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A novel fiber optimization method based on normal distribution function with continuously varying fiber path

C.Y. Kiyono [a,b,*], E.C.N. Silva [a], J.N. Reddy [b]

[a] Department of Mechatronics and Mechanical Systems Engineering, Politechnique School of University of São Paulo, Avenida Professor Mello Moraes, 2231, 05508-030 São Paulo, SP, Brazil

[b] Department of Mechanical Engineering, Texas A&M University, College Station, TX 77843-3123, USA

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ABSTRACT

Tailoring fiber orientation has been a very interesting approach to improve the efficiency of composite structures. For the discrete angle selection approach, previous methods use formulations that requires many variables, increasing the computational cost, and they cannot guarantee total fiber convergence (which is the selection of only one candidate angle). This paper proposes a novel fiber orientation optimization method based on the optimized selection of discrete angles, commonly used to avoid the multiple local minima problem found in fiber orientation optimization methods that consider the fiber angle as the design variable. The proposed method uses the normal distribution function as the angle selection function, which requires only one variable to select the optimized angle among any number of discrete candidate angles. By adjusting a parameter in the normal distribution function, total fiber convergence can be achieved. In addition, a usual problem in fiber angle optimization methods is that because fibers can be arbitrarily oriented, structural problems may exist at the intersection of discontinuous fiber paths. Besides, composite manufacturing technologies, such as Advanced Fiber Placement (AFP), produce better results when fiber paths are continuous. These problems can be avoided by considering continuously varying fiber paths. In the proposed method, fiber continuity is also achieved by using a spatial filter, which improves the fiber path and avoids structural problems. Numerical examples are presented to illustrate the proposed method.

1. Introduction

The increasing availability of composite materials, accessibility to novel manufacturing techniques and novel design, and optimization techniques for these materials have increased the interest of the application of composite materials in structural panels and shell structures for many engineering applications. The optimization concept applied to composite materials allows finding the optimized geometric contours of the laminate, the optimized material distribution, the optimized orientation of fiber paths, as well as optimized mapping regions for the insertion of additional layers of material aimed, for example, to increase the strength of regions with distribution of intense loads. This work is focused on the optimization of the fiber path orientation and thus, only related works are considered.

In the literature, there is no unanimity on the best method for optimization of fiber angles. There are different applicable techniques. The same generally fall in: indirect parameterization related to the use of lamination parameters where intermediate variables are used and lamination sequence is available after a post-processing step [1–4]; and direct parameterization in which the physical description of the laminate is explicit [5–10]. While indirect parameterization imposes difficulties to consider manufacturing constraints, direct parameterization introduces local minima to the design domain. Within the indirect parameterization techniques, lamination parameters introduced by Tsai and Hahn [11] are often used. Thus, Liu and Haftka [1] discuss buckling load maximization of composite panels without stiffeners by using lamination parameters as continuous design variables for fiber angle values equal to 0°, ±45°, and 90°. In another work, IJsselmuiden et al. [2] study the incorporation of Tsai–Wu failure criterion within the solution space of the lamination parameters. Recently, Bohrer et al. [3] explored the use of lamination parameters in the optimization of composite plates subjected to buckling and small mass impact, and Faria [4] proposed a new optimization

* Corresponding author at: Department of Mechatronics and Mechanical Systems Engineering, Politechnique School of University of São Paulo, Avenida Professor Mello Moraes, 2231, 05508-030 São Paulo, SP, Brazil.

E-mail address: ckiyono@usp.br (C.Y. Kiyono).

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technique using lamination parameters to optimize composite structures under buckling subjected to multiple load cases. Although techniques based on lamination parameters can be successfully used in different applications, its use makes difficult to impose manufacturing constraints on the fiber angles, since they do not generate a direct description of the stacking plies, or its sequence.

In later years, the allowable fiber angles have been restricted to a finite set of values 0°, $\pm 45^\circ$, and 90° due to manufacturing. Thus, heuristic optimization algorithms can be considered given the discrete nature of the problem and the presence of multiple local minima. Following this idea, Gürdal et al. [12] employs genetic algorithms for determining the optimized fiber angle. Other studies [13–15] use genetic algorithms to optimize fiber orientations and thicknesses of laminated composite structures to minimize the weighted sum of mass and the deformation of the laminated structure subjected to a Tsai–Wu failure criterion [16]. However, Sigmund [17] questions the advantage of using such algorithms in topology optimization problems. The main advantages arising from the use of genetic algorithm are the fact it is able to find a "global minimum" do not need the gradient calculation, and allow handling discrete variables, however, this comes with a high computational cost. Sigmund then compares the use of the genetic algorithm with an algorithm based on gradients from the point of view of the topology optimization, showing the inefficiency of algorithms not based on gradients in optimization problems with many design variables. Another difficulty of the heuristic algorithms (e.g., genetic algorithms) is the way they deal with constraints. The inclusion of constraints on the problem is solved indirectly by verifying them at each iteration, or by changing the definition of the objective function by adding penalization functions that require the determination of semi-empirical coefficients, making it difficult to use.

Gradient-based optimization methods have also been applied to the optimization of fiber angles. Mota Soares et al. [5] presents the optimization problem whose design variables are the fiber angles and the thickness by using analytical sensitivities. Several objective function combinations and constraints are considered such as displacement, structural strain energy, material volume, natural frequency, buckling load and a specific mechanical stress criteria for composite materials based on the Tsai–Hill criterion. In other interesting work, optimization of reinforcements is performed analogously to fiber angles optimization [6]. In an attempt to make the solution of laminated structure optimization problem more efficient, Bruyneel and Fleury [18] present the optimization of composite structures by using sequential convex programming where they discuss the use of the family of MMA algorithms (Method of Moving Asymptotes). The technique is efficient for optimizing composite structures when the thickness and fiber orientation are simultaneously considered as design variables. In sequence, other works propose to optimize the fiber orientation of laminated composite material to alter the resonance frequencies of the composite panels by taking into account the maximum load capacity of fiber [19–22].

However, to optimize the fiber orientation by using angle values as design variables is a problem that has many local minima and it is highly sensitive with the initial fiber configuration, making it difficult to obtain the optimized solution. Thus, Stegmann and Lund [7] suggest obtaining the optimized angles through an optimization approach based on a material model formed by combining multiple elasticity tensors considering different fiber orientations, and by using an optimization algorithm based on gradients and a penalization coefficient to force solutions with only one angle at each element. This method is called DMO (Discrete Material Optimization). Following, Lund [8] extends this topology optimization formulation for optimization of laminated composite plates and shells structures with respect to linear buckling load subjected to mass constraints. The work of Stegmann and Lund [7] gave rise to a series of works that aims to perform optimization by considering discrete values of angles, which appear as alternatives to the DMO method. Thus, there is a method of parameterization of mechanical properties called "Shape Functions with Penalization" (SFP) [9] which is simpler than the DMO and employs a smaller number of design variables for selecting the optimized laminate orientations with convergence speed and quality comparable to the results obtained by the DMO. However, it considers only laminates with fiber angles equal to 0°, $\pm 45^\circ$, and 90°, typically found in aeronautical applications, for example [9,23]. Gao et al. [10] proposes a parameterization method for the selection problem of fiber orientation called "Bi-Value Coding Parameterization" (BCP). This method generalizes the concept of the SFP shape functions, making it possible to consider in the optimization a large number of allowable discrete orientations or different material candidates, with a number of design variables substantially smaller than DMO, for example. Following, Sørensen et al. [24] present an alternative method called "Discrete Material and Thickness Optimization" (DMTO) which allows the directly optimization of the orientation of the material as well as the thickness of each ply. It employs a parameterization by using interpolation functions with penalties and it is tested in optimization problems involving mass minimization subjected to constraints such as buckling load factors, limited displacement, among others. However, all these discrete material optimization methods suffer from a common issue: the fiber convergence cannot be guaranteed. Because of the nature of their formulations, there is the possibility that the final elasticity tensor is still a mixture of one or more fiber angles, even if applying a large penalization coefficient value. The fiber angle optimization problem has some similarities with the multi-material optimization problem in the sense in the latter the optimization searches for a material property among a set of available discrete property values. In a interesting work, Yin and Ananthasuresh [25] have applied the normal distribution function to the multi-material topology optimization problem for optimal selection of different isotropic materials. They proposed a continuation approach on a penalization coefficient and they guarantee the selection of only one material at the end of the optimization procedure. However, their formulation can generate artificially high stiffness materials during the optimization process, which can possibly lead to an undesired local minimum. Nevertheless, this approach can be used to fiber orientation optimization.

Therefore these previous studies show that there is still room for improvement in optimization techniques applied to composite laminates including the optimization of the fiber orientation considering discrete materials and using gradients algorithms. Finally, in particular, a problem that exists in the fiber orientation optimization is to ensure the continuity of such orientation along the laminate. Some studies in the literature applying fiber orientation optimization have results with fiber discontinuity among finite elements [7–10], which not only makes difficult the post-processing and manufacturing of these designs, but also they cause stress concentrations which makes difficult the convergence in stress minimization problems. Some approaches have been proposed to achieve fiber continuity [26–28], however, they had limitations and disadvantages [29]. Recently the level-set method was successfully applied to address the fiber continuity [29], however, this method shows a great sensitivity with the initial fiber configuration. Brampton et al. [29] propose to solve this problem by using the level set topology optimization solution for isotropic material as the initial fiber configuration, and thus, additional work needs to be done to achieve good fiber orientation solutions with the level set method.

Thus, this paper proposes a novel fiber orientation optimization formulation applied to optimization of composite laminates, considering the optimized selection of discrete fiber angles defined at each element within the mesh. The proposed method is based on the normal distribution function, which has the advantage of using only one variable to select the optimized discrete angle among any number of candidates, can achieve total fiber convergence (unlike the previous discrete material optimization methods), has low sensitivity to the initial fiber configuration, and it includes a technique that ensures the continuity of the fiber orientation. In this work, a similar continuation approach suggested by Yin and Ananthasuresh [25] for the normal distribution function approach is used, and in addition, a normalization function is applied to avoid artificial high stiffness materials faced by their formulation.

This paper is organized as follows. In Section 2, a brief description on the discrete fiber angle optimization methods is presented. In Section 3, the proposed method is explained. In Section 4, the optimization problem is formulated and in Section 5, the sensitivity analysis is presented. In Section 6, it is presented the filtering technique to achieve fiber continuity. In Section 7, some numerical examples are shown and finally, in Section 8, the concluding remarks are inferred.

2. Discrete fiber angle optimization

The DMO (Discrete Material Optimization) [7,8], SFP (Shape Functions with Penalization) [9], and BCP (Bi-value Coding Parametrization) [10] are the most recent optimization methods for optimized discrete fiber angle selection. These methods were proposed as alternatives to the CFAO (Continuous Fiber Angle Optimization) which is known to present the multiple local minima problem where the optimized solution is highly dependent on the initial fiber configuration [7]. The basic concept of these discrete angle methods is to define different candidates and to calculate the effective elastic tensor $\mathbf{C}_e$ as a weighted sum of the elastic tensors $\mathbf{C}_i$ of these candidates, such as

$$\mathbf{C}_e = \sum_{i=1}^{n_c} w_i \mathbf{C}_i \tag{1}$$

where w_i are the weighting functions and n_c is the total number of candidates. In order to produce realistic results regarding physical properties, two conditions must be satisfied [7]

$$0 \leqslant w_i \leq 1 \tag{2}$$

$$\sum_{i=1}^{n_c} w_i = 1 \tag{3}$$

However, in their work, the reason that this approach can overcome the local minima problem is that, at the beginning of the optimization process, all w_i have the same value, and $\mathbf{C}_e$ is a mixture of all candidates. The main objective of these methods is to drive only one w_i to 1, while the other weighting functions must be equal to 0, so that only one candidate is chosen and the effective elastic tensor is equal to the elastic tensor of this candidate. This is the definition of "fiber convergence" in the case of discrete fiber angle optimization, where the candidates are the discrete angles θ_i defined a priori, and $\mathbf{C}_i$ is the rotated elastic tensor of the orthotropic material used. This method can also be applied to the discrete material selection problem, where $\mathbf{C}_i$ is the elastic tensor for each material. The difference among these methods is how to parameterize the weighting functions w_i. For the DMO method, w_i can be written as [7,8]

$$w_i = \frac{\hat{w}_i}{\sum_{k=1}^{n_c} \hat{w}_k}, \quad \text{and} \quad \hat{w}_i = (\vartheta_i)^{p_\theta} \prod_{\substack{j=1 \\ j \neq i}}^{n_c} \left(1 - (\vartheta_j)^{p_\theta}\right) \qquad 0 \leqslant \vartheta_i \leq 1 \tag{4}$$

where ϑ_i is the orientation design variable associated to each candidate angle i, and p_θ is the penalization coefficient used to drive the orientation variables towards 0 or 1 in order to achieve fiber convergence. The normalization in Eq. (4) is defined so that $\sum w_i = 1$. The fiber convergence χ can be measured by the DMO convergence measure proposed by Stegmann and Lund [7], which is essentially the Euclidean norm of the weighting functions times a tolerance level ϵ

$$\chi = \epsilon \sqrt{w_1^2 + w_2^2 + \cdots + w_{n_c}^2} \tag{5}$$

where ϵ is typically 95–99.5%. Thus, if one of the weighting functions w_i is greater or equal to χ, it can be assumed that the method has converged and only one angle has been chosen. It is important to mention that Eqs. (1), (4) and (5) are calculated at each element of the finite element mesh. Although this method is very robust and can achieve good results for fiber angle optimization, there are two main drawbacks. First, even with large values of p_θ, fiber convergence can not be guaranteed, especially at low stress regions because the sensitivity of the objective function with respect to the variables can be very low and no particular angle is chosen. Additionally, depending on the loading condition, there may be the case where a combination of two angles is the optimized solution, which can not be avoided by the DMO method. Second, because this method does not have any mathematical restrictions on the number of candidates that can be used, the total number of orientation variables can be too large, since one variable must be associated to one candidate at each element of the domain. Assuming that n_v is the number of orientation variables for an element, the total number of orientation variables is equal to $N_v = N_e \times n_v$, where N_e is the total number of elements. Thus, the method can have a high computational cost, especially at the sensitivity analysis. To reduce the total number of orientation variables, Stegmann and Lund [7] proposed the patch approach, where some elements are grouped and they share the same effective elastic tensor $\mathbf{C}_e$ and consequently, the same orientation variables. However, they mention that the solution is dependent on the shape of the patches and also requires some expertise of the engineer. Stegmann and Lund [7] have also used the DMO method for combining fiber angle optimization with topology optimization by adding an isotropic polymeric foam material as a candidate representing the void region. They mention that a mass constraint has been applied, however, it is not clear how it is calculated.

In an attempt to improve the discrete fiber angle optimization method, Bruyneel [9] proposed the SFP method that defines the weighting functions as the shape functions from the FEM. The method is presented only for 4 candidates requiring 2 orientation variables per element

$$\begin{aligned} w_1 &= \left[\tfrac{1}{4}(1-\vartheta_1)(1-\vartheta_2)\right]^{p_\theta} \quad w_2 = \left[\tfrac{1}{4}(1+\vartheta_1)(1-\vartheta_2)\right]^{p_\theta} \\ w_3 &= \left[\tfrac{1}{4}(1+\vartheta_1)(1+\vartheta_2)\right]^{p_\theta} \quad w_4 = \left[\tfrac{1}{4}(1-\vartheta_1)(1+\vartheta_2)\right]^{p_\theta} \\ -1 &\leqslant \vartheta_i \leqslant 1 \end{aligned} \tag{6}$$

The size reduction of the optimization problem is promptly noticed, $n_c = 4,\ n_v = 2$. The SFP method can certainly be extended to more than four candidates if other existing finite element shape functions with more nodes are used. Bruyneel [9] has also applied topology optimization to some examples, however, they have used the SIMP method to model the material distribution, since the number of candidates is limited. However, this method also has some drawbacks. If the orientation variables have not yet converged towards their limits, and $p_\theta \neq 1$, then the condition of

Eq. (3) is not satisfied, yielding to non-physical properties. The number of candidates is restricted and must match the number of shape functions used. Higher-order FEM shape functions can have negative values, which is a major problem. Thus, only shape functions of first order elements must be used in order to satisfy condition of Eq. (2). Additionally, if more than 8 candidates are required, more complex shape functions must be formulated to follow the number of candidates. Later, Gao et al. [10] proposed the BCP method as a generalization of the SFP method:

$$w_i = \left[\frac{1}{2^{n_v}}\prod_{k=1}^{n_v}(1 + s_{ik}\vartheta_k)\right]^{p_\theta} \tag{7}$$

where s_{ik} can be calculated as [10]

$$s_{ik} = \begin{cases} 1 & i \in [1, 2^{k-1}] \\ -1 & i \in [2^{k-1}, 2^k] \\ s_{\xi k} & i \in [2^k + 1, 2^{n_v}] \quad \text{where } \xi = 2^{\lceil \log_2 i \rceil} + 1 - i \end{cases} \tag{8}$$

The number of orientation variables per element is equal to $n_v = \lceil \log_2 n_c \rceil$. As indicated by Gao et al. [10], the BCP scheme can interpolate $n_c = \left[2^{(n_v-1)} + 1, 2^{n_v}\right]$ material candidates with n_v orientation variables. However, Eq. (7) must be used to interpolate 2^{n_v} possible candidates, which means that $i = 1, \ldots, 2^{n_v}$. If the actual number of candidates is less than possible number of candidates, $n_c < 2^{n_v}$, then some other material (possibly a void phase) must be defined to fill up the vacant candidates, and constraints must be applied to the orientation variables so that these additional candidates are not chosen. Thus, strictly speaking, $n_c = 2^{n_v}$, which is equivalent to the SFP method in the matter of number of candidates and orientation variables [30]. Additionally, the BCP method presents the same convergence problem as mentioned for the DMO and SFP methods, resulting in regions with mixed materials.

Regardless their efficiency of achieving good results, these 3 methods share the same problems:

- no guarantee of total fiber convergence;
- the number of candidates is related to the number of variables;
- fiber continuity can not be achieved by any known means so far, and a fiber continuity constraint would be too complicated to formulate and implement.

Another more recent method for fiber path optimization has been proposed by Brampton et al. [29] which is based on the level-set method. The fiber paths are defined by the contour lines of a level-set function, and then, these paths are projected as individual fiber angles for each element within the mesh. This method does not suffer from the fiber convergence problem since the angle is already defined by the level-set function. The fiber continuity is also guaranteed because the fibers follow the contour lines. However, the solution of this method is dependent on the initial fiber configuration, as demonstrated by authors [29].

3. Proposed method – NDFO

The fiber optimization method we propose in this work is based on previous discrete methods in the sense that one discrete angle must be chosen within a set of candidates. The main difference is in the parametrization of the weighting functions w_i. Our method has the following advantages over the mentioned methods:

- only ONE orientation variable is associated to ANY number of candidates;
- total fiber convergence can be achieved;
- a filtering technique can be easily implemented to achieve fiber continuity;
- the formulation is straightforward to be implemented.

In order to associate only one orientation variable to any number of candidates, the normal distribution function is used as a parametrization of the weighting functions. The normal distribution function is given by [25]

$$f(\vartheta|x,\sigma) = e^{-\frac{(\vartheta-x)^2}{2\sigma^2}} \tag{9}$$

By changing the parameters σ and x, we can control the width of the curve and the location of the peak, respectively, as shown by the graphs of Fig. 1 Thus, to use the normal distribution function as the weighting function, there are three aspects that need to be addressed. The first aspect is that conditions of Eqs. (2) and (3) must be satisfied. By analyzing Fig. 1 and Eq. (9), only condition of Eq. (2) is satisfied. Failing to satisfy condition of Eq. (3) can produce artificially high stiffness materials during the optimization and can lead to a bad solution [8]. Thus, to satisfy condition of Eq. (3), we propose to use the normalization scheme proposed by Stegmann and Lund [7]

$$w_i = \frac{\hat{w}_i}{\sum_{k=1}^{n_c}\hat{w}_k} \tag{10}$$

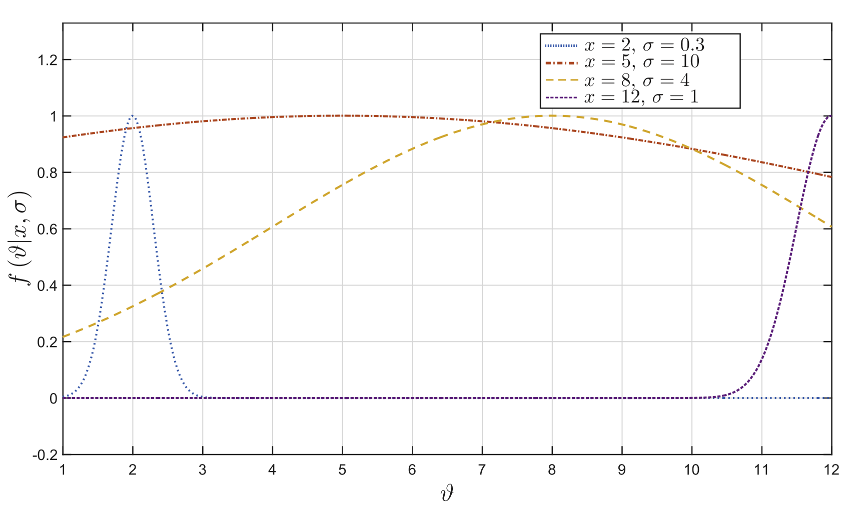


Fig. 1. Normal distribution curves.

Table 1
Influence of σ on the values of w_i.

	$p_\theta = 10$		$p_\theta = 4$		$p_\theta = 0.3$	
	$\vartheta = 2$	$\vartheta = 5$	$\vartheta = 2$	$\vartheta = 5$	$\vartheta = 2$	$\vartheta = 5$
w_1	0.202	0.190	0.212	0.144	0.004	0.000
w_2	0.203	0.197	0.218	0.179	0.992	0.000
w_3	0.202	0.202	0.212	0.210	0.004	0.000
w_4	0.199	0.205	0.193	0.230	0.000	0.004
w_5	0.194	0.206	0.165	0.237	0.000	0.996

The second aspect is to associate the orientation variable to the candidate angle (or material). By defining ϑ as the continuous orientation variable, and i being the integer values of ϑ as the peak locations to denote each candidate i, the weighting function $\hat{w}_i$ can be written as

$$\hat{w}_i = e^{-\frac{(\vartheta - i)^2}{2p_\theta^2}} \quad \text{where } i = 1, \ldots, n_c \tag{11}$$

Finally, the third aspect is the value of σ, which has been replaced by p_θ because it works as a penalization coefficient to control the width of the curve, in order to achieve fiber convergence. For large values, for instance $p_\theta = 10$, the normal function has a low curvature, as it can be seen in Fig. 1. This means that all $\hat{w}_i$, and consequently all w_i, will have close values, and the effective elastic tensor $\mathbf{C}_e$ is a mixture of all candidates. On the other hand, the smaller the value of p_θ, the values of w_i will be more discrete (0 or 1), as represented by the blue line of Fig. 1. Table 1 shows the influence of p_θ on the values of w_i (Eq. (10)) for $i = 1, \ldots, 5$ and two values of ϑ. It can be seen that for $p_\theta = 4$ and $p_\theta = 10$, the values of w_i are very similar around $i = \vartheta$, and for $p_\theta = 0.3$ almost discrete values are achieved for w_i.

Thus, the value of p_θ is very important to avoid the local minima problem and to achieve fiber convergence. We propose applying the continuation scheme on the penalization coefficient, starting p_θ equal to 4 and reducing it gradually until p_θ^{min}. From our tests, p_θ^{min} shows a great influence in achieving total fiber convergence. As mentioned before, the orientation variable varies continuously from 1 through n_c. If ϑ is somewhere between i and $i+1$, the value of p_θ^{min} will dictate whether w_i is discrete or not. However, there is a major problem when ϑ is exactly in the middle of two adjacent integers, $\vartheta = [i + (i+1)]/2$. In this very particular case, no matter the value of p_θ^{min}, their weighting functions will have the same value, $w_i = w_{i+1}$, which characterizes as a mixture of these two candidates, and thus, the proposed method fails to achieve fiber convergence. However, we rely on the fact that the probability that this problem happens is almost zero because of the computation approximations and a very large computer precision (large number of digits). Even if $\vartheta = [i + (i+1)]/2$, it is proposed to manually add a small perturbation to the variable to avoid the singularity. To illustrate the influence of p_θ^{min}, consider a problem with $n_c = 5$ and $\vartheta = 3.5001$ (this value has been chosen because it is very close to the middle of two integers). Fig. 2 shows the graphs of w_i for four different values of p_θ^{min}. It can be seen that the smaller the value of p_θ^{min}, the value of w_i approaches discrete values, 0 or 1. However, if p_θ^{min} is too small, the exponential function can be equal to zero regardless the value of ϑ. Consequently, all $\hat{w}_i$ are also equal to zero and there will be a numerical problem in Eq. (10) (division by zero). The value $p_\theta^{min} = 0.012953$ has been found empirically through exhaustive tests. However, the limit value of p_θ^{min} where the exponential function becomes zero cannot be stipulated because it depends on the precision of the computer.

Because the weighting functions are based on the normal distribution function, we named this method NDFO, which stands for Normal Distribution Fiber Optimization. Although we tested only for fiber angle optimization, this method can be applied to any multi-material optimization problem too.

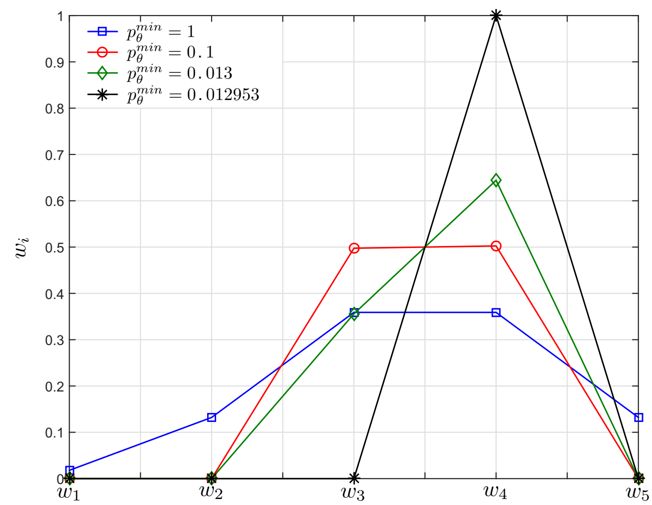


Fig. 2. Values of w_i for $n_c = 5$ and $\vartheta = 3.5001$ and different values of p_θ^{min}.

4. Optimization problem formulation

The response of the structure is obtained by employing the finite element method and assuming linear elasticity, however, the proposed method can be applied to non-linear problems as well. To illustrate the proposed method, the eight node laminated shell element is used to model thin structures made of orthotropic materials. This element is based on the degenerated three-dimensional solid approach and first-order shell theory kinematics [31,32], with 5 degrees of freedom per node (3 translations and 2 rotations) and selective integration to avoid shear locking. A representation of the element is presented in Fig. 3. However, because the proposed method only affects the effective elastic tensor $\mathbf{C}_e$, any element type that considers the use of orthotropic materials can be used. At each element is defined one orientation variable ϑ^e, which must represent only one fiber angle by the end of the optimization process by using the proposed method. This angle is assumed to be constant within the entire element. Thus, the element stiffness matrix $\mathbf{K}^e$, which is dependent on ϑ^e, can be obtained as

$$\mathbf{K}^e(\vartheta^e) = \int \mathbf{B}^T \mathbf{C}_e(\vartheta^e) \mathbf{B} dV^e \tag{12}$$

where $\mathbf{B}$ is the strain–displacement matrix and $\mathbf{C}_e(\vartheta^e)$ is the effective elastic tensor obtained by Eq. (1). Eq. (12) refers to a single layer element to simplify the formulation presented further. For a multi-layer structure, there should be one variable for each layer within each element, and Eq. (12) is applied separately for each layer and then the resulting matrices are summed up to build the element stiffness matrix. By properly assembling the elements

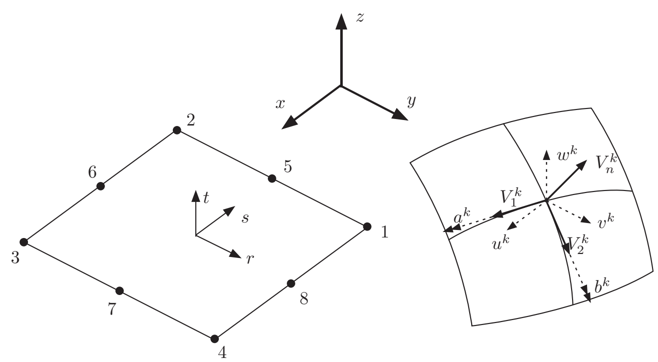


Fig. 3. Representation of the shell element, with natural (r, s, and t) and global (x, y, and z) coordinates, orthonormal basis ($V^k_{1,2,n}$), and degrees of freedom (u^k, v^k, w^k, a^k, and b^k.

stiffness matrices, the global stiffness matrix **K** is obtained and the static equilibrium equation can be written as

$$\mathbf{KU} = \mathbf{F} \tag{13}$$

where **U** and **F** are the global displacement and load vectors. It is implicit that **K** and **U** are functions of ϑ^e.

The objective function is to minimize the total compliance of a structure, $\mathcal{C}$, which can be defined as

$$\mathcal{C} = \mathbf{U}^T\mathbf{KU} = \mathbf{F}^T\mathbf{U} \tag{14}$$

Essentially, the compliance measures the sum of the displacements at the points where the loads are applied. Thus, the optimization problem formulation can be defined as

$$\begin{aligned} \text{Minimize}: \quad & \mathcal{C} = \mathbf{F}^T\mathbf{U} \\ \text{subject to}: \quad & \mathbf{KU} = \mathbf{F} \\ & 1 \leqslant \vartheta^e \leqslant n_c \end{aligned} \tag{15}$$

where the orientation variable can vary continuously from 1 through n_c, which is the number of discrete angle candidates. This problem is solved by the GCMMA optimization algorithm, developed and kindly provided by Prof. Kirster Svanberg [33]. The optimization procedure runs iteratively and follows the flowchart presented in Fig. 4. To update the orientation variables, the GCMMA algorithm requires the differentiation of the objective function with respect to the orientation variables, which is presented in the next section.

5. Sensitivity analysis

The differential of the compliance function with respect to the orientation variables for linear analysis is well known and it can be written as

$$\frac{\partial \mathcal{C}}{\partial \vartheta^e} = -\mathbf{U}^T \frac{\partial \mathbf{K}}{\partial \vartheta^e}\mathbf{U} \tag{16}$$

The differential of the stiffness matrix can be calculated element-wise, since each element has its own orientation variable. Thus, by differentiating Eq. (12) we have

$$\frac{\partial \mathbf{K}^e}{\partial \vartheta^e} = \int \mathbf{B}^T \frac{\partial \mathbf{C}_e}{\partial \vartheta^e}\mathbf{B} dV^e \tag{17}$$

where the differential of the effective elastic tensor $\mathbf{C}_e$ is obtained by differentiating Eq. (1), and then, Eqs. (10) and (11) as

$$\frac{\partial \mathbf{C}_e}{\partial \vartheta^e} = \sum_{i=1}^{n_c} \frac{\partial w_i}{\partial \vartheta^e}\mathbf{C}_i \tag{18}$$

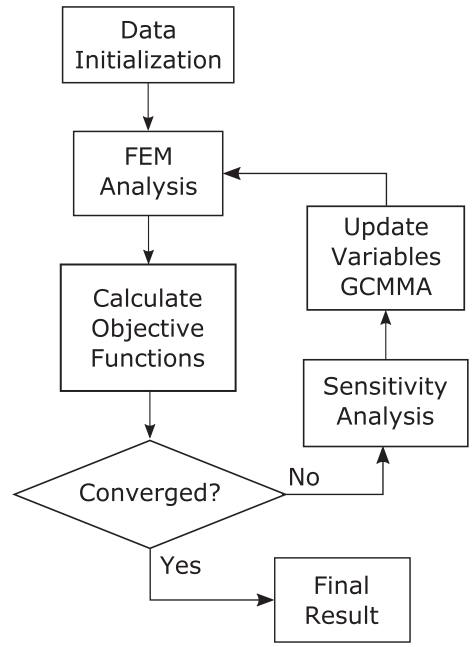


Fig. 4. Flowchart of the iterative optimization process.

$$\frac{\partial w_i}{\partial \vartheta^e} = \frac{1}{\left(\sum_{k=1}^{n_c} \hat{w}_k\right)^2}\left(\frac{\partial \hat{w}_i}{\partial \vartheta^e}\sum_{k=1}^{n_c}\hat{w}_k - \hat{w}_i\sum_{k=1}^{n_c}\frac{\partial \hat{w}_k}{\partial \vartheta^e}\right) \tag{19}$$

$$\frac{\partial \hat{w}_i}{\partial \vartheta^e} = -\frac{\vartheta^e - i}{p_\theta^2} e^{-\frac{(\vartheta^e - i)^2}{2p_\theta^2}} \quad \text{where } i = 1, \ldots, n_c \tag{20}$$

6. Achieving fiber continuity

As mentioned before, fiber continuity is important not only for manufacturing issues, but also to avoid stress concentrations at discontinuous paths. The proposed NDFO method with fiber continuity is represented by NDFO-C. The proposed filter for fiber continuity is a spatial filter based on the projection technique [34,35]. Besides the orientation variables ϑ^e already defined for each element in the mesh, another set of variables must be defined. This new set of variables are called design variables ϕ and they can be assigned anywhere within the mesh. In this work, for simplicity, the design variables are assigned to the centroid of each element, such as the orientation variables. Thus, the orientation variable is now defined as a function of the design variables around it $\vartheta^e = f(\phi_j)$, where $j \in \Omega^e$, and Ω^e is the projection sub-domain defined as a circle with radius r and centered at the centroid of element e. Fig. 5 illustrates the proposed filter.

Thus, we must define the projection function that calculates the orientation variables with respect to the design variables. Because the proposed method requires only one variable per element to describe the optimized angle, the proposed projection function can be a very simple function, such as the mean value of the design variables ϕ_j within Ω^e. The linear projection technique [34] has also been applied as the projection function, however, because it produced practically the same results comparing with the mean function, it was decided that this approach was not included in the manuscript. Thus,

$$\vartheta^e = \frac{\sum_{j\in\Omega^e}\phi_j}{n_j^e} \tag{21}$$

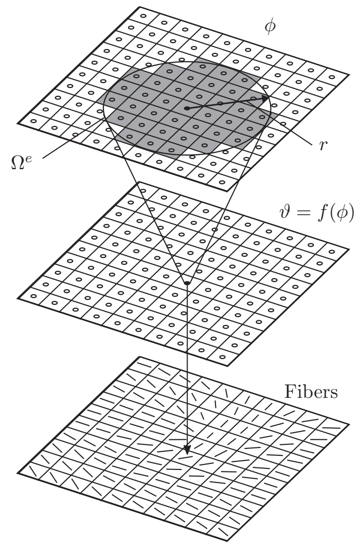


Fig. 5. Illustration of the filtering technique applied to the fibers to achieve fiber continuation.

where n_j^e is the total number of design variables within the subdomain Ω^e. Thus, every equation that is dependent on the orientation variables, is now dependent on the design variables. The sensitivity of the objective function with respect to the design variables can be calculated by the chain rule as

$$\frac{\partial \mathcal{C}}{\partial \phi_j} = \sum_{e \in \Omega_j} \frac{\partial \mathcal{C}}{\partial \vartheta^e} \frac{\partial \vartheta^e}{\partial \phi_j} \tag{22}$$

where $e \in \Omega_j$ means the orientation variables ϑ^e that have been affected by the design variable ϕ_j. Finally, by differentiating Eqs. (10) and (11)

$$\frac{\partial \vartheta^e}{\partial \phi_j} = \frac{\phi_j}{n_j^e} \tag{23}$$

7. Numerical examples

Numerical examples are presented to illustrate the potential of the NDFO method in 3 different cases: a rectangular plate with in-plane load; a square plate with transversal load; and a half-cylinder with vertical load. For each example, results are presented using the CFAO, the BCP, the NDFO, and the NDFO-C methods. The results with fiber continuity are shown with different filter radius to show that the level of continuity can be controlled.

In the CFAO method, the angle of each element θ^e is the variable. For the BCP and NDFO methods, there are 16 candidate angles, equally distributed,

$$\theta = [-78.75^\circ \ -67.5^\circ \ -56.25^\circ \ -45^\circ \ -33.75^\circ \ -22.50^\circ \ -11.25^\circ \ 0^\circ \ \ldots \text{symmetric} \ldots \ 90^\circ] \tag{24}$$

and thus, the BCP method requires 4 variables per element, whereas the NDFO method requires only one. The initial configuration of the design variables has great influence only in the CFAO method. As mentioned before, the BCP and NDFO methods start with a mixture of fiber angles and they will gradually converge to only one fiber angle. Thus, in the CFAO method, the initial angles are equal to $\theta^e = 0^\circ$ for all elements, in the BCP method, all variables starts equal to zero, and in the NDFO method the design variables are equal to $\phi_j = i$ where $\theta_i = 0^\circ$.

The following material properties have been used: $E_1 = 135$ GPa, $E_2 = E_3 = 10$ GPa, $\nu_{12} = \nu_{13} = 0.3$, $\nu_{23} = 0.5$, $G_{12} = G_{13} = 5$ GPa, and $G_{23} = 3$ GPa. For all cases, only one layer has been used, although any number of layers can be used with this method.

In the result figures, the fibers are represented by lines inside each element. The black line means that the fiber convergence has been achieved and there is only one angle in that element. On the other hand, the red lines (refer to the online version with colored figures) represent the non-converged fibers where there is still a mixture of two or more angles. However, red lines appear only for the BCP method. As mentioned before, for these methods, the penalization coefficient p_θ is responsible to improve the convergence of the fibers. For the BCP method, initially $p_\theta = 2$ and increases by unit every 10 iterations until $p_\theta = 8$. For the NDFO method, this coefficient starts equal to $p_\theta = 4$, and after iteration 10, it decreases by 0.1 every 2 iterations until $p_\theta = 0.1$, and then it decreases by 0.01 every 2 iterations until reaching a minimum value equal to $p_\theta = 0.012953$. This minimum value has been found empirically as demonstrated in Section 3.

7.1. Rectangular plate

The first numerical example is the rectangular domain presented in Fig. 6, where $L = 100$ mm. One side is fully clamped and the in-plane load equal to $F = 100$ N is distributed along 50 mm on the indicated region. This domain is modeled using 60x20 elements.

Fig. 7 shows the optimized solution by using the CFAO method. In this case, two results have been obtained by using different initial fiber configurations, $\theta_{ini}^e = 0^\circ$ and $\theta_{ini}^e = 45^\circ$. It can be seen that this method is indeed very susceptible to the local minima problem, where both results are totally different. Additionally, the con-

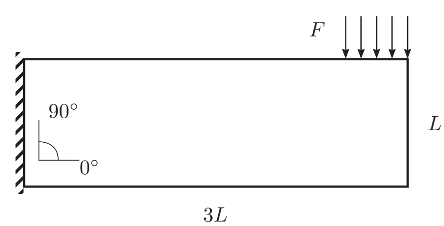


Fig. 6. Rectangular domain for example 1. Left side is clamped.

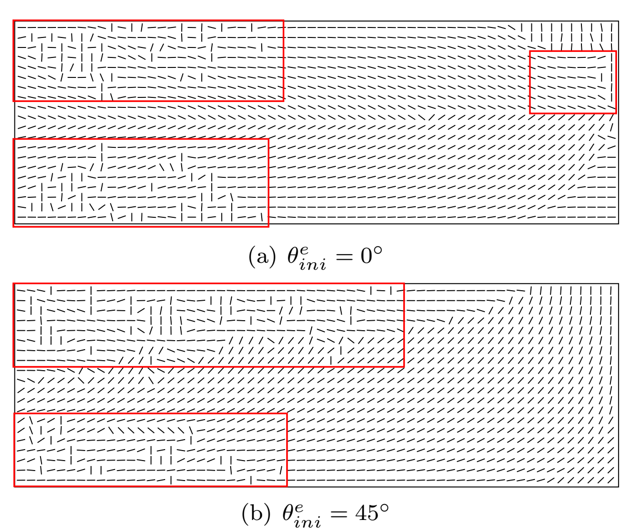
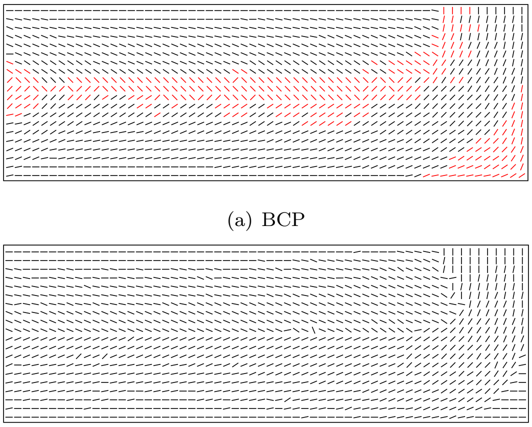


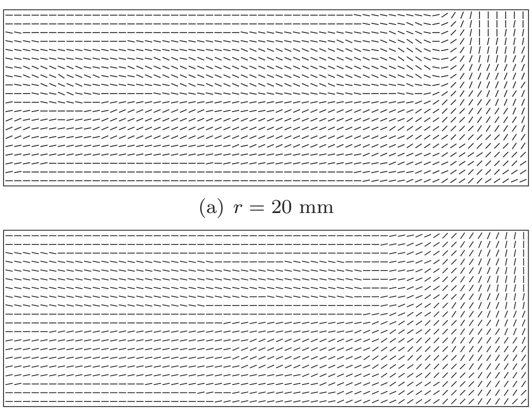
Fig. 7. Solutions for the CFAO method by using different initial fiber configurations. Highlighted areas with local minima problem.



(a) BCP

(b) NDFO

Fig. 8. Solutions for the BCP and NDFO methods. Black and red lines represent converged and non-converged fibers, respectively.



(a) $r = 20$ mm

(b) $r = 50$ mm

Fig. 9. Solutions for the NDFO-C method with two different filter radius.

figuration of the fibers at the highlighted areas also indicates that the solution is trapped in an undesired local minimum. The final compliance in these cases are equal to $\mathcal{C} = 3.49 \times 10^{-3}$ and $\mathcal{C} = 3.23 \times 10^{-3}$, respectively for $\theta^e_{ini} = 0^\circ$ and $\theta^e_{ini} = 45^\circ$.

The results obtained for the BCP and NDFO methods are presented in Fig. 8. These two results are very similar regarding the fibers orientation. In the BCP result, there are still many red lines (non-converged fibers), although the objective function has already converged. For the red lines at the center of the domain, the BCP method was probably looking for a combination of two or more fibers to make this region stiffer. For the red lines at the bottom right corner, the BCP method could not have decided which angle was better because it is a low sensitivity area, i.e., the fibers in that region does not contribute for the stiffness of the structure. The proposed NDFO method has fixed the fiber convergence problem by properly choosing only one angle for every element. The final compliance in these cases are equal to $\mathcal{C} = 2.01 \times 10^{-3}$ (BCP) and $\mathcal{C} = 1.85 \times 10^{-3}$ (NDFO), which are lower than the CFAO cases. The NDFO presents the minimum compliance value because it could achieve total fiber convergence and the fibers are free to assume any angle, i.e., no filter has been applied.

Finally, by using the NDFO-C method, i.e., with the proposed filter to achieve fiber continuity, the optimized results are presented in Fig. 9, where two different filter radius have been used, $r = 20$ mm and $r = 50$ mm. It can be seen that the larger the filter radius is, the smoother is the fiber continuity, which means that the fiber continuity can be controlled by the filter radius. The final compliance in these cases are equal to $\mathcal{C} = 1.97 \times 10^{-3}$ and $\mathcal{C} = 2.19 \times 10^{-3}$, for $r = 20$ mm and $r = 50$ mm, respectively. Because there is a constraint in the arrangement of the fibers, the compliance in these cases are greater than the NDFO solution. However, the compliance of the solution for $r = 20$ mm is still lower than the BCP method, which means that the proposed method could achieve a better result compared to previous methods and still achieve fiber continuity. Table 2 summarizes the compliance values for all cases and Fig. 10 shows the evolution of the compliance values through the iterations.

Table 2
Summary of the compliance values of the solutions obtained for different methods for the rectangle plate example.

CFAO $\theta^e_{ini} = 0^\circ$	CFAO $\theta^e_{ini} = 45^\circ$	BCP	NDFO $r = 20$ mm	NDFO-C $r = 50$ mm	NDFO-C	
$\mathcal{C}$	3.49×10^{-3}	3.23×10^{-3}	2.01×10^{-3}	1.85×10^{-3}	1.97×10^{-3}	2.19×10^{-3}

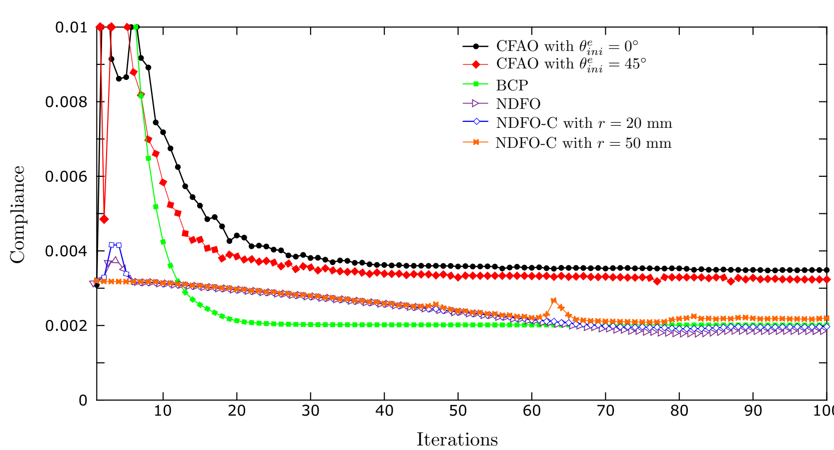


Fig. 10. Evolution of the compliance values through iterations for example 1.

7.2. Square plate

The next numerical example is the square plate domain presented in Fig. 11a, where $L = 100$ mm. All sides are fully clamped and the normal load equal to $F = 10$ N is applied at the center of the plate. This domain is modeled using 40x40 elements. Symmetry is applied to the design variables according to the dotted lines in Fig. 11b.

Fig. 12 shows the optimized solution by using the CFAO, BCP, and NDFO methods. The solutions follow the same pattern, in the center region, the fibers are arranged as circles, and around this center region, the fibers are arranged radially. In the CFAO solution, the symmetry condition has been responsible to avoid the local minima problem, although some few fibers have been wrongly oriented. The final compliance in this case is equal to $\mathcal{C} = 5.97 \times 10^{-4}$. The BCP method has achieved a better fiber configuration solution than the CFAO although there are still some non-converged fibers at the corners. However, the final compliance in this case is equal to $\mathcal{C} = 7.58 \times 10^{-4}$, which is greater than the CFAO compliance. The NDFO method has presented the solution with fibers oriented in a more organized way, with the lowest compliance of all cases $\mathcal{C} = 5.39 \times 10^{-4}$. It is interesting that in this last case the fibers at

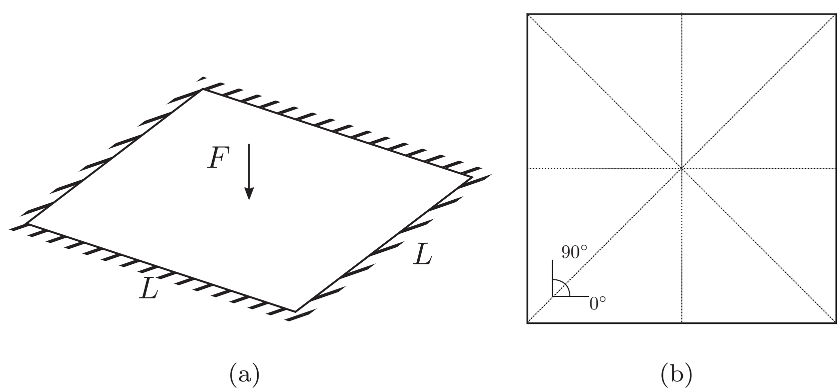


Fig. 11. Square plate domain for example 2 with the four sides clamped. Symmetry is applied to the fiber orientation according to the 4 dotted lines on Fig. (b).

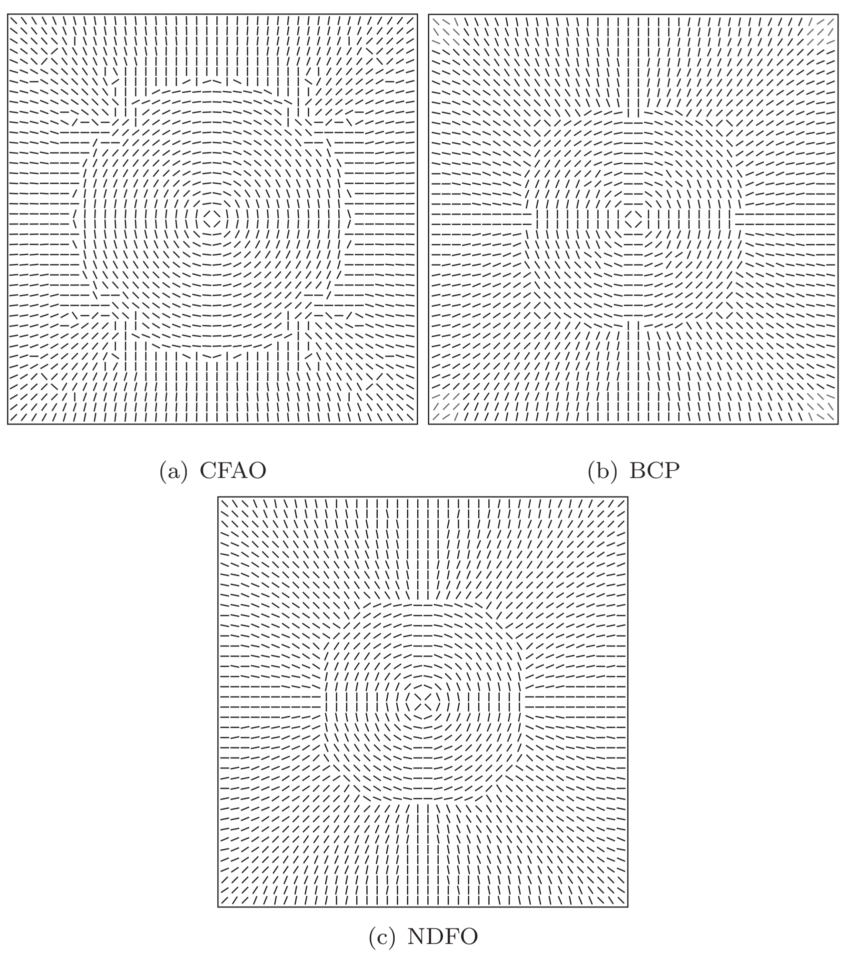
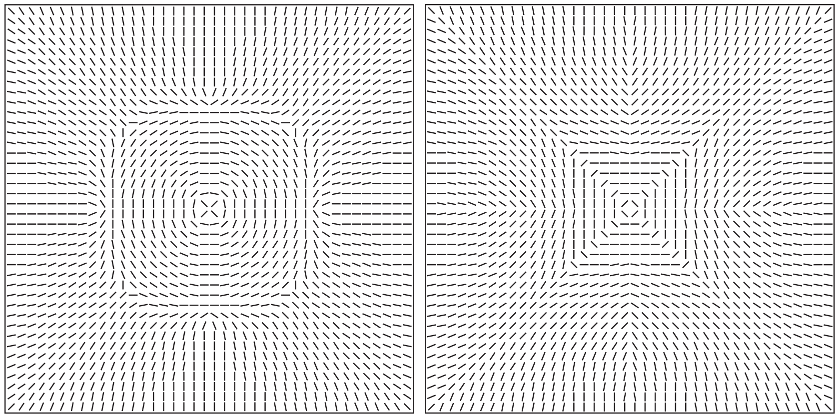


Fig. 12. Solutions for the CFAO, BCP, and NDFO methods. Black and red lines represent converged and non-converged fibers, respectively.



(a) $r = 20$ mm (b) $r = 100$ mm

Fig. 13. Solutions for the NDFO-C method with two different filter radius.

Table 3
Summary of the compliance values of the solutions obtained for different methods for the square plate example.

	CFAO	BCP	NDFO	NDFO-C	NDFO-C $r = 20$ mm	NDFO-C $r = 100$ mm
$\mathcal{C}$	5.97×10^{-4}	7.58×10^{-4}	5.39×10^{-4}	5.54×10^{-4}	7.49×10^{-4}	

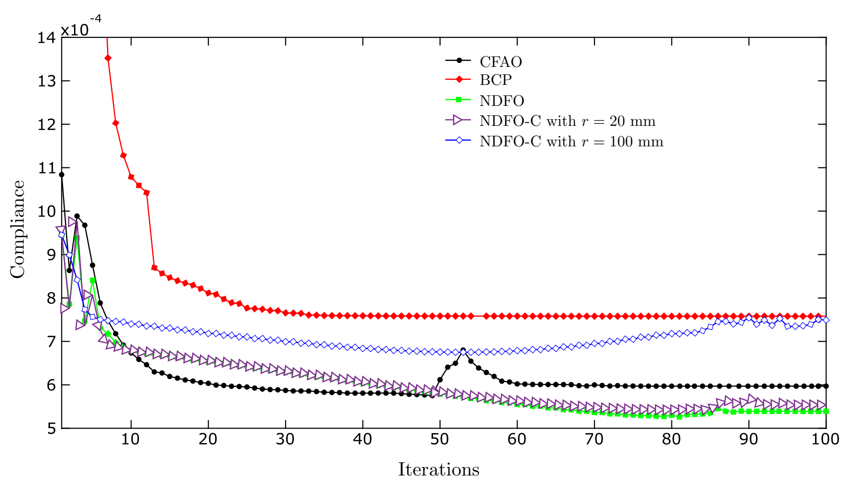
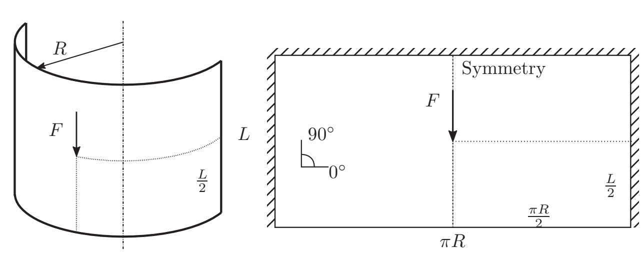


Fig. 14. Evolution of the compliance values through iterations for example 2.

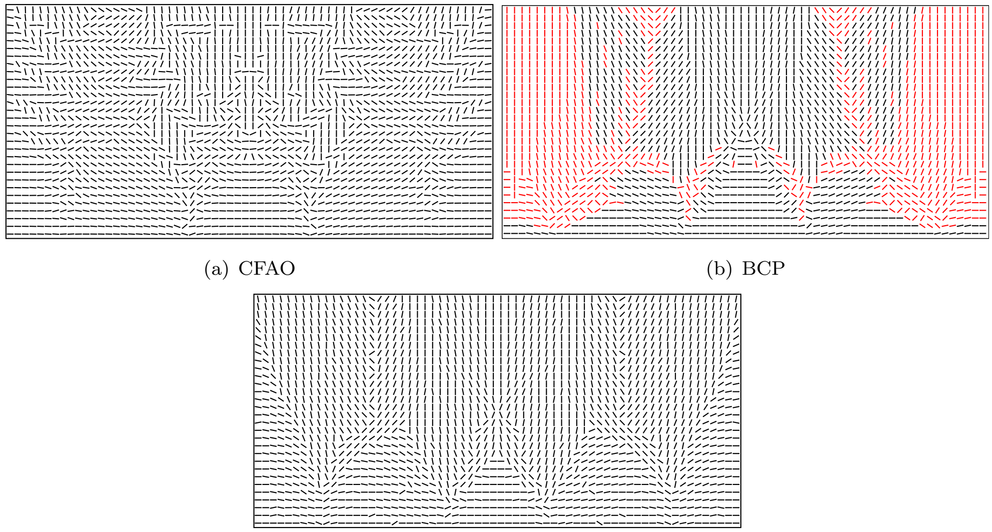
the center of the domain (the loading point) is configured in a cross shape, instead of in circle as the CFAO and BCP solutions. In fact, the cross shaped fibers presents greater stiffness at loading points than the circle shape fibers. Again, the NDFO method has achieved better solution than the previous methods.

By using the NDFO-C method, the optimized results are presented in Fig. 13. Two filter radius have been used, $r = 20$ mm and $r = 100$ mm. Again, the fiber continuity is smoother with larger filter radius. The center cross shape fibers are still present for the smaller filter radius solution, which breaks the fiber continuity although provides lower compliance than the larger filter radius. The compliance for these cases are equal to $\mathcal{C} = 5.54 \times 10^{-4}$ and $\mathcal{C} = 7.49 \times 10^{-4}$. Even for a very large filter radius, the compliance of the proposed method is lower than the BCP solution. Table 3 summarizes the compliance values for all cases and Fig. 14 shows the evolution of the compliance values through the iterations.



(a) (b)

Fig. 15. Square plate domain for example 3. Top and vertical edges are clamped (Fig. (b)). Symmetry is applied to the fiber orientation according to the vertical center dotted line on Fig. (b).



(a) CFAO (b) BCP

(c) NDFO

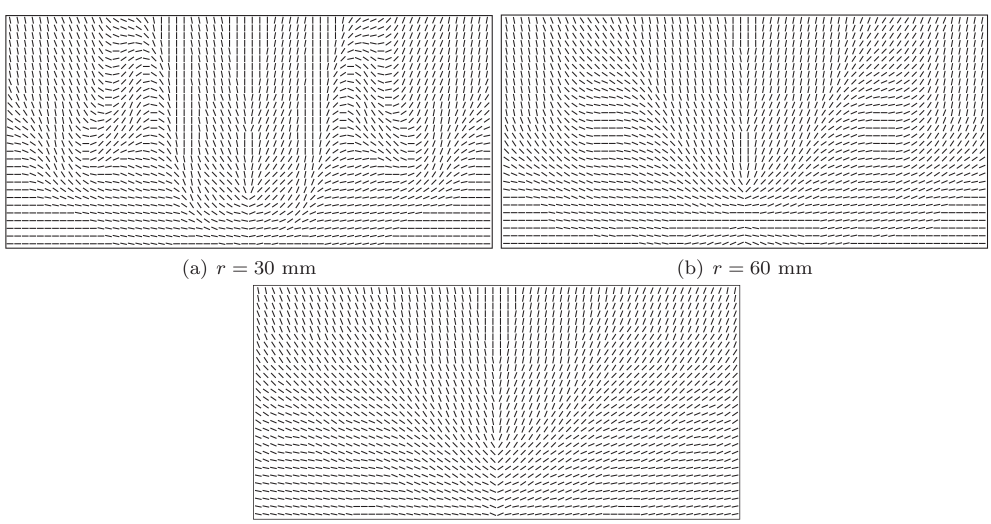
Fig. 16. Solutions for the CFAO, BCP, and NDFO methods. Black and red lines represent converged and non-converged fibers, respectively.

7.3. Half cylinder

The last numerical example is the half-cylinder domain presented in Fig. 15a, where $R = 200$ mm and $L = 300$ mm. Both vertical edges and the top edge are fully clamped, and the vertical load equal to $F = 1$ N is applied at the center of the domain, according to Fig. 15b. Symmetry is applied to the design variables according to the vertical center line in Fig. 15b.

The optimized results for the CFAO, BCP, and NDFO methods are presented in Fig. 16. The solution of CFAO method is clearly a very bad local minimum, which compliance is the greatest $\mathcal{C} = 8.55 \times 10^{-6}$. The solutions of the BCP and NDFO methods are similar when considering the converged fibers of the BCP solution, where almost half fibers have not achieved convergence. The final compliance in these two cases are equal to $\mathcal{C} = 4.23 \times 10^{-6}$ and $\mathcal{C} = 3.94 \times 10^{-6}$, respectively. Again, the proposed NDFO method has obtained a better solution than the other methods.

By using the NDFO-C method, the optimized results are presented in Fig. 17. Three filter radius have been used, $r = 30$ mm, $r = 60$ mm, and $r = 100$ mm. Again, the fiber continuity is smoother with larger filter radius, however, there is an increase in the compliance values. The compliance for these cases are equal to $\mathcal{C} = 4.85 \times 10^{-6}$, $\mathcal{C} = 4.99 \times 10^{-6}$, and $\mathcal{C} = 5.48 \times 10^{-6}$. As expected, these values are greater than the NDFO solution (without filter). Table 4 summarizes the compliance values for all cases and Fig. 18 shows the evolution of the compliance values through the iterations.



(a) $r = 30$ mm (b) $r = 60$ mm

(c) $r = 100$ mm

Fig. 17. Solutions for the NDFO-C method with three different filter radius.

Table 4
Summary of the compliance values of the solutions obtained for different methods for the half cylinder example.

CFAO	BCP	NDFO	NDFO-C	NDFO-C $r = 30$ mm	NDFO-C $r = 60$ mm	$r = 100$ mm
$\mathcal{C}$	8.55×10^{-6}	4.23×10^{-6}	3.94×10^{-6}	4.85×10^{-6}	4.99×10^{-6}	5.48×10^{-6}

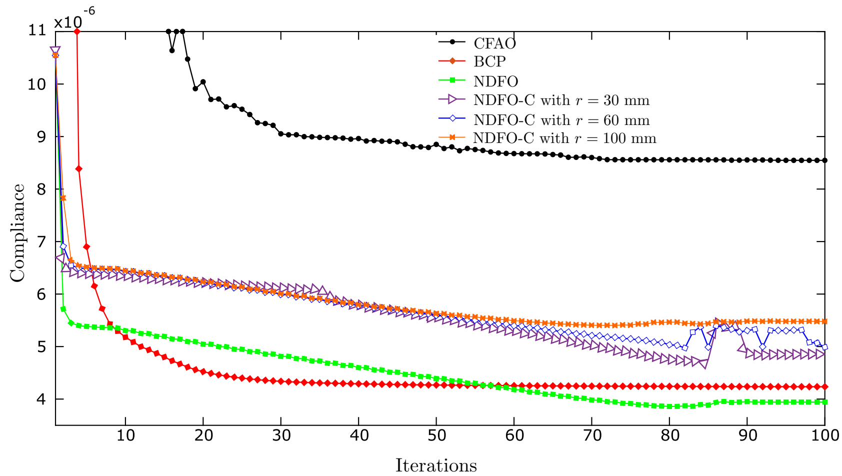


Fig. 18. Evolution of the compliance values through iterations.

8. Concluding remarks

A novel method for optimizing fiber orientation has been proposed, which is based on elemental individual fibers that can assume arbitrary angles. It is also based on the discrete material optimization (DMO) method to find the appropriate angle for each element. The proposed method, called NDFO (Normal Distribution Fiber Optimization) uses the generic form of the normal distribution function to select only one discrete angle among any number of candidate angles. By applying the continuation method to the parameter p_θ, total fiber convergence is achieved. Additionally, fiber continuity is achieved by including a spatial filter, such as the filter proposed in this work, where the level of fiber continuity can be controlled by the size of the filter radius.

Numerical examples have shown that the proposed method has obtained better results than previous methods based on elemental fibers, considering the final configuration of fibers and the value of compliance. With fiber continuity, the fibers are disposed in a more organized way, improving the manufacturability of the solution. However, the compliance is greater than the solutions without the filter. Even though, some results with fiber continuity are still better than the solution from previous methods.

Because the solution of the proposed method is element-based, there is still need for post-processing of the fiber paths before manufacturing process. Thus, additional investigations are needed in order to reduce the gap between numerical results and manufacturing.

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