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# Membrane stress analysis of collapsible tanks and bioreactors

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ABSTRACT

Collapsible tanks, vessels or bioreactors are finding increasing usage in small/medium scale processes because they offer flexibility and lower cost. However, if they are to be used at large scale, they need to be shown capable of handling the physical stress exerted on them. Because of their nonconventional shape and non-uniform pressure distribution, thin shell analysis cannot be used in calculating their stress. Defining curvature in terms of pressure addressed these challenges. Using curvature and numerical analysis, the membrane stress in collapsible tanks designed as bioreactors of volumes between 100 to 1000 $m^3$ were calculated. When the liquid/gas height and static pressure are known, an equation for calculating tension per length was developed. An equation that could calculate the liquid height from the bioreactor's volume, dimensions and working capacity was generated. The equation gave values of liquid height with a maximum deviation of 3% from that calculated by curvature analysis. The stress values from the liquid height and tension equations had a maximum deviation of 6% from those calculated by curvature analysis. The calculated tensile stress in a 1000 $m^3$ collapsible tank was 14.2 MPa. From these calculations, materials that optimize both cost and safety can be selected when designing collapsible tanks.



## 1. Introduction

Collapsible tanks or vessels are vessels made from materials that are flexible, easily deformable, and light. Without a rigid supporting system collapsible tanks normally take the shape of a pillow when filled with fluid, so they are also called pillow or bladder tanks. Collapsible tanks are used as water storage vessels, fuel storage vessels, transportation vessels, sewage tanks, food storage vessels, chemical storage vessels, bioreactors for e.g. ethanol [1] or biogas production [2] etc. The benefits of using collapsible tanks over rigid tanks include process flexibility, ease of transportation, installation, portability and low cost. Collapsible tanks come in sizes suitable for use at small, medium [3] and large scale. Collapsible tanks are usually designed with their end use in view. One common question when using these collapsible tanks especially at larger scale is if the materials used for constructing them can tolerate the stress caused by the high pressure and the large liquid volume. Despite their wide application, there is no scientific publication on how to calculate the stress in these containers to safeguard against failure.

When collapsible tanks are used for storing or processing fluids, they can be designed as pressure vessels. A pressure vessel is a vessel that can withstand the internal pressure that is acting on it [4]. According to its wall thickness, pressure vessels can be classified as thick or thin walled. A pressure vessel is thin walled if the ratio of its wall thickness to its radius is less than or equal to 1/20 [4] . Thin walled vessels offer no resistance to bending, so the stress on it is distributed through its thickness, resulting in only membrane stresses, while thick walled vessels offer resistance to bending, so they have both membrane stresses and bending stresses [4]. Thus, collapsible tanks are thin-walled pressure vessels. Thin shell theory is normally used for calculating the stress in thin walled pressure vessels under constant internal pressure [4]. Using the thin shell theory in calculating the membrane stress in collapsible tanks would not give accurate values. Some reasons for this are; collapsible tanks will not have the same pressure at all points, and they will not have a specific shape at all times because their shape will change with changes in pressure and the volume of fluid in them [5].

Accurate determination of the stresses in any pressure vessel is essential to safeguard against failure [6] which could occur when the membrane stress in the vessel has exceeded the material's intrinsic tolerance values as determined by its Young's modulus [7]. One challenge why the thin shell theory cannot be used to calculate the stress that would act on collapsible tank is that their

*Abbreviations:* DEs, differential equations; WC, working capacity.

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**Nomenclature**

| | |
|---|---|
| k | Curvature |
| s | Arc length |
| $P_0$ | Static pressure |
| h | $Y_-$ liquid height |
| Hg | Gas height |
| α | Directional angle of the tangent to the curve |
| T | Membrane stress force or tension per unit length |
| W | Width of collapsible tank |
| L | Length of collapsible tank |
| p | Perimeter of collapsible tank |
| $A_L$ and A | Liquid and total cross sectional area of collapsible tank |

shape would change with variations in the pressure and volume of the fluid in these tanks. Defining their curvature as a function of the fluid property of interest, which in this case is pressure, helps to overcome this challenge. This also helps in handling the second challenge of non-uniform pressure distribution in collapsible tanks. In this work, the membrane stress associated with collapsible tanks and their geometry was determined using their curvature and numerical analysis. The results gotten from the analysis performed on collapsible tanks can assist in determining their strength, how suitable they would be for large scale purposes, how they can be designed to minimize the stress in them, and what materials are suitable for making them.

## 2. Methods

The geometry of a collapsible tank as determined by curvature is shown in Fig. 1. The stress analysis performed in the direction of the width of a collapsible tank is termed circumferential, while that one performed in the direction of the length is termed longitudinal. The key assumptions used for the analysis performed in this work are; (a) the weight of the material used for constructing the collapsible tank can be neglected, (b) the material is infinitely flexible in the direction of curvature, (c) the collapsible tank membrane stress can be analysed using one plane at a time, and (d) the sum of the static pressures generated by the two directions of curvature gives the overall static pressure acting on the collapsible tank. As failure due to tensile stress has a higher chance of occurring in areas with less material reinforcement [7], curvature analysis performed in this paper best describes regions far from the edges of the collapsible tanks. In these areas, it can be assumed that there would be no interaction of the two orthogonal axes and the joints in the collapsible tank to increase its strength.

### 2.1. The shape of a collapsible tank as defined by its curvature

To find the shape of a collapsible tank, a reference frame was chosen in which position and altitude were considered using the top height of the liquid level as the origin [8]. Every height below the top of the liquid level was negative, while only the gas height (if present) was positive (Fig. 2).

The curvature (k) of any shape or line was defined using Eq. (1), where α is the directional angle of the tangent to the curve and s is the arc length [9]. However, as the pressure is the source of the curvature, the curvature was expressed in terms of the pressure according to Eq. (2), where $P_0$ is the static pressure above the liquid ($N/m^2$), $y_-$ is the liquid height (m) which is defined with respect to the y axis according to Eq. (3), g is gravity ($m/s^2$), ρ (rho) is density ($kg/m^3$) and T is the membrane stress force or tension per unit length (N/m). The differential equations (DEs) relating the curvature, the arc length and the x and y coordinate functions for the curve are shown in Eq. (4) [10].

$$k = \frac{d\alpha}{ds} \tag{1}$$

$$k = \frac{P_0 - y_- g\rho}{T} \tag{2}$$

$$y_- = \begin{cases} y, y \le 0 \\ 0, y > 0 \end{cases} \tag{3}$$

$$\begin{cases} \dfrac{d^2x}{ds^2} = k\dfrac{dy}{ds} \\ \dfrac{d^2y}{ds^2} = -k\dfrac{dx}{ds} \end{cases} \tag{4}$$

### 2.2. Numerical analysis

A system of 4 first order DEs ($U_1(s)$:$U_4(s)$), were defined to reduce the 2 second order DEs in Eq. (4) to their first order forms [11]. The defined functions are shown between Eq. (5)–(8).

$$U_1(s) = x(s) \tag{5}$$

$$U_2(s) = y(s) \tag{6}$$

$$U_3(s) = \frac{dU_1}{ds} = \frac{dx}{ds} \tag{7}$$

$$U_4(s) = \frac{dU_2}{ds} = \frac{dy}{ds} \tag{8}$$

The DEs generated by substituting the defined functions (Eq. (5):(8)) and Eq. (2) into Eq. (4) results in a system of first order DEs as shown in Eq. (9). The resulting system of DEs (see Eq. (10)) was solved numerically using MATLAB® ODE45 solver. The calculations were performed using initial values of $U_1(0) = 0$, $U_2(0) = Hg$, $U_3(0) = 1$, and $U_4(0) = 0$.

$$\begin{cases} \dfrac{dU_3}{ds} = \dfrac{d^2x}{ds^2} = k\dfrac{dy}{ds} = kU_4(s) = \dfrac{P_0 - U_2 g\rho}{T}U_4(s) \\ \dfrac{dU_4}{ds} = \dfrac{d^2y}{ds^2} = -k\dfrac{dx}{ds} = -kU_3(s) = -\dfrac{P_0 - U_2 g\rho}{T}U_3(s) \end{cases} \tag{9}$$

$$\begin{cases} \dfrac{dU_1}{ds} = U_3(s) \\ \dfrac{dU_2}{ds} = U_4(s) \\ \dfrac{dU_3}{ds} = \dfrac{P_0 - U_2 g\rho}{T}U_4(s) \\ \dfrac{dU_4}{ds} = -\dfrac{P_0 - U_2 g\rho}{T}U_3(s) \end{cases} \tag{10}$$

### 2.3. Stress and shape determination

The numerical results from equation 10 was used to analyse the relationship between the membrane stress (T), the static pressure ($P_0$), the gas height (Hg) and the shape of the collapsible tank. The stress (T), static pressure ($P_0$) and gas height (Hg) (see Eq. (10)) were used as control parameters to generate the corresponding collapsible tank shape (x,y). From this shape, other properties of the collapsible tank such as width, liquid depth, and cross sectional area were calculated. The width (W) of the collapsible tank was determined as shown in Eq. (11), when carrying out the circumferential stress analysis.

$$W = 2x(s) \text{ at the point where } \frac{dx}{ds} = 0 (s_1 \text{ in Fig. 2}) \tag{11}$$

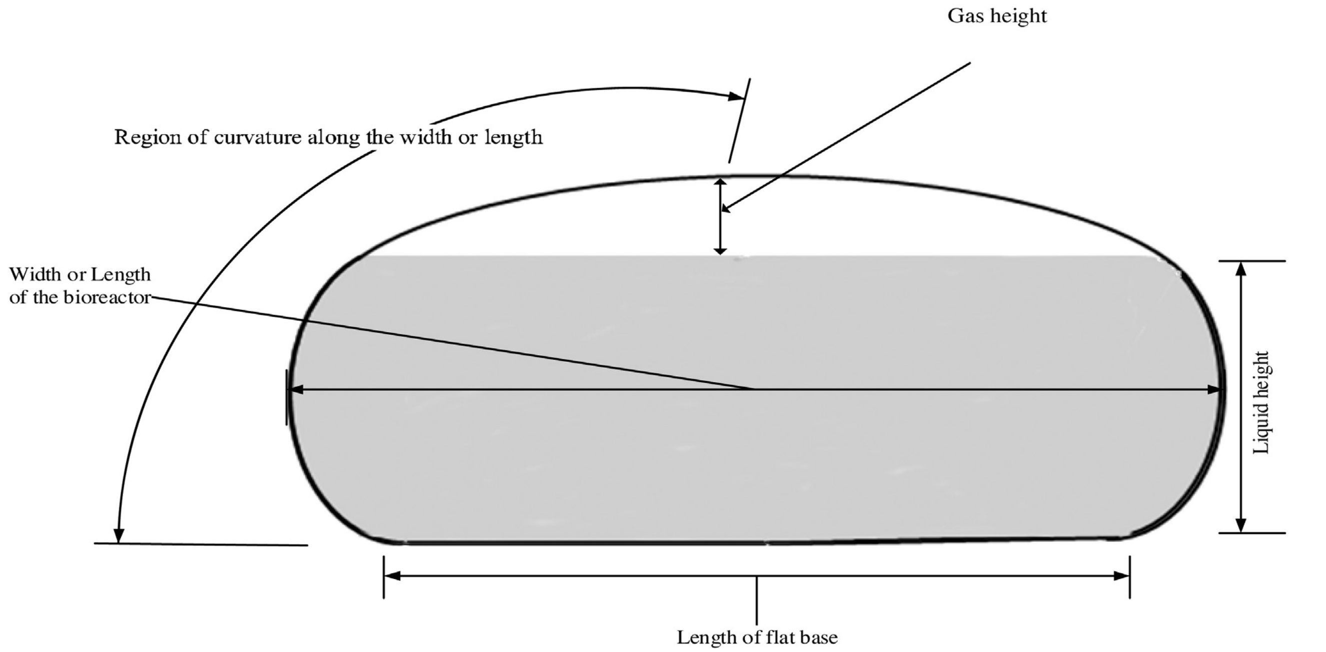


**Fig. 1.** The geometry of a collapsible tank.

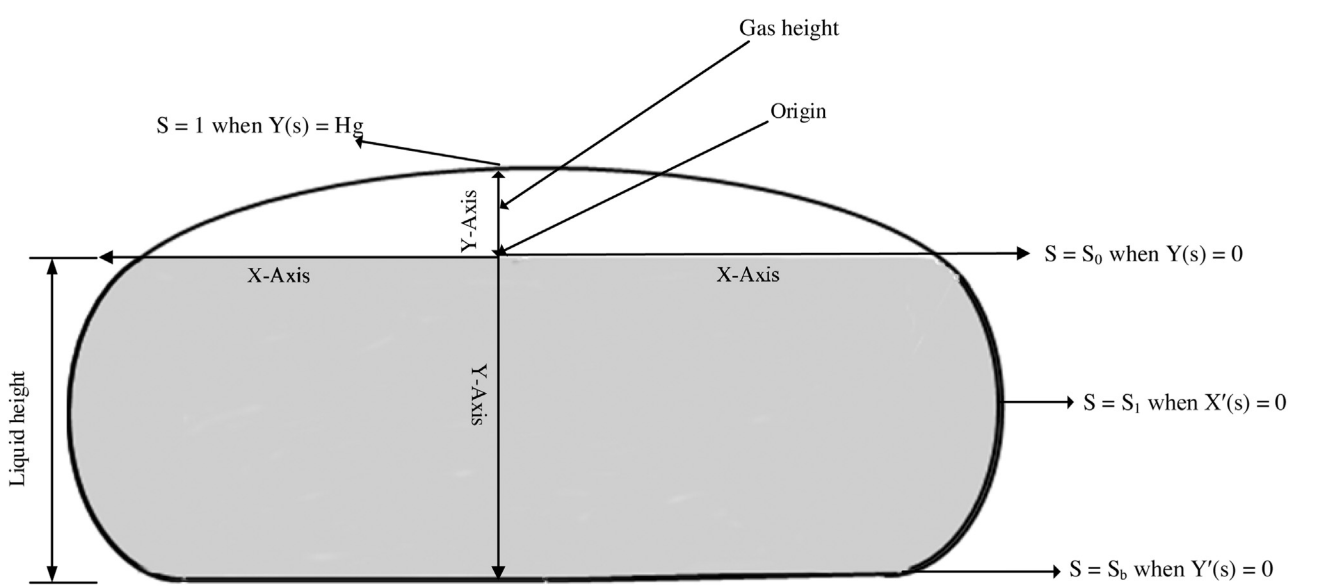


**Fig. 2.** The reference frame used to analyse the collapsible tank.

When the longitudinal stress analysis was carried out, the length of the collapsible tank (L) was determined as shown in Eq. (12).

$$L = 2x(s) \text{ at the point where } \frac{dx}{ds} = 0 \quad (12)$$

The liquid depth (h) for both circumferential and longitudinal stress analysis was calculated using Eq. (13).

$$h = -y(s) \text{ at the point where } \frac{dy}{ds} = 0 \ (s_b \text{in Fig. 2}) \quad (13)$$

The length of the flat base ($L_{base}$) of the collapsible tank for both circumferential and longitudinal stress analysis was determined using Eq. (14).

$$L_{base} = 2x(s) \text{ at the point where } \frac{dy}{ds} = 0(s_b \text{in Fig. 2}) \quad (14)$$

The perimeter (p) of the collapsible tank was calculated by summing the arc length ($s_b$) from the top to the base (i.e. from Hg or s = 1 to $s_b$ in Fig. 2) with the base length ($L_{base}$).

$$p = 2(Ls + L_{base}) \quad (15)$$

The volume per unit length or width of the collapsible tank was determined as the cross sectional area (A) inside the (x,y) curve calculated according to Eq. (16). The last form of Eq. (16) was what was used for the calculations in MATLAB (see Table 1)

$$A = 2\int_{-h}^{Hg} x dy = -2\int_{Hg}^{-h} x dy = -2\int_{1}^{sb} x(s)y'(s)ds \quad (16)$$

The volume of the tank occupied by liquid per unit length or width was determined as the liquid cross sectional area ($A_L$) from

**Table 1**
Result from numerical analysis in the width direction of collapsible tanks from 100 to 1000 $m^3$ and a 30 L bioreactor.

| Specified parameters | | | | Final iterative values | | | Calculated values | | |
|---|---|---|---|---|---|---|---|---|---|
| Volume of bioreactor ($m^3$) | Specified length (m) | Working capacity | Width of bioreactor (m) | Tension per unit length (N/m) | Gas height (m) | Static pressure ($N/m^2$) | Liquid height in the bioreactor (m) | Perimeter of bioreactor (m) | Bioreactor's flat base length (m) |
| 100 | 20 | 0.84 | 7 | 1000 | 0.22 | 58 | 0.63 | 14.57 | 6.63 |
| 200 | 23 | 0.80 | 9 | 1500 | 0.44 | 85 | 0.77 | 18.94 | 8.65 |
| 300 | 35 | 0.80 | 9 | 1500 | 0.44 | 85 | 0.77 | 18.94 | 8.65 |
| 400 | 33 | 0.84 | 11 | 2200 | 0.33 | 72 | 0.94 | 23.44 | 10.75 |
| 500 | 30 | 0.80 | 14 | 2600 | 0.44 | 73 | 1.02 | 28.21 | 13.04 |
| 1000 | 50 | 0.83 | 15 | 3500 | 0.45 | 92 | 1.18 | 30.20 | 13.87 |
| 0.03 | 1.1 | 0.82 | 0.40 | 6 | 0.02 | 10 | 0.05 | 0.84 | 0.37 |

where the liquid region began ($y(s) = 0$ i.e. where $s = s_0$ in Fig. 2) to the base of the collapsible tank ($y(s) = -h$ or $s = s_b$).

$$A_L = 2\int_{-h}^{0} x dy = -2\int_{S0}^{Sb} x(s)y'(s)ds \tag{17}$$

The working capacity of the collapsible tank (WC) was defined as the ratio of liquid cross-sectional area to the total cross-sectional area as shown in Eq. (18).

$$WC = A_L/A \tag{18}$$

Eq. (11), (12),(14)–(17) were multiplied by 2 because the solutions were calculated for the right half of the laterally symmetric cross section (Fig. 2).

The numerical analysis was performed using MATLAB ODE45 solver. Details about how the solver works is not presented here but can be found from literature [12] . The steps taken to perform the analysis from Eq. (7) –(18) in MATLAB are shown in Fig. 3.

### 2.4. Iterative calculation of the stress, static pressure and gas height from specified collapsible tank

For a given cross-sectional area of a collapsible tank (i.e. its volume per unit length or width) and its working capacity, an iterative process was used to find the corresponding tension per unit length or width (T), the gas height (Hg) and the static pressure ($P_0$). T, Hg, and $P_0$ were used as input variables in the iterative process. After determining the solution to the DEs in Eq. (7)–(10), a plot of the solution (x,y) was drawn to ensure the collapsible tank shape was as expected, before the other values (from Eq. (11)–(18)) were calculated. Fig. 3 shows how the analysis was performed with MATLAB. When the cross-sectional area used for the analysis is the volume of the collapsible tank per its length, the analysis is termed circumferential, while when the cross-sectional area is the volume per unit width of the collapsible tank the analysis it is termed longitudinal. Dimensions of collapsible tanks used by FOV Fabrics AB (Borås, Sweden) for making flexible and easily deformable bioreactors were used for the calculations.

For the circumferential membrane stress analysis; the length was assumed constant and the target volume per unit length was calculated. T, Hg and $P_0$ values were defined for target values of A, $A_L$ and W. The steps shown in Fig. 3 as discussed above were placed inside a least square iterative solver of MATLAB (lsqcurvefit), which continued to change the values of T, Hg and $P_0$ until the volume per unit length, the working capacity and width generated were same as the target values. The value of T in the final iterative calculation was the circumferential membrane tension per unit length. The collapsible tank width under these conditions was calculated from Eq. (11) and used for the longitudinal analysis.

For the longitudinal membrane stress analysis; the width was constant and the volume per unit width was calculated. The liquid height (gotten from Eq. (13)) and Hg were additional constrains to the ones used for the circumferential analysis. Iterative steps described for the circumferential stress analysis were performed to give the mean longitudinal tension per unit width.

### 2.5. Calculating the stress in a collapsible tank with known parameters

When the liquid height, gas height and static pressure in a collapsible tank are known, the tension per unit length acting on the collapsible tank can be calculated by integrating the first part of Eq. (9). This part of Equation can be rewritten as;

$$\frac{d^2x}{ds^2} = f[y(s)]\frac{dy}{ds} \quad \text{Where } f[y(s)] = \frac{P_0 - U_2 g\rho}{T} = \frac{P_0 - y(s)g\rho}{T} \tag{19}$$

Multiplying Eq. (19) by ds and integrating gives Eq. (20)

$$\int_{sb}^{s=1} \frac{d^2x}{ds^2} ds = \int_{sb}^{s=1} F[y(s)]\, ds = \int_{-h}^{Hg} f[y(s)]\, dy \tag{20}$$

The left hand side (LHS) of equation 20 gives $-2$ (as $X'(s) = 1$ @ $s = 1$ and $-1$ @ $s = s_b$ see Fig. 2). The integral of the right hand side is shown in Eq. (21)

$$F[y(sb)] - F[y(s=1)] = \frac{-2P_0 h - (-h)^2 g\rho}{2T} - \frac{P_0 Hg}{T} \tag{21}$$

Combining Eq. (21) with the LHS of Eq. (20) and making T subject of the formula gives Eq. (22).

$$T = 0.5P_0(h + Hg) + 0.25\rho g h^2 \tag{22}$$

### 2.6. Comparison of the stress determined by curvature analysis with experimentally measured values from a laboratory scale collapsible tank

The calculated stress value in a 30 L collapsible tank (which is used as a bioreactor) with 1.1 m length was used to compare with the result from numerical curvature analysis. The bioreactor was filled with 25 L of water and sparged with air. The curvature was measured halfway between the top of the bioreactor and the top of the liquid in the bioreactor (i.e. where the curvature is constant) using a flexible retractable measuring tape to measure the change in arc length and a folding ruler used to measure the change in inclined height (which gave the directional change in the angle of the tangent to the curve). The curvature was calculated by dividing the directional change in the angle of the tangent to the curve at the specified point by the corresponding change in the arc length (see Eq. (1)). The static pressure and total pressure were measured using a laboratory made u-tube manometer. The static pressure was the pressure measured on top of the liquid in the bioreactor, while the total pressure was measured at the base of the bioreactor. The gas and liquid height in the bioreactor, and the bioreactor's width and length were measured using a retractable measuring tape. The

```
% rho, and g were specified

rho = 1000; g = 9.81;

% values for T, Hg, P0 were specified

T = 1000; Hg = 0.2; P0 = 100;

% a nameless function holder f, was created to simplify equation 10 and 11

f = @(s,y) [y(3); y(4); (P0/T-rho*g/T*y(2)*(y(2)<0))*y(4); - (P0/T-
rho*g/T*y(2)*(y(2)<0)) * y(3)];

[s y] = ode45(f,[0:0.0001:25],[0,Hg,1,0]);

plot(y(:,1),y(:,2))

% the point at which the curve touches the base

sb = find(y(:,4)>0,1)

% the arc length to the point at which the curve touches the base

s1 = s(sb)

% half the length of the flat base

xb = y(sb,1)

perimeter = 2*(s1+xb)

baselength = 2*xb

% the point on the curve with the maximum width or length

sw = find(y(:,3)<0,1);

Width or length = 2* y(sw,1)

liquidheight = -y(sb,2)

% the point on the curve where the liquid region begins

s0 = find(y(:,2)<0,1);

% Liquid volume per unit length

AL = -2*sum(y(s0:sb,1).*y(s0:sb,4))*0.0001

% The volume per unit length

A = -2*sum(y(1:sb,1).*y(1:sb,4))*0.0001

Working capacity = liqVol/TotVol
```

**Fig. 3.** Numerical analysis with Matlab.

average of the curvature values calculated was used for determining the stress in the bioreactor from Eq. (2). The stress calculated from the previously mentioned experimentally measured parameters in the 30 L bioreactor was compared with the value gotten from the numerical curvature analysis.

## 3. Results and discussion

Collapsible tanks find application in different industries and processes. In the biotechnology industry, they are used for making bioreactors used for biogas production [2], bioethanol production [13], biological production using plant cells [3] etc. especially at small and medium scale. These bioreactors offer flexibility, low cost, easy installation and customized applicability to bio-based production [13]. If these and other benefits derived from using these types of bioreactors can be translated from small and medium scale to the large scale production of bio-products like biofuels, it could assist in increasing the economic competitiveness of the biofuels against fossil based fuels. However, in order to apply these bioreactors or any type of collapsible tank for large scale purposes they need to be able to handle the stresses exacted on them while in operations [14] and during cleaning if needed [15]. The membrane stresses of collapsible tanks meant to be used as bioreactors and that of a 1000 $m^3$ water storage vessel were investigated using curvature and numerical analysis and the results are presented in this section.

### 3.1. *Shape and stress determination in collapsible tanks designed as bioreactors*

Collapsible tanks of volumes between 100–1000 $m^3$ designed as bioreactors for large scale applications were used for stress calculations using numerical analysis.

#### 3.1.1. Circumferential analysis

The results of the analysis performed in the width direction of collapsible tanks from 100 to 1000 $m^3$ are shown in Table 1. The tension per unit length values in the width direction for the collapsible tanks considered were between 1000 N/m and 3500 N/m. It can be observed from Table 1 (see the 200 and 300 $m^3$ collapsible tanks) that collapsible tanks having the same liquid height, static pressure and gas height would have the same tension irrespective of the volume of the collapsible tank or its length, which agrees with Eq. (22). However, the volume, width and length of a collapsible tank or bioreactor are important parameters for its design, cost, and plant layout planning. Because of this and the possibility of controlling the liquid height, static pressure and gas height in the collapsible tank by the volume and dimension of the collapsible tank, a relation between the width, liquid height, volume and working capacity of the bioreactor was calculated from the plot of the results in Table 1. The relation is shown in Eq. (23). It can be seen from the relation that there is a linear relationship between the product of the volume per unit length (A) and the working capacity (WC) of the collapsible tank (i.e. the liquid cross-sectional area in the collapsible tank) and the product of the width (W) and liquid height (h) in the collapsible tank.

$$WC.A = 0.968W.h - 0.0755 \tag{23}$$

From Eq. (23), the liquid height can be calculated from collapsible tanks with specified volume, length, width and working capacity. The calculated liquid height can then be used to determine the tension per unit length acting in the bioreactor using Eq. (22) when $P_0$ and Hg are known.

#### 3.1.2. Longitudinal analysis

The mean tension per unit width in the length direction of collapsible tanks considered was between 1050–4000 N/m. The results from this analysis plotted against those from the circumferential analysis are shown in Fig. 4. There was a linear relationship between the circumferential tension per unit length and the mean longitudinal tension per unit width (Fig. 4). The relation between both stresses is shown in Eq. (24). For the collapsible volumes and dimensions considered, the mean longitudinal tension was higher than the circumferential tension. This is because collapsible tanks tend to have more curvature along their width than along their length, and from Eq. (2) there is an inverse relationship between tension and curvature.

$$T_{Longitudinal} = 1.16 \times T_{Circumferential} - 177.21 \tag{24}$$

### 3.2. Comparing relation equation values with values from numerical analysis

The liquid height relation in Eq. (23) was generated using initially specified length, width, and working capacity values for a specific collapsible tank volume. This relation was examined to determine how well it could estimate the liquid height in different collapsible tanks. For a specified collapsible tank volume, dimensions different from the ones used for generating it were put in it and the liquid height values calculated with it were compared with the values gotten from curvature analysis. Additionally, tension per unit length values was calculated both from Eq. (22) (by using liquid height values from Eq. (23)) and from curvature analysis. The generated liquid height and tension per unit length values from the relations and from curvature analysis for different collapsible tank volumes are shown in Table 2. A maximum difference of 3% (i.e. overestimation or underestimation) in the value of the liquid height calculated from the curvature analysis and from the height relation can be seen (Table 2). In addition, the maximum difference in tension for all cases examined is approximately 6%. This small difference both for the liquid height and tension shows that the height relation estimates very well what the height of liquid would be in a collapsible tank from the dimensions of the collapsible tank.

### 3.3. Comparing experimentally measured values with that from numerical curvature analysis

The stress in a 30 L laboratory scale collapsible tank (that is used as a bioreactor) was calculated from experimentally measured values using its curvature. The average curvature of the bioreactor from its measured directional change in tangent and the change in arc length was 1.78 $m^{-1}$, while the static pressure in the bioreactor was 10 Pa. Using Eq. (2) , the tension per unit length of the bioreactor was calculated to be 5.62 N/m. Performing the numerical analysis for a theoretical 30 L bioreactor as previously described gave the tension per unit length of the bioreactor to be 6 N/m (see Table 1). The liquid height measured from the experimental bioreactor was 5 cm, while that from the numerical curvature analysis was 4.81 cm. The width of the experimental bioreactor was measured to be 36 cm, the corresponding value from the numerical curvature analysis was 39.6 cm. The liquid volume fraction or working capacity in the laboratory bioreactor was 0.833, while numerical analysis gave it as 0.815. Comparing these values shows how well the result from the curvature analysis agrees with the experimentally measured values. From this, it can be expected that numerical curvature analysis can be used to determine the shape and stress in collapsible tanks or bioreactors.

### 3.4. Operating the collapsible tank under different conditions

The collapsible tank or bioreactor can be operated under different conditions. When the collapsible tank is operated at static pressures between 10 and 100 Pa, width to liquid height ratio between 13 and 20, and working capacity between 0.6–1, it can be said to be in a relaxed condition (i.e. normal operating conditions specified by its manufacturer). When operated at static pressure between 1100 and 5000 Pa and with a width to liquid height ratio between 1 and 5, it can be said to be pressurized. When operated at working capacity less than or equal 0.5, static pressure between 15 and 50 Pa and width to liquid height ratio greater than 20, it can be said to be deflated. How these changes can affect the tension and total pressure in the collapsible tank is shown in Table 2 and Fig. 5. The total pressure (used in Fig. 5) was calculated by summing the static pressure with pressure due to liquid height in the same collapsible tank. The results show that the tension per unit length profile (from numerical curvature analysis) and the total pressure profile are similar for the same operating condition in the collapsible tank. This is because the pressure and the liquid height in the collapsible tank are the sources of the tension (Eq. (22)).

Considering the 100 $m^3$ collapsible tank, changing the operating conditions from relaxed to pressurized resulted in 430% increase in tension, while the change from relaxed to deflated resulted in 60% decrease in tension. Considering the 400 $m^3$ collapsible tank, changing from relaxed to pressurized conditions resulted in 430% increase in tension, while changing from relaxed to deflated conditions reduced the tension by 60%. For the 1000 $m^3$ collapsible tank, changing from relaxed to pressurized conditions resulted in 530% increase in tension, while changing from relaxed to deflated conditions resulted in 64% reduction in tension. Using these values, if a bioreactor or collapsible tank would be operated or sterilized at a static pressure up to 5000 Pa, the ultimate tensile strength of its material of construction must be at least 6 times the calculated tension values under normal conditions plus a security margin. If this cannot be attained, the bioreactor or collapsible tank should not be operated or sterilized under these conditions.

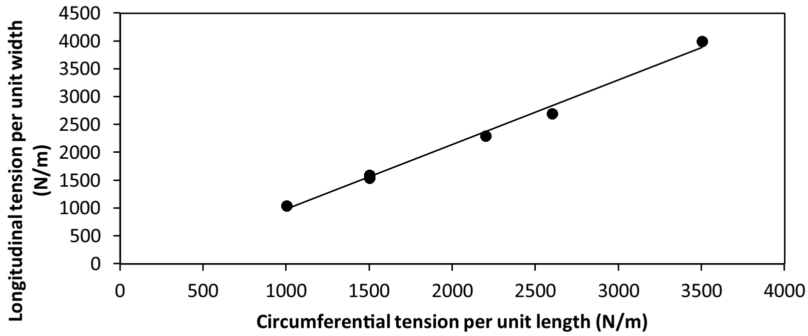


**Fig. 4.** Longitudinal tension against circumferential tension at a particular collapsible tank volume for volumes between 100–1000 $m^3$.

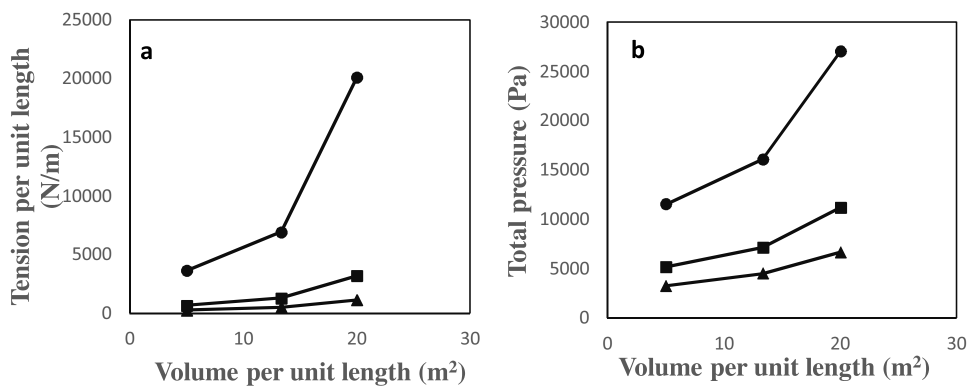


**Fig. 5.** A plot of Tension per unit length from numerical curvature analysis (a) and total pressure (b) against the volume per unit length of collapsible tanks when operated at deflated (▲), relaxed (■) and pressurized (●) conditions.

**Table 2**
Comparison results gotten from the generated relation and from numerical analysis for different bioreactor volumes.

| Volume of bioreactor ($m^3$) | Specified length (m) | Working capacity | Specified width (m) | Liquid height from relation (m) | Liquid height from analysis (m) | Tension per unit length from relation (N/m) | Tension per unit length from numerical analysis (N/m) | Change in liquid height (%) | Change in tension per unit length (%) |
|---|---|---|---|---|---|---|---|---|---|
| 100 | 20 | 0.80 | 8 | 0.526 | 0.523 | 689.97 | 680.97 | −0.67 | 1.32 |
| 100 | 10 | 0.80 | 8 | 1.043 | 1.044 | 2862.33 | 2867.54 | 0.10 | 0.18 |
| 100 | 20 | 0.50 | 8 | 0.333 | 0.329 | 279.60 | 273.43 | −1.15 | 2.26 |
| 100 | 20 | 0.80 | 4 | 1.053 | 1.040 | 3713.23 | 3642.92 | −1.17 | 1.93 |
| 400 | 30 | 0.80 | 15 | 0.740 | 0.726 | 1351.91 | 1301.39 | −1.93 | 3.88 |
| 400 | 15 | 0.80 | 15 | 1.474 | 1.479 | 5545.10 | 5580.81 | 0.33 | −0.64 |
| 400 | 30 | 0.50 | 15 | 0.464 | 0.455 | 537.84 | 517.39 | −1.98 | 3.95 |
| 400 | 30 | 0.80 | 7.5 | 1.480 | 1.526 | 6557.12 | 6920.36 | 3.01 | −5.25 |
| 1000 | 50 | 0.82 | 15 | 1.142 | 1.132 | 3255.33 | 3201.64 | −0.84 | 1.68 |
| 1000 | 50 | 0.50 | 15 | 0.694 | 0.673 | 1221.87 | 1150.63 | −3.14 | 6.19 |
| 1000 | 25 | 0.83 | 15 | 2.278 | 2.205 | 13731.72 | 12905.74 | −3.31 | 6.40 |
| 1000 | 50 | 0.82 | 7.5 | 2.269 | 2.324 | 19369.92 | 20098.94 | 2.35 | −3.63 |
| 1000 | 50 | 0.82 | 30 | 0.567 | 0.584 | 790.14 | 836.92 | 2.84 | −5.59 |

### 3.5. Selection of material for construction of collapsible tanks

The tensile strength of general purpose plastics ranges between 1 and 50 MPa. The tensile strength of plastics can be increased up to 280 MPa when they are combined with some other materials like silicone in the form of composites [16]. Turning conventional plastics into composites increases their strength but also increases their cost. One of the driving forces behind the use of collapsible tanks is their cheap cost in comparison to rigid vessels. To be able to maintain this edge, a good trade-off would be needed so that one can combine less expensive materials with plastics to increase their tensile strength, and ensure the strength of the collapsible tank is optimized to prevent failure. Apart from using composites, other materials can also be used for making collapsible tanks as long as they have properties that meet the tank end use requirement. If the area the collapsible tank is going to occupy is known, the tension that would be acting in the bioreactor can be estimated using Eq. (22) and (23), with the known tension, the type of material that satisfies both the cost and tension criteria can be selected for designing the collapsible tank.

Considering a 1000 $m^3$ collapsible tank that would be used for water storage, if because of area constraint the tank should have a length of 40 m, width of 20 m and working capacity of 0.9, then the liquid height in the tank would be 1.2 m (Eq. (23)) while the circumferential tension per unit length would be 3360 N/m (Eq. (22)) and the mean longitudinal stress would be 3720 (Eq. (24)). If the tank would be made of plasticized polyvinylchloride (PVC) as the base polymeric material then a pressure allowance of factor of 1.5 [14] needs to factored into the calculated stress to prevent failure. If this factor is included and the thickness of the tank is 0.75 mm then the required tensile stress would be 14.2 MPa. However, the ultimate tensile strength of plasticized PVC is between 7 and 25 MPa [16] . If the PVC is plasticized with a material of low cost (e.g. textile) [17] which can increase the tensile strength of the composite higher than 14.2 MPa then failure can be prevented while also reducing cost.

## 4. Conclusion

Curvature analyses performed on collapsible tanks used as bioreactors of volumes between 100 $m^3$ to 1000 $m^3$ gave their shape, and the tension in these tanks was found to be between 300 N/m to 20000 N/m. For the bioreactor volumes considered a relation which could give the liquid height in the bioreactor from the properties of the bioreactor was found. The liquid height calculated by the relation had a maximum deviation of 3% from that calculated from curvature analysis. The tension in a collapsible tank can be minimized by minimizing the static pressure and liquid height in the tank taking the area it would occupy into consideration.

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