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Bollinger bands approach on boosting ABC algorithm and its variants

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ABSTRACT

In this study, a new algorithm that will improve the performance and the solution quality of the ABC (artificial bee colony) algorithm, a swarm intelligence based optimization algorithm is proposed. ABC updates one parameter of the individuals before the fitness evaluation. Bollinger bands is a powerful statistical indicator which is used to predict future stock price trends. By the proposed method an additional update equation for all ABC-based optimization algorithms is developed to speed up the convergence utilizing the statistical power of the Bollinger bands. The proposed algorithm was tested against classical ABC algorithm and recent ABC variants. The results of the proposed method show better performance in comparison with ABC-based algorithm with one parameter update in convergence speed and solution quality.

1. Introduction

Bollinger band (BB) is a technical analysis tool which is invented to predict the future stock prices. BB is used to predict maximum and minimum future prices referring from the past prices. BB indicator consists of three lines: an upper band, a lower band, and middle band which represents the moving average of the past prices. If the stock price catches up on the upper band, then the stock is considered as overbought; if price catches up on the lower band, then it is considered as oversold. In this study, working principles of the BB is applied to improve employed bee performance of the ABC algorithm and its variants by adding a second parameter update rule to ABC based algorithms based on BB. In this work, best fitness valued employed bees are used as data series to calculate the BB.

In nature, most of the animals live in a group which requires collaboration across group members. Each individual exhibits an intelligent behavior according to the role assigned to it inside the group. This kind of social behavior ensures the animal groups to get more food or have a better defense with less energy consumption. Swarm Intelligence is an Artificial Intelligence technique inspired by the social behaviors of insect or animal groups.

Optimization is selecting the best parameters among the all available to maximize or minimize a function. For example, the traveling salesman problem, irregular object allocation problem, vehicle routing problem are some of the best known engineering optimization problems. These problems may have very large search space according to parameter counts, and brute force approaches may take years to find the optimum parameters. So, people looked at the nature to see how nature solves its own optimization problems. One naturally inspired optimization method is genetic algorithms [1]. Genetic algorithm is inspired by natural selection and the theory of evaluation. Another naturally inspired optimization is artificial immune systems by Castro and Zuben [2]. In years following the discovery of genetic algorithms, researchers discovered another method of nature in optimization which is called swarm intelligence. One of the popular swarm intelligence based algorithms is particle swarm optimization algorithm by Elbart and Kennedy [3]. The algorithm is inspired by social behavior of birds flocking, and it is a population based stochastic optimization algorithm that is similar to the other swarm intelligence based algorithms. Ant colony algorithm developed by Dorigo et al. [4] which is a popular intelligence based optimization algorithm simulates the food carrying mechanism in the ant colonies.

Bees are also good example for swarms. Bee system by Lucic and Teodorovic [5], The Bees Algorithm by Pham et al. [6] and the recent method developed by Diwold et al. [7] are the examples for bee inspired optimization algorithms. This study focused on Artificial Bee Colony (ABC) algorithm which is inspired by the foraging behavior of bees. In the algorithm, global search process is realized by three types of the bees. First one is employed bees. Each employee bee represents a possible solution or food source. The second kind of the bees is onlooker bees. The task of that type of bees is to search better solutions around the current solutions. Better food sources or solutions attract more onlooker bees, so they are searching condenses around the better solutions. The last type of bees is scout bees. The task of the scout bees is to find untouched food sources. They search the search space randomly to find a bet-

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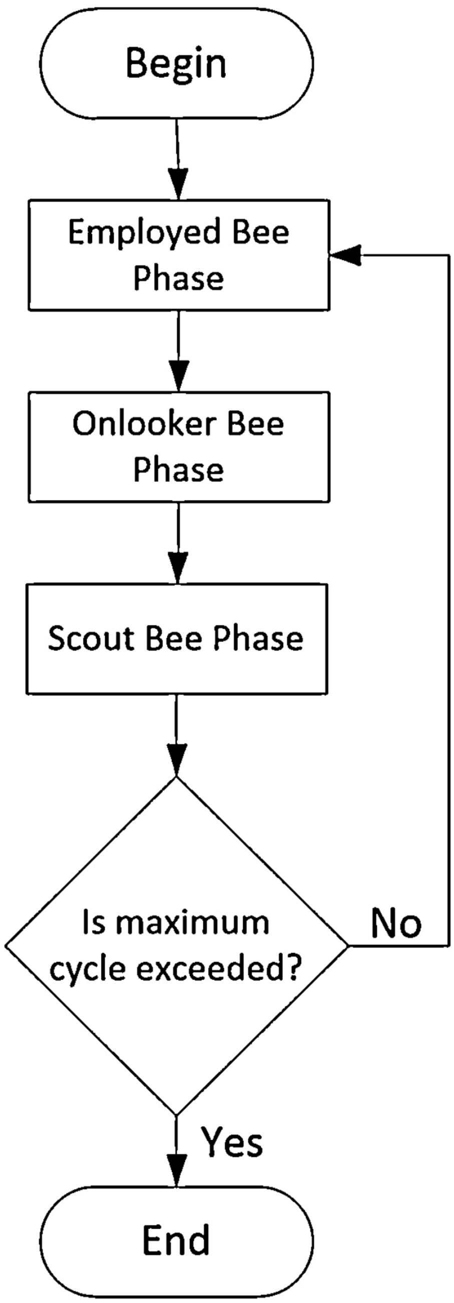

Fig. 1. The ABC algorithm.

ter solution. In this study, the employed bees of ABC algorithm have been investigated to get better solutions with the same fitness evaluation count.

Employed bees and onlooker bees use the same position update procedure. The biggest issue about the update procedure is that it causes slow convergence speed. That because in each phase, one parameter of the solution is updated and then the new solutions are evaluated. In this study, a second parameter update rule is added to employed bee phase to improve the searching ability of the ABC algorithm. The proposed method is applied to the original ABC and four different variants. First of them is G-best guided ABC algorithm (GABC) [8]. This algorithm adds the global best solution information to search equation to let the algorithm search better in the solution space. The second and third algorithms are ABC/Best/1 and ABC/Best/2 algorithms proposed by Gao et al. [9]. These algorithms use the update rule of differential evaluation algorithm and guide the agents to search around the best solution that found in the previous iteration. The last compared algorithm is ABC with variable search strategy (ABCVSS) [10]. The method proposes the integration of multiple solution update rules with ABC to solve optimization problems having different characteristics.

ABC is one of the swarm intelligence based optimization techniques. Its performance is better than most of the swarm based methods. The main problem of the algorithm is its slow convergence speed because it updates only one parameter before every fitness evaluation process. There are many modified versions of the ABC algorithm and similarly all of them apply one update rule before fitness evaluation. In this work an additional update equation has been proposed to speed up the convergence rate and improve the solution quality that uses Bollinger bands indicator (BB) [11] which is originally used to predict the future stock prices.

2. Related works

ABC algorithm was first introduced by Karaboğa in 2005 [12]. The algorithm was first used to solve numerical benchmark functions [13,14]. After ABC appeared in optimization field, some methods which modify ABC have also appeared. One of these studies, named GABC, was developed by Zhu and Kwnong [8]. They changed the neighborhood equation of the algorithm by adding the global best solution. Banharnsakun et al. [15] modified ABC by considering the best-so-far solution of the onlooker bees. Gao and Lui [16] also modified the ABC algorithm and they excluded the probabilistic selection and scout bee from the algorithm. They named the method modified ABC (MABC) and tested against two different ABC based methods on a set of 28 benchmark functions. In [17] authors proposed an improved ABC algorithm based on rank selection and utilized the best-so-far solution. Genetic operators are also used to produce new candidate solutions in ABC [18]. Another improved ABC algorithm combines personal best solution with global best solution [19]. To improve convergence characteristics of ABC, Kıran and Fındık proposed a directed ABC algorithm [20]. Kıran and Fındık also proposed a new method to improve search capabilities of ABC and PSO [21]. Babaoğlu [22] proposed a distribution based update rule to overcome stagnation behavior of the ABC algorithm.

Beyond the numerical benchmark functions, ABC algorithm is also used to solve binary optimization problems [23,24] and discrete optimization problems [25], some complex real problems like automatic voltage regulator [26], vehicle routing problem [27], allocation problem [28], design of filter bank transmultiplexer [29], synthetic radar aperture radar images segmentation [30] and parametric optimization of non-traditional machining process [31]. ABC is used to design digital infinite impulse to solve leaf-constrained minimum spanning tree problem [32] and response filters [33] design. Another application of the ABC is data collection path planning in sparse wireless sensor networks [34].

Thakkar and Kotecha [35] proposed that a Bollinger Bands based proposed a new decentralized cluster head election method for designing of scalable routing protocol with prolonged network lifetime for a wireless sensor network. Ngan and Pang [36] developed a BB based method to detect of defection on patterned fabric and tested the method on both three different patterned fabrics which defect free and defected. They obtained 98.59% accuracy. In [37] Torrisi deal with deadband sampling algorithm for unbounded variables and investigates how cab BB can be used on calculating.

3. The ABC algorithm

Like other swarm based algorithms, ABC algorithm is also an iterative algorithm. Every potential solution is represented by employed bees with D parameters. In each iteration, employed bees

Table 1
Performance of different *a* values.

a values	Function	ABCBB	ABCBEST1BB	ABCBEST2BB	GBESTABCBB	ABCVSSBB
a = 1	F1	**2.55E-16**	**7.74E-51**	**7.73E-40**	**2.19E-16**	**8.82E-93**
	F2	**2.73E-16**	**2.06E-47**	**6.54E-35**	**2.03E-16**	**1.99E-83**
	F3	**2.48E-16**	**1.54E-51**	**8.30E-41**	**2.24E-16**	**2.09E-95**
	F4	**6.68E-18**	**5.86E-119**	**1.01E-71**	**2.99E-18**	**7.61E-172**
	F5	**8.89E-16**	**1.24E-27**	**2.16E-21**	**8.97E-16**	**2.74E-49**
	F6	**0.105947**	**0.2837527**	**0.879017933**	**0.020745407**	**0.00347837**
a = 1.5	F1	4.09E-16	5.04E-47	1.06E-36	3.68E-16	6.82E-82
	F2	3.91E-16	2.21E-43	1.61E-31	4.00E-16	4.51E-80
	F3	3.74E-16	1.35E-47	1.35E-37	3.77E-16	7.28E-92
	F4	1.62E-17	1.36E-110	3.07E-67	8.75E-18	3.72E-166
	F5	1.10E-15	1.11E-25	1.04E-19	1.05E-15	9.00E-48
	F6	0.155581	0.513428867	1.441922333	0.03896216	0.00584378
a = 2	F1	5.03E-16	7.79E-45	6.98E-35	4.49E-16	8.34E-79
	F2	4.84E-16	4.91E-41	4.20E-29	4.39E-16	1.73E-81
	F3	4.78E-16	1.22E-45	9.53E-36	4.36E-16	9.34E-82
	F4	2.35E-17	4.18E-103	2.86E-63	1.74E-17	2.17E-161
	F5	1.24E-15	1.41E-24	1.06E-18	1.11E-15	2.82E-42
	F6	0.255226	0.9203776	1.963883	0.070139203	0.00704355

and onlooker bees update one of those D parameters and calculate the fitness of the new solution (mutant). If the fitness value of mutant solution is better than the original one, then employed bee stores the new solution. ABC algorithm works in three phases and uses a different kind of virtual bee in each phase as shown in Fig. 1. These are employed bees, onlooker bees and scout bees. Employed bees are initialized randomly. The basic principle of the ABC is correcting N solutions with D parameters held by employed bees. If onlooker bees cannot find better solutions near the employed bee, then the employed bee becomes a scout bee to find better solutions in random location in search space. In this study, the method is applied to minimize continuous optimization problems. The problem is finding the best parameter vector X that minimizes the function in D dimensional search space. Each element in vector X has minimum and maximum limit values. If these limits are exceeded, then the exceeding value is set to the corresponding limit. If $x_j^{\max}$ is the maximum limit value of the j$^{\text{th}}$ dimension, $x_j^{\min}$ is the minimum limit value of the j$^{\text{th}}$ dimension, ϕ is a uniformly generated random number in [0,1] and s_{ij} is the j$^{\text{th}}$ parameter of the i$^{\text{th}}$ solution, then the algorithm is initialized as follows:

$$s_{ij} = x_j^{\min} + \phi(x_j^{\max} - x_j^{\min}) \tag{1}$$

Each solution is assigned to an employed bee and the Eq. (1) is applied to all dimensions of all solutions. After initialization, the routine of the algorithm begins with the employed bee phase. In the phase, each employed bee s_i creates a new solution using neighborhood of the s_k in randomly selected dimension by using Eq. (2). The selection of the s_k is full random and is not driven by Eq. (3). The s_k must be different from the current solution ($i \neq k$). If ϕ is a uniformly generated random number in (0,1), then the employed bee generates the new solution as follow:

$$s'_{ij} = s_{ij} + \phi(s_{ij} - s_{kj}) \tag{2}$$

If the new solution s'_{ij} is better than the previous one, then the employed bee holds the new solution. In onlooker bee phase, i$^{\text{th}}$ solution which will be modified is selected according to a roulette wheel method using the Eq. (3). Through the mechanism, employed bees with bigger fitness values have more chance to be selected by onlooker bees.

$$p_i = \frac{f_i}{\sum_{i=1}^{N} f_i} \tag{3}$$

In (3), f_i represents the fitness value of i$^{\text{th}}$ solution and N represents the number of solutions. So solutions with bigger fitness values have more chance to be selected by onlooker bees. The last phase in ABC is scout bee. If a solution cannot be improved for a pre-determined trial count (limit) by employed bees or onlooker bees, this solution is replaced by a random solution just like initialization phase. Thus, it can be said that while employed bee and onlooker bee phases are the exploitation part, the scout bee phase is the exploration part of the ABC algorithm.

4. Bollinger bands in ABC algorithm

4.1. What is Bollinger bands?

BB was invented by John Bollinger in 1980 [11]. BB is a technical analysis tool to predict future prices of the stocks. It uses pre-defined count of past prices and calculates the mean value and standard deviation of the prices to calculate an interval. It assumes that future prices will not exceed these intervals. Investors may decide to buy, sell or keep the shares by looking the position of the current price according to these bands. However only BB is not enough to decide on buying or selling a stock, so many investors use additional indicators (or technical analysis tools). BB consists of three bands. Upper band (UB) is calculated by Eq. (4), lower band (LB) is calculated by Eq. (5) and middle band is the mean value of past prices. A simple BB is illustrated in Fig. 2.

$$UB = M + a \times \sigma \tag{4}$$

$$LB = M - a \times \sigma \tag{5}$$

In Eqs. (4) and (5), M is the mean of a predefined number of past closing prices, a is a constant which usually selected in [1,2] and σ is the standard deviation of past closing prices. To determine the best a value first six benchmark functions in Table 2 were run and the results are illustrated in Table 1. The table shows that to acquire the best performance a should be selected as 1.

4.2. Bollinger bands in ABC

BB is a useful indicator for investors to decide on their investments but how can it be used in ABC algorithm to improve the performance? As mentioned above, ABC algorithm works with three kinds of bees. In short, ABC algorithm exploits the search space by employed bees, onlooker bees and explores the search space by scout bees. Convergence speed of ABC is slow due to the parameter update procedure because it updates only one parameter to find better solutions. At this point BB approach is used to make

Table 2
Benchmark functions.

No of Funct.	Name	Search Range	C	Function
F1	Sphere	[100,100]	US	$f_1(\vec{X}) = \sum_{i=1}^{n} x_i^2$
F2	Elliptic	[100,100]	UN	$f_2(\vec{X}) = \sum_{i=1}^{n} (10^6)^{(i-1)/(n-1)} x_i^2$
F3	SumSquares	[10,10]	US	$f_3(\vec{X}) = \sum_{i=1}^{n} i x_i^2$
F4	SumPower	[10,10]	MS	$f_4(\vec{X}) = \sum_{i=1}^{n} \lvert x_i \rvert^{(i+1)}$
F5	Schwefel2.22	[10,10]	UN	$f_5(\vec{X}) = \sum_{i=1}^{n} \lvert x_i \rvert + \prod_{i=1}^{n} \lvert x_i \rvert$
F6	Schwefel2.21	[100,100]	UN	$f_6(\vec{X}) = \max_i \left\{ \lvert x_i \rvert, 1 \le i \le n \right\}$
F7	Step	[100,100]	US	$f_7(\vec{X}) = \sum_{i=1}^{n} (\lfloor x_i + 0.5 \rfloor)^2$
F8	Quartic	[1.28,1.28]	US	$f_8(\vec{X}) = \sum_{i=1}^{n} i x_i^4$
F9	QuarticWN	[1.28,1.28]	US	$f_9(\vec{X}) = \sum_{i=1}^{n} i x_i^4 + random[0,1)$
F10	Rosenbrock	[10,10]	UN	$f_{10}(\vec{X}) = \sum_{i=1}^{n-1} \left[100 \left(x_{i+1} - x_i^2 \right)^2 + (x_i - 1)^2 \right]$
F11	Rastrigin	[5.12,5.12]	MS	$f_{11}(\vec{X}) = \sum_{i=1}^{n} \left[x_i^2 - 10\cos(2\pi x_i) + 10 \right]$
F12	Non-Continuous Rastrigin	[5.12,5.12]	MS	$f_{12}(\vec{X}) = \sum_{i=1}^{n} \left[y_i^2 - 10\cos(2\pi y_i) + 10 \right]$ $y_i = \left\{ \begin{array}{ll} x_i & \lvert x_i \rvert < \frac{1}{2} \\ \frac{round(2x_i)}{2} & \lvert x_i \rvert \ge \frac{1}{2} \end{array} \right\}$
F13	Griewank	[600,600]	MN	$f_{13}(\vec{X}) = \frac{1}{4000} \sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n} \cos\left(\frac{x_i}{\sqrt{i}} \right) + 1$
F14	Schwefel2.26	[500,500]	UN	$f_{14}(\vec{X}) = 418.98 * \mathrm{n} - \sum_{i=1}^{n} x_i \sin\left(\sqrt{\lvert x_i \rvert} \right)$
F15	Ackley	[32,32]	MN	$f_{15}(\vec{X}) = -20\exp\left\{ -0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2} \right\} - \exp\left\{ \frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i) \right\} + 20 + e$
F16	Penalized1	[50,50]	MN	$f_{16}(\vec{X}) = \frac{\pi}{n}\{10\sin^2(\pi y_1) + \sum_{i=1}^{n-1}(y_i - 1)^2[1 + 10\sin^2(\pi y_{i+1})] + (y_n - 1)^2\} + \sum_{i=1}^{n} u(x_i, 10, 100, 4)$ $y_i = 1 + \frac{1}{4}(x_i + 1) \quad u_{x_i,a,k,m} = \left\{ \begin{array}{ll} k(x_i - a)^m & x_i > a \\ 0 & -a \le x_i \le a \\ k(x_i - a)^m & x_i < -a \end{array} \right.$
F17	Penalized2	[50,50]	MN	$f_{17}(\vec{X}) = \frac{1}{10}\{\sin^2(\pi x_1) + \sum_{i=1}^{n-1}(x_i - 1)^2[1 + \sin^2(3\pi x_{i+1})] +$ $(x_n - 1)^2[1 + \sin^2(2\pi x_{i+1})]\} + \sum_{i=1}^{n} u(x_i, 5, 100, 4)$
F18	Alpine	[10,10]	MS	$f_{18}(\vec{X}) = \sum_{i=1}^{n} \lvert x_i \cdot \sin(x_i) + 0.1 \cdot x_i \rvert$
F19	Levy	[10,10]	MN	$f_{19}(\vec{X}) = \sum_{i=1}^{n-1}(x_i - 1)^2[1 + \sin^2(3\pi x_{i+1})] + \sin^2(3\pi x_1) +$ $\lvert x_n - 1 \rvert[1 + \sin^2(3\pi x_n)]$
F20	Weierstrass	[0.5,0.5]	MN	$f_{20}(\vec{X}) = \sum_{i=1}^{D}(\sum_{k=0}^{k_{\max}}[a^k\cos(2\pi b^k(x_i + 0.5))]) - D\sum_{k=0}^{k_{\max}}[a^k\cos(2\pi b^k 0.5)]$ $a = 0.5, \quad b = 3, \quad k_{\max} = 20$
F21	Schaffer	[100,100]	MN	$f_{21}(\vec{X}) = 0.5 + \frac{\sin^2\left(\sqrt{\sum_{i=1}^{n} x_i^2}\right) - 0.5}{\left(1 + 0.001 * \left[\sum_{i=1}^{n} x_i^2\right]\right)^2}$

Table 2 (*Continued*)

No of Funct.	Name	Search Range	C	Function
F22	Himmelblau	[5,5]	MS	$f_{22}(\vec{X})=\frac{1}{n}\sum_{i=1}^{n}(x_i^4-16x_i^2+5x_i)$
F23	Michalewicz	[0,Π]	MS	$f_{23}(\vec{X})=-\sum_{i=1}^{n}\sin(x_i)\sin^{20}\left(\frac{i*x_i^2}{\pi}\right)$
F24	Shifted Sphere	[100,100]	US	$f_{24}(\vec{X})=\sum_{i=1}^{n}z_i^2 \quad z=x-o$
F25	Shifted Rastrigin	[5.12,5.12]	MS	$f_{25}(\vec{X})=\sum_{i=1}^{n}\left[z_i^2-10\cos(2\pi z_i)+10\right] \quad z=x-o$
F26	Shifted Griewank	[600,600]	MN	$f_{26}(\vec{X})=\frac{1}{4000}\sum_{i=1}^{n}z_i^2-\prod_{i=1}^{n}\cos\left(\frac{z_i}{\sqrt{i}}\right)+1\, z=x-o$
F27	Shifted Ackley	[32,32]	MN	$f_{27}(\vec{X})=-20\exp\left\{-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n}z_i^2}\right\}-\exp\left\{\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi z_i)\right\}$ $z=x-o$
F28	Shifted Alpine	[10,10]	MN	$f_{28}(\vec{X})=\sum_{i=1}^{n}\lvert z_i \cdot \sin(z_i)+0.1 \cdot z_i\rvert \quad z=x-o$

another parameter update by determining a dynamical interval for every parameter of the problem which is calculated for each iteration using BB approach as shown in Fig. 3. BB update is not used for first ten iterations because to find good BBs there should be some good solutions. BB for each dimension are calculated by four steps following:

Fig. 2. Bollinger Bands.

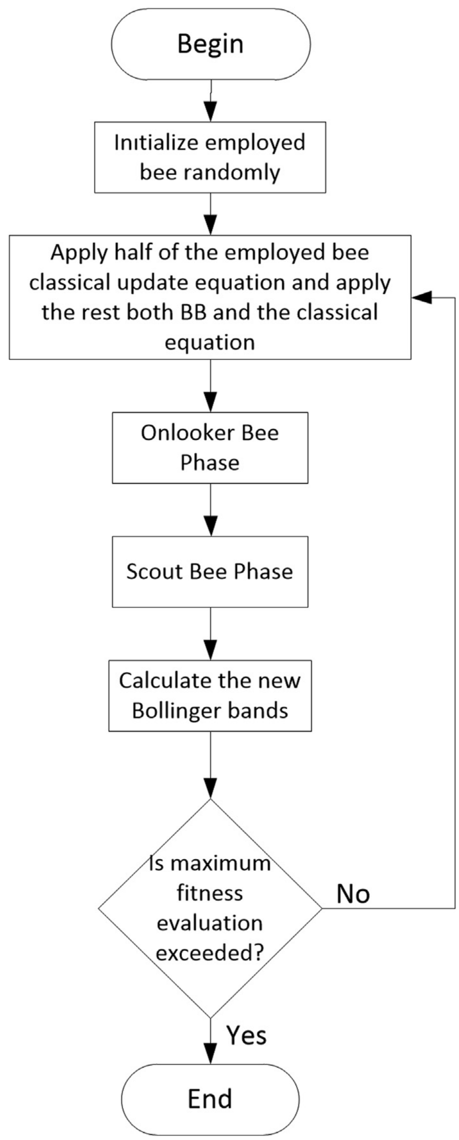

Fig. 3. Bollinger band approach in classic ABC.

1. BB is calculated on data series. For example last 20 days closing prices is used to calculate the average price, standard deviation and BB. To calculate BB for ABC, half of the employed bees (C) with the highest fitness valued are used as data series. So the best 50% of employed bees are copied to "x" array.
2. The mean of each dimension (M) is calculated by Eq. (6).

$$M_j = \frac{\sum_{i=1}^{C} x_{ij}}{C} \tag{6}$$

3. The standard deviation (σ) is calculated for each dimension by Eq. (7)

$$\sigma_j = \sqrt{\frac{\sum_{i=1}^{C} \left(x_{ij} - M_j\right)^2}{C}} \tag{7}$$

4. Upper band (UB) and lower band (LB) are calculated for each dimension of the problem by Eqs. (8) and (9) sequentially.

$$UB_j = M_j + a \times \sigma_j \tag{8}$$

$$LB_j = M_j - a \times \sigma_j \tag{9}$$

Half of the randomly selected employed bee's parameter are updated only in the classical way by Eq. (2) and the rest updated both in the classical way and BB by Eq. (10).

$$s_j = LB_j + (UB_j - LB_j) \times R \tag{10}$$

In Eq. (10), R is a random number in [−1,1].

5. Experiments

Performance comparison is done between each ABC variants with and without BB update rule. Population size and maximum iteration count parameters of ABC variants with and without BB update rule remained the same. So, pop sizes were set to their original values appeared in the corresponding papers as 40, 50, 100, 100 and 40 for ABC, GABC, ABC/Best/1, ABC/Best/2 and ABCVSS respectively. Limit value which controls the transformation of employed bees into scout bees is also calculated as in Eq. (11) [38]:

$$L = N \times D \tag{11}$$

Maximum fitness evaluation count was considered as 1.5E + 5, 3E + 5, and 5E + 5 for 30, 60 and 100 dimensionalities respectively.

Where L is the limit value, N is the employed bee count and D is the dimensionality of the problem. It is common to use only one scout bee for each iteration of the ABC algorithm. Numerical benchmark functions were used to test the accuracy performance of the proposed method. The performance of the method was tested against to ABC based algorithms by a set of functions collected from [39–41]. The set consists of unimodal, multimodal, separable and non-separable functions. In Table 2, these functions were presented and the characteristic of the function in column C was indicated as M for multimodal functions, U for unimodal functions, S for separable functions and N non-separable functions.

In this study, the proposed method was applied to the classical ABC [12], Gbest-guided ABC (GABC) [8], ABC/Best/1, ABC/Best/2 [9] and ABC with variable search strategy (ABCVSS) [10]. Tests were run 30 times independently for each function and each dimensionality. Obtained results were reported as the best, the worst, the mean and the standard deviation in the same Tables (see Tables 3–17). Sign column was drawn from pairwise *t*-test with 0.05 p value to show if the result are statistically significant. If the sign value is " + " that means two results are significantly different and if the value is "-" then it means results are not statistically significant.

Table 18 summarizes all the results above. It shows the count of the better results acquired in the five performance analysis; mean values without pairwise *t*-test, mean values which pairwise *t*-test result is "+", the best and the worst values that methods obtained in 30 independent run and the standard deviation results to show which version of the method is more robust. Each analysis is done on an ABC variant with and without BB update rule.

Convergence graphics for various functions with different D values are drawn in Fig. 4. When it is considered, convergence characteristic of ABC and ABCBB for F1 function with D = 30, there isn't a significant difference until iteration 500, but then until iteration 1500 difference increases. For the same function, it is impossible to say which one is better for ABCBEST1 and ABCBEST1BB for the first 250 iterations but the performance of the ABCBEST1BB is better for further iterations. ABCBEST2 and ABCBEST2BB have the same convergence characteristic with ABCBEST1 and ABCBEST1BB. Performance differences between GABC and GABCBB increase linearly and hit the maximum value at iteration 700. After that point, the performance of the GABC catches up with the GABCBB. Performance differences between ABCVSS and ABCVSSBB increase linearly without any exception.

Table 3
The comparison results of the standard ABC and ABC in which BB approach is applied (ABCBB), D = 30.

	ABC				ABC BB				
Function	Mean	Best	Worst	Std.Dev	Mean	Best	Worst	Std.Dev	Sign
F1	5.18E-16	2.83E-16	6.91E-16	7.57E-17	**2.54E-16**	**1.04E-16**	**3.33E-16**	**5.09E-17**	+
F2	4.98E-16	3.27E-16	5.54E-16	5.41E-17	**2.81E-16**	**2.00E-16**	**4.16E-16**	**4.50E-17**	+
F3	5.13E-16	3.30E-16	7.26E-16	6.19E-17	**2.56E-16**	**1.45E-16**	**3.16E-16**	**3.95E-17**	+
F4	3.19E-17	1.44E-17	5.51E-17	1.10E-17	**5.90E-18**	**1.61E-18**	**1.89E-17**	**3.95E-18**	+
F5	1.31E-15	9.89E-16	1.61E-15	1.50E-16	**8.96E-16**	**7.25E-16**	**9.97E-16**	**9.49E-17**	+
F6	0.7660283	0.315601	1.47647	0.2959416	**0.1081804**	**0.0684435**	**0.189214**	**0.0286474**	+
F7	0	0	0	0	0	0	0	0	+
F8	2.00E-16	6.76E-17	2.85E-16	5.47E-17	**3.82E-17**	**1.70E-17**	**7.41E-17**	**1.55E-17**	+
F9	0.053503	0.0280182	0.0750099	0.0115878	**0.0134305**	**0.006654**	**0.0232036**	**0.0033733**	+
F10	**0.0328451**	**0.000346**	**0.24596**	**0.0584681**	0.0614847	0.00035903	0.598141	0.1237569	–
F11	0	0	0	0	0	0	0	0	+
F12	0	0	0	0	0	0	0	0	+
F13	2.81E-15	0	8.23E-14	1.48E-14	**6.29E-17**	0	**5.55E-16**	**1.06E-16**	–
F14	**2.18E-08**	0	**6.54E-07**	**1.17E-07**	0.0004669	0	0.0138794	0.0024907	–
F15	3.74E-14	2.84E-14	4.26E-14	4.47E-15	**2.11E-14**	**1.42E-14**	**2.84E-14**	**2.58E-15**	+
F16	4.88E-16	2.52E-16	7.38E-16	9.86E-17	**2.31E-16**	**9.90E-17**	**3.23E-16**	**5.35E-17**	+
F17	4.91E-16	3.01E-16	5.55E-16	5.77E-17	**2.35E-16**	**1.69E-16**	**3.18E-16**	**4.34E-17**	+
F18	1.17E-09	**2.30E-15**	1.26E-08	2.64E-09	**7.80E-11**	5.14E-15	**8.43E-10**	**1.83E-10**	+
F19	4.23E-16	2.22E-16	5.24E-16	8.52E-17	**2.09E-16**	**1.11E-16**	**2.84E-16**	**3.80E-17**	+
F20	0	0	0	0	0	0	0	0	+
F21	0.3231326	0.22769	0.396098	0.0414328	**0.1729232**	**0.126991**	**0.227704**	**0.0272323**	+
F22	0	0	0	0	0	0	0	0	+
F23	0	0	0	0	0	0	0	0	+
F24	5.17E-16	3.96E-16	7.31E-16	6.52E-17	**2.66E-16**	**1.47E-16**	**3.25E-16**	**3.89E-17**	+
F25	94704.8	94704.8	94704.8	1.46E-11	**76169.3**	**76169.3**	**76169.3**	1.46E-11	+
F26	0.0003703	0	0.0110288	0.0019793	**3.46E-12**	0	**1.04E-10**	**1.86E-11**	–
F27	19.99928	19.9973	20.0001	**0.0006625**	**19.997617**	**19.9863**	**19.9999**	0.0025815	+
F28	**0.0482223**	**0.0020497**	**0.213659**	**0.0559528**	0.0744657	0.00554561	0.279391	0.0721096	–

Table 4
The comparison results of standard GABC and GABC in which BB approach is applied (GABCBB), D = 30.

	GABC				GABCBB				
Function	Mean	Best	Worst	Std.Dev	Mean	Best	Worst	Std.Dev	Sign
F1	4.47E-16	2.89E-16	5.51E-16	**6.35E-17**	**2.12E-16**	**1.00E-16**	**4.53E-16**	6.90E-17	+
F2	3.88E-16	2.67E-16	5.14E-16	7.80E-17	**1.88E-16**	**8.82E-17**	**3.01E-16**	**5.68E-17**	+
F3	4.46E-16	2.79E-16	5.38E-16	7.37E-17	**2.12E-16**	**9.57E-17**	**4.63E-16**	**7.15E-17**	+
F4	1.66E-17	7.45E-18	2.93E-17	6.35E-18	**2.58E-18**	**3.78E-19**	**1.47E-17**	**2.72E-18**	+
F5	1.35E-15	1.12E-15	1.63E-15	1.50E-16	**9.17E-16**	**7.35E-16**	**9.98E-16**	**8.58E-17**	+
F6	0.2538439	0.154076	0.374448	0.0578231	**0.021585**	**0.0127726**	**0.030231**	**0.0042603**	+
F7	0	0	0	0	0	0	0	0	+
F8	1.28E-16	7.07E-17	2.01E-16	4.11E-17	**1.15E-17**	**2.06E-18**	**2.52E-17**	**4.87E-18**	+
F9	0.0223538	0.0113737	0.0303664	0.0048274	**0.0066536**	**0.00333693**	**0.0144552**	**0.0027007**	+
F10	**0.9044532**	**0.0001259**	**8.12745**	**1.9001926**	5.5709563	0.00126923	69.1378	13.388404	–
F11	0	0	0	0	0	0	0	0	+
F12	0	0	0	0	0	0	0	0	+
F13	0.0002465	0	0.007396	0.0013276	**7.40E-18**	0	**1.11E-16**	**2.77E-17**	–
F14	**3.9484897**	0	**118.438**	**21.260169**	35.531451	0	236.877	62.297435	+
F15	3.17E-14	2.13E-14	3.91E-14	4.30E-15	**1.88E-14**	**1.42E-14**	**2.13E-14**	**2.77E-15**	+
F16	**4.33E-16**	3.11E-16	5.35E-16	**6.61E-17**	4.44E-16	**2.97E-16**	**5.33E-16**	6.92E-17	–
F17	**4.39E-16**	**2.61E-16**	**5.36E-16**	7.56E-17	4.63E-16	3.24E-16	5.37E-16	**6.09E-17**	–
F18	1.50E-07	7.05E-16	2.48E-06	5.26E-07	**4.41E-09**	**5.30E-16**	**1.32E-07**	**2.36E-08**	–
F19	**3.34E-16**	**1.80E-16**	**5.29E-16**	**6.16E-17**	3.99E-16	2.89E-16	5.43E-16	8.02E-17	+
F20	0	0	0	0	0	0	0	0	+
F21	0.2632798	0.0781892	0.312107	0.0473012	**0.1256882**	0.0781892	**0.178223**	**0.0272982**	+
F22	0	0	0	0	0	0	0	0	+
F23	0	0	0	0	0	0	0	0	+
F24	**4.27E-16**	**2.69E-16**	**5.45E-16**	8.09E-17	4.50E-16	2.69E-16	5.47E-16	**7.76E-17**	–
F25	**76556.2**	**76556.2**	**76556.2**	**1.46E-11**	81218.9	81218.9	81218.9	4.37E-11	+
F26	9.47E-16	0	2.74E-14	4.92E-15	**5.55E-17**	0	**1.11E-16**	**5.55E-17**	–
F27	**19.984473**	**19.9614**	**19.9994**	0.0100392	19.998427	19.9937	20.0003	**0.0018042**	+
F28	**0.0011004**	**3.20E-12**	**0.0325029**	**0.0058315**	0.4867656	5.09E-11	2.98425	1.0004962	+

All BB versions of the algorithms are fairly better than classic versions on F6 with D = 60.

In F5 with D = 100,differences between ABC and ABCBB are not clear for first 1700 iteration. From this point forward, differences increase and hit the maximum value at 5800 iteration and then differences decrease and convergence to zero. There is similar convergence characteristic for ABCBEST1 and ABCBEST1BB because for the first 1000 iteration, it is not possible to claim which one convergences faster but after that point, ABCBEST1BB is slightly faster than ABCBEST1. When ABCVSS and ABCVSSBB are analyzed for that function, differences can be seen among the convergence speed regularly increases with iteration.

Table 5
The comparison results of standard ABC/Best/1 and ABC/Best/1 in which BB approach is applied (ABC/Best/1BB), D = 30.

	ABC/Best/1				ABC/Best/1BB				
Function	Mean	Best	Worst	Std.Dev	Mean	Best	Worst	Std.Dev	Sign
F1	3.01E-47	2.07E-48	1.35E-46	3.41E-47	**8.01E-51**	**1.15E-51**	**2.78E-50**	**7.68E-51**	+
F2	4.38E-44	4.99E-45	1.11E-43	3.20E-44	**1.98E-47**	**4.61E-48**	**5.08E-47**	**1.39E-47**	+
F3	9.45E-48	2.55E-48	3.52E-47	8.19E-48	**1.27E-51**	**1.41E-52**	**3.68E-51**	**8.45E-52**	+
F4	1.89E-87	5.41E-95	2.10E-86	5.06E-87	**1.57E-119**	**9.19E-130**	**2.36E-118**	**5.14E-119**	+
F5	2.26E-25	9.60E-26	7.01E-25	1.25E-25	**1.12E-27**	**6.52E-28**	**2.30E-27**	**3.48E-28**	+
F6	2.0075197	1.25935	3.2222	0.4507888	**0.2851943**	**0.182196**	**0.421267**	**0.0492422**	+
F7	0	0	0	0	0	0	0	0	+
F8	2.97E-97	1.32E-99	5.69E-96	1.01E-96	**4.11E-103**	**1.12E-105**	**7.27E-102**	**1.32E-102**	–
F9	0.0217333	0.0121295	0.0292594	0.004562	**0.0104881**	**0.00554089**	**0.0167355**	**0.0026937**	+
F10	**9.7574587**	0.0039189	**76.6548**	**21.678114**	17.88821	**0.00233771**	82.4339	28.418494	–
F11	0	0	0	0	0	0	0	0	+
F12	0	0	0	0	0	0	0	0	+
F13	0.0004931	0	0.007396	0.0018449	**0**	0	**0**	**0**	–
F14	1.27E-12	0	1.82E-12	**8.34E-13**	**8.49E-13**	0	1.82E-12	9.07E-13	–
F15	2.91E-14	2.13E-14	3.20E-14	3.23E-15	**1.61E-14**	**1.42E-14**	**2.13E-14**	**2.38E-15**	+
F16	1.57E-32	1.57E-32	1.57E-32	5.47E-48	1.57E-32	1.57E-32	1.57E-32	5.47E-48	–
F17	1.35E-32	1.35E-32	1.35E-32	0	1.35E-32	1.35E-32	1.35E-32	0	+
F18	3.02E-16	1.70E-26	3.89E-15	9.68E-16	**2.29E-16**	**7.12E-28**	3.89E-15	**8.42E-16**	–
F19	1.35E-31	1.35E-31	1.35E-31	2.19E-47	1.35E-31	1.35E-31	1.35E-31	2.19E-47	–
F20	2.37E-16	0	7.11E-15	1.28E-15	**0**	0	**0**	**0**	–
F21	0.2532718	0.126991	0.345506	0.0630868	**0.1527543**	0.126991	**0.178222**	**0.025478**	+
F22	0	0	0	0	0	0	0	0	+
F23	0	0	0	0	0	0	0	0	+
F24	0	0	0	0	0	0	0	0	+
F25	117694	117694	117694	0	**110591**	**110591**	**110591**	0	+
F26	0.0002465	0	0.007396	0.0013276	**0**	0	**0**	**0**	–
F27	19.99462	19.985	19.9995	**0.0035447**	**19.994087**	**19.9838**	**19.9993**	0.0041375	–
F28	**0.1151638**	2.91E-12	**2.60871**	**0.4724243**	0.6707514	**2.28E-12**	4.25758	1.2060549	+

Table 6
The comparison results of standard ABC/Best/2 and ABC/Best/2 in which BB approach is applied (ABC/Best/2BB) D = 30.

	ABC/Best/2				ABC/Best/2BB				
Function	Mean	Best	Worst	Std.Dev	Mean	Best	Worst	Std.Dev	Sign
F1	5.01E-35	1.25E-35	1.86E-34	3.47E-35	**9.42E-40**	**1.79E-40**	**3.00E-39**	**7.05E-40**	+
F2	1.45E-28	3.08E-30	6.22E-28	1.21E-28	**8.49E-35**	**5.72E-36**	**6.01E-34**	**1.13E-34**	+
F3	6.11E-36	1.21E-36	1.32E-35	3.28E-36	**8.92E-41**	**3.24E-41**	**1.96E-40**	**4.18E-41**	+
F4	1.08E-46	1.18E-53	1.13E-45	2.31E-46	**3.60E-71**	**1.52E-79**	**7.84E-70**	**1.41E-70**	+
F5	1.26E-18	5.60E-19	2.42E-18	4.71E-19	**2.21E-21**	**1.23E-21**	**4.08E-21**	**7.33E-22**	+
F6	3.468686	2.36635	4.47577	0.4900661	**0.8507019**	**0.576305**	**1.17253**	**0.1294331**	+
F7	0	0	0	0	0	0	0	0	+
F8	7.45E-76	1.05E-77	4.18E-75	1.08E-75	**1.91E-84**	**1.27E-85**	**1.15E-83**	**2.84E-84**	+
F9	0.0267316	0.0144635	0.0380936	0.0058678	**0.0130337**	**0.00856425**	**0.0205892**	**0.0031282**	+
F10	**10.897004**	0.0214342	**75.5996**	**15.166298**	17.755531	**0.0189201**	76.8177	24.277711	–
F11	0	0	0	0	0	0	0	0	+
F12	0	0	0	0	0	0	0	0	+
F13	1.03E-08	0	1.79E-07	3.54E-08	**5.30E-10**	0	**5.84E-09**	**1.48E-09**	–
F14	1.82E-12	1.82E-12	1.82E-12	**1.21E-27**	**1.76E-12**	**0**	1.82E-12	3.27E-13	–
F15	3.02E-14	2.84E-14	3.20E-14	**1.78E-15**	**1.95E-14**	**1.42E-14**	**2.13E-14**	2.86E-15	+
F16	1.57E-32	1.57E-32	1.57E-32	5.47E-48	1.57E-32	1.57E-32	1.57E-32	5.47E-48	–
F17	1.35E-32	1.35E-32	1.35E-32	0	1.35E-32	1.35E-32	1.35E-32	0	+
F18	4.43E-13	1.27E-16	5.64E-12	1.22E-12	**3.89E-15**	**2.28E-18**	**3.57E-14**	**7.84E-15**	–
F19	1.35E-31	1.35E-31	1.35E-31	2.19E-47	1.35E-31	1.35E-31	1.35E-31	2.19E-47	–
F20	1.18E-15	0	7.11E-15	2.65E-15	**0**	0	**0**	**0**	+
F21	0.2747748	0.178222	0.373291	0.0447846	**0.1560927**	**0.0781892**	**0.22769**	**0.031197**	+
F22	0	0	0	0	0	0	0	0	+
F23	0	0	0	0	0	0	0	0	+
F24	7.57E-13	0	2.27E-11	4.08E-12	**0**	0	**0**	**0**	–
F25	87759.6	87759.6	87759.6	4.37E-11	**73257.8**	**73257.8**	**73257.8**	**1.46E-11**	+
F26	5.20E-08	0	8.51E-07	1.79E-07	**7.54E-11**	0	**1.63E-09**	**2.97E-10**	–
F27	**19.99791**	**19.9894**	**20.0011**	0.0029975	20.0012	19.9993	20.0022	**0.0006861**	+
F28	0.5778603	0.0083807	3.31554	0.819986	**0.2611473**	**1.03E-06**	**1.87431**	**0.3904684**	–

5.1. Pressure vessel design problem

Pressure vessel is a real world engineering optimization problem. The task is finding the best design parameters of producing 750 ft^3 (21.24 m^3) pressure vessel with working pressure of 3000 psi (20.68 MPa). There are four design parameters illustrated in Fig. 5. L and R can be continuous values but T_h and T_s are expected to be discrete values because they will be multiplied to 0.0625 inch (0.16 cm) [43]. These design parameters have limitations as follows:

$$x_1 \in [1.125, 12.5]$$

$$x_2 \in [0.625, 12.5]$$

Table 7
The comparison results of standard ABCVSS and ABCVSS in which BB approach is applied (ABCVSSBB), D = 30.

	ABCVSS				ABCVSSBB				
Function	Mean	Best	Worst	Std.Dev	Mean	Best	Worst	Std.Dev	Sign
F1	**7.16E-90**	9.58E-114	**2.15E-88**	**3.85E-89**	4.16E-89	**8.16E-126**	1.25E-87	2.24E-88	–
F2	1.37E-75	2.45E-116	4.10E-74	7.36E-75	**1.88E-95**	**1.77E-125**	**5.63E-94**	**1.01E-94**	–
F3	3.91E-72	4.89E-116	1.17E-70	2.10E-71	**1.04E-97**	**1.99E-128**	**3.12E-96**	**5.60E-97**	–
F4	1.18E-132	4.84E-197	3.52E-131	6.32E-132	**3.78E-191**	**1.50E-278**	**1.12E-189**	**0**	–
F5	**2.89E-46**	2.70E-59	**5.39E-45**	**1.07E-45**	2.38E-44	**6.42E-65**	7.14E-43	1.28E-43	–
F6	0.0449123	0.0110827	0.109207	0.0269674	**0.0029627**	**0.00027088**	**0.0340082**	**0.0060245**	+
F7	0	0	0	0	0	0	0	0	+
F8	2.84E-167	1.25E-237	8.52E-166	0	**6.24E-179**	**6.00E-245**	**1.05E-177**	0	+
F9	0.0178912	0.0112214	0.0262358	0.004249	**0.0021894**	**0.00180539**	**0.0024172**	**0.0001554**	+
F10	2.4166021	**0.0005417**	68.1175	12.204265	**0.3537297**	0.00123957	**4.00111**	**0.9930959**	–
F11	**0**	0	**0**	**0**	0.0002793	0	0.0014363	0.0005589	+
F12	**0**	0	**0**	**0**	0.0002261	0	0.0013991	0.0005057	+
F13	**0**	0	**0**	**0**	1.53E-12	0	1.62E-11	4.01E-12	+
F14	**4.24E-13**	**0**	**1.82E-12**	**7.69E-13**	3.85E-05	1.82E-12	6.63E-05	3.14E-05	+
F15	2.43E-14	1.42E-14	3.55E-14	4.77E-15	**2.23E-14**	1.42E-14	**2.84E-14**	**3.17E-15**	–
F16	1.57E-32	1.57E-32	1.57E-32	5.47E-48	1.57E-32	1.57E-32	1.57E-32	5.47E-48	–
F17	**1.35E-32**	1.35E-32	**1.35E-32**	**0**	9.37E-12	1.35E-32	2.81E-10	5.04E-11	–
F18	2.78E-17	4.29E-62	6.11E-16	1.15E-16	**3.83E-51**	**2.67E-70**	**5.76E-50**	**1.26E-50**	–
F19	1.35E-31	1.35E-31	1.35E-31	2.19E-47	1.35E-31	1.35E-31	1.35E-31	2.19E-47	–
F20	0	0	0	0	0	0	0	0	+
F21	0.2738482	0.126991	0.373291	0.0587361	**0.1544761**	**0.0781892**	**0.178222**	**0.0312748**	+
F22	0	0	0	0	0	0	0	0	+
F23	0	0	0	0	0	0	0	0	+
F24	0	0	0	0	0	0	0	0	+
F25	77102.8	77102.8	77102.8	**1.46E-11**	**9.39E-06**	**9.28E-06**	**9.46E-06**	5.25E-08	+
F26	**1.63E-14**	0	**4.90E-13**	**8.79E-14**	2.12E-12	0	5.52E-11	9.97E-12	–
F27	19.992087	19.9845	19.9991	**0.0039303**	**2.0379835**	**0.0433922**	**19.9895**	5.9832388	+
F28	0.0891028	3.20E-12	1.26058	0.3136813	**0.0002808**	**2.84E-12**	**0.0008908**	**0.0003975**	–

Table 8
The comparison results of standard ABC and ABC in which BB approach is applied (ABC BB), D = 60.

	ABC				ABCBB				
Function	Mean	Best	Worst	Std.Dev	Mean	Best	Worst	Std.Dev	Sign
F1	1.18E-15	7.71E-16	1.43E-15	1.68E-16	**5.85E-16**	**3.04E-16**	**7.40E-16**	**1.07E-16**	+
F2	1.10E-15	7.62E-16	1.43E-15	1.50E-16	**6.49E-16**	**4.56E-16**	**9.13E-16**	**1.15E-16**	+
F3	1.18E-15	9.19E-16	1.40E-15	1.18E-16	**5.93E-16**	**4.31E-16**	**7.59E-16**	**1.05E-16**	+
F4	4.14E-17	1.41E-17	6.30E-17	1.33E-17	**1.43E-17**	**1.59E-18**	**3.39E-17**	**6.81E-18**	+
F5	2.87E-15	2.10E-15	3.36E-15	2.87E-16	**1.93E-15**	**1.58E-15**	**2.31E-15**	**1.78E-16**	+
F6	9.676447	4.87469	15.1386	2.45174658	**1.599929667**	**1.19407**	**2.13756**	**0.21324532**	+
F7	0	0	0	0	0	0	0	0	+
F8	4.95E-16	3.05E-16	6.88E-16	7.07E-17	**9.90E-17**	**4.91E-17**	**2.01E-16**	**3.04E-17**	+
F9	0.10145402	0.0605386	0.132163	0.0197882	**0.03139095**	**0.0162323**	**0.0526576**	**0.00729318**	+
F10	**0.09593204**	**0.00072428**	0.628482	**0.13499598**	0.111460571	0.00183809	**0.589908**	0.14986105	–
F11	0	0	0	0	0	0	0	0	+
F12	0	0	0	0	0	0	0	0	+
F13	1.92E-16	0	**5.55E-16**	**1.72E-16**	**8.51E-17**	0	9.99E-16	1.78E-16	+
F14	**2.98E-06**	3.64E-11	**7.78E-05**	**1.40E-05**	3.948015084	3.64E-11	118.438	21.2602565	–
F15	8.44E-14	7.46E-14	1.03E-13	6.97E-15	**4.74E-14**	**3.91E-14**	**5.68E-14**	**4.34E-15**	+
F16	1.17E-15	8.99E-16	1.41E-15	1.36E-16	**5.26E-16**	**3.24E-16**	**7.08E-16**	**7.67E-17**	+
F17	1.18E-15	9.62E-16	1.40E-15	1.08E-16	**5.96E-16**	**4.41E-16**	**7.54E-16**	**9.02E-17**	+
F18	3.54E-07	2.22E-11	7.10E-06	1.31E-06	**2.51E-08**	**5.85E-12**	**2.66E-07**	**5.61E-08**	–
F19	1.07E-15	8.82E-16	1.37E-15	1.24E-16	**4.97E-16**	**3.29E-16**	**5.53E-16**	**5.84E-17**	+
F20	1.75E-14	0	2.84E-14	8.75E-15	**0**	0	**0**	**0**	+
F21	0.47721387	0.459781	0.490268	**0.00796171**	**0.398069033**	**0.345506**	**0.441908**	0.01991378	+
F22	−78.3323	−78.3323	−78.3323	1.42E-14	−78.3323	−78.3323	−78.3323	1.42E-14	–
F23	**−196.3424**	**−196.976**	**−195.716**	**0.30957127**	−196.227467	−196.888	−195.496	0.37585624	–
F24	1.18E-15	8.90E-16	1.42E-15	1.37E-16	**6.15E-16**	**3.81E-16**	**7.65E-16**	**8.59E-17**	+
F25	**133227**	**133227**	**133227**	0	154752	154752	154752	0	+
F26	2.08E-13	0	6.21E-12	1.11E-12	**7.77E-17**	0	**5.55E-16**	**1.38E-16**	–
F27	20.0000433	19.9999	20.0001	**6.16E-05**	**19.99965333**	**19.999**	**20**	0.0002655	+
F28	**0.21566875**	0.0254772	**0.429801**	**0.11677135**	0.329388607	**0.00966542**	0.787067	0.19796369	+

$x_3 \in [0, 240]$

$x_4 \in [0, 240]$

The problem has six penalty functions described below. If one of them exceeded fitness 1e6 added to fitness value:

$$g_1(x) = 0.0193x_3 - x_1 \leq 0$$

$$g_2(x) = 0.00954x_3 - x_2 \leq 0$$

Table 9
The comparison results of standard GABC and GABC in which BB approach is applied (GABC BB), D = 60.

	GABC				GABCBB				
Function	Mean	Best	Worst	Std.Dev	Mean	Best	Worst	Std.Dev	Sign
F1	1.01E-15	7.71E-16	1.22E-15	**1.15E-16**	**5.30E-16**	**2.89E-16**	**9.29E-16**	1.44E-16	+
F2	9.16E-16	6.82E-16	1.16E-15	**1.06E-16**	**4.72E-16**	**2.86E-16**	**9.80E-16**	1.48E-16	+
F3	1.03E-15	7.25E-16	1.20E-15	**1.46E-16**	**5.34E-16**	**3.17E-16**	**9.73E-16**	1.67E-16	+
F4	2.53E-17	1.13E-17	4.41E-17	8.04E-18	**6.49E-18**	**4.41E-19**	**2.39E-17**	**5.03E-18**	+
F5	3.02E-15	2.31E-15	3.42E-15	2.44E-16	**2.06E-15**	**1.43E-15**	**2.33E-15**	**2.12E-16**	+
F6	4.58593267	3.46558	6.24868	0.60179472	**0.7760315**	**0.569763**	**0.981892**	**0.1038181**	+
F7	0	0	0	0	0	0	0	0	+
F8	3.67E-16	2.67E-16	4.96E-16	7.04E-17	**3.97E-17**	**1.78E-17**	**5.89E-17**	**1.17E-17**	+
F9	0.05147336	0.0332858	0.0656904	0.00901728	**0.023801777**	**0.00963142**	**0.036089**	**0.00565488**	+
F10	**0.83472417**	**0.00064944**	**6.85604**	**1.81761645**	13.41401503	0.015487	89.371	24.8132273	+
F11	0	0	0	0	0	0	0	0	+
F12	0	0	0	0	0	0	0	0	+
F13	7.77E-17	0	4.44E-16	8.67E-17	**3.33E-17**	0	**1.11E-16**	**5.09E-17**	+
F14	**29.3208567**	3.64E-11	**118.438**	**49.985939**	72.1789663	3.64E-11	355.318	108.555727	–
F15	7.47E-14	6.39E-14	8.53E-14	**5.06E-15**	**4.45E-14**	**3.20E-14**	**5.68E-14**	5.48E-15	+
F16	**1.04E-15**	**7.67E-16**	**1.20E-15**	**1.17E-16**	1.09E-15	7.69E-16	1.39E-15	1.26E-16	–
F17	1.07E-15	7.73E-16	1.22E-15	**1.24E-16**	**1.02E-15**	**7.39E-16**	**1.21E-15**	1.25E-16	–
F18	2.38E-06	5.24E-14	3.92E-05	7.36E-06	**1.45E-08**	**1.85E-15**	**2.08E-07**	**4.12E-08**	–
F19	**8.66E-16**	**7.28E-16**	**9.88E-16**	**9.21E-17**	1.01E-15	7.33E-16	1.22E-15	1.17E-16	+
F20	7.11E-15	0	2.84E-14	8.80E-15	**0**	0	**0**	**0**	+
F21	0.46459913	0.429723	0.485013	**0.01308796**	**0.3583389**	**0.312103**	**0.396098**	0.02261969	+
F22	**−78.3323**	−78.3323	**−78.3323**	**1.42E-14**	−78.32759	−78.3323	−78.191	0.02536413	–
F23	**−196.3239**	**−196.918**	**−195.811**	**0.24138052**	−196.033967	−196.693	−195.103	0.29921035	+
F24	**1.04E-15**	**6.78E-16**	**1.22E-15**	1.54E-16	1.10E-15	7.68E-16	1.39E-15	**1.22E-16**	–
F25	222062	222062	222062	0	**159567**	**159567**	**159567**	0	+
F26	**5.18E-17**	0	**1.11E-16**	**5.54E-17**	1.04E-16	0	5.55E-16	9.90E-17	+
F27	19.9989	19.9952	20.0004	**0.00125406**	**19.99840333**	**19.9937**	**20**	0.00171862	–
F28	**1.29676888**	**0.06062**	**5.87321**	**1.29321996**	3.844649933	0.688848	7.16552	1.71244089	+

Table 10
The comparison results of standard ABC/Best/1 and ABC/Best/1 in which BB approach is applied (ABC/Best/1 BB) D = 60.

	ABC/Best/1				ABC/Best/1BB				
Function	Mean	Best	Worst	Std.Dev	Mean	Best	Worst	Std.Dev	Sign
F1	4.12E-44	4.32E-45	1.96E-43	3.79E-44	**3.13E-47**	**7.56E-48**	**1.21E-46**	**2.49E-47**	+
F2	1.98E-41	4.68E-42	4.21E-41	1.11E-41	**3.48E-44**	**1.03E-44**	**8.33E-44**	**1.71E-44**	+
F3	1.36E-44	2.92E-45	5.24E-44	1.06E-44	**9.22E-48**	**3.72E-48**	**2.67E-47**	**5.44E-48**	+
F4	5.75E-75	2.89E-82	1.52E-73	2.73E-74	**1.01E-108**	**1.55E-116**	**2.94E-107**	**5.28E-108**	–
F5	8.44E-24	4.74E-24	1.54E-23	2.39E-24	**8.90E-26**	**4.78E-26**	**1.73E-25**	**2.54E-26**	+
F6	20.6103367	16.5313	24.341	2.02221369	**5.394408667**	**4.36253**	**6.79318**	**0.59218676**	+
F7	0	0	0	0	0	0	0	0	+
F8	5.54E-91	5.48E-93	5.06E-90	1.16E-90	**2.04E-96**	**6.78E-98**	**2.30E-95**	**4.07E-96**	+
F9	0.06630456	0.0489904	0.0794348	**0.0071249**	**0.033434697**	**0.0202786**	**0.0518988**	0.00797534	+
F10	**40.7815069**	**0.0028653**	**145.581**	**45.3102923**	61.89414089	0.0758697	159.087	46.4939841	–
F11	0	0	0	0	0	0	0	0	+
F12	0	0	0	0	0	0	0	0	+
F13	0	0	0	0	0	0	0	0	+
F14	3.94793333	3.64E-11	118.438	21.2602716	**3.70E-11**	3.64E-11	**4.37E-11**	**1.90E-12**	–
F15	6.87E-14	5.68E-14	7.82E-14	5.53E-15	**4.26E-14**	**3.91E-14**	**4.97E-14**	**3.04E-15**	+
F16	7.85E-33	7.85E-33	7.85E-33	4.11E-48	7.85E-33	7.85E-33	7.85E-33	4.11E-48	–
F17	1.35E-32	1.35E-32	1.35E-32	0	1.35E-32	1.35E-32	1.35E-32	0	+
F18	2.22E-16	1.07E-23	4.11E-15	7.73E-16	**1.37E-16**	**1.27E-24**	4.11E-15	**7.37E-16**	–
F19	1.35E-31	1.35E-31	1.35E-31	2.19E-47	1.35E-31	1.35E-31	1.35E-31	2.19E-47	–
F20	2.37E-14	0	4.26E-14	8.47E-15	**1.42E-15**	0	**1.42E-14**	**4.26E-15**	+
F21	0.4672646	0.441908	0.482542	**0.01007531**	**0.3698728**	**0.312383**	**0.414668**	0.02186472	+
F22	−78.3323	−78.3323	−78.3323	1.42E-14	−78.3323	−78.3323	−78.3323	1.42E-14	–
F23	**−196.656433**	**−197.021**	**−196.345**	**0.16851284**	−196.3211	−196.88	−195.858	0.24771198	+
F24	0	0	0	0	0	0	0	0	+
F25	154606	154606	154606	0	**144367**	**144367**	**144367**	0	+
F26	3.70E-18	0	1.11E-16	1.99E-17	**0**	0	**0**	**0**	–
F27	**19.9158233**	**19.8843**	**19.963**	0.02348758	19.99590667	19.9912	19.9994	**0.00205539**	+
F28	**0.74892437**	0.00345161	**1.93391**	**0.41924865**	1.120718767	**5.49E-12**	5.05441	1.59305944	–

$$g_3(x) = 750 \times 1728 - \pi x_3^2 x_4 - \frac{4}{3}\pi x_3^3 \leq 0$$

$$g_4(x) = x_4 - 240 \leq 0$$

$$g_5(x) = 1.1 - x_1 \leq 0$$

$$g_6(x) = 0.6 - x_2 \leq 0$$

The objective function is:

$$f(x) = 0.6224 x_1 x_3 x_4 + 1.7781 x_2 x_3^2 + 3.1611 x_1^2 x_4 + 19.84 x_1^2 x_3$$

Table 11
The comparison results of standard ABC/Best/2 and ABC/Best/2 in which BB approach is applied (ABC/Best/2 BB), D = 60.

Function	ABC/Best/2				ABC/Best/2BB				Sign
	Mean	Best	Worst	Std.Dev	Mean	Best	Worst	Std.Dev	
F1	4.20E-33	1.78E-33	1.05E-32	1.97E-33	**1.81E-37**	**7.55E-38**	**3.49E-37**	**7.03E-38**	+
F2	4.77E-27	4.17E-28	1.44E-26	3.50E-27	**9.06E-33**	**1.57E-33**	**2.52E-32**	**5.81E-33**	+
F3	1.02E-33	2.80E-34	2.19E-33	4.53E-34	**4.24E-38**	**2.16E-38**	**9.83E-38**	**1.74E-38**	+
F4	2.21E-39	4.61E-46	4.42E-38	8.17E-39	**1.51E-64**	**2.72E-70**	**2.18E-63**	**5.06E-64**	−
F5	1.51E-17	1.06E-17	2.16E-17	2.61E-18	**3.47E-20**	**2.41E-20**	**4.79E-20**	**6.35E-21**	+
F6	23.86537	19.7834	27.3581	1.86977316	**8.481775**	**6.76461**	**10.5626**	**0.87997065**	+
F7	0	0	0	0	0	0	0	0	+
F8	4.40E-72	5.27E-73	1.18E-71	3.16E-72	**8.04E-80**	**5.92E-81**	**2.87E-79**	**6.97E-80**	+
F9	0.07314533	0.0550955	0.0879056	0.00787086	**0.039434047**	**0.0260597**	**0.0520073**	**0.00607161**	+
F10	**47.6519812**	**0.0435303**	**121.777**	43.3837889	83.9431535	0.0485889	165.569	**35.5296118**	+
F11	0	0	0	0	0	0	0	0	+
F12	0	0	0	0	0	0	0	0	+
F13	1.01E-08	0	2.67E-07	4.80E-08	**3.17E-11**	0	**7.35E-10**	**1.36E-10**	−
F14	0.06385167	4.37E-11	1.91555	0.34385175	**7.01E-05**	**3.64E-11**	**0.00210169**	**0.00037726**	−
F15	7.35E-14	6.39E-14	8.53E-14	4.87E-15	**4.59E-14**	**3.91E-14**	**5.68E-14**	**4.30E-15**	+
F16	7.85E-33	7.85E-33	7.85E-33	4.11E-48	7.85E-33	7.85E-33	7.85E-33	4.11E-48	−
F17	1.35E-32	1.35E-32	1.35E-32	0	1.35E-32	1.35E-32	1.35E-32	0	+
F18	7.70E-11	9.80E-15	9.09E-10	2.23E-10	**9.94E-12**	**8.04E-16**	**1.34E-10**	**3.19E-11**	−
F19	1.53E-31	1.35E-31	5.66E-31	7.76E-32	**1.35E-31**	1.35E-31	**1.35E-31**	**2.19E-47**	−
F20	2.32E-14	0	4.26E-14	1.13E-14	**1.47E-14**	0	**2.84E-14**	**5.78E-15**	+
F21	0.46810117	0.414668	0.482542	**0.01230982**	**0.3842168**	**0.312103**	**0.414668**	0.02490857	+
F22	−78.3323	−78.3323	−78.3323	1.42E-14	−78.3323	−78.3323	−78.3323	1.42E-14	−
F23	−188.822433	−189.795	−188.044	**0.41503202**	−**189.000767**	−**189.82**	−**188.333**	0.42739846	−
F24	0	0	0	0	0	0	0	0	+
F25	183356	183356	183356	0	**124140**	**124140**	**124140**	0	+
F26	2.63E-09	0	7.14E-08	1.28E-08	**4.95E-14**	0	**1.48E-12**	**2.66E-13**	−
F27	**20.0014167**	**19.9999**	**20.0022**	0.0006006	20.00224667	20.0015	20.003	**0.00030847**	+
F28	**2.03460477**	0.740354	**4.70977**	**0.93756271**	2.643362133	**0.245785**	6.17785	1.6160873	−

Table 12
The comparison results of standard ABCVSS and ABCVSS in which BB approach is applied (ABCVSSBB), D = 60.

Function	ABCVSS				ABCVSSBB				Sign
	Mean	Best	Worst	Std.Dev	Mean	Best	Worst	Std.Dev	
F1	4.65E-85	1.48E-117	1.01E-83	1.91E-84	**1.04E-88**	**2.67E-124**	**3.11E-87**	**5.57E-88**	−
F2	6.09E-80	1.41E-111	1.83E-78	3.28E-79	**1.76E-85**	**3.90E-121**	**5.25E-84**	**9.42E-85**	−
F3	5.91E-82	3.31E-115	1.71E-80	3.07E-81	**9.43E-88**	**3.20E-124**	**2.83E-86**	**5.08E-87**	−
F4	1.71E-100	1.81E-206	5.12E-99	9.19E-100	**5.31E-197**	**4.72E-295**	**1.59E-195**	**0**	−
F5	2.11E-42	7.42E-61	4.99E-41	9.20E-42	**3.25E-48**	**1.35E-61**	**9.76E-47**	**1.75E-47**	−
F6	1.51904333	1.06645	2.95714	0.33265873	**0.317105**	**0.14114**	**0.584368**	**0.11766852**	+
F7	0	0	0	0	0	0	0	0	+
F8	2.79E-169	1.31E-233	8.38E-168	0	**1.56E-170**	**1.67E-237**	**4.67E-169**	0	+
F9	0.04530215	0.0218188	0.0772808	0.01086467	**0.000650197**	**0.0005663**	**0.00072101**	**3.90E-05**	+
F10	**2.81969654**	0.00181758	**69.2304**	**12.3677944**	5.151296288	**0.00098095**	72.317	17.4462772	−
F11	**0**	0	**0**	**0**	0.000299249	0	0.00077042	0.00036552	+
F12	**0**	0	**0**	**0**	7.35E-05	0	0.00075801	0.00022072	−
F13	**0**	0	**0**	**0**	4.10E-11	0	1.23E-09	2.21E-10	−
F14	**3.66E-11**	**3.64E-11**	**4.37E-11**	**1.31E-12**	2.61E-05	4.37E-11	3.52E-05	1.44E-05	+
F15	6.04E-14	4.26E-14	7.82E-14	1.05E-14	**5.13E-14**	4.26E-14	**6.39E-14**	**5.08E-15**	+
F16	**7.85E-33**	7.85E-33	**7.85E-33**	**4.11E-48**	8.78E-12	7.85E-33	2.63E-10	4.73E-11	−
F17	1.35E-32	1.35E-32	1.35E-32	0	1.35E-32	1.35E-32	1.35E-32	0	+
F18	4.81E-17	7.08E-58	6.11E-16	1.56E-16	4.81E-17	**4.66E-67**	6.11E-16	1.56E-16	−
F19	1.35E-31	1.35E-31	1.35E-31	2.19E-47	1.35E-31	1.35E-31	1.35E-31	2.19E-47	−
F20	**4.74E-16**	0	**1.42E-14**	**2.55E-15**	1.94E-14	0	2.84E-14	8.59E-15	+
F21	0.47416423	0.451776	0.487077	**0.00903751**	**0.375988733**	**0.345506**	**0.429723**	0.02623959	+
F22	−78.3323	−78.3323	−78.3323	1.42E-14	−78.3323	−78.3323	−78.3323	1.42E-14	−
F23	−199.364767	−**199.56**	−199.008	0.13029548	−**199.3954**	−199.525	−**199.241**	**0.07842517**	−
F24	6.20E-26	0	1.86E-24	3.34E-25	**0**	0	**0**	**0**	−
F25	132238	132238	132238	**0**	**3.97E-06**	**3.93E-06**	**4.00E-06**	1.50E-08	+
F26	**0**	0	**0**	**0**	1.10E-09	0	3.29E-08	5.90E-09	−
F27	19.98889	19.9775	19.9964	**0.00477119**	**2.700554793**	**0.0435171**	**19.9816**	6.77364416	+
F28	0.03189194	6.86E-12	0.566291	0.10955597	**0.000467837**	**4.95E-12**	**0.00054776**	**0.00015644**	−

When the $X = \{x_1, x_2, x_3, x_4\} = \{1.125, 0.625, 58.2901554, 43.6926562\}$ optimum value of $f(x) = 7197.72893$ is obtained.

Proposed approach tested on pressure vessel design problem. Because of the proposed approach can be applied all ABC variants, ABCVSSBB selected because of its performance. Population size is set the same value for ABCVSSBB, SPSO and ABC as 40. Three different stopping criteria used as maximum fitness evaluation count. They were 10.000, 20.000 and 30.000. Initial solutions are created randomly between the limits mentioned above. All methods are run 30 times for each stopping criteria. The parameters for SPSO is as follows. Inertia weight$w = 1/(2 \times \log(2))$. Positive constant parameters$c_1 = c_2 = 0.5 + \log(2)$. Results of ABC and SPSO are

Table 13
The comparison results of standard ABC and ABC in which BB approach is applied (ABCBB), D = 100.

	ABC				ABCBB				
Function	Mean	Best	Worst	Std.Dev	Mean	Best	Worst	Std.Dev	Sign
F1	2.27E-15	1.87E-15	2.55E-15	1.74E-16	**1.12E-15**	**8.89E-16**	**1.44E-15**	**1.31E-16**	+
F2	2.07E-15	1.56E-15	2.50E-15	1.90E-16	**1.21E-15**	**9.62E-16**	**1.41E-15**	**1.36E-16**	+
F3	2.27E-15	2.04E-15	2.54E-15	**1.06E-16**	**1.12E-15**	**7.63E-16**	**1.41E-15**	1.31E-16	+
F4	5.36E-17	1.99E-17	8.83E-17	1.57E-17	**2.41E-17**	**4.26E-18**	**5.66E-17**	**1.39E-17**	+
F5	5.00E-15	4.32E-15	5.43E-15	**2.54E-16**	**3.42E-15**	**2.98E-15**	**4.10E-15**	2.63E-16	+
F6	24.5176133	19.402	30.9252	3.05427744	**6.433809333**	**5.02583**	**7.56485**	**0.62923327**	+
F7	0	0	0	0	0	0	0	0	+
F8	9.52E-16	7.56E-16	1.13E-15	1.07E-16	**2.37E-16**	**1.47E-16**	**3.18E-16**	**4.31E-17**	+
F9	0.1594136	0.100235	0.223165	0.026129	**0.057185797**	**0.0286992**	**0.0789138**	**0.01214566**	+
F10	**0.30354177**	**4.88E-05**	**2.71913**	**0.62086596**	0.712286303	0.00011512	4.11153	1.04239031	–
F11	0	0	0	0	0	0	0	0	+
F12	0	0	0	0	0	0	0	0	+
F13	6.29E-16	0	3.55E-15	6.28E-16	**1.33E-16**	0	**5.55E-16**	**1.53E-16**	+
F14	7.89586667	1.09E-10	118.438	29.5436278	**3.948056239**	1.09E-10	118.438	**21.2602488**	–
F15	1.47E-13	1.28E-13	1.63E-13	8.08E-15	**8.48E-14**	**7.11E-14**	**1.03E-13**	**7.49E-15**	+
F16	2.17E-15	1.59E-15	2.51E-15	1.91E-16	**1.01E-15**	**7.69E-16**	**1.22E-15**	**1.25E-16**	+
F17	2.14E-15	1.82E-15	2.52E-15	1.90E-16	**1.09E-15**	**8.63E-16**	**1.37E-15**	**1.26E-16**	+
F18	3.26E-06	8.84E-09	1.31E-05	3.94E-06	**1.86E-07**	**2.00E-09**	**1.40E-06**	**2.94E-07**	+
F19	2.02E-15	1.65E-15	2.53E-15	2.25E-16	**9.45E-16**	**6.79E-16**	**1.21E-15**	**1.27E-16**	+
F20	8.62E-14	5.68E-14	1.42E-13	2.49E-14	**1.71E-14**	**0**	**5.68E-14**	**1.57E-14**	+
F21	0.49711933	0.494218	0.498384	**0.00093665**	**0.478348233**	**0.466295**	**0.487077**	0.00540197	+
F22	−78.3323	−78.3323	−78.3323	1.42E-14	−78.3323	−78.3323	−78.3323	1.42E-14	–
F23	−289.4298	−291.387	**−287.797**	**0.77420464**	**−289.578233**	**−292.173**	−287.264	1.06752507	–
F24	2.16E-15	1.66E-15	2.50E-15	1.76E-16	**1.22E-15**	**1.07E-15**	**1.44E-15**	**9.07E-17**	+
F25	287045	287045	287045	0	**263872**	**263872**	**263872**	0	+
F26	4.88E-16	0	1.44E-15	3.83E-16	**8.51E-17**	0	**5.55E-16**	**1.24E-16**	+
F27	**20.0000433**	**19.9999**	**20.0001**	6.16E-05	20.00011667	20	20.0002	**4.53E-05**	+
F28	**0.41968813**	0.113199	**0.877103**	**0.21091852**	0.599844573	**0.0955152**	1.07744	0.29270574	+

Table 14
The comparison results of standard GABC and BB approach applied GABC (GABCBB), D = 100.

	GABC				GABCBB				
Function	Mean	Best	Worst	Std.Dev	Mean	Best	Worst	Std.Dev	Sign
F1	1.84E-15	1.36E-15	2.32E-15	**2.29E-16**	**1.10E-15**	**7.30E-16**	**2.07E-15**	3.09E-16	+
F2	1.67E-15	1.42E-15	1.87E-15	**1.25E-16**	**9.70E-16**	**6.73E-16**	**1.66E-15**	2.53E-16	+
F3	1.83E-15	1.56E-15	2.27E-15	**1.90E-16**	**1.03E-15**	**7.36E-16**	**1.44E-15**	1.99E-16	+
F4	3.28E-17	1.40E-17	5.10E-17	8.62E-18	**1.17E-17**	**1.70E-18**	**2.66E-17**	**7.06E-18**	+
F5	5.21E-15	4.76E-15	5.66E-15	**2.50E-16**	**3.54E-15**	**2.76E-15**	**4.11E-15**	3.69E-16	+
F6	16.6113867	13.9547	19.1937	1.3651081	**4.474371333**	**3.65545**	**5.48456**	**0.47090454**	+
F7	0	0	0	0	0	0	0	0	+
F8	7.54E-16	5.44E-16	9.11E-16	8.31E-17	**8.60E-17**	**4.15E-17**	**1.69E-16**	**2.39E-17**	+
F9	0.09418007	0.0789306	0.114706	0.00995145	**0.04471054**	**0.0318954**	**0.0569155**	**0.00662335**	+
F10	**16.1755672**	**0.00110678**	**83.0047**	**30.1199872**	54.48212165	0.0244058	151.903	46.4324024	+
F11	0	0	0	0	0	0	0	0	+
F12	0	0	0	0	0	0	0	0	+
F13	1.67E-16	0	5.55E-16	1.92E-16	**5.92E-17**	0	**1.11E-16**	**5.54E-17**	+
F14	**107.359315**	1.09E-10	473.753	**111.079295**	156.3064167	1.09E-10	473.753	127.010493	–
F15	1.33E-13	1.14E-13	1.46E-13	8.97E-15	**7.99E-14**	**6.75E-14**	**9.95E-14**	**7.93E-15**	+
F16	**1.86E-15**	1.59E-15	**2.09E-15**	**1.58E-16**	2.00E-15	**1.56E-15**	2.31E-15	1.80E-16	+
F17	**1.89E-15**	1.56E-15	**2.31E-15**	**1.96E-16**	1.97E-15	**1.41E-15**	2.48E-15	2.00E-16	–
F18	1.08E-05	9.61E-13	0.00012423	2.37E-05	**5.06E-07**	**3.41E-15**	**9.00E-06**	**1.63E-06**	+
F19	**1.66E-15**	**1.38E-15**	**1.97E-15**	**1.64E-16**	1.97E-15	1.54E-15	2.32E-15	2.02E-16	+
F20	5.97E-14	2.84E-14	1.14E-13	**1.53E-14**	**3.60E-14**	**0**	**8.53E-14**	2.31E-14	+
F21	0.4950875	0.491503	0.497371	**0.00183649**	**0.468203267**	**0.441908**	**0.482542**	0.00868379	+
F22	**−78.3323**	−78.3323	**−78.3323**	**1.42E-14**	−78.32602	−78.3323	−78.2381	0.02349761	–
F23	**−288.021367**	−289.152	**−287.123**	**0.57814188**	−287.841667	**−289.603**	−286.44	0.74609608	–
F24	**1.86E-15**	1.44E-15	**2.28E-15**	**1.90E-16**	1.94E-15	**1.42E-15**	2.48E-15	2.03E-16	–
F25	310553	310553	310553	0	**270559**	**270559**	**270559**	0	+
F26	7.16E-07	0	2.15E-05	3.86E-06	**1.74E-16**	0	**5.55E-16**	**1.81E-16**	–
F27	20.0001333	19.9998	20.0003	**0.00013499**	**19.99865333**	**19.9957**	**20**	0.00131244	+
F28	1.29349967	5.70E-07	**7.16933**	2.06076957	**1.148476913**	**2.42E-09**	7.71678	**1.70118196**	–

directly taken from [44] and combined in Table 19. Comparison results show that ABCVSSBB found the lowest mean cost solution. Also best results and lowest standard deviation values belong to ABCVSSBB. Lowest worst and lowest standard deviation values for 20.000 and 30.000 fitness evaluation belong to ABC algorithm.

5.2. *Comparison of the other algorithms*

In this section proposed approach compared to two state of the art population based methods of artificial algae algorithm (AAA) [44] and galactic swarm optimization algorithm (GSO) [45]. AAA is new bio-inspired metaheurstic optimization method which simu-

Table 15
The comparison results of standard ABC/Best/1 and ABC/Best/1 in which BB approach is applied (ABC/Best/1BB), D = 100.

	ABC/Best/1				ABC/Best/1BB				
Function	Mean	Best	Worst	Std.Dev	Mean	Best	Worst	Std.Dev	Sign
F1	1.75E-42	4.00E-43	7.27E-42	1.35E-42	**1.48E-45**	**4.99E-46**	**3.42E-45**	**6.74E-46**	+
F2	5.88E-40	1.59E-40	1.16E-39	2.78E-40	**1.01E-42**	**3.54E-43**	**2.06E-42**	**4.46E-43**	+
F3	8.14E-43	2.59E-43	1.94E-42	4.26E-43	**6.11E-46**	**2.76E-46**	**1.32E-45**	**2.69E-46**	+
F4	8.48E-60	2.11E-67	1.74E-58	3.27E-59	**1.85E-91**	**6.47E-101**	**4.99E-90**	**8.96E-91**	−
F5	6.43E-23	3.01E-23	1.00E-22	1.53E-23	**6.65E-25**	**3.84E-25**	**9.48E-25**	**1.31E-25**	+
F6	47.0712533	41.8619	51.7455	2.53119856	**20.09584333**	**17.1977**	**24.563**	**1.63254832**	+
F7	0	0	0	0	0	0	0	0	+
F8	2.68E-88	3.12E-90	1.27E-87	3.01E-88	**1.91E-93**	**1.48E-94**	**7.22E-93**	**1.65E-93**	+
F9	0.12708244	0.0958081	0.143466	0.01233349	**0.068045147**	**0.0482497**	**0.0823107**	**0.00832849**	+
F10	**71.0618659**	0.0619981	**231.321**	**64.353106**	126.6604805	**0.0186739**	250.705	65.2559399	+
F11	2.37E-16	0	7.11E-15	1.28E-15	**0**	0	**0**	**0**	−
F12	0	0	0	0	0	0	0	0	+
F13	0	0	0	0	0	0	0	0	+
F14	1.22E-10	1.16E-10	1.31E-10	3.94E-12	**1.14E-10**	**1.09E-10**	**1.16E-10**	**3.43E-12**	+
F15	1.31E-13	1.21E-13	1.46E-13	6.71E-15	**7.88E-14**	**6.75E-14**	**8.53E-14**	**5.03E-15**	+
F16	4.71E-33	4.71E-33	4.71E-33	2.05E-48	4.71E-33	4.71E-33	4.71E-33	2.05E-48	−
F17	1.35E-32	1.35E-32	1.35E-32	0	1.35E-32	1.35E-32	1.35E-32	0	+
F18	1.28E-16	3.65E-22	1.89E-15	3.97E-16	**1.04E-16**	**3.35E-23**	1.89E-15	**3.64E-16**	−
F19	1.35E-31	1.35E-31	1.35E-31	2.19E-47	1.35E-31	1.35E-31	1.35E-31	2.19E-47	−
F20	1.18E-13	8.53E-14	1.42E-13	**1.29E-14**	**4.07E-14**	**2.84E-14**	**8.53E-14**	1.75E-14	+
F21	0.49636457	0.493459	0.49785	**0.00089792**	**0.4769989**	**0.466295**	**0.482542**	0.00472927	+
F22	−78.3323	−78.3323	−78.3323	1.42E-14	−78.3323	−78.3323	−78.3323	1.42E-14	−
F23	−279.2492	−279.904	−278.425	**0.34099662**	**−280.707267**	**−282.105**	**−279.048**	0.6836621	+
F24	0	0	0	0	0	0	0	0	+
F25	**270269**	**270269**	**270269**	0	313514	313514	313514	0	+
F26	0	0	0	0	0	0	0	0	+
F27	**19.98909**	**19.9819**	**19.9957**	0.00345768	19.99831333	19.997	19.9995	**0.00073925**	+
F28	**0.1968534**	**1.31E-11**	**1.9786**	**0.46852018**	0.915916173	0.0246775	6.74318	1.55703083	+

Table 16
The comparison results of standard ABC/Best/2 and ABC/Best/2 in which BB approach is applied (ABC/Best/2BB), D = 100.

	ABC/Best/2				ABC/Best/2BB				
Function	Mean	Best	Worst	Std.Dev	Mean	Best	Worst	Std.Dev	Sign
F1	4.94E-32	2.67E-32	1.04E-31	2.01E-32	**2.91E-36**	**1.15E-36**	**5.71E-36**	**1.08E-36**	+
F2	2.53E-26	9.14E-27	8.54E-26	1.68E-26	**7.97E-32**	**2.15E-32**	**2.45E-31**	**5.70E-32**	+
F3	2.10E-32	6.92E-33	4.22E-32	7.01E-33	**1.07E-36**	**4.89E-37**	**3.09E-36**	**4.88E-37**	+
F4	1.22E-28	6.30E-34	3.36E-27	6.02E-28	**9.44E-52**	**6.77E-59**	**2.73E-50**	**4.89E-51**	−
F5	6.73E-17	4.83E-17	8.60E-17	1.09E-17	**1.51E-19**	**1.01E-19**	**1.96E-19**	**2.25E-20**	+
F6	50.2476133	47.5513	55.0172	1.91788156	**24.79700333**	**21.4253**	**28.1105**	**1.53572905**	+
F7	0	0	0	0	0	0	0	0	+
F8	1.42E-69	1.98E-70	3.71E-69	7.82E-70	**2.52E-77**	**1.76E-78**	**9.28E-77**	**1.95E-77**	+
F9	0.13892613	0.100243	0.166829	0.01469211	**0.076979973**	**0.0520081**	**0.102973**	**0.00952347**	+
F10	**93.4389556**	**0.221364**	**243.435**	66.4982567	153.8051267	84.609	263.265	**44.2671336**	+
F11	0	0	0	0	0	0	0	0	+
F12	0	0	0	0	0	0	0	0	+
F13	6.76E-09	0	2.03E-07	3.64E-08	**2.27E-11**	0	**3.93E-10**	**7.85E-11**	−
F14	3.95144766	1.24E-10	118.438	21.2596275	**0.025166176**	**1.16E-10**	**0.752423**	**0.13504896**	−
F15	1.38E-13	1.28E-13	1.49E-13	**4.54E-15**	**8.68E-14**	**7.82E-14**	**9.24E-14**	5.16E-15	+
F16	4.71E-33	4.71E-33	4.71E-33	2.05E-48	4.71E-33	4.71E-33	4.71E-33	2.05E-48	−
F17	2.32E-32	1.47E-32	4.18E-32	5.94E-33	**1.35E-32**	**1.35E-32**	**1.35E-32**	**0**	+
F18	2.02E-09	2.98E-12	2.18E-08	4.54E-09	**8.33E-10**	**7.37E-14**	**9.01E-09**	**1.99E-09**	−
F19	1.85E-31	1.35E-31	3.81E-31	7.06E-32	**1.35E-31**	1.35E-31	**1.35E-31**	**2.19E-47**	+
F20	1.32E-13	1.14E-13	1.71E-13	1.87E-14	**7.39E-14**	**5.68E-14**	**8.53E-14**	**1.39E-14**	+
F21	0.49714037	0.495946	0.49785	**0.00042776**	**0.480705167**	**0.471615**	**0.487077**	0.00333858	+
F22	−78.3323	−78.3323	−78.3323	1.42E-14	−78.3323	−78.3323	−78.3323	1.42E-14	−
F23	−266.962567	−268.624	−264.868	**0.72385982**	**−268.779067**	**−270.396**	**−266.994**	0.82818472	+
F24	0	0	0	0	0	0	0	0	+
F25	299818	299818	299818	0	**297683**	**297683**	**297683**	0	+
F26	**7.29E-12**	0	**1.35E-10**	**2.58E-11**	8.99E-12	0	2.54E-10	4.56E-11	−
F27	**20.0021567**	20.0017	20.0026	0.00025649	20.00223667	**20.0016**	20.0026	**0.00023591**	−
F28	**6.56218633**	**2.84271**	**11.3774**	**2.16827676**	8.505990333	3.15504	12.8574	2.46616563	+

lates life cycle of the microalgae as an optimization method. GSO is another state of the art optimization method inspired from the motion of universal objects like stars, galaxies and superclusters of galaxies. The motion of these objects are influenced by different gravitational forces.

Comparison results are illustrated in Table 20 for D = 30, Table 21 for D = 60 and Table 22 for D = 100. In Table 20 ABCVSS acquire the best mean values in F1, F3, F4, F10, F11, F18 and F19. For F7, F22, F23 and F24 ABCVSSBB obtained the same mean values with AAA. When the best results are evaluated, GSO outperforms the other algorithms but the weak point of the GSO algorithm appears on the worst values because biggest worst values also belong to GSO algorithm. So GSO has the maximum standard deviation values and

Table 17
The comparison results of standard ABCVSS and ABCVSS in which BB approach is applied (ABCVSSBB), D = 100.

	ABCVSS				ABCVSSBB				
Function	Mean	Best	Worst	Std.Dev	Mean	Best	Worst	Std.Dev	Sign
F1	**3.68E-81**	1.00E-114	**1.07E-79**	**1.93E-80**	2.78E-79	**4.62E-117**	8.33E-78	1.50E-78	–
F2	4.81E-81	3.47E-112	1.44E-79	2.58E-80	**9.16E-88**	**4.78E-115**	**2.59E-86**	**4.64E-87**	–
F3	1.58E-72	4.60E-113	4.75E-71	8.53E-72	**6.23E-87**	**1.00E-124**	**1.87E-85**	**3.35E-86**	–
F4	1.54E-106	3.42E-208	4.61E-105	8.28E-106	**5.51E-153**	**2.85E-278**	**1.65E-151**	**2.97E-152**	–
F5	2.60E-41	7.14E-62	6.48E-40	1.18E-40	**7.20E-51**	**7.29E-63**	**1.95E-49**	**3.50E-50**	–
F6	7.729474	5.8227	9.30231	0.87319246	**2.491777**	**1.66675**	**3.51528**	**0.36936424**	+
F7	0	0	0	0	0	0	0	0	+
F8	3.01E-164	1.72E-235	9.02E-163	0	**1.00E-182**	**2.87E-257**	**2.22E-181**	0	+
F9	0.0771604	0.0599719	0.109372	0.01093744	**0.000262778**	**0.00023401**	**0.00029302**	**1.51E-05**	+
F10	**0.7143642**	**0.00262144**	**5.42065**	**1.26746142**	12.82847028	0.0120785	76.3767	27.0029862	+
F11	**0**	0	**0**	**0**	0.000196021	0	0.00048194	0.00022393	+
F12	**0**	0	**0**	**0**	7.62E-05	0	0.0004812	0.00017052	+
F13	**0**	0	**0**	**0**	7.75E-09	0	2.32E-07	4.17E-08	–
F14	**1.13E-10**	**1.09E-10**	**1.16E-10**	**3.63E-12**	2.12E-05	2.08E-05	2.17E-05	2.10E-07	+
F15	1.11E-13	9.24E-14	1.39E-13	1.21E-14	**9.24E-14**	**7.82E-14**	**1.03E-13**	**6.99E-15**	+
F16	4.71E-33	4.71E-33	4.71E-33	2.05E-48	4.71E-33	4.71E-33	4.71E-33	2.05E-48	–
F17	1.35E-32	1.35E-32	1.35E-32	0	1.35E-32	1.35E-32	1.35E-32	0	+
F18	2.92E-16	1.85E-58	3.00E-15	6.02E-16	**8.88E-17**	**3.53E-64**	**6.11E-16**	**2.08E-16**	–
F19	1.35E-31	1.35E-31	1.35E-31	2.19E-47	1.35E-31	1.35E-31	1.35E-31	2.19E-47	–
F20	**1.52E-14**	**0**	**5.68E-14**	**1.90E-14**	1.02E-13	5.68E-14	1.42E-13	2.02E-14	+
F21	0.49722373	0.49345	0.498525	**0.00128231**	**0.4789659**	**0.466295**	**0.488808**	0.00460161	+
F22	−78.3323	−78.3323	−78.3323	1.42E-14	−78.3323	−78.3323	−78.3323	1.42E-14	–
F23	**−297.759533**	**−299.137**	−294.209	1.25590925	−297.3254	−298.786	**−294.629**	**1.19817847**	–
F24	4.63E-05	0	0.00031736	0.00010842	**1.13E-07**	0	**1.15E-06**	**3.40E-07**	+
F25	288853	288853	288853	**0**	**2.88E-06**	**2.85E-06**	**2.90E-06**	1.12E-08	+
F26	**0**	0	**0**	**0**	1.72E-08	0	5.17E-07	9.29E-08	–
F27	19.9896	19.983	**19.9962**	**0.00375136**	**4.699525383**	**0.0436521**	19.9987	8.43922988	+
F28	0.01835393	1.28E-11	0.215642	0.03867872	**0.000228001**	**1.19E-11**	**0.00029977**	**0.0001142**	+

Table 18
Summarized experiment results.

Dimension	ABC	ABCBB	GABC	GABCBB	ABC/Best/1	ABC/Best/1BB	ABC/Best/2	ABC/Best/2BB	ABCVSS	ABCVSSBB
	Mean Values									
30	3	19	9	13	2	17	2	18	8	13
60	5	19	9	16	4	15	3	18	8	15
100	3	21	8	17	4	15	4	18	9	14
	Mean Values with Sign +									
30	0	17	5	10	1	10	1	12	4	6
60	2	17	5	13	2	11	2	11	3	7
100	2	19	3	15	4	12	2	14	5	9
	Best Values									
30	3	16	7	11	0	14	1	15	2	13
60	3	17	6	14	3	13	2	15	2	14
100	2	19	2	19	3	15	2	17	4	14
	Worst Values									
30	3	19	8	14	2	15	2	17	8	13
60	5	19	9	16	4	14	3	18	8	15
100	4	19	8	16	4	14	3	18	9	14
	Standard Deviation									
30	4	17	7	15	4	14	3	17	10	10
60	7	16	15	9	5	13	3	17	11	11
100	6	17	15	9	5	13	5	16	11	11

ABCBB overperform ABC for almost all function expect F14 for D = 30, 60 and F10, F28 for all D values but the result of F14 is not statistically significant. GABC results are better than GABCBB for F10, F14, F16, F19 and F24 but only F10 is statistically significant for D values. ABC/Best/1 is only better than ABC/Best/1BB for only F10, F28 and F27 for D = 60 and 100. ABC/Best/2 has similar results over ABC/Best/2BB but additionally F26 is better for D = 100 but it is not statistically significant. ABCVSS is better than ABCVSSBB for F1 when D is 30 and 100 but it is not statistically significant. But in F14 ABCVSS is better for all D values and they are all statistically significant. For D = 60 and 100, ABCVSSBB found the optimum value of zero but ABCVSS didn't.
When total results of mean values are examined ABC/ABCBB is 11/59, GABC/GABCBB is 26/46, 'ABC/Best/1'/"ABC/Best/1BB" is 10/47, "ABC/Best/2"/"ABC/Best/2BB" is 9/54 and ABCVSS/ABCVSSBB is 25/42. The biggest difference in best values and worst values appears between ABC and ABCBB with 8/52 and 12/57 consequently. When standard deviation values considered, BB approach improved the robustness of the ABC algorithm because ABC is better on 17 test and ABCBB is better on 50 test. BB approach keeps the robustness same for ABCVSS algorithm because both ABCVSS and ABC algorithm is better in 32 tests.

it is not a stable algorithm. The performance of the AAA algorithm is similar with ABCVSS for D = 30.

In Table 21 ABCVSSBB performs best mean values for F1, F2, F3,F4, F5, F8, F10, F11, F14, F16–19 and F25. To claim that "ABCVSSBB overperforms other algorithms", Table 22 is analyzed. In Table 22 ABCVSSBB has the best mean values for F1-5, F8, F10 −12, F16–18 and F25. When the total of the results are considered ABCVSSBB has 46, AAA has 32 and GSO has 20 best mean results. Results show that ABCVSSBB outperforms the state of the art population based methods.

6. Results and discussion

ABC methods and variants update each solution that represented by employed bees by changing one parameter each time.

Table 19
The comparison result of ABCVSSBB, SPSO and ABC algorithms for pressure vessel problem with 10.000, 20.000 and 30.000 fitness evaluations.

Max. Fit. Ev.	Algorithm	Mean	Best	Worst	Std. Dev.
10000	ABCVSSBB	**7285.05733**	**7201.04**	**7529.34**	**8.2E+01**
	SPSO	7787.81825	7307.80230	8703.37492	3.377E+02
	ABC	7467.07005	7216.87872	7903.67565	1.757E+02
20000	ABCVSSBB	**7270.95733**	**7200.01**	7903.68	**1.291E+02**
	SPSO	7755.77219	7215.65815	8903.28015	3.551E+02
	ABC	7700.73753	7294.37680	**7903.67564**	2.400E+02
30000	ABCVSSBB	**7284.82833**	**7198.33**	7903.68	**1.384E+02**
	SPSO	7709.40529	7218.30817	8470.96724	3.073E+02
	ABC	7463.02094	7204.57448	**7903.67564**	2.315E+02

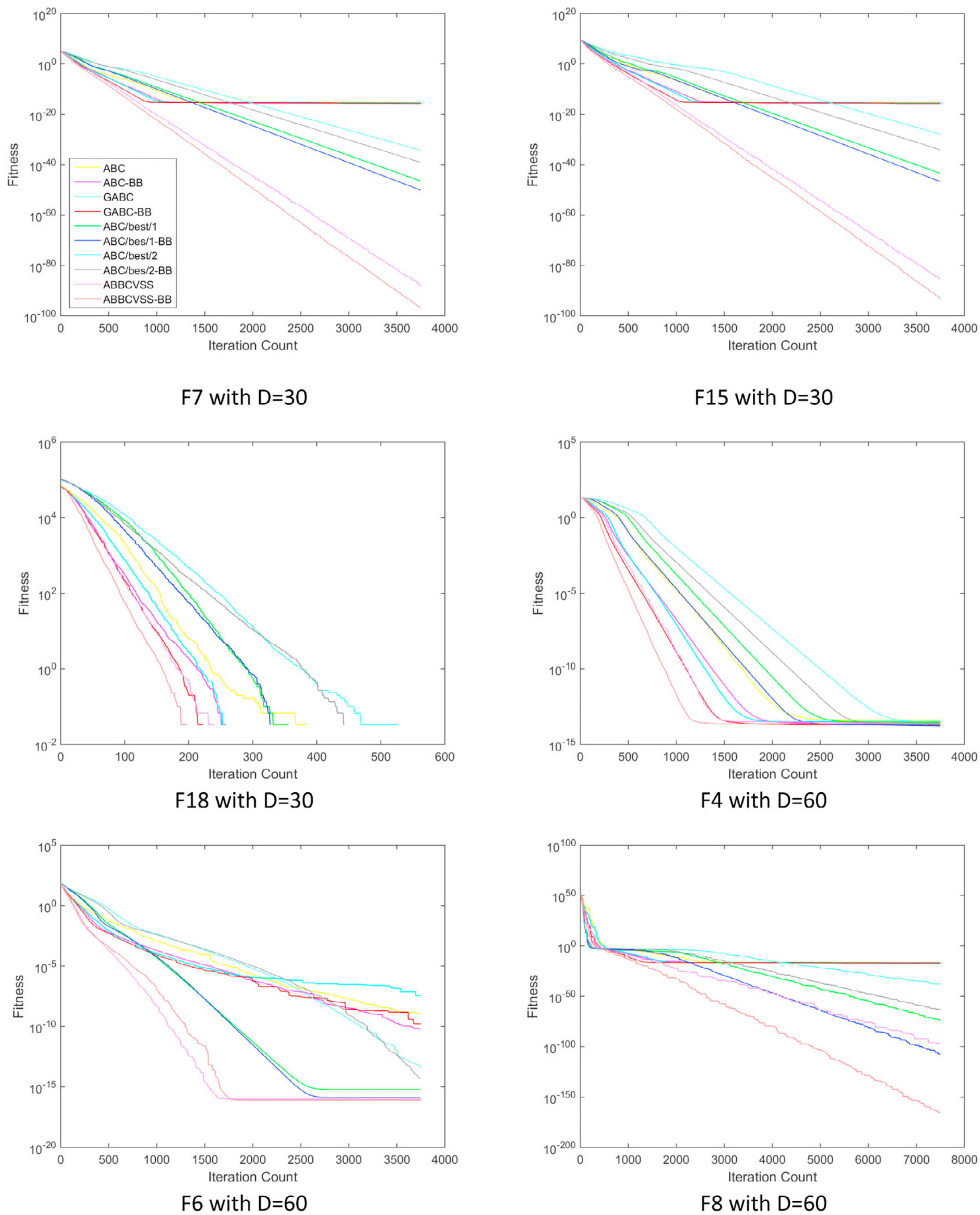

Fig. 4. Convergence characteristic of the BB approach.

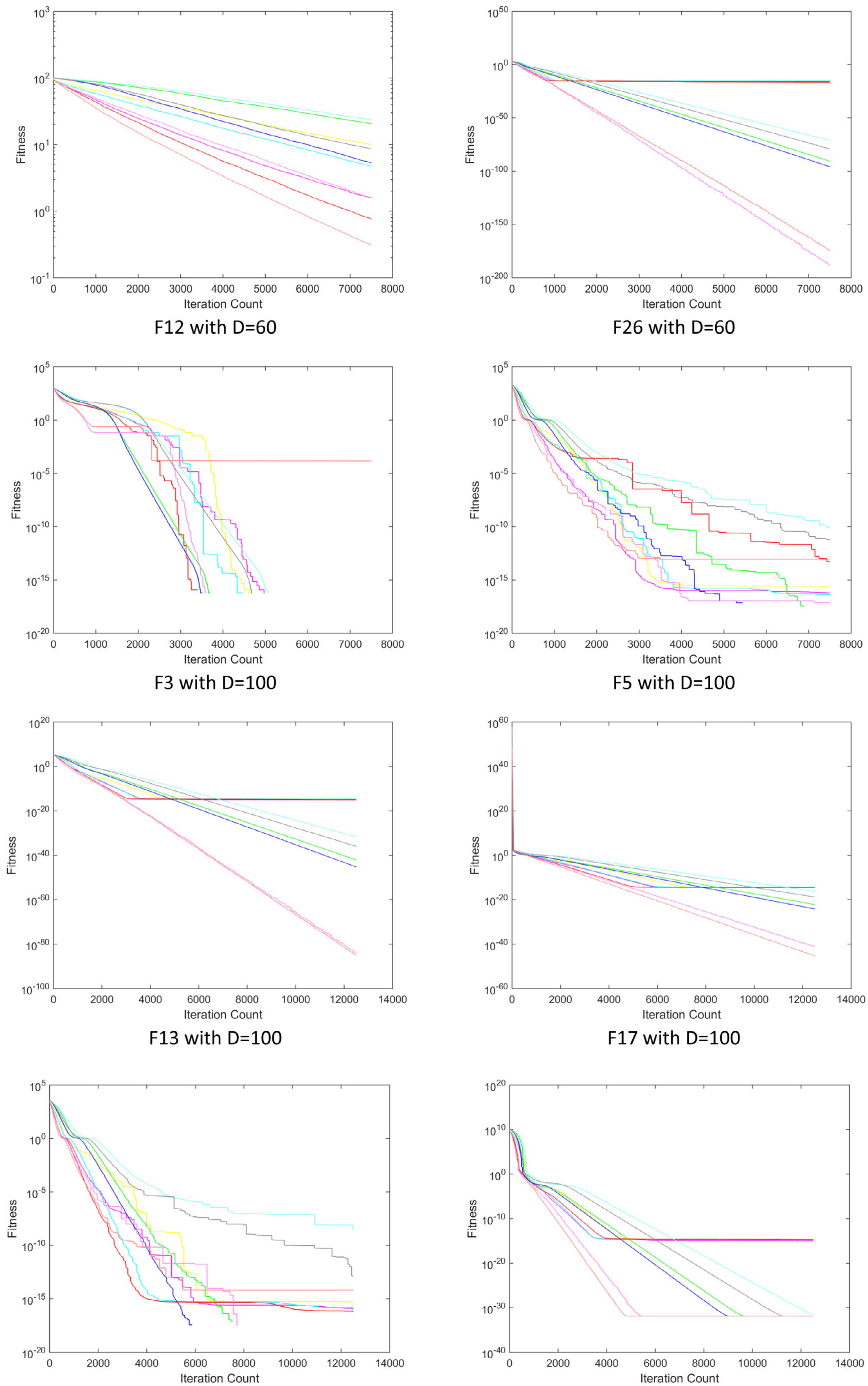

Fig. 4. (*Continued*)

Table 20
The comparison results of standard ABCVSSBB, AAA and GSO, D = 30.

Fun.	ABCVSSBB				AAA					GSO				
	Mean	Best	Worst	Std. Dev.	Mean	Best	Worst	Std.Dev	Sig.	Mean	Best	Worst	Std.Dev	Sig.
F1	1.13E-97	3.20E-128	3.39E-96	6.09E-97	3.06E-30	5.57E-31	1.00E-29	2.28E-30	+	**0**	**0**	**0**	**0**	–
F2	**8.77E-94**	2.95E-122	**2.63E-92**	**4.72E-93**	2.09E-27	7.28E-28	7.12E-27	1.37E-27	+	107333.3333	**0**	3220000	578007.689	–
F3	4.23E-97	1.90E-123	1.23E-95	2.21E-96	2.97E-31	3.09E-32	8.67E-31	2.01E-31	+	**0**	**0**	**0**	**0**	–
F4	**2.83E-160**	5.68E-273	**8.49E-159**	**1.52E-159**	3.40E-108	9.81E-115	2.52E-107	7.18E-108	–	33670	**0**	1000000	179452.124	–
F5	**4.56E-52**	2.43E-66	**8.24E-51**	**1.52E-51**	3.92E-19	1.22E-19	7.54E-19	1.60E-19	+	0.333378596	**0**	10	1.79504655	–
F6	0.0026024	0.00029618	0.0164248	0.0030348	0.3499745	0.231014	0.545211	0.07498593	+	**0**	**0**	**0**	**0**	+
F7	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	–	**0**	**0**	**0**	**0**	–
F8	5.41E-189	4.18E-256	1.62E-187	**0**	9.81E-58	1.02E-59	6.40E-57	1.63E-57	+	**0**	**0**	**0**	**0**	–
F9	0.00218925	0.00192829	0.00244144	**0.00012997**	0.016326753	0.00790568	0.025968	0.00578593	+	**0.0001103**	**3.88E-06**	**0.0011121**	0.00020876	+
F10	**3.89354997**	**1.13E-07**	69.3851	13.4252841	10.55858452	0.00081544	79.1143	15.5442458	–	23.91818156	0.00033412	**29.2914**	**10.6892379**	+
F11	**0.0003431**	**0**	**0.00148911**	**0.00060956**	0.0331653	**0**	0.994959	0.17860061	–	1.502056804	**0**	28.9247	5.85763263	–
F12	0.00018151	**0**	0.00138804	0.00046283	**0**	**0**	**0**	**0**	+	1.133623333	**0**	34.0087	6.10474848	–
F13	1.43E-12	**0**	2.19E-11	5.31E-12	**0**	**0**	**0**	**0**	–	**0**	**0**	**0**	**0**	–
F14	3.72E-05	1.82E-12	6.74E-05	3.25E-05	**2.43E-13**	**0**	**1.82E-12**	**6.18E-13**	+	3058.671333	1258.54	5247.67	1023.56919	+
F15	2.33E-14	1.78E-14	**2.84E-14**	**3.26E-15**	**1.93E-14**	1.07E-14	2.84E-14	4.83E-15	+	0.033232733	**0**	0.445815	0.10815037	–
F16	**1.57E-32**	**1.57E-32**	**1.57E-32**	**2.74E-48**	5.61E-32	1.57E-32	2.29E-31	4.34E-32	+	0.075980476	0.00823518	0.420867	0.07853629	+
F17	8.43E-12	**1.35E-32**	2.53E-10	4.54E-11	**5.77E-31**	2.83E-32	**2.81E-30**	**5.81E-31**	–	1.250829803	0.00103408	2.13986	0.55432667	+
F18	**8.13E-17**	3.66E-69	**1.33E-15**	**3.06E-16**	1.16E-07	7.73E-11	8.00E-07	1.98E-07	+	0.126983463	**0**	3.05693	0.56055399	–
F19	**1.35E-31**	1.35E-31	**1.35E-31**	**2.19E-47**	1.74E-31	**1.35E-31**	3.69E-31	5.81E-32	+	11.79239493	0.00382501	26.38	8.50255155	+
F20	2.37E-16	**0**	7.11E-15	1.28E-15	**0**	**0**	**0**	**0**	–	0.640799933	**0**	8	1.80060682	–
F21	0.14576138	0.0781892	0.22769	0.03785353	0.135934367	0.0781892	0.178222	0.0345228	–	**0.000647727**	**0**	**0.0097159**	**0.00242357**	+
F22	**−78.3323**	**−78.3323**	**−78.3323**	**1.42E-14**	**−78.3323**	**−78.3323**	**−78.3323**	**1.42E-14**	–	−69.61287	−74.4788	−66.0503	1.94385063	+
F23	**−29.61786**	**−29.6309**	**−29.5608**	**0.01712705**	−29.4567133	−29.5732	−29.1554	0.09374677	+	−6.91058233	−8.22637	−5.94777	0.75963782	+
F24	**0**	**0**	**0**	**0**	2.99E-30	1.17E-31	1.39E-29	2.84E-30	+	**0**	**0**	**0**	**0**	–
F25	7.91E-06	7.77E-06	8.01E-06	5.60E-08	**0**	**0**	**0**	**0**	+	**0**	**0**	**0**	**0**	+
F26	9.24E-17	**0**	1.22E-15	3.05E-16	**0**	**0**	**0**	**0**	–	**0**	**0**	**0**	**0**	–
F27	2.03905334	0.0433012	19.9998	5.98648223	**1.98E-14**	1.42E-14	**2.84E-14**	**4.07E-15**	–	0.472586757	**0**	13.8017	2.47568378	–
F28	7.30E-05	3.76E-12	0.00074521	0.00021918	**2.18E-07**	5.31E-10	**1.86E-06**	**4.20E-07**	–	0.149344923	**0**	4.44021	0.79682618	–

Table 21
The comparison results of standard ABCVSSBB, AAA and GSO, D = 60.

	ABCVSSBB				AAA					GSO				
Fun.	Mean	Best	Worst	Std. Dev.	Mean	Best	Worst	Std. Dev.	Sig.	Mean	Best	Worst	Std. Dev.	Sig.
F1	**5.08E-95**	1.93E-124	**1.48E-93**	**2.65E-94**	1.82E-29	4.23E-30	4.96E-29	1.26E-29	+	0.000322824	**0**	0.00968472	0.00173846	–
F2	**4.13E-86**	1.39E-121	**1.22E-84**	**2.19E-85**	1.71E-26	2.17E-27	4.73E-26	1.10E-26	+	1797411.768	**0**	25500000	5575151.71	–
F3	**5.11E-92**	9.39E-120	**1.11E-90**	**2.06E-91**	4.23E-30	5.47E-31	1.48E-29	2.81E-30	+	0.000364769	**0**	0.00996441	0.00179123	–
F4	**8.33E-167**	8.36E-280	**2.50E-165**	**0**	2.14E-106	2.12E-114	3.15E-105	6.70E-106	–	33352.24751	**0**	1000000	179502.009	–
F5	**2.35E-49**	2.50E-66	**2.08E-48**	**5.76E-49**	1.78E-18	8.28E-19	4.75E-18	8.40E-19	+	1	**0**	20	3.95811403	–
F6	0.312309867	0.126431	0.558673	0.09904864	3.550493	2.60905	4.25681	0.46911452	+	**0.012129577**	**0**	**0.26725**	**0.04961776**	+
F7	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	–	**0**	**0**	**0**	**0**	–
F8	**4.93E-175**	1.01E-252	**1.48E-173**	**0**	1.41E-56	1.85E-58	1.01E-55	2.00E-56	+	2.11E-11	**0**	6.06E-10	1.09E-10	–
F9	0.000657282	0.00058203	**0.00072224**	**3.49E-05**	0.03898874	0.01792	0.056369	0.01002492	+	**0.000142219**	**6.66E-06**	0.0025174	0.0004435	+
F10	**7.965455835**	**0.00076096**	75.2154	20.9942115	47.0406703	0.190439	95.1978	32.0475398	+	58.57174	58.2018	**58.8367**	**0.12168537**	+
F11	**0.000199714**	**0**	**0.0007851**	**0.00033139**	0.0331653	**0**	0.994959	0.17860061	–	3.86883102	**0**	57.8494	12.343592	–
F12	0.000144042	**0**	0.00073656	0.00028815	**0**	**0**	**0**	**0**	+	1.915690667	**0**	25.0997	6.26130354	–
F13	5.57E-14	**0**	1.67E-12	3.00E-13	**0**	**0**	**0**	**0**	–	0.068083362	**0**	1.75344	0.31722198	–
F14	**2.75E-05**	4.37E-11	**3.54E-05**	**1.38E-05**	3.94793333	**3.64E-11**	118.438	21.2602716	–	8581.696	2861.38	12194	2305.04486	+
F15	5.11E-14	3.91E-14	6.39E-14	5.62E-15	**3.35E-14**	2.84E-14	**4.26E-14**	**3.27E-15**	+	1.122075733	**0**	15.2851	3.48006667	–
F16	**7.85E-33**	**7.85E-33**	**7.85E-33**	**1.37E-48**	2.11E-31	1.56E-32	4.87E-31	1.05E-31	+	0.117928583	0.041874	0.205836	0.04372556	+
F17	**1.35E-32**	**1.35E-32**	**1.35E-32**	**5.47E-48**	2.82E-30	3.97E-31	7.55E-30	1.76E-30	+	4.737434333	2.95885	5.82892	0.61720844	+
F18	**1.48E-17**	2.57E-64	**2.22E-16**	**5.54E-17**	2.48E-07	1.21E-08	1.22E-06	2.76E-07	+	0.170324415	**0**	2.78386	0.63841998	–
F19	**1.35E-31**	1.35E-31	**1.35E-31**	**2.19E-47**	4.88E-31	**1.35E-31**	1.97E-30	3.57E-31	+	41.81171333	14.948	52.6937	6.84900527	+
F20	1.89E-14	**0**	2.84E-14	8.47E-15	**0**	**0**	**0**	**0**	+	0.747503173	**0**	8.61731	2.16515969	–
F21	0.3766988	0.272741	**0.429723**	0.03647784	0.41908757	0.373291	0.441908	**0.01817084**	+	**0.023443858**	**0**	0.495946	0.09336671	+
F22	−**78.3323**	−**78.3323**	−**78.3323**	**1.42E-14**	−**78.3323**	−**78.3323**	−**78.3323**	**1.42E-14**	–	−70.02212	−75.9433	−66.4105	2.68664035	+
F23	−**59.5798833**	−**59.6187**	−**59.4934**	**0.03304294**	−58.51035	−58.9665	−57.7659	0.32959157	+	−11.6813893	−13.314	−9.69998	0.87285326	+
F24	**0**	**0**	**0**	**0**	2.11E-29	2.63E-30	7.03E-29	1.72E-29	+	1.55E-05	**0**	0.0004415	7.92E-05	–
F25	**5.13E-06**	5.03E-06	**5.18E-06**	**3.17E-08**	0.1326612	**0**	0.994959	0.33822102	+	4.832365011	**0**	86.7741	18.4172612	–
F26	8.73E-14	**0**	2.62E-12	4.70E-13	**0**	**0**	**0**	**0**	–	3.01694	**0**	90.5082	16.2467191	–
F27	4.033178373	0.0434913	19.9955	7.97892775	**3.29E-14**	2.49E-14	**4.26E-14**	**4.20E-15**	+	1.74808896	**0**	18.0676	4.53543615	–
F28	0.007961783	5.43E-12	0.231354	0.04148362	**3.10E-07**	8.48E-09	**1.28E-06**	**3.27E-07**	–	0.43226515	**0**	12.8804	2.31159573	–

Table 22
The comparison results of standard ABCVSSBB, AAA and GSO, D = 100.

Func.	ABCVSSBB				AAA					GSO				
	Mean	Best	Worst	Std.Dev	Mean	Best	Worst	Std.Dev	Sig.	Mean	Best	Worst	Std.Dev	Sig.
F1	**2.13E-80**	4.31E-118	**6.40E-79**	**1.15E-79**	7.14E-29	1.90E-29	2.92E-28	6.44E-29	+	666.6667392	**0**	20000	3590.10986	–
F2	**1.36E-91**	6.69E-114	**2.04E-90**	**4.98E-91**	4.22E-26	7.57E-27	8.70E-26	1.87E-26	+	8609429.147	**0**	146000000	28040849	–
F3	**7.13E-86**	9.96E-121	**2.14E-84**	**3.84E-85**	2.90E-29	3.45E-30	6.30E-29	1.46E-29	+	323.3399669	**0**	9700	1741.20206	–
F4	**4.07E-169**	2.38E-281	**1.22E-167**	**0**	1.21E-97	2.45E-107	3.16E-96	5.67E-97	–	3.33E+44	**0**	1.00E+46	1.80E+45	–
F5	**4.84E-46**	4.92E-65	**1.45E-44**	**2.60E-45**	4.39E-18	1.45E-18	9.99E-18	1.88E-18	+	2.06964758	**0**	32.0412	6.82675517	–
F6	2.639363	1.78976	3.97714	0.56488576	9.049822	6.92255	11.1288	1.0994826	+	**3.11E-06**	**0**	**9.32E-05**	**1.67E-05**	+
F7	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	–	**0**	**0**	**0**	**0**	–
F8	**2.11E-174**	3.16E-247	**5.64E-173**	**0**	8.01E-56	1.07E-56	4.52E-55	1.00E-55	+	0.178957	**0**	5.36871	0.96371294	–
F9	0.00025904	0.0002139	**0.00028695**	**1.61E-05**	0.07681811	0.0417472	0.125505	0.01657351	+	**7.42E-05**	**7.49E-07**	0.00055974	0.00012553	+
F10	**18.30336761**	**0.00370743**	137.685	33.8555136	104.428364	1.33583	177.471	41.6812268	+	98.48931	98.1946	**98.9027**	**0.14695309**	+
F11	**0.00024138**	**0**	**0.00047827**	**0.00022952**	0.23215717	**0**	1.98992	0.49303787	+	6.540935617	**0**	112.796	22.5140737	–
F12	**7.69E-05**	**0**	**0.00046466**	**0.00016845**	0.03333862	**0**	1	0.17950451	–	29.67197167	**0**	300.831	68.665374	+
F13	6.43E-15	**0**	1.93E-13	3.46E-14	**0**	**0**	**0**	**0**	–	0.035228133	**0**	0.592232	0.13100076	–
F14	1.84E-05	1.16E-10	2.18E-05	7.21E-06	**1.11E-10**	**1.09E-10**	**1.16E-10**	**3.22E-12**	+	17244.13533	9958.46	26085.4	4324.61477	+
F15	9.46E-14	7.46E-14	1.10E-13	8.50E-15	**5.08E-14**	3.91E-14	**6.39E-14**	**5.74E-15**	+	0.713216371	**0**	11.9099	2.68307714	–
F16	**4.71E-33**	**4.71E-33**	**4.71E-33**	**0**	2.85E-31	5.78E-32	7.43E-31	1.69E-31	+	0.15874564	0.0627572	0.294962	0.05867599	+
F17	**1.35E-32**	**1.35E-32**	**1.35E-32**	**5.47E-48**	9.33E-30	2.19E-30	3.68E-29	8.25E-30	+	8.637341333	7.70556	9.57985	0.48615669	+
F18	**7.59E-17**	9.71E-63	**6.11E-16**	**1.95E-16**	4.98E-07	2.39E-08	1.99E-06	5.51E-07	+	0.30184132	**0**	4.44021	1.10622511	–
F19	5.01E-06	**1.35E-31**	0.00015034	2.70E-05	**1.31E-30**	4.18E-31	**3.60E-30**	**6.96E-31**	–	81.14568333	65.7646	93.1021	6.8427215	+
F20	9.58E-14	5.68E-14	1.42E-13	2.50E-14	**0**	**0**	**0**	**0**	+	1.334759967	**0**	17.3617	3.59170127	–
F21	0.4763075	0.451776	0.485013	0.00839599	0.4965646	0.49345	0.498226	**0.00114265**	+	**0.002212394**	**0**	**0.0372241**	0.00712271	+
F22	**−78.3323**	**−78.3323**	**−78.3323**	**1.42E-14**	**−78.3323**	**−78.3323**	**−78.3323**	**1.42E-14**	–	−68.42189	−73.4872	−65.5364	2.1773277	+
F23	**−99.5183367**	**−99.6014**	**−99.3591**	**0.05878739**	−97.00751	−97.9402	−95.9721	0.46216453	+	−17.2351067	−19.0592	−15.7794	0.87081345	+
F24	4.03E-08	**0**	1.21E-06	2.17E-07	**6.33E-29**	1.65E-29	**1.43E-28**	**3.40E-29**	–	333.3336095	**0**	10000	1795.05488	–
F25	**2.94E-06**	2.92E-06	**2.96E-06**	**8.14E-09**	0.0663306	**0**	0.994959	0.24818638	–	9.54327	**0**	86.8996	23.944724	+
F26	2.43E-14	**0**	7.28E-13	1.31E-13	**0**	**0**	**0**	**0**	–	**0**	**0**	**0**	**0**	–
F27	6.027129103	0.0436073	19.9961	9.13972484	**5.07E-14**	3.91E-14	**6.04E-14**	**5.72E-15**	+	0.48158689	**0**	14.4393	2.59188251	+
F28	0.000246145	0.0002352	0.00025371	4.60E-06	**5.90E-07**	1.33E-08	**2.03E-06**	**5.32E-07**	+	0.149328667	**0**	4.44021	0.79682841	–

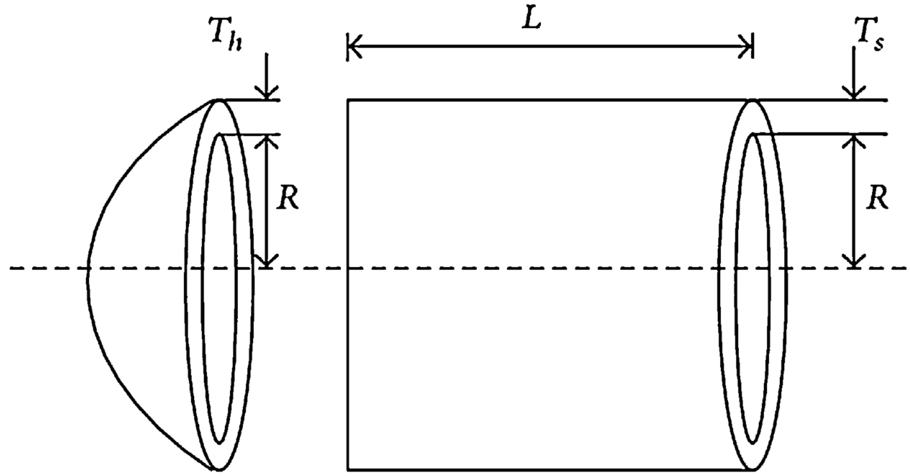

Fig. 5. Cylindrical pressure vessel design problem [42].

The proposed method adds an additional update rule and does not change the original update rule.

The proposed approach shows the best performance on classical ABC and the worst performance on ABCVSS but it is still good on the best and the worst values of both ABC and ABCVSS. Standard GABC has smaller standard deviation results, so the algorithm is more robust than BB version.

Best values found for BB versions of ABC, GABC and ABC/Best/1 increase by dimensionality. Namely, in a high dimensional problem, BB versions of the methods perform better than standard versions. Statistically significant better mean values are increased by dimensionality in GAB BB and ABC/best/1 BB.

Standard derivation is better for BB versions of ABC, ABC/Best/1 and ABC/Best/2. Standard version of GABC has smaller standard deviation values, so it is more robust than BB version. ABCVSS has similar standard deviation values for both versions.

All convergence graphics show that adding additional BB update rule improves the convergence performance of all methods.

7. Conclusion and future works

ABC is the most investigated swarm intelligence based algorithm in recent years. The researchers realized that the weakest point of the algorithm was its updating rule and strategy. Thus they always motivated to find better update rules but adding additional update rule has not been considered as an alternative way. In this study, an additional update equation for ABC algorithm and its variants is proposed using Bollinger bands which is a technical analysis tool to predict maximum or minimum future stock prices. Experimental results show that the proposed update equation can be applied to employed bee phase of all ABC oriented algorithm because it only adds a new parameter update procedure to original algorithms. ABC variants show faster convergence speed and final results on 28 benchmark functions when the proposed approach is applied. As a future work, other technical analysis tools will be considered to improve ABC based algorithms.

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