



# A wireless railway catenary structural monitoring system: Full-scale case study



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## ABSTRACT

Full-scale structural measurements of new and existing railway catenary systems are becoming increasingly important due to continually increasing train speeds and the resulting consequences. Higher speeds lead to increased loads and greater structural dynamic responses, necessitating that both static and dynamic regulations be fulfilled. Sampling directly on railway catenary sections is necessary to assess their structural behaviour. The results can both be analysed directly and be used for validating and/or improving numerical models, which in turn can be used to explore the structural response at higher speeds. This case study presents and explores a newly developed wireless sensor system that includes multiple sensors that can be mounted arbitrarily on any of the wires in a catenary system. All sensors synchronously sample accelerations and rotational velocities over a range of up to 1400 m. This paper shows the results of mounting the developed sensor system and sampling the data of an existing railway catenary section at the Hovin station in Norway. Sampling was performed from both self-excited tests and 140 scheduled train passages. The outputs have been analysed to show that the data can be used to successfully assess railway catenary structural response components.

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## 1. Introduction

Full-scale measurements can significantly contribute to understanding the dynamic behaviour of a structure. The nature of the structure determines which types of measurements are best suited for analysing its behaviour. In addition, the environment (e.g., location, usage) of the structure may also favour or exclude certain types of measurements. It is therefore necessary for a monitoring system to be designed specifically for each type of structure. In the case of railway catenary sections, it is important to assess their behaviour as trains pass through the sections. In addition, the system that is assessed while the trains pass through the sections is a coupled dynamic system consisting of two dynamic systems: the moving pantograph and the spatially stationary catenary section. Uninterrupted contact between the pantograph and the catenary system is essential for maintaining electrical power to the train [1]. The complexity of this interaction and the dynamic responses increase with increasing train speed [2], and speeds above the design speed make the dynamic response even more critical [3]. In general, railway catenary systems are considered lightly damped [1,4,5], so oscillations are present in a substantial time around the actual contact point on a catenary system [1]. Thus, the quality of contact is directly dependent

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on a desired structural behaviour and to the amount of wear. This is of even greater importance on lines where multiple pantographs is used.

Measurements sampled directly on a catenary system are needed to investigate the dynamic behaviour of the catenaries during a complete train passage. Drugge [6] directly sampled displacement time series from a catenary at five points in one span by mounting measuring equipment at both adjacent supports and at three additional poles within the span. Stickland et al. [7] obtained sampled displacement series from the UK East Coast Main Line by using a drawstring potentiometer. Cho et al. [8] and Cho [9] sampled acceleration time series by using a strain-gauge type of accelerometer mounted on the contact line. Multiple wireless sensors were used for full-scale railway catenary damping estimations in a study by Nàvik et al. [10].

This paper explores the development of a new wireless sensor system for examining the structural behaviour of existing railway catenary sections while in operation. The purpose is to develop a monitoring system that can be used both to monitor the structural response of railway catenary systems for a brief period as scheduled trains pass and for sampling responses from the experiments. A short mounting time is critical to be allowed access to the track, so it is necessary to develop a monitoring system that is easy to mount and unmount. Thus, a wireless system was developed that included multiple sensors that are easily fastened to the wires in the catenary system. The sensors measure acceleration and rotational velocity in and around three axes and simultaneously sample data. The data are at once made accessible online through a wireless router connection after the transfer of data to the master unit is complete. Nine sensors are mounted on one railway catenary section at the Hovin station in Norway, all in close vicinity to one pole span. This sensor set-up is used to predict different parameters and responses in one span and create a base line for future monitoring.

It is important to control maximum uplift on soft contact lines, particularly at the pole support, whereas oscillations are a major concern within the span. It is therefore important to assess several points at and between the pole supports. There are a variety of operational modal analysis (OMA) methods that can be used on the output-only data, i.e., the acceleration or rotational velocity time series, for modal parameter estimation [11]. The analysis methods used in this investigation of monitoring system performance are power spectral densities (PSDs) estimated by the Burg method [12], histograms resulting from peak picking from the PSDs, and continuous short-time Fourier transforms [13]. The results include an assessment of the dynamic response measurements sampled by the new monitoring system, identification of the important system frequencies and the energy transfer between frequencies from the pre- to the post-passage part of the time series. The assessment is executed by analysing measurements from simple hand excitations and the more complex train passages at the Hovin station catenary section. A very important result of sampling and analysing measurements from full-scale existing structures is that numerical models can be validated and/or improved so that they describe the reality as close as possible. These models can only then be used to explore properties outside of what is built, such as higher train speeds, more pantographs, bigger uplift force, changes in wear, and if these changes leads to excessive structural vibrations. Many of these structures have a life span longer than 50 years, so a change in wear or point wear would be crucial for the maintenance costs.

## 2. Monitoring system

The developed monitoring system consists of up to ten wireless sensors and one master unit. The sensors consist of a commercial motion-processing unit (MPU) [14], a battery pack and a radio antenna. The MPU units (MPU-6000) combine a microelectromechanical system (MEMS) tri-axis gyroscope, a MEMS tri-axial accelerometer and a Digital Motion Processor™. The units feature six 16-bit analogue-to-digital converters (ADCs) to digitize the accelerometer and gyroscope outputs. They also feature a user-programmable full-scale range of  $\pm 250$ ,  $\pm 500$ ,  $\pm 1000$  and  $\pm 2000^\circ/\text{s}$  for the gyroscope, along with a range of  $\pm 2$  g,  $\pm 4$  g,  $\pm 8$  g and  $\pm 16$  g for the accelerometer [14]. The rechargeable battery in the sensors, a Boston Power Sonata® 5300, was chosen so that the monitoring system could run for at least one week without charging. Each sensor weighs 390 g; an example is shown in Fig. 1.

The sampled data are transferred from the sensors to the master unit through a custom-designed low-power 2.4 GHz radio network. The master unit antenna is an Alfa AOA-2049TM, the antenna on the sensors is a Pulse Electronics W1030, the radio chip is a Nordic nRF24L01 and the microprocessor is a STM32L152. In the current configuration, sampling can be performed at a rate of up to 500 Hz and a time duration of up to 8 min. Data are continuously sampled to flash memory, but data are only transferred when the system is triggered. The system is primarily self-triggered by train passages, i.e., when a threshold value is exceeded, but it can also be manually triggered through the master unit. Thus, data can also be selected for transfer from up to 8 min before triggering. The threshold value can be chosen separately for each degree of freedom. When the value is exceeded in any one sensor, the master unit triggers all sensors at the same time stamp to ensure synchronized data. An internal clock is used in the sensors and is synchronized to the internal clock of the master unit to ensure that the time is correct. The master unit's internal clock is updated to the correct time using internet time. Synchronization is not performed continuously due to possible problems from strong electrical currents that occur as the train passes each sensor. Thus, a time stamp is sent at selected time intervals to all sensors from the master unit. The master unit can either be accessed directly by connecting through a local Wi-Fi or remotely on a website using a wireless wide area network. Data can be downloaded, measuring parameters can be changed and the system can be triggered through both connections. The transferring setup is shown in Fig. 2.

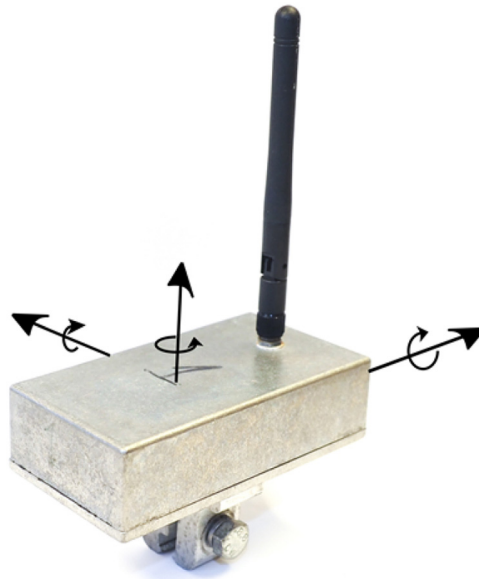


Fig. 1. A sensor with axes directions. Photo: NTNU/Petter N avik.

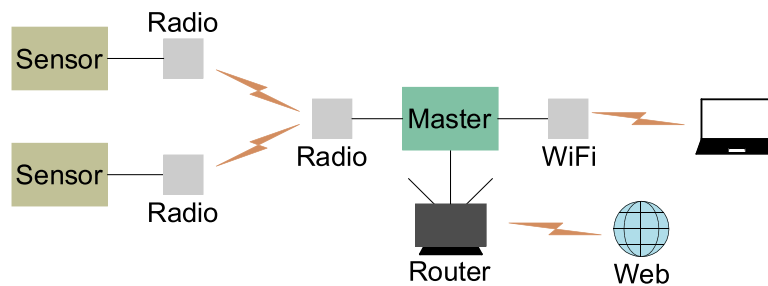


Fig. 2. Setup for the communication and transfer of data.

### 3. Railway catenary systems and pantograph

Railway catenary systems are wire systems that supply power to electrical trains. The major structural components are a contact wire, a messenger wire, droppers, registration arms and brackets, as shown in Fig. 3. The contact wire is the conductive part that transfers the electricity to the train through the pantograph. The primary role of the messenger wire is to carry the contact wire via the droppers, which also enables the desired geometry, stiffness and elasticity of the system. The cantilevered brackets that carry the messenger wire are fastened to the poles along the line. The contact wire is connected to these brackets through the light steady arm. The main function of the light steady arm, aside from being a fastener to the contact wire at the supports, is to obtain the desired horizontal geometry of the contact wire. The contact and messenger wires stretch over many pole spans and are pre-tensioned to obtain the desired vertical geometry. The tension forces are obtained by using a tensioning device consisting of weights and a transmission system. The length of the droppers is also important for the vertical geometry. It is often desirable for the quality of the pantograph-catenary interaction to include a small sag in the vertical height of the contact wire in the middle of each span. For more details on railway catenary systems, see Kiessling et al. [2].

The pantograph is mounted on the roof of the train with the sole purpose of supplying an uninterrupted and reliable energy transfer to the passing train. The contact between the pantograph and contact wire initiates the dynamic response in the catenary system. In this case study, the electricity is transmitted by two bow strips of carbon at the top of the pantograph. This contact is the most critical part of the energy transfer to modern trains [15]. A static and aerodynamic uplift force in the pantograph and a proper catenary design primarily ensures continuous contact.

The major existing catenary systems used in Norway are Tabell 54, System 35, System 20 and System 25. The three latter systems can be used in new railway catenary sections [16]. The type of catenary systems that should be used for new railway sections is thoroughly specified in technical regulations [17]. The criteria that decide which system is to be used are train velocity, allowed type and number of pantographs, train density, and open vs. tunnelled lines [16,17].



Fig. 3. Presentation of important structural components of railway catenary systems. Photo: NTNU/Petter Nāvik [10].

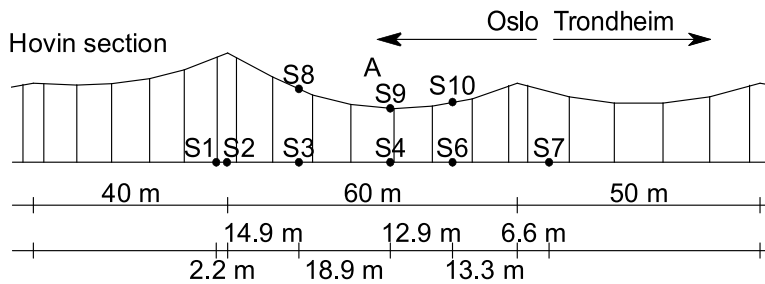


Fig. 4. Vertical geometry of Hovin station railway catenary section “wire 1”.

**Table 1**  
Description of the Hovin section spans and their theoretical natural frequencies [2].

Span length	40 m	45 m	50 m	55 m	60 m
Number of spans	5	2	5	4	9
Length to the outside dropper	2 m	2 m	2 m	2 m	2 m
First symmetric frequency [2]	1.26 Hz	1.12 Hz	1.01 Hz	0.92 Hz	0.85 Hz
First non-symmetric frequency [2]	1.29 Hz	1.15 Hz	1.03 Hz	0.94 Hz	0.86 Hz

#### 4. Case study: railway catenary section at the hovin station

The monitoring system is evaluated through this case study, which is performed on a railway catenary section located at the Hovin station and numbered “wire 1”. The section is part of the Dovre railway line in Norway, the major line between Oslo and Trondheim. It is a 1411-m-long section that utilizes a “System 35” catenary system. The vertical geometry of the section is presented in Fig. 4.

The investigated section is divided in 27 spans with lengths between 40 and 60 m. The first and last spans are not in contact with the pantograph at any time. A more detailed description of the spans of the Hovin section and their theoretical natural frequencies are presented in Table 1. The theoretical natural frequencies are computed in accordance with Kiessling et al. [2]. The horizontal geometry contains four curves from south to north with radii of 4000, 1000, 526 and 1670 m. The speed limit at the section is 100 km/h, except that it is 130 km/h at the largest curve running south.

The instrumentation of the section consists of nine sensors mounted in close vicinity to one span. The sensors are fastened to the wire by tightening a single screw, and in this case study it took approximately 5 min to mount the 9 sensors. Running the service train to the different mounting locations was the most time-consuming process. The distances between the poles and sensors are measured with a laser. A detailed description of where the sensors are mounted is presented in Fig. 5. The mounted monitoring system, as presented in Fig. 6, shows some of the sensors, how they are mounted, the master unit and the wireless wide-area network antenna.



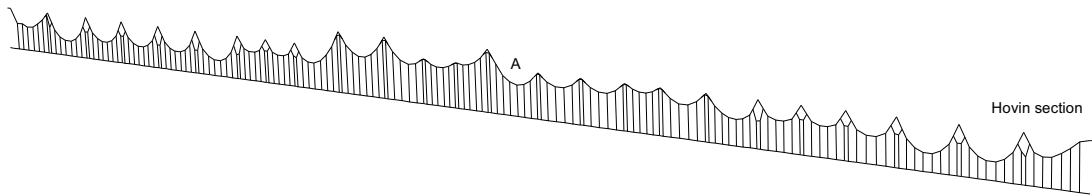


Fig. 5. Instrumentation of Hovin station railway catenary section “wire 1”.



Fig. 6. Mounted monitoring system at the Hovin station: left—sensors; middle—mounting of a sensor; right—master unit and wireless wide-area network antenna. Photo: NTNU/Petter Nàvik.

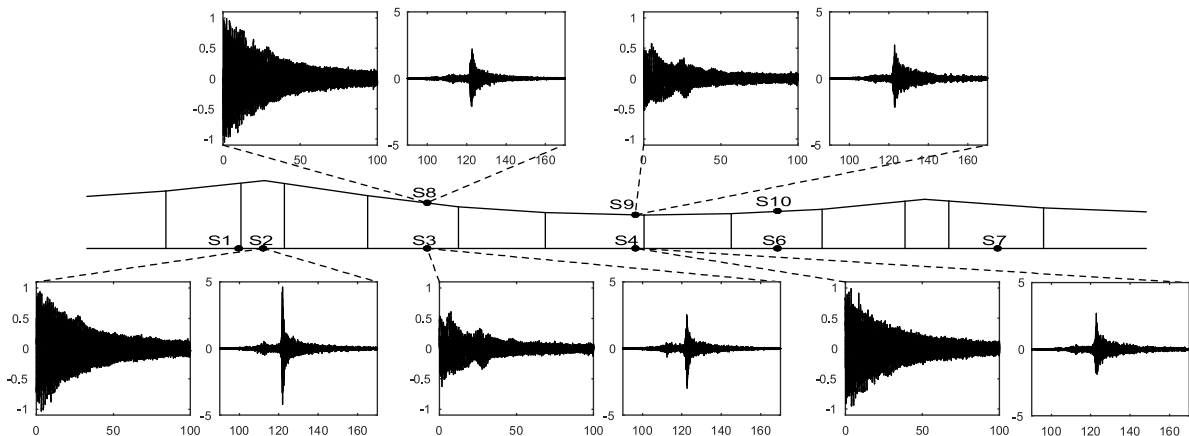
#### 4.1. Dynamic assessment

In evaluating the monitoring system, the sensor performance can be assessed in the frequency domain; this is common for evaluating any dynamic system. The evaluation of the time series will depend on the actual occurrences. It has previously been shown in numerical investigations of catenary systems that different information can be extracted from the different parts of a train passage time series [18,19]. The pre-passage data typically include information about the load frequencies relevant to the section that originate from the pantograph frequencies and the different span-pass frequencies. However, the post-passage data reveal more information on the fundamental system frequencies and damping properties [18].

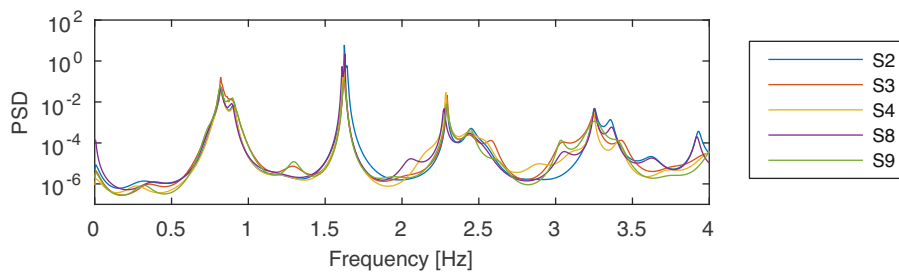
The test results from the Hovin Station section originate from two different types of excitation: simple hand excitations intended to yield the expected behaviour through harmonic excitation and free vibration, and the responses from train passages during operational loading that render a greater complexity in the time series. The data of interest for train passages are not in the single passage but in the catenary system response to the general traffic loading from which it is exposed at any given location. Thus, the train passage data are analysed as the total collection of data. Furthermore, the response from the train passage may also be divided into pre- and post-passage segments. The variation near the peak uplift is evaluated by combining the pre- and post-passage segments with different operational modal analysis tools, such as the power spectral density, the power spectral peak histogram and the spectrograms from the short-time Fourier transform. The combined effort of these different analyses facilitates possibilities for a good validation of the operational performance of the monitoring system for assessing the energy input and transfer between frequencies originating from the train's passage.

##### 4.1.1. Power spectral density (PSD) estimation

There are several methods available for estimating PSDs in structural dynamics. The method chosen to be used for estimating the PSD of pantograph passages depend largely on the passage duration. It is important to be aware that if the frequency distribution and energy transfer in the frequency domain are to be investigated in detail, the current time series will quickly become very short; therefore, it is important to choose the method accordingly [11]. The Burg method is used for PSD estimation in the investigation of the train passage [12]. The Burg PSD estimate is a parametric method based on a pre-chosen order for the employed autoregressive (AR) prediction model. In parametric methods, the signal is assumed to be an output of a linear system driven by white noise, i.e., parametric methods estimate the PSD by initially estimating the parameters of the linear system assumed to generate the signal. The Burg method does not apply a window function to the dataset, as is common in non-parametric methods. The Burg method may be considered better than non-parametric methods when applied to short parts of a recorded time series, such as a train passage. However, it is important to be aware of some of its disadvantages; for example, the Burg method exhibits spectral line splitting, particularly at high signal-to-noise



**Fig. 7.** Sampled acceleration time series from a hand excitation and a train passage for S2, S3, S4, S8 and S9.



**Fig. 8.** Power spectral density of the response from S2, S3, S4, S8 and S9 hand excitation.

ratios. In addition, the Burg method can also introduce spurious spectral peaks in high-order systems, and when estimating sinusoids in noise, it can exhibit a bias that is dependent on the initial phase [20].

#### 4.1.2. Short-time Fourier transform (STFT) and spectrogram analysis

Strictly speaking, the recorded time series during a train passage will be non-stationary and possess time-varying spectral characteristics. An STFT is a helpful tool when analysing the changing characteristics of the process. More complete and precise information about the uplift process can be provided by using an STFT in the analysis of the time series [18]. The calculated spectrogram provides a visual representation of the motion, thus providing a useful time-frequency representation. In the STFT, time series are segmented into time intervals that are sufficiently narrow to be considered stationary [13]. It is important to recognize that the time resolution, i.e., how well two peaks in time can be separated from each other, and the frequency resolution, i.e., how well two spectral components can be separated from each other, cannot be determined arbitrarily because they are both directly related to the time window size used to segment the original time series. It is equally important to consider the amount of overlap between the chosen windows. Avoiding overlap will result in more distinct differences between windows, whereas in overlapping windows, the results will be averaged to reduce random errors. By conducting an analysis for each incremental shift using overlapping windows, the STFT can also be used as a sliding discrete Fourier transform [21]. Finally, the spectrogram from the STFT analysis is achieved by squaring the magnitude. The Burg spectrum method is used to evaluate the frequencies in the short-windowed time series.

#### 4.2. Analysis and results

Data have been collected at the Hovin Station to evaluate and validate the full-scale functionality of the developed monitoring system. The analysed data originate from two different types of excitation. Examples of the time series from several sensors for both the simple hand-excited response (left) and train-passage response (right) are presented in Fig. 7. The hand excitation was performed between the second and third sensors, which can be seen in the time series magnitude mirroring the excited second fundamental frequency of the given span. This is further enhanced by the PSD estimates in Fig. 8, where a large response at the second mode is present, as expected. The PSD estimate also verifies the expected harmonic behaviour of the structure shown by including estimates for all five time series shown in Fig. 7. In this case, all five PSD estimates clearly coincide, showing up to 4 Hz for clarity in Fig. 8. The recorded time series show that the synchronized measurements in the monitoring system work as intended, whereas the harmonic hand excitation shows the correct frequency representation between sensors.

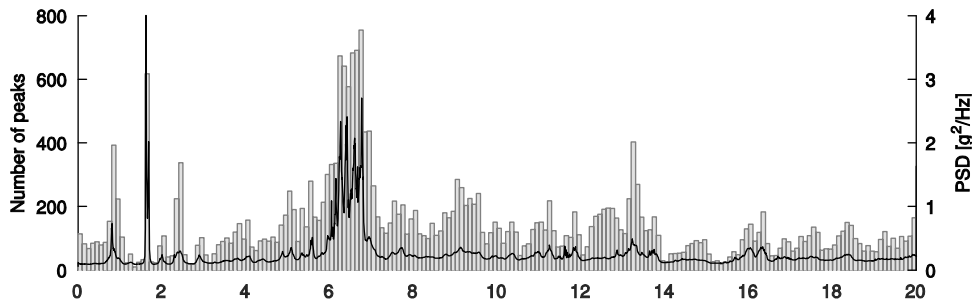


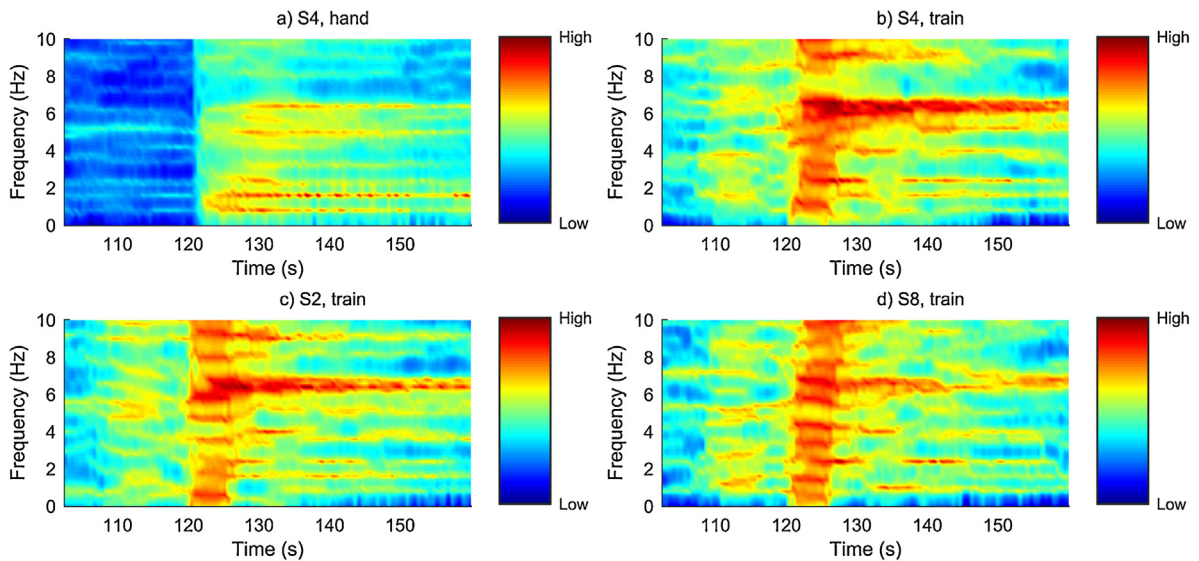
Fig. 9. Total PSD from 140 train passages and 9 sensors and histogram with all peaks picked from individual PSDs.

To test the functionality of the monitoring system in operational conditions, all sensors were left on the catenary for an anticipated monitoring duration to collect a sufficient number of train passages. The loading from the train passages varies due to variations in train type, train speed, the given pantograph and its individual setting, and environmental parameters, such as wind and temperature. All pantographs are assumed to have settings based on the Norwegian design rules but may also contain aleatoric and epistemic uncertainties of any system; in other words, both the train and train speed, as well as the pantograph settings, can cause considerable variation in the catenary dynamic response. Numerical models of catenary systems [18] have shown that the catenary is a slender and flexible system with many similar frequency components that can include small variations. This gives similar frequency components from an eigenvalue analysis, which can be seen to accumulate around given frequencies.

All sampled time series are used to find relevant frequencies that dominate the system response at the current catenary section. The entire time series is included, i.e., the pre-, peri- and post-passage, to show the total frequency content. Due to the variations in train passages, the result can be further clarified by picking the estimated PSD peaks from each passage and presenting their distribution as a frequency function in a histogram. The estimated PSD from all passages and the peak distribution histogram are included in Fig. 9. This represents both the peak and the energy distribution. The importance of the histogram representation becomes evident when remembering that the response data are collected from an in-space stationary system excited by a secondary system (pantograph) in motion. Both the motion in itself and the internal dynamic response of the secondary system will influence the result of the catenary. Thus, the pre-passage data will have a lower response level than the post-passage data. This implies that the PSD estimate will show a low energy content for frequencies dominated by pre-passage responses compared with the post-passage response components. In other words, when a sufficient time duration is used, the monitoring system can provide sufficient data to analyse and identify the frequency components of concern. The frequencies will be location dependent and not merely dependent on the chosen catenary system, i.e., the importance of collecting sufficient data at any trouble span or section. This will enable the possibility of considering either direct actions or actions via numerical modelling and model updating when examining necessary changes.

The oncoming train will create a broad-banded load frequency distribution as it passes the initial spans. Thus, the pre-passage response includes responses from propagating waves and reflections that are one limiting factor for the allowed train speed of a catenary system. The post-passage is different because it reflects the motion introduced by the pantograph uplift, i.e., the sudden impulse and the following free decaying response will be dominated by the fundamental frequencies.

To identify the major frequencies and the frequency transfer zone of the passage, corresponding spectrograms are introduced in Fig. 10b–d showing their characteristics. The important uplift influence zone can be determined based on the time series of the passages and the results from the spectrograms. This provides a good estimation of the part that should be included in the division into the pre- and post-passage analysis. According to the train passage shown in Fig. 10b–d, the monitoring system is triggered by the peak passage and the time series is saved for a given period both before and after passage. The passage occurs at 120 s, with small variations depending on sensor positions and running direction. An influence zone of approximately 5 s is clearly observed, remembering that the time window used in the STFT will also affect the width of the displayed zone. The properties of the remaining part of the time series are then less time variant and correspond better to the conditions of a detailed operational modal analysis, i.e., determining the modal properties and executing the necessary model updating of finite element models. As a result, the previously mentioned techniques included in this investigation also become more distinct. For instance, the lines in the spectrograms in Fig. 10 show that frequency peaks of 1.63 and 2.4 Hz found in Figs. 8 and 9 are typical components in a post-passage segment along with frequencies of approximately 6.7 Hz in Fig. 9, which indicates the frequency band of the pantograph-catenary interaction. The pre-passage does not have as clear lines as the post-passage segment, but some of the dominating frequencies are the components around 0.81 Hz, the pantograph-catenary interaction band, and frequencies between 8 to 14 Hz. The strength of the histogram is clearly present in the latter frequency band, where the lower energy level in the pre-passage becomes weighted such that it render equal importance as the post-passage frequencies. Note that the fundamental modes have a considerably longer duration; several frequency components are excited, but the energy converges toward fundamental modes, rendering the other mode shapes with high damping, whereas the lasting fundamental modes typically have a low damping characteristic. The latter is further



**Fig. 10.** Spectrograms of the frequency response from hand excitation (a) S4 and train passages (b) S4, (c) S2 and (d) S8.

shown by the spectrogram of the simple hand excitation in Fig. 10a, which shows the fundamental frequency components and their energy mitigation after the initial excitation.

## 5. Conclusions

A new monitoring system was described and applied to an assessment of the dynamic behaviour of railway catenary systems. The investigation also provided possible analyses of sampled full-scale measurements using operational modal analysis. A railway catenary section located at the Hovin station in Norway was used as a case study for the monitoring system evaluation. The monitoring system was successfully mounted within a limited time; mounting all sensors took 5 min, which demonstrated the ability to mount the system rapidly in arbitrary locations. The system automatically sampled all train passages for a week—a total of 140 passages, which was more than sufficient to assess and demonstrate that the monitoring system was energy efficient. The results showed that the developed system was capable of monitoring relevant oscillations occurring in the catenary system regardless of the initiation source (e.g., wind, train). To ensure relevant sampling, the monitoring system could be controlled either manually through a wireless wide-area network or by appropriate trigger values. Finally, an operational modal analysis applied to the collected data showed that relevant information could be extracted and dynamic properties could be identified based on the sampled data, thus making it possible to use the modal results in a numerical model updating procedure.

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