



ELSEVIER

Contents lists available at ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

A new hesitant fuzzy QFD approach: An application to computer workstation selection

Q2 Sezi Çevik Onar^a, Gülçin Büyüközkan^b, Başar Öztayşi^a, Cengiz Kahraman^{a,*}

Q3^a Istanbul Technical University, Industrial Engineering Department, Maçka, 34367 Istanbul, Turkey

^b Galatasaray University, Industrial Engineering Department, Ortaköy, 34357 Istanbul, Turkey

ARTICLE INFO

Article history:

Received 21 August 2015

Received in revised form 19 January 2016

Accepted 13 April 2016

Available online xxx

Keywords:

QFD

Hesitant fuzzy sets

HFLTS

Workstation

Design requirement

Customer requirements

TOPSIS

AHP

ABSTRACT

Computer workstation selection is a multiple criteria decision making problem that is generally based on vague linguistic assessments, which represent human judgments and their hesitancy. In this paper, a new fuzzy quality function deployment (QFD) approach is used to effectively determine the design requirements (DRs) of a computer workstation. Hesitant fuzzy linguistic term sets (HFLTS) are innovatively employed to capture the hesitancy of the experts in this approach. More precisely, the proposed new QFD approach is the first study that determines the importance of customer requirements (CRs), the relations between CRs and DRs and the correlations among DRs via HFLTS. Additionally, HFLTS based Analytic Hierarchy Process (AHP) and Technique for Order Performance by Similarity to Ideal Solution (TOPSIS) methods are utilized in the computational steps to select the best computer workstation. A real industrial application is carried out to validate the implementation of the proposed approach.

© 2016 Published by Elsevier B.V.

1. Introduction

A workstation is a customized computer that is designed for specific scientific or technical application. Increasing competition and technological innovation in the industry and business world in general brings about new developments in the workstation design. However, workstations are usually designed arbitrarily with little consideration to the specific needs and requirements of their users. Considering additional benefits of tailor-made workstations that are customized for specific uses and needs, a customer-driven approach in workstation design would benefit companies. Such an approach would not only capture customers' perspectives, but also raise the overall level of their satisfaction level. Quality function deployment (QFD) is a customer-driven tool that is widely used for product planning purposes. It can be beneficial to reach higher levels in customer satisfaction [1,2]. Good design requires consideration of design aspects that clients want and expect. To address this, QFD uses a matrix called House of Quality (HOQ) [3] that translates Customer Needs or Requirements (CRs) into engineering characteristics or Design Requirements (DRs). The HOQ is constructed with the importance weights of each of the CRs, as well as the correla-

tion matrix among DRs and the relationship matrix between CRs and DRs [1–5].

The importance levels of CRs, functional relationships among CRs and DRs, and the assessments of alternatives based on DRs are difficult to express precisely. Although crisp data are needed to design workstations, experts usually prefer to provide their evaluations in linguistic terms. The fuzzy set theory lets these linguistic assessments be incorporated into numerical analyses. The ordinary fuzzy sets have been recently extended to Type 2 fuzzy sets, hesitant fuzzy sets, intuitionistic fuzzy sets, non-stationary fuzzy sets and fuzzy multisets [6]. Hesitant fuzzy sets (HFS), which are developed by Torra [7], allow more than one value for defining the membership value of an element, enabling an expert better express his/her assessment [8]. In this paper, we prefer to use hesitant linguistic term sets (HFLTS) in the development of a new fuzzy QFD approach since HFLTS enable the integration of various linguistic evaluations assigned by experts as an inclusive linguistic interval. HFLTS have been used in several papers in the literature [9–16].

Main features of the proposed hesitant fuzzy QFD approach Q4 are its use of HFLTS in the pairwise comparisons among CRs, relations between CRs and DRs, correlations among DRs and evaluation of alternatives. The weights of the CRs are determined by a hierarchical and pairwise comparison-based approach while the alternatives are ranked by using a hesitant fuzzy TOPSIS method. Besides, we propose a new approach taking the hesitant correla-

* Corresponding author.

E-mail address: kahramanc@itu.edu.tr (C. Kahraman).

tions among DRs into account in the HOQ operations. To the best of our knowledge, there is no QFD study based on hesitant fuzzy sets in the literature and this study is different from the other existing approaches since it considers the experts' hesitations in each phase of the QFD approach.

The remainder of this paper is structured as the following; Section 2 presents basic concepts of QFD and a literature review of fuzzy QFD methodology. In Section 3, the main concepts of HFS and HFTLS are given. Section 4 gives the proposed decision making approach which is based on hesitant fuzzy QFD. In Section 5, a case study is provided to demonstrate the applicability of the proposed method. The last section concludes the paper and gives some perspectives.

2. Literature survey on fuzzy QFD

The overall methodological structure is based on the QFD technique, supported by a hesitant fuzzy set approach, where linguistic data are considered. In the following, first, basic QFD terminology on classical QFD is given. Then a literature review on fuzzy set extensions in QFD is given.

2.1. Quality function deployment (QFD)

QFD is a popular quality method that is developed in the 1960s and 1970s to address design quality challenges to meet better customer expectations [1,2]. QFD is a proven and comprehensive technique that is able to translate CRs into DRs by the so-called is HOQ [3]. The HOQ is the basic structure of QFD and includes the following integral components: the relationship matrix between CRs and DRs, CRs' importance weights, and the correlation matrix for DRs [1–5]. The well-known HOQ approach is depicted in Fig. 1.

The integral elements of the typical HOQ structure shown in Fig. 1 are briefly introduced below:

CRs: Customer requirements are also known as customer attributes, customer needs or demanded quality. The first step for constructing an HOQ is the identification, clarification and specification of customer needs. CRs represent the initial input for the HOQ and highlight those product specifications that should be paid attention to so that the “voice of the customer” is well understood.

DRs: Design requirements are also called product features, engineering attributes, technical attributes, engineering characteristics or substitute quality characteristics. These product requirements are associated with CRs.

CRs' analysis: Not all of CRs have the same level of importance for customers. In order to prioritize the identified CRs, a direct evaluation or different analytical techniques can be adopted.

Relationships matrix between CRs and DRs: The relationship matrix represents the extent to which each DR affects its associated CR. This matrix constitutes the body of the HOQ.

DRs' analysis: The results taken from the previous steps are used to compute the final importance degrees of DRs.

The HOQ is frequently discussed and applied in theoretical and practical literature, as it has the potential to significantly improve the accuracy of the preceding steps. HOQ is oriented towards design and is thus an important resource for designers. Furthermore, it is a tool that can summarize customers' feedback and translate it into a useful information format that can be easily understood and used by design teams.

Companies can enjoy various advantages when applying QFD, as it is customer-oriented, helps to combine large amount of verbal data, brings multifunctional teams together, improves the consensus processes, creates competitive advantage, decreases start-up and engineering costs borne during product development processes, and is usable across a wide range of processes and ser-

vices in different sectors [1–5,17]. Thus, various business areas such as communication, software systems, transportation, electronics, education and research, manufacturing, services, IT and shipbuilding, aerospace, construction, packaging, textile industries and supply chain management make use of the QFD methodology [18–20].

In the next subsection, a literature review on fuzzy set extensions in QFD is given.

2.2. Fuzzy set extensions in QFD

The QFD method is a useful analysis tool that is widely used in product design and development. To deal with challenges related to uncertainty and imprecision in QFD, various researchers have developed many fuzzy QFD approaches by combining the fuzzy set theory with QFD. These approaches include conventional QFD computation methods using fuzzy variables [21,22], fuzzy out-ranking [23], entropy [24], incomplete fuzzy preference relations [25,26], multiple formatted fuzzy preference relations [27,28], fuzzy integral [29,30], fuzzy analytical network process [31,32], fuzzy multicriteria decision making (MCDM) [33,34], fuzzy goal programming [32,34], rough set based approach [35,36] and fuzzy expert systems [37], among others. Interested readers can refer to fuzzy QFD literature survey articles (e.g. [38]) for more detailed information.

Reviewed literature suggests that these fuzzy QFD approaches usually concentrate on obtaining the importance ranking of CRs and/or DRs. However, relatively a small number of papers investigate the selection process based on DRs. Our paper focuses on a DRs-based selection process.

Extended fuzzy set types include type-2 fuzzy sets, hesitant fuzzy sets, intuitionistic fuzzy sets, non-stationary fuzzy sets and fuzzy multisets. It is observed that the extended fuzzy sets are new topics and rarely used as modeling tools in QFD. In one of the first studies, Li [39] applied 2-tuple linguistic representation model under multi-granularity linguistic environment in the construction of HOQ. Ko [40] adopted a 2-tuple linguistic computational approach for constructing HOQ based failure modes and effects analysis, while Li et al. [41] handled software quality evaluation problem based on the geometric aggregation operators with hesitant fuzzy uncertain linguistic information. In another study, Li et al. [41] proposed an intuitionistic fuzzy set theory based QFD approach for the knowledge management system selection problem. In the proposed approach, the linguistic assessment data of HOQ are transformed into intuitionistic fuzzy numbers and the alternatives are prioritized and ranked with the intuitionistic TOPSIS method. Recently, Karsak and Dursun [42] employed a fusion of fuzzy information and 2-tuple linguistic representation model in the QFD to calculate the weights of supplier selection criteria and subsequently the ratings of suppliers.

3. Hesitant fuzzy linguistic term sets (HFLTSS)

Hesitant fuzzy sets (HFSs) are the extensions of fuzzy sets which can solve the difficulties in determining the membership degree of an element [7]. It represents the hesitancy where there are possible values for membership and it is not clear which one is the right value.

Definition 1. A hesitant fuzzy set (HFS) on X , where X is a fixed set, can be defined as follows:

$$E = \{ \langle x, h_E(x) \rangle \mid x \in X \} \quad (1)$$

where $h_E(x)$ denotes membership degrees of the element $x \in X$ to the set E and its values are in $[0, 1]$.

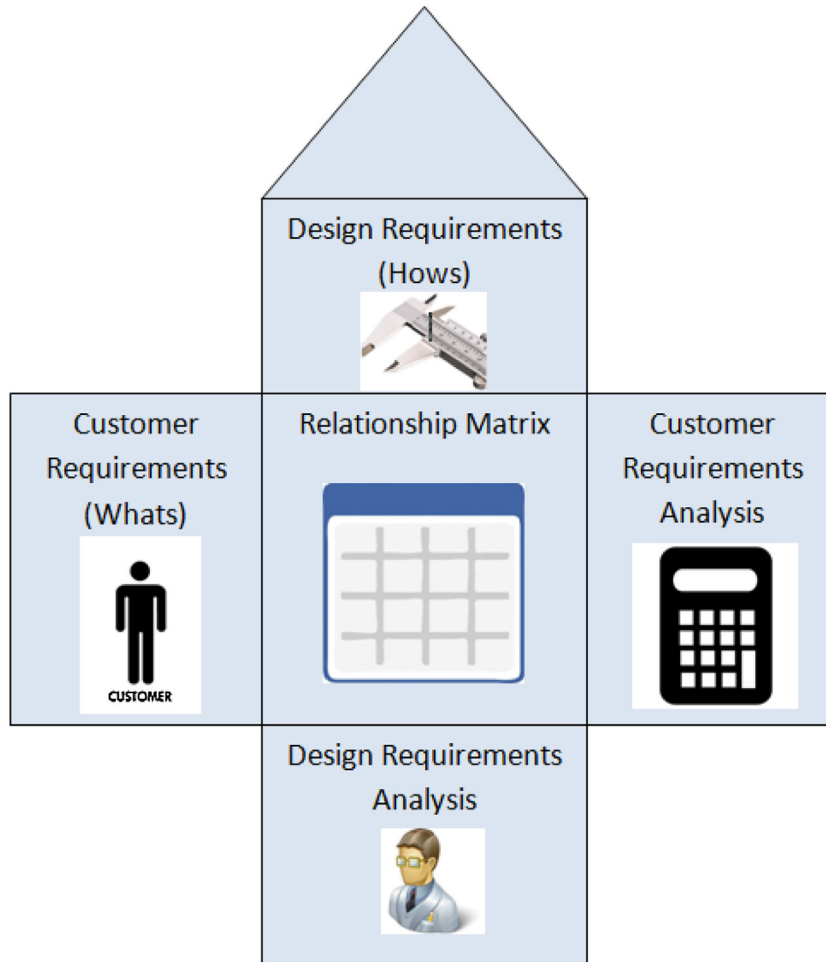


Fig. 1. HOQ in QFD.

190 Hesitant fuzzy sets can be classified as dual hesitant fuzzy sets
191 [43], interval valued hesitant fuzzy sets [44], generalized hesitant
192 fuzzy sets [45], triangular fuzzy hesitant fuzzy sets [46] and hesitant
193 fuzzy linguistic term sets [6].

194 Experts may hesitate while selecting the appropriate linguistic
195 expression. In the classical fuzzy linguistic approaches, a single
196 expression should be selected which limits the experts. Hesitant
197 fuzzy linguistic terms sets (HFLTS) introduced by Rodriguez et al.
198 [6] can be used when the experts hesitate between several linguistic
199 expressions. HFLTS itself, as well as the methods developed for
200 HFLTS enable representing and solving multiple linguistic assess-
201 ments mathematically.

202 Chen and Hong [47] developed a new hesitant multicriteria deci-
203 sion making approach that considers the pessimistic and optimistic
204 attitudes of experts. Lee and Chen [48] proposed new aggrega-
205 tion operators; namely, hesitant fuzzy linguistic weighted average
206 (HFLWA), hesitant fuzzy linguistic weighted geometric (HFLWG),
207 hesitant fuzzy linguistic ordered weighted average (HFLOWA), and
208 hesitant fuzzy linguistic ordered weighted geometric (HFLOWG)
209 operators for aggregating hesitant linguistic term sets and devel-
210 oped a new fuzzy decision making method using these operators.
211 Instead of representing HFLTS with labels or intervals of linguis-
212 tic terms, Wang et al. [49] used linguistic scale functions in the
213 transformation process between qualitative information and quan-
214 titative data. Yavuz et al. [16] developed a HFLTS based multicriteria
215 decision making approach for alternative-fuel vehicle selection
216 and applied their proposed model on a home health care service
217 provider in the USA.

218 Basic definitions on HFLTS can be listed as follows [6,50]:

219 **Definition 2.** An HFLTS, H_S , is an ordered finite subset of consec-
220 utive linguistic terms of a linguistic term set S which can be shown
221 as $S = \{s_0, s_1, \dots, s_g\}$.

222 **Definition 3.** Assume that E_{G_H} is a function that converts linguis-
223 tic expressions into HFLTS, H_S . Let G_H be a context-free grammar
224 that uses the linguistic term set S . Let S_{ll} be the expression domain
225 generated by G_H . This relation can be shown as $E_{G_H} : S_{ll} \rightarrow H_S$.

226 Using the following transformations, comparative linguistic
227 expressions are converted into HFLTSs;

$$228 E_{G_H}(s_i) = \{s_j | s_i \in S\} \tag{2}$$

$$229 E_{G_H}(\text{at most } s_i) = \{s_j | s_j \in S \text{ and } s_j \leq s_i\} \tag{3}$$

$$230 E_{G_H}(\text{lower than } s_i) = \{s_j | s_j \in S \text{ and } s_j < s_i\} \tag{4}$$

$$231 E_{G_H}(\text{at least } s_i) = \{s_j | s_j \in S \text{ and } s_j \geq s_i\} \tag{5}$$

$$232 E_{G_H}(\text{greater than } s_i) = \{s_j | s_j \in S \text{ and } s_j > s_i\} \tag{6}$$

$$233 E_{G_H}(\text{between } s_i \text{ and } s_j) = \{s_k | s_k \in S \text{ and } s_i \leq s_k \leq s_j\} \tag{7}$$

234 **Definition 4.** The envelope of an HFLTS, represented by $env(H_S)$,
235 is a linguistic interval whose limits are obtained by its maximum
236 and minimum values:

$$237 env(H_S) = [H_{S^-}, H_{S^+}], H_{S^-} \leq H_{S^+} \tag{8}$$

where

$$H_{S^-} = \min(s_i) = s_j, s_i \in H_S \text{ and } s_i \geq s_j \forall i$$

$$H_{S^+} = \max(s_i) = s_j, s_i \in H_S \text{ and } s_i \leq s_j \forall i$$

Definition 5. Let $S = \{s_0, s_1, \dots, s_g\}$ be a linguistic term set. A HFLTS, H_S , is defined as an ordered finite subset of consecutive linguistic terms of S :

$$H_S = \{s_i, s_{i+1}, \dots, s_j\} \text{ such that } s_k \in S, k \in \{i, \dots, j\} \quad (9)$$

Definition 6. An ordered weighted average (OWA) operator of dimension n is a mapping OWA: $R^n \rightarrow R$, so that

$$OWA(a_1, a_2, \dots, a_n) = \sum_{j=1}^n w_j b_j \quad (10)$$

where b_j is the j^{th} largest of the aggregated arguments a_1, a_2, \dots, a_n , and $W = (w_1, w_2, \dots, w_n)^T$ is the associated weighting vector satisfying $w_i \in [0, 1], i = 1, 2, \dots, n$ and $\sum_{i=1}^n w_i = 1$.

Definition 7. A triangular fuzzy membership function $\tilde{A} = (a, b, c)$ is used as the representation of the comparative linguistic expressions based on HFLTS H_S , the definition domain of \tilde{A} should be the same as the linguistic terms $\{s_i, \dots, s_j\} \in H_S$. The min and the max operators are used to compute a and c .

$$a = \min \{a_L^i, a_M^i, a_M^{i+1}, \dots, a_M^j, a_R^j\} = a_L^i \quad (11)$$

$$c = \max \{a_L^i, a_M^i, a_M^{i+1}, \dots, a_M^j, a_R^j\} = a_R^j \quad (12)$$

The remaining elements $a_M^i, a_M^{i+1}, \dots, a_M^j \in T$ should contribute to the computation of the parameter b . The aggregation operator OWA will be used to aggregate them:

$$b = OWA_{W^S} (a_M^i, a_M^{i+1}, \dots, a_M^j) \quad (13)$$

4. Hesitant fuzzy QFD: steps of the methodology

Hesitant Fuzzy Sets has the advantage of considering the hesitancy of experts under uncertainty. Neither classical QFD method nor ordinary fuzzy QFD method can handle this hesitancy.

In this section, we will first give the steps of the proposed hesitant fuzzy QFD methodology and then extend the same steps for the design problems having correlations among DRs.

4.1. Steps of the proposed Hesitant Fuzzy QFD

Step 1. Identify and construct the hierarchy of customer requirements as given in Fig. 2. Then determine the design requirements corresponding to customer requirements.

Step 2. Compute the weights of customer requirements

Steps 2.1-2.5 are applied to both the main customer requirements and the sub-customer requirements. The global weights of sub-customer requirements are calculated using steps 2.6-2.7.

Step 2.1: Construct pairwise comparison matrices for customer requirements and obtain the compromised evaluations from the experts using HFLTS. The HFLTS are obtained by utilizing the linguistic terms in Table 1 and context-free grammar; such as between, greater than, less than, at most, at least etc.

Step 2.2: Aggregate and build fuzzy envelope for HFLTS by using the OWA operator, as proposed by Liu and Rodríguez [50]. In this approach, the result of aggregation yields a trapezoidal fuzzy number. First, the scale given in Table 1 is sorted from the lowest (s_0) to the highest (s_g). Assume the experts evaluations vary between two

Table 1
Linguistic scale for hesitant fuzzy AHP.

Linguistic term	s_i	Abb.	Triangular fuzzy number
Absolutely high importance	s_{10}	(AH)	(7,9,9)
Very high importance	s_9	(VH)	(5,7,9)
Essentially high importance	s_8	(ESH)	(3,5,7)
Weakly high importance	s_7	(WH)	(1,3,5)
Equally high importance	s_6	(EH)	(1,1,3)
Exactly low importance	s_5	(EE)	(1,1,1)
Equally low importance	s_4	(EL)	(0,33,1,1)
Weakly low importance	s_3	(WL)	(0,2,0,33,1)
Essentially low importance	s_2	(ESL)	(0,14,0,2,0,33)
Very low importance	s_1	(VL)	(0,11,0,14,0,2)
Absolutely low importance	s_0	(AL)	(0,11,0,11,0,14)

terms i.e. s_i and s_j . Then $s_0 \leq s_i < s_j < s_g$. The parameters of trapezoidal fuzzy membership function $\tilde{A} = (\alpha, \beta, \gamma, \delta)$ are computed as follows:

$$\alpha = \min \{a_L^i, a_M^i, a_M^{i+1}, \dots, a_M^j, a_R^j\} = a_L^i \quad (14)$$

$$\delta = \max \{a_L^i, a_M^i, a_M^{i+1}, \dots, a_M^j, a_R^j\} = a_R^j \quad (15)$$

$$\beta = \begin{cases} a_M^i, & \text{if } i+1=j \\ OWA_{w^2} \left(a_M^i, \dots, a_M^{\frac{i+j}{2}} \right), & \text{if } i+j \text{ is even} \\ OWA_{w^2} \left(a_M^i, \dots, a_M^{\frac{i+j-1}{2}} \right), & \text{if } i+j \text{ is odd} \end{cases} \quad (16)$$

$$\gamma = \begin{cases} a_M^{i+1}, & \text{if } i+1=j \\ OWA_{w^1} \left(a_M^j, a_M^{j-1}, \dots, a_M^{\frac{i+j}{2}} \right), & \text{if } i+j \text{ is even} \\ OWA_{w^1} \left(a_M^j, a_M^{j-1}, \dots, a_M^{\frac{i+j+1}{2}} \right), & \text{if } i+j \text{ is odd} \end{cases} \quad (17)$$

OWA operation given in Definition 6 requires a weight vector. Filev and Yager [51] define first and second types of weights using the α parameter which belongs to the unit interval [0,1]. First type of weights $W^1 = (w_1^1, w_2^1, \dots, w_n^1)$ is defined as:

$$w_1^1 = \alpha_2, w_2^1 = \alpha_2(1 - \alpha_2), \dots, w_n^1 = \alpha_2(1 - \alpha_2)^{n-2} \quad (18)$$

The second type of weights $W^2 = (w_1^2, w_2^2, \dots, w_n^2)$ is defined as:

$$w_1^2 = \alpha_1^{n-1}, w_2^2 = (1 - \alpha_1)\alpha_1^{n-2}, \dots, w_n^2 = 1 - \alpha_1, \quad (19)$$

where $\alpha_1 = \frac{g-(j-i)}{g-1}, \alpha_2 = \frac{(j-i)-1}{g-1}$ and g is the number of terms in the evaluation scale, j is the rank of highest evaluation and i is the rank of lowest evaluation value of the given interval.

Step 2.3: Obtain pairwise comparison matrix (\tilde{C}) composed of aggregated fuzzy numbers in Step 2.2

$$\tilde{C} = \begin{pmatrix} 1 & \tilde{c}_{12} & \dots & \tilde{c}_{1n} \\ \tilde{c}_{21} & 1 & \dots & \tilde{c}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{c}_{n1} & \tilde{c}_{n2} & \dots & 1 \end{pmatrix} \quad (20)$$

where $\tilde{c}_{ij} = (c_{ijl}, c_{ijm1}, c_{ijm2}, c_{iju})$.

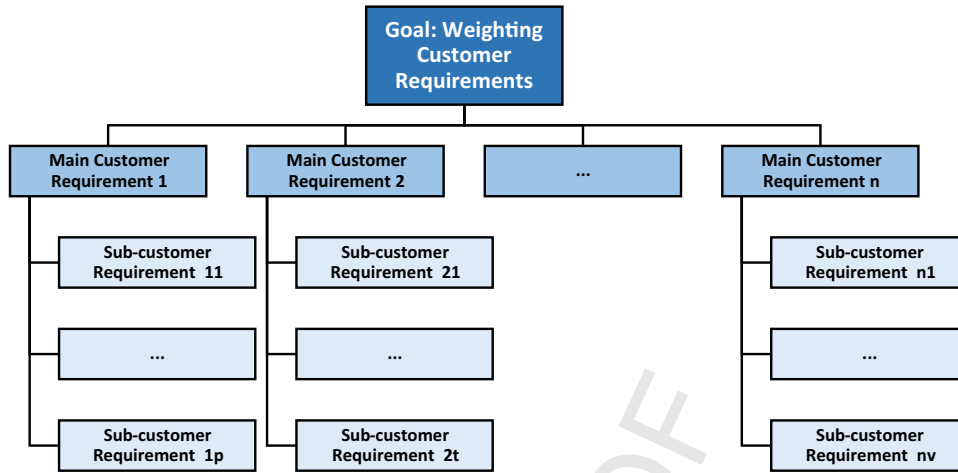


Fig. 2. Hierarchy of customer requirements.

Since the fuzzy envelopes, obtained in previous step are trapezoidal fuzzy numbers, reciprocal values are calculated as follows [8]:

$$\tilde{c}_{ji} = \left(\frac{1}{c_{iju}}, \frac{1}{c_{ijm2}}, \frac{1}{c_{ijm1}}, \frac{1}{c_{ijl}} \right) \quad (21)$$

Step 2.4: Compute fuzzy geometric mean for each row (\tilde{r}_i) of the matrix \tilde{C} using Eq. (22).

$$\tilde{r}_i = (\tilde{c}_{i1} \otimes \tilde{c}_{i2} \dots \otimes \tilde{c}_{in})^{1/n} \quad (22)$$

Step 2.5: The fuzzy weight (\tilde{w}_i^{CR}) of each main customer requirement is calculated using (\tilde{r}_i) values as follows:

$$\tilde{w}_i^{CR} = \tilde{r}_i \otimes (\tilde{r}_1 \oplus \tilde{r}_2 \dots \oplus \tilde{r}_n)^{-1} \quad (23)$$

In this study, the $\tilde{r}_1 \oplus \tilde{r}_2 \dots \oplus \tilde{r}_n$ value is accepted as the maximum parameter of the linguistic term absolutely high importance in Table 1 in order to decrease the deviation in the weights.

This calculation process is same for the sub-customer requirements \tilde{w}_{ij}^{CR} , where j denotes the number of sub-customer requirements belonging to the main customer requirement i .

Step 2.6: Calculate the fuzzy global weights of sub-customer requirements by using Eq. 24.

$$\tilde{w}_{ij}^G = \tilde{w}_i^{CR} \times \tilde{w}_{ij}^{CR} \quad (24)$$

where \tilde{w}_{ij}^G is the global weight of sub-customer requirement ij .

Step 2.7: Defuzzify the trepozoidal fuzzy numbers \tilde{w}_{ij}^G using Eq. (25) and normalize the defuzzified values using Eq. (26).

$$w_{ij}^G = \frac{\alpha + 2\beta + 2\gamma + \delta}{6} \quad (25)$$

$$w_{ij}^N = \frac{w_{ij}^G}{\sum_i \sum_j w_{ij}^G} \quad (26)$$

Step 3. Collect the data for the relations between DRs and CRs by using HFLTS from experts. The HFLTS of relations are obtained by utilizing the linguistic terms in Table 2 and context-free grammar.

Table 2
Linguistic scale for correlations.

Linguistic term	Abb.	Triangular fuzzy number
Absolutely low	AL	(1,2,3)
Very low	VL	(2,3,4)
Low	L	(3,4,5)
Medium	M	(4,5,6)
High	H	(5,6,7)
Very high	VH	(6,7,8)
Absolutely high	AH	(7,8,9)

Step 4. Aggregate HFLTS relations by using the aggregation operator defined in Step 2.2 and obtain relation matrix \tilde{R} with trapezoidal fuzzy numbers as given by Eq. (27).

$$\tilde{R} = \begin{matrix} & DR_1 & DR_2 & \dots & DR_z \\ \begin{matrix} CR_{11} \\ CR_{12} \\ \vdots \\ CR_{nv} \end{matrix} & \begin{bmatrix} \tilde{R}_{111} & \tilde{R}_{112} & \dots & \tilde{R}_{11z} \\ \tilde{R}_{121} & \tilde{R}_{122} & \dots & \tilde{R}_{12z} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{R}_{nv1} & \tilde{R}_{nv2} & \dots & \tilde{R}_{nvz} \end{bmatrix} \end{matrix} \quad (27)$$

Step 5. Obtain weighted relation matrix (\tilde{R}^w) whose elements are obtained by using Eq. (28)

$$\tilde{R}_{ijk}^w = w_{ij}^N \times \tilde{R}_{ijk} \quad (28)$$

Step 6. Obtain fuzzy importance values of DRs by summing the elements in each column of \tilde{R}^w as shown in Eq. (29).

$$\tilde{DR}_k^{Imp} = \sum_i \sum_j \tilde{R}_{ijk}^w \quad (29)$$

where \tilde{DR}_k^{Imp} denotes the fuzzy importance of design requirement k .

Step 7. Using Eqs. (25) and (30), obtain the crisp importance weights of DRs by defuzzifying \tilde{DR}_k^{Imp} and normalizing them.

$$DR_k^N = \frac{DR_k^{Imp}}{\max_{k=1, \dots, z} DR_k^{Imp}} \quad (30)$$

where DR_k^{Imp} and DR_k^N denote the defuzzified and normalized importance values of design requirement k , respectively.

Step 8. Collect the HFLTS from experts to evaluate alternatives with respect to DRs by utilizing the linguistic terms listed in Table 2 and context-free grammar.

Step 9. Apply the hesitant fuzzy TOPSIS method to prioritize the alternatives.

Step 9.1. Aggregate HFLTS evaluations by using the aggregation operator defined in Step 2.2 in order to obtain decision matrix composed of trapezoidal fuzzy numbers.

$$\tilde{D} = \begin{matrix} & DR_1 & DR_2 & \dots & DR_z \\ A_1 & \begin{bmatrix} \tilde{\tau}_{11} & \tilde{\tau}_{12} & \dots & \tilde{\tau}_{1z} \end{bmatrix} \\ A_2 & \begin{bmatrix} \tilde{\tau}_{21} & \tilde{\tau}_{22} & \dots & \tilde{\tau}_{2z} \end{bmatrix} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_L & \begin{bmatrix} \tilde{\tau}_{L1} & \tilde{\tau}_{L2} & \dots & \tilde{\tau}_{Lz} \end{bmatrix} \end{matrix} \quad (31)$$

where $\tilde{\tau}_{ij} = (\tau_{ij}, \tau_{ijm1}, \tau_{ijm2}, \tau_{iju})$.

Step 9.2: Build a normalized decision matrix \tilde{D}^N by using Eq. (33).

$$\tilde{D}^N = \begin{matrix} & DR_1 & DR_2 & \dots & DR_z \\ A_1 & \begin{bmatrix} \tilde{\tau}_{1z}^N & \tilde{\tau}_{12}^N & \dots & \tilde{\tau}_{1z}^N \end{bmatrix} \\ A_2 & \begin{bmatrix} \tilde{\tau}_{21}^N & \tilde{\tau}_{22}^N & \dots & \tilde{\tau}_{2z}^N \end{bmatrix} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_L & \begin{bmatrix} \tilde{\tau}_{L1}^N & \tilde{\tau}_{L2}^N & \dots & \tilde{\tau}_{Lz}^N \end{bmatrix} \end{matrix} \quad (32)$$

$$\tilde{\tau}_{ij}^N = \frac{\tilde{\tau}_{ij}}{\max_{i=1, \dots, L} \tilde{\tau}_{ij}}, j = 1, \dots, z \quad (33)$$

Step 9.3: Obtain the weighted normalized decision matrix \tilde{D}_w^N by using Eq. (35).

$$\tilde{D}_w^N = \begin{matrix} & DR_1 & DR_2 & \dots & DR_z \\ A_1 & \begin{bmatrix} \tilde{\tau}_{w1z}^N & \tilde{\tau}_{w12}^N & \dots & \tilde{\tau}_{w1z}^N \end{bmatrix} \\ A_2 & \begin{bmatrix} \tilde{\tau}_{w21}^N & \tilde{\tau}_{w22}^N & \dots & \tilde{\tau}_{w2z}^N \end{bmatrix} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_L & \begin{bmatrix} \tilde{\tau}_{wL1}^N & \tilde{\tau}_{wL2}^N & \dots & \tilde{\tau}_{wLz}^N \end{bmatrix} \end{matrix} \quad (34)$$

$$\tilde{\tau}_{wij}^N = DR_j^N \times \tilde{\tau}_{ij}^N, i = 1, \dots, L; j = 1, \dots, z \quad (35)$$

Step 9.3: Obtain the weighted normalized decision matrix \tilde{D}_w^N by using Eq. (35).

Step 9.4: Calculate the distances of each alternative from positive $\tilde{A}^+ = (\tilde{v}_1^+, \dots, \tilde{v}_p^+)$ and negative $\tilde{A}^- = (\tilde{v}_1^-, \dots, \tilde{v}_p^-)$ ideal solutions by defining $\tilde{v}_i^+ = (1, 1, 1, 1)$ and $\tilde{v}_i^- = (0, 0, 0, 0)$.

$$d_i^+ = \sum_{j=1}^z d(\tilde{\tau}_{wij}^N, \tilde{v}_i^+) \quad (36)$$

where

$$d(\tilde{\tau}_{wij}^N, \tilde{v}_i^+) = \sqrt{\frac{1}{4} [(1 - \tau_{wji}^N)^2 + (1 - \tau_{wji m1}^N)^2 + (1 - \tau_{wji m2}^N)^2 + (1 - \tau_{wju}^N)^2]} \quad (37)$$

and

$$d_i^- = \sum_{j=1}^z d(\tilde{\tau}_{wij}^N, \tilde{v}_i^-) \quad (38)$$

where

$$d(\tilde{\tau}_{wij}^N, \tilde{v}_i^-) = \sqrt{\frac{1}{4} [(0 - \tau_{wji}^N)^2 + (0 - \tau_{wji m1}^N)^2 + (0 - \tau_{wji m2}^N)^2 + (0 - \tau_{wju}^N)^2]} \quad (39)$$

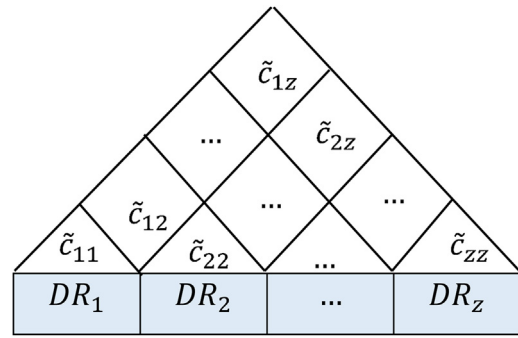


Fig. 3. Correlations among DRs.

Step 9.5: Calculate the closeness coefficient of each alternative and rank the alternatives.

$$CC_i = \frac{d_i^-}{d_i^+ + d_i^-} \quad (40)$$

In the next subsection, we will consider the correlations among design requirements in order to reflect the inner dependencies of DRs on the weighted relation matrix.

4.3. Consideration of correlations among design requirements

In this section we assume that there exist correlations among DRs. In this case, the compromised correlations among DRs, \tilde{c}_{ij} , are expressed by experts using HFLTS based on the linguistic scale that was provided in Table 2. In Fig. 3, the roof of HOQ shows these correlations.

These correlations among DRs are aggregated by using the aggregation operator defined in Step 2.2 to obtain the relation matrix \tilde{C} with trapezoidal fuzzy numbers.

$$\tilde{C} = \begin{matrix} & DR_1 & DR_2 & \dots & DR_z \\ DR_1 & \begin{bmatrix} \tilde{c}_{11} & \tilde{c}_{12} & \dots & \tilde{c}_{1z} \end{bmatrix} \\ DR_2 & \begin{bmatrix} \tilde{c}_{21} & \tilde{c}_{22} & \dots & \tilde{c}_{2z} \end{bmatrix} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ DR_z & \begin{bmatrix} \tilde{c}_{z1} & \tilde{c}_{z2} & \dots & \tilde{c}_{zz} \end{bmatrix} \end{matrix} \quad (41)$$

Normalized relation matrix \tilde{R}^{norm} is formed as follows:

$$\tilde{R}^{norm} = \begin{matrix} & DR_1 & DR_2 & \dots & DR_z \\ CR_{11} & \begin{bmatrix} \tilde{R}_{111}^{norm} & \tilde{R}_{112}^{norm} & \dots & \tilde{R}_{11z}^{norm} \end{bmatrix} \\ CR_{12} & \begin{bmatrix} \tilde{R}_{121}^{norm} & \tilde{R}_{122}^{norm} & \dots & \tilde{R}_{12z}^{norm} \end{bmatrix} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ CR_{nv} & \begin{bmatrix} \tilde{R}_{nv1}^{norm} & \tilde{R}_{nv2}^{norm} & \dots & \tilde{R}_{nvz}^{norm} \end{bmatrix} \end{matrix} \quad (42)$$

where

$$\tilde{R}_{ij,k}^{\text{norm}} = \frac{\sum_{l=1}^z (\tilde{R}_{ij,l} \otimes \tilde{c}_{l,k})}{\sum_{k=1}^z \sum_{l=1}^z (\tilde{R}_{ij,k} \otimes \tilde{c}_{k,l})}, ij = 11, 12, \dots, nv \quad (43)$$

In this case, the weighted relation matrix (\tilde{R}^w) is obtained by using Eq. (44).

$$\tilde{R}_{ijk}^w = w_{ij}^N \times \tilde{R}_{ij,k}^{\text{norm}} \quad (44)$$

The rest of the methodology is composed of the same steps given in Section 4.1.

5. Case study

In this section, three computer workstations are compared by using the proposed hesitant QFD method based on the determined CRs and DRs. First we give the problem definition and implement the proposed method to the computer workstation selection problem. Later, a sensitivity analysis and comparisons with classical and ordinary fuzzy QFD approaches are presented.

5.1. Problem definition

Based on the QFD method, this paper aims to carry out the selection of the most suitable computer workstation that fulfills design requirements determined according to customer expectations. A workstation is a customized computer that is designed for specific scientific or technical application. Such equipment is usually integrated within a local network and has a multi-user operating system that is able to be run by a single person. In the past, every computer connected to the internet was used to be called a workstation. However, this definition of a workstation is a thing of the past, thanks to the technological advancements (mostly due to 3D animations) by certain companies such as Sun Microsystems, Silicon Graphics, Apollo Computer, HP and IBM. Compared to personal computers, workstations are able to provide a higher performance to end users. This improved performance is usually based on the use of higher-end computer components like microprocessor (CPU), graphics processing unit (GPU), physical memory and other parts that ensure multitasking.

5.2. Identification of customer requirements and expectations

The target customer group of workstations includes computer-aided designers, digital content creators, financial services employees, software developers, power office employees, analysts and printmakers.

Computer-aided designers: Professionals in this group are largely occupied with 2D and 3D modeling with the help of computer software. It also includes industrial and mechanical engineers who design specific components, as well as architects and civil engineers who design buildings. This group basically expects high resolution screens and capable graphics cards.

Digital content creator: Various fields can be categorized within this group, such as GPS maps, meteorological maps and multimedia (videos, sound and pictures). Professionals working as digital content creators need multitasking capabilities, a powerful CPU as well as high performing GPUs.

Financial services employees: This profession usually works with financial calculation algorithms that need to be computed fast enough to obtain the results in a short time. This translates into large amounts of physical memory to store data and workhorse microprocessors.

Software Developers: Developers usually tend to work on the go, and prefer mobility over other needs. Therefore their expectations

Table 3
Workstation CRs.

Performance (CR1)	Data processing (CR11) Image processing (CR12) Image production (CR13) Program production (CR14) Gaming (CR15)
Mobility (CR2)	Charging time (CR21) Battery life (CR22) Weight (CR23) Thickness (CR24)
Peripherals (CR3)	Display connectivity (CR31) Universal connectivity (CR32) Adapter (CR33) Sound (CR34) Display (CR35)

Table 4
Pairwise comparisons of the main CRs with respect to the goal.

	Performance	Mobility	Peripherals
Performance	EE	Between EHI and WHI	Between ELI and EHI
Mobility		EE	Between WLI and ELI
Peripherals			EE

are more focused on long battery life, rather than processing power or other specifications.

Power Office employees: People who usually work with Office applications can be categorized within this group. This user profile basically needs a fair level of processor performance and robust computer case.

Analysts: This profession group mainly requires computers with high processing power.

Printmakers: Graphical designers work in front of big screens and need large amounts of GPU power. In addition, software they use usually requires high-capacity physical memory components.

The proposed approach is applied for the workstation selection problem of a large IT company, which includes the entire customer groups mentioned above. A group of three experts has supported the process. The expert group has identified 14CRs in three main dimensions, as shown in Table 3. This corresponds to Step 1 in our proposed approach.

In the computer market, there are several workstation manufacturers. In this study, we considered the following three workstation manufacturers based on the experts opinions:

Company G is a Taiwan-based company established in 1986. The company manufactures motherboards, motherboard components, notebooks, desktop PCs, servers and mobile phones. Company G is one of the top 20 companies in Taiwan and its market capitalization is 133 million USD.

Company H is a large international conglomerate which is based in Palo Alto, California, USA. It manufactures hardware for data processing, printing solutions and digital image products. It is also a software and service provider. In 2002, Company H merged with another international computer company. Its operating systems and microprocessors are well known in the market. Company H also produces servers and workstations and management software.

Company A is one of the leading mobile phone and computer producers. Company A's computers are well known for their capabilities in graphical design related tasks. It started to use Intel chips in all its products. In 2009, Company A announced that they started building their own engineering team to design customized microchips.

Fig. 4 illustrates the HFLTS assessments between DRs and CRs in the House of Quality (HOQ). We give here only a small part of the huge HOQ matrix because of space constraints. The whole details can be found in Tables 4–7 and 10.

Table 5
Pairwise comparisons of the sub-customer requirements with respect to performance.

	Data processing (CR11)	Image processing (CR12)	Image production (CR13)	Program production (CR14)	Gaming (CR15)
Data processing (CR11)	EE	Between ELI and EHI	Between ESLI and ELI	Between EHI and WHI	Between EHI and ESHI
Image processing (CR12)		EE	Between ELI and EHI	Between WLI and EE	ELI
Image production (CR13)			EE	Between ELI and EHI	Between EHI and WHI
Program production (CR14)				EE	Between EHI and WHI
Gaming (CR15)					EE

Table 6
Pairwise comparisons of the sub-customer requirements with respect to mobility.

	Charging time (CR21)	Battery life (CR22)	Weight (CR23)	Thickness (CR24)
Charging time (CR21)	EE	Between ALI and ESLI	Between ESLI and ELI	Between ELI and EHI
Battery life (CR22)		EE	Between EHI and WHI	Between EHI and WHI
Weight (CR23)			EE	Between EHI and ESHI
Thickness (CR24)				EE

Table 7
Pairwise comparisons of the sub-customer requirements with respect to peripherals.

	Display connectivity (CR31)	Universal connectivity (CR32)	Adapter (CR33)	Sound (CR34)	Display (CR35)
Display connectivity (CR31)	EE	Between ELI and EHI	Between ESLI and ELI	Between EHI and WHI	Between EHI and ESHI
Universal connectivity (CR32)		EE	Between WLI and ELI	Between ELI and EHI	Between ELI and EHI
Adapter (CR33)			EE	Between WLI and EE	Between EHI and ESHI
Sound (CR34)				EE	Between WHI and ESHI
Display (CR35)					EE

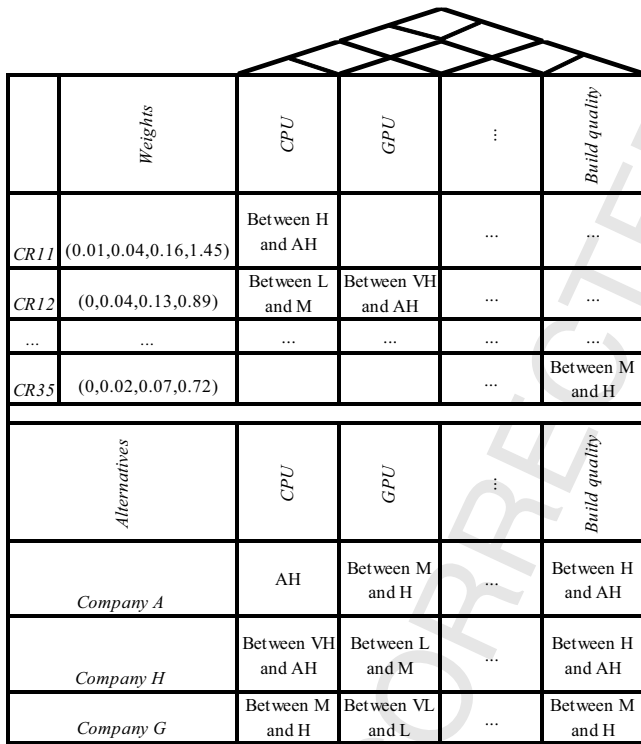


Fig. 4. Assessment using HFLTS in HOQ.

In the next subsection, we implement the proposed method to workstation selection problem.

5.3. Implementation

To compute the weights of CRs (Step 2) we use the following Tables 4-8. Table 4 shows the pairwise comparisons of the main CRs with respect to the goal, filled by the experts' compromised evaluation using HFTLS. Tables 5-7 present the pairwise compar-

isons of the sub-customer requirements with respect to the main CRs Performance, Mobility and Peripherals, respectively.

Applying Steps 2.1-2.5 we obtain Table 8. In order to facilitate the understandability of the approach we give an example of calculations in the following:

Table 8 shows that the pairwise comparison value of Performance and Mobility is calculated as (1, 1, 3, 5). The linguistic evaluations of the experts for this comparison are between "s₆ = Equally High Importance" and "s₇ = Weakly High Importance". The triangular fuzzy numbers associated with the mentioned linguistic terms are (1, 1, 3) and (1, 3, 5), respectively. Using the formulas given in Eqs. (14)-(17), the trapezoidal fuzzy membership function $\tilde{A} = (\alpha, \beta, \gamma, \delta)$ representing the linguistic evaluation is calculated as:

$$\alpha = \min \{ a_L^6, a_L^7, a_M^6, a_M^7, a_R^6, a_R^7 \} = \min \{ 1, 1, 1, 3, 3, 5 \} = 1$$

$$\delta = \max \{ a_L^6, a_L^7, a_M^6, a_M^7, a_R^6, a_R^7 \} = \max \{ 1, 1, 1, 3, 3, 5 \} = 5$$

and since $i + 1 = j$ ($i = 6, j = 7$);

$$\beta = a_M^6 = 1$$

$$\gamma = a_M^7 = 3$$

After determining the pairwise comparison values for each expert evaluation, the normalized weight of each criterion is calculated next. To this end, the geometric mean of each row is calculated. For example, the (0.69, 1, 1.44, 2.47) value in Table 8 is calculated as:

$$(1 \times 1 \times 0.69)^{1/3} = 0.69; (1 \times 1 \times 1)^{1/3} = 1; (1 \times 3 \times 1)^{1/3} = 1.44; (1 \times 5 \times 3)^{1/3} = 2.47$$

Next, the geometric means are summed up. The sum of geometric means given in Table 8 is (1.72, 2.48, 3.88, 5.93) and is obtained as follows:

$$0.68 + 0.34 + 0.69 = 1.72; 1 + 0.48 + 1 = 2.48; 1.44 + 1 + 1.44 = 3.88; 2.47 + 1 + 2.47 = 5.93$$

Table 8

Q13 Pairwise comparison values and normalized weights of the main CRs with respect to the goal.

	Performance	Mobility	Peripherals	Geometric Means	Normalized weight
Performance	(1,1,1,1)	(1,1,3,5)	(0,33,1,1,3)	(0,69,1,1,44,2,47)	(0.12,0.26,0.58,1.43)
Mobility	(0,2,0,33,1,1)	(1,1,1,1)	(0,2,0,33,1,1)	(0,34,0,48,1,1)	(0,06,0,12,0,4,0,58)
Peripherals	(0,33,1,1,3)	(1,1,3,5)	(1,1,1,1)	(0,69,1,1,44,2,47)	(0,12,0,26,0,58,1,43)

Table 9

Weights of the sub-customer requirements.

	Sub-criteria	Relative scores	Global scores	Defuzzified weights	Normalized weights
Performance	Data processing (CR11)	(0.05,0.164,0.283,1.014)	(0.006,0.042,0.165,1.447)	0.311	0.093
	Image processing (CR12)	(0.035,0.166,0.226,0.62)	(0.004,0.043,0.131,0.885)	0.206	0.062
	Image production (CR13)	(0.059,0.208,0.348,1.263)	(0.007,0.054,0.202,1.802)	0.387	0.116
	Program production (CR14)	(0.054,0.133,0.281,0.948)	(0.006,0.034,0.164,1.353)	0.292	0.088
	Gaming (CR15)	(0.033,0.086,0.181,0.498)	(0.004,0.022,0.105,0.711)	0.162	0.049
Mobility	Charging time (CR21)	(0.031,0.078,0.121,0.39)	(0.002,0.01,0.049,0.225)	0.057	0.017
	Battery life (CR22)	(0.149,0.276,0.723,1.509)	(0.009,0.034,0.292,0.873)	0.255	0.077
	Weight (CR23)	(0.076,0.218,0.455,1.031)	(0.004,0.027,0.183,0.596)	0.17	0.051
	Thickness (CR24)	(0.035,0.097,0.198,0.513)	(0.002,0.012,0.08,0.297)	0.08	0.024
Peripherals	Display connectivity (CR31)	(0.048,0.157,0.271,1.025)	(0.006,0.04,0.157,1.462)	0.311	0.093
	Universal connectivity (CR32)	(0.033,0.129,0.212,0.781)	(0.004,0.033,0.123,1.114)	0.239	0.072
	Adapter (CR33)	(0.064,0.24,0.426,1.214)	(0.008,0.062,0.248,1.732)	0.393	0.118
	Sound (CR34)	(0.052,0.159,0.297,1.025)	(0.006,0.041,0.173,1.462)	0.316	0.095
	Display (CR35)	(0.022,0.073,0.113,0.503)	(0.003,0.019,0.066,0.718)	0.148	0.044
	Total			3.327	1

Next, the geometric mean of each row is divided by the sum of geometric means using Eq. (30). The normalized weight for the criterion Performance is obtained as follows:

$$\frac{0.69}{5.93} = 0.12; \quad \frac{1}{3.88} = 0.26; \quad \frac{1.44}{2.48} = 0.58; \quad \frac{2.47}{1.72} = 1.43$$

Steps 2.1–2.5 are repeated to obtain the relative scores in Table 9. Steps 2.6 and 2.7 are applied to calculate the normalized weights of the sub-customer requirements as given in Table 9. As an example calculation, the fuzzy global weight of data processing (CR11) is calculated by using Eq. (24) as follows:

$$\tilde{w}_{11}^G = (0.12, 0.26, 0.58, 1.43) \times (0.05, 0.164, 0.283, 1.014) = (0.006, 0.042, 0.165, 1.447)$$

when we defuzzify the trepezoidal fuzzy numbers \tilde{w}_{11}^G using Eq. (25), we obtained the following result.

$$w_{11}^G = \frac{0.006 + 2 \times 0.042 + 2 \times 0.165 + 1.447}{6} = 0.311$$

To calculate the normalized value 0.093, we first sum the defuzzified values and then the defuzzified value 0.311 is divided by this sum.

$$w_{11}^N = \frac{0.311}{3.327} = 0.093$$

Table 10 presents the relations between design requirements and customer requirements by using the compromised HFLTS obtained from the three experts. Table 11 presents the aggregated relation matrix \tilde{R} between CRs and DRs.

Using Eqs. (14)–(19) and Table 2, the aggregated value for CR11 and CPU relation which corresponds to “Between H and AH” is calculated as (5,6.83,7.17,9).

Table 12 shows the weighted relation matrix (\tilde{R}^w) and the fuzzy importance values of DRs. The weighted correlation for CR11 and CPU relation is calculated as follows:

$$0.093 \times (5, 6.83, 7.17, 9) = (0.47, 0.64, 0.67, 0.84)$$

The crisp importance weights of DRs are given in Table 13. The defuzzified score for CPU is calculated as follows: First, the weighted correlations in the CPU column in Table 12 are summed.

It is found to be $\tilde{DR}_{CPU}^{Imp} = (1.74, 2.36, 2.58, 3.2)$. Then this sum is defuzzified using Eq. (25) to calculate DR_{CPU}^N :

$$DR_{CPU}^N = \frac{1.74 + 2 \times 2.36 + 2 \times 2.58 + 3.2}{6} = 2.47$$

The normalized score is calculated by using Eq. (30).

$$DR_{CPU}^N = \frac{2.47}{2.805} = 0.881$$

Thus, the process for weighting the DRs has been completed. The next step is to apply fuzzy TOPSIS. Table 14 presents the decision matrix including the HFLTS evaluations of the experts.

The evaluation of the alternatives with respect to the DRs is given in Table 15. According to these results, the best workstation alternative is Company H. Even their scores are very close to each other, the second and third alternatives are ranked as Company A and Company G, respectively.

Applying Steps 9.1–9.5 Table 15 is obtained. It presents the evaluation of alternatives with respect to DRs.

When we analyze the values in Table 15, we see that Company A is performing extremely poor in VGA and HDML, while Company G is scoring extremely poor in CPU, GPU and charging power. Company H is the worst in memory but only moderately, which causes it to be the best.

In the next subsection, a sensitivity analysis is given in order to examine the robustness of the given decision.

5.4. Sensitivity analysis

To observe the effects of the possible changes in the weights of the DRs on the computer workstation selection, a sensitivity analysis is conducted. In Fig. 5, one-at-a time sensitivity analysis has been applied. In this figure, the colors blue, orange and grey represent the alternatives A, H and G, respectively. The x-axis represents the criterion weight, while the y-axis represents the scores of alternatives.

In the sensitivity analysis, we change the value of a certain criterion’s weight as the other criteria weights are fixed. Using these new criteria weights, the scores of alternatives are recalculated. The dark red line represents the current weight of the DR. Selec-

Table 10
Identified design requirements and their relations with the customer requirements.

	Design requirements (DRs)												
	CPU	GPU	Memory	Operating system	Charging power	Battery cells	VGA	HDMI	Speakers	Cooling fan	USB	Display	Build quality
CR11	Between H and AH		Between M and VH	Between VL and M						Between VH and AH			
CR12	Between L and M	Between VH and AH	Between VH and AH							Between M and VH			
CR13	Between L and H	Between M and VH	Between AL and L							Between L and M		Between M and H	
CR14	Between VH and AH	Between AL and VL	Between VH and AH	Between AL and L						Between H and VH			
CR15	Between M and VH	Between M and VH	Between M and H	Between H and AH					Between AL and L	Between H and AH			
CR21					Between H and AH	Between M and VH							
CR22					Between L and M	Between VH and AH				Between AL and VL			
CR23						Between L and H						Between AL and VL	Between VH and AH
CR24						Between H and AH				Between L and M		Between H and AH	
CR31							Between VH and AH	Between L and M				Between AL and L	
CR32							Between M and VH	Between M and VH			Between M and H		
CR33					Between H and VH	Between L and M							
CR34									Between H and AH				
CR35												Between M and H	Between M and H

Table 11
Aggregated relation matrix \tilde{R} between CRs and DRs.

	Weight	CPU	GPU	Memory	Operating system	Charging power	Battery Cells	VGA
CR11	0.093	(5,6,83,7,17,9)		(4,5,83,6,17,8)	(2,3,83,4,17,6)			
CR12	0.062	(3,4,5,6)	(6,7,8,9)	(6,7,8,9)				
CR13	0.116	(3,4,83,5,17,7)	(4,5,83,6,17,8)	(1,2,83,3,17,5)				
CR14	0.088	(6,7,8,9)	(1,2,3,4)	(6,7,8,9)	(1,2,83,3,17,5)			
CR15	0.049	(4,5,83,6,17,8)	(4,5,83,6,17,8)	(4,5,6,7)	(5,6,83,7,17,9)			
CR21	0.017					(5,6,83,7,17,9)	(4,5,83,6,17,8)	
CR22	0.077					(3,4,5,6)	(6,7,8,9)	
CR23	0.051						(3,4,83,5,17,7)	
CR24	0.024						(5,6,83,7,17,9)	
CR31	0.093							(6,7,8,9)
CR32	0.072							(4,5,83,6,17,8)
CR33	0.118					(5,6,7,8)	(3,4,5,6)	
CR34	0.095							
CR35	0.044							
	Weight	HDMI	Speakers	Cooling fan	USB	Display	Build quality	
CR11	0.093			(6,7,8,9)				
CR12	0.062			(4,5,83,6,17,8)				
CR13	0.116			(3,4,5,6)		(4,5,83,6,17,8)		
CR14	0.088			(5,6,7,8)				
CR15	0.049		(1,2,83,3,17,5)	(5,6,83,7,17,9)				
CR21	0.017							
CR22	0.077			(1,2,3,4)				
CR23	0.051					(1,2,3,4)	(5,6,83,7,17,9)	
CR24	0.024			(3,4,5,6)		(6,7,8,9)		
CR31	0.093	(3,4,5,6)				(1,2,83,3,17,5)		
CR32	0.072	(4,5,83,6,17,8)			(4,5,83,6,17,8)			
CR33	0.118							
CR34	0.095		(5,6,83,7,17,9)					
CR35	0.044					(4,5,83,6,17,8)	(4,5,83,6,17,8)	

Table 12
Weighted correlation matrix.

	CPU	GPU	Memory	Operating system	Charging power	Battery Cells	VGA
CR11	(0.47,0.64,0.67,0.84)		(0.37,0.54,0.57,0.74)	(0.19,0.36,0.39,0.56)			
CR12	(0.19,0.25,0.31,0.37)	(0.37,0.43,0.5,0.56)	(0.37,0.43,0.5,0.56)				
CR13	(0.35,0.56,0.6,0.81)	(0.46,0.68,0.72,0.93)	(0.12,0.33,0.37,0.58)				
CR14	(0.53,0.62,0.7,0.79)	(0.09,0.18,0.26,0.35)	(0.53,0.62,0.7,0.79)	(0.09,0.25,0.28,0.44)			
CR15	(0.2,0.29,0.3,0.39)	(0.2,0.29,0.3,0.39)	(0.2,0.25,0.29,0.34)	(0.25,0.33,0.35,0.44)			
CR21					(0.09,0.12,0.12,0.15)	(0.07,0.1,0.1,0.14)	
CR22					(0.23,0.31,0.39,0.46)	(0.46,0.54,0.62,0.69)	
CR23						(0.15,0.25,0.26,0.36)	
CR24						(0.12,0.16,0.17,0.22)	
CR31							(0.56,0.65,0.74,0.84)
CR32							(0.29,0.42,0.44,0.58)
CR33					(0.59,0.71,0.83,0.94)	(0.35,0.47,0.59,0.71)	
CR34							
CR35							
Total	(1.74,2.36,2.58,3.2)	(1.12,1.58,1.78,2.23)	(1.59,2.17,2.43,3.01)	(0.53,0.94,1.02,1.44)	(0.91,1.14,1.34,1.55)	(1.15,1.52,1.74,2.12)	(0.85,1.07,1.18,1.42)
	HDMI	Speakers	Cooling fan	USB	Display	Build quality	
CR11			(0.56,0.65,0.74,0.84)				
CR12			(0.25,0.36,0.38,0.5)				
CR13			(0.35,0.46,0.58,0.7)		(0.46,0.68,0.72,0.93)		
CR14			(0.44,0.53,0.62,0.7)				
CR15		(0.05,0.14,0.16,0.25)	(0.25,0.33,0.35,0.44)				
CR21							
CR22			(0.08,0.15,0.23,0.31)				
CR23					(0.05,0.1,0.15,0.2)	(0.26,0.35,0.37,0.46)	
CR24			(0.07,0.1,0.12,0.14)		(0.14,0.17,0.19,0.22)		
CR31	(0.28,0.37,0.47,0.56)				(0.09,0.26,0.29,0.47)		
CR32	(0.29,0.42,0.44,0.58)			(0.29,0.42,0.44,0.58)			
CR33							
CR34		(0.48,0.65,0.68,0.86)					
CR35					(0.18,0.26,0.27,0.35)	(0.18,0.26,0.27,0.35)	
Total	(0.57,0.79,0.91,1.14)	(0.53,0.79,0.84,1.11)	(2.258,3.02,3.63)	(0.29,0.42,0.44,0.58)	(0.92,1.47,1.62,2.17)	(0.44,0.61,0.64,0.81)	

Table 13
Weights of design requirements.

DRs	Defuzzified score	Normalized score
CPU	2.47	0.881
GPU	1.678	0.598
Memory	2.3	0.82
Operating System	0.9817	0.35
Charging power	1.2367	0.441
Battery Cells	1.6317	0.582
VGA	1.1283	0.402
HDMI	0.8517	0.304
Speakers	0.8167	0.291
Cooling Fan	2.805	1
USB	0.4317	0.154
Display	1.545	0.551
Build quality	0.625	0.223

Table 14
Evaluation of alternatives with respect to design requirements.

	CPU	GPU	Memory	Operating System	Charging power	Battery Cells	VGA
Company A	AH	Between M and H	M	AH	Between VH and AH	Between L and H	AL
Company H	Between VH and AH	Between L and M	Between L and M	H	Between VH and AH	Between L and H	AH
Company G	Between M and H	Between VL and L	Between M and H	Between VH and AH	Between AL and VL	Between H and AH	AH
	HDMI	Speakers	Cooling fan	USB	Display	Build quality	
Company A	AL	M	Between M and VH	Between H and VH	Between H and AH	Between H and AH	Between H and AH
Company H	AH	M	Between M and H	Between H and AH	Between H and VH	Between H and VH	Between H and AH
Company G	AH	Between M and H	Between VH and AH	M	M	M	Between M and H

Table 15
Evaluation of alternatives with respect to DRs.

DRs	Weight	Company A	Company H	Company G
CPU	0.881	(7, 8, 8, 9)	(6, 7, 8, 9)	(4, 5, 6, 7)
GPU	0.598	(4, 5, 6, 7)	(3, 4, 5, 6)	(2, 3, 4, 5)
Memory	0.82	(4, 5, 5, 6)	(3, 4, 5, 6)	(4, 5, 6, 7)
Operating system	0.35	(7, 8, 8, 9)	(6, 7, 7, 8)	(6, 7, 8, 9)
Charging power	0.441	(6, 7, 8, 9)	(6, 7, 8, 9)	(1, 2, 3, 4)
Battery cells	0.582	(3, 4.833, 5.167, 7)	(3, 4.833, 5.167, 7)	(5, 6.833, 7.167, 9)
VGA	0.402	(1, 2, 2, 3)	(7, 8, 8, 9)	(7, 8, 8, 9)
HDMI	0.304	(1, 2, 2, 3)	(7, 8, 8, 9)	(7, 8, 8, 9)
Speakers	0.291	(4, 5, 5, 6)	(4, 5, 5, 6)	(4, 5, 6, 7)
Cooling fan	1	(4, 5.833, 6.167, 8)	(4, 5, 6, 7)	(6, 7, 8, 9)
USB	0.154	(5, 6, 7, 8)	(5, 6.833, 7.167, 9)	(4, 5, 5, 6)
Display	0.551	(5, 6.833, 7.167, 9)	(5, 6, 7, 8)	(4, 5, 5, 6)
Build quality	0.223	(5, 6.833, 7.167, 9)	(5, 6.833, 7.167, 9)	(4, 5, 6, 7)
d_i^+		8.496	8.335	8.533
d_i^-		4.666	4.829	4.652
CC_i		0.354	0.367	0.353

tion of alternative H is a robust decision and the changes in the weights of the DRs do not affect the selection of H whereas small changes in the weights of DRs affect the ranking of alternatives A and G. When the weights of the DRs HDMI, battery cells, VGA and speakers become larger than their present values, then the alternative G takes the second rank. Similarly, a slight decrease in the weights of DRs CPU, GPU, charging power and display causes alternative A to take the third order. The most insensitive DRs are USB, Memory, Cooling Fan, and Build Quality since the functions of the alternatives do not have almost any intersection along the axis of the related DR weight.

In the next subsection, we compare our proposed method with both classical QFD and ordinary fuzzy QFD.

5.5. Comparison with the classical QFD and ordinary fuzzy QFD approaches

In this section, we used the classical QFD and ordinary fuzzy QFD approaches for evaluating computer work stations.

For the comparison with classical QFD, the same experts are asked to make a compromise evaluation by using crisp values. The results of our proposed method have been compared with the results of the classical QFD method. Table 16 presents the DRs' crisp weights, the evaluation of each alternative with respect to each DR and the total score of alternatives.

According to the overall result of classical QFD, Alternative H is the best alternative followed by G and A. The selection of the Alternative H remains the same as the result of our proposed method, however the rankings of alternatives G and A are different.

For the comparison with ordinary fuzzy QFD, the same experts are asked to make a linguistic evaluation using scale in Table 2. In order to apply ordinary fuzzy QFD, simple fuzzy additive weighting method is used. Table 17 presents these linguistic evaluations and the scores of alternatives obtained through ordinary fuzzy QFD approach.

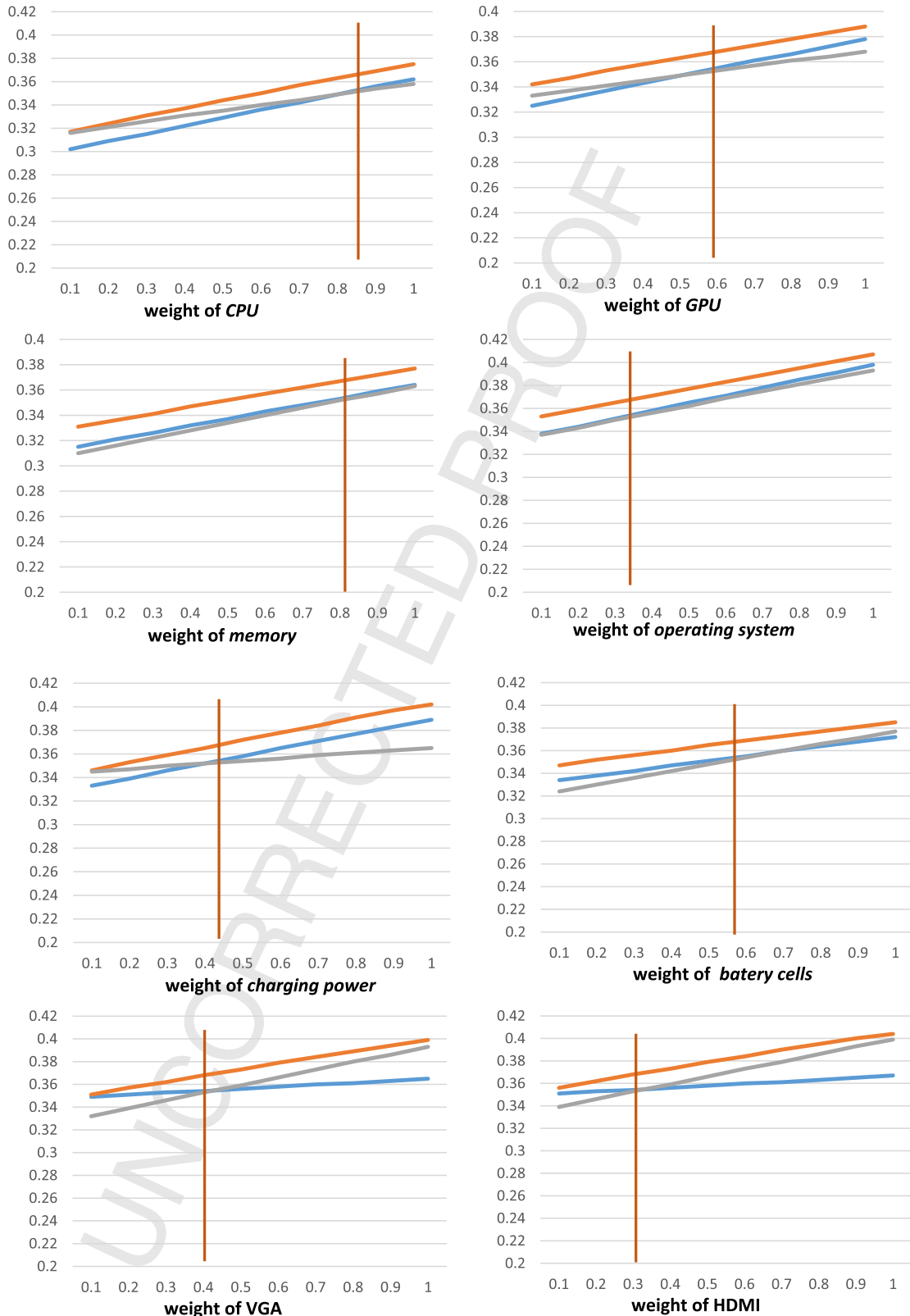
According to the overall result of ordinary fuzzy QFD, Alternative H is the best alternative followed by A. The obtained rank is the same as the rank in classical QFD approach.

634 **Table 18** shows the ranking of the companies with respect to
635 QFD approaches.

636 The experts indicate that the results obtained with the hesitant
637 fuzzy QFD method are more meaningful when compared to the
638 classical and ordinary fuzzy QFD approaches. The differences in the

scores of alternatives come from the hesitant evaluations in the
proposed method. In ordinary fuzzy QFD, experts have to select
one of the linguistic terms falling into the interval evaluations in
Table 14, which forces experts to make a discrete selection whereas
hesitant fuzzy QFD enables aggregated linguistic term sets based on

639
640
641
642
643



Q11

Fig. 5. Sensitivity analysis. (For interpretation of the references to colour in the text, the reader is referred to the web version of this article.)

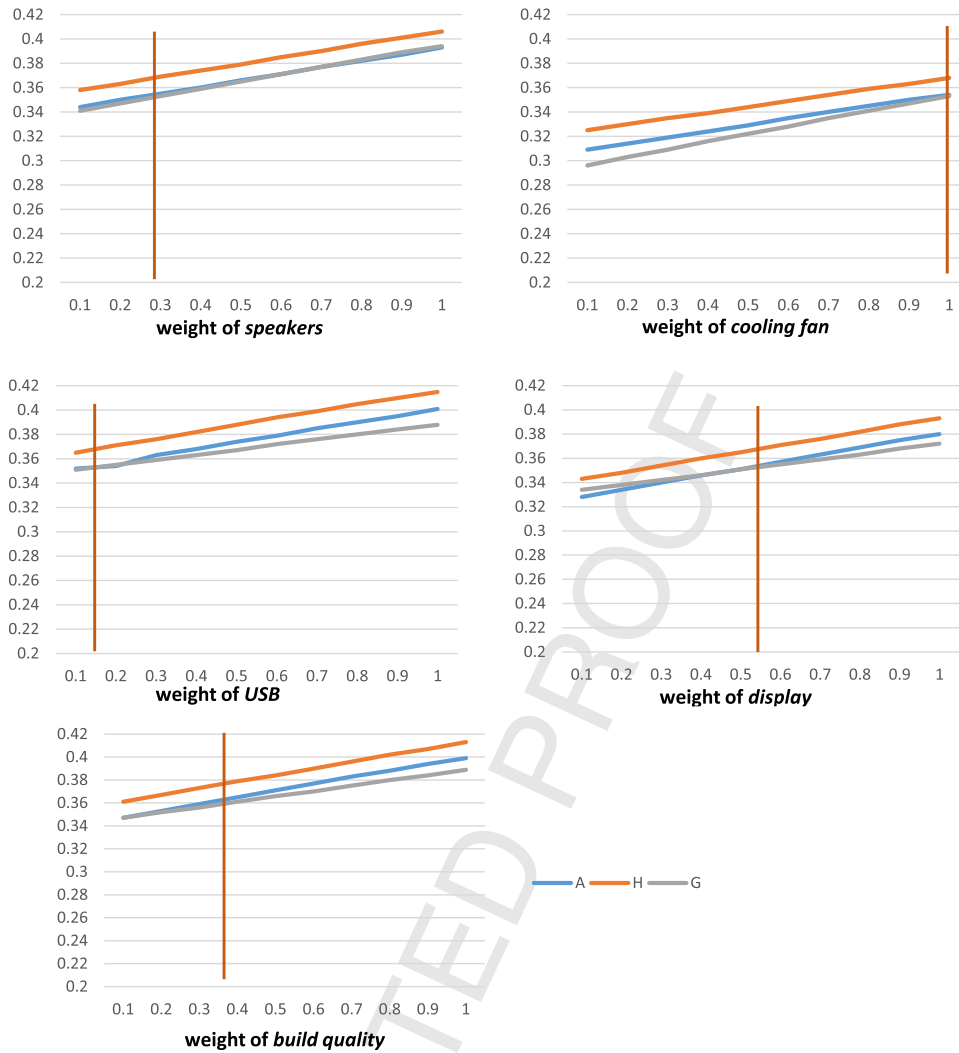


Fig. 5. (Continued)

Table 16
Crisp evaluation.

	Value	Crisp weight				
		5	4	3	2	1
CPU	0.846	◇	◻	▲		
GPU	0.577	◻			▲	◇
Memory	0.769	◻	▲	◇		
Operating system	0.323	◇	▲	◻		
Charging power	0.515	◇			◻	▲
Battery cells	0.769	▲		◇	◻	
VGA	0.323	▲	◻			◇
HDMI	0.254		◻	▲	◇	
Speakers	0.254		▲	◇	◻	
Cooling fan	1.000	▲	◇	◻		
USB	0.131	◻	◇	▲		
Display	0.638	◇	◻	▲		
Build quality	0.323	◇	▲		◻	
	Normalized score					
◇ (A)	0.323					
◻ (H)	0.340					
▲ (G)	0.338					

Table 17
Ordinary fuzzy evaluation.

	Value							
	Crisp weight	AH	VH	H	M	L	VL	AL
CPU	0.846	◇△			▲			
GPU	0.577				◇△		▲	
Memory	0.769			▲	◇△			
Operating system	0.323	◇	▲	△				
Charging power	0.515	△	◇				▲	
Battery cells	0.769		▲	△		◇		
VGA	0.323	△▲						◇
HDMI	0.254	△▲						◇
Speakers	0.254			▲	◇△			
Cooling fan	1.000		▲	△	◇			
USB	0.131	△		◇	▲			
Display	0.638		△	◇	▲			
Build quality	0.323	△	◇	▲				
	Fuzzy scores	Defuzzified scores			Normalized score			
◇ (A)	(30.34,37.062,43.78)	37.06			0.309			
△ (H)	(37.43,44.15,50.88)	44.15			0.368			
▲ (G)	(31.97,38.69,45.41)	38.69			0.323			

Table 18
Ranking of the companies with respect to QFD approaches.

	Company A	Company H	Company G
Hesitant Fuzzy QFD	2	1	3
Classical QFD	3	1	2
Ordinary Fuzzy QFD	3	1	2

644 OWA operator to be used. Therefore, hesitant evaluations provide
645 more flexible and informative representation of uncertainty.

646 **6. Conclusion**

647 Hesitancy is an inherent part of decision making process.
648 Experts generally have difficulty to establish the degree of mem-
649 bership of fuzzy set because of the time pressure, lack of knowledge
650 or data, etc. To overcome these difficulties, the concept of hesitant
651 fuzzy set which permitted the membership degree having a set of
652 possible values can be employed. We have proposed hesitant fuzzy
653 QFD since it can reflect the human’s hesitancy more objectively
654 than the other classical extensions of fuzzy set and applied it to
655 computer workstation selection problem.

656 A computer workstation is a fast and capable individual com-
657 puter for professional use. Usually, companies that need faster
658 microprocessors, larger RAMs and higher speed prefer using
659 computer workstations. Computer workstation selection is a multi-
660 criteria problem under fuzzy environment since experts generally
661 express their evaluations by using linguistic terms. In this paper,
662 this selection process has been supported with a QFD approach,
663 considering the DRs associated with the CRs. The best alternative is
664 determined by weighting the DR scores and calculating the close-
665 ness coefficient for each alternative. A model that considers the
666 effects of correlations among DRs in the computer workstation
667 selection process has also been proposed.

668 These proposed methods enabled us to analyze the vague and
669 imprecise relations between CRs and DRs. The determined weights
670 of CRs, which are obtained with the hesitant fuzzy AHP technique,
671 have been reflected to the workstation selection by using a hesi-
672 tant fuzzy TOPSIS method. Thus, a flexible evaluation process based
673 on HFLTS, which reflects experts’ hesitancies, has been designed.
674 The main contribution of this study is the consideration of experts’
675 hesitancies in each phase of the QFD approach for the first time.
676 The Adapter and image production specifications have been deter-

677 mined as the most important CRs whereas cooling fan and CPU have
678 been determined as the most important DRs.

679 The conducted sensitivity analysis indicated that the best alter-
680 native (H) is not sensitive to the changes in the weights of DRs
681 whereas the rankings of other alternatives (A and G) are sensitive
682 to even the slightest changes in the weights of DRs. Our compara-
683 tive analysis produced a different ranking result due to the ability
684 of the hesitant fuzzy method in handling the uncertainty better.

685 For further research, instead of the OWA operator, other aggre-
686 gation operators such as hesitant interval-valued fuzzy weighted
687 averaging operator or hesitant interval-valued fuzzy ordered
688 weighted averaging operator can be used. We also suggest intu-
689 itionistic fuzzy sets to be used in QFD instead of hesitant fuzzy sets
690 since intuitionistic fuzzy sets can consider both membership and
691 non-membership functions in their definitions.

692 **References**

693 [1] L.P. Sullivan, Quality function deployment: a system to assure that customer
694 needs drive the product design and production process, *Qual. Prog.* (June)
695 (1986) 39–50.
696 [2] Y. Akao, *Quality Function Deployment: Integrating Customer Requirements*
697 *Into Product Design*, Taylor & Francis, 1990.
698 [3] J.R. Hauser, D. Clausing, The house of quality, *Harvard Business Review*
699 (May–June) (1988) 63–73.
700 [4] M.L. Shillito, *Advanced QFD—Linking Technology to Market and Company*
701 *Needs*, Wiley, New York, 1994.
702 [5] B. Prasad, Review of QFD and related deployment techniques, *J. Manuf. Syst.*
703 (1998) 221–234.
704 [6] R.M. Rodriguez, L. Martinez, F. Herrera, Hesitant fuzzy linguistic term sets for
705 decision making, *IEEE Trans. Fuzzy Syst.* 20 (2012) 109–118.
706 [7] V. Torra, Hesitant fuzzy sets, *Int. J. Intell. Syst.* 25 (2010) 529–539.
707 [8] S. Çevik Onar, B. Öztayşi, C. Kahraman, Strategic decision selection using
708 hesitant fuzzy TOPSIS and interval type-2 fuzzy AHP: a case study, *Int. J.*
709 *Comput. Intell. Syst.* 7 (2014) 1002–1021.
710 [9] H. Liao, Z. Xu, X.J. Zeng, J.M. Merigó, Qualitative decision making with
711 correlation coefficients of hesitant fuzzy linguistic term sets, *Knowl. Based*
712 *Syst.* 76 (2015) 127–138.
713 [10] H. Liao, Z. Xu, Approaches to manage hesitant fuzzy linguistic information
714 based on the cosine distance and similarity measures for HFLTSs and their

- application in qualitative decision making, *Expert Syst. Appl.* 42 (2015) 5328–5336.
- [11] J.Q. Wang, J. Wang, Q.H. Chen, H.Y. Zhang, X.H. Chen, An outranking approach for multi-criteria decision-making with hesitant fuzzy linguistic term sets, *Inf. Sci.* 280 (2014) 338–351.
- [12] B. Zhu, Z. Xu, Consistency measures for hesitant fuzzy linguistic preference relations, *IEEE Trans. Fuzzy Syst.* 22 (2014) 35–45.
- [13] N. Zhang, Hesitant fuzzy linguistic information aggregation in decision making, *Int. J. Oper. Res.* 21 (2014) 489–507.
- [14] Z. Zhang, C. Wu, Hesitant fuzzy linguistic aggregation operators and their applications to multiple attribute group decision making, *J. Intell. Fuzzy Syst.* 26 (2014) 2185–2202.
- [15] R.M. Rodríguez, H. Liu, L.A. Martínez, Fuzzy representation for the semantics of hesitant fuzzy linguistic term sets, *Adv. Intell. Syst. Comput.* 277 (2014) 745–757.
- [16] M. Yavuz, B. Oztaysi, S. Çevik Onar, C. Kahraman, Multi-criteria evaluation of alternative-fuel vehicles via a hierarchical hesitant fuzzy linguistic model, *Expert Syst. Appl.* 42 (2015) 2835–2848.
- [17] Y.H. Tseng, C.T. Lin, Enhancing enterprise agility by deploying agile drivers, capabilities and providers, *Inf. Sci.* 181 (2011) 3693–3708.
- [18] L.K. Chan, M.L. Wu, Quality function deployment: a literature review, *Eur. J. Oper. Res.* (2002) 463–497.
- [19] Y. Akao, G.H. Mazur, The leading edge in QFD: Past, present and future, *Int. J. Qual. Reliab. Manag.* 20 (2003) 20–35.
- [20] J.A. Carnevalli, P.C. Miguel, Review, analysis and classification of the literature on QFD-Types of research, difficulties and benefits, *Int. J. Prod. Econ.* 114 (2008) 737–754.
- [21] L.P. Khoo, N.C. Ho, Framework of a fuzzy quality function deployment system, *Int. J. Prod. Res.* 34 (1996) 299–311.
- [22] X.X. Shen, K.C. Tan, M. Xie, The implementation of quality function deployment based on linguistic data, *J. Intell. Manuf.* 12 (2001) 65–75.
- [23] J. Wang, Fuzzy outranking approach to prioritize design requirements in quality function deployment, *Int. J. Prod. Res.* 37 (1999) 899–916.
- [24] L.K. Chan, H.P. Kao, A. Ng, M.L. Wu, Rating the importance of customer needs in quality function deployment by fuzzy and entropy methods, *Int. J. Prod. Res.* 37 (1999) 2499–2518.
- [25] G. Büyüközkan, G. Çifçi, A new incomplete preference relations based approach to quality function deployment, *Inf. Sci.* 206 (2012) 30–41.
- [26] G. Büyüközkan, G. Çifçi, An integrated QFD framework with multiple formatted and incomplete preferences: a sustainable supply chain application, *Appl. Soft Comput.* 13 (2013) 3931–3941.
- [27] G. Büyüközkan, O. Fezizioğlu, Group decision making to better respond customer needs in software development, *Comput. Ind. Eng.* 48 (2005) 2005.
- [28] G. Büyüközkan, O. Fezizioğlu, D. Ruan, Fuzzy group decision making to multiple preference formats in quality function deployment, *Comput. Ind. Eng.* 58 (2007) 392–402.
- [29] C.Y. Tsai, Using fuzzy QFD to enhance manufacturing strategic planning, *J. Chin. Inst. Ind. Eng.* 18 (2003) 33–41.
- [30] O. Fezizioğlu, G. Büyüközkan, An integrated group decision-making approach for new product development, *Int. J. Comput. Integr. Manuf.* 21 (2008) 366–375.
- [31] T. Ertay, G. Büyüközkan, C. Kahraman, D. Ruan, Quality function deployment implementation based on analytic network process with linguistic data: an application in automotive industry, *J. Fuzzy Intell. Syst.* 16 (2005) 221–232.
- [32] C. Kahraman, T. Ertay, G. Büyüközkan, A fuzzy optimization model for QFD planning process using analytic network approach, *Eur. J. Oper. Res.* 171 (2006) 390–411.
- [33] M. Celik, S. Cebi, C. Kahraman, I.D. Er, An integrated fuzzy QFD model proposal on routing of shipping investment decisions in crude oil tanker market, *Expert Syst. Appl.* 36 (2009) 6227–6235.
- [34] Z. Ayag, F. Samanlıoğlu, G. Büyüközkan, A fuzzy QFD approach to determine supply chain management strategies in the dairy industry, *J. Intell. Manuf.* 24 (2013) 1111–1122.
- [35] Y.L. Li, J.F. Tang, K.S. Chin, Y. Han, X.G. Luo, A rough set approach for estimating correlation measures in quality function deployment, *Inf. Sci.* 189 (2012) 126–142.
- [36] Y.L. Li, J.F. Tang, K.S. Chin, X.G. Luo, Y. Han, Rough set-based approach for modeling relationship measures in product planning, *Inf. Sci.* 193 (2012) 199–217.
- [37] C.K. Kwong, Y. Chen, H. Bai, D.S.K. Chan, A methodology of determining aggregated importance of engineering characteristics in QFD, *Comput. Ind. Eng.* 53 (2007) 667–679.
- [38] M. Abdolshah, M. Morad, Fuzzy quality function deployment: an analytical literature review, *J. Ind. Eng.* (2013), Article ID 682532 11.
- [39] M. Li, The extension of quality function deployment based on 2-tuple linguistic representation model for product design under multigranularity linguistic environment, *Math. Probl. Eng.* (2012), Article ID 989284 18.
- [40] W.-C. Ko, Exploiting 2-tuple linguistic representational model for constructing HOQ-based failure modes and effects analysis, *Comput. Ind. Eng.* 64 (2013) 858–865.
- [41] Q. Li, X. Zhao, G. Wei, Model for software quality evaluation with hesitant fuzzy uncertain linguistic information, *J. Intell. Fuzzy Syst.* 26 (2014) 2639–2647. Q9
- [42] E.E. Karsak, M. Dursun, An integrated fuzzy MCDM approach for supplier evaluation and selection, *Comput. Ind. Eng.* 82 (2015) 82–93.
- [43] S. Pushpinder, A new method for solving dual hesitant fuzzy assignment problems with restrictions based on similarity measure, *Appl. Soft Comput.* 24 (2014) 559–571.
- [44] P. Quiros, P. Alonso, H. Bustince, I. Díaz, S. Montes, An entropy measure definition for finite interval-valued hesitant fuzzy sets, *Knowl. Based Syst.* 84 (2015) 121–133.
- [45] G. Qian, H. Wang, X. Feng, Generalized hesitant fuzzy sets and their application in decision support system, *Knowl. Based Syst.* 37 (2013) 357–365.
- [46] D. Yu, Triangular hesitant fuzzy set and its application to teaching quality evaluation, *J. Inf. Comput. Sci.* 10 (2013) 1925–1934.
- [47] S.-M. Chen, J.-A. Hong, Multicriteria linguistic decision making based on hesitant fuzzy linguistic term sets and the aggregation of fuzzy sets, *Inf. Sci.* 286 (2014) 63–74.
- [48] L.W. Lee, S.-M. Chen, Fuzzy decision making based on likelihood-based comparison relations of hesitant fuzzy linguistic term sets and hesitant fuzzy linguistic operators, *Inf. Sci.* 294 (2015) 513–529.
- [49] J. Wang, J.Q. Wang, H.Y. Zhang, X.H. Chen, Multi-criteria decision-making based on hesitant fuzzy linguistic term sets: an outranking approach, *Knowl. Based Syst.* 86 (2015) 224–236 (in press). Q10
- [50] H. Liu, R.M. Rodríguez, A fuzzy envelope for hesitant fuzzy linguistic term set and its application to multicriteria decision making, *Inf. Sci.* 258 (2014) 220–238.
- [51] D. Filev, R.R. Yager, On the issue of obtaining OWA operator weights, *Fuzzy Sets Syst.* 94 (1998) 157–169.