



## On-line constructive fuzzy sliding-mode control for voice coil motors

Chun-Fei Hsu\*, Kai-Yi Wong

*Department of Electrical Engineering, Tamkang University, No. 151, Yingzhuhan Rd., Tamsui Dist., New Taipei City 25137, Taiwan*

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### ABSTRACT

In this paper, a voice coil motor (VCM) featuring fast dynamic performance and high position repeatability is developed. To achieve robust VCM control performance under different operating conditions, an on-line constructive fuzzy sliding-mode control (OCFSC) system, which comprises of a main controller and an exponential compensator, is proposed. In the main controller, a fuzzy observer is used to online approximate the unknown nonlinear term in the system dynamics with on-line structure learning and parameter learning using a gradient descent algorithm. According to the structure learning mechanism, the fuzzy observer can either increase or decrease the number of fuzzy rules based on tracking performance. The exponential compensator is applied to ensure the system stability with a nonlinear exponential reaching law. Thus, the chattering signal can be alleviated and the convergence of tracking error can be speed up. Finally, the experimental results show that not only the OCFSC system can achieve good position tracking accuracy but also the structure learning ability enables the fuzzy observer to evolve its structure on-line.

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### 1. Introduction

The high-precision quick-response linear motion is often required in modern testing systems. If the linear motion is realized using the rotary motors with a mechanical transmission, the mechanical transmission will introduce the backlash and large friction. A voice coil motor (VCM) for automated manufacturing processes is a direct drive motor that uses a permanent magnetic field and coil winding to produce a force proportional to the current applied to the coil winding [1,2]. The fixed part is permanent magnet field and the moving part is coil winding. Because of the characteristics of small size and high-response speed, VCM has been widely used in high-frequency response DDV drives or auto-focus modules of digital cameras [3]. Recently, several control methods for VCMs have been proposed [4–9].

Oboe et al. proposed a PI control with its simple design [1]. Since the control gains of PI control are fixed, the control performance is influenced by the system uncertainties, friction, and load changes of movable parts. Yu et al. proposed an adaptive fuzzy logic PID control to achieve a short response time with fuzzy tuning the control gains [4]. The fuzzy rules should be pre-constructed to achieve the design performance by trial-and-error; however,

this trial-and-error tuning procedure is time-consuming. A fuzzy sliding-mode controller was designed to achieve favorable position control performance [5]; however, it has difficulty in determining suitable membership functions and fuzzy rules. Wu et al. proposed a sliding-mode control system with a state-space observer to improve the control accuracy and robustness [6]; however, the control signal results in the chattering phenomena. Recently, several neural-network-based intelligent control systems had been proposed for VCM control [7–9]. The developed adaptive tuning law only considers the on-line parameter learning of neural networks but doesn't consider the structure adjustment of neural networks. It implies that tracking accuracy would be not satisfied if inadequate hidden nodes of neural network, i.e. too many or too few, are predefined.

Whether a control system is robust against system uncertainties is an important issue for controller performance assessment. It is known that sliding mode control (SMC) system is an effective control approach due to its excellent advantage of strong robustness against model uncertainties, parameter variations and external disturbances [10,11]. However, the chattering problem is one of the most critical handicaps for applying the SMC system to real applications. On the other hand, based on the fuzzy rules base of fuzzy controller having skew-symmetric property, a fuzzy sliding-mode controller (FSMC) system is proposed to offer a significant reduction in rule inferences and simplify the tuning of control parameter [12–14]. Since only one variable (sliding surface) is defined as the input variable for fuzzy rules, the number of the fuzzy rules for

\* Corresponding author.

E-mail addresses: [fei@ee.tku.edu.tw](mailto:fei@ee.tku.edu.tw) (C.-F. Hsu), [\(K.-Y. Wong\).](mailto:400500111@s00.tku.edu.tw)

FSMC is smaller than that for fuzzy control which uses the error and change of error as the input variables. However, obtaining appropriate membership functions and fuzzy rules for the design of FSMC system remains a challenge.

To tune the control parameters including the membership functions and fuzzy rules, some researchers focused on the adaptive fuzzy control (AFC) schemes with on-line parameter learning [15–18]. The AFC systems can tune its controller parameters to achieve favorable performance by designing parameter adaptation laws rather than requiring expert experience. Additionally, by combining the advantages of FSMC system and AFC system, some researchers proposed the adaptive fuzzy sliding-mode control (AFSMC) design methods [19,20]. Thus, not only the controller parameters of AFSMC system can be on-line tuned to achieve favorable performance but also the total number of fuzzy rules in AFSMC systems is greatly reduced compared to existing AFC systems.

Though the control performance in [15–20] is acceptable after controller parameters learning, the number of fuzzy rules should be determined by trial-and-error. It is not an easy task to determine an appropriate number of fuzzy rules. To overcome this problem, some researchers focused on the learning which consists of both structure learning and parameter learning [21–27]. The structure learning is responsible for on-line rule generation and the parameter learning is designed to achieve favorable learning performance. Thus, the number of fuzzy rules can dynamically vary to achieve an economical rule size.

This paper proposes an on-line constructive fuzzy sliding-mode control (OCFSC) system for a VCM to track a periodical position reference with robust characteristics. A fuzzy observer is utilized to on-line approximate the unknown nonlinear term of VCM system dynamics. The structure learning mechanism not only helps automate fuzzy rule generation but also locates good initial rule positions for parameter learning. All adjustable parameters of the OCFSC system are on-line tuned by the gradient descent learning method. Meanwhile, an exponential compensator is applied to ensure the system stability based on a Lyapunov function. Finally, the OCFSC system is implemented on an ARM Cortex-m4 microcontroller for high-performance industrial applications. The experimental results show that not only the fuzzy observer has the admirable property of small fuzzy rules size but also the OCFSC system can achieve favorable control performance such as good parameter variation rejection and good tracking accuracy. Thus, the proposed OCFSC system with low implementation complexity is suitable for real-time VCM control applications.

## 2. Problem formulation of VCM

VCM possesses the advantages of a simple structure, a fast response, and high accuracy. The dynamic equation, which satisfies the Kirchhoff's voltage law and Newton's second law of motion, can be obtained as follows [1,3]

$$v_a(t) = R_a i_a(t) + K_b \dot{x}(t) + L_a \dot{i}_a(t) \quad (1)$$

$$F_t(t) - F_f(t) = (m + M) \ddot{x}(t) + B \dot{x}(t) \quad (2)$$

where  $x(t)$  is the position of the coil winding,  $v_a(t)$  is the input voltage,  $R_a$  is the coil resistance,  $i_a(t)$  is the coil current,  $K_b$  is the back electromotive force coefficient,  $K_t$  is the thrust force coefficient,  $L_a$  is the coil inductance,  $M$  is the mass of the coil winding,  $m$  is the mass of the payload,  $B$  is the viscous coefficient,  $F_f(t)$  is the lumped friction force, and  $F_t(t) = K_t i_a(t)$  is the thrust force. Due to the term  $L_a$  can be neglected; the dynamic equation of VCM represents as

$$\ddot{x}(t) = f(t) + g(t)u(t) + d(t) \quad (3)$$

where  $f(t) = \frac{-(K_t K_b + R_a B)}{(m+M)R_a} \dot{x}(t)$  is the system dynamic,  $g(t) = K_t / (m + M)R_a$  is the control gain,  $d(t) = (-F_f(t)) / m + M$  is the external dis-

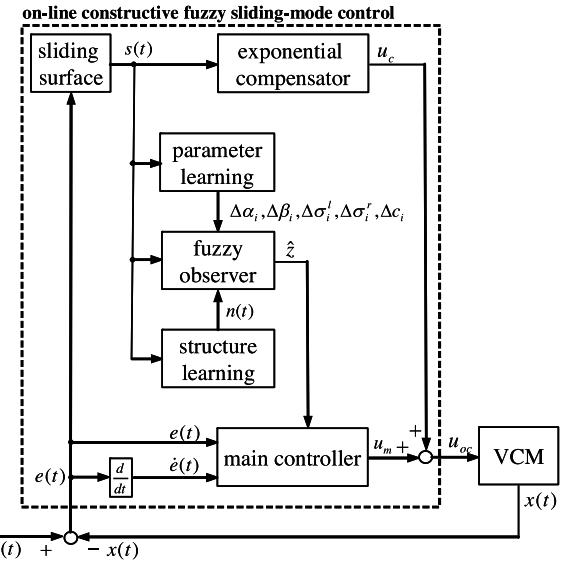


Fig. 1. The OCFSC system for a VCM.

turbance, and  $u(t) = v_a(t)$  is the input voltage. The control objective is to design the position of coil winding  $x(t)$  can track the position command  $x_c(t)$ . Define a tracking error as

$$e(t) = x_c(t) - x(t) \quad (4)$$

Substituting (4) into (3) yields

$$\ddot{e}(t) = z(t) - u(t) \quad (5)$$

where the nonlinear term  $z(t)$  is defined as  $z(t) = \dot{x}_c(t) - (1 - (1/g(t))\ddot{x}(t) - (f(t) + d(t))/g(t))$ . Assuming that all the parameters in the nonlinear term  $z(t)$  are known, an ideal controller is assumed to take the following form [28]

$$u^*(t) = z(t) + k_1 \dot{e}(t) + k_2 e(t) \quad (6)$$

where  $k_1$  and  $k_2$  are positive constants. Imposing the control law  $u(t) = u^*(t)$  upon (5), it follows that

$$\ddot{e}(t) + k_1 \dot{e}(t) + k_2 e(t) = 0 \quad (7)$$

It can be seen that (7) is a Hurwitz polynomial. Thereby, it confirms that the tracking error  $e(t)$  converges to zero asymptotically for any initial condition [28]. Because of the nonlinear term  $z(t)$  is unknown or perturbed, the ideal controller cannot be implemented in real-time VCM control.

## 3. OCFSC system design

This paper proposes an OCFSC system as shown in Fig. 1, i.e.

$$u_{oc}(t) = u_m(t) + u_c(t) = \hat{z} + k_1 \dot{e}(t) + k_2 e(t) + u_c(t) \quad (8)$$

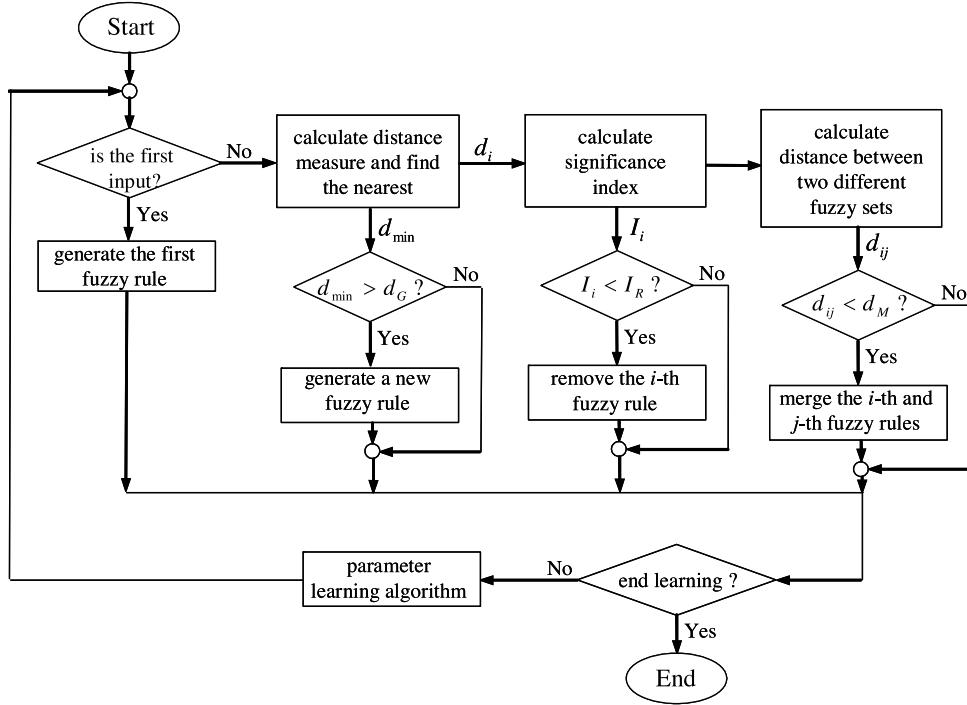
where  $u_m(t)$  is the main controller,  $u_c(t)$  is the exponential compensator, the fuzzy observer  $\hat{z}$  is designed to approximate the unknown nonlinear term  $z(t)$ . A sliding surface, which includes an additional integral term, is defined as

$$s(t) = \dot{e}(t) + k_1 e(t) + k_2 \int_0^t e(\tau) d\tau \quad (9)$$

Assuming that there are  $n(t)$  fuzzy rules in the fuzzy observer at time  $t$ , each fuzzy rule is described as

$$\text{Rule } i : \text{IF } s(t) \text{ is } F_i, \text{ THEN } \hat{z} \text{ is } \alpha_i s(t) + \beta_i \quad (10)$$

where  $F_i$  represents fuzzy sets of  $s(t)$  and  $z_i = \alpha_i s(t) + \beta_i$  is the Takagi-Sugeno-Kang (TSK) type consequent part that is learned

**Fig. 2.** Flowchart of the on-line structure learning.

from the parameter learning. In this paper, the fuzzy set  $F_i$  selects an asymmetric Gaussian function to upgrade the learning capability as follow [29,30]

$$\phi_i = \begin{cases} \exp\left(-\frac{(s(t) - c_i)^2}{(\sigma_i^l)^2}\right), & \text{for } -\infty < s(t) \leq c_i \\ \exp\left(-\frac{(s(t) - c_i)^2}{(\sigma_i^r)^2}\right), & \text{for } c_i < s(t) < \infty \end{cases} \quad (11)$$

where  $\sigma_i^l$  is the left-sided deviation,  $\sigma_i^r$  is the right-sided deviation and  $c_i$  is the mean of the asymmetric Gaussian function in the  $i$ -th fuzzy rule, respectively. The fuzzy observer output is obtained as

$$\hat{z} = \sum_{i=1}^{n(t)} z_i \phi_i \quad (12)$$

**Remark 1.** There are two types of AFC systems, namely direct and indirect. Only one fuzzy system is used under the direct scheme but two fuzzy systems are required under the indirect one. The proposed OCFSC system is of the indirect type, but it is worth noting that we need only one fuzzy system to on-line estimate the unknown nonlinear term  $z(t)$ . Thus, the OCFSC system admits less computation complexity compared with traditional indirect AFC systems.

### 3.1. On-line structure learning

There is no fuzzy rule in the fuzzy observer initially. For the first input data  $s(0)$ , the parameters of the initial fuzzy rule is given as

$$\alpha_1 = \beta_1 = 0 \quad (13)$$

$$\sigma_1^l = \sigma_1^r = \bar{\sigma} \quad (14)$$

$$c_1 = s(0) \quad (15)$$

where  $\bar{\sigma}$  is a parameter specified by designers. For each subsequent piece of input data  $s(t)$ , the fuzzy observer will not generate a new

fuzzy rule but update the parameters of existing fuzzy rules if input data  $s(t)$  falls within the existing fuzzy set. Define the distance measure between the new input data and the existing membership function centers as

$$d_i = |s(t) - c_i|, \quad \text{for } i = 1, 2, \dots, n(t) \quad (16)$$

Find the nearest distance measure defines as

$$d_{\min} = \min_{1 \leq i \leq n(t)} d_i \quad (17)$$

If  $d_{\min} > d_G$  is satisfied, where  $d_G$  a pre-given threshold, it means that there does not exist any fuzzy rule near the new input data. It needs to generate a new fuzzy rule according to the current input data  $s(t)$ . The new fuzzy rule is generated with initial parameters as follows

$$\alpha_{n(t)+1} = \beta_{n(t)+1} = 0 \quad (18)$$

$$\sigma_{n(t)+1}^l = \sigma_{n(t)+1}^r = \bar{\sigma} \quad (19)$$

$$c_{n(t)+1} = s(t) \quad (20)$$

To reduce the number of fuzzy rules, a significance index of the  $i$ -th fuzzy rule is determined as

$$I_i(t+1) = \begin{cases} I_i(t) \exp(-\tau_1), & \text{if } d_i > d_{th} \\ I_i(t), & \text{if } d_i = d_{th} \\ I_i(t) [2 - \exp(-\tau_2 (1 - I_i(t)))], & \text{if } d_i < d_{th} \end{cases} \quad (21)$$

where  $\tau_1$  and  $\tau_2$  are the designed constant. The initial value of significance index is set to 1 for each fuzzy rule. If  $I_i \leq I_R$  is satisfied, where  $I_R$  a pre-given threshold, then the  $i$ -th fuzzy rule will be removed. Further, two fuzzy rules should be merged into a new fuzzy rule if they are similar to each other. A distance between two different fuzzy sets is defined as

$$d_{ij} = |c_i - c_j|, \quad \text{for } i \neq j \quad (22)$$

If the value of the distance  $d_{ij}$  is too small, this means the two fuzzy sets are closed to each other. It implies that if  $d_{ij} \leq d_M$  is satis-

**Table 1**  
Characteristic comparison.

Controller	Stability Proof	Parameter Learning	Structure Learning	Robustness
PD control	No	No	No	good
FSMC system	No	No	No	middle
AWFC system	Yes	Yes	No	excellent
OCFSC system without exponential compensator	No	Yes	Yes	excellent
OCFSC system with exponential compensator	Yes	Yes	Yes	excellent

fied, where  $d_M$  is a pre-given threshold, the  $i$ -th and  $j$ -th fuzzy rules should be merged. In this case, the parameters of the new fuzzy rules are given as

$$\alpha_{new} = \alpha_i + \alpha_j \quad (23)$$

$$\beta_{new} = \beta_i + \beta_j \quad (24)$$

$$\sigma_{new}^l = \max(\sigma_i^l, \sigma_j^l) \quad (25)$$

$$\sigma_{new}^r = \max(\sigma_i^r, \sigma_j^r) \quad (26)$$

$$c_{new} = \frac{c_i + c_j}{2} \quad (27)$$

The flowchart of the structure learning is shown in Fig. 2. The developed structure learning not only helps automate fuzzy rule generation but also locates good initial rule positions for parameter learning.

**Remark 2.** On-line constructive fuzzy system has been widely adopted for the control of complex dynamic systems owing to its good generalization capability and structure adaptation. Most studies don't consider that two or more similar fuzzy sets with high similarity degree can be merged. This paper proposes a structuring learning mechanism for generation, removal and merging of fuzzy rules with simple computation.

### 3.2. On-line parameter learning

The parameter learning phase occurs concurrently with the structure learning. Imposing the control law  $u = u_{oc}$  into (5) and using (9), it is obtained that

$$\ddot{e} + k_1\dot{e} + k_2e = z(t) - \hat{z} - u_c = \dot{s}(t) \quad (28)$$

Multiplying both sides of (28) by  $s(t)$  gives

$$s(t)\dot{s}(t) = s(t)(z(t) - \hat{z} - u_c) = s(t) \left( z(t) - \sum_{i=1}^{n(t)} z_i \phi_i - u_c \right) \quad (29)$$

The parameter learning algorithm is derived to minimize the learning performance  $s(t)\dot{s}(t)$  for achieving fast convergence of  $s(t)$ . The parameter tuning laws  $\Delta\alpha_i$  and  $\Delta\beta_i$  according to the gradient descent method are given by [31–33]

$$\Delta\alpha_i = -\eta_\alpha \frac{\partial s(t)\dot{s}(t)}{\partial \alpha_i} = -\eta_\alpha \frac{\partial s(t)\dot{s}(t)}{\partial z_i} \frac{\partial z_i}{\partial \alpha_i} = \eta_\alpha s(t)^2 \phi_i \quad (30)$$

$$\Delta\beta_i = -\eta_\beta \frac{\partial s(t)\dot{s}(t)}{\partial \beta_i} = -\eta_\beta \frac{\partial s(t)\dot{s}(t)}{\partial z_i} \frac{\partial z_i}{\partial \beta_i} = \eta_\beta s(t) \phi_i \quad (31)$$

where  $\eta_\alpha$  and  $\eta_\beta$  are positive learning rates. The fuzzy rules is updated according to the following equation

$$\alpha_i(t+1) = \alpha_i(t) + \Delta\alpha_i \quad (32)$$

$$\beta_i(t+1) = \beta_i(t) + \Delta\beta_i \quad (33)$$

Only tune the parameters of consequence part of the fuzzy rules will inevitably limit the approximation ability of the fuzzy observer. For improving the generalization capability of fuzzy observer, the

full-tuned parameter learning laws  $\Delta\sigma_i^l$ ,  $\Delta\sigma_i^r$  and  $\Delta c_i$  can be obtained as [31–33]

$$\Delta\sigma_i^l = -\eta_l \frac{\partial s(t)\dot{s}(t)}{\partial \sigma_i^l} = -\eta_l \frac{\partial s(t)\dot{s}(t)}{\partial \phi_i} \frac{\partial \phi_i}{\partial \sigma_i^l} = \eta_l s(t) \frac{2(s(t) - c_i)^2}{(\sigma_i^l)^3} \phi_i \quad (34)$$

$$\Delta\sigma_i^r = -\eta_r \frac{\partial s(t)\dot{s}(t)}{\partial \sigma_i^r} = -\eta_r \frac{\partial s(t)\dot{s}(t)}{\partial \phi_i} \frac{\partial \phi_i}{\partial \sigma_i^r} = \eta_r s(t) \frac{2(s(t) - c_i)^2}{(\sigma_i^r)^3} \phi_i \quad (35)$$

$$\Delta c_i = -\eta_c \frac{\partial s(t)\dot{s}(t)}{\partial c_i} = -\eta_c \frac{\partial s(t)\dot{s}(t)}{\partial \phi_i} \frac{\partial \phi_i}{\partial c_i} = \begin{cases} \eta_c s(t) \frac{2(s(t) - c_i)}{(\sigma_i^l)^2} \phi_i, & \text{for } -\infty < s(t) \leq c_i \\ \eta_c s(t) \frac{2(s(t) - c_i)}{(\sigma_i^r)^2} \phi_i, & \text{for } c_i < s(t) < \infty \end{cases} \quad (36)$$

where  $\eta_l$ ,  $\eta_r$  and  $\eta_c$  are positive learning rates. The asymmetric Gaussian function are updated as follows

$$\sigma_i^l(t+1) = \sigma_i^l(t) + \Delta\sigma_i^l \quad (37)$$

$$\sigma_i^r(t+1) = \sigma_i^r(t) + \Delta\sigma_i^r \quad (38)$$

$$c_i(t+1) = c_i(t) + \Delta c_i \quad (39)$$

### 3.3. Stability analysis

Since the number of fuzzy rules in the fuzzy observer is finite, there exists an approximation error between the system dynamic  $z(t)$  and the fuzzy observer  $\hat{z}$ , i.e. [34]

$$z(t) = \hat{z} + \varepsilon(t) \quad (40)$$

where  $\varepsilon(t)$  denotes the approximation error. It can be bounded by  $0 \leq |\varepsilon(t)| \leq E$  where  $E$  is a positive constant. Substituting (40) into (28) yields

$$\dot{s}(t) = \varepsilon(t) - u_c \quad (41)$$

In this paper, the exponential compensator  $u_c$  is applied as [35,36]

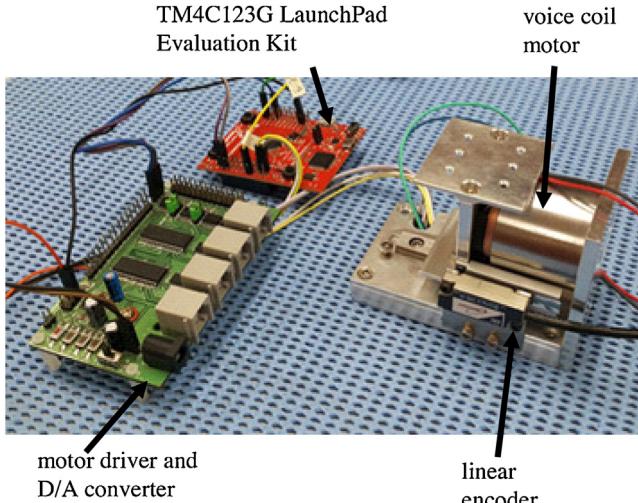
$$u_c = \frac{E}{a + (1-a)e^{-b|s(t)|}} \operatorname{sgn}(s(t)) \quad (42)$$

where  $a$  is a strictly positive constant that is less than one and  $b$  is a strictly positive integer. We can see that if  $|s(t)|$  increases, the term  $(E/a + (1-a)e^{-b|s(t)|})$  converges to  $E/a$  which is greater than  $E$ . It implies that the exponential compensator increases the control gain and the attraction of the sliding surface will be faster. Therefore, the exponential compensator can dynamically vary its control gain between  $E$  and  $E/a$  to speed up the convergence speed. Imposing (42) into (41), we can obtain that

$$\dot{s}(t) = \varepsilon(t) - \frac{E}{a + (1-a)e^{-b|s(t)|}} \operatorname{sgn}(s(t)) \quad (43)$$

To guarantee the stability of the OCFSC system, consider a Lyapunov function candidate in the following form as

$$V(t) = \frac{1}{2}s(t)^2 \quad (44)$$



**Fig. 3.** Experimental setup.

Differentiating (44) with respect to time and using (43), we have that

$$\begin{aligned} \dot{V}(t) &= s(t)\dot{s}(t) = \varepsilon(t)s(t) - \frac{E}{a + (1-a)e^{-b|s(t)|}}|s(t)| \leq |\varepsilon(t)||s(t)| - \frac{E}{a + (1-a)e^{-b|s(t)|}}|s(t)| \\ &\leq |\varepsilon(t)||s(t)| - E|s(t)| \leq -(E - |\varepsilon(t)|)|s(t)| \end{aligned} \quad (45)$$

Since  $\dot{V}_1(t)$  is negative semidefinite, that is  $V_1(t) \leq V_1(0)$ , it implies that  $s(t)$  is bounded. Let function  $\mathcal{E}(t) = (E - |\varepsilon(t)|)|s(t)| \leq -\dot{V}_1(t)$ , and integrate  $\mathcal{E}(t)$  with respect to time, then it is obtained that

$$\int_0^t \mathcal{E}(\tau) d\tau \leq V_1(0) - V_1(t) \quad (46)$$

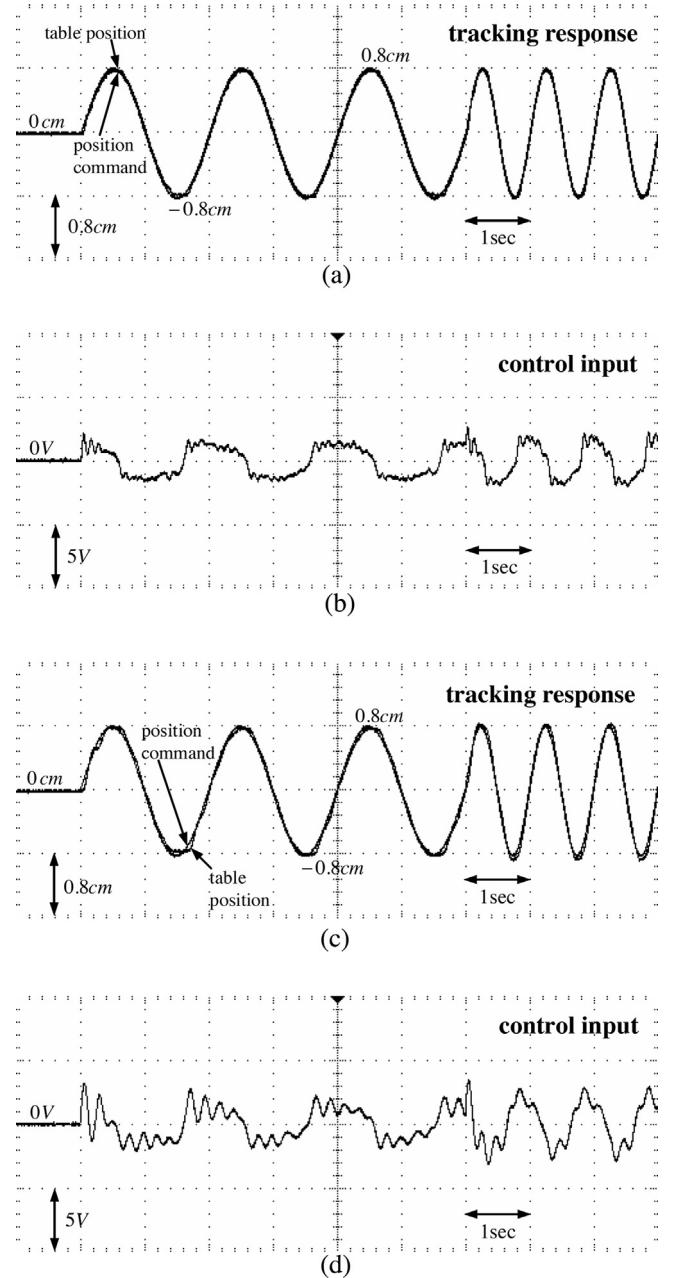
Because  $V_1(0)$  is bounded and  $V_1(t)$  is nonincreasing and bounded, the following result obtains

$$\lim_{t \rightarrow \infty} \int_0^t \mathcal{E}(\tau) d\tau < \infty \quad (47)$$

By Barbalat's Lemma,  $\lim_{t \rightarrow \infty} \mathcal{E}(t) = 0$ , that is  $s(t) \rightarrow 0$  as  $t \rightarrow \infty$ . According to Lyapunov theorem [28], the stability of the OCFSC scheme for a VCM can be guaranteed if the condition of  $|\varepsilon(t)| \leq E$  can be satisfied.

#### 4. Experimental results

A hardware experimental setup for real-time VCM control via a microcontroller approach is shown in Fig. 3. The TM4C123G LaunchPad Evaluation Kit is a low-cost evaluation platform for ARM® Cortex™-M4F-based microcontrollers from Texas Instruments [37]. The LaunchPad includes a M4C123GH6PMI microcontroller with floating point unit, Pulse Width Modulation module, Quadrature Encoder Interface module, Serial Peripheral Interface module and etc. On the software side, we use Code Composer Studio which is an integrated development environment for all of Texas Instruments embedded processor families. It includes an optimizing C/C++ compiler, source code editor, project build environment, debugger, profiler, and many other features [37]. To investigate the effectiveness of the proposed control system, two conditions are tested here. One is the nominal condition and the other is the payload condition by adding a 0.36 kg weigh payload on the coil winding. A comparison among the PD control [1], the FMSMC system [5], the adaptive wavelet fuzzy CMAC (AWFC) sys-

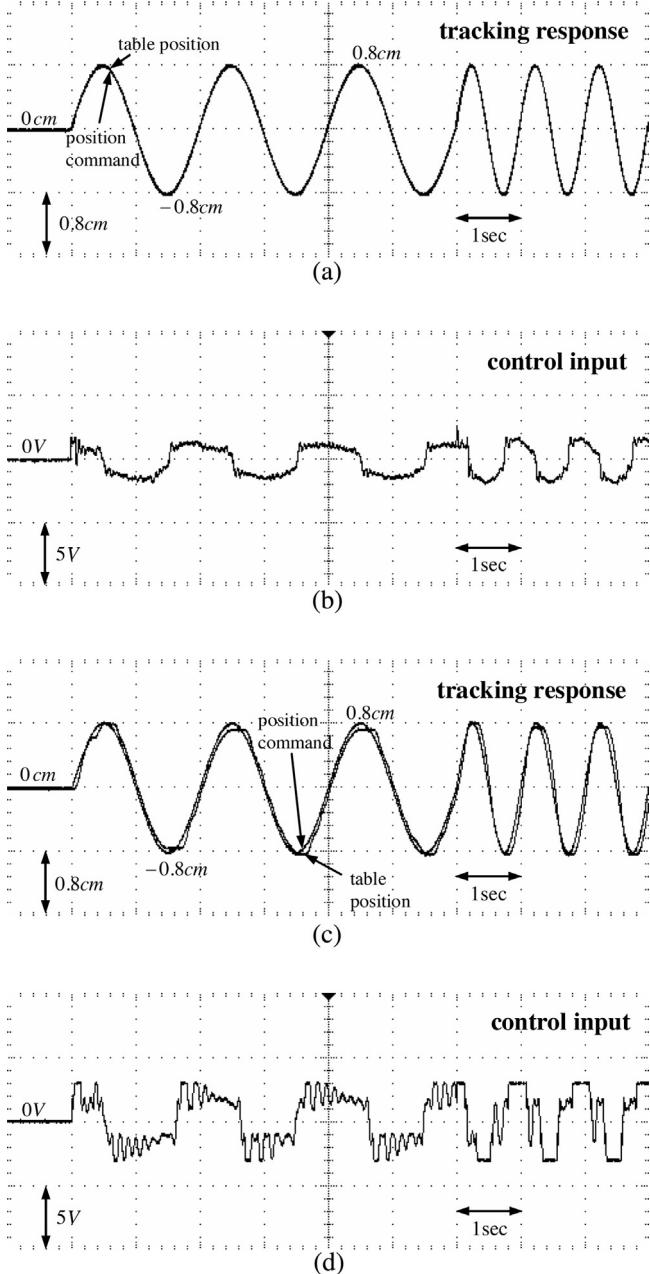


**Fig. 4.** Experimental results of the PD control.

tem [7], and the OCFSC system is made. First, the PD control [1] is applied to the VCM. The PD control is given in the following form

$$u_{pd} = 7.2e + 3.8\dot{e} \quad (48)$$

where the control gains are chosen by trial-and-error. The experimental results of the PD control are shown in Fig. 4. Under the nominal condition, the tracking response  $x(t)$  and the control input  $u(t)$  are shown in Fig. 4(a) and (b), respectively. Meanwhile, under the payload condition, these terms are shown in Fig. 4(c) and (d), respectively. The experimental results show that favorable control performance can be achieved even under frequency change of the position command. Though the accurate position tracking control performance can be achieve, the control gains should be pre-constructed by a tuning procedure.

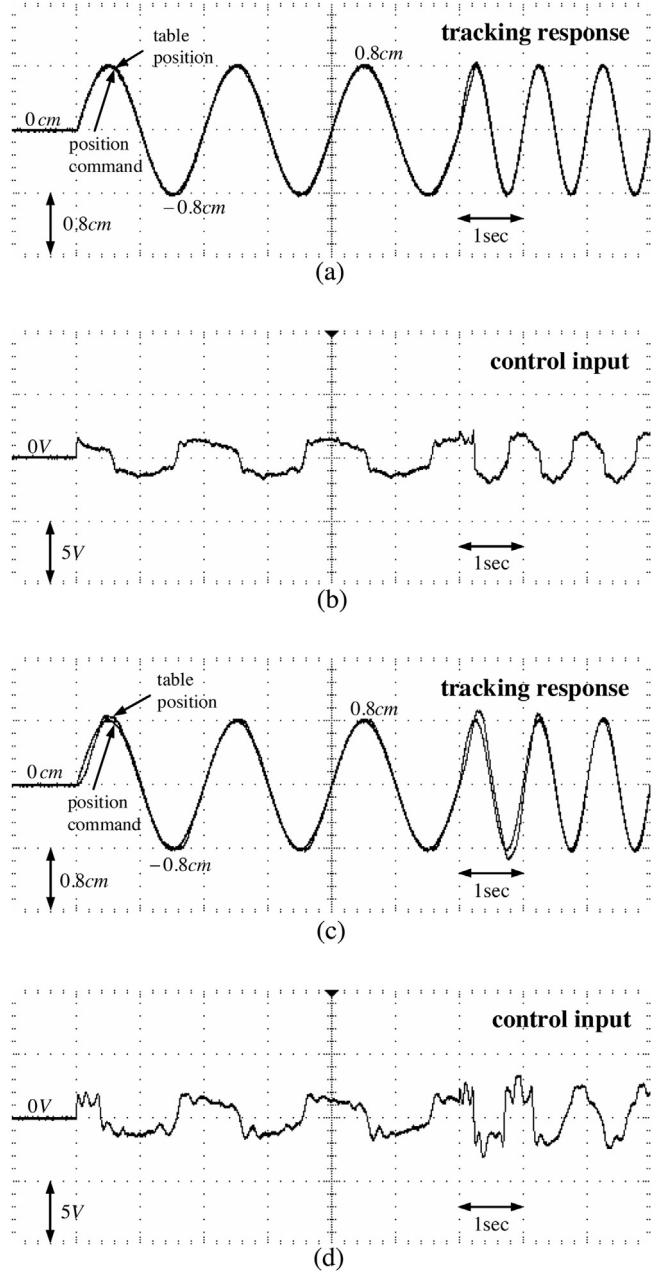


**Fig. 5.** Experimental results of the FMSMC system.

Secondly, the FMSMC system [5] is applied to the VCM again. The fuzzy rules are given in the following form

$$\text{Rule}_i : \text{IF } s(t) \text{ is } F_i, \text{ THEN } u(t) \text{ is } \alpha_i, \quad i = 1, 2, \dots, 7 \quad (49)$$

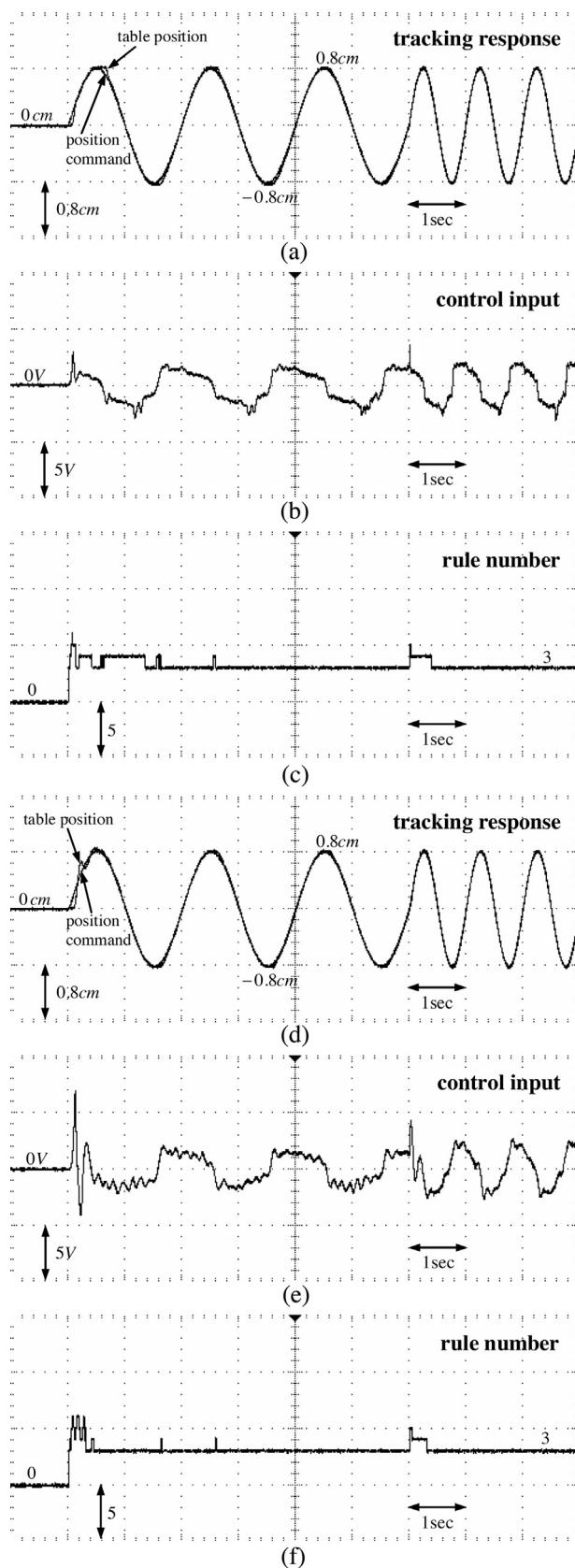
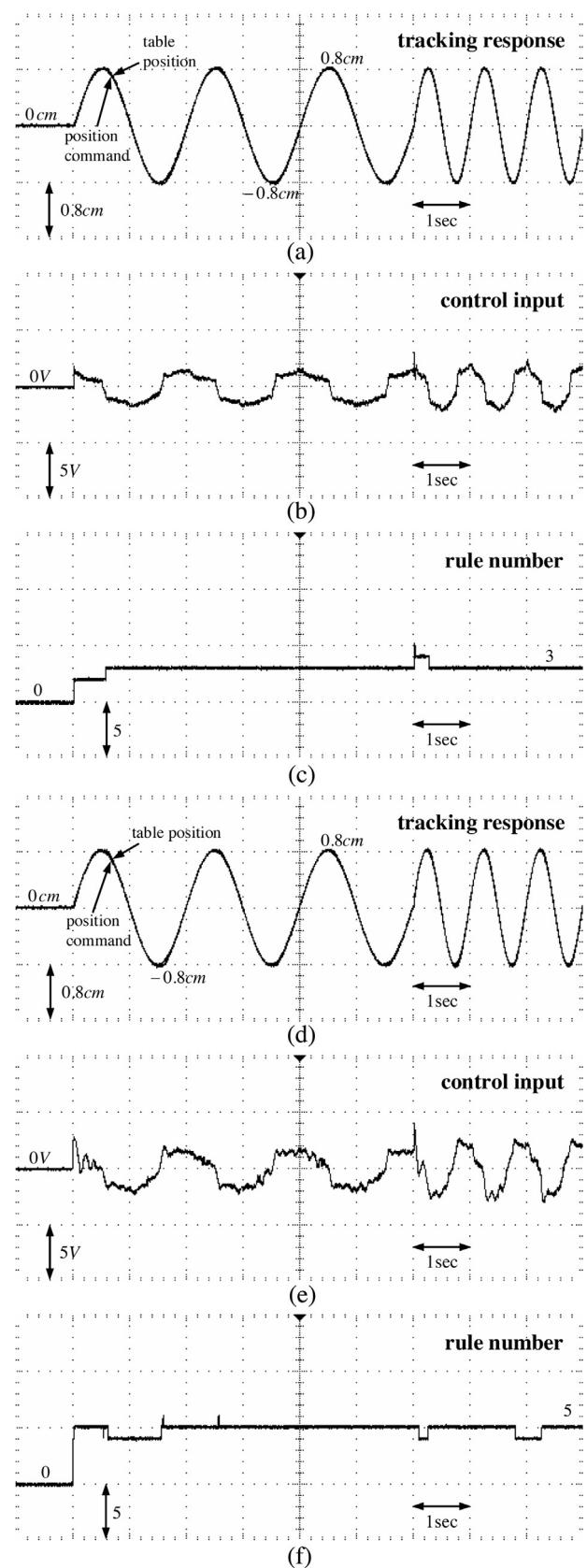
where, in the  $i$ -th rule,  $F_i$  and  $\alpha_i$  are the fuzzy sets of  $s(t)$  and  $u(t)$ , respectively. The fuzzy rules should be constructed using the basic idea that if the state is far away the sliding surface, then a large control input should be applied, and if the state is near the sliding surface then a small control input should be applied. The experimental results of the FMSMC system are shown in Fig. 5. The tracking responses  $x(t)$  are shown in Fig. 5(a) and (c), and the control inputs  $u(t)$  are shown in Fig. 5(a) and (d) for nominal condition and payload condition, respectively. Though the accurate control performance can be achieved under the nominal condition, the tracking performance will be gradually deteriorated under the payload condition. It is difficult to tune a fuzzy rules base that can cope with wide system uncertainties.



**Fig. 6.** Experimental results of the AWFC system.

Next, the AWFC system [7] is applied to the VCM again. A fuzzy CMAC is designed to imitate an ideal controller and a fuzzy compensator is designed to ensure the system stability. The experimental results of the AWFC system are shown in Fig. 6. The tracking responses  $x(t)$  are shown in Fig. 6(a) and (c), and the control inputs  $u(t)$  are shown in Fig. 6(a) and (d) for nominal condition and payload condition, respectively. Though the control performances of the AWFC system are acceptable, the convergence speed of tracking error is slow. Meanwhile, the learning algorithm only considers the parameter learning of the neural network yet it does not consider the structure learning of the neural network. There is a trade off between the control accuracy and the number of hidden neurons. It cannot avoid using too large number of hidden neurons, but the computation loading is heavy.

Finally, the proposed OCFSC system is applied to the VCM again. The learning consists of both structure learning and parameter

**Fig. 7.** Experimental results of the OCFSC system without exponential compensator.**Fig. 8.** Experimental results of the OCFSC system with exponential compensator.

learning. The controller parameters are selected as  $k_1 = 2$ ,  $k_2 = 1$ ,  $\eta_\alpha = 0.01$ ,  $\eta_\beta = 0.001$ ,  $\eta_l = \eta_r = \eta_c = 0.001$ ,  $E = 0.1$ ,  $a = 0.2$ ,  $b = 1$ ,  $\bar{\sigma} = 1$ ,  $d_G = 0.5$ ,  $d_{th} = 1.5$ ,  $\tau_1 = \tau_2 = 0.01$ ,  $I_R = 0.01$  and  $d_M = 0.01$ . These values are chosen through some trials to achieve favorable control performance. To show the effectiveness of the exponential compensator, the experimental results of the OCFSC system without exponential compensator are shown in Fig. 7. Under the nominal condition, the tracking response  $x(t)$ , the control input  $u(t)$  and the number of fuzzy rules  $n(t)$  are shown in Fig. 7(a)–(c), respectively. Meanwhile, under the payload condition, these terms are shown in Fig. 7(d)–(f), respectively. The experimental results show that the OCFSC system without exponential compensator can achieve accurate tracking ability and the fuzzy observer has the admirable property of small fuzzy rules size and high learning accuracy.

Further, the experimental results of the OCFSC system with exponential compensator are shown in Fig. 8. The tracking responses  $x(t)$  are shown in Fig. 8(a) and (d), the control inputs  $u(t)$  are shown in Fig. 8(b) and (e), and the numbers of the fuzzy rule  $n(t)$  are shown in Fig. 8(c) and (f) for nominal condition and payload condition, respectively. Comparing with Fig. 7, it shows that the exponential compensator not only can achieve faster convergence speed of the tracking error but also can improve the dynamic structural adaptation. It is worth noting that the OCFSC system with exponential compensator can be widely adopted for the control of complex dynamic systems owing to its good generalization capability, structure adaptation, and simple computation.

## 5. Conclusions

The purpose of this paper is to develop an OCFSC system to possess high-accuracy position tracking performance for a VCM. A fuzzy observer uses TSK-type fuzzy rules, which is constructed by simultaneous structure and parameter learning, to on-line approximate the unknown nonlinear term of system dynamics. After structure learning, parameters are tuned using the gradient descent algorithm and an exponential compensator is applied to ensure the system stability based on a Lyapunov function. On the basis of the structure and parameter learning, the experimental results are provided to verify the validity of the proposed OCFSC scheme in real-time VCM control applications. Further, a comparison of control characteristics among the PD control, the FSMC system, the AWFC system and the OCFSC system is summarized in Table 1. It shows that the OCFSC system is more suitable to control the VCM since the on-line structure learning and parameter learning are applied.

Further works on the OCFSC system include: (1) consider auto-tuning of the learning rates in the parameter learning law to increase the convergence of the closed-loop system; (2) consider a method such as singular value decomposition [38] to reduce a fuzzy rules after structure and parameter learning.

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