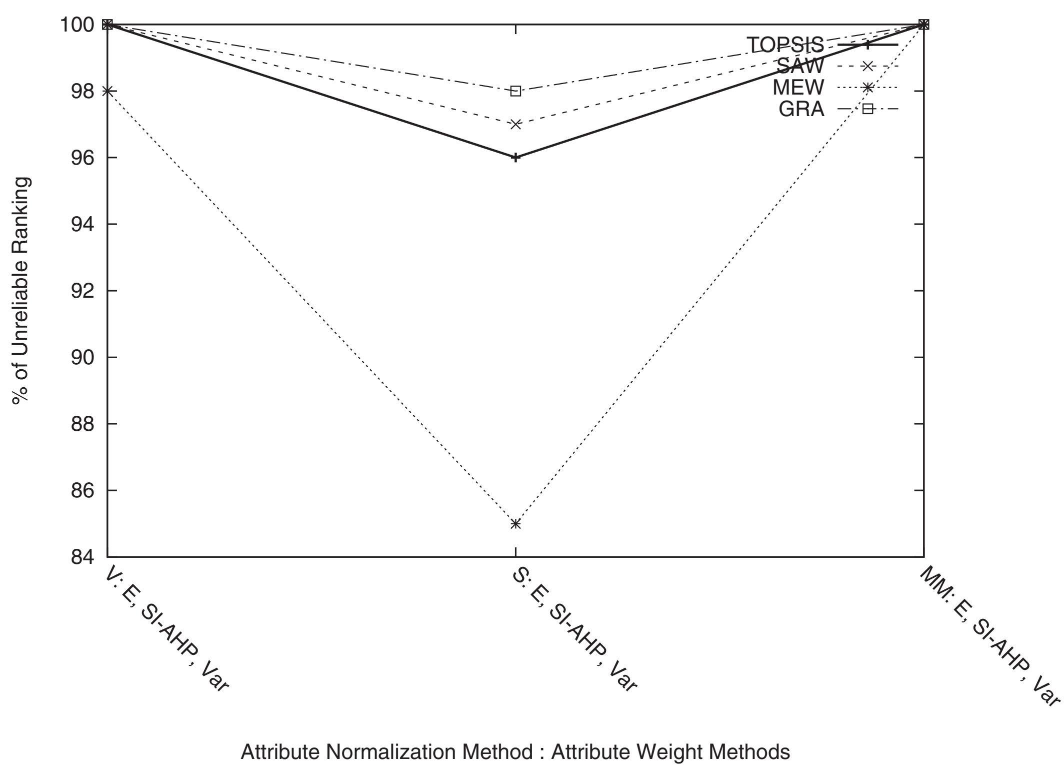

Fig. 7. The percentage unreliable ranking of the classical MADM methods for 10,000 combinations of a_{ij} (**Note -** S: Sum, V: Vector, MM: Max-Min normalization and E: Entropy, SI-AHP, Var: Variance weight methods).

combinations of e_j with the 0% rank reversal problem. The rank count of the individual networks shown in Table 13 follows the Eqs. 10 and 11, and Eq. 12 for 0% of the rank reversal problem.

In addition to the above, an extensive MATLAB simulation is carried out to compare classical MADM methods and SI-MAAR with respect to the percentage of invalid a_{ij} (Fig. 4), the percentage of a_{ij} with the rank reversal problem (Fig. 5) and the percentage of unreliable networks ranking (Figs. 6 and 7) for 10,000 different combinations of a_{ij}. The percentage of a_{ij} with the rank reversal problem is computed against the valid number of a_{ij}. Also, the percentage of unreliable networks ranking is computed for (i) different attribute weight method with respect to the attribute normalization methods (Fig. 6) and (ii) attribute normalization method with respect to the attribute weight methods (Fig. 7).

As shown in Fig. 4, the percentage of invalid a_{ij} (i.e a_{ij} with $x_{ij} = \infty$) in the proposed SI-MAAR method is zero compared to the classical MADM methods. Similarly, Fig. 5 illustrates, the percentage of the rank reversal problem with the proposed SI-MAAR method is zero compared to the classical MADM methods. As shown in Fig. 5, among the classical MADM methods, rank reversal problem is less in the MEW method.

Furthermore, Fig. 6 illustrates the percentage of unreliable ranking of the classical MADM methods for 10,000 different combinations of a_{ij} with respect to the attribute weight method to the different attribute normalization methods. As shown in Fig. 6, the percentage of unreliable ranking of the classical MADM methods is more than 97%, whereas, in the proposed SI-MAAR method is zero (not shown in Fig. 6).

Similarly, Fig. 7 illustrates the percentage of unreliable ranking of the classical MADM methods for 10,000 different combinations of a_{ij} with respect to the attribute normalization method to the different attribute weight methods. As shown in Fig. 7, among the classical MADM methods, the percentage of unreliable ranking is less in the MEW method. Moreover, with the proposed SI-MAAR, the percentage of unreliable ranking is zero in comparison with the classical MADM methods.

Hence, it is demonstrated that the proposed Simplified and Improved Multiple Attributes Alternate Ranking (SI-MAAR) method is the simplest and most improved network ranking method for VHD in heterogeneous wireless networks. SI-MAAR is a simple method with less complex and unambiguous network ranking steps when compared to the classical MADM methods: TOPSIS, SAW, MEW and GRA. Moreover, the proposed SI-MAAR method is completely eliminating unreliability in network rank and the rank reversal problem of the classical MADM methods. Hence, the worst time complexity of the proposed SI-MAAR for n networks of m attributes (including both the benefit and cost attribute) is $O(m*n)$.

Finally, SI-MAAR method is one of the best replacements for the existing classical MADM methods, not only applicable to the field of real heterogeneous wireless networks, but also to the other fields where classical MADM methods are widely used in real life.

5. Conclusions and future work

In comparison with the classical MADM methods: TOPSIS, SAW, MEW and GRA, the proposed Simplified and Improved Multiple Attributes Alternate Ranking (SI-MAAR) method eliminated the attribute normalization and weight calculation method's dependency, resulting in reliable network rank for the seamless handover without the rank reversal problem. The MATLAB simulation carried out in 1000 and 10,000 different combinations of network attributes, and 1000 different combinations of the expected attributes, justified network attribute sensitivity, 100% network rank reliability with 0% rank reversal problem of the SI-MAAR method when compared to the classical MADM methods.

The future work in this direction can be carried out by verifying SI-MAAR method using network simulator and test-bed for the performance metrics like number of handovers, handover delay, QoS of the real-time, non-real-time and interactive applications, and the percentage of user satisfaction in the heterogeneous wireless networks.

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