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OM²DNC: Opportunistic Max²-Degree Network Coding for wireless data broadcasting☆

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ABSTRACT

In wireless broadcasting systems, multiple retransmissions are commonly employed to guarantee the correct reception of each packet. The traditional hybrid automatic retransmission request protocol retransmits one packet per slot. Hence, a large amount of retransmissions is required to correctly receive all the packets, which leads to low spectrum efficiency. In this paper, we propose an Opportunistic Max²-Degree Network Coding (OM²DNC) based wireless broadcasting (WBC) protocol. Specifically, to reduce the overall number of retransmissions, lost packets of different user equipments (UEs) are combined by performing Max-Degree network coding (NC) at the access node. Then, NC combined packets are broadcasted by utilizing the Max-Degree based opportunistic retransmission protocol. At each UE, the lost packet can be recovered by using the proposed joint network recursive systematic convolution decoder. Theoretical analyses and simulation results show that the average number of transmissions performance of the proposed OM²DNC based WBC protocol outperforms that of the traditional NC based WBC protocols.

1. Introduction

Next generation broadband wireless access (BWA) networks such as WiMAX and Long Term Evolution (LTE) are expected to provide efficient, affordable and ubiquitous internet access for data multimedia broadcasting applications such as stock price, real-time traffic and weather information, high-definition (HD) digital television (TV) video streaming and video conferencing. To ensure users' satisfaction, the Quality of Service (QoS) provided by BWA networks should be comparable to that of wired data broadcasting networks. Although the QoS framework for the next generation BWA networks has been designed to support the requirements of multimedia applications, the inherent error-prone nature of the wireless channel often results in high packet error rate (PER) and hence degrades the system reliability. To transmit information reliably over wireless channel, many approaches are employed, such as Forward Error Correcting (FEC), Automatic Retransmission reQuest (ARQ) and Hybrid Automatic Retransmission reQuest (HARQ) [1]. Among these approaches, FEC suffers from low transmission efficiency and ARQ faces large delay and low throughput caused by severe fading channel. HARQ, which combines both FEC and ARQ techniques, can contribute to an efficient utilization of the available resources.

1.1. Motivation and related work

HARQ has now become a fundamental tool of BWA networks, as it can significantly improve the reliability of wireless link. However, HARQ still encounters challenge in wireless broadcasting applications. As identical information is transmitted from one source to many receivers, it is impossible for each receiver to successfully receives all packets all the time. The traditional HARQ protocol retransmits one packet per slot.[1] Therefore, HARQ[2] may require large amount of retransmissions in broadcasting scenario to ensure

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[1] We use the term packet because no matter what exactly is retransmitted in a particular HARQ protocol, a Protocol Date Unit (PDU) in MAC layer is produced and delivered to PHY layer, a resource block (or slot, in LTE the length of a resource block is 500 us) of system must be allocated to the terminal. Therefore in the view of channel occupation, both traditional HARQ (that retransmits information bits and redundancy bits) and HARQ with Incremental Redundancy (HARQ-IR) (that retransmits only redundancy bits) need to retransmit at least one packet.

[2] Though advanced HARQ schemes, i.e., HARQ-IR and HARQ with Chase Combine (HARQ-CC) [2], achieve significant performance improvement, the potential of being combined with network coding makes traditional HARQ Type-I scheme a better choice. For this reason, in this paper we mainly focus on traditional HARQ

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correct reception for every packet in all the receivers, which leads to low bandwidth efficiency. In order to improve the transmission efficiency for wireless broadcasting (WBC) system, recently, network coding (NC) [3–7] has been applied to WBC system. By broadcasting, we refer to the scenario where a common sender serves multiple receivers[3] with the same information. In NC based WBC scenario, lost packets or additional naive packets are NC combined before transmitted. According to the way NC combined packets are utilized, we could divide existing NC based WBC schemes into two types, i.e., *wait-and-decode network coding* (WDNC) [7–10] and *instantly decodable network coding* (IDNC) [14–20].

1.1.1. WDNC based WBC

Typical WDNC applications include fountain codes and random linear. For WDNC based WBC, received packets that are not instantly decodable are stored for future decoding opportunity [7–10]. With WDNC, perfect throughput could be achieved, however, the receivers can decode all the lost packets only if they have received sufficient number of mixed packets, this kind of retransmission strategy usually has a greater computational complexity and incurs additional delay. In recent work, the trade-off between performance, delay and complexity is addressed to facilitate the application of network coding in multimedia services [11,12].

1.1.2. IDNC based WBC

IDNC refers to that the received packets are decoded only at their reception instant and cannot be stored for future decoding. For XORing NC based WBC, NC is performed on lost packets which are carefully selected to ensure the decodability at the receiver [14–19]. A dual-XOR HARQ retransmission scheme for wireless broadcasting is proposed in [21], which introduces an additional XOR operation between two lost packets from the individual receiver. In [22], a XORing network coding combining (XNCC) and distributed Turbo coding based type-III HARQ protocol is proposed for wireless broadcasting system. The broadcasting efficiency can be significantly improved by introducing the network and channel coding gain. The above literatures [8–10,14–19,21] address the problem of NC based retransmission for WBC system. Unfortunately, different lost packets are NC combined and retransmitted without considering the *degree* of each NC packet.

1.2. Contributions

In this paper, an Opportunistic Max^2-Degree Network Coding (OM^{2}DNC) based WBC protocol is proposed to improve the overall system spectrum efficiency. At the access node (AN), lost packets of different user equipments (UEs) are combined by performing Max-Degree NC. Then, NC combined packets are broadcasted by utilizing the Max-Degree based opportunistic retransmission protocol. At each UE, the lost packet can be recovered by using the proposed joint network recursive systematic convolution (RSC) decoder (JNRD). Theoretical analyses and simulation results show that the average number of transmissions (ANTs) performance of the proposed OM^{2}DNC based WBC protocol is superior to that of the maximum clique selection algorithm (MCSA) [16], traditional index NC (INC) [15] protocol and XNCC [22] protocol in severe fading channels.

2. System model and parameters

We consider a WBC system, where a wireless AN[4] broadcasts a frame of λ packets to a set $\mathcal{R} = \{R_1, R_2, \ldots, R_M\}$ of M UEs within its coverage, and the UEs try to receive packets transmitted from BS. BS adopts TDMA protocol to broadcast these packets. During a TDMA slot, only one packet could be transmitted. In total, λ TDMA time slots are needed to broadcast λ data packets, and this λ time slots is referred to as *Broadcast Phase*. After decoding packets, UEs send feedback to report whether packets have been successfully decoded. After *Broadcast Phase*, BS starts *Retransmission Phase*, in which the BS retransmits packets and UEs send feedback to request retransmission of lost packets until all the packets are correctly received.

2.1. Mathematical description

2.1.1. Signal transmission

We assume that the original WBS frame $\mathbf{A}$ consisting of λ information vectors, i.e., $\mathbf{A} = \{\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_\lambda\}$. A transmission information vector could be denoted by the XORing of a set of original. Let $\mathcal{C}$ denote the *candidate set*, which is defined as an index set of original packets that can be XORing NC combined. For the information vector set $\mathbf{A}_\mathcal{C} = \{\mathbf{a}_i \mid i \in \mathcal{C}\}$, the AN combines all the information vectors by using NC, i.e., $\mathbf{a}_\mathcal{C} = \mathbf{a}_i \oplus \mathbf{a}_j \oplus \cdots \oplus \mathbf{a}_k$, where $\oplus$ denotes the XOR operator. Specially, for broadcast phase, the candidate set is the corresponding original information vector itself. The information vector is encoded by the RSC encoder yielding the codeword $\mathbf{b}_\mathcal{C} = \mathrm{RSC}(\mathbf{a}_\mathcal{C})$, where $\mathrm{RSC}(\ \cdot\ )$ denotes the RSC encoder. The code bits $b_\mathcal{C}(i)$ are mapped to symbols $x_\mathcal{C}(i) \in \mathbb{X}$ of the modulation alphabet $\mathbb{X}$ and broadcasted to the UEs. In case of block fading channel, the received signal vector at the lth UE can be expressed as

$$\mathbf{y}_{l,\mathcal{C}} = h_l \sqrt{P_s}\mathbf{x}_\mathcal{C} + \mathbf{n}_l \tag{1}$$

where $\mathbf{x}_\mathcal{C}$ denotes the transmitted signal packet and $\mathbf{n}_l$ is the noise vector whose elements are identically independent distributed (i.i.d) zero-mean Gaussian random variables with variance $\sigma_{n_l}^2$. P_s denotes the BS's transmission power. The coefficient h_l, $\forall\{l\}$ denotes complex zero-mean circular symmetric Gaussian distributed variable with variance $\sigma_{h_l}^2$, i.e., $h_l \sim \mathcal{CN}(0, \sigma_{h_l}^2)$.

2.1.2. Signal reception

After receiving packets, UEs try to decode these packets, and whether a packet is decoded successfully can be determined by calculating the CRC bits attached in the tail of the packet. We assume that the information vector $\mathbf{a}_i$ is lost at the lth UE. Based on the received NC combined signal $y_{l,\mathcal{C}}(m)$, the LLRs for the participating coded bit $b_i(n)$ can be calculated as [23],

$$\begin{aligned} L_{\mathrm{Dem}}^{l}(b_i(n)) &= \mathrm{sign}\left(\hat{x}^m_{\mathcal{C}\backslash\{i\}}(n)\right) \ln \frac{\Pr\left\{ b_i(n) = 0 | y_{l,\mathcal{C}}(m), \hat{h}_{l,\mathcal{C}} \right\}}{\Pr\left\{ b_i(n) = 1 | y_{l,\mathcal{C}}(m), \hat{h}_{l,\mathcal{C}} \right\}} \\ &= \mathrm{sign}\left(\hat{x}^m_{\mathcal{C}\backslash\{i\}}(n)\right) \ln \frac{\sum_{x\in\mathbb{X}, b_i(n)=0} \exp\left(-\frac{\left|y_{l,\mathcal{C}}(m) - \hat{h}_{l,\mathcal{C}}\sqrt{P_s}x\right|^2}{\sigma_{n_l}^2} \right)}{\sum_{x\in\mathbb{X}, b_i(n)=1} \exp\left(-\frac{\left|y_{l,\mathcal{C}}(m) - \hat{h}_{l,\mathcal{C}}\sqrt{P_s}x\right|^2}{\sigma_{n_l}^2} \right)} \end{aligned} \tag{2}$$

where $\hat{h}_{l,\mathcal{C}}$ is the estimate of the channel coefficient $h_{l,\mathcal{C}}$ [25], $\mathrm{sign}(\cdot)$ denotes the sign function, $\backslash$ denotes the relative complement (also termed as set difference) operator, $\mathrm{sign}(\cdot)$ denotes the sign function, $\hat{x}^m_{\mathcal{C}\backslash\{i\}}(n) = [x_{\mathcal{C}\backslash\{i\}}(n)]_m$ denotes the modulated signal portion of $x_{\mathcal{C}\backslash\{i\}}(n)$ associated with the coded bit

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Type-I scheme where unsuccessfully received packets (information bits and redundancy bits) are threw away without further use.

[3] The number of receivers should not be too large for the threat of feedback implosion problem. For the scheme for broadcasting scenario with massive receivers, see [20].

[4] Such as a base station (BS) in a LTE or WiMAX cell.

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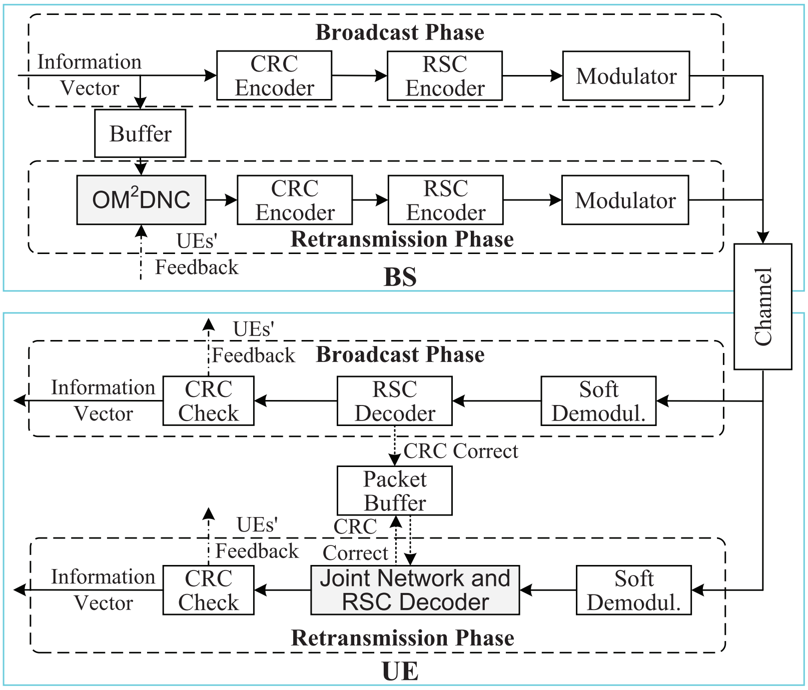

Fig. 1. The block diagram of the proposed OM^{2}DNC based wireless broadcasting protocol.

$b_{\mathcal{C}\backslash\{i\}}(m)$ and $\text{sign}(\hat{x}^m_{\mathcal{C}\backslash\{i\}}(n)) \in \{1, -1\}$. To calculate the probability of $b_i(n) = 0$ (or $b_i(n) = 1$), all possible symbols $x \in \mathbb{X}$ related to $b_i(n)(i) = 0$ (or $b_i(n) = 1$) need to be considered. After getting the LLRs $L^l_{\text{Dem}}(b_i(n))$, the RSC decoding can be done by using the maximum-likelihood decoder or the maximum a posteriori (MAP) decoder [24]. In this way, the soft value of the desired coded bit $b_i(m)$ can be obtained from the received combined signal.

2.2. Assumptions

In this paper, we will consider the following assumptions:

- In both initial broadcast and retransmission phases, all the packets have the same fixed length and use the same modulation scheme.
- In both initial broadcast and retransmission phases, BS adopts TDMA protocol to broadcast these packets. During a TDMA slot, only one packet could be transmitted. We assume that the channel coefficient keeps constant during a TDMA slot, while independent with others in different slots, i.e. the channel is a block fading channel and $\mathbb{E}[h_{l,k}h^*_{m,n}] = \sigma^2_{h_{l,k}}\delta_{l,m}\delta_{k,n}$. $\mathbb{E}[\cdot]$ denotes the expectation, $(\cdot)^*$ denotes the complex conjugate and $\delta_{i,j}$ denotes the Kronecker delta function. Hence, the packet losses at different UEs are uncorrelated. Due to the randomness of UEs' location, we assume the distribution of channel between BS and different UEs are same, i.e., $\sigma^2_{h_{l,k}} = \sigma^2_h, \forall\{l, k\}$.
- The BS knows the instantaneous states of all the packets at each UE. Each UE sends an acknowledgement signal (ACK) whenever the packet is received correctly, or else it sends a negative acknowledgement signal (NAK). For simplicity we assume that all the ACK/NAKs are instantaneous and never lost, i.e. the feedback channel is error free and has no delay.

3. The proposed OM^{2}DNC based WBC protocol

The proposed OM^{2}DNC based WBC protocol includes two phases to ensure reliable packets reception: *Broadcast Phase* and *Retransmission Phase*.

- *Broadcast Phase*: As illustrated in Fig. 1 (BS side), the BS first attaches FEC bits and CRC bits in the tail of the information vector. Then, the BS broadcasts the RSC encoded and modulated packets to the UEs. As illustrated in Fig. 1 (UE side), each UE receives packets and tries to decode these packets. Whether a packet is successfully decoded is determined by the CRC attached in the packet tail. Each UE sends a feedback to the BS after RSC decoding and CRC checking. Specifically, each UE sends an ACK signal whenever the packet is received correctly, or else it sends a NAK signal. If the CRC checking is correct, the received packet will be stored in the buffer. The decoding states of λ *naive packets* at UE m could be represented as a λ-dimensional decoding vector $\boldsymbol{d}_m$, that is
$$\boldsymbol{d}_m = [d^{(1)}_m, d^{(2)}_m, \ldots, d^{(\lambda)}_m]^T \tag{3}$$
where $d^{(n)}_m \in \{0, 1\}$ denotes the decoding state of the nth naive packet at UE m, i.e., $d^{(n)}_m = 0$ if the nth naive packet is correctly received; otherwise, $d^{(n)}_m = 1$.
- *Retransmission Phase*: As illustrated in Fig. 1 (BS side), the BS first forms an NC combined retransmission packet by using the proposed OM^{2}DNC strategy. Information vectors of different lost packets are NC combined by several information vectors and could be denoted as a linear combination of all the information vectors over $GF(2)$, i.e., the inner product of a coefficient vector and information vectors. The ith NC combined vector in the kth round of retransmission is given as
$$\begin{aligned}\boldsymbol{c}_{k,i} &= \boldsymbol{r}_{k,i} \cdot \boldsymbol{A}\\ &= [r^{(1)}_{k,i}, r^{(2)}_{k,i}, \ldots, r^{(\lambda)}_{k,i}][\boldsymbol{a}_1, \boldsymbol{a}_2, \ldots, \boldsymbol{a}_\lambda]^T\end{aligned} \tag{4}$$
where $1 \le k \le \mathcal{K}, 1 \le i \le \mathcal{I}_k$. $r^{(n)}_{k,i} \in \{0, 1\}$, $\boldsymbol{a}_n$, $\mathcal{K}$ and $\mathcal{I}_k$ denote the nth element of vector $\boldsymbol{r}_{k,i}$, the nth information vector, the total number of retransmission rounds and the number of network coded packets in kth round of retransmission, respectively. After passing the CRC encoder, RSC encoder in sequence, the network coded retransmission packets are broadcasted to the UEs by the BS.

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After receiving a retransmission packet, UE m tries to obtain its own lost packet by using prior correctly received naive packets. Whether the received retransmission packet $\boldsymbol{c}_{k,i}$ could help recover a lost packet at UE m or not could be determined by checking the inner product of $\boldsymbol{r}_{k,i}$ and $\boldsymbol{d}_m$, denoted as $D_m^{\boldsymbol{r}_{k,i}}$, which represents the number of lost packets contained in $\boldsymbol{c}_{k,i}$ for UE m. $D_m^{\boldsymbol{r}_{k,i}}$ can be given as

$$D_m^{\boldsymbol{r}_{k,i}} = \boldsymbol{r}_{k,i} \cdot \boldsymbol{d}_m = \sum_{n=1}^{\lambda} r_{k,i}^{(n)} d_m^{(n)} \tag{5}$$

If $D_m^{\boldsymbol{r}_{k,i}} = 0$ or $D_m^{\boldsymbol{r}_{k,i}} > 1$, the recovered information vector contains none lost packet or more than one lost packets corresponding to UE m. If $D_m^{\boldsymbol{r}_{k,i}} = 1$, UE m could obtain a lost packet after correctly decode the received retransmission packet. UE binary combine the correctly received information vectors that are contained in $\boldsymbol{c}_{k,i}$ (according to $\boldsymbol{r}_{k,i}$), and the recovered information vector of lost packet could relaxed as

$$\begin{aligned} t_{k,i} &= [(\boldsymbol{r}_{k,i} \odot \bar{\boldsymbol{d}}_m) \cdot \boldsymbol{A}] + \boldsymbol{c}_{k,i} \\ &= [(\boldsymbol{r}_{k,i} \odot \bar{\boldsymbol{d}}_m) \cdot \boldsymbol{A}] + [(\boldsymbol{r}_{k,i} \odot \bar{\boldsymbol{d}}_m) \cdot \boldsymbol{A} + (\boldsymbol{r}_{k,i} \odot \boldsymbol{d}_m) \cdot \boldsymbol{A}] \\ &= (\boldsymbol{r}_{k,i} \odot \boldsymbol{d}_m) \cdot \boldsymbol{A} \end{aligned} \tag{6}$$

where $\odot$ denotes Hadamard (Schur) product operation and $\bar{\cdot}$ denotes bitwise NOT (or complement) operation.

As illustrated in Fig. 1 (UE side), each UE demodulates the received NC combined retransmission packet by using the proposed JNRD decoder and then feedbacks to the BS after CRC checking. If the CRC checking is correct, the received packet will be stored in the buffer.

We assume that the BS transmits a WBS frame $\mathcal{X}$, which consisting of λ packets $\mathcal{X} = \{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_\lambda\}$, in the broadcast phase. In this paper, a $M \times \lambda$ *feedback matrix* **T** is applied to store the state (correctly received or not) of all the packets at each UE. The (i, j)th entry of the matrix **T** $(1 \le i \le M,\ 1 \le j \le \lambda)$ indicates whether the ith UE has received packet $\mathbf{x}_j$ or not, i.e., $\mathbf{T}(i, j) = 0$ if $\mathbf{x}_j$ is correctly received by the ith UE, otherwise $\mathbf{T}(i, j) = 1$. The ith column of matrix **T** represents the states whether the ith packet is received successfully or not by all the UEs. The ith row of matrix **T** represents the states whether all the packets are received successfully or not by the ith UE. The flowchart of the proposed OM^{2}DNC based WBC protocol is given in Fig. 2.

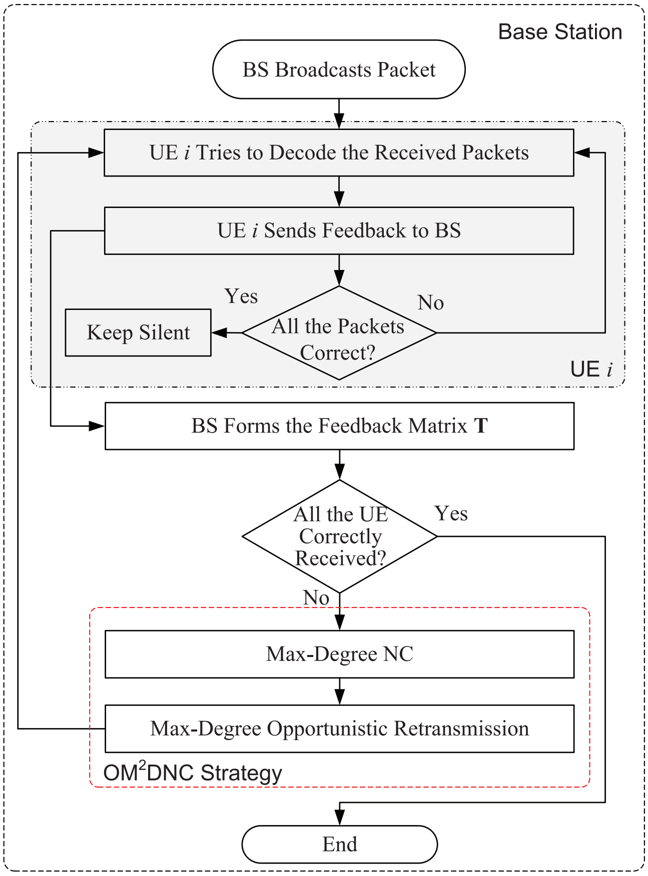

Fig. 2. Flowchart of the proposed OM^{2}DNC based WBC protocol.

		Packet ID									
		P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}
UE ID	R_1	0	1	1	1	1	0	0	0	0	0
	R_2	0	0	0	0	0	1	1	0	0	1
	R_3	0	0	0	1	0	1	0	0	1	1
	R_4	1	1	1	0	0	0	1	0	0	0
	R_5	0	0	1	0	0	0	0	0	1	1

Fig. 3. The example of feedback matrix **T**.

3.1. OM^{2}DNC strategy

In this paper, the OM^{2}DNC strategy is proposed to guarantee the efficiency and decodability of the network coded packets. In the *Retransmission Phase*, BS forms a *feedback matrix* **T** based on UEs' HARQ feedback information, as shown in Fig. 3. When a packet is successfully received by one UE, the corresponding position of the *feedback matrix* **T** will be marked as "0", otherwise "1". Hence, matrix **T** can convey two meanings: (1) whether the packet is lost; (2) the index of UE that lost the packet. In order to guarantee the efficiency and decodability of the network coded retransmission packets, a set of lost packets to perform NC combining, named as candidate set (CS), is generated based on the following two principles: (1) at most one lost packet from each UE is contained in the CS, which is to ensure the decodability of the network coded retransmission packet in each UE's network decoder; (2) as many as lost packets of different UEs are contained in the CS.

The proposed OM^{2}DNC strategy consists of two phases: Phase I, Max-Degree NC; Phase II, Max-Degree opportunistic retransmission.

In Phase I, Max-Degree XORing NC is applied to form retransmission packets. Each retransmission packet is obtained by performing XORing NC on a CS. The CS of lost packets is stored in the *CodingList* and the corresponding UEs' indexes of the lost packets are stored in the *UserList*. The *UserList_Temp* is a temporary list. To determine the CS of lost packets, a Max-Degree based two-step solution is designed:

- *Step I: Selecting the first packet (Lost packet with maximum degree).* Removing a column in the *feedback matrix*$\mathbf{T}$, where the associated packet of this column is correctly received by all UEs, such as packet ID: 8 in Fig. 3. The packet ID with maximum *degree* (number of "1"s in its corresponding column, such as packet ID: 3 in Fig. 3) is stored to *CodingList*. The maximum *degree* is stored in *MaxDegree* and the indexes of UEs which lost the corresponding packet are stored in *UserList*. Removing the corresponding column in the *feedback matrix* **T**.
- *Step II: Updating the CodingList and UserList.* For the ith, $i \in \{1, 2, \ldots, \lambda\}$, column of matrix **T**, if the *degree* of the ith column is equal to *MaxDegree*, then store the corresponding UE IDs in *UserList_Temp*. Comparing *UserList_Temp* with *Userlist*, if there is no intersection, updating *UserList* through *UserList* $=$ *UserList* $\cup$ *UserList_Temp*, updating *CodingList* through *CodingList*

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Algorithm 1 The proposed OM^{2}DNC strategy.

```
Input: $M \times \lambda$ Feedback Matrix $\mathbf{T}$
Export: NC Combined Packets
  for $i = 1$ To Limited Retransmission Number do
    .............................. Phase I: ..................................
    while $sum(\mathbf{T}(:)) \neq 0$ do
      · sum_per_column=$sum(\mathbf{T},1)$
      · MaxDegree = max(sum_per_column)
      · Step I: Select the first packet (lost packet with Maximum Degree)
      · Step II: Update the CodingList and UserList
    end while
    .............................. Phase II: ..................................
    · Select the CSs with Maximum Degree $\mathbf{C}_{max}$
    · Perform XORing NC on lost packets in CodingList corresponding to $\mathbf{C}_{max}$
  end for
```

=*CodingList* $\cup$ i and removing the corresponding column in the *feedback matrix* $\mathbf{T}$. Otherwise, *UserList* = *UserList* and *CodingList* =*CodingList*. If $(i+1) \leq \lambda$, then empty *UserList_Temp* and repeat *Step II* for the $(i+1)$th column of matrix $\mathbf{T}$. Otherwise, if $i = \lambda$, then empty *UserList_Temp*, let *MaxDegree*=*MaxDegree*−1, $i = 1$. If *MaxDegree* > 0, then repeat *Step II* for the ith column of matrix $\mathbf{T}$, otherwise, repeat *Step I* to form another CS.

Example: Applying the Max-Degree XORing NC to the feedback matrix $\mathbf{T}$ given in Fig. 3, we can obtain the following four CSs: $\mathbf{C}_1$={3, 6}, $\mathbf{C}_2$={2, 10}, $\mathbf{C}_3$={4, 7}, $\mathbf{C}_4$={1, 5, 9}. Hence, the number of NC combined packets of the proposed scheme is four ($P_3 \oplus P_6$, $P_2 \oplus P_{10}$, $P_4 \oplus P_7$, $P_1 \oplus P_5 \oplus P_9$), whereas five NC combined packets ($P_1 \oplus P_4$, $P_2 \oplus P_6$, $P_3 \oplus P_7$, $P_5 \oplus P_9$, P_{10}) are formed for the scheme in [14–18], and nine retransmissions are required for traditional ARQ.

In Phase II, NC combined packets are broadcasted by utilizing the Max-Degree based opportunistic retransmission protocol. We assume that all the lost packets form S CSs $\mathbf{C}_j$, $j = 1, 2, \ldots, S$. The CSs with maximum degree $\mathbf{C}_{\max}$ can be obtained by

$$\mathbf{C}_{\max} = \arg\max_{\mathbf{C}_j} \sum_{k \in \mathbf{C}_j} \sum_{m=1}^{M} \left(\mathbf{T}(m,k) \sum_{u=1}^{\lambda} \mathbf{T}(m,u) \right) \tag{7}$$

where M denotes the number of UEs and λ is the number of transmitted packets. Then, the BS combines the lost packets in *CodingList* corresponding to CSs $\mathbf{C}_{\max}$ by XORing NC and retransmit the NC combined packet.

Upon receiving the feedback from UEs, the BS updates the feedback matrix $\mathbf{T}$ and starts the next round of OM^{2}DNC strategy based on the updated matrix $\mathbf{T}$. The OM^{2}DNC strategy is shown in Algorithm 1, where M and λ denote the number of UEs and packets, respectively.

Since each UE can only recover at most one lost packet by utilizing one NC combined retransmission packet, the number of total retransmissions is no less than the maximum number of lost packets among all the UEs. Hence, we note that the minimum number of NC combined packets by using Max-Degree NC equals to the maximum number of lost packets of all UEs, which is obvious for the case of two UEs.

3.2. Joint network-RSC decoding

After receiving a retransmission packet and the corresponding *CodingList*, the lth UE determines whether the received retransmission packet contains its own lost packet. If the desired lost packet of the lth UE is contained in the received NC combined packet, then the lth UE pushes out the corresponding prior successfully received information packets from their packet buffer and XORs them. The combined information packet passes CRC encoder and RSC encoder in sequence, which is the same as what has been done in the BS. The output of the RSC encoder $\mathbf{I}_C^l$ is the combination of the packets which are contained in *CodingList* and successfully received by the lth UE, namely *local generated combination packet*.

We assume that the soft-demodulator generates the Log-likelihood ratio (LLR), namely soft information, for each bit of the combined information packet $\mathbf{C}^l = \mathrm{RSC}\left(\mathbf{I}_L^l \oplus \mathbf{I}_C^l\right) = \mathrm{RSC}\left(\mathbf{I}_L^l\right) \oplus \mathrm{RSC}\left(\mathbf{I}_C^l\right)$, where $\mathbf{I}_L^l$ denotes the information of the lth UE's lost packet. The soft information of the lth UE's lost packet can be obtained by a soft-decision network decoder, which removes the *local generated combination packet* $\mathbf{I}_C^l$ from the soft information of the retransmission packet. Let $\mathbf{C}^l(n)$ and $L[\mathbf{C}^l(n)]$ denote the nth bit of the information $\mathbf{C}^l$ and its LLR value, then the LLR value of $\mathrm{RSC}\left(\mathbf{I}_L^l\right)$ can be obtained from (2) by canceling the effect of $\mathrm{RSC}\left(\mathbf{I}_C^l\right)$ as:

$$\begin{aligned} L[\mathrm{RSC}(\mathbf{I}_L^l)](n) &= \log \frac{\Pr\{[\mathrm{RSC}(\mathbf{I}_L^l)](n) = 1\}}{\Pr\{[\mathrm{RSC}(\mathbf{I}_L^l)](n) = 0\}} \\ &= \begin{cases} L[\mathbf{C}^l](n), \ [\mathrm{RSC}(\mathbf{I}_C^l)](n) = 0 \\ -L[\mathbf{C}^l](n), \ [\mathrm{RSC}(\mathbf{I}_C^l)](n) = 1 \end{cases} \end{aligned} \tag{8}$$

Hence, by using the soft-decision network decoder, the codewords which are prior known at the UE can be removed before RSC decoding. The soft information obtained by using the soft-decision decoder are applied to RSC decoding firstly and then passes the CRC decoder. If successful decoding, the received packet will be stored in the packet buffer.

4. Performance analysis

In this section, the performance of the proposed OM^{2}DNC based WBC protocol is theoretically analyzed. In [13], it was shown that the optimal solution of index network coding problem is an NP-hard problem. Hence, a lower bound of ANTs can be obtained through theoretical analysis. Suppose that there are M UEs with PERs P_m, $m \in \{1, \ldots, M\}$, in the *Broadcast Phase* and the *Retransmission Phase*. We have the following results:

Proposition 1. The ANTs lower bound of the NC based HARQ protocol with M UEs is

$$\Xi_{N1} = \max_{i=\{1,\ldots,M\}} \left\{ (1 - P_i)^{-1} \right\} \tag{9}$$

Proof. For the NC based WBC protocol, the number of transmissions needed to successfully deliver a packet to an UE with the PER P_i is a random variable Y. We have

$$P[Y \leq k] = 1 - P[Y > k] = 1 - P_i^k \tag{10}$$

and

$$\begin{aligned} P[Y = k] &= \left(1 - P_i^k\right) - \left(1 - P_i^{k-1}\right) \\ &= P_i^{k-1}(1 - P_i) \end{aligned} \tag{11}$$

The ANTs per successful packet is

$$\begin{aligned} E[Y] &= \sum_{k=1}^{\infty} kP[Y = k] \\ &= 1 - P_i + \sum_{k=2}^{\infty} k\left(P_i^{k-1}(1 - P_i)\right) \\ &= (1 - P_i)^{-1} \end{aligned} \tag{12}$$

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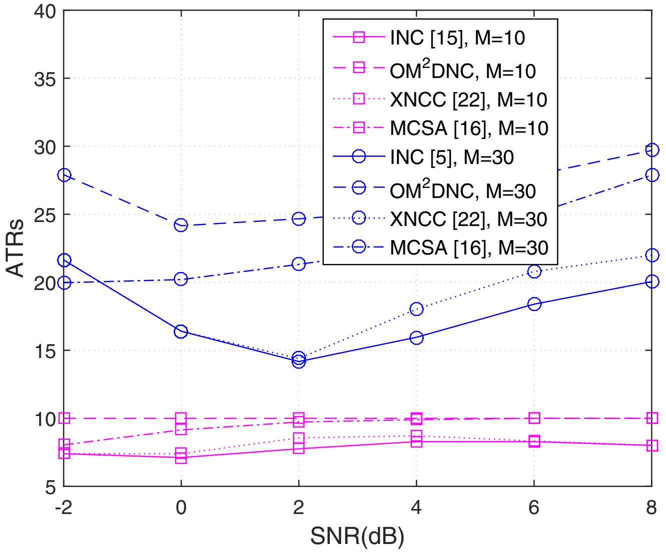

Fig. 4. ATRs versus SNR. $\lambda = 100$ and $L_C = 400$.

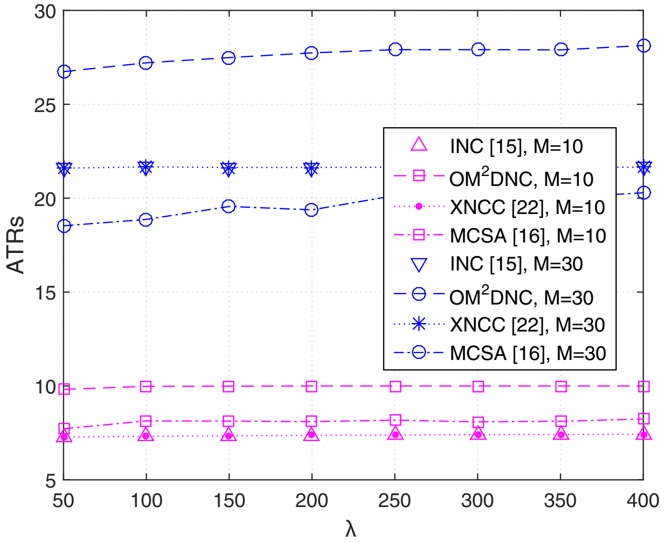

Fig. 5. ATRs versus λ. $L_C = 400$ and SNR $= -2$dB.

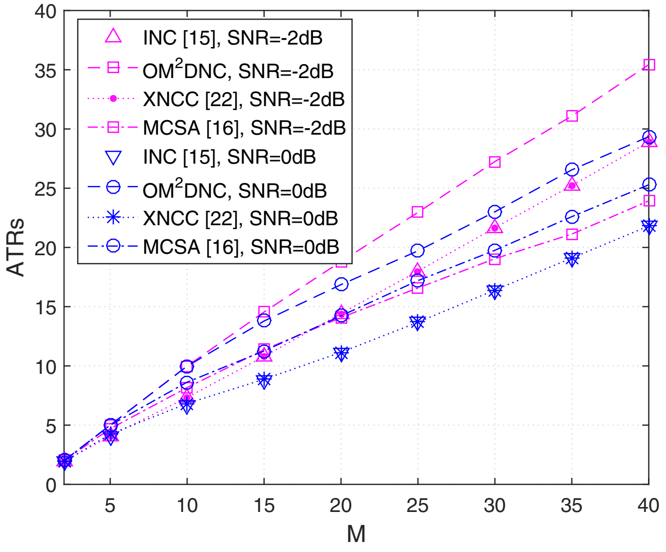

Fig. 6. ATRs versus M. $\lambda = 100$, $L_C = 400$.

Thus the ANTs of NC based WBC protocol with M UEs is

$$\Xi_{N1} = \max_{i=\{1,\cdots M\}} \left\{(1-P_i)^{-1}\right\} \quad \square \tag{13}$$

5. Simulation results

In this section, we carry out numerical simulations to evaluate the performance of the proposed OM^{2}DNC based WBC protocol. We assume that the channels between BS and UEs subject to same complex Gaussian distribution $\mathcal{CN}(0, 1)$ and BS's transmission power P_s is normalized. The channel code at BS is a rate R=1/2 RSC code with constraint length $K = 5$. The feedforward generator is F=37 and the feedback generator is $B = 21$ (both in octal). The length of information packet is L_I bits (containing CRC bits), and the length of coded packet is L_C bits.

Three existing schemes are adopted here for comparison, i.e., INC [15], MCSA [16], and XNCC [22]. In INC [15], original packets are divided into two subsets, i.e., Φ_1 and Φ_2 under two conditions: (1) any packet in Φ_2 could be combined decodably with at least one packet in Φ_1, and (2) all lost packets at the receivers with most lost packets is contained in Φ_1. Packets in Φ_1 are ordered according to the number of packets in Φ_2 that could be separately decodably combined with them, from small to large. According to the order, each packet in Φ_1 is decodably combined with packets in Φ_2 by sequence, and packets in Φ_2 failed to be combined are decodably combined among each other. Combination results are then scheduled to be retransmitted. In MCSA [16], each lost packet at each receiver is denoted as a vertex in a graph, where two vertexes are connected if the corresponding packets are lost in a same receiver or owned by different receivers. Each vertex is allocated with an initial weight proportional to the number of lost packets at the corresponding receiver. This weight is then updated as the sum of initial weight of connected vertexes. The vertex with maximum weight is selected out to add into the clique, and remain vertexes connected to this vertex form a new graph accordingly. This procedure continues until the newly formed graph is empty, and then the output clique is regarded as a candidate set to produce a coded packet.

Fig. 4 shows the simulation results of average targeted receivers (ATRs) versus SNR. Here, a receiver is regarded as a targeted receiver for a coded packet when coded packet could help the receiver recover a lost packet. From the figure, we could find that proposed OM^{2}DNC based WBC protocol achieves a significant ATRs performance gain when compared with the MCSA [16], INC [15] protocol and XNCC [22] protocol, respectively. That means in our proposed scheme, each coded packet could serve more receivers and therefore more NC gain could be achieved.

Fig. 5 shows the ATRs performance as a function of λ at SNR=−2dB. We can see from the figure that the proposed OM^{2}DNC strategy outperforms MCSA [16], INC [15] protocol and XNCC [22] protocol on the ATRs performance, respectively. Moreover, the ATRs performance can be observed as less influenced by the number of original packets as the curves of each scheme is almost flat.

Fig. 6 shows the ATRs versus the number of UEs M. As shown in the figure, the ATRs increases as the number of UEs M increases. The proposed OM^{2}DNC strategy outperforms MCSA [16], INC [15] protocol and XNCC [22] protocol on the ATRs performance, respectively.

Fig. 7 shows the simulation results of ANTs versus SNR. The numerical results of the theoretical analyses in (9) is also depicted for comparison. We can see from Fig. 7 that the exact ANTs performance of the proposed OM^{2}DNC strategy derived by Monte

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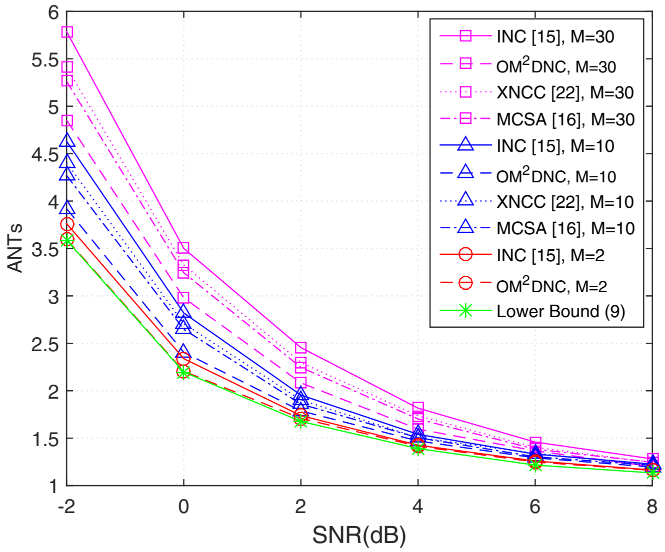

Fig. 7. ANTs versus SNR. $\lambda = 100$ and $L_c = 400$.

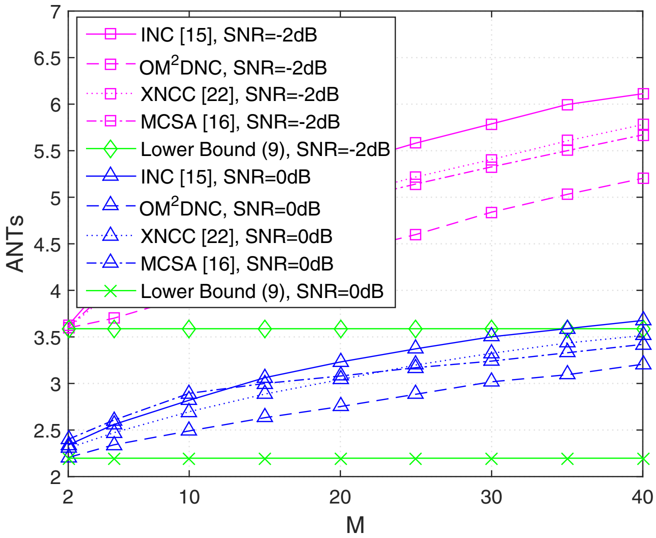

Fig. 9. ANTs versus M. $\lambda = 100$, $L_c = 400$.

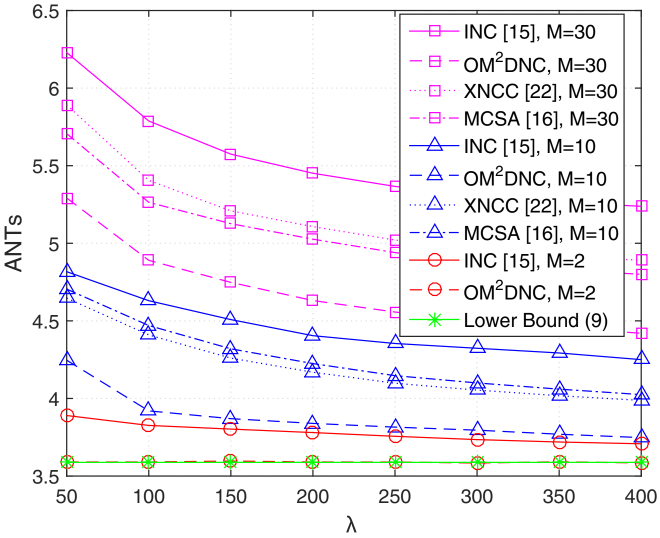

Fig. 8. ANTs versus λ. $L_c = 400$ and SNR=$-$2dB.

Carlo simulation for $M = 2$ matches well with the theoretical analyses in (9). Meanwhile, simulation results show that the proposed OM^{2}DNC based WBC protocol achieves a significant ANTs performance gain when compared with the MCSA [16], INC [15] protocol and XNCC [22] protocol, respectively.

Fig. 8 shows the ANTs performance as a function of λ at SNR $=$ $-$2dB. We can see from Fig. 8 that the proposed OM^{2}DNC strategy outperforms MCSA [16], INC [15] protocol and XNCC [22] protocol on the ANTs performance, respectively. We can see from Fig. 8 that the ANTs performance of the proposed OM^{2}DNC strategy obtained by Monte Carlo simulation for $M = 2$ matches well with the theoretical analyses in (9). Moreover, ANTs performance floor can be observed in the large λ region when $M = 10$ and 30, which suggests the inherent performance gap between the proposed OM^{2}DNC strategy and the optimal index network coding solution.

Fig. 9 shows the ANTs versus the number of UEs M. As shown in the figure, the ANTs increases as the number of UEs M increases. The proposed OM^{2}DNC strategy outperforms MCSA [16], INC [15] protocol and XNCC [22] protocol on the ANTs performance, respectively. Meanwhile, simulation results show that the performance gap between the proposed OM^{2}DNC based WBC protocol and the theoretical lower bound in (9) increases as the increases of the number of UEs M.

From the above simulation comparison, it is clear that our proposed scheme achieve significant performance gain compared with existing schemes. It is nontrivial to give a brief explanation on why better performance could be achieved in our scheme and how to go further towards the lower bound. Priority plays an important role in coded packets determination. In general, two priorities are indeed important in order to achieve higher performance gain. One priority is to prioritize the receiver that demands most packet. Since retransmission packets suffer loss, NCB could roughly be regarded as a finite Markov decision procedure, with the numbers of lost packets at receivers regarded as state in the state space. Therefore to recover lost packets is to transform the current state to the all-zero state. Since only at most one step could be taken after receiving one coded packet at each receiver, it is obvious to prioritize the receiver that need the most packets (for the purpose to transform to all-zero state faster). The other priority is to prioritize the packet that are needed most by receivers. In this way, there will be more chances for lost packets to be decodably combined, thus higher NC gain would be achieved. The performance advance of our scheme verifies the above priorities. Thus it is not clear yet that how to make a tradeoff between the two priorities. Further effort could be made to build a new rule that makes better balance between the two priorities, thus better performance would be expected to be achieved.

6. Conclusions

In this paper, an OM^{2}DNC based WBC protocol is proposed to increase the bandwidth efficiency of wireless multimedia broadcasting systems. The advantages of the proposed scheme over the traditional NC based retransmission schemes are shown through theoretical analyses and simulations. Comparing with traditional NC based protocol, the proposed OM^{2}DNC based WBC protocol can significantly reduce the number of transmissions per successful packet, whose gains is attributed to the potential advantage of the OM^{2}DNC strategy.

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