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AN HIERARCHICAL LOCATION–ALLOCATION MODEL FOR PRIMARY HEALTH CARE DELIVERY IN A DEVELOPING AREA

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Abstract—Location–allocation models can play an important role in making primary health care facilities more accessible to rural populations in the developing world. Traditional models, however, have failed to deal realistically with the fact that health care systems are hierarchical in nature, and that benefits and utilization decline with distance.

In this paper, an hierarchical location–allocation model in which benefits accrue to facility level and decline exponentially with distance is presented as a possible approach to ameliorating problems of rural accessibility to health care in Third World settings. The model is subjected to sensitivity analysis with reference to data for Salcette Taluka, Goa, India. The analysis suggests that the traditional P-median model may be a much less appropriate solution to the problem than a simple strategy of locating facilities from the highest to the lowest level in centers of strictly decreasing population.

Key words—hierarchical facility systems, primary health care, developing world, Salcette, Goa, India

INTRODUCTION

The importance of providing efficient primary health care in developing countries is widely recognized, yet this goal has proved to be an elusive one in many countries. Warren *et al.* indicate that "at least 70% of the Ghanaian population is without easy access to the formal health care system despite large public expenditures" [1]. Particularly troublesome is the imbalance between the provision of urban and rural facilities, which, in the case of Nigeria, Okafor [2] believes:

> can be corrected by investing more in the rural sector and by locating hospitals in strategic rural settlements that can maximize the accessibility of rural populations to facilities.

Rushton [3] has argued that location–allocation models can play an important role in attempts of authorities to "expand the number of health service sites and to distribute health workers more widely". The approach to these models which he espouses, however, fails to address adequately two important characteristics of medical facility systems in developing areas.

In the first place, **rural primary health care systems in developing areas are hierarchically organized** [4–6]. Rushton recognizes this fact, but fails to address the benefits which accrue to a facility's position in an hierarchy. The systems which he recommends are based upon the P-median model which aims to locate facility systems in such a way that aggregate demand weighted, patron-facility distance is minimized. Algebraically, this goal can be expressed as:

$$\text{MINIMIZE: } Z = \sum_{i}^{N} P_i \min_{j \in A} d_{ij} \qquad (1)$$

where:

- P_i is the population of demand zone i.
- N is the number of demand zones.
- j is a potential facility site.
- A is the set of facility locations.
- d_{ij} is the distance between demand zone i and potential facility location j.

In its single-level form, this model is by far the most widely used location–allocation approach. Applications to hierarchical medical facility systems have been piecemeal level-by-level efforts [7–9]. Hodgson [10] demonstrated that this approach may produce inferior results to simultaneously locating all levels of a hierarchical system so as to:

$$\text{MINIMIZE: } Z = \sum_{k}^{K} U_k \sum_{i}^{N} P_i \operatorname{MIN}_{j \in A} d_{ij} \qquad (2)$$

where:

- k is the hierarchical level of the facility.
- K is the number of hierarchical levels.
- U_k is the proportion of total use which accrues to facilities at level k.

This model treats facilities as being a part of a successively-inclusive hierarchy, by which is meant a system in which facilities at any given level offer all services offered by facilities of all lower orders.

Secondly, whether single- or multi-level, **the assumptions upon which the P-median model are based are unrealistic.** These are that all patrons will attend the closest facility and that minimizing the distance from the nearest facility is an appropriate goal. This is related to the first difficulty, for implicit is the assumption that neither attendance patterns nor benefits are affected by a facility's level in the hierarchy. Studies [6, 11, 12] have shown that distance does have a profound mitigating effect on facility attendance "in rural Third-World settings where most patients travel on foot and few have access to motorized transportation . . . " [13]. It is not safe to assume, as does the P-median model, that all potential patrons are served simply because facilities have been located. The P-median model's treatment of the disbenefits accruing to distance as linear is questionable. If benefit may be inferred from attendance patterns, consider Stock's findings which ". . . suggest that utilization patterns exhibit a highly regular pattern of exponential distance decay" [13].

Furthermore, the P-median model assumes that no benefit accrues to low-level services being offered at higher-level facilities. This assumption is familiar to geographers, being fundamental to Christaller's [14] Central Place Theory which assumes that a patron will seek a lower-order service at the closest center at which it is offered regardless of that center's rank in the hierarchy. Its familiarity need not imply its credibility, however. In the real world, a high-order facility and a low-order facility both offering a particular lower level of service might not appear equally attractive to a prospective patron. The higher-order facility may have more staff, and hence a lower expected waiting time; the patient may feel that the higher-order facility can perform the low-level service better (that a doctor used to dealing with serious problems will bandage the finger better); or a trip to the facility may be combined with a shopping trip, for which the larger center may be more attractive. This reasoning concurs with Stock's observation that "quality of service appears to be the second most important determinant of distance decay in the study area. Higher-order facilities are more successful than dispensaries in attracting patients from longer distances" [13].

The P-median model, then, is unrealistic in it's assumption of all-or-nothing, least-distance attendance and in its ignoring the advantages that higher-order facilities might offer to patrons seeking lower-order services. Hodgson [15] presented a model which modifies patron behavior and facility benefit based on a facility's level in the hierarchy. In this paper, the model is applied and examined within a real-world setting.

A MODEL WITH ALLOCATIONS BASED ON FACILITY SERVICE LEVEL

There is, in the geographical literature, considerable evidence that a service center's size, as well as its location, will affect attendance patterns within a system of facilities. This notion has been formalized in a number of market area models [16, 18], but there has been considerable resistance to incorporating it into location–allocation models. Although medical facilities in developing areas may not be subject to the same types of forces modeled by marketing specialists, it is worthwhile considering models which account for differential attractiveness based on facility level.

Batty [18] proposed a form of Reilly's [17] *Law of Retail Gravitation* which combines a size/attractiveness factor with a negative exponential distance decay function, allocating patrons to the center offering the greatest value of:

$$S_j^\alpha \exp(-\beta d_{ij}) \tag{3}$$

where:

S_j is the size (or level of service) of a facility at j.
α is a parameter describing the effect of size upon attractiveness.
β is a parameter describing the effect of distance upon attractiveness.

The perceived benefit of a patron who attends the facility offering the greatest value of (3), may be inferred to be maximized when (3) is maximized. Further, aggregate patron perceived benefit in using facilities at level k is:

$$B_k = \sum_i^N P_i \max_{j \in A} S_j^\alpha \exp(-\beta d_{ij}) \tag{4}$$

An appropriate successively-inclusive hierarchical location–allocation model would:
MAXIMIZE:

$$Z = \sum_k^K B_k = \sum_k^K U_k \sum_i^N P_i \max_{j \in A} S_j^\alpha \exp(-\beta d_{ij}) \tag{5}$$

The model infers benefit from patron behavior, attributing benefit to the size of a facility and disbenefit to the distance it is from the patron. The parameter α describes the degree to which size affects benefit, and β describes the degree to which distance does.

Applied to a set of hypothetical problems [15], this model produced intuitively pleasing and logical location sets and hierarchical service structures. Sensitivity analysis indicated that the results vary considerably with variation in α and β, however. Attendance data are difficult to obtain, even in the developed world, and the consequent difficulty in estimating values for these interaction parameters casts doubt upon the utility of the model in the developing world. In this study, the model is subjected to sensitivity analysis in a real-world setting to demonstrate what it can tell us about other approaches to hierarchical facility location which are less demanding of data.

The analysis is based on the premise that the negative exponential model, with some (undetermined) parameters, can adequately describe the attendance patterns at, and benefits accruing from, an hierarchical system of facilities. This is not a particularly contentious assumption. The differential drawing power arising from size is nullified for $\alpha = 0.0$, in which case allocation will be to the nearest facility regardless of size. The negative exponential form describes the relationship between many physical or geographical phenomena and distance. In any case, as β decreases from severe to more gentle distance decay, the linear benefit/distance function peculiar to the P-median model is approximated (Fig. 1). Correlation of 100 values of $\exp(-d_{ij})$ with d_{ij} values of 0.2, 0.4, . . . , 20 (Table 1) confirms this tendency to increasing linearity. It is, however, particularly interesting to ascertain what can be learned about hierarchical location–allocation models for parameters *not* approximating the P-median model.

The analysis was conducted for a range of values

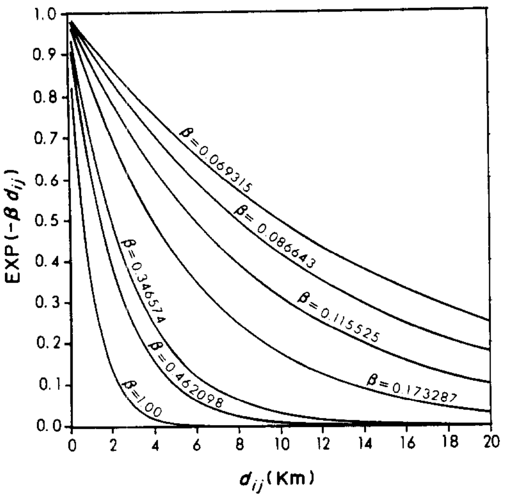

Fig. 1. Increasing linearity of the benefit function as β is relaxed.

Table 1. Correlation of exp $(-\beta d_{ij})$ with d_{ij} for $d_{ij} = 0.2, 0.4, \ldots, 20$ km)

β	r
1.00	−0.520
0.462098	−0.714
0.346574	−0.786
0.173287	−0.919
0.115525	−0.960
0.086643	−0.976
0.069315	−0.985

of α and β, some of which were selected arbitrarily, and others which were based on simple formulae. The value $\beta = 1.0$ was originally employed in debugging the program, and represents an extreme decay in facility influence over distance. Six other values of β were selected on the basis that they would halve a facility's benefit at regular distance intervals appropriate to the size of the study area (Table 2). The value $\alpha = 0.0$ results in size being of no consequence whatever, the assertion of P-median modeling. Two facilities offering the same service will split the market for that service 50/50 regardless of their size. Where $\alpha = 1.0$, attractiveness is strictly proportional to size: twice the size, twice the benefit. The intermediate values provide a range of factors of the benefit which would be derived from a facility of double size. For example, $\alpha = 0.1375$ describes a situation in which a facility of twice the size offers 1.1 times the benefit (Table 3).

The effects of changing values of α and β may be

Table 2. β Values and distances at which facility benefits are halved

β	Distance (km)
0.462098	1.5
0.346574	2.0
0.173287	4.0
0.115525	6.0
0.086643	8.0
0.069315	10.0

Table 3. α Values and factors by which facility benefits are increased through a doubling of size

α	Factor
0.0	1.0
0.137504	1.1
0.263034	1.2
0.378512	1.3
0.485427	1.4
0.584963	1.5
0.678072	1.6
0.765535	1.7
0.847997	1.8
0.925999	1.9
1.00	2.0

viewed through an examination of the relationship between benefit and distance (Fig. 2). Each of the three curves represents a different level of a $K = 3$ hierarchy. These levels are represented by arbitrary values of S_j, in particular, S_1 (highest level, top curve) = 6, $S_2 = 3$, and $S_3 = 1$. The benefit values portrayed in each plot have been scaled to a proportion of the highest value.

For $\alpha = 0.0$ the three curves would be coincident, as facility size plays no role in benefit. For low, non zero values of α (0.1375), size has little effect on benefit, and the curves are quite close together (Fig. 2A, 2B). Where α is high (1.0), facility size *is* important and each curve is much higher than that of successively lower-order facilities. Combined with relatively shallow distance decay ($\beta = 0.693$; Fig. 2C) a high-level facility at 20 km is as beneficial as a middle-order one at roughly half the distance, and more so than a low-level one at any lower distance. This last case describes a situation in which low-level facilities would receive no patronage whatever, and would be redundant within the hierarchical facility structure.

With larger values of β, benefits decrease more dramatically with distance, and the effect of facility size is reduced somewhat, enhancing relatively the attractiveness of the lower-level facilities. In contrast with the above no-patronage scenario, values of $\alpha = 1.0$ and $\beta = 1.0$ (Fig. 2D), create a situation in which the benefit of the highest-order facility at 4 km, although higher than that of the lower orders, is roughly the same as the middle-level facilities at 3 km and the lowest-order ones at 2 km. Under these circumstances, some utilization of the lower-order centers together with higher utilization of the high-order ones would be expected.

DEMONSTRATING THE MODEL: A CASE STUDY

The study area investigated is Salcette Taluka, Goa, India, as modeled by Hodgson and Valadares [19]. The taluka (an administrative area akin to a county in the U.S.A.), is roughly 15 km from east to west and 25 km from north to south (Fig. 3). Near-neighbor villages are walking distance apart, longer trips are commonly made by bicycle, group-taxi, or bus. Twenty-seven urban and village agglomerations serve as focal points for demand zones, the largest of which, Margao, is Goa's major commercial center.

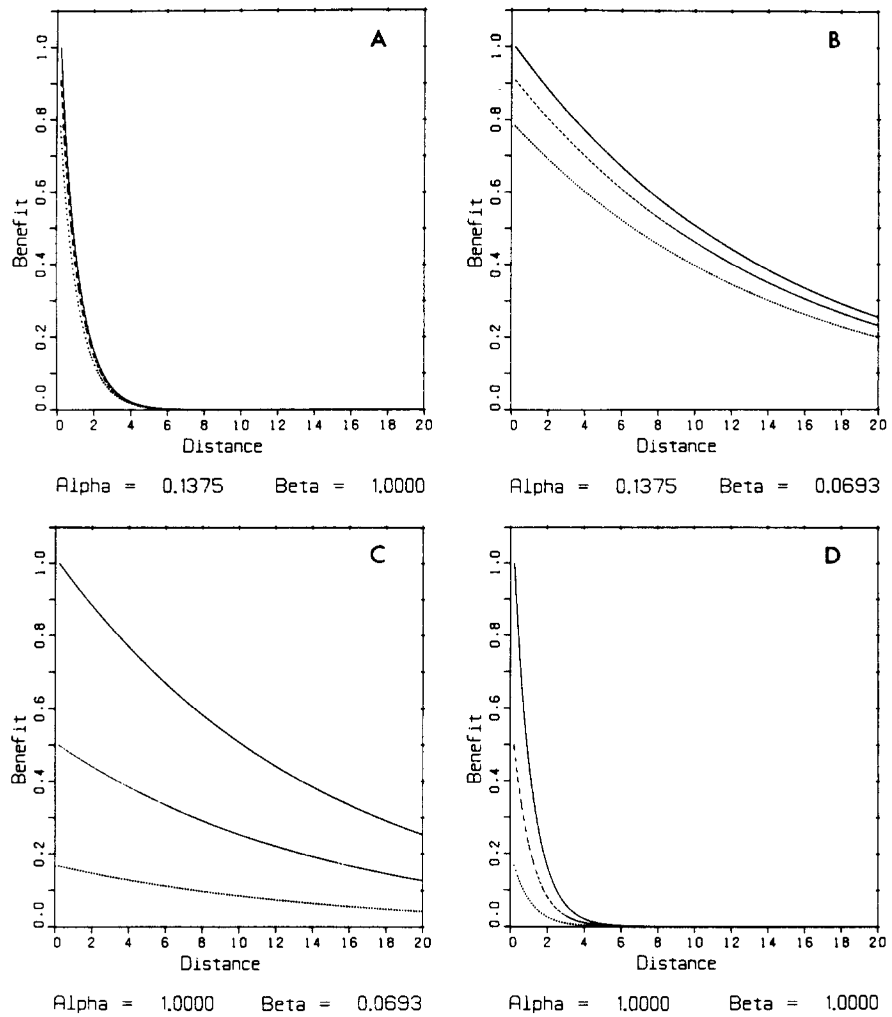

Fig. 2. The effect of varying α and β on the benefit function.

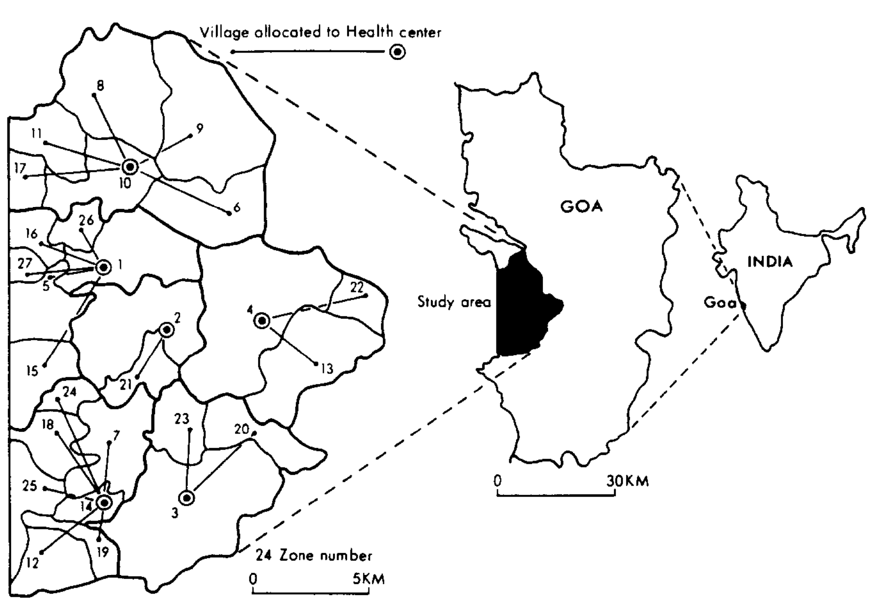

Fig. 3. The study area: Salcette Taluka, Goa, India. Portrayed is the single-level P-median solution of Hodgson and Valadares [25].

Table 4. The data base

Node	Population	Health center
1 Margao	37,956	Urban
2 Navelim, Davorlim, Dicarple, Talaulim, Aquem	14,726	—
3 Cuncolim, Varoda, Talvarda	12,395	Rural
4 Curtorim, Sao Jose de Areal	10,319	—
5 Benaulim, Cana, Adsulim	7,804	—
6 Raia, Rachol	6,268	—
7 Chinchinim, Deussue	5,953	Rural
8 Verna, Nagoa	5,861	Twice a week
9 Loutolim, Camurlim	5,473	—
10 Nuvem	5,151	—
11 Majorda, Utorda, Calata	5,094	—
12 Velim	5,352	—
13 Chandor, Guirdolim, Cavorim	4,127	—
14 Assolna	3,354	—
15 Varca	3,110	Twice a week
16 Colva, Vanelim	3,042	—
17 Betalbatim, Gonsua	2,583	—
18 Carmona	2,539	—
19 Ambelim	2,342	—
20 Paroda, Mulem	2,018	—
21 Dramapur, Sirlim	1,876	—
22 Macasana	1,741	—
23 Sarzora	1,676	—
24 Orlim	1,542	—
25 Cavelossim	1,532	Twice a week
26 Seraulim, Ducolna	1,116	—
27 Sernabatim	1,064	—

Demand values, P_i, are estimated by 1971 census populations (Table 4). Distances, d_{ij}, are measured in straight lines. The health care system modeled has one urban health center at Margao, two rural health centers at Cuncolim and Chinchinim, and three twice-a-week centers in smaller villages.

The model is demonstrated through comparison with three other systems. The first is the actual system in place in Salcette (ACTUAL), and the second is the P-MEDIAN hierarchical system generated by equation (1). The SIZE-DOWN system results from simply locating facilities, from the highest to the lowest level, in centers of strictly decreasing population. It represents a nonsystems approach to attempting to provide accessibility to as many persons as possible.

The model generates an optimal system of the same general structure as in Salcette, with one high-, two middle-, and three low-level facilities. Relative usages, U_k, are estimated as being proportional to the number of places offering the particular level of service, i.e. 0.1, 0.3 and 0.6 from high to low. Service levels (sizes, S_j) are estimated at 6, 3 and 1 on the basis that a six-day facility offers thrice the service of a two-day one, and then, purely hypothetically, that an urban facility offers twice a rural facility's level of service. One can only speculate on the effect these choices might have upon the analysis. It is likely that different values of U_k and S_j would result in different values of α and β being used in the analysis, but unlikely that the general results would differ greatly.

Clearly the performance of the models under assessment will depend on the values of α and β. Where β is high, that is, where benefits decline severely with distance, there is an impetus to locate facilities close to high demand, in the larger demand centers. Where α is also high, this impetus is strengthened, and the simple SIZE-DOWN approach can be expected to give good results. Where α is low, facility size is of little importance and system considerations might result in higher-order facilities appearing in more strategically located, smaller, demand centers. Coupled with lower distance decay (lower β), a more uniform coverage such as that produced by the P-median model might result.

The sensitivity analysis, involving 11 values of α and 7 values of β, generates 77 optimal solutions. It is clearly impossible to map or discuss all of them, but two types of solution stand out in importance. Certain values of α and β produce the same locations as do the P-MEDIAN or SIZE-DOWN solutions. One of each such solution is mapped and tabulated to demonstrate the nature of their allocation patterns and hierarchical service structures. This provides some feel for the nature of solutions, allowing the overall results of the sensitivity analysis to be considered in much more aggregate form.

Values of $\alpha = 0.0$, $\beta = 0.0495$ produced the same facility locations and allocation patterns as the P-MEDIAN model. In this solution, not actually presented as part of the sensitivity analysis, the highest-level facility is located in Margao (1), the second-order ones in centers 4 and 14, and the lowest-level ones in centers, 2, 3 and 10 (Fig. 4). At the second and third levels, allocation areas are quite uniform, typical of the P-median model. These values of the interaction parameters result in a somewhat poorly structured hierarchy (Table 5) in the sense that a higher-order facility serves fewer persons than does a lower-order facility at a particular service level. For example, the third-order facility at center 10 provides low-order service to 30,430 persons, more than does either second-order facility. Moreover this facility provides 25% more people with low-order service than the facility at center 4 provides with middle-level service. This unbalanced hierarchical structure results from the P-median allocation rule's inability to simulate human behavior realistically, and is evidence of its unsuitability in locational modeling of this type.

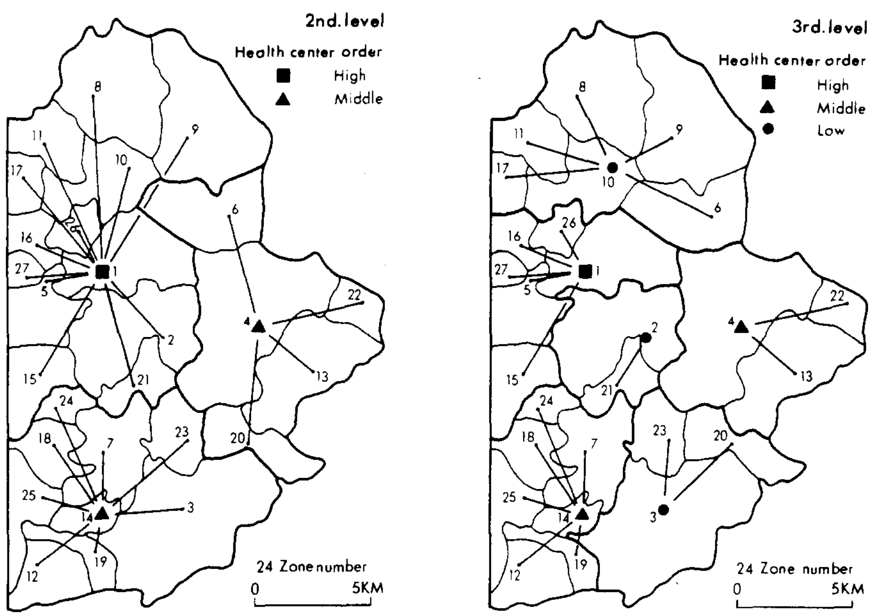

Fig. 4. The P-MEDIAN result, $\alpha = 0.0$, $\beta = 0.0495$.

All runs in which $\beta = 1.0$ led to the SIZE-DOWN result. Allocation patterns varied somewhat over the range of α values. Where $\alpha = 0.9260$, they appear to be quite lopsided (Fig. 5) in contrast with the more balanced allocations normally associated with P-median solutions. The hierarchical structure of the SIZE-DOWN solution (Table 5), has no facility serving more persons at any level than does one of a higher order, a reasonable outcome of an hierarchical process. Even if two facilities offering the same level of service were equally attractive regardless of their size, strongly imbalanced structures do not ring true. A two-day a week facility, for instance, would likely be elevated to normal rural health center status, operating all week, were it to attract large numbers of patrons. It is not reasonable to expect a low-order center to have a large service population, which is a situation forced upon some such centers by the P-MEDIAN solution's tendency toward uniform spacing and its insistence on least-distance allocation.

The overall results of the sensitivity analysis are considered in aggregate form, through comparison of objective function values. For each pair of interaction parameters, the optimal solution is generated and the value of the objective function, Z^*, is recorded. Particular solutions are evaluated on the negative exponential objective function (equation 4), and this particular value, Z^p made available for comparison with the optimal one. For example, the optimal solution, for $\alpha = 0.485427$ and $\beta = 0.173287$ has facilities located as:

Level 1: at 1
Level 2: at 3 and 4
Level 3: at 8, 9, 12.

Table 5. Level-by-level service structure of two solutions

Level of Service	Level of Facility	$\alpha = 0.00$, $\beta = 0.0495$ 'P-median result' Center	Population served Absolute	%	$\alpha = 0.9260$, $\beta = 1.00$ 'Size-down result' Center	Population served Absolute	%
High	1	1	156,014	100	1	156,014	100
Middle	1	1	94,856	60.8	1	84,522	54.2
	2	4	24,473	15.7	2	34,331	22.0
		14	36,685	23.5	3	37,161	23.8
Low	1	1	54,092	34.7	1	57,761	37.0
	2	4	16,187	10.4	2	18,144	11.6
		14	22,614	14.5	3	37,161	23.8
	3	2	16,602	10.6	4	16,187	10.4
		3	16,089	10.3	5	15,020	9.6
		10	30,430	19.5	6	11,741	7.5

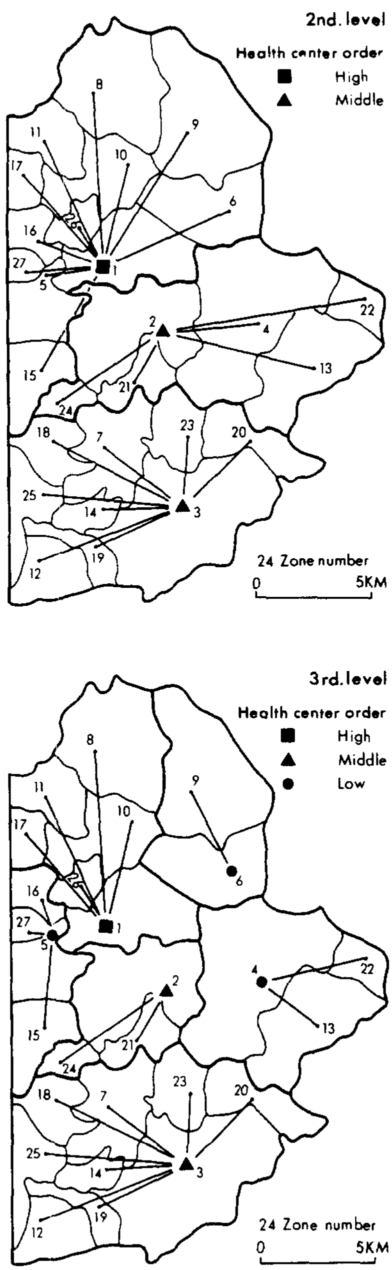

Fig. 5. The SIZE-DOWN result, $\alpha = 0.9260$, $\beta = 1.0$.

and scores $Z^* = 166{,}546$ on the negative exponential function. The ACTUAL system has facilities located as:

Level 1: at 1
Level 2: at 3 and 7
Level 3: at 8, 15, and 25.

and scores a value of $Z^p = 159{,}436$ on the negative exponential model.

The index used for comparison is termed a 'shortfall'. Since the location–allocation procedure is a maximization, lower values of the objective function imply inferior solutions. The shortfall value simply measures, in percentage terms, the degree to which a particular solution's objective function, Z^p, falls below the optimal solution's Z^*.

$$\text{SHORTFALL} = 100\,(Z^* - Z^p)/Z^* \qquad (6)$$

In the case of the values calculated above, the shortfall value is: 100 (166,546 − 159,436)/166,546 = 4.27.

The results of the sensitivity analysis are tabulated, (Table 6) and require some interpretation. In the first place, values are not tabulated for any combination of α and β leading to any facility in the ACTUAL system serving no patrons. Such a circumstance was discussed in relation to the choice of α and β values, in considering the case where $\alpha = 1.0$ and $\beta = 0.0693$. In that situation, low-level facilities never offer greater benefit than middle- or higher-level ones at any distance. This situation actually arose in 26 of the Salcette runs. Such results are dismissed as being unreasonable on the grounds that the actual Salcette facilities are used at all levels, and that if the model's general form is correct, appropriate values of α and β would produce no redundant facilities in the ACTUAL system.

Except where distance decay is very steep ($\beta = 1.0$) or very shallow ($\beta = 0.0495$) as in the particular case discussed above, neither the SIZE-DOWN nor P-MEDIAN solutions result. It is clear, however, that there is a tendency for the P-median model to score better for low β in combination with low α as predicted in earlier discussion. High values of both parameters produce solutions which are more closely related to the SIZE-DOWN model. Higher values of α in conjunction with lower values of β produce the no-service solutions omitted from the table.

Hodgson and Valadares [19] claimed, using a single-level P-median model, that the ACTUAL Salcette system was inefficient by a factor of 1.57 times. Considering the system as a hierarchy with the same values of U_k employed here, the P-median model suggested that it was inefficient by a factor of 1.28. The present analysis suggests that the ACTUAL system's apparent low efficiency is an artifact of using an inappropriate location model. The negative exponential location–allocation model, for many values of α and β, suggests that the P-MEDIAN solution is not much better than the ACTUAL system. In two isolated cases ($\alpha = 0.9266$ or $\alpha = 1.0$, $\beta = 0.3466$) the ACTUAL system outperforms the P-median model. With these parameters, considerable locational emphasis is placed on being close to a large demand point, and the ACTUAL system benefits from its facilities being in larger places. This is not to conclude that under real world parameter values the ACTUAL system would outperform the P-median model. It does, however, detract from the Hodgson and Valadares conclusion that it is considerably inferior to the P-median solution.

CONCLUSION

In this study, a location–allocation model which attributes higher spatial drawing power to higher-order places within a successively-inclusive hierarchy of facilities is implemented. The model, however, will accommodate a situation where facility level has no effect on attendance patterns or benefits, if necessary,

Table 6. Percentage shortfalls of three location schemes

			1.00	0.4621	0.3466	0.1733	0.1155	0.0866	0.0693
α	Factor solution		0.69	1.50	2.0	4.0	6.0	8.0	10.0
		Actual	20.67	17.12	15.12	10.42	7.88	6.31	5.27
0.0	1.0	Size-down	**0.0**	**0.41**	**0.94**	2.11	2.07	1.84	1.64
		P-median	9.19	5.65	3.89	**1.19**	**0.44**	**0.16**	**0.06**
		Actual	18.70	15.09	13.30	8.20	5.50	4.03	3.03
0.1375	1.1	Size-down	**0.0**	**0.29**	**1.02**	1.73	1.57	1.46	1.29
		P-median	9.76	6.02	4.44	**1.23**	**0.42**	**0.37**	**0.38**
		Actual	17.03	13.54	11.72	6.56	3.95	3.03	2.40
0.2630	1.2	Size-down	**0.0**	**0.43**	**1.11**	1.77	1.53	1.79	1.91
		P-median	10.09	6.42	4.78	**1.74**	**1.17**	**1.61**	**1.36**
		Actual	15.60	12.26	10.42	5.23	3.44		
0.3785	1.3	Size-down	**0.0**	**0.60**	**1.30**	**1.70**	**2.06**		
		P-median	10.24	6.71	5.11	2.26	2.31		
		Actual	14.37	11.17	9.33	4.27	2.99		
0.4854	1.4	Size-down	**0.0**	**0.79**	**1.42**	**1.57**	2.57		
		P-median	10.28	6.92	5.38	2.72	**2.05**		
		Actual	13.30	10.24	8.38	3.89			
0.5850	1.5	Size-down	**0.0**	**0.93**	**1.53**	**1.98**			
		P-median	10.23	7.06	5.56	3.24			
		Actual	12.37	9.43	7.53	3.56			
0.6781	1.6	Size-down	**0.0**	**1.03**	**1.59**	**2.34**			
		P-median	10.12	7.12	5.65	3.04			
		Actual	11.56	8.71	6.78				
0.7655	1.7	Size-down	**0.0**	**1.13**	**1.35**				
		P-median	9.97	7.13	5.67				
		Actual	10.83	8.07	6.13				
0.8480	1.8	Size-down	**0.0**	**1.22**	**1.17**				
		P-median	9.79	7.10	5.64				
		Actual	10.19	7.49	5.57				
0.9260	1.9	Size-down	**0.0**	**1.18**	**1.03**				
		P-median	9.59	7.03	5.60				
		Actual	9.62	6.97	5.07				
1.00	2.0	Size-down	**0.0**	**1.04**	**0.93**				
		P-median	9.39	6.93	5.55				

Bold indicates best solution.

through an appropriate choice of parameters. A facility's importance is treated as declining exponentially with distance, a functional form which has been shown to describe many physical and human phenomena. As a natural extension, benefit is abstracted from a facility's drawing power. Thus, benefit may accrue to a facility's level within the hierarchy as well as to its location.

The model has been subjected to sensitivity analysis over a range of size- and distance-effect parameters for data from Salcette Taluka, Goa, India. With certain values of these parameters, the traditional *P*-median model appears to provide an optimal solution. This result is characterized by unbalanced hierarchical structures, however, suggesting that these values, and hence the *P*-median model, are inappropriate. Under other values, the SIZE-DOWN system, in which facilities are located from the highest to the lowest level in centers of strictly decreasing population, results. This solution is characterized by spatially eccentric allocation patterns, which if efficient, are certainly less equitable than the *P*-MEDIAN result. For many values of α and β, the ACTUAL system is not substantially worse than the *P*-MEDIAN, and in two cases, it appeared to be marginally better.

Further work is required, both in terms of model development and in terms of data input. In the case of modeling, the all-or-nothing allocation procedure is questionable, and the use of a model with probabilistic allocation procedures such as Hodgson's [20] Consumers' Welfare approach should be investigated. It would also be of great interest to seek more equitable solutions through the introduction of center-type models based on the negative exponential allocation rule. Wedding the maximum coverage approaches of Moore and ReVelle [21] and Bennett *et al.* [22] with negative exponential distance decay might prove to be fruitful. With respect to data requirements, calibration of the attendance model with real world interaction data is essential. Such data are notoriously difficult to obtain for developing countries, but the payoffs could be substantial.

Relative to the *P*-median model, the negative exponential model is complex in its formulation and data requirements. The results of this study suggest the possibility that even the *P*-median model may be more complex than necessary, that a simple SIZE-DOWN strategy may produce better results. If calibration of the model with real-world interaction data confirms this finding, the immense effort will be justified, for it will have led to the finding that a simple intuitive strategy is superior to the *P*-median model. This would be extremely good news from the point of view of real-world implementation, for a map and a table of populations would be the only tools required to locate spatially efficient medical facility systems. If this paper serves as an incentive for others to examine attendance patterns within actual health facility systems in the developing world, its significance will extend far beyond the presentation of a somewhat esoteric operations research model.

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