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Review

Thermal analysis of FGM plates – A critical review of various modeling techniques and solution methods

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ABSTRACT

Functionally Graded Materials (FGMs) are the advanced materials in the field of composites, which can resist high temperatures and are proficient in reducing the thermal stresses. In recent decades, significant investigations are reported in the predicting the response of FGM plates subjected to thermal loads. This paper presents a comprehensive review of developments, applications, various mathematical idealizations of materials, temperature profiles, modeling techniques and solutions methods that are adopted for the thermal analysis of FGM plates. An attempt has been made to classify the various analytical and numerical methods used for the stress, vibration and buckling analyses of FGM plates under one-dimensional or three-dimensional variation of temperature with constant/linear/nonlinear temperatures profiles across the thickness. An effort has been made to focus the discussion on the various research studies carried out till recently for the thermal analysis of FGM plates. Finally, some important conclusions and the suggestions for future directions of research in this area are presented. It is felt that this review paper will serve the interests of all the academicians, researchers and engineers involved in the analysis and design of FGM plates.

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1. Introduction

Functionally Graded Materials (FGMs) are the advanced materials in the group of composites, formed by continuous gradation of two or more constituent phases over a specified volume. FGMs can also be defined as a material which posses gradual variation in properties due to material heterogeneity. This gradation of properties can be tailored properly inorder to achieve optimized characteristics of each components. The material property variation can be either unidirectional or multidirectional, and also it can be continuously or discontinuously graded from one surface to another. Commonly used FGMs are continuously graded in one particular direction. But in all types of FGMs smooth transition in thermomechanical properties is ensured, thereby mitigating problems due to delamination and cracking.

FGMs were mainly designed and developed to resist high temperature gradients. Therefore in recent decades, thermal response of these structural members is extensively investigated by researchers. Many analytical studies have been reported on the analysis of FGMs and a review of those developments till the year 1995 relating to thermoelastic problems have been discussed by Tanigawa [1]. It was found that the thermoelastic behaviour of FGM can be predicted more accurately through precise modeling of material nonhomogeneity and thermal field. Various techniques that are involved in modeling the microstructure dependent thermophysical properties of FGM were discussed by Markworth et al. [2]. Based on review, some modeling methods were recommended for additional studies and hence few approaches for the improvement were also suggested. The behaviour of crack-tip fields in an FGM were dealt by Jin and Batra [3] and its fracture related problems were summarized based on crack-bridging concept and rule of mixture. Carrera presented exhaustive comparative studies using more than 40 two dimensional theories in assessing the global and local response of multilayered plates and shells subjected to mechanical [4] and thermomechanical loads [5]. Further studies were extended by the same author [6] towards establishing the accuracy of the theory developed called Carrera's Unified Formulations (CUF) by comparing with various Classical Plate Theories (CPT), Equivalent Single Layer Theories (ESL), Layer-Wise Theories (LW), Zig-Zag Theories (ZZ) and Mixed Refined Variational Theories (RMVT). An overview of various theories and finite elements that have been developed from last thirty years were discussed in [7] for laminated plates and shells. Most of these theories were later extended for the modeling and analysis of FGM.

Jha et al. [8] presented a review on aero thermo elastic and vibration analysis on FGMs since 1998, in which historical developments and its applications were also discussed. A critical review of two dimensional and three dimensional theories based on analytical and numerical methods; and its solution techniques that are employed for the stress, vibration and buckling analyses of FGM plate subjected to mechanical and thermomechanical loads were discussed by Swaminathan and Naveenkumar [9]. Various theories like CPT, First Order Shear Deformation Theory (FSDT), Third Order Shear Deformation Theory (TSDT), Higher Order Shear Deformation Theory (HSDT), Simplified Theories, Mixed theories, three dimensional elasticity theories and CUF based models that are used for modeling and analysis of FGM plates and shells has been reviewed by Thai and Kim [10]. An overview of various semi-analytical numerical methods like Finite Layer Method, State Space Method (SSM), Asymptotic Method, Sampling Surface Method (SAS), that are adopted for the analysis of laminated composite and sandwich functionally graded elastic/piezoelectric materials plates and shells was presented by Wu and Liu [11] for various combination of boundary conditions and micromechanical schemes.

In recent years, significant research works have been published in analysis of FGM plates exposed to thermal environments. But, none of the above articles discussed the developments in the analysis and design of FGM plates subjected to thermal loads alone. Therefore, in the present article an attempt has been made to discuss the various theories and solution methods available for the stress, vibration and buckling analyses of FGM plates subjected to thermal loads. Each sub sections are distinctly classified based on the method of analysis adopted (exact, analytical and numerical) and the type of thermal load (constant, linear and nonlinear) considered. Also, historical developments, applications and various mathematical idealizations (material and temperature profiles) adopted for the analysis of FGM plates were also dealt in separate sections.

1.1. Development of FGM

The concept of gradation in material composition was first proposed for composites and polymeric materials by Shen and Bever [12] in 1972. Most of these materials were used as coating materials, inorder to improve the bonding strength and to reduce thermal stresses. The first practical application of FGMs was carried out at National Aerospace Laboratories of Japan in 1984 to create square shells for the base of fuselage and hemispherical bowls for nosecones of a space plane. In general, aerospace structures are supposed to perform at target service temperatures of 2100 K and temperature gradient of 1600 K across a cross section of less than 10 mm [13]. There are no industrial materials available at present which can withstand such high-temperature gradients without losing their structural integrity. Hence, the unique idea of gradation of material composition was devised by using a heat-resistant ceramic material on high-temperature side and tough metals with high-thermal conductivity on the other side, and

Table 1
Mechanical properties of various ceramics and metals used in FGM plates.

		Young's modulus of elasticity (GPa)	Thermal conductivity ($W\,m^{-1}\,K^{-1}$)	Thermal coefficient of expansion ($10^{-6}\,K^{-1}$)	Density (Mg/m^3)
Ceramics	Zirconia	100–250	2.3–12.2	1.7–2.7	5–6.15
	Alumina	215–413	12–38.5	4.5–10.9	3.5–3.98
	Aluminum nitride	302–348	4.3–5.6	60–177	2.92–3.33
	Boron carbide	362–472	3.2–9.4	17–42	2.35–2.55
	Silicon	140–180	84–100	7–8	2.28–2.38
	Silicon carbide	90–137	7.9–11	3.8–20.7	4.36–4.84
	Silicon nitride	166–297	1.4–3.7	10–43	2.37–3.25
	Tungsten carbide	600–686	4.5–7.1	28–88	15.25–15.88
Metals	Cast Irons	80–150	40–72	11–13	6.9–7.35
	Stainless steels	189–203	12–25	16–18	7.85–8.1
	Aluminum alloys	68–82	76–240	21–24	2.5–2.9
	Copper alloys	112–148	160–390	16.2–21.6	8.93–8.94
	Lead alloys	12.5–15	23–44	19–29.3	10–11.4
	Magnesium alloys	42–47	50–160	25.2–27.1	1.74–1.95
	Nickel alloys	150–245	65–90	9–16	7.75–8.65
	Titanium alloys	90–120	7–15	8.4–9.36	4.4–4.8
	Zinc alloys	68–95	100–140	23–28	4.95–7

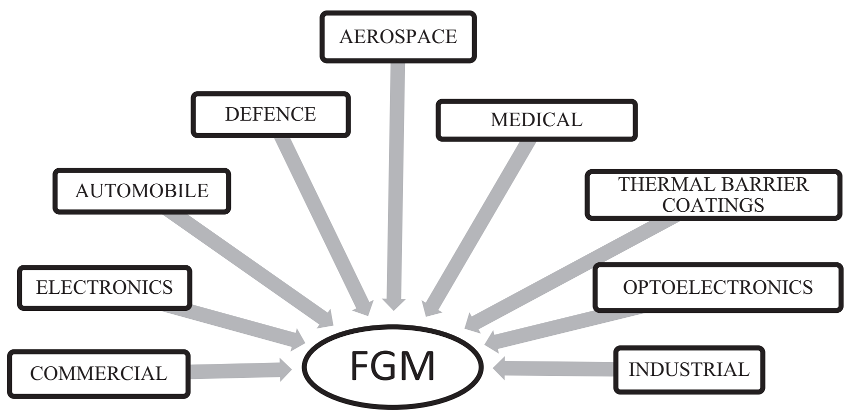

Fig. 1. Applications of FGM.

thereby gradually varying the composition from ceramic to metal. These Functionally Graded Materials have the advantage of the physical and chemical properties of both the materials, thereby increasing the bond strength and reducing the interfacial stress, thermal stress and crack driving forces [14]. Some of the commonly used metals and ceramics in FGMs with their mechanical properties are listed in Table 1.

FGMs can be classified into two types based on the distribution of constituent phases, namely continuous or discontinuous (stepwise or layered) gradation of materials. Also, based on manufacturing techniques these can be further grouped as thin and bulk FGMs [15,16]. Thin FGMs are manufactured by Physical Vapor Deposition (PVD), Self propagating High temperature Synthesis (SHS) method, Chemical Vapor Deposition (CVD), etc., which are generally used as surface coating material. While bulk FGMs are consolidated to form a volumetric bulk material and are manufactured using Powder Metallurgical (PM) technique, Solid Free Foam (SFF) technique, Centrifugal Casting method, etc. A comprehensive review of various processing techniques adopted to fabricate an FGM and its successful applications to numerical simulations were discussed by Kieback et al. [17] and Gasik [18]. A remarkable improvement in the production methods has reduced the manufacturing cost and hence these FGMs are finding widespread applications in various sectors.

1.2. Applications of FGMs

FGMs were basically developed for aero space structures operating under high temperature fluctuations. But recently, FGMs are gaining potential applications in different areas due to its flexibility in making a particular composite material according to the requirement. Significant applications of FGMs in various sectors are shown in Fig. 1 below.

In the first application of space-plane project, FGMs like SiC/C, Ni-based alloy/ZrO_2, TiC/Ni were manufactured by CVD process. These ceramic-metal FGMs were capable of withstanding high temperature fluctuations, thermal shocks and stress concentrations at the interfaces [13]. Also, FGMs can act as a thermal barrier system and are found in insulation of combustion chambers, rocket engine components and exhaust wash structures of space vehicles. FGMs with TiAl/SiC fibers are used in Heat exchange panels, rocket nozzles, spacecraft truss structure, nose caps and leading edge of missiles and space shuttle. FGMs in space craft truss structure which can withstand huge mass of 200 metric tons with high temperature resistant and high gravity gradient features. Carbon nanotube (CNT) FGMs are thermally stable and exhibit excellent mechanical properties with high toughness, hardness, abrasion resistance, flexural strength which can be used in both high and low temperature zones. Most of the helicopters, fighter jets, defence tanks, weapons and armor suits are made of FGMs. These posses good damping properties with thermal and chemical inertness and hence used in Fuselage tanks, stabilizers, rotor blades, aircraft wings, cryogenic propellant tanks, gas turbine engines, nozzles and compressor components of fighter planes and helicopters. Ultra light weight FGMs are used in defence sector to develop weapon platform, armor plates, barrier materials, bullet proof jackets, etc. Military submarine components like sonar domes composite piping systems are made with Glass/Epoxy FGM, propulsion shafts with Carbon/Glass fiber FGM, Cylindrical pressure hulls with Graphite/Epoxy FGM and diving cylinders with Al/SiC FGM.

Medical applications include the replacement of living tissues in human body with biopolymer FGMs. Orthopedic and dental implants are usually composed of Collagen Hydroxyapatite (HAP) and titanium alloys. High density Polyethylene with a graded biopolymer coating is used in orthopedic implants like total hip, shoulders and knee joint replacements [19,20]. Nanohydroxyapatite reinforced polyvinyl alcohol (nanoHA/PVA) gels are used as an artificial articular cartilage repair material [21]. Ti-29Nb-13Ta-4.6Zr (TNTZ) with graded microstructure is used as dental implants to reconstruct the masticatory function when tooth root is completely lost or extracted [22].

In photoelectronic devices the refractive index modulation, diffusion length, energetic band gap and other properties can be adjusted using material gradation technique thereby enhancing the absorption capabilities and generation efficiencies. Hence, these are widely used in antireflective layers, optical fibers, optical lenses, photo-detectors, solar cells, optical sensors, semiconductor devices, computer circuit boards, cellular phones. FGMs embedded with piezoelectric layers are used in shape memory alloys.

Automotive parts require high strength with resistance to crack, fracture and thermal shocks. FGM with Al/SiC are used as engine cylinder liners, flywheels, drive shafts and racing car breaks. Diesel engine pistons are made of SiCw/Al-alloy and leaf springs with Al/C FGMs. Few others include motor cycle drive sprockets, pulleys, shock absorbers, radiator end caps, etc. Most of the forming tools, cutting tools, forging and machine tools are manufactured using FGM. Few examples include lathe, drill press, broaching machine, gear shaper, hone etc.

FGM is also used as a coating material which inturn reduces heat loss from engine exhaust system components like turbocharger casings, exhaust headers, exhaust manifolds, tail pipes and down pipes, thereby reducing consumption of coolant [16,23]. Turbine wheel blades of gas turbine engine operating at 40,000 rpm are coated with TiAl/SiC FGM to provide thermal barrier. Also, anti-abrasion sports equipments like tennis rackets, baseball cleats, sports shoes, racing bicycle frames, etc are developed based on the property of relaxation of stresses. Some of the commonly used FGMs include razor blades, cutting tools, eye glass frames, helmets, X-ray tables, automobile fuel tanks and pressure vessels, wind turbine blades, MRI scanner parts and cryogenic tubes, laptop cases, titanium watches, window glasses, camera tripods, etc. Various FGMs with their significant properties and applications are listed in Table 2.

In recent years, FGMs are found to be most advantageous over conventional structural materials and layered composites due to its continuous change in characteristic property. Though they have widespread applications in various sectors, there are few difficulties which have to be resolved by further research in this area. Mathematical modeling of the graded materials plays a vital role in predicting the accurate behaviour of FGM plate. Though experimental investigation methods are available to predict the individual thermo-physical material properties, microscopic studies have to be performed and quantitative relations have to be established for accurate evaluation of physical and thermal properties of graded materials. These relations are inturn used with various theories for analytical or numerical evaluation of various FGM plate responses. In this paper, an attempt has been made to review various theories and mathematical idealizations techniques used for the stress, vibration and buckling analyses of FGM plates subjected to thermal loads.

2. Material properties and temperature profiles

FGMs are formed by, gradually changing the arbitrary composition of two or more constituent phases of materials. This can be achieved by continuously changing the volume fraction distribution of constituent materials from one to another, thereby simultaneously changing the thermal and mechanical properties. This nonhomogeneity in material property can be mathematically idealized, by assuming the FGM specimen to be homogeneous, and then defining the variation of mechanical properties according to constitutive relations. Generally these FGMs are designed to suit high temperature exposure conditions and hence most of the commonly used FGMs are made of ceramic and metal. Several micromechanical models that are used in modeling the material property variations and temperature distributions in an FGM plate were discussed in the next sections.

2.1. Material property idealization

FGMs often possess variety of microstructures by gradually changing the volume fraction of constituent phases called as gradation of materials. The micromechanical modeling of FGMs are based on approximate evaluation of volume fraction and shape of the dispersed phase or phases [24]. A review of several micromechanical models and its accuracy in estimation of mechanical properties, for a single layered and a multi layered FGMs have been dealt in [25–30]. In the next sections, various homogenization techniques that are used for defining the spatial distribution of

Table 2
FGMs with their significant properties and applications [14,16].

FGM	Significant property	Application
Al_2O_3/Al-alloy	Thermal barrier and anti-wear machine parts	Rocket nozzle, wings and engine casting
E-glass/Epoxy	Hardness and damping resistance	Brake rotors, solar domes, Composite piping systems
Al/SiC	Hardness and toughness	Combustion chambers, racing car breaks, Engine cylinder liners, Flywheels
$BaTiO_3$/Si	Signal loss control at high frequency	Dielectric motors
Carbon/Epoxy	Lightweight and good damping properties	Helicopter components viz. landing gear doors, heat exchanger panels, engine parts
Al alloy/Carbon nanotubes (CNT)	Lightweight and high stiffness	Artificial ligaments, MRI scanner spares, eye glass frames, dentistry parts, musical instruments
TiAl/SiC, SiC/C	Temperature and shock resistant coatings	Heat exchange panels, rocket nozzle, spacecraft truss structure, solar panels, reflectors, turbine wheel blades
Graphite/Epoxy	High strength to stiffness ratio, good radiation resistant and reduces thermal distortions	Cylindrical pressure hulls, space telescopes, cryogenic tanks, satellite antennas
Al/Al_2O_3, WC/Co,	Heat, wear and tear resistance property	Machine and forming tools, cutting tools, artificial bones
SiCw/Al-alloy	Thermal resistance, chemical inertness, Hardness and toughness	Storage cylinders, diesel engine pistons, racing bicycle and vehicle frames

constituent phases, and hence methods for evaluating the effective material properties across the plate thickness are discussed.

2.1.1. Power law function (P-FGM)

This method of estimation is based on linear rule of mixture and has been extensively used in the literature to investigate the response of FGM structures [31–41]. For example, Apalak and Gunes [33] studied thermal residual stresses in FGM plates, while Swaminathan and Naveenkumar [38] carried out stability analysis of FGM sandwich plates. This is one of the simplest and well established model in which, the material properties are graded across the thickness (h) of the FGM plate from bottom (z = –h/2) metal rich surface to top (z = h/2) ceramic rich surface by changing the volume fraction of the constituents. The position dependent variation of material property across the thickness direction is given by,

$$E_z = E_m + (E_c - E_m)V_f^p \tag{1a}$$

$$\alpha_z = \alpha_m + (\alpha_c - \alpha_m)V_f^p \tag{1b}$$

$$k_z = k_m + (k_c - k_m)V_f^p \tag{1c}$$

$$V_f = \left(\frac{z}{h} + \frac{1}{2}\right) \tag{1d}$$

where, E, α and k are the modulus of elasticity, thermal coefficient of expansion and thermal conductivity respectively. V_f represents volume fraction of ceramic phase and p is a power law index or material property gradient index. The subscripts c and m represents the constituents of ceramic and metal respectively.

2.1.2. Mori-Tanaka scheme (MT-FGM)

Mori-Tanaka method of estimation is most applicable for regions with graded microstructure which have well defined continuous matrix phase and randomly distributed spherical particulate phase. This method accounts the effect of elastic fields among neighboring inclusions and its interactions with the constituents. The effective bulk modulus K_z and shear modulus G_z are evaluated using the relations given by [42–45]

$$\frac{K_z - K_m}{K_c - K_m} = \frac{V_f^p}{1 + (1 - V_f^p)\left(\frac{K_c - K_m}{K_m + \frac{4}{3}G_m}\right)} \tag{2a}$$

$$\frac{G_z - G_m}{G_c - G_m} = \frac{V_f^p}{1 + (1 - V_f^p)\left(\frac{G_c - G_m}{G_m + f_m}\right)} \tag{2b}$$

$$f_m = \frac{G_m(9K_m + 8G_m)}{6(K_m + 2G_m)} \tag{2c}$$

The effective values of Young's Modulus of elasticity E_z and Poisson's ratio ν are calculated based on effective Bulk modulus K_z and shear modulus G_z and are related as,

$$E_z = \frac{9K_zG_z}{3K_z + G_z} \tag{3a}$$

$$\nu = \frac{3K_z - 2G_z}{2(3K_z + G_z)} \tag{3b}$$

The effective heat conductivity k_z [46] and coefficient of thermal expansion α_z [47–49] are determined using the following relation,

$$\frac{k_z - k_m}{k_c - k_m} = \frac{V_f^p}{1 + (1 - V_f^p)\left(\frac{k_c - k_m}{3k_m}\right)} \tag{4a}$$

$$\frac{\alpha_z - \alpha_m}{\alpha_c - \alpha_m} = \frac{\frac{1}{K_z} - \frac{1}{K_m}}{\frac{1}{K_c} - \frac{1}{K_m}} \tag{4b}$$

where, the volume fraction distribution V_f, is assumed according to power law function p. The subscripts m and c represents the thermo-mechanical properties of metal and ceramic constituents respectively.

Though several homogenization methods are available for the estimation of effective material properties, Mori-Tanaka scheme and Power law estimation were most often used for the analysis in the recent years. Analytical and numerical investigations have been carried out in [50–53] and comparative studies are also discussed in [54–57] for various types of FGM structures.

2.1.3. Sigmoidal function (S-FGM)

This method is most applicable in layered FGM plates i.e., FGM plate with ceramic at the centre and graded to metal on both sides or vice versa. In such cases, if single power law function is used stress concentrations will appear at the interfaces where the material is continuous but changes rapidly. Therefore smooth distribution of stresses at the interfaces is ensured by defining the volume fraction using two power law functions, which is also called as Sigmoidal function [58–62]. The variation of volume fraction of the constituents from the middle plane to the top surface (Vf_1) and bottom surface to the middle plane (Vf_2) of the plate is expressed as,

$$V_{f1} = 1 + \left(\frac{z}{h} - \frac{1}{2}\right)^p \quad \text{for} \quad 0 \leqslant z \leqslant h/2 \tag{5a}$$

$$V_{f2} = \left(\frac{z}{h} + \frac{1}{2}\right)^p \quad \text{for} \quad -h/2 \leqslant z \leqslant 0 \tag{5b}$$

Further, the effective material properties across the plate thickness is computed using power law function based on rule of mixture by using appropriate volume fraction function across the thickness direction. Also, optimal composition of constituents can be achieved by varying power law parameter.

2.1.4. Exponential function (E-FGM)

This idealization method for FGM is most common in studies related to fracture mechanics. Several investigators adopted exponential gradation of materials for the analysis of FGM plates [63–68] and sandwich plates [69,70]. Through the thickness variation of thermoelastic material properties are represented as,

$$E_z = E_m e^{\left(\frac{z}{h}\ln\left(\frac{E_c}{E_m}\right)\right)} \tag{6a}$$

$$\alpha_z = \alpha_m e^{\left(\frac{z}{h}\ln\left(\frac{\alpha_c}{\alpha_m}\right)\right)} \tag{6b}$$

$$k_z = k_m e^{\left(\frac{z}{h}\ln\left(\frac{k_c}{k_m}\right)\right)} \tag{6c}$$

where E_z, α_z and k_z are the modulus of elasticity, thermal coefficient of expansion and thermal conductivity across the transverse direction z, of an FGM plate with thickness h.

2.1.5. Temperature dependent properties (TD)

FGMs are mainly designed for structural members to suit high temperature environments. Therefore, the effect of temperature plays a vital role in evaluating the behaviour of FGM. Touloukian [71] proposed a new method of evaluating the material properties (P) of ceramics and metals depending upon the exposure temperatures (T) and is expressed as,

$$P = P_0\left(\frac{P_{-1}}{T} + 1 + P_1T + P_2T^2 + P_3T^3\right) \tag{7}$$

where P_0, P_1, P_2 and P_3 are constants in the cubic fit of the material property and temperature. The temperature dependent variation of modulus of elasticity, thermal conductivity and thermal coefficient of expansion are evaluated for ceramics and metals. Thus the effective properties that are dependent on both temperature and position (z) are estimated using simple rule of mixtures and are given by,

$$P_{eff}(z,T) = P_m(T) + [P_c(T) - P_m(T)]V_f \tag{8}$$

where V_f is the volume fraction of the ceramic phase evaluated using power law function p and is independent of temperature. The effective material properties can be evaluated for various forms of temperatures across the thickness direction namely, constant, linear and nonlinear variations.

Recent studies in [72–74] focused on assessing the response of FGM structures using coupled relations that includes the effect of both temperature and position. Also, few researchers [41,75–77] discussed the significant difference between coupled and uncoupled problems. While, Lee and Kim [59] revealed the influence of homogenization schemes in estimating the material properties by using coupled form of Mori-Tanaka scheme.

2.2. Temperature variation

Thermal analysis of FGM can be carried out by appropriate modeling the distribution of temperature. Generally, the material properties in FGMs vary across the thickness of the plate while in-plane will be homogeneous. Therefore, the variations in temperature are assumed to occur only in the thickness direction and are evaluated in different ways. Some of the most commonly used methods available in the literature are discussed in the following sections.

2.2.1. Constant and linear variation

This type of temperature distribution has been discussed in [33,74,76,78,79] for the analysis of FGM plates and shells. The temperature at top (T_1) and bottom (T_0) surfaces were assumed to be same in constant distribution, while different in linear distribution. Through the thickness variation of temperature is expressed as,

$$T_z = T_0 + (T_1 - T_0)\left(\frac{z}{h} + \frac{1}{2}\right) \tag{9}$$

where T_Z is the temperature at any point across the plate thickness (h) through the coordinate direction z.

2.2.2. Heat conduction equation

The nonlinear variation of temperature is usually obtained from the solution of heat conduction equation. The heat equation in three dimensional form is discussed in [80,81] and is expressed as,

$$k_x \frac{\partial^2 T}{\partial x^2} + k_y \frac{\partial^2 T}{\partial y^2} + \frac{\partial k_z}{\partial z}\frac{\partial T}{\partial z} + k_z \frac{\partial^2 T}{\partial z^2} + q = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{10}$$

where q is the internal heat source or heat flux. The use of above equation for the transient response of various types of FGMs were discussed by Praveen and Reddy [31]. The thermal equilibrium of FGMs, called as steady state responses (without heat flux, q) are evaluated by setting the right hand side to zero. Also, static analysis (with or without heat flux, q) can be performed by neglecting the term ($\partial T/\partial t$) i.e., rate of change in temperature with respect to time. Most of the studies on static analysis of FGM were carried out either with [52,82] or without [70] considering the effect of internal heat source.

Most of the two dimensional thermo-elastic studies were carried out using one dimensional heat conduction equation [41,83–86]. This is due to the assumption of material homogeneity in the plane of the plate and varies only in the thickness direction. The static steady state one dimensional heat equation without heat flux is given by,

$$-\frac{d}{dz}\left(k_z \frac{dT_z}{dz}\right) = 0 \tag{11}$$

The solution for one-dimensional and three dimensional heat conduction equation can be obtained by using relevant boundary conditions and temperatures at the surfaces. Thus, through the thickness variation of temperature can be evaluated for different material property idealizations that are discussed in the earlier sections.

2.2.3. Polynomial distribution

In recent studies, the temperature profile (T_z) in the thickness direction was defined according to layerwise description of displacement field. The higher order terms of the displacement field are also included to capture the effect of nonlinear distribution. Polynomial form of temperature variation across the plate thickness is expressed as,

$$T_z = T_1 + \left(\frac{z}{h}\right)T_2 + \frac{\Psi(z)}{h}T_3 \tag{12}$$

where T1, T2 and T3 are the thermal load terms across the plate thickness and $\psi(z)$ is the higher order displacement term, which can be assumed zero for models based on First order Shear Deformation Theory (FSDT) and Classical Laminated Plate Theory (CLPT). Zenkour and coauthors [87–89] used higher order terms of Reddy's Third Order Shear Deformation Theory (R-TSDT) and Sinusoidal Shear Deformation Theory (SSDT) to determine the value of $\psi(z)$, and obtained the corresponding nonlinear polynomial form temperature profiles. Similar works have been carried out by Tounsi and coauthors [90–92] and Mantari and Granados [93] for the analysis of FGM plates and FGM sandwich plates. Akbarzadeh et al. [94] considered polynomial form of temperature field with four variables of thermal load based on Third Order Shear Deformation Theory (TSDT) and is given by,

$$T_z = T_1 + \left(\frac{z}{h}\right)T_2 + \left(\frac{z}{h}\right)^2 T_3 + \left(\frac{z}{h}\right)^3 T_4 \tag{13}$$

The stress, vibration and buckling analyses of FGM plates using the various material idealizations and different temperature profiles discussed in the preceding sections are presented in the following sections.

3. Thermal stress analysis

In this section, various theories adopted for the stress analysis of FGM plates subjected to thermal loads are discussed based on the various temperature field namely three dimensional variation, one dimensional nonlinear variation and one-dimensional constant and linear variation. Most of the exact solutions were solved analytically and are based on three-dimensional variation of temperature, whereas few studies were also reported using one-dimensional temperature profiles. Studies related to one-dimensional variation of temperatures are discussed under two subclasses namely, analytical and numerical methods.

3.1. Three dimensional temperature variation

Three dimensional elasticity solutions using 3D temperature profiles are found to be most accurate methods for the thermal analysis of FGM plates. Various research works have been carried out on analytical evaluation of 3D stresses, strains and displacements for a thermally stressed FGM plates with different types of edge support and plate geometry. Most of the theories used for the analysis of FGM plates are extensions of laminated plate theories. Mian and Spencer [95] developed 3D elasticity solutions for an inhomogeneous plate across the thickness direction based on the two dimensional solutions of classical thin plate theory for the equivalent plate. Theoretical formulations and solution methods were presented for rectangular, cylindrical polar coordinates systems and radially symmetric systems. Reddy and Cheng adopted asymptotic technique for square FGM plates [96] and smart FGM plates with piezo-electric actuators [97] subjected to thermal and mechanical loads. It was found that the assumption of constant transverse displacement across the plate thickness is not valid for plates subjected to thermal loads and the maximum longitudinal compressive stress appears at the top surface of the plate. Wang et al. [98] obtained asymptotic solutions for thin FGM plates subjected to sudden change in temperature at the boundary based on Lord and Shulman theory (L–S theory) with power law distribution of material properties. Vel and Batra [54] used power series method to study the thermomechanical deformations of a simply supported rectangular plate, in which the material properties were evaluated using either self consistent method or Mori-Tanaka method or the combination of both. Parametric studies were performed and a comparative study with CLPT, FSDT and TSDT models was presented. Significant difference was observed in displacements and stresses obtained using exact solutions and 2-D plate theories. It was observed that the results obtained using different material schemes agree qualitatively but differ quantitatively. Later this work was extended to study transient thermal stresses [99], in which the transient longitudinal stress was found to be nearly eight times greater than the steady state value for rapid time dependent surface temperature condition. Ootao and Tanigawa [100] adopted exponential variation of material properties to study the transient thermal stresses in a simply supported rectangular FGM plate subjected to partial heat supply. The work was extended to study the plate behaviour under non-uniform heat supply [63,101]. Analytical formulations were developed using heat conduction, Laplace and Finite cosine transformations and the solutions were obtained using series expansion of Bessel functions. It was found that the most precise evaluation of transverse stresses is possible in transient state. Further, Ootao and Ishihara [102] adopted piece wise exponential law to study the response of two and three layered rectangular plate models. It was concluded that the maximum value of thermal stresses can be reduced using multi layered FGM plate instead of single layered FGM plate in transient analysis. Xu et al. [103] presented exact thermo-elastic solutions for a simply supported exponentially graded rectangular plate with variable thickness using double fourier sinusoidal series expansions. Thermo elastic solutions for circular FG plates subjected to axisymmetric loads were studied by Jabbari et al. [104].

Ying et al. [105] developed semi-analytical solutions for a FGM plate with one pair of opposite edges simply supported using State Space Method (SSM). Solutions for MT-FGM plates were obtained using Levy's method and Differential Quadrature Method (DQM), which makes it feasible to treat non-simply supported edges. Alibeigloo [106] obtained three dimensional elasticity solutions for an exponentially graded rectangular plates with simply supported edges using Fourier series and SSM technique. Further, it was extended for solid and annular circular FGM plates [107] and for sandwich circular plate with a layer of FGM core [70]. Analytical solutions for various support conditions were obtained using DQM technique. It was observed that, the neutral surface and middle surface will not coincide with each other and it depends on the variation of young's modulus of elasticity across the plate thickness. Liu and Zhong [108] presented Peano-Baker series solution for an orthotropic simply supported and isothermal functionally graded rectangular plate based on SSM method. It was observed that the material gradient distribution significantly affects the total stiffness, deflection and temperature field distribution of the plate.

3.2. One-dimensional nonlinear temperature variation

Various researchers have adopted nonlinear or parabolic variation of temperatures across the plate thickness which are either based on heat conduction equation or polynomial functions. The related studies are discussed under two categories namely, analytical and numerical methods. The various analytical methods using either two or three dimensional theories are presented under analytical methods whereas the finite element and the meshless methods are dealt under numerical methods.

3.2.1. Analytical methods

Tanigawa [109] used transient heat conduction problem to study the associated thermal stresses in a nonhomogeneous FGM plate. Thereafter optimization problems were discussed inorder to obtain optimum material composition for the purpose of reducing the thermal stress distribution. Stress intensity factors for a crack in an E-FGM plate subjected to prescribed surface temperature [110] and thermal shock [111] were studied. It was found that, the crack close to heating side of the strip will be more likely to be unstable than that of the cooling side. Noda [83] discussed crack propagation path, thermal stresses and thermal stress intensity factors in FGM plates subjected to steady state temperature fields and thermal shocks. Significant decrease in stresses was observed by adopting a precise and appropriate gradation of material properties. Ravichandran [112] studied the effect of residual thermal stresses that arises during the fabrication of FGM system. Both temperature dependent and independent gradation forms were evaluated for continuous and discrete change in composition across the thickness direction. It was found that, the residual stresses can be decreased by adopting multiple layers of FGM (greater than 11) plates with constant composition and thickness. Bouchafa et al. [113] employed exponential gradation of material properties to evaluate residual thermal stresses in Al_2O_3-Ni FGM system. It was concluded that, the magnitude of stresses increases by adding a fully ceramic or metal surface in the middle of graded regions and hence should be kept minimum to avoid cracking at interfaces.

A new higher order micromechanical theory for FGM plates "HOTFGM" which includes both local and global effects have been developed by Aboudi et al. [114,115]. The accuracy of the method has been established and proved to be efficient tool for the analysis of FGM plates. Theoretical formulations were developed in Cartesian coordinate system with temperature gradient across the thickness direction. The theory was extended to incorporate inelastic and TD response of the constituent phases [116] and by applying partial homogenization scheme normal to the functionally graded direction [117]. Further it was extended to develop a two dimensional frame work for modeling FGM plates by enabling nonuniform spacing of fibers in two directions [118].

Reddy [119] developed analytical formulations and Navier's solutions for a simply supported FGM plates which accounts for the effects of thermomechanical coupling, time dependency and geometric nonlinearity. The response of plates has been studied using R-TSDT and FSDT subjected to various loading and boundary conditions. Woo and Meguid [120] studied large deflection in FGM plates subjected to steady state temperature field under different types of thermomechanical loads. The nonlinear bending responses were investigated by Shen [121], for simply supported FGM plates with or without peizoelectric actuators and are subjected to combined thermal and electrical loads. Yang and Huang [122] obtained asymptotic solutions for nonlinear transient response of FGM plates with initial geometric imperfections using improved perturbation technique. It was found that, the effect of heat conduction and temperature dependency could not be neglected while evaluating the response of FGM plates. Brischetto et al. [123] presented Unified Formulations (UF) for a simply supported rectangular plate subjected to steady state thermal loads. The accuracy with the exact solutions was achieved by using higher order expansion terms across the thickness coordinate. Asymptotic solutions were formulated in [124] for an infinite sandwich plate with double sided FGM coatings under convective boundary conditions to investigate transient heat conduction and thermal stresses. Some design rules were suggested for FGM cutting tools to resist high thermal shocks. Based on layerwise theory and perturbation technique, Tahani and Mirzababaee [125] derived solutions for cylindrical bending of FGM plates subjected to mechanical, thermal and thermomechanical loads. Significant increase in the magnitude of transverse deflection was observed due to the effect of geometric nonlinearity. Zenkour used SSDT to formulate solutions using power law function [88] and exponential function [87] for FGM plates subjected to hygro-thermo-mechanical loads. Polynomial form of temperature profile was assumed to evaluate static bending response of the plate and hence the effects of both temperature and moisture concentrations were discussed. Further, the studies were extended for FG sandwich plates [89] and the results obtained from SSDT were compared with R-TSDT, FSDT and CLPT. Tounsi and co-authors adopted a new four variable refined theory to study the bending behaviour of FGM plates subjected to thermo-mechanical loads with [126] or without [92,127] resting on elastic foundations. Further, the work was extended for FGM sandwich plates [90,91]. The accuracy of theory was established by comparing the results with various other theories that were reported in the literature. Mantari and Granados [93] obtained analytical solutions using quasi-3D hybrid type HSDT with six unknowns for FGM sandwich plates subjected to generalized nonlinear temperature field. The deflections and stresses were found to be highly sensitive to polynomial forms adopted to define the temperature field. Recently, Swaminathan and Sangeetha [128] presented analytical formulations and solutions for an FGM plate using HSDT with twelve degrees of freedom which includes the effects of both transverse shear and normal deformations. Parametric studies were performed to study the displacements and stresses in an FGM plate subjected to nonlinear thermal loads. Ramos et al. [129] investigated thermoelastic response of a simply supported rectangular FGM sandwich plates using CUF, which includes several shear strain shape functions related to non-polynomial displacement fields like sinusoidal, hybrid and hyperbolic deformation theories. Later, the author proposed a new set of plate theories and by performing comparative studies it was revealed that, these theories are effective in predicting similar results as that of classical polynomial based functions.

Cheng and Batra [130] presented three dimensional solutions using Mori-Tanaka estimation and asymptotic scheme to study the thermomechanical deformations of a rigidly clamped FGM elliptical plate. Behravan discussed thermo-elastic solutions for exponentially graded circular/annular plates subjected to asymmetric loads [131] and circular plates resting on gradient hybrid foundation [132] using semi-analytical methods. The influence of thermal gradient index on displacement and stresses were found to be more than the elastic grading indices. Fallah and Nosier adopted FSDT to study linear [133] and nonlinear [134] behaviour of FGM circular plates with various clamped and simply supported boundary conditions subjected to asymmetric thermo-mechanical loads. Zhang and Zhou [135] used higher order shear deformation theory with multi-term Ritz method to evaluate the nonlinear bending response of FGM circular plates. Temperature dependent material properties were assumed according to power law function and solutions obtained for three cases of temperature fields. Kiani [136] analyzed steady state thermo-elastic response of rotating functionally graded nanoplates using surface elasticity approach. Static bending problems in a thermally stressed annular and circular micro plates were discussed in [137] using DQM. The effect of influencing factors like applied loads, plate geometry ratios and material inhomogeneity were discussed.

3.2.2. Numerical methods

Tanaka et al. [43] presented a new thermoelastic material design called Mori-Tanaka's theory to reduce thermal stresses in FGM plates. Incremental finite element methods (FEM) with direct sensitivity analysis and optimization techniques were used to arrive at the optimal volume fraction of phases in FGM. Further, the improved solutions were discussed in [42], which considered multiple design parameters and accounted volume fraction dependence of material properties to estimate the micro structural behaviour. The differences in optimized stresses predicted using Mori-Tanaka's theory was found to be small, because it predicts a lesser value of ceramic rich FGM phase. Reddy and Chin [73] performed parametric studies on FGM cylinders and plates subjected to abrupt thermal loads using power law function. Thermomechanical coupling effects were discussed by comparing the results of coupled and uncoupled formulations. The static and dynamic thermo-elastic responses for FGM plates including geometric nonlinear effects were discussed in [31] and for FGM cylinders exposed to rapid heating conditions in [138]. Finite element models based on R-TSDT was developed by Reddy [119], which accounted the time dependency effect along with thermomechanical coupling and geometric nonlinearity. Cho and Oden [139] presented parametric studies for thermally stressed FGM plates using Crank-Nicolson-Galerkin scheme. Significant difference was observed in the thermal characteristics of transient and steady-state responses. Apalak [33] adopted eight noded isoparamteric multilayered finite element to study the effect of thermal residual stresses and discussed the method to determine optimum material composition to reduce the effect of stresses. Muliana [140] introduced a new micromechanical modeling method by idealizing the spatial distribution of spherical particles in a homogeneous matrix for predicting the thermo-visco-elastic response of FGM plates. Static and dynamic characteristics of Reissner–Mindlin plates were studied using Non-Uniform Rational B-spline (NURBS) based isogeometric FEM [141] and cell-based smoothed FEM with discrete shear gap technique [142] for MT-FGM plates. Gulshan et al. [143] used R-TSDT with C^0 continuous isoparametric Lagrangian finite element with seven degrees of freedom at each node to study the static response of FGM skew plates. Kulikov and Plotnikova [144] dealt with the implementation of sampling surfaces method and state space method to study the response of laminated FGM plates. In three dimensional stress problems, convergence of results was observed by introducing sampling surfaces inside each layer at chebyshev polynomial nodes. Thai et al. [145] presented shear and normal deformation theory with four unknown variables to predict the static and dynamic response of FGM isotropic and sandwich plates using both Mori-Tanka and power law homogenization schemes. FEM formulations were developed using NURBS based Isogeometric Analysis (IGA) and were discretized using C^1 continuity of displacement field. The accuracy of the model has been established by comparing with various other models.

In recent decades, many investigations have been reported based on meshless or meshfree based interpolation methods. Dai et al. [146] presented dynamic response of FGM plates with piezoelectric sensors and actuators based on FSDT and by using element free Galerkin method and Moving Least Square (MLS) method with C^1 continuity. Golmakani and Kadkhodayan [147] studied nonlinear large deflections of circular and annular plates using power law function for Temperature Dependent (TD) and Temperature Independent (TID) variation of material properties across the plate thickness. The solutions for FSDT model were obtained using dynamic relaxation method (DRM) and finite difference discretization technique. It was concluded that, the value of deflections and stresses predicted for plates with simply supported edges are greater than for plates with clamped edges. Wu et al. [148] used meshless collocation method based on the differential reproducing kernel (DRK) inter-polation technique for the analysis of multilayered FG electro-thermo-elastic plates. Edge cracking of FGM plates subjected to thermal shock has been discussed by Burlayenko [149]. A meshless local Petrov–Galerkin (MLPG) approach has been adopted by Sladek et al. [150] for the bending analysis of circular piezoelectric FGM plates under static and transient dynamic mechanical and thermal loads. The physical quantities were approximated using Moving Least Square method and Local integral equations. It was concluded that, the mechanical deflection and electric potential are highly influenced by gradation of thermal expansion coefficient rather than other parameters.

3.3. One-dimensional constant and linear temperature variation

Few researchers carried out stress analysis of FGM plates subjected to either constant or linearly varying temperatures across the plate thickness. Some of the articles dealing with the evaluation of both linear and nonlinear temperature profiles are already discussed in the previous sections and hence for brevity they are not dealt here again. Based on the method of analysis, the reported research studies are grouped under two sections namely analytical and numerical methods.

3.3.1. Analytical methods

Fukui et al. [151] studied thermal stresses for a thick walled FGM tubes subjected to uniform thermal loads. It was found that the, distribution of stresses and strains mainly depends upon the gradation of components in the radial direction. Tsukamoto [55] presented combined micro- and macro-mechanical approaches, for the analyses of an FGM plate subjected to transient thermal stresses. In-plane and out-of-plane micro stresses were derived using Eshelby's equivalent inclusion method and Mori–Tanaka's mean-field approximation, and the macro-mechanical analysis was carried out based on the CLPT. The significance of considering inelastic deformations for elastic, elastic-plastic and elastic-plastic-creep analysis has been discussed. Chung and Chang [152] obtained Fourier series solutions for a simply supported rectangular plates using CLPT by considering P-FGM, E-FGM and S-FGM material property variations. It was observed that S-FGMs are most sensitive to the variations in the ratio of thermal coefficient of expansions than that of P-FGM and E-FGM plates.

Shen [153] considered TD material properties to study the nonlinear bending response of a simply supported rectangular FGM plate. Governing equations were derived for R-TSDT and the solutions were obtained using mixed Galerkin-perturbation technique. This work was extended by Yang and Shen [154] for combined mechanical and thermal loads with different boundary conditions. It was found that the material gradation, volume fraction, temperature variation, plate geometry, boundary conditions, transverse shear deformations have significant effect in determining the nonlinear bending behaviour of FGM plates, while the effect of in-plane constraints were negligible. Matsunaga [79] adopted higher order shear deformation theory for the thermal stress analysis of a simply supported FG plate subjected to constant and linear variation of thermal loads. The effect of geometric nonlinearity in FGM plate was examined by Kumar et al. [155]. It was observed that nonlinear deflections were more in plates subjected to thermal loads. Analytical and numerical models were developed by Sadowski et al. [156] to study the response of structural elements under thermal and mechanical loads. Accuracy of solutions obtained by analytical formulations and finite element methods were found to be in good correlation with each other.

3.3.2. Numerical methods

Apalak [33] discussed the effect of residual stresses in FGM plates using 3D eight noded isoparamteric multilayered finite element with three degrees of freedom at each node and with 2500 layers through the plate thickness. Bhandari and Purohit [62] adopted eight-node quadratic Lagrange element and performed comparative studies on FGM plates subjected to thermomechanical loads for different material gradient laws (P-FGM, S-FGM and E-FGM) and boundary conditions (simply supported, clamped, free and combined) using FSDT. The deflections and stresses were found to remain closer to each other with increase in power law parameter in S-FGM plates than that of P-FGM and E-FGM plates. Natarajan and Manickam [157] considered QUAD-8 shear flexible element to study the bending behaviour of sandwich plates using zig-zag functions. The accuracy of HSDT with 13 degrees-of-freedom at each node was established by comparing the results with other lower order models with 11, 9 and 5 degrees of freedom at each node. Sadowski et al. [156] investigated mechanical and thermal responses of structural elements using finite element method for an airplane made of FGM. Comparative studies were performed with simplified analytical models and a good conformability between the two methods was established. Bui et al. [158] developed FEM formulations to study the static bending behaviour of heated FGM plates using TSDT and power law function. Parametric studies were performed for ZrO_2/SUS_30_4, Al_2O_3/SUS_30_4 and Si_3N_4/SUS_30_4 plates with different shape configurations. Due to the nonlinear behaviour of constituent phases and thermal expansion coefficients, the response predicted by ZrO_2/SUS_30_4 was found to be quite different from other materials.

4. Thermal vibration analysis

Most of the advanced applications of FGMs are found in supersonic and hypersonic space vehicles, where very high temperature gradients are attained in short period of time. Therefore the dynamic behaviour of these structural members in thermal environments is one of the significant areas of research. Most of the investigations are focused on evaluating the free and forced vibration frequencies of FGM plates by solving the eigenvalue problem. Though some of the studies are reported on three-dimensional elasticity solutions, they were found to use one-dimensional variation of temperature profiles. Also, most of the studies were focused on evaluating vibrational frequencies under different temperature conditions. Hence in this section, various temperature profiles are grouped together and are discussed under the same section namely: one-dimensional constant, linear and nonlinear temperature variation.

4.1. One-dimensional constant, linear and nonlinear temperature variation

In this section, the various investigations carried out for the free and forced vibration analysis in FGM plates using both three dimensional and two dimensional theories are discussed. The different analytical solution methods used are dealt under analytical methods while finite element and meshless method are discussed under numerical methods.

4.1.1. Analytical methods

Free vibration and transient response of initially stressed FGM plates subjected to impulsive lateral patch load without or resting on Pasternak type elastic foundations were studied using CLPT by Yang and Shen [159]. The solutions for free vibration analysis were obtained from 1-D Differential Quadrature approximation and

Galerkin method, while modal superposition method was adopted for transient analysis. The linear and nonlinear vibration characteristics of shear deformable plates were discussed in [160,161]. Higher order shear deformation theories were used to investigate the geometric nonlinear dynamic responses of FGM plates with or without peizoelectric layers in thermal environments [162–164]. Xia and Shen [165] investigated small and large amplitude free vibrations of thermally post-buckled hybrid FGM plates with piezoelectric actuators. The effect of heat conduction and temperature dependency of material properties were discussed and it was observed that, the natural frequencies decreases with the increase in power law parameter and temperature. Shen and Wang [166] evaluated the performance of Voigt model and Mori–Tanaka model for nonlinear free vibration of FGM plates resting on elastic foundations. Comparative studies revealed that the difference caused by these two methods was found to be negligible as compared to the differences due to different solution methods or plate theories.

Kim [75] adopted TSDT and Rayleigh-Ritz method to investigate frequency characteristics of FGM plates subjected to two types of nonlinear temperature distributions across the thickness. Random free vibration of FGM laminates were dealt by Kitipornchai et al. [167] using semi-analytical method and first order perturbation technique. The second order frequencies were obtained from randomness in thermo-elastic properties of plate and were found to be greatly influenced by temperature, boundary conditions and layup schemes. Benachour et al. [168] used a new four variable refined theory to study the free vibration response of FGM plates. Accuracy of the theory was established by comparing with first order and other higher order theories. The geometric nonlinear dynamic responses was investigated by Hao et al. [169] for FGM plates subjected to in-plane and transversal excitations in time dependent thermal environments. Akbarzadeh et al. [94] obtained analytical solutions based on coupled thermoelastic assumptions for FGM plates subjected to lateral thermal shock. The effect of coupling was in the form of damping because of which the vibration amplitude decreased and the frequency increased with increase in time. Chakraverty and Pradhan [170] investigated free vibration of E-FGM plates subjected to different combination of boundary conditions within the framework of Classical Plate Theory and Rayleigh–Ritz method. The frequency parameter was found to decrease with increase in temperature gradient for all edge conditions and thermal state considered. Hong [171,172] presented thermal vibrations and transient response of a simply supported Terfenol-D FGM plates mounted with magnetostrictive layers using generalized differential quadrature method. Fazzolari [173] carried out free vibration and stability analysis of FGM sandwich plates subjected to three types of thermal loads. A new advanced Hierarchical Trigonometric Ritz Formulation was developed using refined higher order plate models. The model characteristics of P-FGM and S-FGM plates with temperature dependent properties were investigated in [174] and it was observed that the natural frequencies were found to be of lesser magnitude in S-FGM plates. Duc and Cong [175] presented nonlinear thermal dynamic response of thick FGM plates resting on elastic foundations using FSDT and stress functions without using Volmir's assumptions. The studies were extended for R-TSDT [176] for uniform and nonlinear rise in temperatures. The impact of Pasternak foundations was found to be much better than Wrinkler foundation on dynamic behaviour of thick FGM plates in thermal environments. Sobhy [177] illustrated hygrothermal vibration of FGM sandwich plates resting on Winkler–Pasternak elastic foundations. It was concluded that the fundamental natural frequency of FGM sandwich plate decreases with increase in volume fraction index.

Li and Lu [178] derived exact solutions based on three dimensional linear theory of elasticity to investigate the free vibration response of FGM rectangular plates with various support conditions. The natural frequencies obtained from Chebyshev–Ritz method were found to decrease with increase in volume fraction index for all types of boundary condition considered. Farid et al. [179] presented formulations and solutions based on three dimensional elasticity theory for simply supported FGM curved panels resting on two parameter elastic foundations, subjected to uniform and nonuniformly distributed thermal loads. Nie and Zhong [180] studied free and forced vibrations of exponentially graded circular plates using semi-analytical solutions based on state space method. Three dimensional free vibration of FGM plates with general boundary conditions were discussed in [181,182] using Rayleigh-Ritz method for temperature dependent and independent material properties. Similar investigations based on 3-D elasticity theory were carried out using Chebyshev–Ritz method [183,184] and using differential quadrature method [185] to investigate free vibration of FGM annular plates with different boundary conditions. The effect of temperature rise, geometric parameters and gradation index were investigated and it was observed that natural frequencies found to decrease with increase in temperature rise. Ansari et al. [186] demonstrated vibration characteristics of FGM nanoplates in prebuckling domain. The effect of surface stresses was included in classical plate theory by implementing Gurtin–Murdoch elasticity theory. The effect of surface stresses was found to be most predominant in nanoplates with lesser plate thickness. Eshraghi et al. [137] introduced new solution methods based on modified couple stress theory and DQM, to study free vibration of FGM annular and circular micro plates subjected to thermal loads. When the plates were under the influence of initial thermal stress, the first dimensionless natural frequency was found to decrease with increase in surface temperature.

4.1.2. Numerical methods

Liew and co-authors [187,188] developed finite element formulations based on FSDT for active control of FGM plates integrated with piezoelectric sensors/actuators. The work was later extended to implement element free kp-Ritz method by Zhoa et al. [189], in which the influence of parametric variations were discussed. The effect of geometric nonlinearity in FGM rectangular and skew plates were studied by Sundararajan et al. [190] based on Mori-Tanaka scheme with temperature dependent properties. Nonlinear thermal vibrations in pre- and post- buckled regions were discussed for FGM plates by Park and Kim [191] and for FGM plates resting on elastic Pasternak foundation by Taczala et al. [192] based on FSDT. The nonlinear frequency ratio was found to decrease initially and then increases with increase in volume fraction index. Malekzadeh and Alibeygi [193] adopted geometric mapping technique with differential quadrature method. Natarajan et al. [142] adopted cell-based smoothed finite element method with discrete shear gap technique. Parandvar and Farid [194] used a new finite element modal reduction method called modified system identification base (MSIB) method to solve free vibration equations of motion for FGM plates. In all the above three papers, temperature dependent material properties along with power law function was used. Valizadeh et al. [141] used FSDT and NURBS based Bubnov-Galerkin iso-geometric finite element method to study the linear free flexural vibration and supersonic flutter analysis of FGM plates. It was observed that, metallic volume fraction increases with increase in volume fraction index, due to which there is a reduction in stiffness of the material structure and hence vibration and flutter frequencies also reduces. Jari et al. [195] investigated nonlinear thermo-mechanical free vibration based on R-TSDT and NURBS-based isogeometric approach with C^1 continuity. Alijani et al. [196] studied dynamic response of FGM plates using FSDT adopting nonlinear finite element method and multi-modal energy approach with 30 degrees-of-freedom. Similar studies were carried out by Parandvar and Farid [197] using HSDT and by reducing the number of degrees-of-freedom to 4. This was achieved by using modal reduction method along with shooting method and numerical time integration technique. Bui et al. [158] evaluated natural frequencies of FGM plates with different configurations based on TSDT. It was observed that, the natural frequencies of FGM plates possess similar behaviours, regardless of the constituent materials being used to form the plates.

Fakhari et al. [198] studied nonlinear natural frequencies and dynamic responses of FGM plates with surface bonded peizoelectric layers based on HSDT. The effect of applied voltage was found to be negligible in predicting the response of the plate. Prakash and Ganapathi [199] analyzed asymmetric free vibration response of circular FGM plates using three-noded shear flexible plate element based on the field-consistency principle subjected to uniformly distributed thermal loads. Kiani and Eslami [200] derived the nonlinear temperature distribution for circular FGM plates using Crank–Nicolson method based on hybrid iterative central finite difference technique. The effect of arbitrary thermal shock, geometric nonlinearity and temperature dependency was investigated using Hamilton's principle and Newton– Raphson–Newmark iterative method. Natarajan and Manickam [157] investigated free flexural vibrations of sandwich plates using QUAD-8 shear flexible element and HSDT model with 13 and 11 degrees-of-freedom. The responses were studied for two types of plates configurations namely, FGM face sheets with homogeneous core and the other with homogeneous face sheets with FGM core. Layerwise finite element formulations were developed for the same configurations by Pandey and Pradyumna [201], to study the dynamic response of FG sandwich plates subjected to nonlinear thermal loads and temperature dependent material properties. The studies were carried out using both Mori-Tanaka scheme and rule of mixtures. It was observed that, the natural frequencies obtained using both the methods were found to be lesser for sandwich plates with thinner FGM layer as compared with that of the plates having thick FGM layer.

5. Thermal buckling analysis

When FGMs are exposed to high temperature fields, their structural integrity will be lost and becomes geometrically unstable. Buckling is a stability state of a structural element, at which the stable flat equilibrium configuration of the plate begins to deform and starts bending. The in-plane thermal load which creates instability in the structure is called critical buckling temperature. If this temperature is further increased, the growth of deflections and stresses will accelerate and eventually leads to the failure of structure. Though many research studies have already been reported on thermal stability of FGM plates using two dimensional models, three dimensional exact solutions are not yet reported in the literature. In the following sections, FGM plates subjected to constant, linear and nonlinear variations of temperature distribution are discussed. The various analytical and numerical methods are dealt in subsections.

5.1. One dimensional nonlinear temperature variation

Studies related to thermal buckling of FGM plates subjected to nonlinearly varying thermal loads are discussed based on the method of analysis adopted and are classified into two sections namely analytical and numerical methods.

5.1.1. Analytical methods

Lanhe [202] derived equilibrium and stability equations for moderately thick simply supported FGM plates based on FSDT. Analytical solutions were obtained for uniform temperature rise and steady state temperature across the plate thickness. Javaheri and Eslami [203] carried out buckling analysis of rectangular FGM plates under four types of thermal loads. Critical buckling temperatures evaluated using HSDT model was found to be the most accurate, whereas the same was overestimated by CLPT. Shariat and Eslami studied thermal buckling analysis of FGM imperfect plates based on CLPT [204] and FGM thick rectangular plates based on TSDT [34] for nonlinear and linear composition of constituent materials respectively. The buckling temperature was found to be higher for nonlinear variation as compared to uniform temperature rise across the thickness of the plate. Tung and Duc [205] employed Galerkin method to study buckling response of FGM plates with geometric imperfections based on CLPT. The work was extended to study the postbuckling response of shear deformable FGM plates resting on elastic foundations based on HSDT [206]. The influence of geometric parameters and in-plane boundary conditions were discussed. Bouazza [207] constructed nonlinear stability equations based on FSDT using power law function. The effect of transverse shear deformation in predicting the critical buckling temperature was found to be significant in thick plates with large aspect ratio. Zhang and Zhou [208] used the concept of physical neutral surface and HSDT to investigate thermal postbuckling response of FGM plates resting on elastic foundations. Nonlinear approximate solutions were obtained using Multi-term Ritz method for various boundary conditions. Post buckling behaviour was found to be different for different types of thermal loads and boundary conditions. Lee et al. [209] discussed the thermal buckling behaviour of FGM plates based on neutral surface concept. Temperature dependent material properties with heat transfer effects and FSDT model were used. Significance of this concept was observed in models with large difference of Young's modulus, because in such cases neutral surface will be located away from the mid-surface. Due to which the critical buckling temperature and other thermal related behaviours of FGM plates will be affected. Shen [210] studied thermal postbuckling response of simply supported FGM plates with geometric mid-plane symmetry. Material properties were assumed to be temperature dependent and two cases of temperature field across the plate thickness i.e., in-plane non-uniform parabolic variation and steady state heat conduction were considered. For geometrically perfect plates under the case of heat conduction, the post buckling path was no longer of the bifurcation type. Zenkour and Sobhy [211] adopted sinusoidal shear deformation theory to study the buckling behaviour of symmetric FGM sandwich plates subjected to nonlinear thermal load. Sobhy [177] introduced a new four variable shear deformation plate theory to investigate the hygrothermal buckling behaviour of sandwich FGM plates resting on Winkler–Pasternak elastic foundations. Several types of plate configurations were considered in the analysis and the accuracy of the model was established by comparing with various other models that are already reported in the literature. Fazzolari and Carrera [212] employed hierarchical trigonometric Ritz formulation to evaluate the critical buckling temperature of FGM isotropic and sandwich plates using quasi 3D-Equivalent single layer and zig-zag plate models that were developed on the basis of CUF. The critical temperatures were found to be the most accurate in these models compared to CLPT, FSDT and HSDT, and the highest values were observed in a fully clamped FGM plate.

Najafizadeh and Eslami [213] studied symmetrical buckling behaviour of solid circular plates subjected to thermal loads based on Kirchhoff plate theory. Najafizadeh and Heydari investigated thermal buckling of FGM circular plates based on TSDT [214] and FSDT [215] subjected to uniform and nonlinear variation of thermal loads. It was observed that the critical buckling temperature predicted by TSDT was found to be more accurate, whereas it was over-predicted by FSDT and CLPT models. Nonlinear post buckling behaviour of solid FGM circular plates for various boundary conditions was studied by Fallah and Nosier [134] based on First order Von-karman theory. Bifurcation-type buckling was observed in clamped FGM plates with immovable edges in radial direction, while in simply supported FGM plates snap-through buckling behaviour was observed under thermal and thermo-mechanical loads respectively. Kiani and Eslami [216] solved thermal instability problem for heated thin annular FGM plates resting on elastic foundations based on CLPT for simply supported, free and clamped boundary conditions. Later, the work was extended to study the linear and nonlinear stability behaviour of rotating circular FGM plates based on CLPT and Touloukian model [217]. Snap through phenomenon was observed in rotating FGM plates which were buckled under in-plane thermal loads. Further, Ghiasian et al. [218] studied the response of shear deformable temperature dependent circular/annular plates for various support conditions. In most of the cases discussed, the fundamental buckling pattern of symmetrically heated annular plates was found to be asymmetric. Zhang [219] determined the post buckling patterns in elliptical FGM plates using HSDT and Ritz method for various boundary conditions. The bifurcation type of buckling was not observed in FGM plates subjected to thermal loads and with immovable clamped edges. Mansouri and Shariyat [220] discussed biaxial buckling response of orthotropic FGM auxetic plates in hygrothermal environments based on R-TSDT and DQM. Ashoori and Sadough [221] extended modified couple stress theory for the analysis of heated annular size-dependent FGM plates resting on elastic foundations and subjected to thermo-mechanical loads. It was concluded that, Bifurcation-type buckling was observed only in plates with clamped edges.

5.1.2. Numerical methods

Three dimensional thermal buckling analysis of FGM plates was first presented by Na and Kim [222] using finite element model with 18 noded solid element for TD Material properties. Thermal stability of clamped FGM plates was discussed under uniform, linear and sinusoidal temperature rise across the thickness. The effect of time dependent temperature rise were dealt using Crank–Nicolson method [223]. Further, buckling and postbuckling response of FGM plates were investigated using Green–Lagrange non- linear strain-displacement relation for TID material properties [224] based on 3-D finite element method and FSDT respectively. Significance of volume fraction index in predicting the response of FGM plates was discussed and also, highest value of buckling temperature was observed in sinusoidal temperature rise. Prakash et al. [225] discussed the influence of neutral surface position in predicting the stability of FGM plates exposed to high thermal conditions based on FSDT and Newton–Raphson method. The thermal stress resultants were found to cross the mid-surface of the plate, under critical temperature gradient and exhibiting snap-through behaviour. Ghannadpour et al. [226] discretized FGM plate into some finite strips to derive the fundamental equilibrium equations based on CLPT and principle of minimum potential energy to study buckling response of FGM plate. The above semi-analytical method was extended to predict the post buckling behaviour of simply supported plates subjected to different types of thermal loads and boundary conditions [227]. Critical buckling temperature of simply supported FGM plates was found to be least, as compared to other support conditions. Lal et al. [228] presented direct iterative based stochastic finite element method with randomness in system properties to study the second order statistics of post buckling responses for FGM plates with TID and TD material properties. The nonlinear formulations were developed based on HSDT using C^0 nonlinear finite element method and mean centered first order perturbation technique. Thermal buckling characteristics of FGM plates were studied by Valizadeh et al. [141] using FSDT and NURBS based isogeometric finite element method. Natarajan et al. [142] adopted a cell-based smoothed finite element method with discrete shear gap technique and Mori-Tanaka homogenization method with temperature dependent properties to compute thermal buckling of FGM plates with circular cutout at the centre. The accuracy of the method and its insensitivity to shear locking were discussed. Tran et al. [229] adopted NURBS-based isogeometric finite element approach in combination with HSDT to formulate equilibrium and stability equations of FGM plates subjected to various types of thermal loads. The work was further extended to study nonlinear buckling response of FGM plates using temperature dependent material properties [230]. Reduction in plate stiffness was observed due to thermal membrane effect and hence nonlinear deflections were found to be greater than the linear ones. Jari et al. [195] used higher order NURBS which satisfies C^1continuity criteria as the basis functions for HSDT model, which resolved the shear locking problems in stiffness formulations. Mirsalehi et al. [231] examined buckling characteristics of thin FGM microplate using spline finite strip method, based on modified couple stress theory for various support conditions. Parametric studies were performed and it was observed that, critical buckling temperature of plate reduces with increase in length.

Ganapathi and Prakash [232] carried out buckling analysis of simply supported FGM skew plates using eight-noded C^0 shear flexible quadrilateral plate element based on consistency approach and FSDT. Material properties were assumed according to power law function and nonlinear temperature using heat conduction equation. This work was extended to study the geometrical nonlinear postbuckling response of FGM skew plates under different boundary conditions and Mori-Tanaka scheme with temperature dependent material properties [233]. The existence of bifurcation-type buckling was investigated using shear deformable finite element approach and direct iterative technique. The thermal load carrying capacity of the plate was found to increases with increase in the skew angle. Malekzadeh [234] derived three dimensional equilibrium equations for an arbitrary straight-sided quadrilateral FGM plates with TD properties based on geometric mapping technique and differential quadrature method. Buckling behaviour of FGM plates with various shapes and boundary conditions were discussed. Thai et al. [145] presented a simple four-unknown shear and normal deformations theory for the analyses of FGM isotropic and sandwich plates using NURBS based isogeometric approach with C^1 continuity of displacement field. It was observed that, instead of using polynomial functions any other function which can satisfy the free conditions of shear stresses at top and bottom surface of the plate can be used.

Recently, meshfree methods have been applied for various investigations due to its flexibility in defining the nodal positions. Jaberzadeh et al. [235] adopted element-free Galerkin method and constructed the shape functions using moving least squares approximation to investigate the buckling response of FG skew and trapezoidal plates with different boundary conditions based on CLPT. Zhang et al. [236] presented thermal buckling of different types of E-FGM plates using local Kriging meshless method. The discrete eigen-value equations were developed using FSDT model and local Petrov–Galerkin weak-form formulation combined with shape functions having the Kronecker delta function. Buckling characteristics under various combinations of thermal loads and boundary conditions were discussed and it was observed that for all the parameters considered the critical buckling temperature was higher for linear variation of thermal field.

5.2. One dimensional constant and linear temperature variation

In the following sections, the various research works reported till recently for the thermal buckling of FGM plates and FGM sandwich plates subjected to constant and linear distribution of thermal loads across the thickness of the plate are discussed under analytical and numerical methods.

5.2.1. Analytical methods

Postbuckling response of FGM plates subjected to in-plane and transverse loads were investigated using semi-analytical approach based on CLPT by Yang and Shen [74]. The formulations accounted both for the geometric nonlinearity and temperature dependency of material properties for various combinations of edge supports with or without resting on Pasternak-type elastic foundation. Postbuckling characteristics were highly influenced by volume fraction, geometric parameters, boundary conditions and foundation stiffness. Bouazza et al. [237] derived stability equations for a simply supported FGM plate based on FSDT and Von Karman's nonlinear equations. Four different types of volume fractions were assumed according to power law function, namely linear, quadratic, cubic, and inverse quadratic variation across the thickness of the plate. It was observed that, the difference in critical buckling temperature between the linear and the cubic cases increases with the increase in plate aspect ratio. It was also concluded that the transverse shear deformation has considerable effect on the critical buckling temperature. Matsunaga [238] obtained critical buckling temperatures for simply supported FGM plates based on quasi-static linear thermo-elasticity theory and two dimensional global higher order theories. The material properties were assumed to obey power law function and the Navier's solution technique was used to solve the eigenvalue problem, where coupling effect was neglected. Chen et al. [239] used average stress method to evaluate buckling temperatures of a simply supported hybrid FGM plates subjected to initial stresses and thermal loads. It was concluded that, thermal buckling coefficient decreases due to initial compressive stress and increases with initial tensile stress. Zenkour and Sobhy [211] investigated thermal buckling of several types of symmetric sandwich FGM plates with isotropic core based on sinusoidal shear deformation plate theory. Accuracy of the critical temperatures obtained from present model was established by comparing the results with CLPT, FSDT and HSDT. Geometrical nonlinear thermal postbuckling behaviour of FGM sandwich plates resting on elastic foundation were studied by Tung [240] based on FSDT and Galerkin method for both TD and TID material properties. The influence of geometric imperfection, tangential edge constrains and temperature dependence of material properties were considered in evaluating the buckling and postbuckling responses of plates subjected to uniform external pressure, thermal and thermomechanical loads. Hong et al. [241] presented an iterative analytical procedure based on FSDT to investigate the nonlinear stability of eccentrically stiffened thick S-FGM plates with TD material properties. Later, the same method was extended by incorporating higher order modeling method based on R-TSDT [242]. It was found that, the higher order model was effective in considering the influence of temperature on both FGM plates and stiffeners.

Najafizadeh and Hedayati [215] obtained closed form solutions for FGM circular plates based on FSDT and energy method. Ansari et al. [186] demonstrated thermal buckling response of FGM nanoplates including the effect of surface stresses in prebuckling domain. It was observed that, critical buckling temperature was found to decrease with increase in material gradient index. Yang et al. [243] presented thermomechanical post buckling response of FGM cylindrical panels with initial geometric imperfections. Formulations were developed using classical shell theory and semi-analytical solutions were obtained using DQM. It was observed that the buckling temperature was over-predicted, if temperature dependency of material properties were neglected. Yaghoobi et al. [244] investigated the buckling response of hybrid laminates based on FSDT and the solutions for a two layered FGM plate integrated with surface bonded piezoelectric actuators were obtained using Levy type solution and power series Frobenius method. The thermal buckling strength and temperatures were found to increase with the application of negative voltage on the actuator layers, while decreased by considering temperature dependency effects.

5.2.2. Numerical methods

Nonlinear finite element equations based on FSDT were developed to investigate the post buckling response in FGM plates in thermal environments by Park and Kim [191] and in hygrothermal environments by Lee and Kim [245]. Significant differences in the post buckling pattern between FGM plates and isotropic plates were discussed with both TD and TID type of variations in material properties. Zhao et al. [246] investigated the buckling behaviour in solid plates and plates with square/circular holes at the centre based on FSDT and element-free kp-Ritz method for P-FGM plates. It was concluded that the influence of volume fraction exponent and hole size predominantly affects the critical buckling temperature. Jalali [247] examined thermal stability of sandwich circular plates with variable thickness using pseudo-spectral method. It was concluded that, the thermal buckling factor increases with increase in volume fraction index and decrease in core-to-face sheet thickness ratio. Yu et al. [248] investigated buckling response of FGM plates with internal defects like cracks and cutouts, using extended isogeometric analysis method with level sets and FSDT. It was observed that, the value of critical buckling temperature rise was higher in plates with cutouts or defects located closer to the boundary of the plate than the cutouts at the centre.

6. Future directions for research

FGMs have revolutionized the field of modern material science by incorporating graded microstructure and have proved to serve as a multi-functional material in handling extreme conditions most effectively. The flexibility in design and its performance under thermal environments are the main reasons for its potential applications in various sectors. Therefore, modeling and analysis of FGMs are extremely important for the development of this emerging new material. Though remarkable investigations are reported, there are few areas which need further extensions in research work.

1. 3D exact solutions reported for thermal stress analysis of FGM plates are developed using three dimensional distribution of temperature field. There is also a need to develop such a solution for the thermal vibration and buckling analyses of FGM plates.
2. A large number of two dimensional theories assume transverse deformations to be linear, which is not a valid assumption for thermal analysis. Therefore, 2-D computational models have to be developed by including higher order transverse displacement terms for accurate evaluation of thermal responses.
3. Computational techniques based on numerical methods have to be developed for 3-D analysis of FGMs, so that time involved and cost incurred for the analysis can be reduced. The finite element and meshless methods are more flexible in handling various plate geometries and boundary conditions.
4. Most of the numerical methods reported in the literature were based on FSDT. Hence, an attempt has to be made to develop a reliable finite element formulations and solutions using HSDT. Also the accuracy of various computational models for plates with complex geometry and boundary conditions including geometric nonlinear effects has to be evaluated.

5. The analytical formulations and solutions have to be developed for nonlinearly varying thermal loads using higher order displacement models, including geometric nonlinear effects. Also, the effect of temperature dependency has to be studied for complex geometries and boundary conditions.
6. Though most of the 2-D theories have discussed power law variation of material properties, three dimensional thermo-elastic solutions have not been reported yet.
7. The current studies have limited to evaluation of temperature independent properties. A focused attention is required for the development of temperature dependent analysis for various geometries including nonlinear effects.
8. Evaluations of most appropriate temperature distribution, for the development of analytical models have to be studied precisely for accurate evaluation of plate deformations.

7. Conclusions

A review of progressive developments in the thermal stress, vibration and buckling analyses of single/multi-layered FGM and sandwich plates are presented. Most of the theories/computational models employed for the analysis of composite laminates/isotropic plates are extended for FGM plates. Three dimensional elasticity solutions were found to be the most accurate solutions for thermal analysis of FGM plates. Due to the mathematical complexity involved the studies are restricted to analytical methods with only simply supported edges or clamped edges. Most of the three dimensional solutions that are reported on stress analysis of FGM plates have adopted three dimensional distribution of temperature variations for thermal analysis, whereas vibration and buckling analyses limits to one-dimensional variation of temperature distribution. Hence, exact solutions for thermal vibration and buckling analyses are still not available. Exact three dimensional solutions using power law function has not been dealt. Also, 3D solutions including geometric nonlinear effects and temperature dependent variations in material properties are not reported yet. The computational effort and cost involved is very high for 3D analysis and hence development of most accurate solution methods based on approximate methods/2D theories is gaining attention by researchers.

Most of the 2D plate theories like CLPT, FSDT, HSDT, TSDT, CUF, non-polynomial based refined plate theories, etc., have considered only the effect of transverse shear deformation in predicting the global responses of FGM plates, while only few displacement models have focused on including the effect of both transverse shear and transverse normal deformations. It has to be noted that the assumption of constant transverse deflection in 2D plate theories were found to be invalid for thermal analysis of FGM plates and thus the effect of higher order transverse displacement terms have to be employed for accurate prediction of responses. Among the available 2D theories, FSDT, R-TSDT and HSDT have been discussed widely in literature for various plate geometries, temperatures, initial imperfections and boundary conditions. Few semi-analytical computational models are also developed for the thermal analysis of FGM plates based on state-space method, asymptotic method, DRQ technique, etc. These methods are extended for numerical methods like FEM and meshless methods, and hence found to be consistent.

The micromechanical modeling methods adopted for the estimation of effective material properties for these graded composites plays a crucial role in determining the overall response of the plate. Among several homogenization methods, Mori-Tanaka method was found to consider the effect of neighboring inclusions and its interaction with the constituent phases. Inspite of that, the responses predicted from various idealization schemes were found to agree qualitatively but differ quantitatively. Also FGMs can resist high temperatures and hence the thermal responses need not necessarily intermediate between metal and ceramic plate and are found to be much higher.

The distribution of temperature in an FGM plate significantly affects the stress resultants induced due to thermal load. Appropriate gradation scheme and temperature profiles can reduce the thermal stresses and stress intensity factors. Therefore, residual stresses can be nullified by using multilayered FGMs with constant composition and thickness. Precise evaluation of thermo-elastic responses in graded materials is only possible by using temperature dependent constituent material properties along with heat conduction equation for the temperature distribution. The differences between the deflections and stresses computed using temperature dependent and temperature independent material properties increases with thermal gradient and material gradient index. Also higher inclusion of metal causes deterioration of thermal resistance and degradation of material stiffness. Due to thermal membrane effect, the geometric nonlinear deflections were found to be greater than linear ones. Hence, under thermal loading conditions FGM plates with clamped edges were found to be more stable than plates with other boundary conditions and are capable of neutralizing bifurcation type of instability.

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