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Resource allocation for OFDMA-based multicast cognitive radio networks using a Stackelberg pricing game

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ABSTRACT

The resource allocation problem for orthogonal frequency-division multiple-access (OFDMA)-based multicast cognitive radio networks is investigated under the spectral activities of primary users (PUs). The interactions between PUs and secondary users (SUs) are modelled using a Stackelberg game where the PUs are the leaders while the SUs are the followers. Using an efficient pricing framework, the PUs who are the licensed spectrum owners compete to lease their subcarriers to the SUs. The competition among PUs is modelled using a non-cooperative game in which they greedily adjust the pricing coefficients to harvest maximum profits while maintaining tolerable interference. Based on the pricing coefficients, the SUs autonomously form coalitions and collectively adjust their received power so as to access more subcarriers at affordable costs. Two disjoint algorithms are proposed to facilitate successful transactions between PUs and SUs so that Stackelberg equilibrium can be achieved where both the PUs and SUs can obtain maximum payoffs. Simulation results demonstrate that the proposed scheme outperforms the conventional unicast and multicast schemes in cognitive radio networks while achieving a near-optimal performance comparable to the exhaustive search scheme.

1. Introduction

Spectrum scarcity is one of the major bottlenecks of high-quality and delay-sensitive wireless services. Nonetheless, it is revealed in a report by the Federal Communications Commission (FCC) [1] that spectrum access is in fact a more significant problem than spectrum scarcity, mainly due to the rigid spectrum management policies which restrict potential wireless users to acquire spectrum. Intuitively, spectrum utilization can be improved considerably by making it possible for an unlicensed user to access a spectrum band licensed to a primary network (PRN) based on certain criteria. This has led to the invention of cognitive radio (CR) which is viewed as a viable future communication technology for improving spectral efficiency. According to [2], the software-defined radio based CR system is an intelligent wireless communication system that is capable of detecting available channels in a wide spectrum and adjusting the transmission or reception parameters accordingly to allow coexistence of licensed or primary users (PUs) and unlicensed or secondary users (SUs).

The adoption of CR technology in multicast systems is initiated by the apparent lack of spectrum due to growing demand of multicast services [2]. This promising technology can potentially alleviate spectrum scarcity in multicast systems by allowing multicast SUs to opportunistically access the spectrum licensed to PUs. Since PUs have access priority, SUs are required to exert minimal effect on PUs. The protection of PUs is necessary because no PRN will be willing to share its spectrum with a multicast CR network (MCRN) if the secondary users' activities on the licensed band are detrimental to the PUs. Therefore, the main task of the MCRN is to ensure that SUs can maximize the spectrum utilization under the constraints of multiple PUs' interference temperatures [3]. In order to efficiently utilize the valuable spectrum, the orthogonal frequency-division multiple-access (OFDMA) technique is adopted in MCRNs and an efficient radio resource management (RRM) scheme for subcarrier and power allocation (SPA) is employed [4–10].

Over the past decade, substantial efforts have been devoted to designing efficient RRM schemes for OFDMA-based CRNs, particularly for unicast communications [4]. Recently, due to the explosive growth of mobile applications which fuel the demand for wireless multicast services especially wireless streaming and internet-protocol television (IPTV), RRM for MCRNs has garnered immense research interest. In [5], the authors modelled the multicasting problem in MCRNs where they considered PUs' maximal interference and solved the problem using a subgradient update algorithm. Resource allocation for OFDMA-based MCRNs which consid-

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ers scalable video transmission using H.264 has been studied in [6]. In this work, integer programming is adopted for subcarrier allocation to different SUs with consideration of interference tolerance of PUs. Besides, RRM of MCRNs is formulated in [7] by taking the maximization of the expected sum rate of cognitive multicast groups as the design objective and an efficient joint SPA technique is proposed. Subsequently, the work in [8] optimizes real-time video multicast in CRNs where fined grained scalability is used to encode each video into a base layer and an enhancement layer to accommodate heterogeneous channels. This work has been extended to a mesh MCRN [9] where an assistance strategy for relaying is proposed to reduce the effect of channel heterogeneity. In [10], the optimization problem of MCRNs is solved using a Lagrangian dual decomposition approach by developing an asymptotically optimal joint SPA algorithm.

Undeniably, many design and implementation issues exist in the realization of an MCRN, particularly the issue on how to achieve “peaceful” coexistence between PUs and multicast SUs on different subcarriers. The unregulated and self-organized nature of MCRNs makes the RRM problem very challenging. One feasible approach to model the interaction between PUs and SUs is to use a pricing mechanism. Apparently, pricing for CRNs has been embraced in the work carried out in [11] as an effective tool for creating policies for resource sharing between PUs and SUs. In [12], the authors proposed a spectrum leasing framework under which the PUs are rewarded for allowing SUs to operate in their licensed bands. Notably, the work in [13] formalized the profit maximization problem which can be solved using stochastic dynamic programming. Successively, the authors explored the price dynamics in a competitive market consisting of multiple service providers [14] while a recent work [15] considered the competition among multiple PUs in an attempt to sell spectrum. In contrast, the study in [16] focused on the competition among multiple SUs to acquire licensed bands and this work was extended in [17] to study spectrum trading across multiple PUs and multiple SUs. In addition, the work in [18] studied the investment and pricing decisions of a network operator under spectrum supply uncertainty. In [19], a spectrum allocation scheme is modelled using the hybrid game model based on reputation instead of pricing, which deals with multiple PUs and SUs coexisting and sharing the spectrum. The work in [20] manages to address a Stackelberg game model with pricing in which individual users attempt to hierarchically access to the wireless spectrum while maximizing their energy efficiency. Besides, the authors in [21] study the database-assisted dynamic access network where spectrum brokers compete to provide service for SUs with different quality of service (QoS) demands and budget. The interaction among the BSs and SUs is characterized as a two-stage Stackelberg game which is able to yield optimal profits for BSs and SUs. Likewise, the work in [22] investigates spectrum procurement and pricing which utilizes the differentiated pricing among the heterogeneous SUs to improve the profit of the CR networks. The spectrum procurement and pricing is modelled as a five-stage Stackelberg game to analyze the optimal decisions for CR networks by using backward induction.

Unlike the aforementioned schemes which model the SUs as selfish users, however, in the current work, the SUs are regarded as altruistic wireless users who tend to cooperate with each other to maximize spectral efficiency. In this paper, resource allocation in MCRNs is formulated as a clustering problem to alleviate the effect of channel heterogeneity, but clustering optimization is complicated by the introduction of interference temperature which is used by PUs to control spectrum usage of SUs. The interaction between PUs and SUs can be captured using a Stackelberg game [23,24] where the PUs (leaders) who are the spectrum owners attempt to lease their spectrum to the SUs (followers). Spectrum leasing between PUs and SUs is regulated by a non-cooperative pricing framework under which the PUs greedily adjust their pricing coefficients to garner maximum profit while keeping the interference below a threshold. Under this pricing architecture, the cooperation among SUs is modelled using a coalitional game where the SUs wisely form coalitions and collectively adjust their power to acquire more subcarriers from PUs at minimal costs. To achieve Stackelberg equilibrium (SE) [23], two disjoint algorithms are proposed to enable successful transactions between PUs and SUs so that the proposed games can reach their respective equilibria at which all PUs and SUs are satisfied with their payoffs.

The rest of this article is organized as follows. In Section 2, we outline the system model of a hybrid system comprising a PRN and MCRN. In this section, the interference temperature model for MCRN is also described. Section 3 formulates the cognitive Stackelberg game with pricing (CSGP) which consists of a non-cooperative price adjustment game (NPAG) and a multicast coalitional game with pricing (MCGP) for PRN and MCRN, respectively. An algorithm that guides the CSGP to achieve its SE is proposed in Section 4 and its complexity is analyzed. Simulation results and performance analysis are presented in Section 5. We end the article with some concluding remarks in Section 6.

2. System model and problem formulation

Consider a hybrid network comprising a PRN and a MCRN as illustrated in Fig. 1, both are single-cell OFDMA-based networks that co-exist within the same geographical area. In the PRN, there are M licensed PUs with each PU $m \in \mathcal{M} = \{1, \ldots, M\}$ receiving distinct unicast traffic on the licensed spectrum in the downlink from a primary BS (PBS). At the same time, the MCRN consists of a secondary BS (SBS) accommodating K SUs where only the downlink multicast transmission is considered. As shown in Fig. 1, the channel gains of different links are defined as follows.

- $|h_{k,\,n}|^2$ denotes the channel gain of the communication link from the SBS to the kth SU on the nth subcarrier,
- $|g^P_{k,n}|^2$ denotes the channel gain of the interference link from the PBS to the kth SU on the nth subcarrier,
- $|g^S_{m,n}|^2$ denotes the channel gain of the interference link from the SBS to the mth PU on the nth subcarrier.

For brevity, $|g^P_{k,n}|^2$ and $|g^S_{m,n}|^2$ are be generally known as “interference gains” which will be used throughout this paper.

In this hybrid network, the PUs are licensed to operate on a specific frequency band which is partitioned into N orthogonal subcarriers with each subcarrier exclusively assigned to one PU at a time. Thus, all PUs can simultaneously receive data from the PBS without any internal interference caused within the PRN. At the same time, the SUs are permitted to access the licensed spectrum if the interference created externally by SUs is tolerable to the PRN. If spectrum access by SUs is detrimental to data transmission of the PUs, the latter will automatically discontinue spectrum sharing with the MCRN. As a result, the transmit power of the SBS must be carefully controlled so as to exert minimal interference effect on the PUs while ensuring satisfactory QoS at the receivers of SUs. In this context, it is also assumed that the PUs and SUs do not have any prior knowledge of each other’s spectrum utilization.

2.1. System model for OFDMA-based multicast cognitive radio networks

In the MCRN, the SBS transmits G downlink traffic flows to one distinct multicast group of SUs. Let $\mathcal{K}_g$ denote the user set of the gth multicast group corresponding to the gth traffic flow. For simplicity, it is assumed that each user only belongs to one multicast group, so that $\mathcal{K}_g \cap \mathcal{K}_h = \emptyset$, $g \neq h$, $g, h \in \mathcal{G}$ where $\mathcal{G} = \{1, 2, \ldots, G\}$ is the multicast group set. However, the proposed method is still

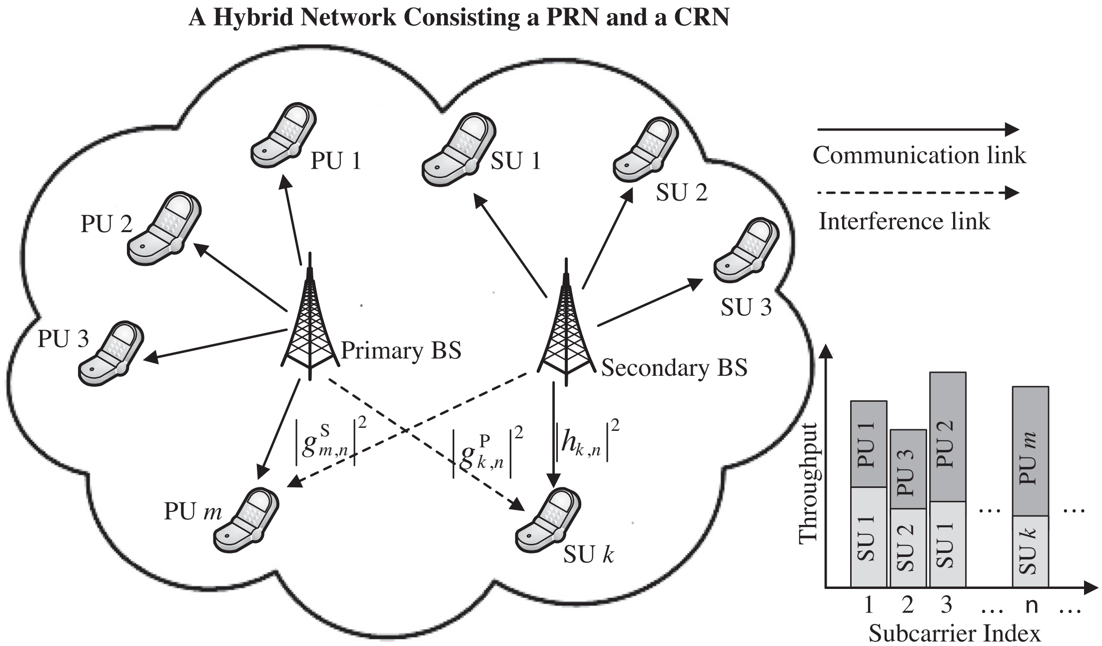

Fig. 1. Illustration of a hybrid network with dynamic spectrum sharing.

applicable by omitting this assumption. The cardinality of $\mathcal{K}_g$, denoted as $|\mathcal{K}_g|$, represents the number of users in the gth group. This model is applicable for both unicast and multicast systems where the gth group is unicast if $|\mathcal{K}_g| = 1$ whereas it is multicast if $|\mathcal{K}_g| > 1$. All multicast users belong to the set $\mathcal{K} = \cup_{g=1}^{G}\mathcal{K}_g$ and $K = \sum_{g=1}^{G}|\mathcal{K}_g|$ is the total number of users in the multicast system.

The coexistence of PUs and SUs may cause interference to the kth SU induced by signals from the PBS destined to the mth PU when both the PBS and SBS transmit on the nth subcarrier. This interference is denoted as $|g_{k,n}^P|^2 p_{m,n}^P$ where $p_{m,n}^P$ is the power used by the PBS to transmit data to the mth PU on the nth subcarrier. Thus, the signal to interference-plus-noise ratio (SINR) at the receiver of the kth SU on the nth subcarrier can be expressed as

$$\mathrm{SINR}_{k,n} = \frac{|h_{k,n}|^2 p_{k,n}^S}{\left|g_{k,n}^P\right|^2 p_{m,n}^P + w\eta_0} \tag{1}$$

where $p_{k,n}^S$ is the transmit power by the SBS to the kth SU on the nth subcarrier. From (1), it is shown that the SINR of a SU not only relies on its own subchannel gains, but it is also affected by the interference gains from the PBS to the SU. For instance, if a SU with good channel conditions is located very close to the PBS, it will still receive a low SINR due to being exposed to high interference from the PBS.

In conventional multicast transmission, the SBS involuntarily uses the lowest data rate among all the SUs within the group to ensure reliable decodability of data for all the SUs [25]. If the SBS transmits multicast data to the gth multicast group with a power $p_{g,n}^S$ on the nth subcarrier, the achievable data rate of the group is always constrained by the lowest data rate among all SUs within the group, typically determined by the lowest channel gain as

$$\gamma_{g,n} = \min_{k\in\mathcal{K}_g}\gamma_{k,n} = \min_{k\in\mathcal{K}_g}\frac{|h_{k,n}|^2}{\left|g_{k,n}^P\right|^2 p_{m,n}^P + w\eta_0} \tag{2}$$

For a fixed desired bit error rate (BER) performance, the achievable data rate for the gth group on the nth subcarrier is denoted as

$$r_{g,n} = \sum_{k\in\mathcal{K}_g}\alpha_{g,n} r_{k,n} = w|\mathcal{K}_g|\log_2\left(1 + \gamma_{g,n} p_{g,n}^S\right) \tag{3}$$

where $\alpha_{g,n} = 1$ if the nth subcarrier is allocated to the gth group, otherwise $\alpha_{g,n} = 0$. Using (3), the aggregate data rate (ADR) for the system can be written as

$$r_T = w\sum_{g=1}^{G}\sum_{n=1}^{N}\alpha_{g,n}|\mathcal{K}_g|\log_2\left(1 + \gamma_{g,n} p_{g,n}^S\right) \tag{4}$$

In [26], a multicast scheme is proposed to maximize the throughput by allocating subcarriers to multicast flows with the maximal ADR. This scheme does not ensure fair access to system resources because the groups with higher channel gains and/or larger sizes always dominate usage of subcarriers, thus depriving the fairness for groups with lower channel gains and/or smaller sizes. Therefore, the authors in [27] proposed a fair multicast scheme which guarantees a minimum number of subcarriers to be assigned to individual groups based on their respective channel conditions and group sizes. However, the aforementioned schemes underutilize system resources because multiuser diversity of OFDMA is not exploited efficiently. This issue has been investigated in [25] which demonstrates that if a multicast system is constrained to transmit at the least user's rate decodable by all users within a group, the multicast system would saturate the capacity when the number of users increases in Rayleigh and Ricean fading environments. In order to effectively explore multiuser diversity, the G multicast groups can be partitioned into $S > G$ subgroups where subcarriers are dynamically assigned to each subgroup based on their channel gains to maximize the ADR. Let $\mathcal{S} = \{1, 2, \ldots, S\}$ be the subgroup set, the new multicast clustering problem is formulated as [28]

$$\max_{\{\alpha_{s,k},\beta_{s,n},p_{s,n}\}}\sum_{s=1}^{S}\sum_{n=1}^{N}\sum_{k=1}^{K}\alpha_{s,k}\beta_{s,n} w\log_2\left(1 + \frac{\gamma_{s,n}p_{s,n}}{\Theta w\eta_0}\right) \tag{5}$$

$$\text{subject to } G \le S \le K \tag{6}$$

$$\sum_{s=1}^{S}\sum_{n=1}^{N} p_{s,n} \le P_{BS}^{\max},\ p_{s,n} \ge 0, \forall n \in \mathcal{N} \tag{7}$$

$$\alpha_{s,k}, \beta_{s,n} \in \{0,1\}, \forall s \in \mathcal{S}, \forall k \in \mathcal{K}, \forall n \in \mathcal{N} \tag{8}$$

$$\sum_{s=1}^{S} \alpha_{s,k} = 1, \sum_{s=1}^{S} \beta_{s,n} = 1, \forall k \in \mathcal{K}, \forall n \in \mathcal{N} \tag{9}$$

$$\alpha_{s,j} + \alpha_{s,k} \le 1, j \ne k, j \in \mathcal{K}_g, k \in \mathcal{K}_h \tag{10}$$

$$\sum_{n=1}^{N} \beta_{s,n} \ge \beta_s^{\min}, \forall s \in \mathcal{S} \tag{11}$$

In (5), $\gamma_{s,n} = \min_{k \in \mathcal{K}_s} |h_{k,n}|^2$, $s \in \mathcal{S}$ where $\mathcal{K}_s$ is the user set of the sth subgroup. The value of S is not pre-determined and is dynamically adjusted according to (6) based on the channel gains of the multicast users. If $S = K$, the system adopts the conventional unicast scheme (CUS) whereby every user is clustered into its respective group, with each group receiving data on its own allocated subcarriers and power. Besides, $S = G$ corresponds to the conventional multicast scheme (CMS) which clusters all users into their respective multicast groups and every multicast group receives its own data using different subsets of subcarriers and power. In addition, the conditions on power allocation are expressed in (7) in which the total transmit power of the BS is limited by $P_{BS}^{\max}$. In (8), $\alpha_{s,k}$ and $\beta_{s,n}$ are binary variables representing the allocation of the kth user and the nth subcarrier to the sth subgroup, respectively. Condition (9) ensures that a specific user and subcarrier can only be assigned to one subgroup at a time. Besides, constraint (10) prevents two users belonging to different multicast groups from being assigned to the same subgroup. To ensure fairness, condition (11) ensures that every multicast group obtains a minimum number of subcarriers denoted as $\beta_s^{\min}$. Obviously, the formulation given in (5)–(11) is a non-deterministic polynomial-time (NP)-hard combinatorial optimization problem where determining its optimal solution within a given time is very challenging. Performing a direct exhaustive search at the BS would incur a prohibitive computational burden, which is not feasible due to rapid variations of the wireless channel. Therefore, suboptimal algorithms with lower complexity and acceptable performance are usually preferable for practical implementation.

The aforementioned clustering model allows SUs from the same multicast group to autonomously form their own subgroups with their chosen members to improve their data rates. In this clustering framework, the objective of the SUs is to maximize (5) through subgroup formation while generating tolerable interference to the PUs. Therefore, it is essential to have an efficient interference model for the PRN to accurately manage the external interference generated by the MCRN as well as to regulate SPA in the MCRN so that the QoS for all PUs is not jeopardized.

2.2. Interference model for OFDMA-based multicast cognitive radio networks

The most popular criterion to quantify interference in spectrum sharing between PRN and MCRN is the interference temperature [3]. Formally, the interference temperature is defined as the radio frequency (RF) power measured at the PU and is used to provide an accurate measurement of the acceptable RF interference in a frequency band. Any transmission in the MCRN on a licensed band is considered detrimental to a PU if the interference generated by the former exceeds the interference temperature constraint (ITC). In other words, the licensed frequency band could only be made available to SUs provided the ITC is not exceeded. Let T_g denote the interference temperature of a channel with a bandwidth w and central frequency f_c, T_g is generally expressed as $T_g(f_c, w) = P_g(f_c, w)/\kappa w$ where $P_g(f_c, w)$ is the average interference power centered at f_c covering a bandwidth w and κ is the Boltzmann's constant.

In this work, the generalized interference temperature model studied in [3] is adopted where no prior information about the RF environment is available and hence a licensed signal cannot be identified in the presence of interference and noise. In this model, the interference temperature is measured at some points but not at the PUs. In other words, this model is frequency band-based and has the same interference temperature threshold on each frequency band. Under this generalized model, the ITC can be written as $\sum_{n=1}^{N} |g_{m,n}^S|^2 p_{g,n}^S + I_n \le \kappa w T_g = I_{th}$ where T_g is the interference temperature threshold on each subcarrier with a bandwidth w, I_n is the interference power sensed on the nth subcarrier at the measurement point and I_{th} is the maximum interference power acceptable to PUs [3]. Note that the ITC is per-subcarrier-based and each subcarrier can only be used by at most one multicast group at one time, thus the ITC can be simplified to

$$\left|g_{m,n}^S\right|^2 p_{g,n}^S + I_n \le I_{th} \tag{12}$$

The MCRN is authorized to access the nth subcarrier if the condition in (12) is met. Hence, the SUs need to select appropriate subcarriers and power to achieve the target SINR without generating excessive interference to the PUs. Thus, it is assumed that the SUs are endowed with the capability of adapting to their respective environment by making timely changes to the operating parameters, e.g., frequency and power.

The willingness of PUs to share their subcarriers with SUs and the interaction among SUs to gain access to licensed subcarriers can be modelled using a spectrum leasing framework where SUs attempt to acquire subcarriers from the PRN through a leasing process based on certain criteria.

3. Stackelberg game formulation

A dynamic spectrum leasing architecture is proposed in which PUs that own the spectrum rights willingly and actively share their spectrum with multicast SUs. The PUs have the freedom to lease their spectral bands to the SUs and this leasing implies that the SUs need to pay credits at certain prices to the PUs. The credits obtained are used by the PUs to regulate the coalition formation (CF) and SPA for the SUs so that the PUs can achieve an optimal tradeoff between spectrum utilization and received interference level. Indeed, the price is expected to be proportional to the amount of spectrum leased by the PUs and the amount of interference generated by the SUs.

In this framework, each PU is regarded as a self-interested spectrum owner who greedily adjusts the spectrum price to maximize its own profit. Hence, a non-cooperative price adjustment game (NPAG) is proposed for PUs to compete in spectrum leasing. Under this pricing scheme, the clustering problem for the MCRN can be modelled as a multicast coalitional game with pricing (MCGP) where multicast SUs attempt to acquire subcarriers by adjusting their power so that the prices of subcarriers are affordable. In the MCGP, the SUs can act cooperatively by forming coalitions with others to improve their buying power so as to obtain more subcarriers and/or use higher power. The CF implies that the SUs within a coalition can use the same set of subcarriers while the cost of spectrum access is equally borne by all the SUs in that coalition. The adoption of a pricing scheme in the NPAG and MCGP induces some strategic interdependence between the PUs and SUs which is modelled using a Stackelberg game.

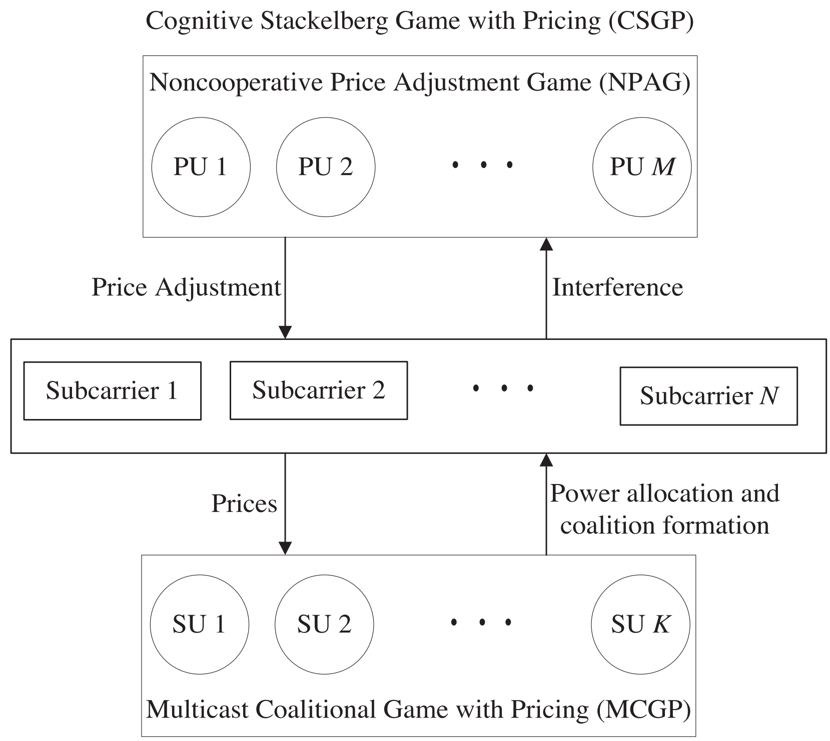

Fig. 2. A cognitive Stackelberg game under a pricing framework for M PUs (leaders) playing the NPAG and K SUs (followers) playing the MCGP.

Definition 1. A cognitive Stackelberg game with pricing (CSGP) is a strategic hierarchical game denoted as $CSGP = \langle \mathcal{M}, \mathcal{K}, \pi^P, \pi^S \rangle$ where $\mathcal{M}$ is the finite set of players containing the PUs as the leaders of the game while $\mathcal{K}$ is the finite set of players containing the SUs as the followers who move based on the actions of the leaders. The payoff functions for the PUs and SUs are denoted as π^P and π^S, respectively, and the functions are linked to each other under a common pricing framework.

The CSGP is illustrated in Fig. 2 in which the NPAG and MCGP are coupled under a common pricing framework. It is demonstrated that the PUs in the NPAG disjointedly adjust the prices of all subcarriers and subsequently broadcast them to the SUs. Based on this pricing model, the SUs cooperatively form coalitions and adjust their powers in order to acquire subcarriers at acceptable costs. In return, the interference created by the MCRN is measured at the PUs and the interference level indicates the total revenue (sum of all credit payments made by the SUs) for the PRN. In order to maximize the total revenue, the PUs constantly adjust the prices of subcarriers based on the interference received. This price adjustment is iterative and will continue until Stackelberg equilibrium (SE) is achieved.

3.1. Multicast coalitional game with pricing for OFDMA-based MCRNs

The cooperative behaviors among multicast SUs in the priced-based SPA can be investigated using the MCGP, which is a subgame of the CSGP. In fact, the pricing architecture introduced by the PRN is used to regulate the CF and SPA among the SUs so that the interference incurred by the MCRN does not exceed the ITC. Formally, the MCGP is defined as follows:

Definition 2. The multicast coalitional game with pricing (MCGP) is a K-player cooperative game denoted as $MCGP = \langle \mathcal{K}, \pi^S \rangle$ where $\mathcal{K}$ is a finite set of players (i.e., multicast SUs) and π^S is a real-valued net payoff function (the difference between the payoff function v and cost function c) such that $\pi^S(\mathcal{S}_i) \in \mathbb{R}_+$ for all $\mathcal{S}_i \subseteq \mathcal{K}$ with $\pi^S(\emptyset) = 0$.

In the MCGP, every SU attempts to maximize its own net payoff by forming coalitions with others and collectively adjusting its received power in order to acquire more subcarriers at minimal costs. In general, the net payoff of coalition $\mathcal{S}_i$ can be defined as the difference between its payoff and the cost of forming the coalition, i.e.,

$$\pi^S(\mathcal{S}_i) = v(\mathcal{S}_i) - c(\mathcal{S}_i) \tag{13}$$

where $v(\mathcal{S}_i)$ is the payoff function for coalition $\mathcal{S}_i$ which is formulated as a function of its achievable data rate while $c(\mathcal{S}_i)$ is the cost function for coalition $\mathcal{S}_i$ which is the sum of all the payments made by coalition $\mathcal{S}_i$ to the PUs for spectrum leasing. Essentially, the net payoff function of coalition $\mathcal{S}_i$ in (13) can be expanded as

$$\begin{aligned}\pi^S(\mathcal{S}_i) &= \sum_{k=1}^{K}\sum_{n=1}^{N}\alpha_{i,k}\beta_{i,n}\Big(w\log_2\big(1+\gamma_{i,n}p^S_{i,n}\big) - \upsilon\Omega^m_{i,n}\left|g^S_{m,n}\right|^2 p^S_{i,n}\Big)\\ &= \sum_{n=1}^{N}\beta_{i,n}w|\mathcal{S}_i|\log_2\big(1+\gamma_{i,n}p^S_{i,n}\big) - \sum_{n=1}^{N}\beta_{i,n}\upsilon\Omega^m_{i,n}\left|g^S_{m,n}\right|^2 p^S_{i,n}\end{aligned} \tag{14}$$

where the jth and kth SUs are said to form coalition $\mathcal{S}_i$ if their CF strategies are $\alpha_{i,j} = 1$ and $\alpha_{i,k} = 1$, for $\forall j, k \in \mathcal{S}_i$, $j \neq k$. In (14), υ is a non-negative unit conversion constant and $\Omega^m_{i,n}$ is the pricing coefficient (price per unit interference) broadcast by the mth PU to charge coalition $\mathcal{S}_i$ for gaining access the nth subcarrier. Naturally, $\Omega^m_{i,n}$ is always proportional to the interference received by the mth PU due to the multicast traffic from the SBS destined to coalition $\mathcal{S}_i$ on the nth subcarrier. The amount of interference generated by the MCRN can be quantified as $|g^S_{m,n}|^2 p^S_{i,n}$, which is normally measured at the PUs and subsequently fed back to the MCRN for pricing computation. Without loss of generality, the total net payoff for

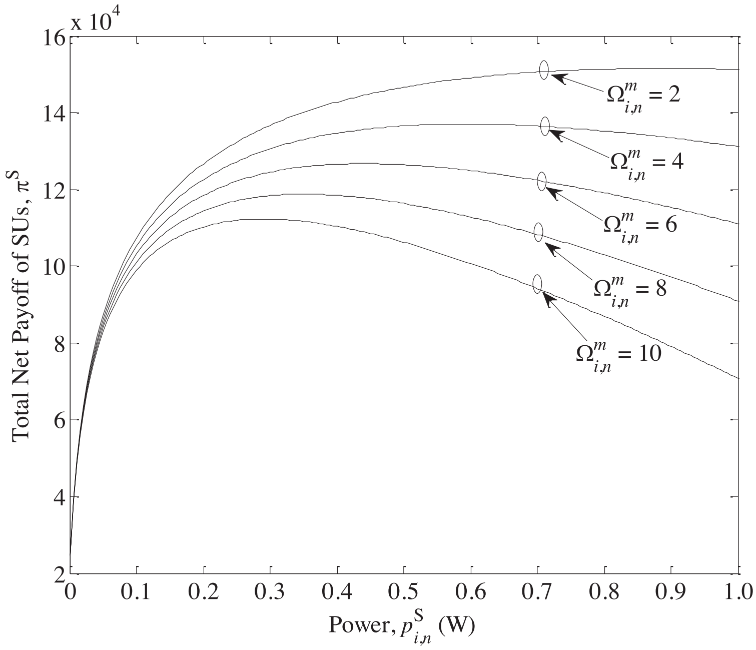

Fig. 3. The payoff function versus power for different pricing coefficients.

the MCRN can be determined as

$$\pi^S(\mathcal{S}) = \sum_{i=1}^{|\mathcal{S}|} \pi^S(\mathcal{S}_i) \tag{15}$$

Obviously, the total net payoff in (15) represents the total welfare of all coalitions in the MCGP. In this context, it is assumed that maximizing (15) can always guarantee maximum net payoffs to all SUs. For simplicity, only one multicast group is considered here and the maximization problem in (5)-(11) can be simplified for the MCGP as

$$(MCGP) \max_{\{\alpha_{s,k},\beta_{s,n},p^S_{i,n}\}} \left(\sum_{i=1}^{|\mathcal{S}|}\sum_{n=1}^{N} \beta_{i,n} w|\mathcal{S}_i|\log_2\left(1+\gamma_{i,n}p^S_{i,n}\right) - \sum_{i=1}^{|\mathcal{S}|}\sum_{n=1}^{N} \beta_{i,n}\upsilon\Omega^m_{i,n}\left|g^S_{m,n}\right|^2 p^S_{i,n} \right) \tag{16}$$

$$\text{subject to } \sum_{i=1}^{|\mathcal{S}|}\sum_{n=1}^{N} p^S_{i,n} \le P^{\max}_{SBS} \tag{17}$$

$$\sum_{i=1}^{|\mathcal{S}|} \alpha_{i,k} = 1, \sum_{i=1}^{|\mathcal{S}|} \beta_{i,n} = 1, \forall i \in \mathcal{S}, \forall n \in \mathcal{N} \tag{18}$$

$$\sum_{n=1}^{N} \beta_{i,n} \ge \beta_i^{\min}, \forall i \in \mathcal{S} \tag{19}$$

where $P^{\max}_{SBS}$ is the maximum transmit power available for the SBS. Apparently, the maximization problem in (16)–(19) shows that CF and SPA are intertwined to each other under a common pricing framework. To reduce complexity, the intertwined problem can be solved explicitly by decomposing the problem into three subproblems, i.e., a) power allocation problem, b) coalition formation problem, and c) subcarrier allocation problem.

As shown in (14), the net payoff function is a strictly concave function with respect to the received power on a particular subcarrier. This can be proven using the following second derivative of $\pi^S(\mathcal{S}_i)$ with respect to $p^S_{i,n}$,

$$\frac{\partial^2 \pi^S(\mathcal{S}_i)}{\partial\left(p^S_{i,n}\right)^2} = -\frac{w(\gamma_{i,n})^2}{\left(1+\gamma_{i,n}p^S_{i,n}\right)^2} \le 0 \tag{20}$$

From (20), there always exists an optimal power level which maximizes the net payoff function under a fixed pricing model.

To better appreciate this feature, the concavity of the function is depicted in Fig. 3 where the curves of the total net payoffs versus different power levels are illustrated for different pricing values. Evidently, Fig. 3 demonstrates that the net payoff function exhibits downward concavity in which maximization of the net payoff can be done by finding the optimal power level. Furthermore, it is also observed in Fig. 3 that the total net payoff of SUs decreases when the subcarrier price increases. The subcarrier price increase prevents SUs from receiving data at high power, which results in a lower net payoff. To obtain the power level that maximizes the total payoff, the power allocation strategy can be summarized in the following proposition.

Proposition 1. ***(Power allocation):*** *For a given price $\Omega^m_{i,n}$, power should be allocated to coalition $\mathcal{S}_i$ for the nth subcarrier in a waterfilling manner based on the following power update algorithm:*

$$p^S_{i,n} = \left(\Delta_n - \frac{1}{\gamma_{i,n}}\right)^+ \tag{21}$$

where $\Delta_n = w/\upsilon\Omega^m_{i,n}|g^S_{m,n}|^2$ is the water level for the nth subcarrier.

Proof. For a given price $\Omega^m_{i,n}$, to find the optimal power allocation for coalition $\mathcal{S}_i$ that maximizes its payoff, take the first derivative of (14) with respect to $p^S_{i,n}$ and this produces

$$\frac{\partial \pi^S(\mathcal{S}_i)}{\partial p^S_{i,n}} = \frac{\alpha_{i,k}\beta_{i,n}w\gamma_{i,n}}{1+\gamma_{i,n}p^S_{i,n}} - \upsilon\Omega^m_{i,n}\left|g^S_{m,n}\right|^2 = 0 \tag{22}$$

If the nth subcarrier and the kth SU are assigned to coalition $\mathcal{S}_i$ such that $\alpha_{i,k} = 1$ and $\beta_{i,n} = 1$, (22) can be expressed as

$$\frac{w\gamma_{i,n}}{1+\gamma_{i,n}p^S_{i,n}} - \upsilon\Omega^m_{i,n}\left|g^S_{m,n}\right|^2 = 0 \tag{23}$$

Rearranging (23) results in

$$p_{i,n}^{S} = \left(\frac{w}{\upsilon \Omega_{i,n}^{m} \left| g_{m,n}^{S} \right|^2} - \frac{1}{\gamma_{i,n}} \right)^{+} \quad (24)$$

where $\Delta_n = w/\upsilon\Omega_{i,n}^{m}|g_{m,n}^{S}|^2$ is the water level for the nth subcarrier. □

Note that coalition $\mathcal{S}_i$ cannot access the spectrum if the licensed spectrum is priced higher than what the coalition can afford. It is manifested in (24) that if the received power is equal to or less than zero on the nth subcarrier due to an unacceptable pricing of the nth subcarrier, the SBS will turn off transmission to coalition $\mathcal{S}_i$ on this subcarrier. In such a scenario, the nth subcarrier is said to be inactive for coalition $\mathcal{S}_i$. On the contrary, if the price of the subcarrier is acceptable to all members of coalition $\mathcal{S}_i$, the coalition will strive to acquire the subcarrier to improve its net payoff.

Proposition 2. *The nth subcarrier is active for coalition $\mathcal{S}_i$,* i.e., $\pi_n^S(\mathcal{S}_i) > 0$ if and only if $\upsilon\Omega_{i,n}^{m}|g_{m,n}^{S}|^2 < w\gamma_{i,n}$.

Proof. For a given price $\Omega_{i,n}^{m}$, the nth subcarrier is inactive for coalition $\mathcal{S}_i$ if $p_{i,n}^{S} \leq 0$ and hence $\pi_n^S(\mathcal{S}_i) = 0$. In contrast, the nth subcarrier is active for coalition $\mathcal{S}_i$ if $\pi_n^S(\mathcal{S}_i) > 0$ and $p_{i,n}^{S} > 0$. Substituting (24) into (14) gives

$$\pi_n^S(\mathcal{S}_i) = w\left(|\mathcal{S}_i| \log_2\left(\frac{w\gamma_{i,n}}{\upsilon\Omega_{i,n}^{m}\left|g_{m,n}^{S}\right|^2} \right) - 1 + \frac{\upsilon\Omega_{i,n}^{m}\left|g_{m,n}^{S}\right|^2}{w\gamma_{i,n}} \right) > 0 \quad (25)$$

The inequality in (25) holds due to the fact that $\log_2(x) + 1/x > 1$ for $x > 1$. Therefore, the nth subcarrier is said to be active for coalition $\mathcal{S}_i$ if condition (25) is met. To fulfil condition (25), the following condition should be imposed

$$\frac{w\gamma_{i,n}}{\upsilon\Omega_{i,n}^{m}\left|g_{m,n}^{S}\right|^2} > 1 \quad (26)$$

Rearranging (26) gives $\upsilon\Omega_{i,n}^{m}|g_{m,n}^{s}|^2 < w\gamma_{i,n}$ which implies that $p_{i,n}^{S} > 0$. □

Based on Proposition 2, the PUs can intentionally escalate the price of subcarriers to prevent the SUs from accessing their subcarriers if the SUs produce harmful interference to the PUs on these subcarriers. Since the payoff of a coalition is always constrained by the least channel gain, a subcarrier is likely to be inactive for a coalition if one of the members within the coalition has a very low channel gain such that condition (26) is violated. Therefore, rational SUs with high channel gains in that coalition may unilaterally deviate from the coalition to avoid paying unnecessary charges for subcarrier access. In other words, rational SUs will only join a coalition if the price of subcarriers is sufficiently low, thus allowing them to be allocated more power to improve their net payoff.

Proposition 3. ***(Coalition formation):*** *For a given price $\Omega_{i,n}^{m}$, the kth SU should join coalition $\mathcal{S}_i$ such that $\alpha_{i,k} = 1$, $i = \arg\max_{x\in\mathcal{S}} \sum_{n=1}^{N} \beta_{x,n}\gamma_{x,n}/\Omega_{x,n}^{m}$.*

Proof. To find the CF strategy for the kth SU which can maximize (15), substitute (24) into (16) and this gives

$$\pi^S(\mathcal{S}_i) = \sum_{k=1}^{K}\sum_{n=1}^{N} \alpha_{i,k}\beta_{i,n} w\left(\log_2\left(\frac{w\gamma_{i,n}}{\upsilon\Omega_{i,n}^{m}\left|g_{m,n}^{S}\right|^2} \right) - 1 + \frac{\upsilon\Omega_{i,n}^{m}\left|g_{m,n}^{S}\right|^2}{w\gamma_{i,n}} \right) \quad (27)$$

Based on Proposition 2, an active nth subcarrier requires $\upsilon\Omega_{m,n}|g_{m,n}^{s}|^2 < w\gamma_{i,n}$. This shows that $\pi^S(\mathcal{S}_i)$ is an increasing function with respect to $\gamma_{i,n}/\Omega_{i,n}^{m}$ if $\upsilon\Omega_{m,n}|g_{m,n}^{s}|^2 < w\gamma_{i,n}$. Therefore, in order to maximize (15) in the MCGP, the kth SU should join coalition $\mathcal{S}_i$ which provides the highest net payoff. The CF strategy of the kth user can be given by $\alpha_{i,k} = 1$ where

$$\begin{aligned} i &= \arg\max_{x\in\mathcal{S}} \sum_{n=1}^{N} \beta_{x,n}\left(\log_2\left(\frac{\gamma_{x,n}}{\Omega_{x,n}^{m}} \right) - 1 + \frac{\Omega_{x,n}^{m}}{\gamma_{x,n}} \right) \\ &= \arg\max_{x\in\mathcal{S}} \sum_{n=1}^{N} \beta_{x,n}\log_2\left(\frac{\gamma_{x,n}}{\Omega_{x,n}^{m}} \right) \end{aligned} \quad (28)$$

The strategy in (28) can be further simplified as $i = \arg\max_{x\in\mathcal{S}} \sum_{n=1}^{N} \beta_{x,n}\gamma_{x,n}/\Omega_{x,n}^{m}$. □

In this context, the fair payoff distribution is adopted where the SUs within a coalition will equally share the payoff and bear the same cost. Therefore, a SU tends to join coalition $\mathcal{S}_i$ rather than coalition $\mathcal{S}_j$ if it can either gain a higher payoff or pay a lower price after joining coalition $\mathcal{S}_i$. This scenario can be expressed as

$$\frac{v(\mathcal{S}_i)}{|\mathcal{S}_i|} - \frac{v(\mathcal{S}_j)}{|\mathcal{S}_j|} \geq \frac{c(\mathcal{S}_i)}{|\mathcal{S}_i|} - \frac{c(\mathcal{S}_j)}{|\mathcal{S}_j|} \quad (29)$$

From (16) and (29), it is shown that the merger of any two coalitions not only improves the payoff, but also increases the cost due to the usage of more subcarriers. If the improvement in payoff is higher than the price increase, then the two coalitions will willingly merge to obtain a higher net payoff. However, the CF does not always guarantee a higher net payoff because the merger of any two coalitions may cause the new coalition to acquire inactive subcarriers due to the lower $\gamma_{g,\,n}$ achieved. As a result, the MCGP is not necessarily superadditive and cohesive. Instead, the SUs tend to form smaller coalitions in high-priced environments so that the costs of subcarriers are affordable.

Proposition 4. ***(Subcarrier allocation):*** *For a given price $\Omega_{i,n}^{m}$, the nth subcarrier should be allocated to coalition $\mathcal{S}_i$ such that $\beta_{i,n} = 1$, $i = \arg\max_{x\in\mathcal{S}} (\gamma_{x,n}/\Omega_{x,n}^{m})^{|\mathcal{S}_x|}$.*

Proof. To find the subcarrier allocation for the kth SU which can maximize (15), substitute (24) into (16) and this gives

$$\pi_n^S(\mathcal{S}_i) = |\mathcal{S}_i| w\left(\log_2\left(\frac{w\gamma_{i,n}}{\upsilon\Omega_{i,n}^{m}\left|g_{m,n}^{S}\right|^2} \right) - 1 + \frac{\upsilon\Omega_{i,n}^{m}\left|g_{m,n}^{S}\right|^2}{w\gamma_{i,n}} \right) \quad (30)$$

Similar to the proof given in Proposition 3, the nth subcarrier should be allocated to coalition $\mathcal{S}_i$ such that $\beta_{i,n} = 1$ where

$$\begin{aligned} i &= \arg\max_{x\in\mathcal{S}} |\mathcal{S}_x|\left(\log_2\left(\frac{\gamma_{x,n}}{\Omega_{x,n}^{m}} \right) - 1 + \frac{\Omega_{x,n}^{m}}{\gamma_{x,n}} \right) \\ &= \arg\max_{x\in\mathcal{S}} |\mathcal{S}_x| \log_2\left(\frac{\gamma_{x,n}}{\Omega_{x,n}^{m}} \right) \end{aligned} \quad (31)$$

The strategy in (31) can be further simplified as $i = \arg\max_{x\in\mathcal{S}} (\gamma_{x,n}/\Omega_{x,n}^{m})^{|\mathcal{S}_x|}$. □

Propositions 1, 3 and 4 demonstrate that the CF and SPA strictly rely on the subcarrier prices whereby a lower pricing value motivates CF to rake in more subcarriers and higher power, thus garnering a higher net payoff for every SU. In other words, the net payoff of a coalition is a decreasing function of $\Omega_{i,n}^{m}$, which is proven in the following proposition:

Proposition 5. *The net payoff function of coalition $\pi_n^S(\mathcal{S}_i)$ is a strictly decreasing function of $\Omega_{i,n}^{m}$ on the nth subcarrier if* $\upsilon\Omega_{i,n}^{m}|g_{m,n}^{s}|^2 < w\gamma_{i,n}$.

Proof. To investigate the impact of prices on the payoff function of coalition $\mathcal{S}_i$, take the first derivative of (16) with respect to $\Omega_{i,n}^m$

$$\frac{\partial \pi_n^S(\mathcal{S}_i)}{\partial \Omega_{i,n}^m} = -\frac{w}{\upsilon \Omega_{i,n}^m} + \frac{\left|g_{m,n}^S\right|^2}{\gamma_{i,n}} \tag{32}$$

The first derivative in (32) shows that the payoff of coalition $\mathcal{S}_i$ is a decreasing function of $\Omega_{i,n}^m$ when $\upsilon\Omega_{i,n}^m < w\gamma_{i,n}/|g_{m,n}^s|^2$. From Proposition 2, it is shown that the nth subcarrier is active in coalition $\mathcal{S}_i$ when $\upsilon\Omega_{i,n}^m < w\gamma_{i,n}/|g_{m,n}^s|^2$. Therefore, it can be concluded that the coalition on an active subcarrier always has a decreasing payoff when $\Omega_{i,n}^m$ increases. □

It is noteworthy that any changes of price by the PUs may cause instability to the MCGP and all the SUs need to play the MCGP again to attain a new equilibrium state. Due to variations in pricing coefficients, the equilibrium of the MCGP is not unique and there may exist infinitely many equilibria for different sets of pricing coefficients. Therefore, in order for the MCGP to reach equilibrium state, the NPAG must first achieve NE in which a set of optimal fixed pricing coefficients can be provided for the MCGP. In other words, the MCGP will only attain a unique equilibrium if the NPAG achieves NE.

3.2. Non-cooperative price adjustment game for OFDMA-based PRNs

In PRNs, PUs can lease their subcarriers to the coalitions formed by SUs to gain profit through an efficient pricing mechanism. The price adjustment by the PUs can be modelled using a non-cooperative game termed NPAG where each PU selfishly adjusts the pricing coefficients of its allocated subcarriers to maximize its own profit. In fact, the NPAG is a subgame of the CSGP which can be formally defined as follows:

Definition 3. Let $NPAG = \langle \mathcal{M}, \{\mathbf{\Omega}^m\}, \{\pi_m^P(.)\}\rangle$ denote the non-cooperative price adjustment game (NPAG) where $\mathcal{M}$ is the index set of the non-cooperative players referred to as the selfish PUs, $\mathbf{\Omega}^m$ denotes the pricing strategies available for the mth PU while $\pi_m^P(.)$ is the profit function for the mth PU.

The pricing strategy of the mth PU denoted by $\mathbf{\Omega}^m$ can be represented as an $|\mathcal{S}| \times N$ matrix

$$\mathbf{\Omega}^m = \begin{bmatrix} \Omega_{1,1}^m & \Omega_{1,2}^m & \cdots & \Omega_{1,N}^m \\ \Omega_{2,1}^m & \Omega_{2,2}^m & \cdots & \vdots \\ \vdots & \vdots & \ddots & \Omega_{|\mathcal{S}|-1,N}^m \\ \Omega_{|\mathcal{S}|,1}^m & \cdots & \Omega_{|\mathcal{S}|,N-1}^m & \Omega_{|\mathcal{S}|,N}^m \end{bmatrix} \tag{33}$$

where each row of the matrix $\mathbf{\Omega}^m$ contains the pricing coefficients of all the subcarriers for a particular coalition. Since a subcarrier is only exclusively assigned to one PU shared by one coalition, $\mathbf{\Omega}^m$ has zero entries on the subcarriers which are not assigned to that PU. In general, $\Omega_{i,n}^m$ is a non-negative pricing coefficient which quantifies the willingness of the mth PU to lease its nth subcarrier to coalition $\mathcal{S}_i$. For example, if the interference gain between the SBS and the mth PU using the nth subcarrier is small or the PU is far from the SBS, $\Omega_{i,n}^m$ can be set to a low value to make the subcarrier more affordable to the SUs. Nevertheless, if the PU is more sensitive to interference, or the interference gain between the SBS and PU is large or the SBS is close to the PU, $\Omega_{i,n}^m$ can be set to a high value to prevent the SBS from transmitting at high power. In general, the profit garnered by the mth PU on the nth subcarrier can be expressed as

$$\pi_{m,n}^P = \upsilon \Omega_{i,n}^m \left|g_{m,n}^S\right|^2 p_{i,n}^S \tag{34}$$

The total profit obtained by the mth PU on all of its allocated subcarriers can be given as

$$\pi_m^P = \sum_{n=1}^{N} \delta_{m,n}\beta_{i,n}\upsilon\Omega_{i,n}^m\left|g_{m,n}^S\right|^2 p_{i,n}^S \tag{35}$$

where $\beta_{i,n} = 1$ means that the nth subcarrier is leased to coalition $\mathcal{S}_i$ and $\delta_{m,\,n}$ is the binary subcarrier indicator which has $\delta_{m,n} = 1$ if the nth subcarrier is assigned to the mth PU, otherwise $\delta_{m,n} = 0$. Based on (35), the total profit for the PRN is given by

$$\pi^P = \sum_{n=1}^{M}\sum_{n=1}^{N} \delta_{m,n}\beta_{i,n}\upsilon\Omega_{i,n}^m\left|g_{m,n}^S\right|^2 p_{i,n}^S \tag{36}$$

The total profit for the PRN can be defined as the aggregate income from the collection of payments made by SUs. In the NPAG, the main objective of the PRN is to maximize the total profit through an efficient pricing scheme. Therefore, the NPAG can be defined as the following maximization problem:

$$(NPAG)\ \max_{\Omega_{i,n}^m} \sum_{n=1}^{M}\sum_{n=1}^{N} \delta_{m,n}\beta_{i,n}\upsilon\Omega_{i,n}^m\left|g_{m,n}^S\right|^2 p_{i,n}^S \tag{37}$$

$$\text{subject to } \left|g_{m,n}^S\right|^2 p_{g,n}^S + I_n \le I_{th} \tag{38}$$

where the PUs are tasked to adjust their subcarrier pricing to maximize their profit function in (37) subject to the ITC in (38). Since subcarrier allocation for the PRN is not the main focus of the present work, thus it is assumed that subcarriers are pre-allocated to the PUs based on certain criteria where every PU must acquire at least one subcarrier. Since each PU is allocated with a separate set of subcarriers, the pricing strategy of one PU is independent of others, but competition always exists among the PUs to lease their subcarriers to the SUs.

Proposition 6. *For a given power $p_{i,n}^S$, the pricing coefficient $\Omega_{i,n}^m$ for coalition $\mathcal{S}_i$ on the nth subcarrier can be adjusted by the mth PU based on the pricing strategy*

$$\Omega_{i,n}^m = \sqrt{\frac{\xi_{m,n}\gamma_{i,n}}{\upsilon\kappa\left|g_{m,n}^S\right|^2}} \tag{39}$$

where $\xi_{m,\,n}$ is a non-negative Lagrangian multiplier which can be adjusted to obtain the optimal pricing coefficient subject to the ITC.

Proof. To find the optimal pricing strategy of the mth PU which can maximize its total profit on the nth subcarrier, the formulation in (37) and (38) can be derived using Lagrangian relaxation as

$$\mathcal{L}\left(\Omega_{i,n}^m, \xi_{m,n}\right) = \sum_{m=1}^{M}\sum_{n=1}^{N} \upsilon\Omega_{i,n}^m\left|g_{m,n}^S\right|^2 p_{i,n}^S - \sum_{m=1}^{M}\sum_{n=1}^{N}\xi_{m,n}\left(\frac{\left(\left|g_{m,n}^S\right|^2 p_{i,n}^S + I_n\right)}{\kappa w} - T_g\right) \tag{40}$$

Substituting $p_{i,n}^S$ obtained in (24) into (40) yields

$$\mathcal{L}\left(\Omega_{i,n}^m, \xi_{m,n}\right) = \sum_{m=1}^{M}\sum_{n=1}^{N}\left(w - \frac{\upsilon\Omega_{i,n}^m\left|g_{m,n}^S\right|^2}{\gamma_{i,n}}\right) - \sum_{m=1}^{M}\sum_{n=1}^{N}\xi_{m,n}\left(\frac{1}{\kappa\upsilon\Omega_{i,n}^m} - \frac{\left|g_{m,n}^S\right|^2}{\kappa w\gamma_{i,n}} + \frac{I_n}{\kappa w} - T_g\right) \tag{41}$$

Taking the derivative of (41) with respect to $\Omega_{i,n}^m$ gives

$$\frac{\partial\mathcal{L}}{\partial\Omega_{i,n}^m} = -\frac{\left|g_{m,n}^S\right|^2}{\gamma_{i,n}} + \frac{\xi_{m,n}}{\kappa\upsilon\left(\Omega_{i,n}^m\right)^2} = 0 \tag{42}$$

Table 1
The NPAG algorithm.

1. Initialization:
Let $t = 1$, define the initial values for ϕ and $\xi_{m,n}(0)$, $\forall m \in \mathcal{M}$, $\forall n \in \mathcal{N}$.
2. Price adjustment: (for all *M* PUs)
a) The mth PU measures its received interference on its allocated subcarriers and estimates $[|g^S_{m,n}|^2 p^S_{i,n}]_{n\in\mathcal{N}_m}$.
b) The mth PU calculates $D_{m,n}(t) = T_g - (|g^S_{m,n}|^2 p^S_{i,n} + I_n)/\kappa w$ based on the received interference, $\forall n \in \mathcal{N}_m$. If $D_{m,n}(t) \le 0$, $\forall n \in \mathcal{N}_m$, the mth PU broadcasts a stop message and go to step 3, otherwise go to Step 2c).
c) The mth PU updates $\xi_{m,n}(t+1) = (\xi_{m,n}(t) - \phi D_{m,n}(t))^+$ for $D_{m,n}(t) > 0$and let $\xi_{m,n}(t+1) = \xi_{m,n}(t)$ for $D_{m,n}(t) \le 0$, $\forall n \in \mathcal{N}_m$.
d) The mth PU broadcast $[\xi_{m,n}]_{n\in\mathcal{N}_m}$ and $[|g^S_{m,n}|^2 p^S_{i,n}]_{n\in\mathcal{N}_m}$ to all coalitions in the MCGP.
e) The coalitions receive $[\xi_{m,n}]_{n\in\mathcal{N}_m}$ and perform CF and SPA using the MCGP algorithm in Table 2. All PUs enter the sensing mode. Once the beacons from SUs are received, proceed to Step 2a).
3. Termination:
The algorithm ends with solution $(\xi_{m,n})*$, $\forall n \in \mathcal{N}_m$.

Table 2
The MCGP algorithm.

[1] Initialization:
Let $t = 1$, $|\mathcal{S}| = |\mathcal{K}|$, $\alpha_{i,k} = 1, \forall i = k$, $\alpha_{i,k} = 0, \forall i \ne k$, $k \in \mathcal{K}$, $i \in \mathcal{S}$. Determine $\mathcal{S}_i = \{k | k \in \mathcal{K}, \alpha_{i,k} = 1\}$, $\forall i \in \mathcal{S}$. Next, construct $\boldsymbol{c} = \{\boldsymbol{c}_1, \boldsymbol{c}_2, \ldots \boldsymbol{c}_c\}$ and let the initial power level be $p^S_{i,n}(0) = P^{\max}_{SBS}/N$, $\forall n \in \mathcal{N}$.
[2] Subcarrier allocation:
a) The nth subcarrier is assigned to coalition $\mathcal{S}_i$ based on Proposition 2, $\forall n \in \mathcal{N}$,
b) Let $\mathcal{N}_i = \{n | n \in \mathcal{N}, \beta_{i,n} = 1\}$, $\forall i \in \mathcal{S}$.
c) The SBS transmits beacons with $p^S_{i,n}(t)$ on the nth subcarrier, $\forall n \in \mathcal{N}$. The PUs receive the beacons and perform price adjustment using the NPAG algorithm in Table 1. All SUs enter the sensing mode. Once coalition$\mathcal{S}_i$ receives $[\xi_{m,n}]_{n\in\mathcal{N}_i}$ and $[|g^S_{m,n}|^2 p^S_{i,n}]_{n\in\mathcal{N}_i}$ broadcast by the PUs, proceed to Step 3. If a stop message is received, go to Step 4. Otherwise, let $t = t + 1$ and go to Step 2.
[3] Coalition formation (for all *S* coalitions):
a) Coalition $\mathcal{S}_i$ computes $p^S_{i,n}(t)$ based on (24). Subsequently, coalition $\mathcal{S}_i$ computes $\Omega^m_{i,n}$ based on (39) and computes $\pi^S(\mathcal{C}_j)$, $\forall \mathcal{C}_j \in \mathcal{C}^t$ based on
$\pi^S(\mathcal{C}_j) = \sum_{\mathcal{S}_i\in\mathcal{C}_j} \pi^S(\mathcal{S}_i) = \sum_{\mathcal{S}_i\in\mathcal{C}_j} w(|\mathcal{S}_i| \log_2(\frac{w\gamma_{i,n}}{\Omega^m_{i,n}|g^S_{m,n}|^2}) - 1 + \frac{\Omega^m_{i,n}|g^S_{m,n}|^2}{w\gamma_{i,n}})$
b) To move from $\mathcal{C}^t_j$ to $\mathcal{C}^{t+1}_j$, compute $\Omega^m_{i',n}$ and $\pi^S(\mathcal{C}_j)$, $\forall \mathcal{C}_j \in \mathcal{C}^{t+1}$ where i' indicates the indices of all possible mergers of coalitions in $\mathcal{C}^{t+1}$. Find $(\mathcal{C}^{t+1}_j)^* = \arg\max_{\mathcal{C}_j\in\mathcal{C}^{t+1}} \pi^S(\mathcal{C}_j)$ and let $\mathcal{C}^{t+1}_j = (\mathcal{C}^{t+1}_j)^*$.
c) If $\pi^S(\mathcal{C}^{t+1}_j) > \pi^S(\mathcal{C}^t_j)$, move from $\mathcal{C}^t_j$ to $\mathcal{C}^{t+1}_j$ and the new coalition $i' = i \cup j$ is created as $\mathcal{S}_{i'} = \mathcal{S}_i \cup \mathcal{S}_j \in \mathcal{C}^{t+1}_j$, $\mathcal{N}_{i'} = \mathcal{N}_i \cup \mathcal{N}_j$, $\mathcal{S} = \mathcal{S} - \{i, j\} + \{i'\}$. Otherwise, remain in $\mathcal{C}^t_j$.
[4] Power allocation:
Power is waterfilled to the nth subcarrier based on Proposition 1, $\forall n \in \mathcal{N}$ and go to Step 5.
[5] Termination:
The algorithm ends with $(\mathcal{C}^{t+1}_j)^*$ and $(p^S_{i,n})^*$.

Rearranging (42) provides the pricing strategy for the mth PU as in (39). It is also shown in (42) and that (39) is the optimal pricing coefficient for the mth PU that maximizes its profit function on the nth subcarrier. □

Proposition 6 shows that $\Omega^m_{i,n}$ is proportional to the smallest channel gain $\gamma_{i,n}$, which implies that the coalitions with high channel gains can afford to pay higher prices. Once a larger coalition is formed, $\Omega^m_{i,n}$ needs to be reduced because the new coalition may not possess the same buying power due to the lower $\gamma_{i,n}$ achieved. If the price is not decreased on a newly formed coalition, the coalition has to use a lower received power, resulting in lower profit for the PUs. In other words, a reduction in $\Omega^m_{i,n}$ not only improves the net payoff for the SUs, but also yields a higher profit for the PUs.

Proposition 7. *The payoff π^P_m for the mth PU is a decreasing function of $\Omega^m_{i,n}$.*

Proof. Substituting (24) into (35) yields

$$\pi^P_m = \sum_{n=1}^{N} \upsilon\Omega^m_{i,n}\left|g^S_{m,n}\right|^2 \left(\frac{w}{\upsilon\Omega^m_{i,n}\left|g^S_{m,n}\right|^2} - \frac{1}{\gamma_{i,n}}\right) = \sum_{n=1}^{N}\left(w - \frac{\upsilon\Omega^m_{i,n}\left|g^S_{m,n}\right|^2}{\gamma_{i,n}}\right) \tag{43}$$

Taking the derivative of π^P_m with respect to $\Omega^m_{i,n}$ yields

$$\frac{\partial\pi^P_m}{\partial\Omega^m_{i,n}} = -\frac{\upsilon\left|g^s_{m,n}\right|^2}{\gamma_{i,n}} < 0 \tag{44}$$

From (44), it is shown that the function π^P_m always increases as $\Omega^m_{i,n}$ decreases. □

Proposition 7 reveals that the PUs can improve their payoff by reducing $\Omega^m_{i,n}$ so that the SUs can access the licensed subcarriers at higher power. Note that, in some systems, the PUs may cheat on the prices to induce the SUs to pay more than necessary. This problem does not exist in this pricing model because the prices charged by the PUs are proportional to the resulting interference caused by the SUs. If a PU attempts to charge a high price to a coalition, it will cause the coalition to decrease its received power on the subcarriers and eventually lowers the payoff.

Since the pricing strategy of a PU is independent of others, every PU can disjointedly adjust the prices of their allocated subcarriers without observing the actions of other PUs. Propositions 6 and 7 show that the pricing strategy is a decreasing function of $\Omega^m_{i,n}$ where a reduction in $\Omega^m_{i,n}$ improves the profits of the PUs albeit inducing higher interference to the PUs. Based on the pricing update strategy derived in (39), $\xi_{m,n}$ plays a crucial role in providing an optimal trade-off between profit and interference. In order to obtain an optimal trade-off, $\xi_{m,n}$ is iteratively decreased with an adaptive step size. Let $\xi_{m,n}(t+1) = (\xi_{m,n}(t) - \phi(t)D_{m,n}(t))^+$ be the subgradient update method where $\phi(t) > 0$ is the update step while $D_{m,n}(t) = I_{th} - (|g^S_{m,n}|^2 p^S_{g,n} + I_n)$. By using this method, the optimal $\Omega^m_{i,n}$ can be obtained by finding the optimal $\xi_{m,n}$ subject to the ITC. Once $D_{m,n} \le 0, \forall m \in \mathcal{M}$, $\forall n \in \mathcal{N}$, the NPAG is said to have achieved NE without further price adjustment. In this scenario, the fixed optimal pricing coefficients allow the MCGP to attain equilibrium as well. Once the NPAG and MCGP simultaneously reach their respective equilibria, the CSGP is said to have achieved SE.

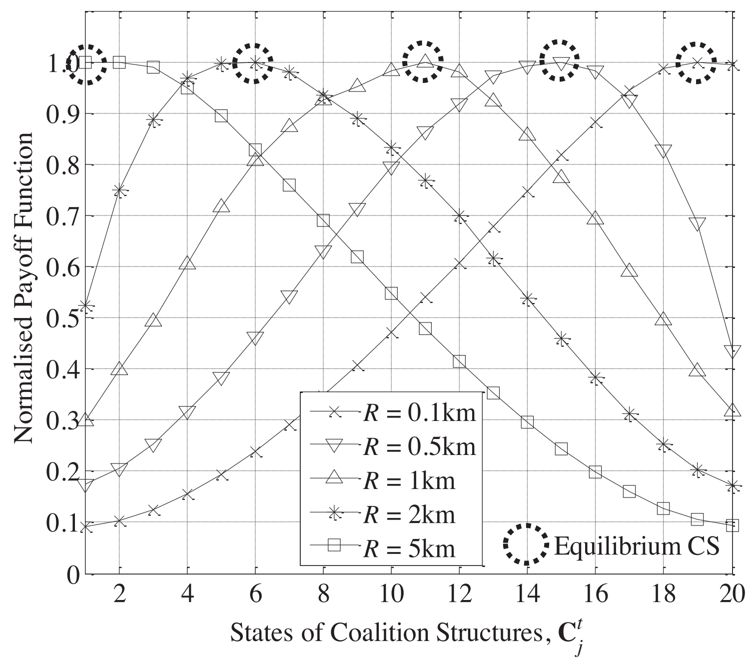

Fig. 4. Normalized payoff function versus states of CS for different coverage radii among 20 multicast users with 128 subcarriers.

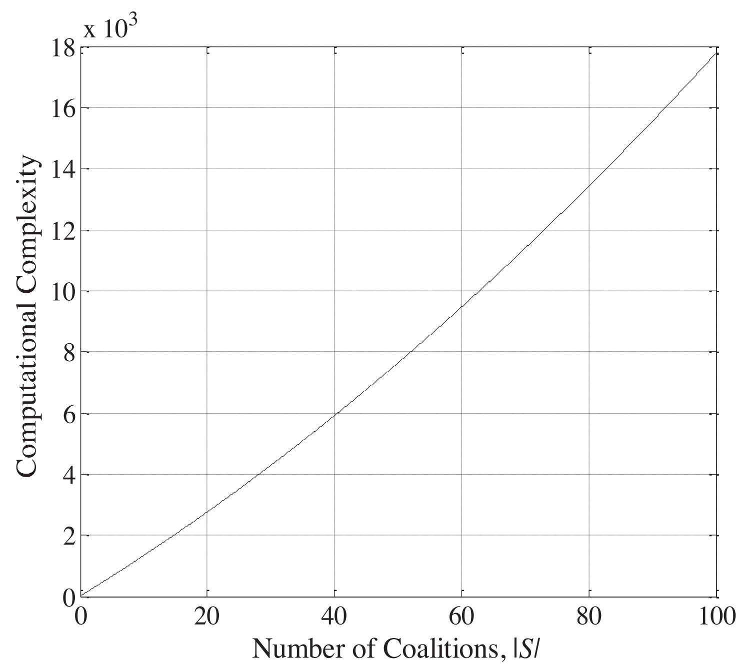

Fig. 5. Computational complexity versus number of coalitions for a 64-subcarrier MCRN.

4. Algorithm for the CSGP and complexity analysis

The CSGP consists of two disjoint algorithms, i.e., the NPAG and MCGP algorithms which are summarized in Tables 1 and 2, respectively. The CSGP algorithm starts by allowing SUs in their singletons to transmit distinct beacons on their pre-allocated subcarriers using equally-assigned power. The PUs in the NPAG measure the power of the beacons and start to price the subcarriers pre-assigned to them. Since subcarrier allocation is not the main focus of this work, no subcarrier allocation will be performed in the NPAG. Based on the subgradient update method, the PUs iteratively update $\xi_{m,n}$ subject to the ITC. At every iteration, the PUs broadcast their respective $\xi_{m,n}$ as well as the channel state information (CSI) to the SUs. The SUs compute the pricing coefficients for its current and other possible mergers of coalitions upon receiving $\xi_{m,n}$ and the CSI before entering into negotiation for CF.

Using the transition model for CF proposed in [28], only two coalitions are allowed to merge at each iteration. Therefore, the SUs in negotiation can provide feedback to the SBS which will instruct the merger of the two coalitions capable of producing the highest net payoff. Before reaching the equilibrium state, the SUs are allowed to explore possible states of CF denoted as $\mathcal{C}$. For simplicity, $\mathcal{C}$ is divided into $|\mathcal{S}|$ states where $\mathcal{C}_j$, $j = 1, 2, \ldots, |\mathcal{C}|$ with the same cardinality are grouped in the same state. The objective

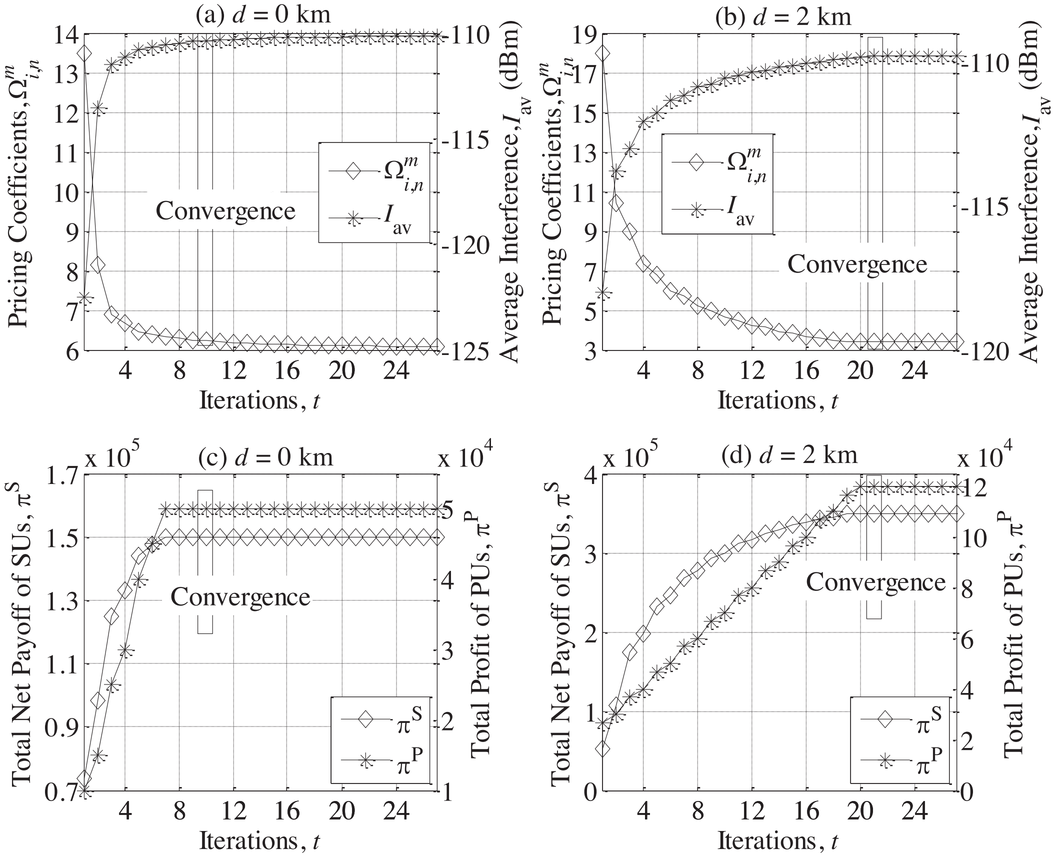

Fig. 6. Convergence results of the MCGP and NPAG for different values of *d*.

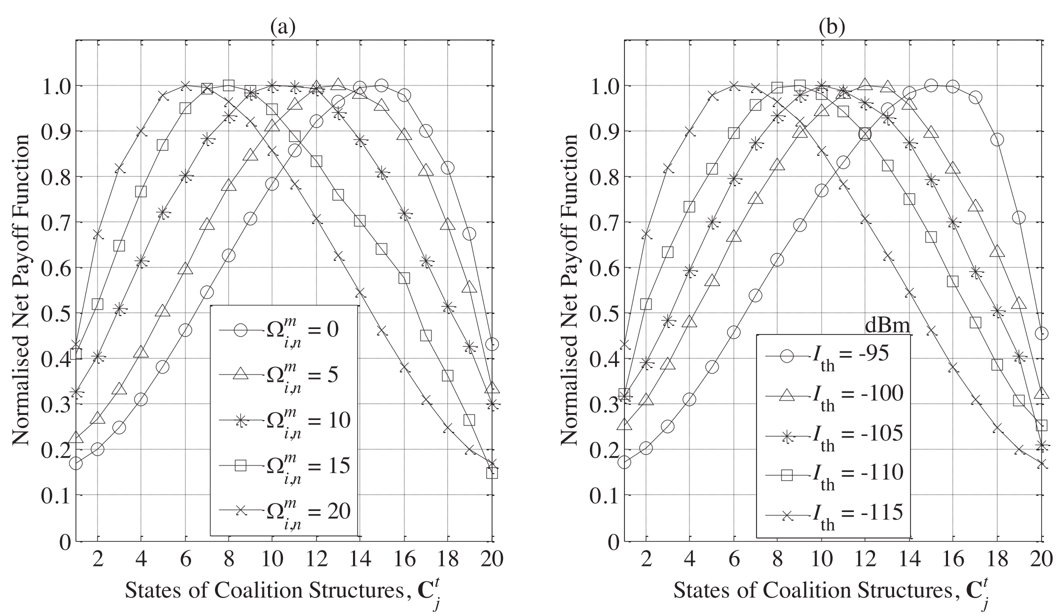

Fig. 7. Normalized payoff function versus states of coalition structures for different pricing coefficients and ITC.

of the MCGP is to maximize the total payoff by finding a coalition structure (CS) such that

$$\mathcal{C}_j^* = \arg\max_{\mathcal{C}_j \in \mathcal{C}} v\big(\mathcal{C}_j\big) \tag{45}$$

Without loss of generality, the CF process is executed until no further price adjustments are made by the PUs. At this point, the NPAG and MCGP simultaneously reach their respective equilibria and the CSGP is said to have achieved SE.

Since the NPAG and MCGP are the subgames of the CSGP, the existence and uniqueness of SE for the CSGP can be shown by independently proving the existence of unique Nash equilibria (NEs) in both the NPAG and MCGP [?].

Proposition 8. *A unique NE exists in the NPAG if* $\pi_m^P(\Omega_{i,n}^m)$ *is strictly concave in* $\Omega_{i,n}^m$ *for all m and n* [23].

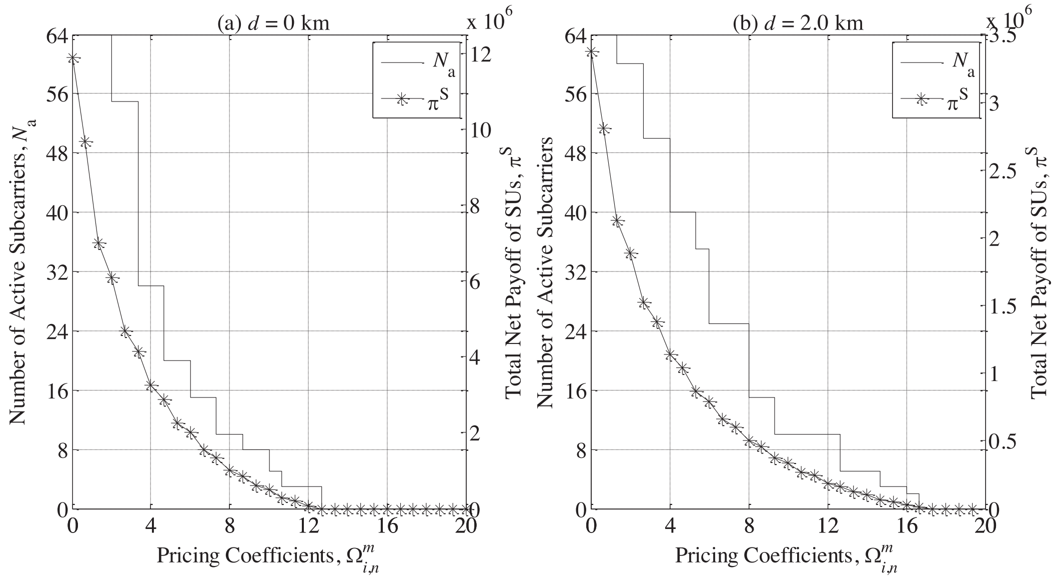

Fig. 8. Number of active subcarriers and total payoff of SUs versus pricing coefficients for $d=0$ km and $d=2$ km.

Proof. To show the concavity of $\pi^P_m(\Omega^m_{i,n})$, taking the second derivative of (41) with respect to $\Omega^m_{i,n}$ results in

$$\frac{\partial^2 \mathcal{L}}{\partial \left(\Omega^m_{i,n}\right)^2} = -\frac{2\xi_{m,n}}{\kappa \upsilon \left(\Omega^m_{i,n}\right)^3} \tag{46}$$

Since $\xi_{m,\,n}$, κ, υ and $\Omega^m_{i,n}$ are non-negative, $\partial^2\mathcal{L}/\partial(\Omega^m_{i,n})^2$ denoted in (46) is always less than zero. This implies that $\pi^P_m(\Omega^m_{i,n})$ is always continuous and strictly concave in $\Omega^m_{i,n}$ over a compact set, which has a unique solution. Hence, if all PUs selfishly act according to the proposed NPAG algorithm in Table 1, the NPAG will converge to a unique NE which guarantees an optimal pricing for PUs. □

Proposition 9. *A unique NE exists in the MCGP if $v(\mathcal{C}_j)$ in the MCGP shows a concavity property when the CF process moves forward* [28].

Proof. The concavity property of $v(\mathcal{C}_j)$ for the MCGP can be proven by showing the curves of $v(\mathcal{C}_j)$ as a function of the states of CS for different coverage radii (denoted as R). The coverage radius is one of the factors that determine the equilibrium CS. Therefore, it is observed in Fig. 4 that the equilibrium CS is achieved at different states of CS for different coverage radii.

It is demonstrated in Fig. 4 that the payoff function proposed in (16) is concave when the CS moves forward. When the CF process moves forward (from singleton to grand coalition), the SUs who receive the highest payoff at the current CS will be reluctant to move for further CF. Therefore, the MCGP is said to have achieved NE in which every SU is satisfied with the received payoff in the equilibrium CS. From Fig. 4, it is also proven that the NE which exists in the MCGP is unique and the MCGP will converge to this unique NE if all SUs cooperatively act based on the proposed MCGP algorithm in Table 2. □

In order to reduce the signaling complexity, the PRN and MCRN can exchange information (e.g., pricing coefficients, CSI and SPA strategies) via the communications between PBS and SBS. In the MCGP, users are required to negotiate with one another to find their best partners to form the coalitions that are beneficial to all users within the coalitions. The negotiation involves information exchange mainly in the form of CSI of the users on each subcarrier. The negotiation process described here can be achieved using a common control channel where users can exchange messages to perform the proposed distributed CF. The signaling complexity of the MCGP is similar to that of the coalitional game proposed in [28], which has been proven feasible for practical implementation. Other than that, the computational complexity of the MCGP for one iteration is denoted as $\mathcal{O}(|\mathcal{S}|(N+(|\mathcal{S}|-1)/2))$ where $|\mathcal{S}|(|\mathcal{S}|-1)/2$ evaluations of function (45) are needed for CF and $N|\mathcal{S}|$ evaluations based on Proposition 4 are required for subcarrier allocation. The computational complexity of the MCGP algorithm is evaluated analytically as a function of $|\mathcal{S}|$ for a 64-subcarrier MCRN as illustrated in Fig. 5. It is noticed in Fig. 5 that the computational complexity of the MCGP grows quadratically with the number of coalitions, where the complexity for one iteration can be further expressed as $\mathcal{O}(|\mathcal{S}|^2)$. The low computational complexity facilitates practical implementation of the MCGP, even in a MCRN with a large number of SUs. In addition, it is worth mentioning that the complexity of the MCGP reduces when the CF process moves forward because the size of CS reduces when two coalitions are merged at every iteration. Therefore, the MCGP appears to be more favorable and feasible than the exhaustive search scheme which has a complexity of $\mathcal{O}(|\mathcal{S}|^N)$.

5. Simulation and numerical results

A hybrid network comprising a PRN and a MCRN each with a cell radius of 2 km is considered. The PBS and SBS are placed at the center of the cell within which the PUs and multicast SUs are uniformly and randomly distributed around their respective BSs. The distance d between the PBS and SBS is adjusted within 2 km to vary the interference gains. The PBS and SBS disjointedly deploy an isotropic transmitter each with a maximum power of 30 dBm while the background noise power is assumed to be −140 dBm. In this hybrid network, 20 PUs and 20 SUs are simulated to share 64 subcarriers. For performance comparison, the cognitive grouping genetic algorithm (CGGA) [29], the cognitive CMS (CCMS) [5] and the cognitive CUS (CCUS) [30] are also simulated in this hybrid network.

First, strategic interdependence between PUs and SUs is analyzed in Fig. 6 where $I_{th} = -110dBm$ is used for all subcarriers. Fig. 6(a) and (b) show that the PUs in the NPAG iteratively reduce the value of $\xi_{m,\,n}$ using the subgradient update method to find the

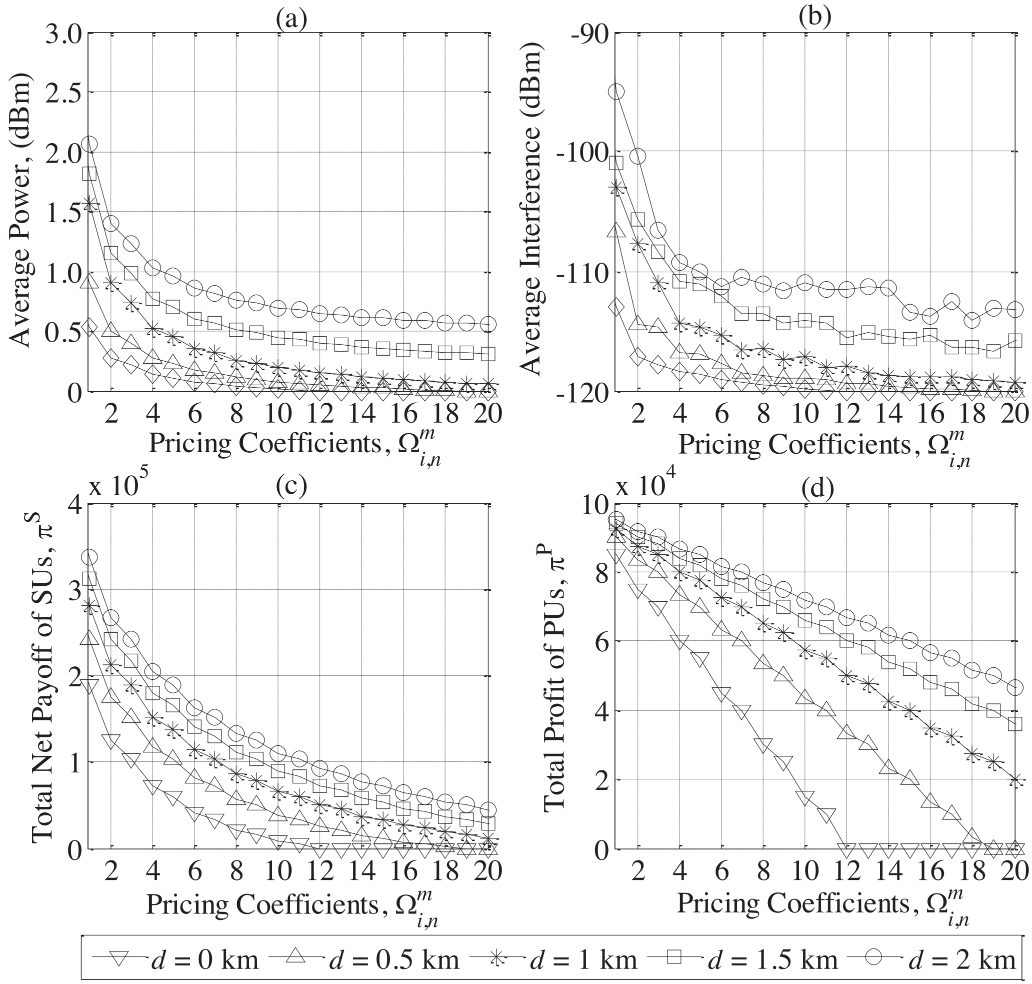

Fig. 9. The impact of pricing coefficients on the total net payoff, total profit, power of SUs and resulting interference on PUs.

optimal $\Omega_{i,n}^{m}$ such that the ITC is not exceeded. The exponential reduction of $\Omega_{i,n}^{m}$ motivates the SUs in the MCGP to form larger coalitions and use higher power, subsequently producing higher interference to the PRN. Constrained by the ITC, the average interference power received by the PUs eventually converges to a value below $I_{th} = -110dBm$. Further price reduction is avoided by the PUs because this move will violate the ITC in (38) where rational PUs are not willing to compromise their QoS for a higher profit. In the system with $d = 0$ km, it is shown in Fig. 6(a) and (c) that both the NPAG and MCGP reach equilibrium at the 9th iteration where maximum payoff and profit can be obtained. Nevertheless, Fig. 6(b) and (d) illustrate that the NPAG and MCGP in the system with $d = 2$ km can only reach equilibrium at the 21st iteration. Both games converge much slower in this scenario because the SUs tend to move forward to form larger coalitions and use higher power. The tendency to form larger coalitions is mainly due to the lower interference gains obtained in the system with $d = 2$ km, hence the subcarriers are more affordable for SUs if a larger coalition is formed.

In Fig. 7, the strategies of CF in the MCGP are shown to be highly dependent on the values of $\Omega_{i,n}^{m}$ and I_{th}. As shown in (2), forming larger coalitions may result in lower $\gamma_{i,\,n}$, which yields a lower payoff due to more inactive subcarriers acquired by the newly formed coalitions. The tendency of SUs toward forming larger coalitions depends on the condition of $\upsilon\Omega_{i,n}^{m}|g_{m,n}^{S}|^2 < w\gamma_{i,n}$ where a decrease in $\gamma_{i,\,n}$ due to the CF must be compensated by a decrease in $\Omega_{i,n}^{m}$ so that this condition is met to activate a subcarrier. Therefore, it is evident in Fig. 7(a) that a higher payoff can be obtained by forming larger coalitions if $\Omega_{i,n}^{m}$ is reduced. More precisely, the SUs are motivated to form larger coalitions if the reduction in $\Omega_{i,n}^{m}$ is greater than the decline in $\gamma_{i,\,n}$ after CF. Moreover, it is demonstrated in Fig. 7(b) that SUs tend to form larger coalitions in the MCRN with a higher ITC. If the PUs can tolerate higher interference, the SUs are allowed to use higher power, which makes the subcarriers more affordable for the SUs when they form larger coalitions.

The effect of pricing coefficients on the performance of the MCGP is investigated in Fig. 8. It is noticed that the number of active subcarriers drops drastically when $\Omega_{i,n}^{m}$ increases, this is because the SUs could not afford to obtain subcarriers from the PUs. Correspondingly, the net payoffs of the SUs exhibit a similar exponential declining trend where the SUs suffer from payoff degradation due to the acquisition of more inactive subcarriers. For successful spectrum leasing, coalitions should be formed on active subcarriers where $\upsilon\Omega_{i,n}^{m}|g_{m,n}^{S}|^2 < w\gamma_{i,n}$ so that a positive net payoff can be obtained by the SUs from this leasing. Notably, it is observed in Fig. 8 that no active subcarriers are available when $\Omega_{i,n}^{m} = 13$ and $\Omega_{i,n}^{m} = 17$ for the systems with $d = 0$ km and $d = 2$ km, respectively. For $d = 0$ km, the high interference gains between PUs and SUs make the condition $\upsilon\Omega_{i,n}^{m}|g_{m,n}^{S}|^2 < w\gamma_{i,n}$ more stringent due to the rise in $|g_{m,n}^{S}|^2$ and the decrease in $\gamma_{i,\,n}$. This reduces the purchasing power of the SUs and therefore no sub-

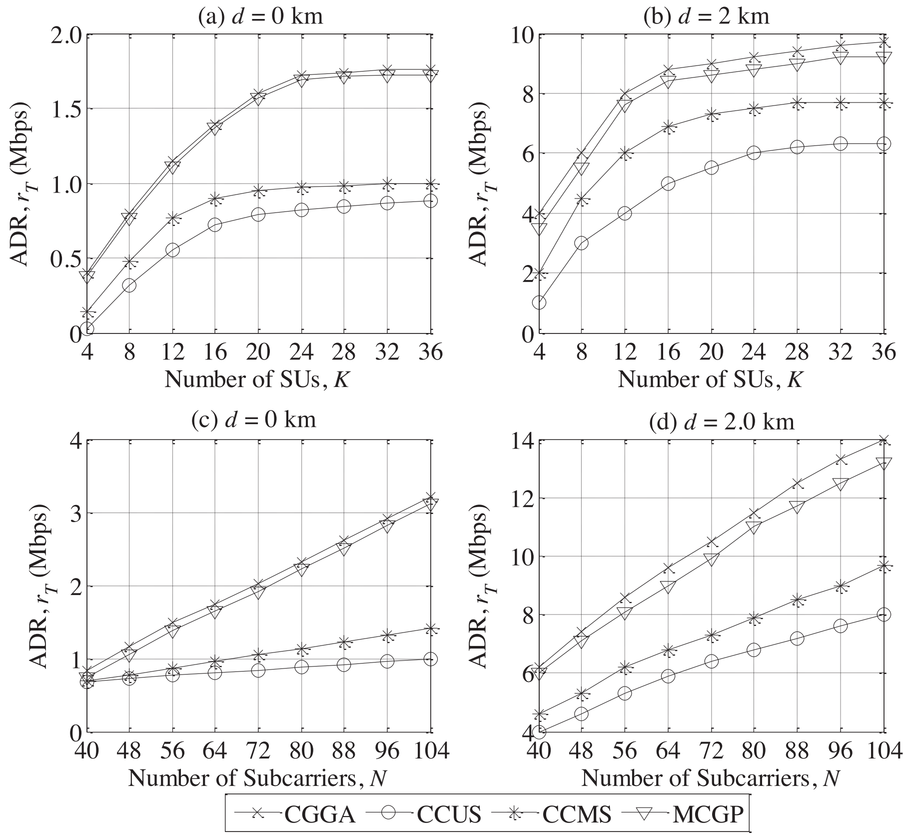

Fig. 10. Impact of pricing coefficients on the total net payoff, total profit, power of SUs and the resulting interference on PUs.

carrier can be afforded at $\Omega_{i,n}^m \geq 13$. Any leasing by the SUs beyond the price of $\Omega_{i,n}^m = 13$ will violate the condition conjectured in Proposition 2, which results in $p_{i,n} \leq 0$ and $\pi_n^S \leq 0$. On the other hand, the buying power of the SUs is improved in the system with $d = 2$ km where a price up to $\Omega_{i,n}^m = 17$ can be afforded to acquire active subcarriers, mainly due to the lower interference gains attained.

In Fig. 9(a), the average power allocated to each coalition is shown to be inversely proportional to $\Omega_{i,n}^m$, which implies that the SUs are restricted from using high power to receive on high-priced subcarriers so that the resulting interference is within the ITC. It is demonstrated in Fig. 9(b) that the average interference also exhibits a decreasing trend which corresponds to the power curves in Fig. 9(a). For a fixed $\Omega_{i,n}^m$, a higher power can be used for a system with larger d due to the lower interference gains but this also creates higher interference to the PRN. Fig. 9(c) and (d) verify Propositions 5 and 7 that the payoff of SUs and the profit of PUs decrease with $\Omega_{i,n}^m$. From Fig. 9, the maximal payoff and profit can be obtained by finding the optimal $\Omega_{i,n}^m$ subject to the ITC. For instance, if the ITC is $I_{th} = -110dBm$, the optimal pricing coefficient is found to be $\Omega_{i,n}^m = 8$, which gives a total net payoff of 1.5×10^5 and a total profit of 8×10^4 for the system with $d = 2$ km.

Next, Fig. 10 shows the comparison between the CGGA, CCUS, CCMS and MCGP in terms of ADR for systems with different values of K, N and d. In Fig. 10(a) where $d = 0$ km, the MCGP performs similar as the CGGA. When $K > 24$, the MCGP can attain an improvement of 44% and 55% compared to the CCMS and CCUS, respectively. When $d = 2$ km, the multicast environment becomes less hostile with lower interference gains. Therefore, the performance loss of the CCUS and CCMS compared to the MCGP reduces. It is shown in Fig. 10(b) that the MCGP still achieves 19% and 32% improvement over the CCMS and CCUS, respectively. In this simulation, the system capacity is saturated when $K > 24$ due to limited subcarriers ($N = 64$) and the constraint of $\gamma_{i,n}$. In Fig. 10(c) and (d), it is noted that the MCGP is more spectrally efficient than the CCUS and CCMS. When N increases, the performance gap between the MCGP and CCMS increases, implying that the MCGP can exploit multiuser diversity more efficiently. In general, the MCGP shows near-optimal performance as compared to the CGGA because the price of anarchy [23] of the MCGP always approximates the value of 1 in all the cases shown in Fig. 10.

Finally, Fig. 11 compares the CGGA, CCUS, CCMS and MCGP in terms of ADR as a function of ITC. It is shown in Fig. 11(a) that the MCGP achieves a substantial improvement of 42% and 53% in terms of ADR compared to the CCMS and CCUS, respectively. In the system with d=0 km, the CCMS which clusters all SUs in one group may acquire many inactive subcarriers due to a very low $\gamma_{i,n}$, resulting in a low ADR. Unlike the CCMS, the MCGP adaptively partitions the SUs into smaller coalitions so that the subcarriers are more affordable to every SU, thus preventing inactive subcarriers from entering the coalitions and hence increasing the ADR. Particularly, the performance gap between the CCMS and MCGP becomes smaller as d increases because the CCMS is now more efficient as the interference gains have become smaller. However, the MCGP still outperforms the CCMS because the SUs are allowed to form coalitions and adjust their received power based on the pricing coefficients of PUs. In Fig. 11(a)–(d), the ADR curves of all four schemes saturate when the ITC is sufficiently large. When $I_{th} > -90dBm$, the ADRs of all schemes remain almost unchanged because the ITC has become an inactive constraint.

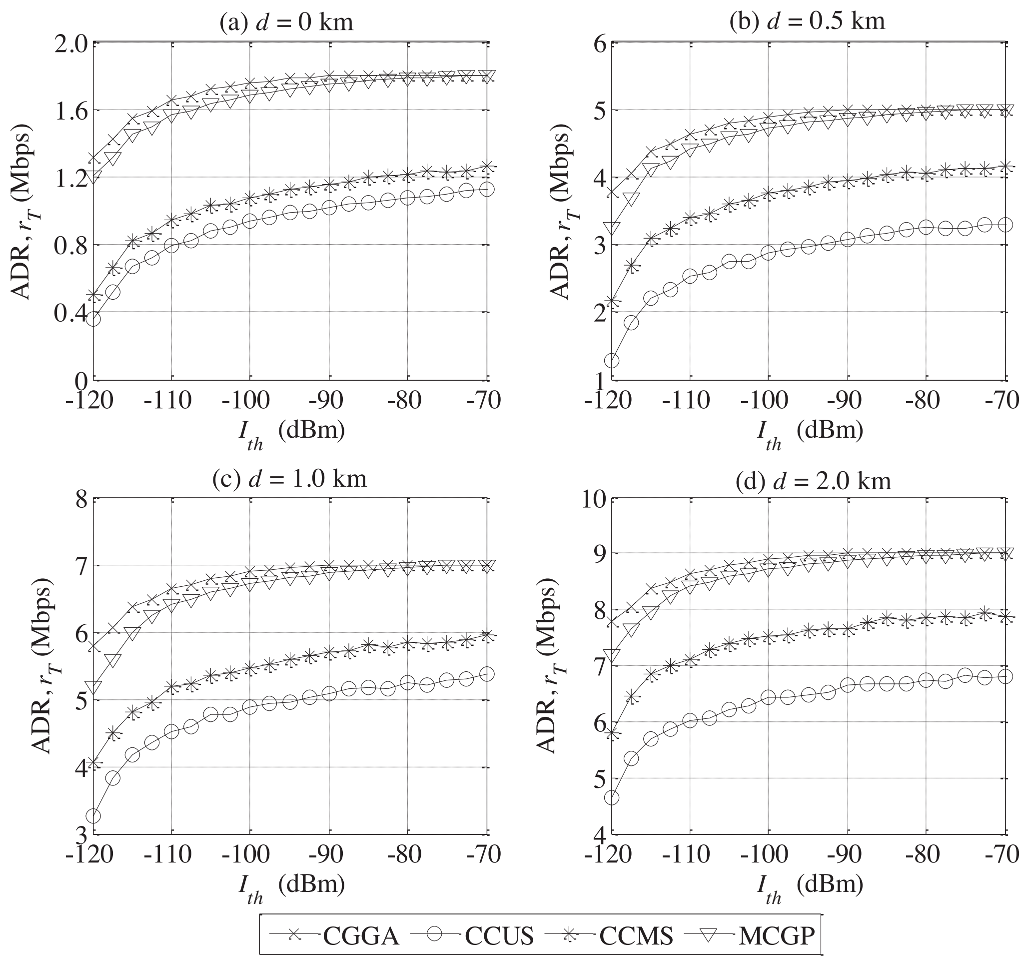

Fig. 11. Performance comparison of the MCGP with the CGGA, CCUS and CCMS in terms of ADR as a function of ITC for different values of *d*.

6. Conclusion

In this paper, an efficient spectrum leasing framework is proposed to model spectrum sharing between PUs and SUs in an OFDMA-based multicast cognitive radio network. An adaptive pricing mechanism is introduced for the PUs to regulate the CF and SPA among SUs so that the interference created by SUs is below the ITC. The interaction between PUs and SUs is modelled using the CSGP where PUs who are the leaders compete among themselves in the NPAG to lease their subcarriers to the SUs. Being the followers, the SUs cooperatively form coalitions in the MCGP to acquire more subcarriers at minimal costs. The pricing strategies of the PUs play a vital role in deciding the CF and SPA. It is noticed that the PUs tend to reduce the pricing coefficients if the ITC allows it because this move can improve the total profit. Besides, lower pricing coefficients also improve the net payoff of the SUs by encouraging them to form larger coalitions and use higher received power. Finally, simulation results show that the MCGP significantly outperforms the conventional schemes particularly in systems with high interference gains.

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