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# Enhanced accuracy of fuzzy time series model using ordered weighted aggregation

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ABSTRACT

Accuracy is one of the most vital factors when dealing with forecast using time series models. Accuracy depends on relative weight of past observations used to predict forecasted value. Method of aggregation of past observations is significant aspect in time series analysis where determination of next observation depends only on past observations. Previous research on fuzzy time series for forecasting treated fuzzy relationship equally important which might not have properly reflected the importance of each individual fuzzy relationship in forecasting that introduced inaccuracy in results. In this paper, we propose ordered weighted aggregation (OWA) for fuzzy time series and further design forecasting model signifying efficacy of the proposed concept. Objective of using fuzzy time series is to deal with forecasting under the fuzzy environment that contains uncertainty, vagueness and imprecision. OWA is utilized to generate weights of past fuzzy observations; thereby eliminating the need for large number of historical observations required to forecast value. OWA weights are determined by employing regularly increasing monotonic (RIM) quantifiers on the basis of fuzzy set importance using priority matrix. Experimental study reveals how OWA coalesced with fuzzy time series for designing of forecasting model. It can be observed from comparative study that use of OWA considerably reduces mean square error (MSE) and average forecasting error rate (AFER). Robustness of proposed model is ascertained by demonstrating its sturdy nature and correctness.



## 1. Introduction

Effectiveness of timely action and adequate preparedness in real time system can be greatly enhanced through forecasting. In past few decades of research and growth, numerous concepts & techniques have been proposed to decipher efficient forecasting. Prediction using time series analysis is one of the oldest and most reliable techniques to prophesy future outcome. Techniques for time series analysis can be broadly divided into two categories: conventional approach also known as statistical techniques and non-conventional approach. Conventional techniques confide on identifying behavior of time series. Box-Jenkins or Auto Regressive Integrated Moving Average (ARIMA), Exponential Smoothing and Multiple Regressions are most widely used statistical methods [41]. These are straightforward and easy to interpret but have several restrictions and drawbacks. Foremost drawback of conventional techniques is inaccuracy of prediction and numerical instability. Due to heavy computational burden, these techniques converge slowly and may diverge in certain cases. Most of them are designed particularly for specific problems without a wide range applicability in other domains. In contrast, non-conventional techniques have been implemented successfully in numerous disciplines. These techniques make fewer assumptions about internal structure of the system and rely on input-output relationships to describe the behavior of time series. Field of time series forecasting is nowvastly different from what it was 20 years ago. It has grown up massively with the advent of greater computing power. More mature soft computing approaches have been proposed to forecast uncertain and vague data. Artificial Neural Network is being used in designing of prediction models due to vast development in the area of artificial intelligence. Garg et al. [1] performed extensive logical survey on implementation of forecasting method using artificial neural network. However, Artificial Neural Network could not generate efficient predictors because of its drawbacks like: (1) it has large training time (2) it can only utilize numerical data pairs (3) It traps in local minima that it deviates from optimal performance. Another soft computing technique which has recently received attention is fuzzy based approach.

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Initial work of Zadeh [2,3] on fuzzy set theory has been applied in several diversified areas. In fuzzy treatment, linguistic values or fuzzy sets are utilized to approximate the desired output rather than numbers. Immense work has been done on forecasting problems using fuzzy time series [4–27]. Primary reason for fuzzy time series popularity is that it can relate trend and cyclic component in fuzzy logical relationship. Hence, it can utilize historical data more effectively. Forecasting using fuzzy time series has been emanated as an intelligent approach in the domain where information is vague and imprecise. Moreover, fuzzy time series can handle situations where neither viewing of trend is possible nor visualization of patterns in time series is handled. Substantial work has been done using fuzzy time series for real time forecasting problems.

Fuzzy time series definitions were proposed by Song and Chissom. Song and Chissom presented the concepts of time variant and time invariant time series [4,5]. It was applied on the time series data of University of Alabama to forecast enrollments. Song and Chissom [6] also proposed an average auto correlation function as a measure of dependency. Chen [7,8] presented simplified arithmetic operations in place of max-min composition operations which were used by Song & Chissom and then, designed high order fuzzy time series forecasted model. Hunrag [9,10], Hwang and Chen [11], Lee Wang and Chen [12], Li and Kozma [13] developed a number of fuzzy forecasting methods with some variations. Singh [14,15] developed forecasting models using computational algorithm. Lee et al. proposed a fuzzy candlestick pattern to improve forecasting results [16]. Then, a multivariate heuristic model was proposed to achieve highly complex matrix computations [17]. Work on determination of length of interval of fuzzy time series was done [18]. [19] performed generalization of forecasting model. Jilani [20] proposed multivariate high order fuzzy time series based forecasting method for car road accidents. Forecasting model based on event discretization function was presented [21] and same was used for forecasting of average length of stay of patient [22]. Subsequently, Garg [22–24] also proposed optimized forecasting models. Garg [25] developed fuzzy based model to forecast number of outpatient visits in hospital.

List of significant improvements of our proposed model over aforementioned fuzzy time series models are summarized as: 1) Conventional time series models such as ARIMA, ARCH,GAR CH, etc can be designed only after making some assumption. These models cannot be used to deal with nonlinear relationship. However, proposed model can capture nonlinearity easily. (2) Almost all previous fuzzy time series models considered fuzzy logical relationship equally important. In proposed model, priority matrix is created to define importance of each fuzzy set in fuzzy time series like importance of each criterion in multiple criterion based decision problems. (3) Past forecasting methods utilized differences of time series data as the universe of discourse. However, increasing and decreasing rate of time series cannot be captured from difference alone. Proposed model eliminates subtlety of the universe of discourse by determining percentage change of data. (4)Previous studies did not consider method of aggregation of past observations although it is a significant factor in designing of forecasting models where next prediction depends only on past observations. Henceforth, proposed model employed OWA for aggregation of data.

It has been observed that various studies have been proposed to perform effective aggregation and these have resulted in significant achievements as well. However, no one explored the use of OWA to aggregate fuzzy time series observations. Prime Objective of proposed model is to capitalize the potency of OWA in fuzzy time series based forecasting model to accomplish higher forecasting accuracy.

Application of forecasting exists in almost every domain be it healthcare or meteorological or financial or agricultural or economical. It is also concurred that in every real life situation, irrespective of its domain, there will always be more than one factor influencing the stats. Moreover, mostly these factors are complex and it is very difficult to estimate their impact on the real time data/stats. For this reason only, time series model came into consideration i.e. to predict the future value in comparatively easier but accurate way; in other terms method which reduces the attached complexities of various external factors. Further, research is being done to enhance the forecasting accuracy of time series models and to build models that can be applied in every sphere of life. In this direction only, its applicability is demonstrated on three spheres of life (healthcare, financial and educational). Proposed model can be applied in other areas as well. Outpatient visits application has been selected for detailed demonstration since healthcare is one of the most important and key aspect of our life.

This study is organized into ten sections. Section 1 has introduction on history of fuzzy time series, OWA efficacy. Section 2 is discussion of related work on the evolution of OWA concepts and OWA based fuzzy time series forecasting models. Section 3 highlights the key features of statistical and fuzzy approach of time series analysis, reveals the concept of fuzzy time series and OWA respectively. Section 4 describes the steps for designing of proposed forecasting model in detail. Also, concise algorithm and its computational complexity are discussed. Section 5 demonstrates usage of proposed model in application domain (predicting outpatient visit in hospital) in detail as an experimental study. Section 6 displays the impact of partitioning of intervals. Section 7 presents impact of order of model and type of defuzzification method. Section 8 discusses the impact of considering OWA weights. Section 9 evaluates and compares the results of proposed forecasting model with previous fuzzy time series and conventional forecasting models and reveals its performance. In Section 10, accurateness of the proposed method is tested on TAIEX stock exchange data and enrollment data of University of Alabama. Section 11 checks the robustness of the proposed method. Section 12 has conclusion and future work.

## 2. Related work

Preceding research on fuzzy time series for forecasting problems considered fuzzy relationship equally important which might not have properly reflected the importance of each individual fuzzy relationship in forecasting [42]. Employed the concepts of entropy discretization and a fast Fourier Transform algorithm for designing of forecasting model. An ant colony optimization and auto-regression based fuzzy time series model was given [43]. This model was used to trade the actual data of Taiwan capitalization weighted stock index. Chen et al. [44] presented forecasting model based on fuzzy time series and fuzzy variation group to forecast daily Taiwan stock exchange index (TAIEX). A high order fuzzy time series model was proposed based on adaptive expectation method and entropy based partitioning [45]. This model produced decision rules and an efficient high order fuzzy time series model to forecast internet stock. Sun et al. [46] presented a novel forecasting method based on fuzzy sets and multivariate fuzzy time series to predict stock index future prices. Yu [26] used concept of recurrent relationship to generate forecasting model and recommended that different weights must be assigned to fuzzy relationships. Expectation and Grade-Selection method based on transitional weight were developed for calculating weights [27]. This model designed weight transition matrix on basis of findings of visible and hidden linguistic value. This model was success to a certain extent to reduce forecasting error. However this model required large number of historical data for training process which was too complex to maintain. Yager [30] introduced a class of function to generate weights and concept of ordered weighted aggregation (OWA)operators. A flexible aggregation characteristic of OWA accounted for its wider use in various domains. Subsequently, Yager [31] introduced the concept of smoothing of time series by Ordered Weight Aggregation (OWA) operators. Sufyan [32] applied OWA successfully to aggregate user feedback to enhance web search quality of a document. Sadiq [33] utilized OWA to find out the overall status of environmental systems by taking data aggregations of environmental indices. During the same time, frequency moving average and ordered weighted aggregation based time series model (MA-OWA) was proposed to forecast air quality by daily O3 concentration [34]. This technique utilized OWA operator to aggregate multiple lag periods in single value by use of different situation parameters. Lagrange multipliers were applied to transforms Yager's OWA equation to a polynomial equation [47]. This approach was used to solve constrained optimization problem and to compute the optimal weighting vector. The mapping of OWA dimension n is done by giving the concept of weighting vectors [48]. Two significant characterizing measures were given with respect to weighting vector W of an OWA [49]. These were "Orness" and dispersion of aggregation. A refined OWA algorithm was proposed to determine OWA weights [50]. Different alpha values ranging from 0.5–1.0 were utilized to generate different sets of OWA weights. Another fusion based adaptive network fuzzy time series model was proposed to employ ordered weighted averaging operator(OWA) and to fuse high ordered data into aggregated values of single attributes [51]. Chang [52] developed weighted fuzzy rules based forecasting model by determining the number of each fuzzy relation. This paper has proposed OWA based model to forecast fuzzy time series data; where OWA is utilized to generate weights of past fuzzy observations thereby eliminating the need for large number of historical observations required to forecast a value. In this model, next value is predicted from OWA aggregation of past three fuzzy observations and importance of each fuzzy set is produced by creating priority matrix.

## 3. Review of time series analysis, fuzzy time series and OWA studies

This section has been divided into three subsections. Subsection 3.1 is introduction of time series analysis. It also elucidates the difference between statistical and fuzzy time series analysis. Subsection 3.2 has the review of fuzzy time series studies and subsection 3.3 unearths the concept of OWA operators in detail.

### 3.1. Time series analysis: statistical vs. fuzzy approach

Objective of time series analysis can be stated succinctly as follows: given a sequence up to time t, x(1), x(2)_,. . ., x(t), find the continuation x(t + 1), x(t + 2). . .x(t + 3). Observations can either be spatiotemporal or temporal data that is usually stumbled upon in variety of areas. A distinctive characteristic of time series is that data cannot be generated independently; their diffusion varies with time which is controlled by trend and cyclic components. Time series analysis consists of two steps: (1) building a model that represents a time series, and (2) using the model to predict (forecast) future values. A time series is represented by a mathematical model Y(t) = F(t) + R(t). Here, F(t) represents an ordered or systematic part known as signal component and R(t) presents a random part known as noise component. Nevertheless, observation of these two components cannot be done separately. Generally, stochastic processes $\{R_t, t \in I\}$ are used to deal with the random component of time series that is a random function of time and depends upon the structure of $R_t$. It becomes purely random process if occurring noise is independent of time ($E\{X_t\} = m$). Systematic part has non-random nature that is a deterministic function of time series. Numerous functions like Fourier series, low degree polynomials and periodic functions are used to analyze and study the characteristics of the deterministic part of time series such as trend, cyclic and seasonal component. Structure of $R_t$ classifies time series model in stationary or non-stationary form. Markov process, moving average (MA), autoregressive (AR) and autoregressive moving average (ARMA) processes are used to analyze and handle stationary time series [35]. Box-Jerkin method based on class of model; also called integrated autoregressive moving average (ARIMA) model is used to deal with non-stationary time series [36]. Box-Jenkins extended their research by modifying ARIMA model in general seasonal integrated autoregressive moving average (SARIMA) model. On the other hand, fuzzy time series models have performed designing of time series model under the fuzzy environment. Time series data in fuzzy environment contains uncertainty, vagueness and imprecision, which makes these two approaches act differently in their philosophy. Concepts of stationary and non-stationary time series in statistical time series are being here dealt as time invariant fuzzy time series and time variant fuzzy time series. Further, fuzzy time series analysis deals with fuzzy logical relations in time series data rather than random and non-random functions in case of usual time series analysis. In many real life situations, it is hard to harvest a trend or cycle component in time series observations. Thus, these components can be analyzed by using the fuzzy time series methods.

### 3.2. Fuzzy time series

Fuzzy set theory is an intellectual quest in which the philosophy of mathematics, abstraction and idealization are combined. Fuzzy set theory provides a strict mathematical framework in which vague conceptual phenomena can be studied precisely and rigorously. Fuzzy set theory and fuzzy system applications are applied to build a fuzzy logic interface that comprises of universe of discourse, fuzzification, knowledge based decision making phenomena and defuzzification process. Fuzzy time series is applicable when process is dynamic and historical data is fuzzy sets. Some basic fuzzy concepts are presented in own discourse to make present method self-contained. For this, [1–26] are viewed.

#### 3.2.1. Fuzzy set

Fuzzy set is a pair (A, m) where A is a set and m: A→ |0,1|. For a finite set $A = \{x_1, \ldots, x_n\}$, fuzzy set (A, m) is often denoted by $\{(m(x_1)/x_1), \ldots, (m(x_n)/x_n)\}$. For each, x &1028; A, m(x) is called the grade of membership of x in (A, m). Let x &1028; A, then x is not included in the fuzzy set (A, m) if m(x) = 0, x is fully included if m(x) = 1 and x is called fuzzy member if 0 < m(x) < 1. The set x &1028; A | m(x) > 0 is called the support of (A, m) and the set x &1028; A | m(x) = 0 is called its kernel.

#### 3.2.2. Fuzzy time series

Let Y(t) (t = . ., 0,1,2,. .), subset of real numbers, be the Universe of Discourse on which fuzzy sets $f_j(t)$ are defined. If F(t) is collection of $f_1(t)$, $f_2(t)$,. ., then F(t) is called a fuzzy time-series defined on Y(t).

#### 3.2.3. Fuzzy relationship

Assume that F(t) is a fuzzy time-series and F(t) = F(t − 1) * R(t − 1, t), where R(t − 1, t) is a fuzzy relation and "*" is the max–min composition operator. Then, F(t) is caused by F(t − 1) and it is denoted as "F(t − 1) → F(t)", where F(t − 1) and F(t) are fuzzy sets. Let $F(t-1) = A_i$ and $F(t) = A_j$. Relationship between two consecutive observations, F(t) and F(t − 1), referred to as a fuzzy logical relationship (FLR), can be denoted by $A_i \to A_j$, where $A_i$ is called the left-hand side (LHS) and $A_j$ the right-hand side (RHS) of the FLR. All fuzzy logical relationships can be further grouped together into different fuzzy logical relationship groups according to the same left-hand sides of the fuzzy logical relationship. For example, there are two fuzzy logical relationships with the same left-hand side

($A_i$): $A_i \rightarrow A_{j1}$ and $A_i \rightarrow A_{j2}$. These two fuzzy logical relationships can be grouped into a fuzzy logical relationship group.

#### 3.2.4. Time—invariant/time-variant fuzzy time series

Suppose F(t) is caused by F(t − 1) only, and F(t) = F(t − 1) * R(t − 1, t). For any t, if R(t − 1, t) is independent of t, then F(t) is named as time-invariant fuzzy time-series, otherwise time-variant fuzzy time-series.

#### 3.2.5. Order of fuzzy time series

Fuzzy time series order can be determined from its definition itself. Suppose, F(t + 1) is caused only by F(t) or F(t − 1) or F(t − 2) . . . or F(t − m) (m > 0). This relation can be expressed by

$$F(t+1) = F(t) \cup F(t-1) \ldots F(t-m) * R_{\circ}(t+1, t-m)(m > 0) \qquad (1)$$

Eq. (1) is called first order series of F (t + 1)

If F(t + 1) is caused by F(t) and F(t − 1) . and F(t − n) simultaneously. This relation can be expressed as:

$$F(t+1) = F(t) \cap F(t-1) \ldots F(t-n) * R_{\circ}(t+1, t-n)(n > 0) \qquad (2)$$

Eq. (2) is call $n^{th}$ order model of F(t). Fuzzy relationship is represented by $A_{i1}, A_{i2}, \ldots, A_{in} \rightarrow A_j$, here $F(t-n) = A_{i1}$, $F(t-n+1) = A_{i2}$ and so on $F(t-1) = A_{in}$. This relationship is called $n^{th}$ order fuzzy time series model.

### 3.3. Ordered weighted aggregation (OWA) concept

Sometimes formulation of multiple parameters based decision problems cannot be done accurately by either aggregation of pure "ANDing" or pure "ORing" of parameters. These types of problems require aggregation that lies between these two extremes. OWA is one such type of operator that allows us to easily adjust degree of aggregation. OWA operators were first introduced by Yager [30]. This generalized averaging operator bounds aggregation range by the maximum (ORing) and the minimum (ANDing) operators. OWA aggregation operator can be defined as

$$F(X_1, \ldots, X_n) = \sum_{i=1}^{n} w_i * b_i \qquad (3)$$

here, F is an OWA operator of n dimensions having $X_1, X_2, \ldots, X_n$ as its n arguments. $b_i$ is $i^{th}$ largest element from the collection of arguments $X_1, X_2, \ldots, X_n$

$w_i$ is $i^{th}$ collection of weight of arguments such that $w_i \in [0,1]$ and $\sum w_i = 1$, $w_i$ weights are associated with ordered position of arguments rather than particular argument $X_i$.

Subsequent examples briefly illustrate this concept.

Example 1: Let F be an OWA operator of three dimensions $X_1 = 0.6, X_2 = 0.3$, $X_3 = 0.7$ and weights are (0.45, 0.3, 0.25) implying $w_1 = 0.45$, $w_2 = 0.3$ and $w_3 = 0.25$.

Since, $X_3$ is largest among $X_1$, $X_2$ and $X_3$. Hence, $b_1 = X_3$ i.e. 0.7. Similarly, $b_2 = 0.6$ and $b_3 = 0.3$. Using Eq. (3), F(0.6,0.3, 0.7) can be evaluated as

F (0.6, 0.3, 0.7)) = 0.7*0.45 + 0.6*0.3 + 0.3*0.25 = 0.57

Example 2: In criteria based problems, where decision is based on importance of each criterion, OWA approach is very helpful. Let, the criteria $X_i$ be twice as importance as criteria $X_j$ and $X_j$ be thrice as importance as criteria $X_k$. Given that weights are = ($w_1$, $w_2$, $w_3$). F ($X_i$, $X_j$, $X_k$) can be evaluated as:

$$F(X_i, X_j, X_k) = \{X_i * w_1 + X_j * w_2 + X_k * w_3\} Since, X_i > X_j > X_k$$

In order to use OWA, we must determine weight of arguments of problem domain. Various methods of weight generation are available in the literature [30,31,37,38]. Our problem domain is fuzzy time series. Therefore, we took up then regularly increasing monotone (RIM) quantifiers [37] denoted by Q(r) to determine weights of linguistic arguments of fuzzy time series.

**Table 1**
RIM(Q) vs β.

| β | Quantifiers Q | β | Quantifiers Q |
|---|---|---|---|
| β = 0 | At least one | β = 1 | Half |
| β = 0.1 | At least few | β = 2 | Most |
| β = 0.5 | A few | β = ∞ | All |

#### 3.3.1. RIM quantifiers

Yager [30] in 1988 introduced a class of functions to generate weights using regularly increasing monotonic (RIM) quantifiers. RIM was proposed to determine OWA weights by using linguistic quantifiers such as: "there exists" $Q_*(r)$(OR) and "for all" $Q^*(r)$(AND). Thus for any RIM quantifier Q(r), the limit $Q_*(r) <= Q(r) <= Q^*(r)$ holds true. Also, Q(r) must satisfy two properties: (a) Q(0) = 0 and (b) r∈[0,1]. OWA weights with m number of criterion can be determined using RIM quantifier as follows [37].

$$w_i = Q\left(\frac{i}{m}\right) - Q\left(\frac{i-1}{m}\right) \quad \text{i} = 1, 2 \ldots \text{m} \qquad (4)$$

The RIM quantifiers can continuously change its values between $Q_*(r)$ and $Q^*(r)$. This generates a family of RIM quantifiers. This family of RIM quantifiers can be defined by parameterized class of fuzzy subsets [37]. It can be further defined as: Q(r) = rβ where, r ≥ 0.

For β = 1; Q(r) = r (a linear function). This is called unitor quantifier.

For β → ∞; $Q^*(r)$ (and type). This is universal quantifier, implying most of the criterion must be satisfied for acceptable solution.

For β → 0; $Q_*(r)$ (or-type). This is called existential quantifier, implying some of criterion must be satisfied for acceptable solution.

Eq. (4) can be redefined as

$$w_i = Q\left(\frac{i}{m}\right)^{\beta} - Q\left(\frac{i-1}{m}\right)^{\beta} \quad \text{i} = 1, 2 \ldots \text{m} \qquad (5)$$

here, β is a degree of polynomial. For β = 1, the uniform distribution of weights takes place. It means that equal weight is assigned to each criterion i.e. $w_i = 1/n$, where n is number of criterion. For β < 1, the RIM quantifier acts like "or-type" operator. For β > 1, the RIM quantifier acts like "and-type" operator. All definitions of β in terms of quantifier are given in Table 1.

In fuzzy time series, usually most of the criterion must be satisfied to obtain a solution. Here "criterion" is past observation and "solution" is predicted value. "n − 1" past values are required to predict next value for $n^{th}$ fuzzy time series. Since, most of the criterion must be satisfied, "most" RIM quantifiers is best suited for aggregation of past n fuzzy values. RIM "most" quantifier is defined as $Q(r) = r^2 (\beta = 2)$ [38].

Subsequent example briefly illustrates this concept of RIM quantifiers:

Example 3: Given that m = 3 is number of criterion and "most" linguistic quantifiers is used. Weights ($w_1$, $w_2$, $w_3$) are calculated using Eq. (5) For i = 1 to 3as:

$w_1 = Q(1/3) - Q(0/3) = (1/3)^2 - (0/3)^2 = (0.33)^2 = 0.1089$

$w_2 = Q(2/3) - Q(1/3) = (2/3)^{2-}(1/3)^2 = (0.66)^2 - (0.33)^2 = 0.3267$

$w_3 = Q(3/3) - Q(2/3) = (3/3)^2 - (2/3)^2 = (1)^2 - (0.66)^2 = 0.5644$

## 4. Proposed—OWA based fuzzy time series model

In $n^{th}$ order fuzzy time series forecasting model, $F_{t+1}$ is driven by $F_t$, $F_{t-1}$ and $F_{t-2} \cdots F_{t-(n-1)}$ i.e. n past observations are required to forecast value. Aggregation of past observations must be carried out on basis of their relative weight to predict $F_{t+1}$. OWA operator is used to perform effective aggregation of past observations. OWA weight of each past observation is calculated by determining importance of respective fuzzy set in the system. this is hybrid algorithm, amalgamation of OWA with Fuzzy is presented. How-

**Table 2**
Fuzzy set vs Linguistic Variable.

| Fuzzy Set | Linguistic Variable | Fuzzy Set | Linguistic Variable |
|---|---|---|---|
| $F_1$ | poor outpatient visits | $F_5$ | Very good outpatient visits |
| $F_2$ | below average outpatient visits | $F_6$ | Excellent outpatient visits |
| $F_3$ | Average outpatient visits | $F_7$ | Extraordinary outpatient visits |
| $F_4$ | Good outpatient visits | | |

ever, fundamental steps are same and none of the steps/methods or techniques have been chosen randomly. Every aspect has been fully understood and justified with proper reasoning. Outline for fundamental steps for designing of fuzzy time series models is

1) Define universe of discourse
2) divide U in equal no of intervals
3) Fuzzification
4) Define FLR
5) Determined forecasted value
6) Defuzzification

Universe of discourse is defined first and then, U is divided in equal no of intervals using random partitioning method. Subsequently fuzzification is done using membership function. Thereafter 4th step of fuzzy time series is implemented by designing 3rd fuzzy time series. Significance of considering 3rd order is explained in Section 4. So in order to determine weights, priority matrix is designed to find out importance of each fuzzy set. Subsequently, weighted centroid method is applied in defuzzication process as written in step 6. It is proved from Table 2 that why weighted centroid defuzzification is taken.

Detailed description of each steps of OWA based fuzzy time series model to forecast value $F_{t+1}$ is:

Step 1: This step is usually carried out as the first step towards making universe of discourse suitable for numerical evaluation by associating time series data of different times. It eliminates subtlety of the universe of discourse. Various forecasting methods have utilized the differences of time series data as the universe of discourse[14,20]. However, increasing and decreasing rate of time series cannot be captured from difference alone. Hence the designed model broadly used the percentage of time variant parameter to forecast the value. Since value of parameter is dependent on time with a characteristics of time variant relation. Therefore, it is called a time variant time series.

Calculate rate of change (RoC) of time series data using Eq. (6)

$$RoC_{t+1} = ((X_{t+1} - X_t) \div X_t) * 100 \tag{6}$$

here, X(t + 1) is value at time t + 1 index and X(t) is actual value at time t index. RoC is the rate of change of value from time t to t + 1.

Step 2: Define Universe of Discourse $U = [D_{min} - D_1, D_{max} + D_2]$, here $D_{max}$ and $D_{min}$ are maximum and minimum values of RoC respectively. $D_1$ and $D_2$ are positive real values to partition U in equal length of intervals say; $u_1, u_2, u_3 \ldots u_n$ of equal lengths.There are numerous methods to divide U. When we change the way of division; it will affect the forecasting results in fuzzy time series. Mostly used partitioning methods are explained below. Also, illustrated is their impact on the model. Subsequently, reason to use random portioning is stated.

There are numerous methods to partitioning the intervals as:

(1) Random Partitioning
(2) distribution based partitioning
(3) Average based length Partitioning

Random Partitioning: Random partition of U is commonly practiced when the Universe of discourse is divided equally by the length chosen into n number of intervals. The length must not be larger than length the universe of discourse. Suppose, we have following time series data: 6,10,12,6,4.

Case 1: Universe of Discourse is not divided

U = [4,12] if Universe discourse is not divided then according to [3,4] models, MSE is 8.8.

Case 2: If length of interval is set as 4 then u1 = [3,5], u2 = [5,7], u3 = [7,9], u4 = [9,11], u5 = [11,13] then MSE is 4.5.

Hence, random partition of U in different length of intervals results in different forecasting results.

Distribution based partitioning: The fluctuations in time series can be represented by absolute value of first differences of any two consecutive data. Hence heuristic can reflect atleast half of first differences. Distribution based length is calculated according to the distribution of first differences of data. Algorithms for distribution length can be referred to [2].

Suppose, we have same above time series data: 6,10,12,6,4.

First differences are 4,2,6,2. The average of first differences is 14

From base mapping table [2], the base for length of intervals is 10

The number of first difference larger than 10 is 0, larger than 4 is 1 and larger than 2 is 2.

Because 2 is largest length, we can have, which is still smaller than at least half the first differences, 2 is chosen as the length of intervals

length of interval is set as 2 then $u_1$ = [3,5], $u_2$ = [5,7], $u_3$ = [7,9], $u_4$ = [9,11], $u_5$ = [11,13] then MSE is 4.5.

Average Length based partition: Average of first differences may not necessarily fulfil the heuristic. The average based length is set to one half of the average of the first differences. Algorithms for average length based partition can be referred to [2].

Suppose, we have same above time series data: 6,10,12,6,4.

1) First differences are 4,2,6,2. The average of first differences is 14
2) Take half of the average based length which is 7
3) From base mapping table [6], the base for length of intervals is 1
4) Round the length 7 by the base 1 which is 1, so 1 is chosen as the length of intervals
5) Length of interval is set as 2 then $u_1$ = [4,5], $u_2$ = [5,6], $u_3$ = [6,7], $u_4$ = [7,8], $u_5$ = [8,9] $u_6$ = [9,10], $u_7$ = [10,11], $u_8$ = [11,12] then MSE is 0.25

Proposed model is designed for random partitioning method.

Step 3: Define seven fuzzy set as $F_i$ (where i = 1 to 7) based on the linguistic variables in the Universe of Discourse. This practice is called fuzzification of the time series data where fuzzy set $F_i$ denotes a linguistic value of the RoC represented by a fuzzy set.

If, Universe of discourse has more linguistic variables then number of intervals will be decided accordingly. A fuzzy set of U can be defined as: $F_i = f_{Fi}(u_1)/u_1 + f_{Fi}(u_2)/u_2 + f_{Fi}(u_3)/u_3 + \ldots\ldots f_{Fi}(u_n)/u_n$.

Where $f_{Fi}$ is membership function of fuzzy set $F_i$. $f_{Fi}: U \to [0,1]$, $u_k$ is the element of fuzzy set $F_i$. $f_{Fi}(u_k)$ is the degree of belongingness of $u_k$ to $F_i$. $f_{Fi}(u_k) \in [0,1]$ where $1 \le k \le n$. Chen [4] gave the concept of deciding membership function and creating fuzzy sets on basis of their belongingness.

Define seven fuzzy sets as linguistic variables as in Table 2.

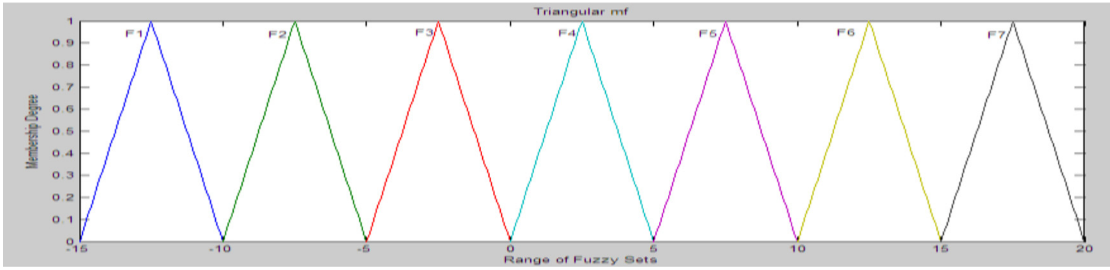


**Fig. 1.** Trinagular *mf.*

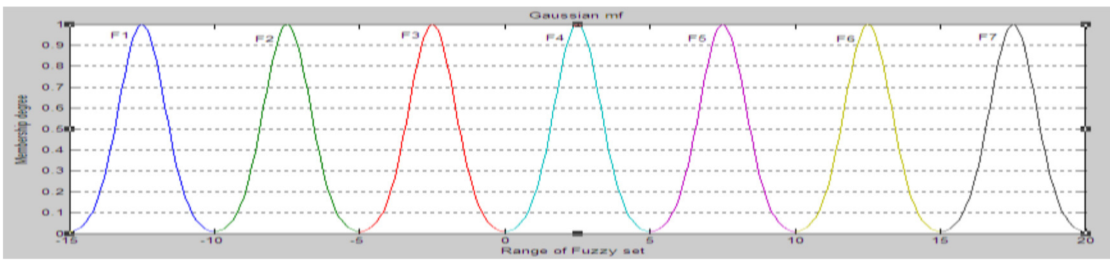


**Fig. 2.** Gaussian *mf.*

The membership grades ofabove fuzzy sets which are assigned to linguistic values can be defined as:

$F_1 = 1/u_1 + 0.5/u_2 + 0/u_3 + 0/u_4 + 0/u_5 + 0/u_6 + 0/u_7$
$F_2 = 0.5/u_1 + 1/u_2 + 0.5/u_3 + 0/u_4 + 0/u_5 + 0/u_6 + 0/u_7$
$F_3 = 0/u_1 + 0.5/u_2 + 1/u_3 + 0.5/u_4 + 0/u_5 + 0/u_6 + 0/u_7$
$F_4 = 0/u_1 + 0/u_2 + 0.5/u_3 + 1/u_4 + 0.5/u_5 + 0/u_6 + 0/u_7$
$F_5 = 0/u_1 + 0/u_2 + 0/u_3 + 0.5/u_4 + 1/u_5 + 0.5/u_6 + 0/u_7$
$F_6 = 0/u_1 + 0/u_2 + 0/u_3 + 0/u_4 + 0.5/u_5 + 1/u_6 + 0.5/u_7$
$F_7 = 0/u_1 + 0.5/u_2 + 0/u_3 + 0/u_4 + 0/u_5 + 0.5/u_6 + 1/u_7$

The Proposed model will work same for each type of membership function. Since Membership function represents degree of truth as an extension of valuation. In other words, it represents linguistic term by a number in unit interval. There are various ways to symbolize membership degree $u_T(x) = P$ such as likelihood view, random set view, similarity view, utility view and measurement view. In our case, likelihood view is satisfied in which majority of population satisfies a particular linguistic term. It implies linguistic variable denote highest membership degree which is irrespective of variation in degree. Since, proposed model is employed for highest degree of membership and not for variation in degree of membership; hence each membership function will work same for proposed model.

Example: Let us consider three different type of membership functions (*mf*) like triangular, Gaussian and trapezoidal. Figs. 1–3 portray their respective curves. In all figures, X axis represents interval/range of a membership function and y axis represents membership degree. Now consider RoC sample for year 2002/02 is −8.28 as shown in Table 4, Triangular *mf* generates fuzzy set $F_2$ for this particular value as in Fig. 1. Gaussian *mf* also produces fuzzy set $F_2$ as shown in Fig. 2. Similarly, from Trapezoidal, same fuzzy set $F_2$ is obtained as in Fig. 3. From here, It is proved that belongingness of RoC to a fuzzy set depends only upon the interval not on the variation of membership degree. In our paper, we have worked on belongingness to a fuzzy set not on the variation of degree and hence, result will be same irrespective of membership function used.

According to our proposed model, U should be partitioned into number of intervals same as given linguistic variable's. Basically, we have laid foundation of proposed model in such a way that number of fuzzy sets same as number of linguistic variable. The author has chosen triangular membership function due to its simplicity. It is easy to use and apply in fuzzy time series model. Any other membership function can also be considered while designing of fuzzy time series model like trapezium membership function or Gaussian membership function. Result will be same in case membership function changes. Moreover, algorithm will perform same if the membership function is in different shapes or even continuous functions. Since, every linguistic variable represents each fuzzy set that shows highest membership degree irrespective of type of *mf.*

Step 4: Priority matrix is created to define the importance of each fuzzy set in fuzzy time series like importance of each criterion in multiple criterion based decision problems. Count the number of RoC that fall in each fuzzy set. Now importance $\mu$ is assigned to each fuzzy set on basis of count of RoC. Arrange all fuzzy set in ascending order on basis of number of RoC as $\text{Count}(F_i) < \text{Count}(F_k)$

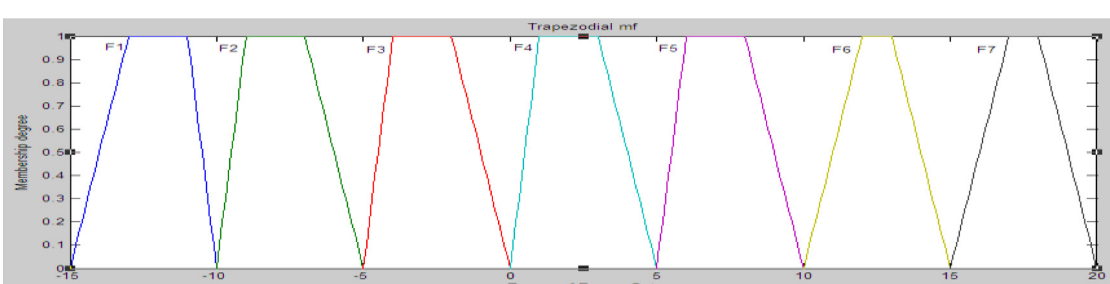


**Fig. 3.** Trapezoidal *mf.*

**Table 3**
Structure of priority matrix.

| $\mu(F_i)$ | $\mu(F_k)$ | $\mu(F_j)$ | ... | $\mu(F_n)$ |
|---|---|---|---|---|
| 1/n | 2/7 | 3/n | ... | n/n |

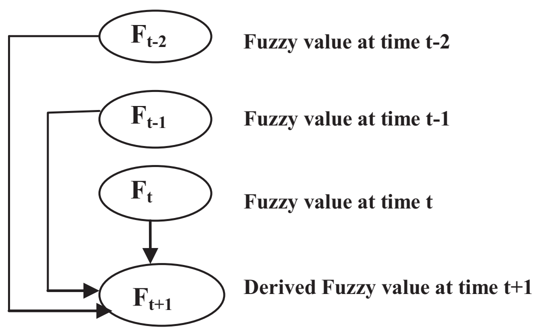


**Fig. 4.** 3rd Order Fuzzy Time Series Model.

...< Count ($F_n$). Then $\mu(F_i) = 1/n$, $\mu(F_k) = 2/n$ .... $\mu(F_n) = n/n$. Now create a priority matrix as shown in Table 3:

Step 5: Define fuzzy logical relationship (FLR) for 3rd order time series model in form of $F_t \rightarrow F_{t+1}$, $F_{t-1} \rightarrow F_{t+1}$, $F_{t-2} \rightarrow F_{t+1}$. It can be written as $F_t, F_{t-1}, F_{t-2} \rightarrow F_{t+1}$. This means that prediction at a particular period of time depends on past three observations.

3rd order model is used to forecast value at any time t + 1. Intent of using 3rd order fuzzy time series model is to obtain FLR, free from ambiguities while ensuring low complexity. In this context, an ambiguity exists if two or more FLR have same fuzzy values/elements/sets. This implies that these FLR are not unique. This decreases the forecasting accuracy of model. Use of high-order fuzzy time series can overcome drawback of ambiguities that generally exists in low order fuzzy time series model [8]. However, higher order time series induces complexity and reduces efficiency of model. In our problem domain, to avoid ambiguities, at least three past fuzzy time series

observations are required to forecast value at time t + 1. For simplicity and appropriateness, we took 3rd order time series in our study. If, n is the total number of time series data then n − 3 FLR in form of rules are generated by $F_t, F_{t-1}, F_{t-2} \rightarrow F_{t+1}$. Fig. 4 implies that prediction at time t + 1 depends on previous value at time t, t − 1 and t − 2 as:

Similarly, 4th, 5th and other higher order model can be described.

Step 6: In order to predict $F_{t+1}$, we must aggregate $F_t, F_{t-1}, F_{t-2}$ (3rd order fuzzy time series model). In this study, we used OWA for aggregation. In order to use OWA, we must calculate weights of past values. We used importance of past fuzzy set in RIM quantifiers to calculate OWA weight as described.

Let $F_1, F_2, \ldots, F_n$ be n linguistic variables and importance associated with these linguistic variables be $\mu_1, \mu_2, \ldots \mu_n$ as determined in previous steps. Eq. (5) can be redefined for calculation of OWA weights, on basis of importance of past fuzzy observation as:

$$w_j = Q(S_j/T)^{\beta} - Q(S_{j-1}/T)^{\beta}\ \mathrm{j} = 1, 2, \ldots.\mathrm{n} \quad (7)$$

here, β=2, $S_j = \sum_{k=1}^{j} \mu_k$ and $T = \sum_{j=1}^{n} \mu_j$

Properties of Eq. (7) are:

a) $w_i$ is ith collection of weight of arguments such that $w_i \in [0,1]$.
b) Sum of weights $\sum w_i = 1$, $w_i$ weights are associated with ordered position of arguments rather than particular argument $X_i$.
c) Here, β is a degree of polynomial. For β = 1, the uniform distribution of weights takes place. It means that equal weight is assigned to each criterion i.e. $w_i = 1/n$, where n is number of criterion. For β < 1, the RIM quantifier acts like "or-type" operator. For β > 1, the RIM quantifier acts like "and-type" operator.

Further to simplify the Eq. (7), the variables Sj and Tj represents $\sum_{k=1}^{j} \mu_k$ and $\sum_{j=1}^{n} \mu_j$ respectively. Here, $\mu_k$ and $\mu_j$ are importance of fuzzy sets in fuzzy time series model.

Subsequent example illustrates the steps for calculation of weights using RIM quantifiers.

Example 4: Let $F_1$, $F_2$ and $F_3$ be three fuzzy sets. Importance associated with these fuzzy set is $\mu(F_1) = 0.5$, $\mu(F_2) = 0.7$, $\mu(F_3) = 0.4$ and fuzzy quantifier used is "most" (β = 2)

$$T = \sum_{j=1}^{n} \mu(A_i) \Rightarrow (0.5 + 0.7 + 0.4) \Rightarrow 1.6$$

$$\Rightarrow (0.5/1.6)^2 - (0/1.6)^2 \Rightarrow 0.0976$$

$$w_2 = Q((S_2)/1.6)^2 - Q(S_1/1.6)^2 \Rightarrow (1.2/1.6)^2 - (0.5/1.6)^2 \Rightarrow 0.4649$$

$$w_1 = Q(S_1/1.6)^2 - Q(S_0/1.6)^2 w_3 = Q((S_3)/1.6)^2 - Q((S_2)/1.6)^2$$

$$\Rightarrow (1.6/1.6)^2 - (1.2/1.6)^2 \Rightarrow 0.4375$$

$$\sum_{j=1}^{n} w_i \Rightarrow 0.0976 + 0.4649 + 0.4375 = 1$$

Step 7: "DRoC" stands for defuzzified rate of change. Primarily, time series data is fuzzified after calculating their rate of change. Thereafter, their relative weights are determined. Henceforth, defuzzification is performed to attain forecasting value, since every fuzzified value must be defuzzified. There are numerous defuzzification methods like centroid and many others [6–8,20]. Other defuzzification methods can also be considered but accuracy of forecasting models varies with method of defuzzification method. The main defuzzification methods are the Max method, the centroid method and height method.

- The Maxima method, which computes the value whose membership degree is highest
- The centroid method, which determine the centre of gravity of fuzzy number
- The weighted centroid, which calculates the baby centre of membership function kernels weighted by the corresponding membership degrees.
- The average method: which determine average of membership function.

We have chosen weighted centroid for defuzzification which is further improved version of centroid method since it reduces MSE and AFER to a certain extent.Fuzzy logical relationship can be written as $F_t, F_{t-1}, F_{t-2} \rightarrow F_{t+1}$. Where $F_t, F_{t-1}, F_{t-2}$ are fuzzy sets at time t, t − 1 and t − 2 and maximum membership values of these fuzzy sets occur at intervals $u_t$, $u_{t-1}$, $u_{t-2}$ respectively, and mid points of $u_t$, $u_{t-1}$, $u_{t-2}$ are $m_t$, $m_{t-1}$, $m_{t-2}$respectively.

The defuzzified value of RoC at t + 1 ($DRoC_{t+1}$) can be calculated as:

$$DRoC_{t+1} = [m_t, m_{t-1}, m_{t-2}] * [w_i, w_j, w_k]^T$$
$$\Rightarrow (m_t * w_i) + (m_{t-1} * w_j) + (m_{t-2} * w_k) \quad (8)$$

Since, in 3rd order fuzzy time series forecasting model, $F_{t+1}$ is most effected by $F_t$ follow by $F_{t-1}$ and then $F_{t-2}$. Hence, ordered sequence of fuzzy set is $\{F_t, F_{t-1}, F_{t-2}\}$. Let $m_t$, $m_{t-1}$, $m_{t-2}$ be mid points of fuzzy intervals $F_t$, $F_{t-1}$ and $F_{t-2}$ respectively.

$W_i$, $W_j$, $W_k$ are respective weights of fuzzy sets determined according to their importance.

Fuzzy logical relationship can be written as $F_t, F_{t-1}, F_{t-2} \rightarrow F_{t+1}$.

Step 8: Forecasted value at t + 1 is calculated using Eq. (9)

$$F_{val} = ((DRoC_{t+1} * X_t) \div 100) + X_t \quad (9)$$

*4.1. Algorithm code for steps illustrated in above proposed model*

Input: A time series data = ($X_{t1,}$ $X_{t2,}$. . .· · ·$X_{tn}$)
Output: Forecasting data = ($F_{t4,}$. . .· · ·$F_{tn}$)
For t = 1 to. . . . . .n(end of time series data)
Compute$RoC_{t+1} = ((X_{t+1} - X_t) \div X_t) * 100$
*End For*
For i = 1 to number of fuzzy sets
Obtained the priority matrix
*End For*
For t = 3 to. . . . . .n(end of time series data)
Obtained FLR in the form of: $F_t$, $F_{t-1}$, $F_{t-2} \rightarrow F_{t+1}$
Compute: for each FLR at t = 3 to n. . .

$$w_j = Q(S_j/T)^{\beta} - Q(S_{j-1}/T)^{\beta}$$

here, $\beta = 2$, $S_j = \sum_{k=1}^{j} \mu_k$ an
*End For*
For t = 3 to. . . . . .n(end of time series data)
Compute $DRoC_{t+1} = [m_t, m_{t-1}, m_{t-2}] * [w_i, w_j, w_k]^T$
*End For*
For t = 1 to. . . . . .n(end of time series data)

$$F_{val} = ((DRoC_{t+1} * X_t) \div 100) + X_t$$

*End For*

*4.2. Computational complexity of algorithm for proposed model*

Determination of RoC for time series data will take approx O(n). Let us assume that defining of U and formation of fuzzy set will take time equal to number of intervals that is L. Further each RoC is written as fuzzy set in O(n). Creation of priority matrix takes constant time P. Further, writing of 3rd FLR will take 3n time. Subsequently, determination of weights for 3rd order requires constant time 3W. DRoC and Fval calculation requires O(n).

TotalComputationalcomplexity

$$= O(n) + L + O(n) + P + O(3n) + 3W + O(n) + O(n)$$

$$=> 7O(n) + L + P + 3W => 7O(n) + C$$

Here, C = L + P + 3 W (L is no of intervals, P is no of fuzzy set, W is weights)

Computational complexity can be written in standard notion as = aO(n) + c where a and c are constants.

The computational complexity of [7,26,27], models is also linear and similar to our model i.e. aO(n) + c. However difference is in value of constants "a" and "c". Value of a and c will be in increasing order for proposed model, [7,26,27], respectively.

Additionally, 2nd order FLR is written as $F_t$, $F_{t-1} \rightarrow F_{t+1}$, 3rd order FLR is written as $F_t$, $F_{t-1}$, $F_{t-2} \rightarrow F_{t+1}$ and 4th order as $F_t$, $F_{t-1}$, $F_{t-2}$, $F_{t-3} \rightarrow F_{t+1}$. From this equation, it can be observed that order of FLR will increase the constant factor marginally. Hence, order of FLR does not have huge impact on computational complexity. It is immensely driven by number of time series data.

## 5. Application in healthcare: forecasting of outpatient visit

Forecasting the number of outpatient visit can not only influence patient waiting time but also improves the coordination of care [28,29]. Yu [26] predicted the number of outpatient visits by designing of weighted Fuzzy Time Series Model. Likewise, Cheng [27] proposed fuzzy time series based on weighted transitional

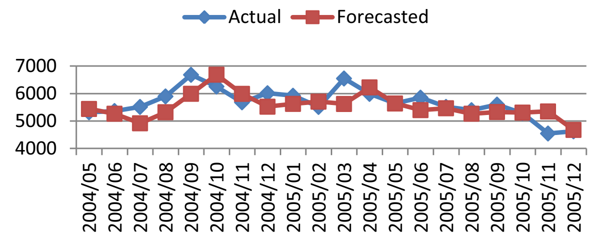


**Fig. 5.** Actual vs Forecasted (Outpatient).

matrix for forecasting the number of outpatient visits Therefore, we considered this application domain to demonstrate potential of our proposed model. Same historical outpatient data has been used in previous work [26,27] to evaluate the proposed model. Here, number of outpatient visits is seen as time series data. Our objective is to analyze the time series data so that forecasting for next month can be done on basis of data of past months in advance. Thus, health care administrator can keep record of number of outpatient visits on monthly basis.

In this study we used the dataset for year 2004 and 2005 of outpatient visits [26,27] collected from the department of internal medicine of a hospital to verify our proposed model. Random partitioning of intervals is used to divide Universe of Discourse. Triangular membership function [40] is considered for fuzzification process and weighted centroid defuzzification method is utilized for forecasting of outpatient data.

Computation of each step for outpatient data is shown below for sturdy understanding of proposed model.

Step 1: RoC of outpatient visit in hospital for each month is shown in Table 4.

Step 2: Obtain $D_{min} = -13.98$, $D_{max} = 18.79$ from Table 4. Define Universe of Discourse U = [−15, 20], where $D_1 = 1.02$, $D_2 = 1.21$. Equal intervals are $u_1$ = [−15 −10], $u_2$ = [−10 −5], $u_3$ = [−50], $u_4$ = [05], $u_5$ = [510], $u_6$ = [1015], $u_7$ = [15,20]

Step 3: Fuzzification of data as shown in Table 5.

Step 4: Number of RoC fall in each fuzzy set is calculated. Count($F_1$) = 2, Count($F_2$) = 8, Count($F_3$) = 2, Count($F_4$) = 5, Count($F_5$) = 4 Count($F_6$) = 1, Count($F_7$) = 1.Further arrange fuzzy set in ascending order on basis of RoC numbers i.e. $F_7 < F_6 < F_3 < F_1 < F_5 < F_4 < F_2$. Priority matrix is designed as Table 6.

Step 5: Fuzzylogicalrelationship (FLR) for 3rd order is determined as in Table 7.

Step 6: OWA weights are determined for each fuzzy logical relationship calculated in step 5. FLR at time 2004/05 is $F_1$, $F_5$, $F_2 \rightarrow F_2$. Hence weights $w_1$, $w_2$ and $w_3$ are determined for $F_1$, $F_5$ and $F_2$ respectively. Importance of $\mu(F_1)$, $\mu(F_5)$ and $\mu(F_2)$ is 4/7, 5/7 and 7/7 respectively from priority matrix (Table 6).

$\{b_1, b_2, b_3\} = \{\mu(F_1), \mu(F_5), \mu(F_2)\} = \{0.57, 0.71, 1\}$
$w_1 = (0.57/2.28)^2 - (0/2.28)^2 = 0.0625$,
$w_2 = (1.28/2.28)^2 - (0.57/2.28)^2 = 0.25390625$,
$w_3 = (2.28/2.28)^2 - (1.28/2.28)^2 = 0.6835$,
All FLR weights are given in Table 8.

Step 7 and 8: Forecasted RoC and outpatient visit ($F_{val}$) are shown in Table 9.

Fig. 5 shows the existing trends of forecasted outpatient visits obtained by proposed method and actual value. It can be observed from Fig. 5 that forecasted values computed by the proposed model are significantly in closed accordance to the actual productions

**Table 4**
RoC.

| Month | Outpatient | RoC (%) | Month | Outpatient | RoC (%) |
|---|---|---|---|---|---|
| 2004/01 | 6519 | | 2005/01 | 5920 | −1.53027 |
| 2004/02 | 5979 | −8.28348 | 2005/02 | 5512 | −6.89189 |
| 2004/03 | 6322 | 5.736745 | 2005/03 | 6548 | 18.79536 |
| 2004/04 | 5666 | −10.3765 | 2005/04 | 5987 | −8.5675 |
| 2004/05 | 5318 | −6.1419 | 2005/05 | 5638 | −5.8293 |
| 2004/06 | 5364 | 0.864987 | 2005/06 | 5851 | 3.777935 |
| 2004/07 | 5513 | 2.777778 | 2005/07 | 5514 | −5.7597 |
| 2004/08 | 5895 | 6.929077 | 2005/08 | 5395 | −2.15814 |
| 2004/09 | 6682 | 13.3503 | 2005/09 | 5598 | 3.762743 |
| 2004/10 | 6254 | −6.40527 | 2005/10 | 5284 | −5.60915 |
| 2004/11 | 5681 | −9.16214 | 2005/11 | 4545 | −13.9856 |
| 2004/12 | 6012 | 5.826439 | 2005/12 | 4624 | 1.738174 |

**Table 5**
Fuzzification.

| Month | Outpatient | RoC (%) | Fuzzy | Month | Outpatient | RoC (%) | Fuzzy |
|---|---|---|---|---|---|---|---|
| 2004/01 | 6519 | | | 2005/01 | 5920 | −1.53027 | $F_3$ |
| 2004/02 | 5979 | −8.28348 | $F_2$ | 2005/02 | 5512 | −6.89189 | $F_2$ |
| 2004/03 | 6322 | 5.736745 | $F_5$ | 2005/03 | 6548 | 18.79536 | $F_7$ |
| 2004/04 | 5666 | −10.3765 | $F_1$ | 2005/04 | 5987 | −8.5675 | $F_2$ |
| 2004/05 | 5318 | −6.1419 | $F_2$ | 2005/05 | 5638 | −5.8293 | $F_2$ |
| 2004/06 | 5364 | 0.864987 | $F_4$ | 2005/06 | 5851 | 3.777935 | $F_4$ |
| 2004/07 | 5513 | 2.777778 | $F_4$ | 2005/07 | 5514 | −5.7597 | $F_2$ |
| 2004/08 | 5895 | 6.929077 | $F_5$ | 2005/08 | 5395 | −2.15814 | $F_3$ |
| 2004/09 | 6682 | 13.3503 | $F_6$ | 2005/09 | 5598 | 3.762743 | $F_4$ |
| 2004/10 | 6254 | −6.40527 | $F_2$ | 2005/10 | 5284 | −5.60915 | $F_5$ |
| 2004/11 | 5681 | −9.16214 | $F_2$ | 2005/11 | 4545 | −13.9856 | $F_1$ |
| 2004/12 | 6012 | 5.826439 | $F_5$ | 2005/12 | 4624 | 1.738174 | $F_4$ |

**Table 6**
Priority matrix.

| $\mu(F_7)$ | $\mu(F_6)$ | $\mu(F_3)$ | $\mu(F_1)$ | $\mu(F_5)$ | $\mu(F_4)$ | $\mu(F_2)$ |
|---|---|---|---|---|---|---|
| 1/7 | 2/7 | 3/7 | 4/7 | 5/7 | 6/7 | 7/7 |

**Table 7**
Fuzzy Relationships.

| Month | Fuzzy | FLR | Month | Fuzzy | FLR |
|---|---|---|---|---|---|
| 2004/01 | | – | 2005/01 | $F_3$ | $F_5,F_2,F_2 \rightarrow F_3$ |
| 2004/02 | $F_2$ | – | 2005/02 | $F_2$ | $F_3,F_5,F_2 \rightarrow F_2$ |
| 2004/03 | $F_5$ | – | 2005/03 | $F_7$ | $F_2,F_3,F_5 \rightarrow F_5$ |
| 2004/04 | $F_1$ | – | 2005/04 | $F_2$ | $F_7,F_2,F_3 \rightarrow F_2$ |
| 2004/05 | $F_2$ | $F_1,F_5,F_2 \rightarrow F_2$ | 2005/05 | $F_2$ | $F_2,F_7,F_2 \rightarrow F_2$ |
| 2004/06 | $F_4$ | $F_2,F_1,F_5 \rightarrow F_4$ | 2005/06 | $F_4$ | $F_2,F_2,F_7 \rightarrow F_4$ |
| 2004/07 | $F_4$ | $F_4,F_2,F_1 \rightarrow F_4$ | 2005/07 | $F_2$ | $F_4,F_2,F_2 \rightarrow F_2$ |
| 2004/08 | $F_5$ | $F_4,F_4,F_2 \rightarrow F_5$ | 2005/08 | $F_3$ | $F_2,F_4,F_2 \rightarrow F_3$ |
| 2004/09 | $F_6$ | $F_5,F_4,F_4 \rightarrow F_6$ | 2005/09 | $F_4$ | $F_3,F_2,F_4 \rightarrow F_4$ |
| 2004/10 | $F_2$ | $F_6,F_5,F_4 \rightarrow F_2$ | 2005/10 | $F_5$ | $F_4,F_3,F_2 \rightarrow F_5$ |
| 2004/11 | $F_2$ | $F_2,F_6,F_5 \rightarrow F_2$ | 2005/11 | $F_1$ | $F_5,F_4,F_3 \rightarrow F_1$ |
| 2004/12 | $F_5$ | $F_2,F_2,F_6 \rightarrow F_5$ | 2005/12 | $F_4$ | $F_1,F_5,F_4 \rightarrow F_4$ |

## 6. Impact of partitioning of intervals

Proposed model is run for partitioning methods as discussed in step 4 of Section 4 and MSE's are calculated respectively as shown in Table 10.

It can be observed from Table 10 that optimum way to divide U is average based partitioning method for proposed model. However, random partitioning of intervals is best suited from the designing perspective of proposed model. Since, key point in deciding effective lengths of intervals is that they should be no too large or small. When effective length is too large there will be no fluctuations in the fuzzy time series. On the other side, when length is too small, the meaning of fuzzy time series will be diminished. In order to reflect fluctuations properly and to keep fuzzy time series meaningful, the heuristic is set in such a way that at least half of fluctuations in the time series are reflected by effective lengths of intervals. In our proposed model, U should be partitioned into number of intervals same as given linguistic variable's. Basically, we have laid foundation of proposed model in such a way that number of fuzzy sets same as number of linguistic variable which asserted us to take random partitioning approach. Random partition of U is chosen accordance with the number of linguistic variables $F_1$, $F_2$,. ., $F_n$. This means that U is divided into number equal to number of linguistic variables.

## 7. Impact of order of model and type of defuzzification

### 7.1. Impact of order of model

We have studied the impact of taking various orders of model. Initially, we took $2^{nd}$ order of model. It was very easy to design

**Table 8**
OWA weights.

| Month | Fuzzy | FLR | OWA Weights |
|---|---|---|---|
| 2004/01 | | – | – |
| 2004/02 | $F_2$ | – | – |
| 2004/03 | $F_5$ | – | – |
| 2004/04 | $F_1$ | – | – |
| 2004/05 | $F_2$ | $F_1,F_5,F_2 \rightarrow F_2$ | $w_1 = 0.06$, $w_2 = 0.26$, $w_3 = 0.68$ |
| 2004/06 | $F_4$ | $F_2,F_1,F_5 \rightarrow F_4$ | $w_1 = 0.19$, $w_2 = 0.28$, $w_3 = 0.53$ |
| 2004/07 | $F_4$ | $F_4,F_2,F_1 \rightarrow F_4$ | $w_1 = 0.12$, $w_2 = 0.46$, $w_3 = 0.42$ |
| 2004/08 | $F_5$ | $F_4,F_4,F_2 \rightarrow F_5$ | $w_1 = 0.1$, $w_2 = 0.3$, $w_3 = 0.6$ |
| 2004/09 | $F_6$ | $F_5,F_4,F_4 \rightarrow F_6$ | $w_1 = 0.09$, $w_2 = 0.33$, $w_3 = 0.58$ |
| 2004/10 | $F_2$ | $F_6,F_5,F_4 \rightarrow F_2$ | $w_1 = 0.02$, $w_2 = 0.27$, $w_3 = 0.71$ |
| 2004/11 | $F_2$ | $F_2,F_6,F_5 \rightarrow F_2$ | $w_1 = 0.25$, $w_2 = 0.16$, $w_3 = 0.59$ |
| 2004/12 | $F_5$ | $F_2,F_2,F_6 \rightarrow F_5$ | $w_1 = 0.19$, $w_2 = 0.58$, $w_3 = 0.23$ |
| 2005/01 | $F_3$ | $F_5,F_2,F_2 \rightarrow F_3$ | $w_1 = 0.07$, $w_2 = 0.33$, $w_3 = 0.6$ |
| 2005/02 | $F_2$ | $F_3,F_5,F_2 \rightarrow F_2$ | $w_1 = 0.04$, $w_2 = 0.24$, $w_3 = 0.72$ |
| 2005/03 | $F_7$ | $F_2,F_3,F_5 \rightarrow F_5$ | $w_1 = 0.22$, $w_2 = 0.22$, $w_3 = 0.56$ |
| 2005/04 | $F_2$ | $F_7,F_2,F_3 \rightarrow F_2$ | $w_1 = 0.01$, $w_2 = 0.52$, $w_3 = 0.47$ |
| 2005/05 | $F_2$ | $F_2,F_7,F_2 \rightarrow F_2$ | $w_1 = 0.22$, $w_2 = 0.06$, $w_3 = 0.72$ |
| 2005/06 | $F_4$ | $F_2,F_2,F_7 \rightarrow F_4$ | $w_1 = 0.22$, $w_2 = 0.65$, $w_3 = 0.13$ |
| 2005/07 | $F_2$ | $F_4,F_2,F_2 \rightarrow F_2$ | $w_1 = 0.09$, $w_2 = 0.33$, $w_3 = 0.58$ |
| 2005/08 | $F_3$ | $F_2,F_4,F_2 \rightarrow F_3$ | $w_1 = 0.12$, $w_2 = 0.3$, $w_3 = 0.58$ |
| 2005/09 | $F_4$ | $F_3,F_2,F_4 \rightarrow F_4$ | $w_1 = 0.04$, $w_2 = 0.35$, $w_3 = 0.61$ |
| 2005/10 | $F_5$ | $F_4,F_3,F_2 \rightarrow F_5$ | $w_1 = 0.14$, $w_2 = 0.18$, $w_3 = 0.68$ |
| 2005/11 | $F_1$ | $F_5,F_4,F_3 \rightarrow F_1$ | $w_1 = 0.13$, $w_2 = 0.49$, $w_3 = 0.38$ |
| 2005/12 | $F_4$ | $F_1,F_5,F_4 \rightarrow F_4$ | $w_1 = 0.07$, $w_2 = 0.29$, $w_3 = 0.64$ |

**Table 9**
Forecasted RoC (Outpatient).

| Month | Outpatient | RoC(%) | Fuzzy | FLR | DRoC(%) | $F_{val}$ |
|---|---|---|---|---|---|---|
| 2004/01 | 6519 | | | – | | |
| 2004/02 | 5979 | −8.28348 | $F_2$ | – | | |
| 2004/03 | 6322 | 5.736745 | $F_5$ | – | | |
| 2004/04 | 5666 | −10.3765 | $F_1$ | – | | |
| 2004/05 | 5318 | −6.1419 | $F_2$ | $F_1,F_5,F_2 \rightarrow F_2$ | −4.00391 | 5439.139 |
| 2004/06 | 5364 | 0.864987 | $F_4$ | $F_2,F_1,F_5 \rightarrow F_4$ | −0.99609 | 5265.028 |
| 2004/07 | 5513 | 2.777778 | $F_4$ | $F_4,F_2,F_1 \rightarrow F_4$ | −8.33045 | 4917.155 |
| 2004/08 | 5895 | 6.929077 | $F_5$ | $F_4,F_4,F_2 \rightarrow F_5$ | −3.51108 | 5319.434 |
| 2004/09 | 6682 | 13.3503 | $F_6$ | $F_5,F_4,F_4 \rightarrow F_6$ | 1.634948 | 5991.38 |
| 2004/10 | 6254 | −6.40527 | $F_2$ | $F_6,F_5,F_4 \rightarrow F_2$ | 0.073964 | 6686.942 |
| 2004/11 | 5681 | −9.16214 | $F_2$ | $F_2,F_6,F_5 \rightarrow F_2$ | −4.23469 | 5989.162 |
| 2004/12 | 6012 | 5.826439 | $F_5$ | $F_2,F_2,F_6 \rightarrow F_5$ | −2.8125 | 5521.222 |
| 2005/01 | 5920 | −1.53027 | $F_3$ | $F_5,F_2,F_2 \rightarrow F_3$ | −6.46122 | 5623.552 |
| 2005/02 | 5512 | −6.89189 | $F_2$ | $F_3,F_5,F_2 \rightarrow F_2$ | −3.63333 | 5704.907 |
| 2005/03 | 6548 | 18.79536 | $F_7$ | $F_2,F_3,F_5 \rightarrow F_5$ | 1.966667 | 5620.403 |
| 2005/04 | 5987 | −8.5675 | $F_2$ | $F_7,F_2,F_3 \rightarrow F_2$ | −4.93802 | 6224.659 |
| 2005/05 | 5638 | −5.8293 | $F_2$ | $F_2,F_7,F_2 \rightarrow F_2$ | −5.83333 | 5637.758 |
| 2005/06 | 5851 | 3.777935 | $F_4$ | $F_2,F_2,F_7 \rightarrow F_4$ | −4.27778 | 5396.819 |
| 2005/07 | 5514 | −5.7597 | $F_2$ | $F_4,F_2,F_2 \rightarrow F_2$ | −6.6 | 5464.834 |
| 2005/08 | 5395 | −2.15814 | $F_3$ | $F_2,F_4,F_2 \rightarrow F_3$ | −4.5 | 5265.87 |
| 2005/09 | 5598 | 3.762743 | $F_4$ | $F_3,F_2,F_4 \rightarrow F_4$ | −1.23047 | 5328.616 |
| 2005/10 | 5284 | −5.60915 | $F_5$ | $F_4,F_3,F_2 \rightarrow F_5$ | −5.21484 | 5306.073 |
| 2005/11 | 4545 | −13.9856 | $F_1$ | $F_5,F_4,F_3 \rightarrow F_1$ | 1.22449 | 5348.702 |
| 2005/12 | 4624 | 1.738174 | $F_4$ | $F_1,F_5,F_4 \rightarrow F_4$ | 2.877778 | 4675.795 |

fuzzy logic relationships because forecasting depends only on two past observations.However, a lot of ambiguities arisen while designing of $2^{nd}$ order fuzzy relationships. As a result, we determined that forecasting accuracy of proposed model in term of MSE & AFER was not convincing as compared to existing models as shown in Table 11. Subsequently, model designed for $3^{rd}$ order that means, forecasting depends on three past observations. There were no ambiguities among fuzzy relationships. Moreover, results are promising in form of MSE and AFER than other methods. There-after, for $4^{th}$ order, accuracy is better but complexity increased. To make our model simple and efficient, we implemented it for $3^{rd}$ order.

### 7.2. Impact of defuzzification method

Comparative study of different defuzzification method for same outpatient data has been shown in below Table 12.

**Table 10**
Comparison of different Partitioning Method.

| Method | Random partition | Distribution based Partition | Average based Partition |
|---|---|---|---|
| MSE | 165755 | 185673 | 137428 |

**Table 11**
Impact of Order of Model.

| Order of Model | MSE | AFER(%) |
|---|---|---|
| $2^{nd}$ order | 343688.6 | 7.890893 |
| $3^{rd}$ order | 165755 | 5.14 |
| $4^{th}$ order | 136786.7 | 4.32 |

**Table 12**
Impact of type of defuzzification.

| Month | Outpatient | $F_{val}$ | $F_{val}$ | $F_{val}$ | $F_{val}$ |
|---|---|---|---|---|---|
| | | Centroid | Weighted | Average | Max |
| | | | Centroid | Method | Method |
| 2004/01 | 6519 | | | | |
| 2004/02 | 5979 | | | | |
| 2004/03 | 6322 | | | | |
| 2004/04 | 5666 | | | | |
| 2004/05 | 5318 | 5429.917 | 5439.139 | 5429.917 | 6090.95 |
| 2004/06 | 5364 | 5096.417 | 5265.028 | 5096.417 | 5716.85 |
| 2004/07 | 5513 | 5051.1 | 4917.155 | 5051.1 | 5498.1 |
| 2004/08 | 5895 | 5467.058 | 5319.434 | 5467.058 | 5650.825 |
| 2004/09 | 6682 | 5845.875 | 5991.38 | 5845.875 | 6042.375 |
| 2004/10 | 6254 | 6849.05 | 6686.942 | 6849.05 | 7517.25 |
| 2004/11 | 5681 | 6201.883 | 5989.162 | 6201.883 | 7035.75 |
| 2004/12 | 6012 | 5633.658 | 5521.222 | 5633.658 | 6391.125 |
| 2005/01 | 5920 | 5861.7 | 5623.552 | 5861.7 | 6462.9 |
| 2005/02 | 5512 | 5870.667 | 5704.907 | 5870.667 | 6364 |
| 2005/03 | 6548 | 5466.067 | 5620.403 | 5466.067 | 5925.4 |
| 2005/04 | 5987 | 6711.7 | 6224.659 | 6711.7 | 7693.9 |
| 2005/05 | 5638 | 6036.892 | 5637.758 | 6036.892 | 7034.725 |
| 2005/06 | 5851 | 5684.983 | 5396.819 | 5684.983 | 6624.65 |
| 2005/07 | 5514 | 5607.208 | 5464.834 | 5607.208 | 5997.275 |
| 2005/08 | 5395 | 5284.25 | 5265.87 | 5284.25 | 5651.85 |
| 2005/09 | 5598 | 5260.125 | 5328.616 | 5260.125 | 5529.875 |
| 2005/10 | 5284 | 5458.05 | 5306.073 | 5458.05 | 5737.95 |
| 2005/11 | 4545 | 5416.1 | 5348.702 | 5416.1 | 5680.3 |
| 2005/12 | 4624 | 4507.125 | 4675.795 | 4507.125 | 4885.875 |
| | | MSE= 246030.7 | MSE = 165755 | MSE = 245031.6 | MSE = 679328 |

## 8. Impact of OWA weight

All past fuzzy observations are assigned equal weight, this implies that assign $w_i = w_j = w_k = 1$ in Eq. (8). Accuracy of forecasting system gets degraded. From Table XIII, it can be seen that utilizing OWA weights for past observations reduced MSE to 165755 by almost 50% of MSE that is 246030.7 obtained by equal weights. Thus it is proved that accuracy of system is improved to a greatextent by determining relative weight of past observations.

## 9. Performance evaluation

In this section, we assessed the forecasting efficacy of our proposed model;on number of outpatient visits and juxtaposed the results with previous selective models [26,27,7] on same benchmark data. To verify, we too have used outpatient data of same year 2005. Forecasting accuracy is measured by average forecasting error rate (AFER) and mean squared error (MSE)by following equations which are defined by Eqs. (10) and (11):

$$AFER = (\sum_{t=1}^{n} (|A_t - F_t|/A_t))/n * 100\% \quad (10)$$

$$MSE = \sum_{t=1}^{n} (A_t - F_t)^2/n \quad (11)$$

Here, n is total number of time series data, $F_t$ is forecasted value and $A_t$ is actual value of time series data at time t.

### 9.1. Performance comparison with fuzzy times series models

Lower value of MSE and AFER are measure of higher forecasting accuracy. From the results of Table 14, MSE and AFER are lowest in case of proposed model and clearly indicating its superiority over than Cheng, Yu's, Chen model and other existing soft computing models.

Figs. 6 and 7 display MSE and AFER of all selected models for outpatient visits graphically.

#### 9.1.1. Discussion

From Table 13, Figs. 6 and 7, it can be seen that MSE and AFER are lowest in case of proposed method and clearly indicating its transcendence over existing Yu [26], Chen [7] and Cheng [27] forecasting models for number of outpatient visits. Proposed method utilized rate change of time series data rather than data itself. Subsequently, FLR are defined on basis of impact of past fuzzy observation on current fuzzy observation. It used 3rd order time series model, sufficient to remove ambiguities while preserving efficiency. Uniqueness of presented model is priority matrix that determined the importance of each fuzzy observation in the system. On the other hand, existing fuzzy time series model did not count the importance of each past fuzzy observation to predict next value. They treated all past fuzzy observation equally. So, dominant reason of accurateness of proposed method is to generate OWA weight for each past observation that contributes in prediction of next value on basis of importance of corresponding fuzzy value in the system.

### 9.2. Comparison of proposed method with conventional techniques for forecasting

The ARIMA model is a precise forecasting model for short time periods. Nevertheless, limitation of ARIMA model is that a large amount of historical data (at least 50 and preferably 100 or more) is required. However, sometimes, it is crucial to forecast future situations using little data in a short span of time due to factors of uncertainty. Proposed model utilized the concept of measurement error to deal with the differences between estimators and observations. The benefit of proposed model makes it possible for decision makers to predict possible situations based on fewer observations

**Table 13**
OWA Weights/Equal Weights.

| Month | Outpatient | $F_{val}$ ($w_1 = w_2 = w_3$) | SE ($w_1 = w_2 = w_3$) | $F_{val}$ (OWA weights) | Proposed-SE (OWA weights) |
|---|---|---|---|---|---|
| 2004/01 | 6519 | | | | |
| 2004/02 | 5979 | | | | |
| 2004/03 | 6322 | | | | |
| 2004/04 | 5666 | | | | |
| 2004/05 | 5318 | 5429.917 | 12525.34 | 5439.139 | 14674.58 |
| 2004/06 | 5364 | 5096.417 | 71600.84 | 5265.028 | 9795.509 |
| 2004/07 | 5513 | 5051.1 | 213351.6 | 4917.155 | 355031.7 |
| 2004/08 | 5895 | 5467.058 | 183134.1 | 5319.434 | 331276.1 |
| 2004/09 | 6682 | 5845.875 | 699105 | 5991.38 | 476955.7 |
| 2004/10 | 6254 | 6849.05 | 354084.5 | 6686.942 | 187439 |
| 2004/11 | 5681 | 6201.883 | 271319.4 | 5989.162 | 94963.97 |
| 2004/12 | 6012 | 5633.658 | 143142.4 | 5521.222 | 240863.2 |
| 2005/01 | 5920 | 5861.7 | 3398.89 | 5623.552 | 87881.7 |
| 2005/02 | 5512 | 5870.667 | 128641.8 | 5704.907 | 37212.98 |
| 2005/03 | 6548 | 5466.067 | 1170580 | 5620.403 | 860436.8 |
| 2005/04 | 5987 | 6711.7 | 525190.1 | 6224.659 | 56481.65 |
| 2005/05 | 5638 | 6036.892 | 159114.6 | 5637.758 | 0.058403 |
| 2005/06 | 5851 | 5684.983 | 27561.53 | 5396.819 | 206280.5 |
| 2005/07 | 5514 | 5607.208 | 8687.793 | 5464.834 | 2417.296 |
| 2005/08 | 5395 | 5284.25 | 12265.56 | 5265.87 | 16674.56 |
| 2005/09 | 5598 | 5260.125 | 114159.5 | 5328.616 | 72567.63 |
| 2005/10 | 5284 | 5458.05 | 30293.4 | 5306.073 | 487.2194 |
| 2005/11 | 4545 | 5416.1 | 758815.2 | 5348.702 | 645937 |
| 2005/12 | 4624 | 4507.125 | 13659.77 | 4675.795 | 2682.722 |
| | | | MSE= 246030.7 | | MSE = 165755 |

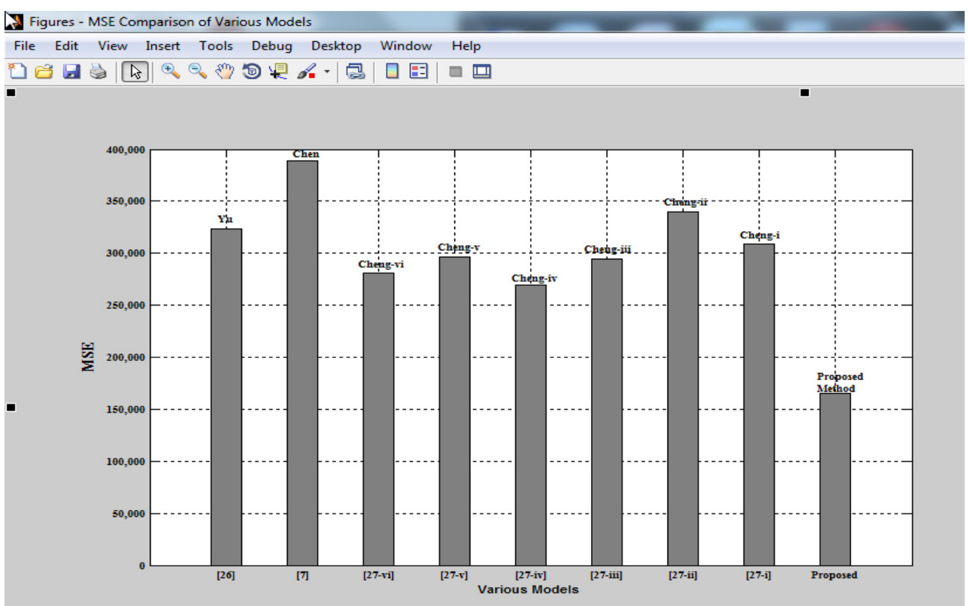


**Fig. 6.** MSE Compaison (Outpatient).

**Table 14**
Comparison of AFER & MSE (Outpatient).

| Method Name | Concept used | AFER (%) | MSE |
|---|---|---|---|
| Proposed – Method (OWA-TSM) | Fuzzy time series and OWA | 3.06 | 165755 |
| Cheng et al.-(i) [27] | Equal triangle G-S Method | 7.79 | 308823 |
| Cheng et al.-(ii) [27] | Equal triangle Exp-Method | 8.49 | 339317 |
| Cheng et al.-(iii) [27] | TFA G-S Method | 7.52 | 294147 |
| Cheng et al.-(iv) [27] | TFA Exp Method | 7.48 | 269189 |
| Cheng et al.-(v) [27] | MEPA G-S Method | 8.00 | 296103 |
| Cheng et al.-(vi) [27] | MEPA Exp Method | 7.88 | 280579 |
| Chen [7] | High Order FTS | 9.77 | 388571 |
| Yu [26] | Fuzzy-Recurrence | 8.6 | 322971 |

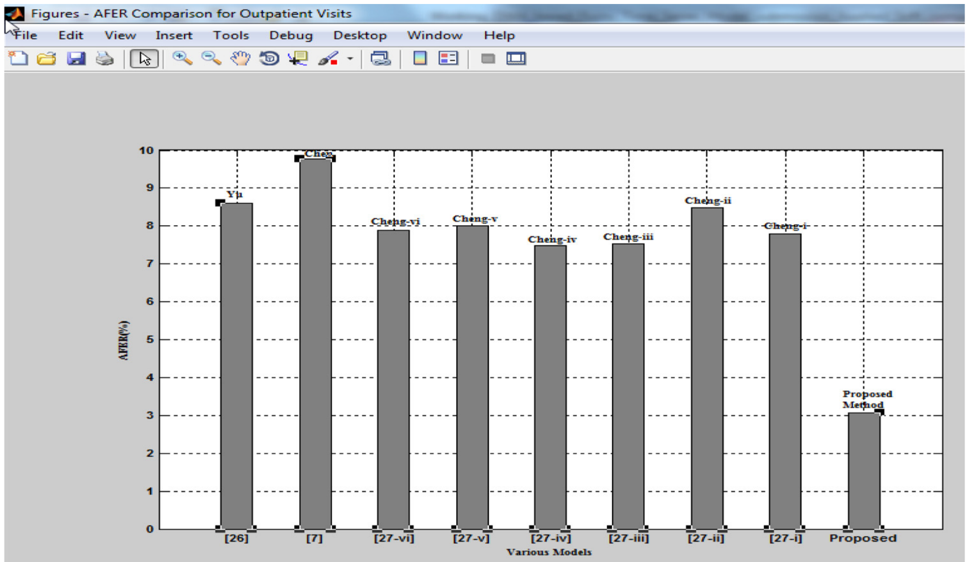


**Fig. 7.** (a) AFER Comparison (Outpatient).

**Table 15**
Performance Comparison between MA/ARIMA/Proposed.

| Name of Model | MSE | AFER (%) |
|---|---|---|
| Moving Average | 4370448 | 5.98% |
| Simple exponential smoothing model | 318355.095 | 2.97% |
| Brown's exponential smoothing model | 272886.54 | 2.68% |
| ARIMA(1,0,0) model | 271069.55 | 2.76% |
| ARIMA(1,0,1) model | 267447.86 | 2.78% |
| Proposed Model | 165755 | 3.06% |

than the ARIMA model. The MSE and AFER have been juxtaposed of our proposed model with MA/ARIMA as shown in Table 14.

## 10. Extensive applications

Accurateness and applicability of proposed model has been tested & proved in other applications domains as well (Table 15).

### 10.1. Forecasting of TAIEX stock data

The financial forecasting is very prevalent for academy researchers and common investors because if accurate predictions are made then myriad profit can be attained. Stock fund and financial analysts predict price activity in the stock market on the basis of either the assistance of stock analyzing tools or their professional knowledge. Stock analysts make every effort to discover ways to predict stock price accurately. However, forecasting stock returns are difficult because market volatility needs to be captured in an implemented and used model. In stock market forecasting, high-order time-series model utilize the previous several periods of stock prices. TAIEX stock market data from year 2000–2003 is used for the new empirical analysis & experimental study to demonstrate the efficiency of proposed model. The data can be downloaded from http://finance.yahoo.com. The data from January to December for each year are used for forecasting. In addition, results from the TAIEX forecasting are also compared with the other models on same benchmark data. The root of mean square error (RMSE) as defined in Eq. (12) is taken as the evaluation metric to inspect forecasting performance of proposed model. Basically, random partition of U is exercised when the Universe of discourse is divided equally by the length chosen into n number of intervals. Triangular membership function is applied to fuzzify the data. Result will be same for any type of membership function. 3rd order fuzzy logic relationship are employed and centroid defuzzification method is used for defuzzification process Since same metric is taken by others models for same data. The comparative study of proposed model to other models on same TAIEX data is shown in Table 16.

RMSE can be defined as:$RMSE = \sqrt{\sum_{t=1}^{n}(A_t - F_t)^2/n}(12)$

**Table 16**
Performance Comparison for TAIEX Stock Data.

| Models | 2000 | 2001 | 2002 | 2003 | Average |
|---|---|---|---|---|---|
| AR(1) | 375 | 515 | 98 | 97 | 271.25 |
| AR(2) | 376 | 506 | 99 | 92 | 268.25 |
| Chen [3] | 176 | 148 | 101 | 74 | 124.75 |
| Yu [8] | 191 | 167 | 75 | 66 | 124.75 |
| Yu and Hurang [9] | 150 | 99 | 79 | 59 | 96.75 |
| Chang et al. [10] | 173 | 119 | 61 | 53 | 101.5 |
| M.Y.Chen(3rd Order) [11] | 108 | 88 | 60 | 42 | 74.5 |
| Quiesn et al. [12] | 131.53 | 112.59 | 60.33 | 51.54 | 84.75 |
| Cheng et al. [13] | 130 | 120 | 66 | 55 | 92.75 |
| Proposed Model | 128.3 | 119.45 | 69.56 | 61.34 | 94.65 |

**Table 17**
Comparison of MSE & AFER (OWA-OTSM:Enroll).

| Method | MSE | AFER |
|---|---|---|
| Song Chissom [5] | 775687 | 4.38% |
| Song [6] Chissom – adv | 407507 | 3.11% |
| Chen [7] | 321418 | 3.12% |
| Hwang chen & lee [11] | 226611 | 2.45% |
| Hunarg [9] | 227194 | 2.39% |
| Proposed Method | 174390.9 | 1.653621% |

### 10.2. Forecasting of enrollment data of university of Alabama

Further, we evaluated our proposed model in education domain on Enrolment data of University of Alabama and compared the results with previous prediction models [5–7,11,9] on same bench mark data to demonstrate the performance of our method. Details of parameters for enrolment data is: Random partition of U is exercised, Triangular membership function is applied for fuzzify the data, 3rd order fuzzy logic relationships are employed and centroid defuzzification method is used for defuzzification process. Forecasting accuracy is measured by average forecasting error rate (AFER) and mean squared error (MSE) are shown in Table 17.

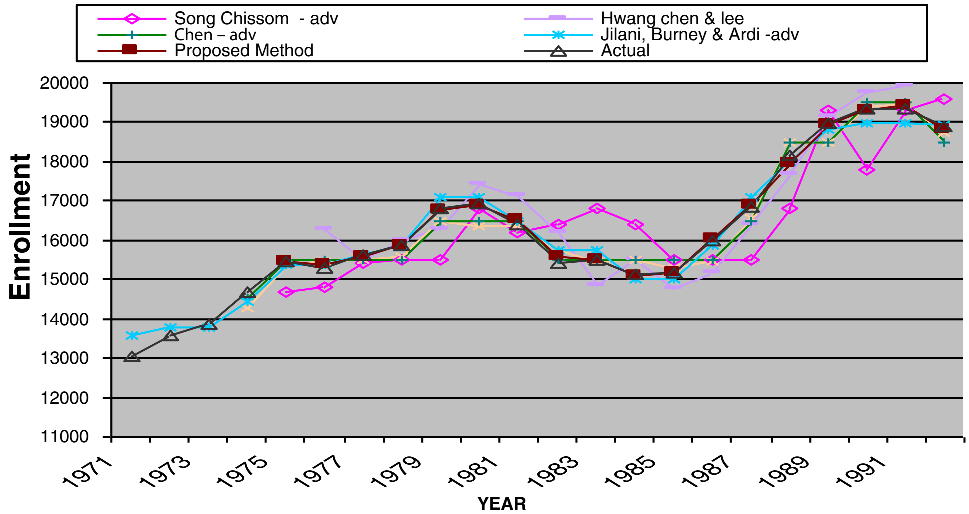


**Fig. 8.** Actual vs Forecasted (OWA-OTSM:Enroll).

**Table 18**
Randomly increased of Outpatient by 5%.

| Month | Outpatient | RoC (%) | Fuzzy | OWA Weights | | | DRoC (%) | $F_{VAL}$ | SE= $(A_i - F_i)^2$ | $\|A_i - F_i\|/A_i$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | $W_1$ | $W_2$ | $W_3$ | | | | |
| 2004/01 | 6519 | | | | | | | | | |
| 2004/02 | 5979 | −8.28 | $F_2$ | | | | | | | |
| 2004/03 | 6322 | 5.74 | $F_5$ | | | | | | | |
| 2004/04 | 5949.3 | −5.9 | $F_2$ | | | | | | | |
| 2004/05 | 5318 | −10.61 | $F_1$ | 0.15 | 0.22 | 0.63 | −4.923 | 5656.426 | 114531.83 | 6.364 |
| 2004/06 | 5364 | 0.86 | $F_4$ | 0.01 | 0.44 | 0.56 | −1.354 | 5245.985 | 13927.442 | 2.2 |
| 2004/07 | 5513 | 2.78 | $F_4$ | 0.18 | 0.07 | 0.75 | −9.413 | 4859.072 | 427621.24 | 11.862 |
| 2004/08 | 5895 | 6.93 | $F_5$ | 0.21 | 0.64 | 0.15 | 1.021 | 5569.272 | 106098.9 | 5.526 |
| 2004/09 | 6682 | 13.35 | $F_6$ | 0.06 | 0.33 | 0.61 | 1.875 | 6005.531 | 457609.97 | 10.124 |
| 2004/10 | 6254 | −6.41 | $F_2$ | 0.05 | 0.24 | 0.71 | 0.666 | 6726.481 | 223238.08 | 7.555 |
| 2004/11 | 5681 | −9.16 | $F_2$ | 0.25 | 0.26 | 0.49 | −2.296 | 6110.413 | 184395.75 | 7.559 |
| 2004/12 | 6012 | 5.83 | $F_5$ | 0.17 | 0.51 | 0.32 | −1.064 | 5620.553 | 153230.46 | 6.511 |
| 2005/01 | 5920 | −1.53 | $F_3$ | 0.05 | 0.32 | 0.63 | −6.759 | 5605.633 | 98826.401 | 5.31 |
| 2005/02 | 5787.6 | −2.24 | $F_3$ | 0.1 | 0.22 | 0.68 | −3.73 | 5699.156 | 7822.297 | 1.528 |
| 2005/03 | 6548 | 13.14 | $F_7$ | 0.13 | 0.38 | 0.49 | 1.76 | 5889.474 | 433657.06 | 10.057 |
| 2005/04 | 5987 | −8.57 | $F_2$ | 0.03 | 0.31 | 0.66 | −3.507 | 6318.365 | 109802.95 | 5.535 |
| 2005/05 | 5638 | −5.83 | $F_2$ | 0.25 | 0.16 | 0.59 | −3.418 | 5782.342 | 20834.713 | 2.56 |
| 2005/06 | 5851 | 3.78 | $F_4$ | 0.19 | 0.57 | 0.23 | −1.641 | 5545.502 | 93329.295 | 5.221 |
| 2005/07 | 5514 | −5.76 | $F_2$ | 0.09 | 0.33 | 0.58 | −6.6 | 5464.834 | 2417.296 | 0.892 |
| 2005/08 | 5664.75 | 2.73 | $F_4$ | 0.12 | 0.3 | 0.58 | −4.5 | 5265.87 | 159105.25 | 7.041 |
| 2005/09 | 5598 | −1.18 | $F_3$ | 0.1 | 0.37 | 0.53 | −1.683 | 5569.422 | 816.694 | 0.511 |
| 2005/10 | 5284 | −5.61 | $F_2$ | 0.08 | 0.3 | 0.63 | −5.247 | 5304.278 | 411.188 | 0.384 |
| 2005/11 | 4772.25 | −9.68 | $F_2$ | 0.15 | 0.29 | 0.56 | 0.478 | 5309.278 | 288399.5 | 11.253 |
| 2005/12 | 4624 | −3.11 | $F_3$ | 0.14 | 0.41 | 0.46 | 2.5 | 4891.556 | 71586.347 | 5.786 |
| | | | | | | | | | MSE= 107250.75 | AFER= 5.548249 |

Fig. 8 demonstrates actual versus forecasted enrolment data. From the results of Table XVI, MSE and AFER are lowest in case of proposed model, clearly indicating its superiority over other soft computing models. Previous fuzzy based forecasting model [5–7,11,9] did not count the importance of relative position of past fuzzy observation to predict next value. However, our model generates weights for past observations so that most recent observations can be assigned relatively higher weight resulting in more accurate predicted value.

## 11. Robustness of proposed model

Robustness of proposed model is tested in this section. It is examined and proved that proposed method still performs well if the historical time series data are not precise. Two different cases have been taken to check the robustness of the model.

### 11.1. Case 1

Aforethought the same situation for checking the robustness of the forecasting model which is exercised by Song and Chissom [4], Chen [7] and Tsuar [39]. Increase the historical time series data by 5% of randomly selected years. Here, we selected years randomly are: 2004/4, 2005/2, 2005/8 and 2005/11. The Table 18 illustrates actual values; weights generated and forecasted value after increasing the 5% value on some selected year. MSE = 107250.75 & ranges of forecasting errors are from 0.384% to 11.862% and average error is 5.548249% against the MSE = 165755 & ranges of forecasting errors are from 0.00428% to 17.683% and the average error 3.06%. This represents the robustness of the proposed method by proving that the mean square error does not grow with increasing value oftime series data. In fact, in this case, eventually abatement of mean square error took place. Moreover, range of forecasting errors are also decreased.

**Table 19**
Randomly increase of Outpatient by 7% & decrease by 6%.

| Month | Outpatient | RoC (%) | Fuzzy | OWA Weights | | | DRoC (%) | $F_{VAL}$ | SE= $(A_i - F_i)^2$ | $\|A_i - F_i\|/A_i$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 2004/01 | 6519 | | | $W_1$ | $W_2$ | $W_3$ | | | | |
| 2004/02 | 5979 | −8.28 | $F_2$ | | | | | | | |
| 2004/03 | 6322 | 5.74 | $F_5$ | | | | | | | |
| 2004/04 | 6062.62 | −4.10 | $F_3$ | | | | | | | |
| 2004/05 | 5318 | −12.28 | $F_1$ | 0.08 | 0.30 | 0.63 | −3.44 | 5853.98 | 287278.36 | 10.08 |
| 2004/06 | 5042.16 | −5.19 | $F_2$ | 0.07 | 0.29 | 0.64 | 0.66 | 5352.86 | 96536.01 | 6.16 |
| 2004/07 | 5513 | 9.34 | $F_5$ | 0.19 | 0.28 | 0.53 | −8.22 | 4627.56 | 784003.08 | 16.06 |
| 2004/08 | 5895 | 6.93 | $F_5$ | 0.12 | 0.46 | 0.42 | −1.65 | 5421.91 | 223812.72 | 8.03 |
| 2004/09 | 6682 | 13.35 | $F_6$ | 0.10 | 0.30 | 0.60 | 1.50 | 5983.59 | 487778.91 | 10.45 |
| 2004/10 | 6254 | −6.41 | $F_2$ | 0.02 | 0.31 | 0.67 | −0.36 | 6658.14 | 163325.68 | 6.46 |
| 2004/11 | 5681 | −9.16 | $F_2$ | 0.22 | 0.14 | 0.64 | −4.66 | 5962.84 | 79434.66 | 4.96 |
| 2004/12 | 6012 | 5.83 | $F_5$ | 0.19 | 0.57 | 0.23 | −2.81 | 5521.22 | 240863.17 | 8.16 |
| 2005/01 | 5920 | −1.53 | $F_3$ | 0.09 | 0.33 | 0.58 | −6.15 | 5642.26 | 77138.40 | 4.69 |
| 2005/02 | 5897.84 | −0.37 | $F_3$ | 0.08 | 0.30 | 0.63 | −2.67 | 5761.95 | 18465.92 | 2.30 |
| 2005/03 | 6548 | 11.02 | $F_6$ | 0.10 | 0.29 | 0.61 | 3.11 | 6081.00 | 218093.13 | 7.13 |
| 2005/04 | 5627.78 | −14.05 | $F_1$ | 0.03 | 0.31 | 0.66 | −3.51 | 6318.37 | 476908.03 | 12.27 |
| 2005/05 | 5638 | 0.18 | $F_4$ | 0.13 | 0.17 | 0.70 | −3.37 | 5438.25 | 39900.31 | 3.54 |
| 2005/06 | 5499.94 | −2.45 | $F_3$ | 0.11 | 0.49 | 0.40 | 2.38 | 5771.99 | 74010.93 | 4.95 |
| 2005/07 | 5899.98 | 7.27 | $F_5$ | 0.17 | 0.27 | 0.56 | −5.76 | 5182.93 | 514161.32 | 12.15 |
| 2005/08 | 5395 | −8.56 | $F_2$ | 0.18 | 0.43 | 0.38 | −3.16 | 5713.35 | 101345.44 | 5.90 |
| 2005/09 | 5598 | 3.76 | $F_4$ | 0.15 | 0.37 | 0.48 | −1.96 | 5289.26 | 95317.51 | 5.52 |
| 2005/10 | 4966.96 | −11.27 | $F_1$ | 0.04 | 0.36 | 0.61 | −5.37 | 5297.33 | 109141.81 | 6.65 |
| 2005/11 | 4545 | −8.50 | $F_2$ | 0.08 | 0.17 | 0.75 | −0.84 | 4925.15 | 144511.21 | 8.36 |
| 2005/12 | 4947.68 | 8.86 | $F_5$ | 0.25 | 0.37 | 0.38 | 0.59 | 4571.67 | 141385.71 | 7.60 |
| | | | | | | | | | 167531.6 | 6.145651 |

### 11.2. Case 2

At this juncture, we performed more arduous testing of the model in circumstances of non-accuracy in time series data set. Assuming same time series data but increased number of outpatient visits by 7% of the randomly selected years 2004/04, 2005/02, 2005/07 & 2005/12 and decreasing by 6% of randomly selected years 2004/06, 2005/04, 2005/08, 2005/10 and leaving unchanged for the remaining years. The forecasted and actual numbers of outpatient visits are shown in Table 19. MSE = 167531.6 & ranges of forecasting errors are from 2.30% to 16.08% and average error is 6.145651% against the MSE = 165755 & ranges of forecasting errors from 0.00428% to 17.683% and the average error 3.06%. Of course, very small inaccuracy occurred into system as compared to accuracy calculated by proposed system. Nevertheless, still very smaller than Yu [26], Cheng [27] and Chen [7] models.

## 12. Conclusion and future work

The paper attempted to present novel concept of forecasting by amalgamating OWA operators with fuzzy time series to predict number of outpatient visit and ascertained that use of OWA operators is very effective in time series analysis.The proposed concept commenced the designing of proposed model by determining rate change of time series data. So that, increasing and decreasing rate of time series can be captured. The paper proved that accuracy depends on relative weight of past observations and method of aggregation of past observation is important factor where next prediction depends only on past observations. The concept of priority matrix is given whose task is to calculate the priority of each fuzzy set on the basis of number of occurrence of fuzzy set. Henceforth, concept of priority matrix can be utilized in devising new fuzzy time series model. The paper derived 3rd order fuzzy rules. Intent of using 3rd order fuzzy time series model is to obtain FLR, free from ambiguities. Weighted centroid aggregation method has been applied to defuzzify the value. An experiment study is performed to forecast number of outpatient visits due to its significance in health care domain. Values of MSE and AFER obtained by this research indicate the weight determination of fuzzy relationship using OWA can improve prediction accuracy by a considerable factor. We have proved that joint consideration of OWA and fuzzy time series is success to enhance the efficiency of forecasting model.It can be seen that proposed model has shown a considerable improvement in accuracy over conventional models like ARIMA, MA, ARCH, etc. From the comparative study, MSE and AFER are lowest in case of proposed method, clearly indicating its transcendence over existing like Yu, Chen and Cheng forecasting models for number of outpatient visits. In addition, impact of order of model and OWA has been carried out. Also, robustness of proposed model is proved by increasing and decreasing time series data randomly. It is examined and proved that proposed method still performs well if the historical time series data isnot precise. Extensive experiment is performed to forecast TAIEX stock market data. Presented fuzzy-OWA based forecasting model can be used to deal with future trends in health care domain.Presented model can also be considered as a strong standard methodology for planning and management in health care and other domain also.

In the future work, other stock data sets such as (HSI http://www.hsi.com/HSI-Net/); NASDAQ(http://www.nasdaq.com/)to validate proposed model. Genetic algorithms can be further explored to optimize parameters of fuzzy time series such as membership functions, length of interval and number of intervals. The accuracy of proposed model can be improved by employing other machine learning methods such SVM and rough sets.

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## References

[1] B. Garg, M.M.S. Beg, A.Q. Ansari, B.M. Imran, Inferential historical survey of time series predication using artificial neural network, in: Proc. 3rd International Conference on Data Management, Ghaziabad, India, 2010, pp. 158–172.

[2] L.A. Zadeh, The concept of a linguistic variable and its application to approximate reasoning-II, Inf. Sci. 8 (4) (1975) 301–357.

[3] L.A. Zadeh, Fuzzy sets, Inf. Control 8 (1965) 333–353 (Pg/).
[4] Q. Song, B.S. Chissom, Fuzzy time series and its models, Fuzzy Sets Syst. 54 (1993) 269–277 (Pg).
[5] Q. Song, B.S. Chissom, Forecasting enrollments with fuzzy time series: part II, Fuzzy Sets Syst. 62 (1994) 1–8.
[6] Q. Song, A note on fuzzy time series model selection with sample autocorrelation functions, an international, J. Cybern. Syst. 34 (2003) 93–107 (Pg).
[7] S.M. Che, Forecasting enrollments based on fuzzy time series, Fuzzy Sets Syst. 81 (1996) 311–319.
[8] S.M. Chen, Forecasting enrollments based on high order fuzzy time series, Int. J. Cybern. Syst. 33 (2002) 1–16.
[9] K. Huarng, Effective lengths of intervals to improve forecasting in fuzzy time series, Fuzzy Sets Syst. 12 (2001) 387–394.
[10] K. Huarng, Heuristic models of fuzzy time series for forecasting, Fuzzy Sets Syst. 123 (2002) 369–386.
[11] J.R. Hwang, S.M. Chen, C.H. Lee, Handling forecasting problems using fuzzy time series, Fuzzy Sets Syst. 100 (1998) 217–228.
[12] L.W. Lee, L.W. Wang, S.M. Chen, Handling forecasting problems based on two-Factors high-Order time series, IEEE Trans. Fuzzy Syst. 14 (2006) 468–477.
[13] H. Li, R. Kozma, A dynamic neural network method for time series prediction using the KIII model, Proceedings of the 2003 International Joint Conference on Neural Networks (2003) 347–352.
[14] S.R. Singh, A robust method of forecasting based on fuzzy time series, International J. Appl. Math. Comput. 188 (2007) 472–484.
[15] S.R. Singh, A computational method of forecasting based on fuzzy time series, Int. J. Math. Comput. Simul. 79 (2008) 539–554.
[16] C.H.L. Lee, A. Lin, W.S. Chen, Pattern discovery of fuzzy time series for financial prediction, IEEE Trans. Knowl. Data Eng. 18 (2006) 613–625.
[17] K.H. Huanrg, T.H.K. Yu, Y.W. Hsu, A multivariate heuristic model for forecasting, IEEE Trans. Syst. Man Cybern. 37 (2007) 836–846.
[18] U. Yolcu, E. Egrioglu, R. Vedide, R. Uslu, M.A. Basaran, C.H. Aladag, A new approach for determining the length of intervals for fuzzy time series, Appl. Soft Comput. 9 (2009) 647–651.
[19] W. Qiu, X. Liu, H. Li, A generalized method for forecasting based on fuzzy time series, Expert Syst. Appl. 38 (2011) 10446–10453.
[20] T.A. Jilani, S.M.A. Burney, C. Ardil, Multivariate high order fuzzy time series forecasting for car road accidents, Int. J. Comput. Intell. 4 (2007) 15–20.
[21] B. Garg, M.M.S. Beg, A.Q. Ansari, B.M. Imran, Fuzzy Time Series Prediction Model, Communications in Computer and Information Science, 141, Springer−Verlag, Berlin Heidelberg, 2011, pp. 126–137 (ISBN978-3-642-19423-8).
[22] B. Garg, M.M.S. Beg, A.Q. Ansari, B.M. Imran, Soft Computing Model to Predict Average Length of Stay of Patient, Communications in Computer and Information Science, 141, Springer Verlag, Berlin Heidelberg, 2011, pp. 221–232 (ISBN978-3-642-19423-8).
[23] B. Garg, M.M.S. Beg, A.Q. Ansari, Enhanced accuracy of fuzzy time series predictor using genetic algorithm, in: Third IEEE World Congress on Nature and Biologically Inspired Computing (NaBIC2011), Spain, 2011, pp. 273–278.
[24] B. Garg, M.M.S. Beg, A.Q. Ansari, Employing genetic algorithm to optimize OWA-Fuzzy forecasting model, in: Third IEEE World Congress on Nature and Biologically Inspired Computing (NaBIC2011), Spain, 2011, pp. 285–290.
[25] B. Garg, M.M.S. Beg, A.Q. Ansari, A new computational fuzzy time series model to forecast number of outpatient visits, in: Proc. 31 St Annual Conference of the North American Fuzzy Information Processing Society (NAFIPS 2012), University of California at Berkeley, USA August 6–8, 2012, pp. 1–6.
[26] H.K. Yu, Weighted fuzzy time series models for TAIEX forecasting, Physica 349 (2005) 609–624.
[27] Ching-Hsue Cheng, Jia-Wen Wang, Chen-Hsun Li, Forecasting the number of outpatient visits using a new fuzzy time series based on weighted-Transitional matrix, Expert Syst. Appl. 34 (2008) 2568–2575.
[28] E.M. Knod, R.J. Schonberger, Operations Management-Meeting Customers' Demands, 7th ed., McGraw Hill international, 2001.
[29] W. Goldman, J. McCulloch, M.R. Stur, Costs and use of mental health services before and after managed care, Health Aff. (Millwood) 17 (2) (1998) 40–52.
[30] R.R. Yager, On ordered weighted averaging aggregation operators in multi criteria decision making, IEEE Trans. Syst. Man Cybern. SMC-18 (1) (1988) 183–190.
[31] R.R. Yager, Time series smoothing and OWA aggregation, IEEE Trans. Fuzzy Syst. 16 (No-4) (2008).
[32] M.M.S. Beg, User feedback based enhancement in web search quality, Int. J. Inf. Sci., Elsevier 170 (2005) 153–172.
[33] R. Sadiq, S. Tesfamariam, Developing environmental indices using fuzzy numbers ordered weighted averaging (FN-OWA) operators, Int. J. Stochastic Environ. Res. Risk Manage. 22 (No. 1) (2008) 494–505.
[34] C.H. Cheng, S.F. Huang, Forecasting of ozone concentration using frequency MA-OWA model, in: IEEE International Conference on System, Man and Cybernetics, San Antonio, TX, USA, 2009.
[35] G.E.P. Box, G.M. Jenkins, G. Reinsel, Time Series Analysis, Forecasting, and Control, Holden-Day, Inc., San Francisco Calif, 1994 (ISBN-13: 978-0130607744).
[36] P.J. Brockwell, R.A. Davis, Introduction to Time Series & Forecasting, Springer-Verlag, 2002 (ISBN 0-387-95351-5).
[37] R.R. Yager, Quantifier weighted averaging using OWA operators, Int. J. Intell. Syst. 11 (1996) 49–73.
[38] R.R. Yager, D.P. Filev, Parameterized andlike and orlike OWA operators, Int. J. Gen. Syst. 22 (1996) 297–316.
[39] R.C. Tsaur, J.C.O. Yang, H.F. Wang, Fuzzy relation analysis in fuzzy time series model, Comput. Math. Appl. 49 (2005) 539–548 (Pg).
[40] W. Pedrycz, Why triangular membership function, IEEE Trans. Fuzzy Set Syst. 64 (1994) 21–30.
[41] G.P. Box, G.M. Jenkins, Time Series Analysis: Forecasting and Control, Holden-day Inc., San Francisco CA, 1976.
[42] M.-Y. Chen, B.-T. Chen, Online fuzzy time series analysis based on entropy discretization and a fast fourier transform, Appl. Soft Comput. 14 (2013) 156–166.
[43] Q. Cai, D. Zhang, W. Zheng, S.C.H. Leung, A new wieghted fuzzy time series forecasting model combined with ant colony optimization and auto-regression, Knowl. Based Syst. 74 (2015) 61–68.
[44] S.M. Chen, C.D. Chen, TAIEX forecasting based on fuzzy time series and fuzzy variation groups, IEEE Trans. Fuzzy Syst. 19 (2011) 1–12.
[45] Mu-Yen Chen, A high-order fuzzy time series forecasting model for internet stock trading, Future Gener. Comput. Syst. 37 (2014) 461–467.
[46] B. Sun, H. Guo, H.R. Karimi, Y. Ge, S. Xiong, Prediction of stock index futures prices based on fuzzy sets and multivariate fuzzy time series, Neurocomputing 151 (2015) 1528–1536.
[47] R. Fullerand, P. Majlender, An analytic approach for obtaining maximal entropy OWA operator weights, Fuzzy Sets Syst. 124 (2001) 53 (Pg57).
[48] S.B. Ahn, On the Properties of OWA operator weights function with constant level of Orness, IEEE Trans. Fuzzy Syst. 14 (2006) 511–515.
[49] J.R. Cheng, C.H. Cheng, MCDM aggregation model by ME-OWA and ME-OWGA operators, international journal of uncertainty, Fuzziness Knowl. Based Syst. 14 (2006) 421–443.
[50] C.H. Cheng, J.W. Wang, M.C. Wu, OWA weighed based clustering method for classification problems, Expert Syst. Appl. 36 (2009) 4988–4995.
[51] C.H. Cheng, L.Y. Wei, J.W. Liu, T.L. Chen, OWA based ANFIS model for TAIEX forecasting, Econ. Model. 30 (2013) 442–448.
[52] J.R. Chang, A weighted fuzzy time series model on adoptive OWA operators, International Conference on Uncertainty Reasoning and Knowledge Engineering 2 (2011) 94–97 (ISBN:978-1-4244-9984-7).