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An extended force density method for form finding of constrained cable nets

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ABSTRACT

The force density method (FDM) is a classical method used in linear and nonlinear form. The linear approach presents a quick tool for finding cable net new shapes by solving a set of linear equilibrium equations for certain topology, boundary conditions and assumed cables force density. The nonlinear approach was introduced to solve cable nets under constraints (assigned certain distance between nodes, limit force or unstressed length in some elements). Any type of constraint introduces nonlinearity.

This paper studied the prestressed cable nets and the loaded cable nets. For prestressed cable nets, coordinate constraints to all nodes of the cable net are introduced to modify the shape after graphically examining the preliminary shape. This preliminary shape resulted from linear analysis of assumed distribution of cable force densities. For analyzing cable nets under different load cases, the first load case is analyzed to achieve the coordinate constraints assigned to nodes. Analysis results are node coordinates, cable forces and lengths. Young's modulus and areas of cables are used to calculate the unstressed length of all cables using materialization equations, those lengths are used as constraint in the analysis of other load cases. Forces in all cables under different load cases/combinations are calculated. By using this approach, design of cable net under static load is simplified.

Introduction

Cable nets are structures consisting of cables or bars connected by pins or hinges, where cable elements can carry axial forces only. They have several advantages over traditional construction such as, light weight, high stiffness, elastic behavior and innovative forms. They also allow natural light in case of roofs. Its form depends on its forces and vice versa. This means strong relationship between force and form. Accordingly it could not be dealt with as conventional structures. The cable net shapes, whether pretensioned or not, should be analyzed to get the initial form which is a basic form for the design. Before 1970's, the only possible way was to build a physical model and use photogrammetric tool to measure the deformation under external loads. Linkwitz and Scheck (1971) [8], and Scheck (1974) [5] introduced the force density method. It is a simple and very powerful tool to get the initial form for a given cable connectivity, fixed points and cable force densities by using a system of linear equations to get the exact form and improve it to get the desired shape. Frank Baron (1971) [4] introduced nonlinear analysis of cable structures with the finite element method using stiffness matrix including geometrical matrix. Pellegrino and Calladine (1986) [13], Pellegrino (1990) [14] and Pellegrino (1993) [15] used singular value decomposition

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in mixed structure analysis. Dynamic Relaxation method developed by Day (1965) [2] to analyze concrete pressure vessels, Rayleigh (1967) [11] used the DR concept to obtain static equilibrium at the steady state.

Since its introduction, the form finding using FDM has been extensively researched. Some of the research points are topological mapping method [3], equal distribution of static or quasi-static life loads in the composites retainers of a foot bridge [1], equating sum of coordinate difference/length = 0 to obtain minimal way net [9], tension structures with integrated, linear, actively bent elements [16] and form-finding of cable domes making full use of symmetry [17], mixed cable and strut [6], tensegrity [7], membrane structures using force density and stress density and Dome erection analysis [18].

The great advantage of the FDM is that it is the only method among form findings that does not need cable geometry in advance. Thus, it enables the creation of lots of structural shapes based on their topology by converting the nonlinear relation between force and form to linear relation between cable force densities (force/length) and free joints coordinates.

Computation model

Cable nets are geometrically nonlinear and elastically linear. They sustain large displacements with small deformation in elements. Force density analysis assumes that cables are weightless and their joints are pinned. The analysis may be classified into linear and nonlinear.

The linear analysis is applied when a certain topology, known coordinates of fixed nodes and force densities in elements are known. The stability equations of force densities in Cartesian Coordinates forming a set of linear equations. Those coordinates enable the display of cable net and enable for improvement of this shape by changing fixed point location or position and/or force densities. Distribution of force densities not their values governs the form.

The nonlinear analysis comes from any limitation or constraints applied to cable nets. The constraints are typically not applied to all elements or nodes. The number of nonlinear equations is usually less than the number of cables or nodes and there might be more than one solution. Among those solutions, the one that achieves the constraints with a given tolerance is chosen. The previous researches dealt with fixed distance between nodes, limited force and unstressed length of some cables [5] and reactions [10,12] with defined values in chosen fixed nodes constraints.

Also, minimum way net is a cable net in which forces in all cables are equal. In this analysis, after calculating the cable force using LFDM, all forces are restrained to a chosen value and nonlinear analysis is performed.

In this research, the coordinate constraints are used to enhance and control the form of the initially found form from the LFDM. The coordinate constraints are divided into three different functions in x, y and z or more depending on coordinate combinations. The NLFDM with least square method is used to analyze the constrained cable net. The coordinate constraint is used in both prestressed and loaded cable nets. For loaded cable net, the coordinate constraint is used to control the deformation under loads in first load case and, in the following load cases the unstressed length calculated from the first load case is used as a constraint for the other load cases/combinations.

Force density method

FDM utilizes a specified net topology, fixed point coordinates and force densities of elements to get the coordinates of the free joints at equilibrium state. In this research, plan graph at z equal zero is used to define the topology of the net. Only fixed joints coordinates should be exactly defined. The notations used in describing stability equations are consistent with those used by Scheck (1974) as follows:

Lower case is a vector whereas upper case is a matrix. The same vector symbol in upper case refers to a diagonal matrix having vector elements as its diagonal. Element joints are i and j.

Stability equation

Fig. 1 shows system axis and coordinates. In this system only three stability equations exist and sum of forces in the three Cartesian directions are zero.

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0 \tag{1}$$

Stability equations at a free joint K are as follows:

$$S_a \cos(a,x) + S_b \cos(b,x) + S_c \cos(c,x) + S_d \cos(d,x) = P_x \tag{2a}$$

$$S_a \cos(a,y) + S_b \cos(b,y) + S_c \cos(c,y) + S_d \cos(d,y) = P_y \tag{2b}$$

$$S_a \cos(a,z) + S_b \cos(b,z) + S_c \cos(c,z) + S_d \cos(d,z) = P_z \tag{2c}$$

Where S_a is the force in element a, cos (a, x) is the cosine of the angle between element a and X axis, and P_x is the x component of the applied force at K. Similar equations are derived for Y and Z directions.

Substituting cosine by the normalized projection length, Equations 2 become:

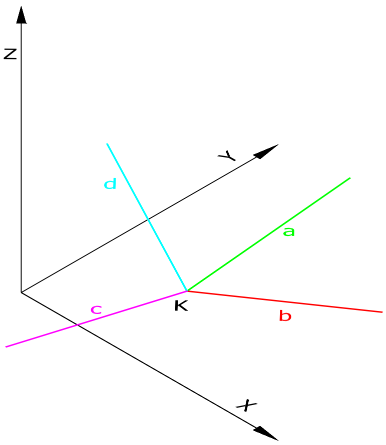

Fig. 1. System axes and coordinates.

$$S_a(x_{ai}-x_{aj})/L_a+S_b(x_{bi}-x_{bj})/L_b+S_c(x_{ci}-x_{cj})/L_c+S_d(x_{di}-x_{dj})/L_d=P_x \tag{3a}$$

$$S_a(y_{ai}-y_{aj})/L_a+S_b(y_{bi}-y_{bj})/L_b+S_c(y_{ci}-y_{cj})/L_c+S_d(y_{di}-y_{dj})/L_d=P_y \tag{3b}$$

$$S_a(z_{ai}-z_{aj})/L_a+S_b(z_{bi}-z_{bj})/L_b+S_c(z_{ci}-z_{cj})/L_c+S_d(z_{di}-z_{dj})/L_d=P_z \tag{3c}$$

The above equations are nonlinear. To linearize them, q is introduced as force density S/L. If q and P are known, the result is a set of linear equations in x, y, and z and it can be formed as follows:

$$q_a(x_{ai}-x_{aj})+q_b(x_{bi}-x_{bj})+q_c(x_{ci}-x_{cj})+q_d(x_{di}-x_{dj})=P_x \tag{4a}$$

$$q_a(y_{ai}-y_{aj})+q_b(y_{bi}-y_{bj})+q_c(y_{ci}-y_{cj})+q_d(y_{di}-y_{dj})=P_y \tag{4b}$$

$$q_a(z_{ai}-z_{aj})+q_b(z_{bi}-z_{bj})+q_c(z_{ci}-z_{cj})+q_d(z_{di}-z_{dj})=P_z \tag{4c}$$

A matrix C_s is defined to be used in obtaining the final equilibrium solution. Matrix C_s is a matrix that describes the element connectivity. So it has m rows and n_s columns, m is the number of elements and n_s is the number of free nodes (n) plus the number of fixed nodes (n_f). Fixed nodes should come after free nodes in the node numbering. All elements of C_s are zero except nodes connecting elements take a value 1 or -1 as follows:

$$c(j,i)\text{of element}: \begin{matrix} 1 & \text{for element first node} \\ -1 & \text{for element second node} \end{matrix}$$

By numbering fixed nodes at the end, C_s is divided into C and C_f (C_s = [C| C_f]).

The coordinate difference vectors are u, v, and w

$$u=Cx+C_f x_f \tag{5a}$$

$$v=Cy+C_f y_f \tag{5b}$$

$$w=Cz+C_f z_f \tag{5c}$$

From Eqs. (4a–c and 5a–c) we get:

$$C^t Q(Cx+C_f x_f)=P_x \tag{6a}$$

$$C^t Q(Cy+C_f y_f)=P_y \tag{6b}$$

$$C^t Q(Cz+C_f z_f)=P_z \tag{6c}$$

Eqs. (6a–c) are linear in x, y and z unknown coordinates.

Prestressed cable net

In case of prestressed cable net the stability equation will be:

$$C^t Q C x = -C^t Q C_f x_f \tag{7a}$$

$$C^t Q C y = -C^t Q C_f y_f \tag{7b}$$

$$C^t Q C z = -C^t Q C_f z_f \tag{7c}$$

Defining the Gaussian transformation matrix of C ($D = C^t Q C$), it could be formed directly from force densities, diagonals are the sum of all force densities connected to the node and off diagonals are negative value of element force density connecting the two nodes.

$D_f = C^t Q C_f$, it has n rows and n_f columns, it could be formed directly by setting negative value of cable force density connecting free node to fixed node, and zero to all other matrix elements.

Multiplying both sides of Eqs. (7a–c) by D^{-1}, we get:

$$x = -D^{-1} D_f x_f \tag{8a}$$

$$y = -D^{-1} D_f y_f \tag{8b}$$

$$z = -D^{-1} D_f z_f \tag{8c}$$

Linear Eqs. (8a–c) are solved to get x, y and z.

General constraint

Constraints are expressed as a function of coordinates and force densities which should be satisfied with an acceptable tolerance. Since coordinates are function of force densities, constraints are function of force densities. Assume r is the number of constraint of n nodes or of m elements.

$$g_i(x, y, z, q) = 0 \quad i(i = 1 : r; r < m) \tag{9}$$

For all r condition

$$g(x, y, z, q) = 0 \quad g(x(q), y(q), z(q), q) = 0 \quad g(q) = 0 \tag{10}$$

The starting set of force densities q^0 will not satisfy the constraint conditions, to solve for q the equations are nonlinear and are solved by iteration using Δq.

$$g(q0) + \frac{\partial g(q^0)}{\partial q} \Delta q = 0 \tag{11}$$

By using:

$$\text{Jacobian matrix } G^T = \frac{\partial g(q^0)}{\partial q} \tag{12}$$

$$\text{and misfit } r = -g(q^0) \tag{13}$$

And substituting Eqs. (12) and (13) in Eq. (11) we get:

$$G^T \Delta q = r \tag{14}$$

The system of r equations is less than the number of nodes/elements. So, it is under determinate and has m−r solutions. The solution that satisfies the minimum sum of Δq square and in the same time minimum misfit value is chosen.

$$\Delta q . \Delta q^T \rightarrow \text{min.} \qquad r . r^T \rightarrow \text{min.} \tag{15}$$

Eq. (14) will be:

$$G^T G \Delta q = G r \tag{16}$$

Then

$$\Delta q = G (G^T G)^{-1} r \tag{17}$$

For the next iteration

$$q^{(new)} = q^{(old)} + \Delta q \tag{18}$$

$q^{(new)}$ is used to get new coordinate Eqs. (6a–c). Compose G^T to get $q^{(new)}$ and check for convergence or maximum number of iterations allowed by the program.

Coordinate constraint

Scheck (1974) introduced node distance, force and length constraints, Malerba (2012) introduced the reaction component constraint.

Coordinate constraint assumes a certain position x, y, and z or one/two or combination of them to some nodes or to all nodes.

$$g(x) = 0 \qquad g(y) = 0 \qquad g(z) = 0 \tag{19}$$

$g(x + y) = 0$ as an example for coordinate combination function.
Taking z coordinate function as an example:

$$\frac{\partial g(q)}{\partial q} = \frac{\partial g(q)}{\partial z} \cdot \frac{\partial z}{\partial q} \tag{20}$$

Using Eqs. (5c and 6c)

$$C^t W q = P_z \tag{21}$$

dq and dz should keep the equilibrium of the system

$$d(C^t W q) = \frac{\partial (C^t W q)}{\partial q} \mathrm{d}q + \frac{\partial (C^t W q)}{\partial z} \mathrm{d}z \tag{22}$$

$$\frac{\partial (C^t W q)}{\partial q} = C^t W \tag{23}$$

$$\frac{\partial (C^t W q)}{\partial z} = \frac{\partial (C^t Q w)}{\partial z} = C^t Q \frac{\partial w}{\partial z} = C^t Q C \tag{24}$$

$$C^t Q \frac{\partial w}{\partial z} = C^t Q C \tag{25}$$

$$\frac{\partial z}{\partial q} = -D C^t W \tag{26}$$

$$\frac{\partial g(z)}{\partial z} = 1 \tag{27}$$

Substitute Eqs. (26) and (27) into Eq. (20)

$$\frac{\partial g(z)}{\partial q} = -D^{-1} C^t W \tag{28}$$

And the same for x and y

$$\frac{\partial g(x)}{\partial q} = -D^{-1} C^t U \tag{29}$$

$$\frac{\partial g(y)}{\partial q} = -D^{-1} C^t V \tag{30}$$

When $r < n_s$, G^T could be formed directly by considering only the constrained nodes, so G^T has dimension rxm.
The Jacobian matrix of the whole coordinate constraints is obtained by sequentially add rows of each class after another.

$$\text{For} \quad g = x + y \quad \frac{\partial g(x, y)}{\partial q} = -D^{-1} C^t U - D^{-1} C^t V \tag{31}$$

Materialization

In order to extend the force density method to be used for cable net analysis for different load cases, the system should be materialized to maintain the unstressed length of each element constant. This is done by applying Hooke's law to the equilibrium set of forces in the elements where first load case is applied. During materialization, the system is no longer linear. The set of nonlinear equations in coordinates and force densities needs to be linearized as previously mentioned in General Constraint.

Unstressed constraint

Hooke's Law $E = \frac{\text{Stress}}{\text{Strain}}$
where E is Young's Modulus

$$E = \frac{S}{A} \Big/ \frac{\Delta L}{L} \tag{32}$$

Substituting $S = q \times L$, $\Delta L = L - L_u$ and $EA = H$ in Eq. (32), the result is $qL = H\frac{L-L_u}{L_u}$, using this result to get:

$$L_u = \frac{H}{H+S} L \tag{33}$$

Eq. (33) is used to calculate the unstressed length for all cable net elements of the final equilibrium resulting from first load case. Then this length is considered as a constraint for the other load cases.

$$G^T = -\overline{L_u^2}\overline{H}^{-1} - \overline{L_u^2}\overline{H}^{-3}(\overline{UC}D^{-1}CU + \overline{VC}D^{-1}CV + \overline{WC}D^{-1}CW) \tag{34}$$

Eq. (34) is the Jacobian matrix of unstressed length constraint.

Program

A computer program is developed to implement the equations formulated in the above sections. The extended force density method (EFDM) can solve two system of cable nets; prestressed and loaded nets. For both systems, the topology and coordinates of fixed nodes of the net (the free nodes are drawn in $z = 0$ plane) are read from drawing exchange format (dxf). Fig. 2 outlines a flowchart for the procedure, identifying the solution algorithm for the prestressed net as well as the loaded net.

For the prestressed cable net, the distribution of cable force densities is read from a text file, the program form the branch node matrix C_s, matrix D and D_f, and solve the set of linear equations 8 to get the free nodes coordinates. The resulted form is displayed after forming dxf file to investigate the shape and enhance it. The enhancement is done by changing free node coordinates. Changes made to the net are transferred to the program as coordinate constraints which could be applied to all nodes. Now the stability equations are no longer linear, but underdetermined nonlinear. Least square method is used to solve these equations to get the force densities in all elements and calculate the cable lengths and forces.

For the loaded cable net, the shape is known in advance. The force densities and loads on the free nodes are read for each load case from text file. For the first load case the linear set of equations 6 is solved to get the initial equilibrium. Coordinate constraints are applied to the initial form, a set of nonlinear equations is solved and dxf file is formed. Then lengths, unstressed lengths and forces are calculated. For the following load cases, the unstressed length calculated from the first load case is set as a constraint for all load cases.

Program verification

To verify the linear part of the program two finite element programs were used; SAP2000 and STAAD.Pro. A rectangular cable net covering an area of 8×6 m is analyzed. Nodes on perimeter are fixed and the entire net is in zero z plane. All free nodes are loaded by 0.5 ton concentrated load in gravity direction, the force divided by length from each program output is used as force density input in EFDM to compare the z coordinate. Fig. 3 shows a plan of the net used for verification and identifies the free nodes in one quarter of the mesh. Table 1 compares nodal displacements in 'z-direction.

From Table 1 it could be seen that the maximum difference between z computed by EFDM, and both SAP2000 and STAAD are 0.50505% and 0.37422%, respectively.

Prestressed unloaded example: Star net

Fig. 4 shows a plan view and node numbering of a star cable net. The net has 36 nodes and 63 members. The perimeter nodes (14 nodes) are fixed, whereas an interior node (6) is z coordinate constrained and all free nodes z coordinate are proportionally constrained also.

The used unit is tonf-m. The nodes 25–36 are fixed, all nodes are in zero z plane except 23, 24, 28 and 36 are in plane $z = 3$ m and node 6 has z constraint of 2 m and all free nodes z coordinate are proportionally constrained too.

By using any value of equal force densities in all elements and solving according to Eq. 7 to get the initial equilibrium position, the z constraint is extended to include z of all free nodes. The iteration stopped after 6 iterations or $\Delta q.\Delta q^T < 0.00001$ and $r.r^T < 1.0E-7$. Fig. 5 shows the LFDM and NLFDM equilibrium position. Fig. 6 shows quarter of the net z position and force densities.

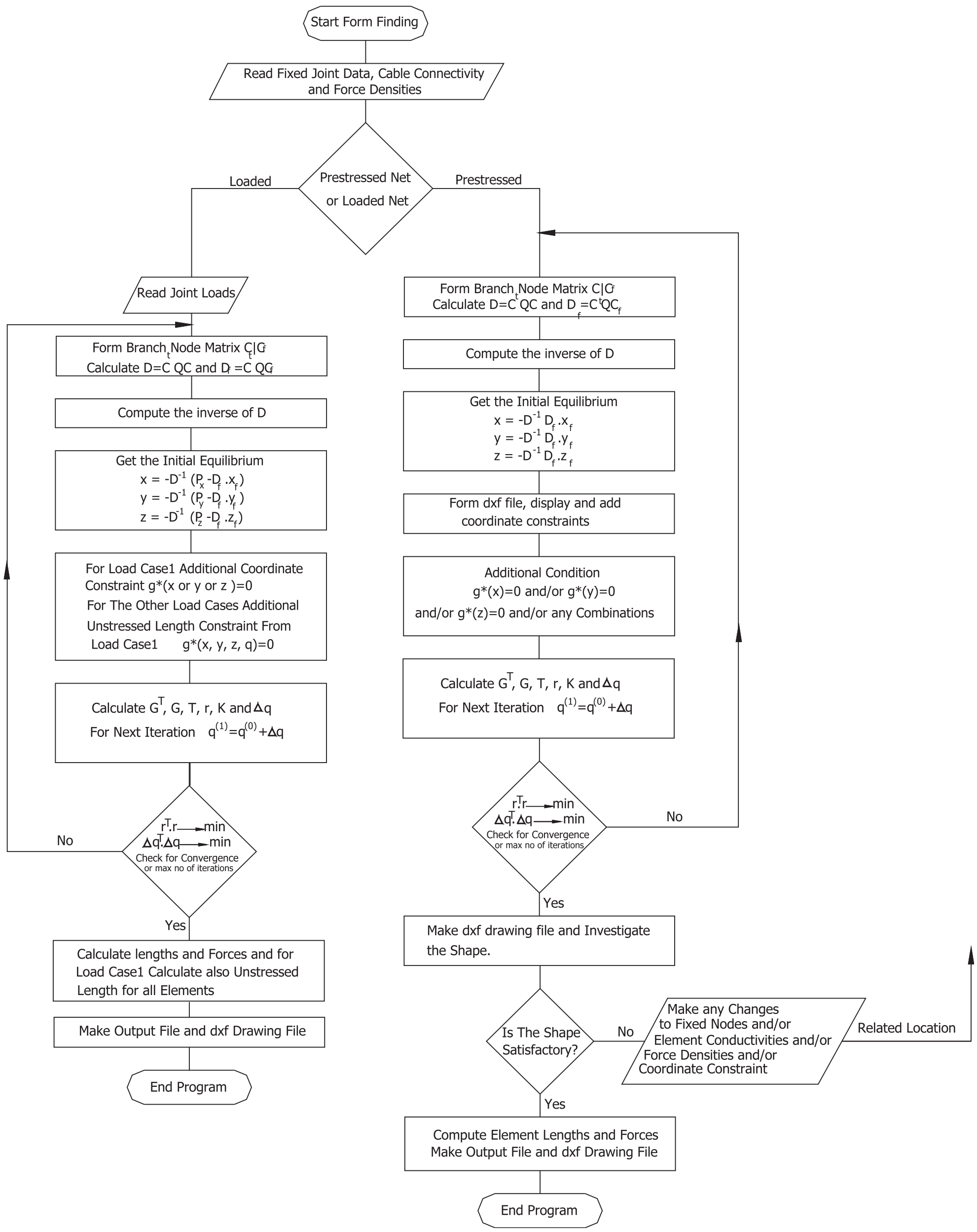

Fig. 2. Program flow chart.

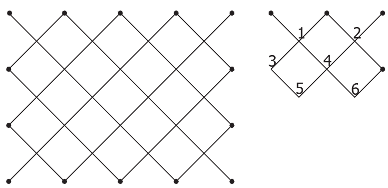

Fig. 3. Cable net for program verification.

Table 1
Program verification nodes *z* coordinate.

Node	EFDM *z*	SAP2000 *z*	% Diff.	EFDM *z*	STAAD *z*	% Diff.
1	−0.06999	−0.06964	**0.50505**	−0.06707	−0.06723	0.24856
2	−0.04489	−0.04484	0.09824	−0.04294	−0.04310	**0.37422**
3	−0.11786	−0.11746	0.33938	−0.11300	−0.11313	0.11823
4	−0.09376	−0.09355	0.23091	−0.09018	−0.09035	0.18918
5	−0.12633	−0.12603	0.23589	−0.12128	−0.12141	0.11049
6	−0.06314	−0.06294	0.30647	−0.06083	−0.06091	0.13481

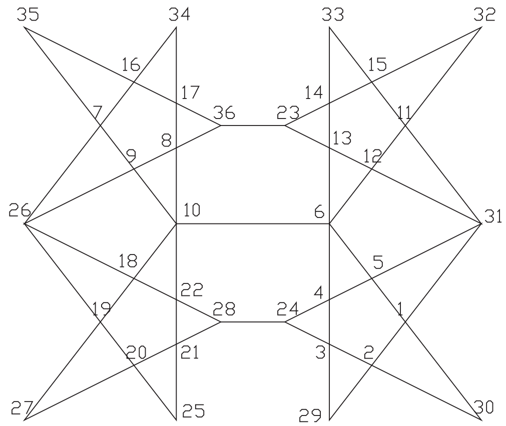

Fig. 4. Plan view and program node numbering of the star net.

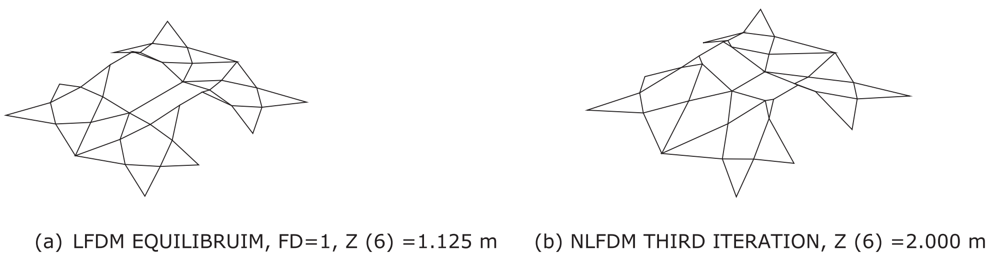

Fig. 5. Prestressed unloaded star net initial and *z* constrained equilibrium shapes.

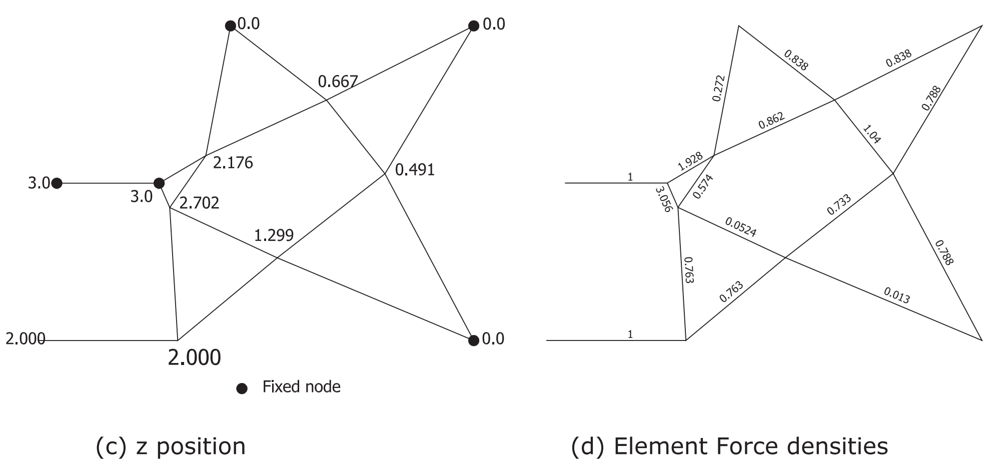

Fig. 6. Quarter of the star net *z* position and force densities.

Loaded net examples

Four examples are presented to demonstrate the capabilities of the proposed algorithm to handle loaded nets. For each example, a figure illustrates node numbering and element numbering. The first table identifies the displacements at four distinct equilibrium states. Symbol a is the equilibrium for load case 1, b is for load case 1 under *z* constraint, c and d are for load cases 2 and 3 respectively based on the unstressed length obtained from b. The second table shows the unstressed length and the corresponding force densities for b, c and d.

Star cable net

It is the same cable net in Section "Prestressed unloaded example: Star net" except nodes 23, 24, 28 and 36 are free. A force density of 5 ton/m is assumed for all elements. The first load is assumed to be 0.4 tons, second load case is 0.54, the third one is 0.7 in gravity direction applied to all free nodes. Fig. 7 shows the nodes and members numbering of a quarter. Table 2 identifies the equilibrium states *z* coordinate and table 3 presents the unstressed length and corresponding force densities for the three load cases.

First the solution determines the initial equilibrium state using LFDM, then, constrain the *z* position of all free nodes by setting the maximum deflection to −0.14 m and the program take the ratio of the maximum *z* to the virtual one and proportionally get the other free nodes *z* constraint. From the final equilibrium position of the first load case, the unstressed length of all elements is the constraint for the following load cases.

Rhombic cable net

A rhombic cable net covers an area of 10 × 10 m. Force density of 5 ton/m is assumed for all elements and three load cases are identified with 0.7, 0.54 and 0.4 tons in gravity direction assumed for all nodes. Maximum deflection is assumed equal to (−0.14 m). Fig. 8 shows a quarter of a rhombic cable net. Table 4 identifies the equilibrium states *z* coordinate and table 5 presents the unstressed length and corresponding force densities of the quarter.

The analysis results shows that node *z* coordinate in load cases b, c and d are slightly different due to the unstressed length constraint applied. It also demonstrates that the unstressed length of each element in all load cases is the same. The resulted force densities in c and d equal to the force densities in b multiplied by the ratio of the applied load. This relation is important and introduce new point to study.

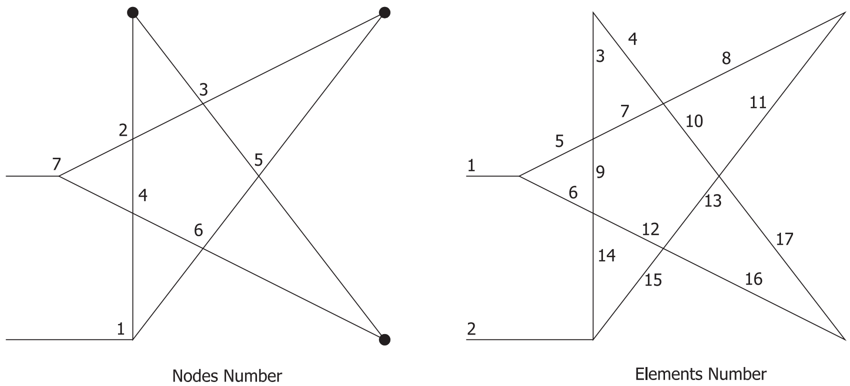

Fig. 7. Nodes and elements numbering of quarter of the star cable net.

Table 2
Nodes (*z*) coordinates.

Element	*z* Coordinate m			
	a	b	c	d
1	−0.1658	−**0.1411**	−0.1443	−0.1472
2	−0.1270	−0.1081	−0.1085	−0.1091
3	−0.0686	−0.0584	−0.0595	−0.0606
4	−0.1707	−0.1452	−0.1468	−0.1483
5	−0.0674	−0.0573	−0.0589	−0.0603
6	−0.1210	−0.1029	−0.1052	−0.1072
7	−0.1888	−0.1607	−0.1623	−0.1638

Table 3
Element unstressed length and force density.

Element	Unstressed length m			Force density ton/m		
	b	c	d	b	c	d
1	3.6159	3.6159	3.6159	5.9414	7.8856	10.0599
2	6.4742	6.4742	6.4742	5.7563	7.3810	9.1334
3	2.7020	2.7020	2.7020	5.8991	8.1858	10.8649
4	3.1429	3.1429	3.1429	5.9319	7.7444	9.7402
5	2.2087	2.2087	2.2087	6.0729	8.0596	10.2818
6	2.4533	2.4533	2.4533	5.2245	6.9370	8.8499
7	2.5136	2.5136	2.5136	5.5923	7.4632	9.5678
8	3.8255	3.8255	3.8255	5.5983	7.4160	9.4465
9	2.6828	2.6828	2.6828	5.2770	7.3766	9.8493
10	3.0847	3.0847	3.0847	5.0281	6.5663	8.2570
11	4.3759	4.3759	4.3759	5.6895	7.2557	8.9304
12	2.4912	2.4912	2.4912	6.1929	8.3170	10.7271
13	2.8567	2.8567	2.8567	5.5346	7.0109	8.5738
14	2.7447	2.7447	2.7447	5.6435	7.8207	10.3673
15	3.2231	3.2231	3.2231	5.5398	7.0034	8.5481
16	3.6920	3.6920	3.6920	6.0355	7.9573	10.0988
17	4.2600	4.2600	4.2600	5.7807	7.4947	9.3666

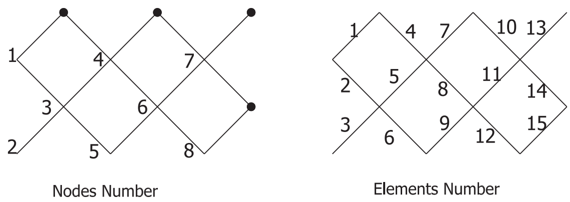

Fig. 8. Nodes and elements number of one-quarter of the rhombic cable net.

Table 4
Nodes (z) coordinates.

Element	z Coordinate m			
	a	b	c	d
1	−0.1543	−0.0792	−0.0806	−0.0791
2	−0.2737	**−0.1405**	−0.1422	−0.1403
3	−0.2387	−0.1225	−0.1243	−0.1223
4	−0.1406	−0.0721	−0.0730	−0.0720
5	−0.2461	−0.1263	−0.1284	−0.1261
6	−0.1836	−0.0942	−0.0956	−0.0941
7	−0.0809	−0.0415	−0.0423	−0.0414
8	−0.1268	−0.0651	−0.0659	−0.0650

Table 5
Element Unstressed length and force density.

Element	Unstressed length m			Force density ton/m		
	b	c	d	b	c	d
1	1.4164	1.4165	1.4164	9.7745	7.5528	5.5903
2	1.4149	1.4149	1.4149	9.7745	7.4114	5.6004
3	1.4143	1.4143	1.4143	9.7745	7.5944	5.5593
4	1.4160	1.4161	1.4160	9.7745	7.4754	5.5762
5	1.4151	1.4151	1.4151	9.7745	7.5945	5.5593
6	1.4142	1.4142	1.4142	9.7745	7.4113	5.6005
7	1.4160	1.4161	1.4160	9.7745	7.5945	5.5593
8	1.4144	1.4144	1.4144	9.7745	7.4754	5.5763
9	1.4146	1.4146	1.4146	9.7745	7.5526	5.5905
10	1.4148	1.4148	1.4148	9.7745	7.4875	5.5951
11	1.4152	1.4152	1.4152	9.7745	7.5526	5.5905
12	1.4145	1.4145	1.4145	9.7745	7.4754	5.5763
13	1.4148	1.4148	1.4148	9.7745	7.5526	5.5905
14	1.4148	1.4148	1.4148	9.7745	7.4875	5.5951
15	1.4157	1.4157	1.4157	9.7745	7.5374	5.5717

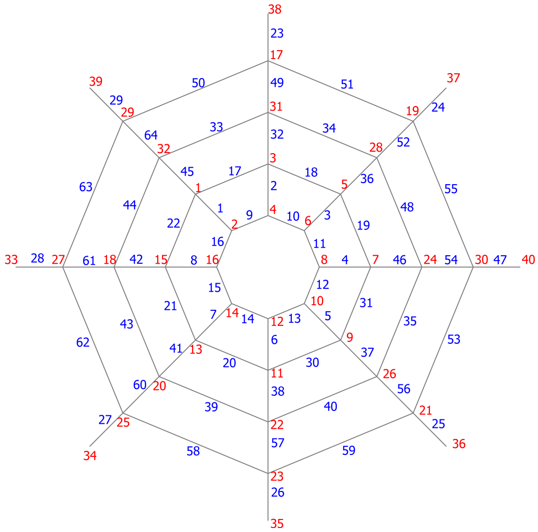

Fig. 9. Plan view and program numbering of circular net.

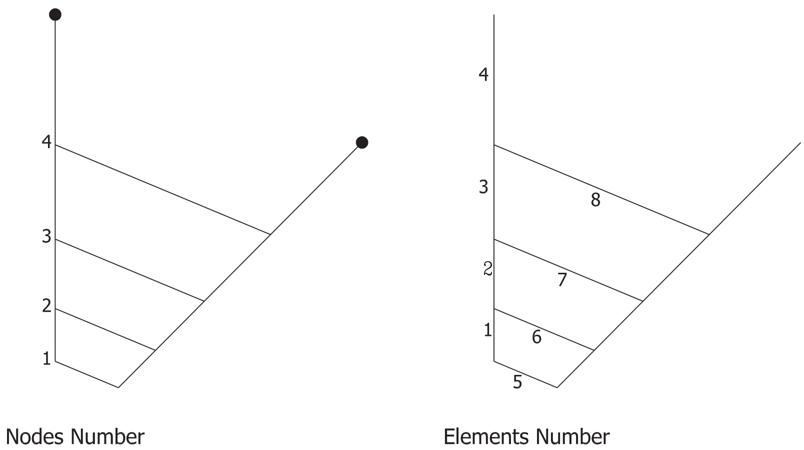

Fig. 10. Nodes and elements number for 1/8 of the circular net.

Table 6
Nodes (z) coordinates.

Element	z Coordinate m			
	a	b	c	d
1	−0.50000	−0.30000	−0.30053	−0.30105
2	−0.45000	−0.27000	−0.27042	−0.27083
3	−0.35000	−0.21000	−0.21041	−0.21080
4	−0.20000	−0.12000	−0.12038	−0.12074

Circular cable net

Fig. 9 shows a circular cable net with read node and element numbers from dxf file. It covers an area of radius 19.66 m. Outer nodes are fixed. Force density is assumed 1 ton/m for radial elements, 3 ton/m for diagonals and 5 ton/m for inner radials. Load cases are 0.15, 0.20 and 0.25 tons in gravity direction for all free nodes. Maximum deflection is assumed equal to (−0.30) m. Fig. 10 shows node and element numbers given to one eighth of the cable net to show results. Table 6 identifies the equilibrium states z coordinate and table 7 presents the unstressed length and corresponding force densities for all load cases.

Table 7
Element unstressed length and force density.

Element	Unstressed length m			Force density ton/m		
	b	c	d	b	c	d
1	1.18732	1.18732	1.18732	5.00000	6.64225	8.27314
2	1.56454	1.56454	1.56454	5.00000	6.66497	8.32891
3	2.12490	2.12490	2.12490	5.00000	6.66498	8.32874
4	2.93378	2.93378	2.93378	5.00000	6.64585	8.28209
5	1.55081	1.55081	1.55081	5.00000	6.64223	8.27310
6	2.45927	2.45927	2.45927	1.00000	1.34733	1.70097
7	3.65584	3.65584	3.65584	1.00000	1.33301	1.66566
8	5.28071	5.28071	5.28071	1.00000	1.31910	1.63187

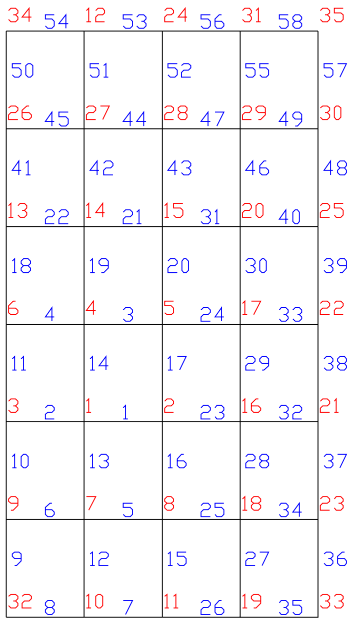

Fig. 11. Plan view and program numbering of orthogonal net.

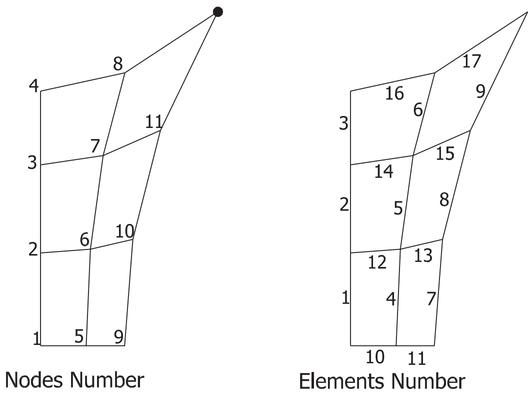

Fig. 12. Node and element numbers of quarter of the orthogonal net.

Table 8
Nodes (z) coordinates.

Element	z Coordinate m			
	a	b	c	d
1	−0.9048	**−0.5000**	−0.5005	−0.5010
2	−0.8460	−0.4675	−0.4679	−0.4684
3	−0.6811	−0.3764	−0.3767	−0.3770
4	−0.4506	−0.2490	−0.2492	−0.2494
5	−0.8635	−0.4772	−0.4777	−0.4782
6	−0.7991	−0.4416	−0.4420	−0.4425
7	−0.6138	−0.3392	−0.3395	−0.3398
8	−0.3430	−0.1896	−0.1897	−0.1898
9	−0.7512	−0.4151	−0.4156	−0.4161
10	−0.6731	−0.3720	−0.3724	−0.3728
11	−0.4320	−0.2387	−0.2390	−0.2393

Table 9
Element unstressed length and force density.

Element	Unstressed length m			Force density ton/m		
	b	c	d	b	c	d
1	2.0854	2.0854	2.0854	3.9137	5.2819	6.8446
2	1.9769	1.9769	1.9769	3.8537	5.2013	6.7407
3	1.6627	1.6627	1.6627	3.5740	4.8254	6.2558
4	2.1686	2.1686	2.1686	4.1355	5.5815	7.2332
5	2.1276	2.1276	2.1276	4.0521	5.4694	7.0885
6	1.9277	1.9277	1.9277	3.8893	5.2512	6.8080
7	2.3963	2.3963	2.3963	6.7013	9.0316	11.6856
8	2.5305	2.5305	2.5305	6.8252	9.1986	11.9017
9	2.9708	2.9708	2.9708	7.0810	9.5434	12.3479
10	1.0267	1.0267	1.0267	3.1993	4.3132	5.5828
11	0.8691	0.8691	0.8691	2.8728	3.8717	5.0094
12	1.1256	1.1256	1.1256	3.3912	4.5719	5.9174
13	0.9848	0.9848	0.9848	3.1617	4.2611	5.5132
14	1.4292	1.4292	1.4292	3.9840	5.3712	6.9520
15	1.4139	1.4139	1.4139	3.7931	5.1123	6.6148
16	1.9483	1.9483	1.9483	7.1900	9.7074	12.5850
17	2.5119	2.5119	2.5119	7.4364	10.0402	13.0165

For circular shape assumed force densities for radial elements should be more than the force densities of circular elements to get reasonable flat shape. This example results shows the same points as the rhombic cable net.

Orthogonal cable net

Fig. 11 shows an orthogonal cable net of 8 × 15 m with the program node and element numbering. Only corner nodes are fixed. Force density is assumed equal to 2 ton/m for inner elements and 4 ton/m for outer elements. Load cases are assumed equal to 0.4, 0.54 and 0.70 tons in gravity direction for all free nodes, maximum deflection is assumed (−0.50 m). Fig. 12 shows the node numbers and element numbers to show results for one-quarter of the cable net. Table 8 identifies the equilibrium states z coordinate and table 9 presents the unstressed length and corresponding force densities for all load cases.

For shapes with fixed nodes only at corners, the more the ratio of the assumed force densities between outer and inner elements, the more flat the shape is. The results show the same points as the rhombic cable net and circular cable net.

In all examples, the results show that unstressed length is kept unchanged while the load change.

Conclusion

This research presents the procedure and computer program to determine cable net form by setting coordinate constraints to the initial equilibrium state of assumed force densities, fixed nodes and given distribution of force densities. The coordinates are the main variable which controls the form of cable net of a certain topology. For loaded cable nets the coordinate constraint is applied only in the first load case and the unstressed length is calculated. For the remaining load cases the first load case unstressed length is the constraint applied to all net elements. The illustrative examples demonstrate the capabilities of the program, thus simplifying the design of cable nets under static loads.

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Further reading

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