

Entropy bound for the photon gas in noncommutative spacetime



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ABSTRACT

Motivated by the doubly special relativity theories and noncommutative spacetime structures, thermodynamical properties of the photon gas in a phase space with compact spatial momentum space is studied. At the high temperature limit, the upper bounds for the internal energy and entropy are obtained which are determined by the size of the compact spatial momentum space. The maximum internal energy turns out to be of the order of the Planck energy and the entropy bound is then determined by the factor (V/l_{pl}^3) through the relevant identification of the size of the momentum space with Planck scale. The entropy bound is very similar to the case of Bekenstein–Hawking entropy of black holes and suggests that thermodynamics of black holes may be deduced from a saturated state in the framework of a full quantum gravitational statistical mechanics.

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1. Introduction

Existence of a minimum length scale, below which no other length could be observed, is suggested by all quantum gravity candidates such as string theory and loop quantum gravity [1,2]. It is then widely believed that a non-gravitational theory which contains a universal minimal length scale will be arisen at the flat limit of quantum gravity. Although the standard relativistic quantum mechanics does not take into account any minimal length scale, the flat limit of quantum gravity will be reduced to this standard theory in the continuum limit (if it exist) in the light of correspondence principle. In this respect, the deformation of standard relativistic quantum mechanics seems to be necessary in order to take into account a quantum gravity length scale. The first attempt in this direction goes back to the seminal work of Snyder who formulated a discrete Lorentz-invariant spacetime in which the spacetime coordinates appear to be non-commuting operators [3]. On the other hand, it is well-known that the transition from non-relativistic to relativistic and also classical to quantum mechanics are obtained by stabilizing deformations of two unstable algebraic structures, say, Lie algebra of Galilean group and Poisson algebra, to two robust stable Lorentz and Heisenberg algebras. However, when Lorentz and Heisenberg stable algebras are put to-

gether, the resultant algebra for the relativistic quantum mechanics turns out to be unstable. Thus, it is natural to explore a stable algebra for the relativistic quantum mechanics [4]. Evidently, the spacetime coordinates also become non-commuting operators for the resultant stable algebra of the relativistic quantum mechanics and a parameter with dimension of length naturally arises in this setup [5]. One could then consider this noncommutative stable algebra to be a candidate for the flat limit of quantum gravity by the relevant identification of the associated deformation parameter with universal minimal length. The non-commutativity of spacetime coordinates is also suggested by string theory at fundamental level [6]. The direct consequence of the non-commuting spacetime coordinates is the modification of the Heisenberg uncertainty principle (see for instance [7]). In this respect, one could also study the effects of a universal minimal length to the classical (special relativity) and non-relativistic (quantum mechanics) limit of relativistic quantum mechanics. For instance, inspired by string theory, generalized uncertainty relations are suggested [8] which support the existence of a minimal length through a nonzero uncertainty in position measurement [9]. This deformed quantum mechanics is very similar to the one that is inspired by non-relativistic subalgebra of Snyder noncommutative algebra [10]. Furthermore, from the fact that a minimal length in one inertial frame may be different in another observer's frame through the Lorentz–Fitzgerald contraction in special relativity, the doubly special relativity (DSR) theories are investigated in order to take into account an observer-independent minimum length (or maximal energy) in special relativity framework [11]. The Lorentz invariance then may be

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considered only as an approximate symmetry which will be broken at the Planck scale. However, it could be possible to formulate a Lorentz-invariant DSR theory through a nonlinear action of Lorentz group on four-momentum space [12]. Recently, it is found that the DSR theories can be realized from maximally symmetric curved four-momentum spaces and the observer-independent scale is determined by the constant curvature of the corresponding four-momentum space in this setup [13]. The duality of curved momentum space and noncommutative spacetime is also shown in quantum geometry setup [14]. Generally, deformed noncommutative spacetimes could be realized from deformed κ -Poincaré algebra on the associated κ -Minkowski noncommutative spacetime [15]. For instance, the Lorentz-violating algebra that is investigated in Ref. [11] and a Lorentz-invariant version that is proposed later in Ref. [12], could be realized from the different bases of κ -Poincaré algebra in a unified picture [16]. Also, it is shown that the Snyder algebra [3] and the stable algebra of relativistic quantum mechanics [5] can be obtained from a ten-dimensional phase space, with curved geometry for four-momentum space, through the symplectic reduction process [17]. In these respects, curved four-momentum spaces found their robust role in *relative locality principle* [18]. Furthermore, in the absence of a full quantum theory of gravity, the DSR theories suggest some quantum gravity corrections usually as a ratio of Planck length to the characteristic length scale of the system under consideration for some standard well-known systems. Although these effects are usually so small, they could be detected in principle in future high energy physics experiments or even in ultra-high energy cosmic rays showers (see Ref. [19] and references therein). In this respect, the DSR theories provide some potentially detectable effects for quantum gravity proposal.

The direct consequence of the curved four-momentum space or noncommutative spacetime is the deformation to the dispersion relation and the associated density of states [20–22]. Since density of states determines the number of microstates in statistical mechanics, the thermodynamical properties of the physical systems in these setups would be significantly different from the standard ones. Indeed, existence of a minimal length and maximal momentum (or maximal energy), as an extra information for the system under consideration, change the standard uniform probability distribution of microstates at high energy regime. In recent years, the effects of minimal length and maximal momentum on thermodynamical systems are studied in many contexts. For instance, thermodynamical properties of some physical systems in noncommutative spaces are studied in Ref. [23]. For the case of generalized uncertainty principle, see Refs. [24,25]. Thermodynamics of relativistic systems are studied in the context of DSR theories [21]. Moreover, it is well-known that black holes obey the thermodynamical laws and emit thermal emission (the so-called Hawking radiation) similar to the black body radiation [26]. In the absence of a full quantum theory of gravity, approaches to quantum gravity proposal such as string theory and loop quantum gravity reveal some thermodynamical aspects of the black hole physics [27,28]. Interestingly, the candidates for flat limit of quantum gravity are capable to reproduce some important thermodynamical results that are common between these alternative approaches. For the thermodynamics of the black holes in noncommutative spaces see Ref. [29]. The effects of the generalized uncertainty principle on black holes thermodynamics are studied in Ref. [30]. Therefore, black holes may be the most interesting thermodynamical systems that is expected to be considered in the framework of quantum gravitational statistical mechanics [31]. In this streamline, we study thermodynamical properties of a system composed of photon gas in noncommutative spacetime, as a semiclassical flat limit of quantum gravity. This study has the potential to open new windows on the statistical origin of black hole thermodynamics. The results

show that the thermodynamical properties of photon gas will be saturated at high temperature in noncommutative spacetime. We also find an interesting analogy between black hole entropy and entropy of saturated photon gas.

The structure of the paper is as follows: In Section 2, we present the stable (noncommutative) algebra of relativistic quantum mechanics and we consider its non-relativistic subalgebra in order to find the deformed density of states in this setup. In Section 3, we obtain the modified partition function by means of the deformed density of states. Using the partition function, we study thermodynamical properties of the photon gas in noncommutative spacetime in Section 4. Section 5 is devoted to the summary and conclusions.

2. Noncommutative spacetime: deformed density of states

In the context of stability theory, special theory of relativity is understood as a transition from the unstable Lie algebra of Galilean group, the kinematical group of non-relativistic quantum mechanics, to the stable algebra of Lorentzian group. Moreover, passage from classical to quantum mechanics is also appeared to be a transition from unstable Poisson algebra to the stable Heisenberg one. It was then pointed out that the fundamental theories of nature may also seem to be stable in the sense that they do not change in a qualitative manner under a small change of the associated parameters [4,32]. However, when Lorentz and Heisenberg stable algebras are put together in relativistic quantum mechanics framework, the resultant algebra turns out to be unstable. It is then natural to explore a stable algebra for the relativistic quantum mechanics through consideration of structural stability. In this regard, the minimal candidate for such a stable algebra of relativistic quantum mechanics is found as [5]

$$\begin{aligned} [J_{\mu\nu}, J_{\rho\sigma}] &= i(J_{\mu\sigma}\eta_{\nu\rho} + J_{\nu\rho}\eta_{\mu\sigma} - J_{\nu\sigma}\eta_{\mu\rho} - J_{\mu\rho}\eta_{\nu\sigma}), \\ [J_{\mu\nu}, p_\rho] &= i(p_\mu\eta_{\nu\rho} - p_\nu\eta_{\mu\rho}), \\ [J_{\mu\nu}, x_\rho] &= i(x_\mu\eta_{\nu\rho} - x_\nu\eta_{\mu\rho}), \\ [x_\mu, x_\nu] &= -i\epsilon\frac{J_{\mu\nu}}{\kappa^2}, \quad [p_\mu, x_\nu] = i\eta_{\mu\nu}\mathcal{I}, \quad [p_\mu, p_\nu] = 0, \\ [x_\mu, \mathcal{I}] &= i\epsilon\frac{p_\mu}{\kappa^2}, \quad [p_\mu, \mathcal{I}] = 0, \quad [J_{\mu\nu}, \mathcal{I}] = 0, \end{aligned} \quad (1)$$

where $\eta_{\mu\nu} = \text{diag}(+1, -1, -1, -1)$ is the Minkowski metric, $\epsilon = \pm 1$, and \mathcal{I} is the nontrivial operator that replaces the trivial center of the standard Heisenberg algebra [33]. Clearly, the spacetime coordinates x_μ become non-commuting operators and the deformation parameter κ with dimension of inverse of length naturally arises in this setup which could be identified with a minimal length scale. The deformed algebra (1) is Lorentz-invariant through the preservation of the commutation relation for generators $J_{\mu\nu}$ and the vector nature of (x_μ, p_μ) . A full representation of the stable algebra (1) by differential operators in a five-dimensional manifold with commutative coordinates ξ_A and flat metric $\eta_{AB} = \text{diag}(+1, -1, -1, -1, \epsilon)$ is given by [34]

$$\begin{aligned} x_\mu &= \frac{i}{\kappa} \left(\xi_\mu \frac{\partial}{\partial \xi^4} - \epsilon \xi^4 \frac{\partial}{\partial \xi^\mu} \right), & p_\mu &= i \frac{\partial}{\partial \xi^\mu}, \\ J_{\mu\nu} &= i \left(\xi_\mu \frac{\partial}{\partial \xi^\nu} - \epsilon \xi_\nu \frac{\partial}{\partial \xi^\mu} \right), & \mathcal{I} &= \frac{i}{\kappa} \frac{\partial}{\partial \xi^4}. \end{aligned} \quad (2)$$

In order to obtain the deformed density of states associated to the deformed algebra (1), we restrict ourselves to a subalgebra containing $\{x^i, p^i, \mathcal{I}\}$ with $i = 1, 2, 3$, that replaces the standard Heisenberg algebra in this setup. Also we fix $\epsilon = -1$ since, as we will see, this choice leads to the compactification of the spatial momenta space and a universal spatial maximal momentum arises for this case. For a fixed i , in x -basis, the representation of this

subalgebra is given by

$$x^i = x, \quad p^i = \kappa \sin\left(\frac{1}{i\kappa} \frac{d}{dx}\right), \quad \mathcal{I} = \cos\left(\frac{1}{i\kappa} \frac{d}{dx}\right). \quad (3)$$

The states $|p\rangle = e^{ikx}$ would be eigenvectors of p^i with eigenvalues

$$p(k) = \kappa \sin(k/\kappa). \quad (4)$$

Considering vanishing boundary condition on a box for $|p\rangle$, gives $k_n = \frac{\pi}{L} n$, with $n \in \mathbb{Z}$. Using the fact that $dn = \frac{dn}{dp} dp$, from the relation (4), to first order of approximation the deformed density of states in this setup will be (see Ref. [35] for more details),

$$g(p)dp = \frac{V}{2\pi^2} \frac{p^2 dp}{\sqrt{1 - (p/\kappa)^2}}, \quad (5)$$

which is not exact since it is obtained in the spirit of the subalgebra (3). An important result, which is also clear from the relation (4), is that there is an spatial maximal momentum as $p < \kappa$ in this setup. Also, the deformed density of states (5) shows that the phase space volume expands at high momentum regime. Thus, the main result of this section is that: the stable algebra of relativistic quantum mechanics (1) suggests the deformation to the density of states as the relation (5). The standard state density could be recovered in the limit of $\kappa \rightarrow \infty$.

The deformed density of states (5) is suggested also in the context of DSR theories [36], non-relativistic Snyder noncommutative space [37], and polymerized phase space [38]. The question then arises: what is the origin of these coincidence? Let us more elaborate on this point. The space of four momentum p_μ in (1) is indeed determined by the de Sitter geometry [5]. On the other hand, the space of momentum in Snyder original noncommutative spacetime [3] has also de Sitter geometry. The topology of the de Sitter space is $R \times S^3$. Identifying R with the space of the energy and S^3 with the space of spatial momenta, a universal maximal momentum (which signals the existence of a UV cutoff) naturally arises which is completely determined by the radius of the three-sphere S^3 , or equivalently with curvature of the de Sitter four-momentum space. More precisely, compactness of S^3 topology is the origin of natural UV cutoff in all of these models [39]. For instance, for the case of non-relativistic Snyder model defined in Ref. [10], the space of spatial momenta is the three sphere S^3 . So there is no surprise when the measure on the momentum space gets the same modification (5) in Ref. [37]. Thus, the density of states (5) underlies on the S^3 compact topology and consequently a universal maximal momentum $p < \kappa$, as an ultraviolet cutoff for the system under consideration, naturally arises in this setup. This maximal momentum is completely determined by the radius of the three-sphere. As we will show in the next sections, existence of a maximal momentum leads to the saturation of thermodynamical properties of the statistical systems at the high temperature regime.

3. Partition function

In this section, we obtain the modified partition function by means of the deformed density of states (5) in order to study thermodynamics of photon gas in noncommutative spacetime.

The number of accessible microstates for a physical system determines the associated thermodynamical properties in the context of statistical mechanics. The microstates, however, is determined only by quantum mechanics and there is no classical statistics in essence. More precisely, for a system at temperature T , the quantum partition function for a single-particle state is defined as

$$\mathcal{Z}_1 = \sum_{\varepsilon_i} e^{-\varepsilon_i/T}, \quad (6)$$

where $\{\varepsilon_i\}$ are the single-particle energy states which are the solution of Hamiltonian eigenvalue problem. From the single-particle partition function one could find the total partition

function and all the thermodynamical properties of the system under consideration then could be easily obtained from the total partition function. Solving the Hamiltonian eigenvalue problem in noncommutative spacetime, however, is not an easy task at all due to the complicated form of the representations of the operators in this setup (see for instance Ref. [40]). Even if one solves the Hamiltonian eigenvalue problem, performing the summation over the microstates' energies in relation (6) to obtain an analytic expression for the associated quantum partition function may be difficult. Nevertheless, one could approximate the summation over the energies by the integral over the phase space volume by means of the density of states. The advantage of this method is that, it is not need to solve the Hamiltonian eigenvalue problem in noncommutative setup. Indeed, one could work with the Hamiltonian function together with the deformed density of states (5) on the associated noncommutative phase space. However, one should note that this semiclassical approximation will be coincided with full quantum consideration at high temperature regime and the full quantum considerations then preserve their importance for the low temperature phenomenons such as the Bose-Einstein condensation. Thus, all the noncommutativity effects are summarized in the deformed density of states (5) and one could replace the summation over the microstates' energies as

$$\sum_{\varepsilon} \rightarrow \frac{V}{(2\pi)^3} \int_{|p| \leq \kappa} \frac{d^3 p}{\sqrt{1 - \frac{p^2}{\kappa^2}}}, \quad (7)$$

where V is the spatial volume. Using the semiclassical approximation (7) in the relation (6), the canonical partition function will be

$$\mathcal{Z}_1(T, V) = \frac{1}{(2\pi)^3} \int d^3 x \int_{|p| \leq \kappa} \frac{e^{-H(x,p)/T} d^3 p}{\sqrt{1 - (p/\kappa)^2}}. \quad (8)$$

For a photon gas confined in a volume V at temperature T with Hamiltonian function $H(p) = p$, the associated modified partition function for the single-particle state could be easily deduced from the relation (8) as

$$\mathcal{Z}_1(T, V) = \frac{V}{(2\pi)^3} \int_0^\kappa \frac{e^{-p/T} p^2 dp}{\sqrt{1 - (p/\kappa)^2}} = \frac{V}{(2\pi)^3} f(T), \quad (9)$$

where we have defined

$$f(T) = -\frac{\kappa^4}{3} T + \frac{\pi \kappa^2}{2T} \left\{ \left[I_1(\kappa T) + \kappa T I_2(\kappa T) \right] - \left[\mathbf{L}_1(\kappa T) + \kappa T \mathbf{L}_2(\kappa T) \right] \right\}, \quad (10)$$

with I_n and \mathbf{L}_n are the modified Bessel functions and modified Struve functions of rank n respectively. In the limit of low temperature, the noncommutativity effects will be removed and we find $f(T) = 8\pi T^3$ and the relation (9) reduces to the standard non-deformed partition function for the photon gas $\mathcal{Z}_1(T \rightarrow 0, V) \approx 8\pi V (T/h)^3$. The total partition function for a non-localized system such as photon gas, with which we are interested in this paper, can be obtained from the standard definition

$$\mathcal{Z}_N(T, V) = \frac{1}{N!} \left[\mathcal{Z}_1(T, V) \right]^N, \quad (11)$$

where N is the number of photons and the Gibb's factor is also considered. Substituting the single-particle state partition function (9) into the definition (11) gives the total partition function for the photon gas as

$$\mathcal{Z}_N(T, V) = \frac{V^N}{(2\pi)^{3N}} \frac{f(T)^N}{N!}. \quad (12)$$

Having the total partition function (12) in hand, we are able to study thermodynamics of the photon gas in noncommutative

spacetime. Considering the maximal spatial momentum κ to be of the order of the Planck momentum (energy) as $\kappa \sim p_{pl}$, it is natural to expect that the deviation from the standard thermodynamical results for a photon gas emerges at the high temperature regime (about the Planck temperature) where the noncommutative effects play significant role.

4. Thermodynamics of photon gas

First of all, we obtain the Helmholtz free energy F which can be obtained from the standard definition as

$$F = -T \ln [Z_N(T, V)] = -NT \left[1 + \ln \left(\frac{Vf}{8\pi^3 N} \right) \right], \quad (13)$$

in which we have used the Stirling's formula $\ln(N!) \approx N \ln N - N$ for large N . The thermal pressure of the photon gas will be

$$P = - \left(\frac{\partial F}{\partial V} \right)_{T, N} = \frac{NT}{V}. \quad (14)$$

Therefore, the familiar form of the equation of state for the ideal gasses

$$PV = NT, \quad (15)$$

is also preserved in this setup. It is the well-known result that the form of equation of state (15) is held for both of the relativistic and non-relativistic ideal gases in standard Maxwell–Boltzmann statistics. It seems that this form also remains unchanged for both of the relativistic and non-relativistic ideal gases in the presence of an ultraviolet cutoff (see for instance Ref. [25] for the case of generalized uncertainty principle).

4.1. Entropy bound

The entropy could be obtained from the Helmholtz energy (13) by definition as

$$S = - \left(\frac{\partial F}{\partial T} \right)_{N, V} = N \left(1 + \ln(V/8\pi^3 N) + (T \ln f)' \right), \quad (16)$$

where a prime denotes the derivative with respect to the temperature. The behavior of the entropy versus the temperature is plotted in Fig. 1. It is clear from Fig. 1 that the entropy increases with a slower rate in noncommutative case (the solid line) than the standard one (the dashed line). Unlike the entropy of the standard photon gas, the entropy approaches to a maximum value at very high temperature regime in the noncommutative setup. The maximum entropy bound for the photon gas is

$$S \leq S_{\max} = C_1(N) + N \ln \left(\frac{V}{l_{pl}^3} \right), \quad (17)$$

where $C_1(N) = N + N \ln \left(\frac{\kappa_0^3}{8\pi^3 N} \right)$ and also we have substituted $\kappa = \kappa_0 p_{pl} = \kappa_0/l_{pl}$ with κ_0 is the dimensionless parameter and l_{pl} is the Planck length. The dimensionless parameter $\kappa_0 \sim \mathcal{O}(1)$ determines the boundary at which the noncommutative effects will become important and it should be fixed only by the experiments (see Ref. [41] in which some upper bounds for this dimensionless parameter are obtained in different contexts). The entropy bound (17) shows that the photon gas is saturated at high temperature regime in noncommutative spacetime and the entropy could not increase by increasing the temperature in this regime. More precisely, the entropy bound (17) for the photon gas originates from the existence of an ultraviolet cutoff in this setup which also, as we shall see, leads to an upper bound for the internal energy.

Appearance of the factor (V/l_{pl}^3) in the relation (17) for the maximum entropy bound, signals the discreteness of the space with respect to the Planck length at high temperature limit in this setup.

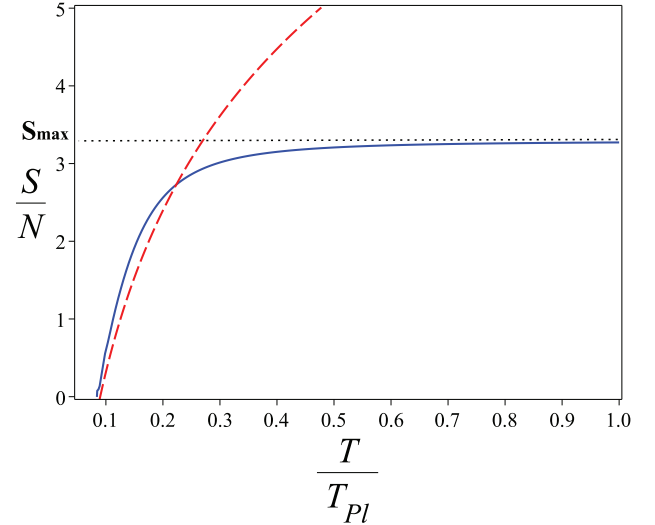


Fig. 1. Entropy versus the temperature. The solid line represents the entropy of the photon gas in noncommutative spacetime with maximal momentum for the spatial momenta and the dashed line corresponds to the non-deformed case. At the high temperature regime, the entropy increases with a slower rate in noncommutative spacetime with respect to the corresponding non-deformed case. At high temperature, the entropy approaches to a maximum value (17) which is originated from the existence of an ultraviolet cutoff (maximal momentum corresponds to a minimal length) in noncommutative framework. The figure is plotted for $T_{pl} = 1$, where T_{pl} is the Planck temperature.

This result, in some sense, is similar to the case of the entropy of the black holes. The Bekenstein–Hawking entropy for black holes is given by the relation $S_{BH} = (A/4l_{pl}^2)$, where A is the horizon area of black hole [26]. Then, the number of microstates is precisely determined by the horizon area and the fundamental area l_{pl}^2 . In other words, each bit of information is proportional to the fundamental (Planck) area l_{pl}^2 . In the case of the photon gas in noncommutative setup, the entropy bound (17) shows that the number of microstates at high temperature regime is precisely determined by the factor (V/l_{pl}^3) , i.e., one bit of information is proportional to l_{pl}^3 . In other words, similar to the black holes, the entropy at high energy regime (where quantum gravitational effects play the central role) precisely is determined by the accessible spatial volume together with the ultraviolet cutoff (Planck length) for the systems. Many attempts have been done to find the microscopic origin of black holes entropy [27,28,42]. Then, in the light of this result, it may be possible that black holes thermodynamics will emerge from a saturated state in a full quantum gravitational statistical mechanics framework. In the absence of such a theory, the gravitational statistical theories such as the generally covariant statistical mechanics may open a new window on this issue [31].

4.2. Internal energy and specific heat

The internal energy of the photon gas will be

$$U = -T^2 \left[\frac{\partial}{\partial T} \left(\frac{F}{T} \right) \right]_{N, V} = NT^2 \frac{f'}{f}. \quad (18)$$

Substituting $f \sim T^3$ for the low temperature regime, immediately gives the standard result $U = 3NT$ for the photon gas. The internal energy versus the temperature is plotted in Fig. 2. It is clear from Fig. 2 that, at high temperature regime, the internal energy of the photon gas in noncommutative framework (the solid line) increases with a smaller rate with respect to the non-deformed case (the dashed line). Interestingly, there is an upper bound, of the order of the Planck energy, for the internal energy of the photon gas

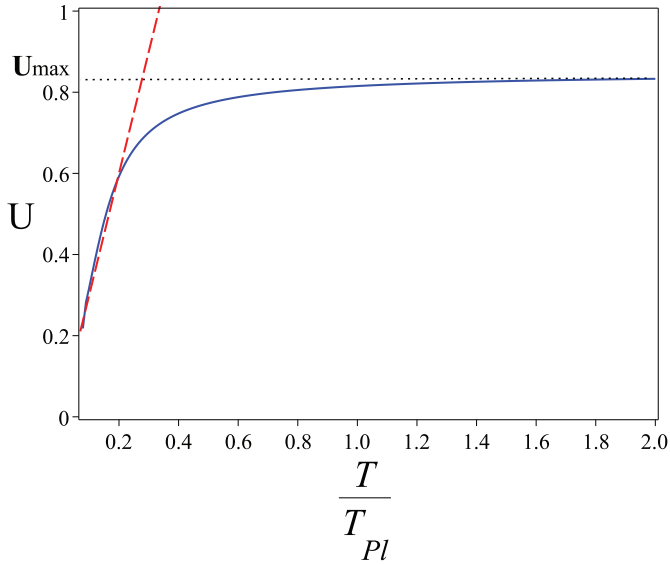


Fig. 2. Internal energy of the photon gas versus the temperature. The solid line represents the internal energy in noncommutative setup and the dashed line corresponds to the standard case. Clearly, noncommutative effects dominate when the temperature approaches the Planck temperature and the internal energy of the photon gas then increases with a much less rate in this regime. The system will be finally saturated and the internal energy approaches to its maximum value (19) which is of the order of Planck energy. This feature also originates from the existence of an ultraviolet cutoff in this setup.

in noncommutative spacetime as follows

$$U \leq U_{\max} = \left(\frac{8}{9\pi\kappa_0} \right) E_{pl}. \quad (19)$$

This result also originates from the fact that there is an ultraviolet cutoff $|p| \leq \kappa$ for the system under consideration in noncommutative spacetime.

The specific heat then will be

$$C_v = \left(\frac{\partial U}{\partial T} \right)_v = NT^2 \left(\frac{f''}{f} - \frac{f'^2}{f^2} + \frac{2f'}{Tf} \right). \quad (20)$$

The specific heat versus the temperature is plotted in Fig. 3. Unlike the standard photon gas with constant specific heat $C_v = 3N$ (for a fixed N), the specific heat in noncommutative framework becomes temperature-dependent at high energy regime (see Fig. 3). As it is clear from Fig. 3, $C_v(T \rightarrow \infty) = 0$ which shows that the photon gas saturates at high temperature limit (around the Planck temperature), when the noncommutative effects become significant and the system cannot access a higher energy scale by increasing the temperature.

5. Summary and conclusions

Existence of a minimal length, preferably of the order of Planck length, is suggested by quantum gravity candidates such as string theory and loop quantum gravity. Although a full quantum theory of gravity is not formulated yet, it is widely believed that a non-gravitational theory that admits a minimal length scale would be emerged at the flat limit of quantum gravity. Evidently, such a theory could be achieved through the deformation of the algebraic structure of the standard relativistic quantum mechanics in such a way that spacetime coordinates become non-commuting operators. Recently, in the context of doubly special relativity theories, it was shown that this issue could be also realized from a curved four-momentum space with constant curvature such as the de Sitter geometry with topology $\mathbf{R} \times \mathbf{S}^3$. Identifying \mathbf{R} with the space of

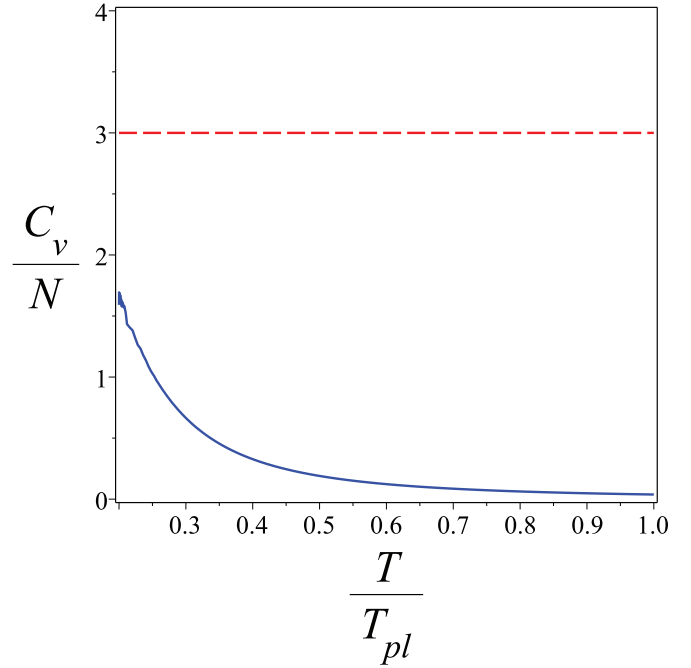


Fig. 3. Specific heat versus the temperature. The solid line represents the heat capacity of the photon gas in noncommutative space and the dashed line corresponds to the non-deformed case. While the specific heat is constant (for a fixed N) for the standard photon gas, it becomes temperature-dependent at high temperature regime in noncommutative spacetime. Indeed, the photon gas saturates at the high energy regime and then the specific heat tends to zero in this regime. In other words, the system cannot access a higher energy scale by increasing the temperature.

energy and \mathbf{S}^3 with the space of spatial momenta, a universal maximal momentum (corresponds to a minimal observer-independent length scale) naturally arises which is completely determined by the radius of three-sphere or equivalently with curvature of the de Sitter four-momentum space. The deformation to the dispersion relation and density of states are the direct consequence of these setups. Since the number of microstates is determined by the density of states, the significant effects on the thermodynamical properties of the physical systems will be arisen in these setups. In this paper, after obtaining the modified partition function by means of the deformed density of states in spacetime with stable noncommutative algebra, we have studied the thermodynamical properties of the photon gas in this setup. The results show that the entropy of the photon gas increases with a smaller rate at the high temperature regime in noncommutative setup in comparison with the standard non-deformed case. Also, the entropy approaches to a maximum entropy bound around the Planck temperature. The number of accessible microstates associated to the resultant entropy bound is totally determined by the factor (V/l_{pl}^3) which is qualitatively very similar to the black holes' Bekenstein-Hawking entropy $S_{BH} = (A/l_{pl}^2)$ in which the number of accessible microstates for a black hole is precisely determined by the factor (A/l_{pl}^2) , where A is the black hole horizon area. In other words, similar to the case of black holes, the entropy for the photon gas in high temperature regime is precisely determined by the accessible spatial volume and an ultraviolet cutoff in noncommutative spacetime. This result suggests that thermodynamics of black holes may be obtained from a saturated state in the framework of a full quantum gravitational statistical mechanics. Furthermore, the internal energy of the photon gas also gets a finite maximum value, of the order of the Planck energy, at very high temperature regime. The associated specific heat then tends to zero at high temperature regime since the photon gas is saturated in this regime and could

not access to a higher energy scale by increasing the temperature. Although we have studied the thermodynamics of a particular system (the photon gas) in noncommutative spacetime, having upper bounds for the entropy and internal energy are generic properties since they originate from the existence of an ultraviolet cutoff in this setup.

References

- [1] D.J. Gross, P.F. Mende, Nucl. Phys. B 303 (1988) 407; D. Amati, M. Ciafaloni, G. Veneziano, Phys. Lett. B 216 (1989) 41; M. Kato, Phys. Lett. B 245 (1990) 43; L. Garay, Int. J. Mod. Phys. A 10 (1995) 145.
- [2] C. Rovelli, L. Smolin, Nucl. Phys. B 331 (1990) 80; C. Rovelli, L. Smolin, Nucl. Phys. B 442 (1995) 593; A. Ashtekar, J. Lewandowski, Class. Quantum Gravity 14 (1997) A55; A. Ashtekar, J. Lewandowski, Class. Quantum Gravity 21 (2004) R53.
- [3] H. Snyder, Phys. Rev. 71 (1947) 38.
- [4] M. Flato, Czech J. Phys. B 32 (1982) 472; L.D. Faddeev, Asia-Pac. Phys. News 3 (1988) 21.
- [5] R.V. Mendes, J. Phys. A: Math. Gen. 27 (1994) 8091.
- [6] E. Witten, Nucl. Phys. B 268 (1986) 253; N. Seiberg, E. Witten, JHEP 09 (1999) 032.
- [7] E. Carlen, R.V. Mendes, Phys. Lett. A 290 (2001) 109.
- [8] M. Maggiore, Phys. Lett. B 319 (1993) 83; M. Maggiore, Phys. Rev. D 49 (1994) 5182.
- [9] A. Kempf, G. Mangano, R.B. Mann, Phys. Rev. D 52 (1995) 1108; A. Kempf, G. Mangano, Phys. Rev. D 55 (1997) 7909; K. Nozari, A. Etemadi, Phys. Rev. D 85 (2012) 104029.
- [10] S. Mignemi, Phys. Rev. D 84 (2011) 025021; S. Mignemi, Class. Quantum Gravity 29 (2012) 215019.
- [11] G. Amelino-Camelia, Int. J. Mod. Phys. D 11 (2000) 35; G. Amelino-Camelia, Nature 418 (2002) 34.
- [12] J. Magueijo, L. Smolin, Phys. Rev. Lett. 88 (2002) 190403; J. Magueijo, L. Smolin, Phys. Rev. D 67 (2003) 044017.
- [13] J. Kowalski-Glikman, Phys. Lett. B 547 (2002) 291; J. Kowalski-Glikman, S. Nowak, Class. Quantum Gravity 20 (2003) 4799; F. Girelli, E.R. Livine, D. Oriti, Nucl. Phys. B 708 (2005) 411; J. Kowalski-Glikman, Int. J. Mod. Phys. A 28 (2013) 1330014.
- [14] S. Majid, Lecture Notes in Physics, 541, Springer, 2000, p. 227.
- [15] J. Lukierski, H. Ruegg, A. Nowicki, V.N. Tolstov, Phys. Lett. B 264 (1991) 331; S. Majid, H. Ruegg, Phys. Lett. B 334 (1994) 348.
- [16] J. Kowalski-Glikman, S. Nowak, Phys. Lett. B 539 (2002) 126.
- [17] F. Girelli, T. Konopka, J. Kowalski-Glikman, E.R. Livine, Phys. Rev. D 73 (2006) 045009.
- [18] G. Amelino-Camelia, L. Freidel, J. Kowalski-Glikman, L. Smolin, Phys. Rev. D 84 (2011a) 084010; G. Amelino-Camelia, L. Freidel, J. Kowalski-Glikman, L. Smolin, Gen. Relativ. Gravit. 43 (2011b) 2547.
- [19] G. Amelino-Camelia, Living Rev. Relativ. 16 (2013) 5.
- [20] G. Amelino-Camelia, S. Majid, Int. J. Mod. Phys. A 15 (2000) 4301.
- [21] J. Kowalski-Glikman, Phys. Lett. A 299 (2002) 454; X. Zhang, L. Shao, B.Q. Ma, Astropart. Phys. 34 (2011) 840; S. Das, S. Ghosh, D. Roychowdhury, Phys. Rev. D 80 (2009) 125036; S. Das, D. Roychowdhury, Phys. Rev. D 81 (2010) 085039; N. Chandra, S. Chatterjee, Phys. Rev. D 85 (2011) 045012; G. Amelino-Camelia, N. Loret, G. Mandanici, F. Mercati, Int. J. Mod. Phys. D 21 (2012) 1250052.
- [22] M. Arzano, G. Gubitosi, J. Magueijo, G. Amelino-Camelia, Phys. Rev. D 92 (2015) 024028.
- [23] O.F. Dayi, A. Jellal, Phys. Lett. A 287 (2001) 349; F.G. Scholtz, J. Govaerts, J. Phys. A 41 (2008) 505003; W.H. Huang, K.W. Huang, Phys. Lett. B 670 (2009) 416; A. Shariati, M. Khorrami, A.H. Fatollahi, J. Phys. A 43 (2010) 285001; M.A. Gorji, K. Nozari, B. Vakili, Phys. Rev. D 89 (2014) 084072.
- [24] S.K. Rama, Phys. Lett. B 519 (2001) 103; M. Lubo, Phys. Rev. D 68 (2003) 125004; K. Nozari, B. Fazlipoor, Gen. Relativ. Gravit. 38 (2006) 1661; T. Fityo, Phys. Lett. A 372 (2008) 5872; P. Wang, H. Yang, X. Zhang, JHEP 08 (2010) 043; D. Mania, M. Maziašvili, Phys. Lett. B 705 (2011) 521; P. Wang, H. Yang, X. Zhang, Phys. Lett. B 718 (2012) 265.
- [25] B. Vakili, M.A. Gorji, J. Stat. Mech. (2012) P10013.
- [26] J.D. Bekenstein, Phys. Rev. D 7 (1973) 2333; J.D. Bekenstein, Phys. Rev. D 9 (1974) 3292; S.W. Hawking, Nature 248 (1974) 30; S.W. Hawking, Commun. Math. Phys. 43 (1975) 199.
- [27] G.t. Hooft, Nucl. Phys. B 335 (1990) 138; E. Witten, Phys. Rev. D 44 (1991) 314; L. Susskind, J. Uglum, Phys. Rev. D 50 (1994) 2700; A. Strominger, C. Vafa, Phys. Lett. B 379 (1996) 99; G.T. Horowitz, A. Strominger, Phys. Rev. Lett. 77 (1996) 2368; J.M. Maldacena, arxiv:hep-th/9607235; J.M. Maldacena, A. Strominger, E. Witten, JHEP 9712 (1997) 002.
- [28] C. Rovelli, Phys. Rev. Lett. 77 (1996) 3288; A. Ashtekar, J. Baez, A. Corichi, K. Krasnov, Phys. Rev. Lett. 80 (1998) 904; M. Domagala, J. Lewandowski, Class. Quantum Gravity 21 (2004) 5245.
- [29] P. Nicolini, A. Smailagic, E. Spallucci, Phys. Lett. B 632 (2006) 547; Y.S. Myung, Y.-W. Kim, Y.J. Park, JHEP 0702 (2007) 012; K. Nozari, S.H. Mehdipour, Class. Quantum Gravity 25 (2008) 175015; R. Banerjee, B.R. Majhi, S. Samanta, Phys. Rev. D 77 (2008) 124035; R. Banerjee, B.R. Majhi, S.K. Modak, Class. Quantum Gravity 26 (2009) 085010; C. Bastos, O. Bertolami, N.C. Dias, J.N. Prata, Phys. Rev. D 80 (2009) 124038.
- [30] F. Scardigli, Phys. Lett. B 452 (1999) 39; A.J.M. Medved, E.C. Vagenas, Phys. Rev. D 70 (2004) 124021; G. Amelino-Camelia, M. Arzano, Y. Ling, G. Mandanici, Class. Quantum Gravity 23 (2006) 2585; Y.S. Myung, Y.W. Kim, Y.J. Park, Phys. Lett. B 645 (2007) 393; K. Nozari, A.S. Sefiedgar, Phys. Lett. B 635 (2006) 156; A. Bina, S. Jalalzadeh, A. Moslehi, Phys. Rev. D 81 (2010) 023528; L. Xiang, X.Q. Wen, JHEP 0910 (2009) 046.
- [31] C. Rovelli, Phys. Rev. D 87 (2013) 084055; G. Chirco, T. Josset, C. Rovelli, Class. Quantum Gravity 33 (2016) 045005.
- [32] M. Gerstenhaber, Ann. Math. 79 (1964) 59; A. Nijenhuis, R.W. Richardson, Bull. Am. Math. Soc. 72 (1966) 1; A. Nijenhuis, R.W. Richardson, Bull. Am. Math. Soc. 73 (1967a) 175; A. Nijenhuis, R.W. Richardson, J. Math. Mech. 17 (1967b) 89.
- [33] We work in units $\hbar = c = k_B = 1$, where \hbar , c , and k_B are the Planck constant, speed of light in vacuum, and Boltzmann Constant respectively.
- [34] R.V. Mendes, Phys. Lett. A 210 (1996) 232.
- [35] R.V. Mendes, Eur. Phys. J. C 42 (2005) 445; R.V. Mendes, Eur. Phys. J. C 72 (2012) 2239.
- [36] L. Freidel, E.R. Livine, Class. Quantum Gravity 23 (2006) 2021.
- [37] S. Mignemi, R. Strajn, arxiv: 1501.01447 [hep-th].
- [38] A. Corichi, T. Vukašinac, Phys. Rev. D 86 (2012) 064019; M.A. Gorji, K. Nozari, B. Vakili, Phys. Rev. D 90 (2014) 044051.
- [39] K. Nozari, M.A. Gorji, V. Hosseinzadeh, B. Vakili, Class. Quantum Gravity 33 (2016) 025009; C. Rovelli, F. Vidotto, Phys. Rev. D 91 (2015) 084037.
- [40] R.V. Mendes, J. Math. Phys. 41 (2000) 156.
- [41] S. Das, E.C. Vagenas, Phys. Rev. Lett. 101 (2008) 221301; P. Pedram, K. Nozari, S.H. Taheri, JHEP 1103 (2011) 093; S. Jalalzadeh, M.A. Gorji, K. Nozari, Gen. Relativ. Gravit. 46 (2014) 1632; S. Ghosh, Class. Quantum Gravity 31 (2014) 025025; K. Nozari, M. Khodadi, M.A. Gorji, Europhys. Lett. 112 (2015) 60003.
- [42] J.D. Bekenstein, Phys. Rev. D 12 (1975) 307; S.W. Hawking, Phys. Rev. D 13 (1976) 191; V.P. Frolov, I. Novikov, Phys. Rev. D 48 (1993) 4545; F. Larsen, F. Wilczek, Phys. Lett. B 375 (1996) 3; S.N. Solodukhin, Phys. Rev. D 54 (1996) 3900; K.V. Krasnov, Gen. Relativ. Gravit. 30 (1998) 53; S. Hod, Phys. Rev. Lett. 81 (1998) 4293; S. Carlip, Phys. Rev. Lett. 82 (1999) 2828; A. Sen, Gen. Relativ. Gravit. 40 (2008) 2249.