Applied Soft Computing 49 (2016) 238–247

Contents lists available at ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

A model for failure mode and effects analysis based on intuitionistic fuzzy approach

Hossein Sayyadi Tooranloo[a,*], Arezoo sadat Ayatollah[b]

[a] Assistant Professor of Management Faculty, Vali-e-Asr University of Rafsanjan, Rafsanjan, Iran
[b] Master Science of Information Technology Management, University of Science and Arts, Yazd, Iran

ARTICLE INFO

Article history:
Received 7 November 2015
Received in revised form 18 July 2016
Accepted 26 July 2016
Available online 12 August 2016

Keywords:
FMEA
Intuitionistic fuzzy logic
Group decision-making

ABSTRACT

Failure mode and effects analysis (FMEA) is one of the most powerful methods in the field of risk management and has been widely used for improving process reliability in manufacturing and service sector. High applicability of FMEA has contributed to its applications in many research domains and practical fields pertaining risk assessment and system safety enhancement. However, the method has also been criticized by experts due to several weaknesses and limitations. The current study proposed a novel model for failure mode and effects analysis based on intuitionistic fuzzy approach. This approach offers some advantages over earlier models as it accounts for degrees of uncertainty in relationships among various criteria or options, specifically when relations cannot be expressed in definite numbers. The proposed model provides a tool to evaluate the failure modes, while dealing with vague concepts and insufficient data. The proposed model was tested in a case study examining the failure modes for quality of internet banking services.

1. Introduction

Businesses in service and support sector often face intensified competition, changes in customers' demands and expectations, and technological developments. To encounter such challenges, service providers need to increase their commitment toward eliminating service defects and improving their performance to gain customer satisfaction and maintain their market share. One of the methods used by organizations to maintain service quality and achieve competitive advantage is "Failure mode and effects analysis" [26]. First introduced by NASA in 1963 to assess system reliability requirements, FMEA has been used as a powerful technique for the analysis of safety and reliability of products and processes across a wide range of businesses such as aerospace, nuclear, automobile and pharmaceutical industries [30,25]. FMEA has been applied by safety and reliability engineers to identify the important functions in which a failure can lead to undesirable outputs and create customer dissatisfaction [48]. The technique aims to detect and prioritize potential failure modes by assessing an index called Risk Priority Number (RPN). The index is constructed as the product of three concepts, namely the probability of failure occurrence (O), severity of failure (S), and failure detection (D) [48]. Probability of occurrence refers to the likelihood of occurrence of a cause/mechanism. Severity indicates the potential impact of failure on parts, sub-systems, or customers. And, detection refers to the capability of detecting potential causes/mechanisms before the occurrence of a failure [39]. These three factors are estimated by experts on a scale of 1–10. RPN is a measure of failure risk. Therefore, it can be used to rank failures and prioritize required actions. In the course of calculating RPN, probability indices for severity and occurrence are used directly, whereas detection index is used inversely. Hence, a higher value for RPN index indicates a more critical failure for which corrective actions should be given higher priority. FMEA can provide some measures to reduce the likelihood of faults/failures and help users to determine the key design features and processes that require special control [42]. However, RPN has been widely criticized for several reasons as the following [13,47,11,15,16,35,45]:

- Risk factors are difficult to be accurately assessed.

* Corresponding author at: Assistant Professor of Management Faculty, Vali-e-Asr University of Rafsanjan, Rafsanjan, Iran.
E-mail addresses: h.sayyadi@vru.ac.ir (H. Sayyadi Tooranloo), Arezoo.ayatollah@yahoo.com (A.s. Ayatollah).

http://dx.doi.org/10.1016/j.asoc.2016.07.047
1568-4946/

- Various combinations of O, S and D may produce exactly the same RPN, while the value of their hidden risks may be quite different. For example, when two different scenarios have O, S and D values of (2, 3, 2) and (1, 3, 4) respectively, the RPN values for both scenarios are equal to 12. However, the hidden risks of these combinations are not necessarily identical. This may lead to a waste of resources and time. Additionally, in some cases a high-risk event may remain unnoticed.
- Relative importance of O, S and D are not addressed in FMEA. These risk factors are assumed to be equally important, which does not apply to practical application of FMEA.

Growing complexity in decision making increases uncertainty levels which necessitates the use of a more comprehensive and powerful tool built upon the theory of fuzzy sets (i.e. Fuzzy FMEA). By using fuzzy concepts, assessors can use linguistic terms in the form of verbal expressions to evaluate the risk factors for each item of failure, and then relate these expressions to appropriate membership functions to provide a better and more accurate analysis for the scores of failure modes [24,29,44,55].

In 1986, Atanassov extended the theory of fuzzy sets and introduced the concept of intuitionistic fuzzy sets. Since then, theory of intuitionistic fuzzy sets has become increasingly popular [62] to deal with uncertainty [43]. In this paper, theory of intuitionistic fuzzy sets is used to evaluate the failure criteria for quality of internet banking services using a proposed FMEA technique based on intuitionistic fuzzy approach.

2. Theoretical framework

2.1. Fuzzy FMEA

Fuzzy FMEA provides a tool that works best with vague concepts and in the lack of sufficient information [27]. Using fuzzy theory is essential when dealing with some degrees of uncertainty in relationships among various criteria or when relations cannot be expressed in the form of definite numbers. Fuzzy FMEA has been applied by several earlier studies to assess risk [19]. For example, Chang et al. [20] used grey theory for FMEA. Their study first used fuzzy expressions such as very low, low, medium, high and very high to evaluate O, S and D, and then applied grey relational analysis to determine the risk ratings of potential causes. By performing the grey relational analysis, fuzzy expressions were converted to definitive values, and the lowest levels of O, S and D were defined as the standard series. Data regarding these three factors for each potential cause was seen as comparative series and grey relational coefficients and degree of grey relation were compared against the standard series under the rules of grey theory. The highest degree of grey relation indicated minimal effect of potential cause [22].

Braglia et al. also proposed a multi-criteria decision making approach called fuzzy TOPSIS for FMECA. As a well-known multi-criteria decision-making method, TOPSIS is based on the idea that the best decision should have minimum distance from the positive ideal and maximum distance from the negative ideal. The fuzzy TOPSIS approach provides the possibility of evaluating risk factors (O, S and D) and their relative importance using triangular fuzzy numbers [15,16]. Bowles and Peláez proposed a fuzzy logic-based approach to prioritize failures in a FMEA system [14]. This approach used verbal expressions to describe O, S, D and the risks of failure. In this approach, the relationships between risk and O, S, D were described using fuzzy *if-then* rules obtained from experts' opinion. Garcia et al. proposed a fuzzy data envelopment analysis approach combined with fuzzy sets to determine the rating of failure modes [33]. Chen and Kuo calculated fuzzy RPN by using fuzzy ordered weighted geometric averaging (FOWGA) operator [23]. Similarly, Wang et al. proposed a new definition for fuzzy RPN by using fuzzy weighted geometric mean (FWGM).

Fuzzy RPN can also be calculated using alpha-cut sets, linear programming model and defuzzification through center of gravity method, to obtain the final ranking of failure modes [54]. Kutlu and Ekmekçioğlu [64] proposed a hybrid approach based on TOPSIS and AHP in a fuzzy setup to analyze failure modes. Their study used the fuzzy AHP method to determine the weight of risk factors. After assigning the weights and generating the failure items decision matrix for risk factors, fuzzy TOPSIS was performed to prioritize the failure modes. The study by Liu et al. [65] developed a model based on fuzzy VIKOR techniques to assess and prioritize risk factors. It used linguistic terms and corresponding fuzzy numbers to determine the weight of risk factors based on expert opinions. Then, the fuzzy ordered weighted decision matrix for factors of failure modes was calculated and the VIKOR technique was used to prioritize failure modes. In another attempt [40], investigated the applications of fuzzy FMEA to improve procurement processes of a hospital. They concluded that fuzzy FMEA technique could properly solve problems associated with traditional FMEA and could be useful for exploring potential failure modes and their effects. Finally, the study by Rafie and Samimi [63] proposed a hybrid approach comprising fuzzy rules and neural network to evaluate the RPN in FMEA. It used fuzzy rules to determine severity (S) and detection (D), while occurrence (O) was determined using neural network.

2.2. Intuitionistic fuzzy set (IFS)

Intuitionistic fuzzy set (IFS) is one of the generalizations from the fuzzy sets theory [59]. Out of several higher-order fuzzy sets, IFS has been found to be more capable of dealing with vagueness. First introduced by Atanassov [3], IFS can be viewed as an alternative approach to conventional fuzzy set in dealing with cases with insufficient information. Fuzzy sets only consider the degree of acceptance, whereas IFS is characterized by both a membership function and a non-membership function so that the sum of both values is less than one [4]. Intuitionistic fuzzy sets have been used across different fields of science, including the studies by Atanassov [4–7], Atanassov and Gargov [8], Szmidt and Kacprzyk [50], Buhaescu [17], Ban [9], Deschrijver and Kerre [28] and Stoyanova [49].

Definition 1. Assume reference set $X = \{x_1, x_2, x_3, \ldots\}$. In this case, set A which is a subset of X is an Atanassov's intuitionistic fuzzy set defined as below:

$$A = \left\{ \prec x\,,\ u_A(x)\,,\ \nu_A(x) \succ \forall x \in X \right\} \tag{1}$$

In the above definition, $u_A(x)$, $\nu_A(x)$ are degree of membership and non-membership respectively, which are defined as $u_A(x) : x \to [0, 1]$, $\nu_A(x) : x \to [0, 1]$ and satisfy $0 \le u_{ij}(x) + \nu_{ij}(x) \le 1$. In addition, for each $x \in X$, intuitionistic index π_x is defined as $\pi_x = 1 - u_x - \nu_x$ [4].

Definition 2. Based on Atanassov, $\left(u_{ij}(x), v_{ij}(x), \pi_{ij}(x)\right)$ is an intuitionistic fuzzy number that satisfies the following conditions:

$$\mu_{ij}(x) \in [0,1],\ \nu_{ij}(x) \in [0,1],\ \pi_{ij}(x) \in [0,1],\ 0 \le \mu_{ij}(x) + \nu_{ij}(x) \le 1,\ \pi_{ij}(x) = 1 - \mu_{ij}(x) - \nu_{ij}(x) \tag{2}$$

In this paper, intent of intuitionistic fuzzy numbers is Atanassov's intuitionistic fuzzy numbers.

Although intuitionistic fuzzy number is similar (in appearance) to triangular fuzzy number (a, b, c), it is quite different. Triangular fuzzy number is a convex normal fuzzy set with a membership function in which$(a \prec b \prec c)$, whereas an intuitionistic fuzzy number is a point in three-dimensional space constructed by axes $u_{ij}(x), v_{ij}(x), \pi_{ij}(x)$ [51]. Atanassov and Gargov [8] and Gau and Buehrer [34] have described intuitionistic fuzzy number (0.50,0.20,0.30) as a scenario where votes in favor of adoption are 0.5, votes against it are 0.2 and abstained votes are 0.30. In this context, the following relationship holds true:

$$\mu_{ij}^{\beta}(x) + \nu_{ij}^{\beta}(x) \le 1,\ 0 \le \mu_{ij}^{\alpha}(x) \le \mu_{ij}^{\beta}(x) \le 1,\ 0 \le \nu_{ij}^{\alpha}(x) \le \nu_{ij}^{\beta}(x) \le 1 \tag{3}$$

These numbers are better suited to deal with uncertainty and provide a more logical mathematical framework to deal with inexact facts and incomplete information [61]. Some of the operators and relationships between these numbers are provided as the following. For simplicity's sake, these numbers are expressed as $\left[u_{ij}(x), v_{ij}(x), \pi_{ij}(x)\right]$ where $u_{ij}(x)$, $v_{ij}(x)$ and $\pi_{ij}(x)$ are numbers in the range of [0,1].

Definition 3. Assume intuitionistic fuzzy numbers $A = \left\{\langle x, \mu_A(x), \nu_A(x) | x \in X\rangle\right\}$,$A_1 = \left\{\langle x, \mu_{A1}(x), \nu_{A1}(x) | x \in X\rangle\right\}$, $A_2 = \left\{\langle x, \mu_{A2}(x), \nu_{A2}(x) | x \in X\rangle\right\}$, and the real number n. According to De et al. [66] and Atanassov [4] the following relationships are defined:

$$\bar{A} = \left\{\langle x, \nu_A(x), \mu_A(x) \middle| x \in X\rangle\right\} \tag{4}$$

$$\begin{aligned} A_1 \cap A_2 &= \left\{\langle x, \min\left\{\mu_{A_1}(x), \mu_{A_2}(x)\right\}, \max\left\{\nu_{A_1}(x), \nu_{A_2}(x)\right\} \middle| x \in X\rangle\right\} \\ A_1 \cup A_2 &= \left\{\langle x, \max\left\{\mu_{A_1}(x), \mu_{A_2}(x)\right\}, \min\left\{\nu_{A_1}(x), \nu_{A_2}(x)\right\} \middle| x \in X\rangle\right\} \end{aligned} \tag{5}$$

$$A_1 + A_2 = \left\{\langle x, \mu_{A_1}(x) + \mu_{A_2}(x) - \mu_{A_1}(x) \cdot \mu_{A_2}(x), \nu_{A_1}(x) \cdot \nu_{A_2}(x) \middle| x \in X\rangle\right\} \tag{6}$$

$$A_1 \cdot A_2 = \left\{\langle x, \mu_{A_1}(x) \cdot \mu_{A_2}(x), \nu_{A_1}(x) + \nu_{A_2}(x) - \nu_{A_1}(x) \cdot \nu_{A_2}(x) \middle| x \in X\rangle\right\} \tag{7}$$

$$nA = \left\{\langle x, 1 - (1 - \mu_A(x))^n, (\nu_A(x))^n \middle| x \in X\rangle\right\} \tag{8}$$

$$A^n = \left\{\langle x, (\mu_A(x))^n, 1 - (1 - \nu_A(x))^n \middle| x \in X\rangle\right\} \tag{9}$$

Where n is a Positive integer

3. Design of risk factor assessment model based on FMEA approach in an intuitionistic fuzzy environment

There has been much discussion about the easy and accurate determination of risk factors of failure occurrence (O), Severity (S) and detectability (D). Since verbal evaluation has an approximate nature, it can be concluded that the theory of intuitionistic fuzzy sets is suitable to deal with the uncertainty of such estimates and to achieve more accurate results. On this basis, we propose a group decision-making model that utilizes TOPSIS technique to evaluate failure modes based on FMEA model in an intuitionistic fuzzy environment. The linguistic terms and their corresponding intuitionistic fuzzy numbers, which are used in this study to evaluate the risk factors, are shown in Table 1. Assume that there are n modes of failure $FM_i(1, \ldots, n)$ which are evaluated by FMEA team composed of k members $TM_k(1, \ldots, k)$ for the risk factors $C = \left\{O, S, D\right\}$, based on intuitionistic fuzzy linguistic variables presented in this table.

In the following sections, we discuss the steps of intuitionistic fuzzy TOPSIS, which is aimed at assessing the failure modes based on risk factors.

Step 1. determining the weight of decision-makers:

Assume that the decision-making team is composed of k members and the importance of decision-makers is expressed based on linguistic terms and intuitionistic fuzzy numbers listed in Table 2.

As such, if we assume that $D_k = \left\{\mu_k, \nu_k, \pi_k\right\}$ is an intuitionistic fuzzy number representing the importance of k-th decision-maker, the weight of the k-th decision maker can be calculated as follows:

$$\lambda_k = \frac{\left(\mu_k + \pi_k\left(\frac{\mu_k}{\mu_k + \nu_k}\right)\right)}{\sum_{k=1}^{n}\left(\mu_k + \pi_k\left(\frac{\mu_k}{\mu_k + \nu_k}\right)\right)} \qquad \sum_{k=1}^{n}\lambda_k = 1 \tag{10}$$

Step 2. Constructing the aggregate intuitionistic fuzzy decision matrix based on decision-makers' opinions:

Table 1
intuitionistic fuzzy linguistic variables.

Detectability		Failure severity probability		Failure	occurrence probability
Intuitionistic fuzzy number	Linguistic term	Intuitionistic fuzzy number	Linguistic term	Intuitionistic fuzzy number	Linguistic term
(1,0)	Absolutely Impossible	(1,0)	Risky Without Warning	(0.9,0.1)	Very High
(0.9,0.1)	Highly Unlikely	(0.9,0.1)	Risky With Warning	(0.75,0.1)	High
(0.8,0.1)	Unlikely	(0.8,0.1)	Very High	(0.5,0.45)	Medium
(0.7,0.2)	Very Low	(0.7,0.2)	High	(0.35,0.6)	Low
(0.6,0.3)	Low	(0.6,0.3)	Medium	(0.1,0.9)	Very Low
(0.5,0.4)	Medium	(0.5,0.4)	Low		
(0.4,0.5)	Relatively High	(0.4,0.5)	Very Low		
(0.25,0.6)	High	(0.25,0.6)	Insignificant		
(0.1,0.75)	Very High	(0.1,0.75)	very Insignificant		
(0.1,0.9)	Absolutely Possible	(0.1,0.9)	None		

Table 2
Linguistic terms for rating the importance of criteria and the decision makers.

IFNs	Linguistic terms
(0.9,0.1)	Very Important
(0.75,0.2)	Important
(0.5,0.45)	Medium
(0.35,0.6)	Unimportant
(0.1,0.9)	Very Unimportant

Assume that $R^{(k)} = \left(r_{ij}^{k}\right)_{m\times n}$ is the intuitionistic fuzzy decision matrix and $\lambda = \left\{\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_k\right\}$ is the weight of each decision maker; so that $\sum_{k=1}^{l}\lambda_k = 1$, $\lambda_k \in [0, 1]$ holds true. In the process of group decision-making, all individual decisions should be integrated into an intuitionistic fuzzy decision matrix. IFWA operator proposed by Xu [56] can be used for this purpose. In this case $R = \left(r_{ij}^{k}\right)_{m\times n}$ and we have:

$$
\begin{aligned}
r_{ij} &= IFWA_{\lambda}\left(r_{ij}{}^{(1)}, r_{ij}{}^{(2)}, \ldots, r_{ij}{}^{(l)}\right)\\
&= \lambda_1 r_{ij}{}^{(1)} \oplus \lambda_2 r_{ij}{}^{(2)} \oplus \lambda_3 r_{ij}{}^{(3)} \oplus \ldots \oplus \lambda_l r_{ij}{}^{(l)}\\
&= \left[1-\prod_{k=1}^{l}\left(1-\mu_{ij}^{(k)}\right)^{\lambda_k}, \prod_{k=1}^{l}\left(\upsilon_{ij}^{(k)}\right)^{\lambda_k}, \prod_{k=1}^{l}\left(1-\mu_{ij}^{(k)}\right)^{\lambda_k} - \prod_{k=1}^{l}\left(\upsilon_{ij}^{(k)}\right)^{\lambda_k}\right]
\end{aligned}
\tag{11}
$$

So that: $r_{ij} = \left(\mu_{A_i}\left(x_j\right), \upsilon_{A_i}\left(x_j\right), \pi_{A_i}\left(x_j\right)\right)$ $(i = 1, 2, \ldots, m\ ,\ j = 1, 2, \ldots, n)$
Aggregate intuitionistic fuzzy decision matrix is as follows:

$$
R = \begin{bmatrix}
\left(\mu_{FM_1}(O), \upsilon_{FM_1}(O), \pi_{FM_1}(O)\right) & \left(\mu_{FM_1}(S), \upsilon_{FM_1}(S), \pi_{FM_1}(S)\right) & \left(\mu_{FM_1}(D), \upsilon_{FM_1}(D), \pi_{FM_1}(D)\right)\\
\left(\mu_{FM2}(O), \upsilon_{FM2}(O), \pi_{FM2}(O)\right) & \left(\mu_{FM2}(S), \upsilon_{FM2}(S), \pi_{FM2}(S)\right) & \left(\mu_{FM2}(D), \upsilon_{FM2}(D), \pi_{FM2}(D)\right)\\
\left(\mu_{FM3}(O), \upsilon_{FM3}(O), \pi_{FM3}(O)\right) & \left(\mu_{FM3}(S), \upsilon_{FM3}(S), \pi_{FM3}(S)\right) & \left(\mu_{FM3}(D), \upsilon_{FM3}(D), \pi_{FM3}(D)\right)\\
\vdots & \ddots & \vdots\\
\left(\mu_{FM_n}(O), \upsilon_{FM_n}(O), \pi_{FM_n}(O)\right) & \left(\mu_{FM_n}(S), \upsilon_{FM_n}(S), \pi_{FM_n}(S)\right) & \left(\mu_{FM_n}(D), \upsilon_{FM_n}(D), \pi_{FM_n}(D)\right)
\end{bmatrix} \rightarrow
\begin{bmatrix}
r_{1O} & r_{1S} & r_{1D}\\
r_{2O} & r_{2S} & r_{2D}\\
r_{3O} & r_{3S} & r_{3D}\\
\vdots & \ddots & \vdots\\
r_{nO} & r_{nS} & r_{nD}
\end{bmatrix}
$$

Step 3. determining the weight of risk factors

A valid argument against the traditional FMEA is that the weights of risk factors are considered to be equal, resulting in the same RPN for different values of risk factors. Therefore, a weight must be determined for each risk factor. Suppose that each decision maker expresses his/her opinion about the importance of each risk factor using linguistic terms listed in Table 1. If we assume $w_j^k = \left(\mu_j^{(k)}, \upsilon_j^{(k)}, \pi_j^{(k)}\right)$ as the intuitionistic fuzzy number assigned to j-th criterion based on the opinion of n-th decision-maker, then the weight of risk factors can be calculated using IFWA operator as shown below:

$$\begin{aligned} w_j &= IFWA_\lambda\left(w_j^{(1)}, w_j^{(2)}, \ldots, w_j^{(l)}\right) \\ &= \lambda_1 w_j^{(1)} \oplus \lambda_2 w_j^{(2)} \oplus \lambda_3 w_j^{(3)} \oplus \ldots \oplus \lambda_l w_j^{(l)} \\ &= \left[1 - \prod_{k=1}^{l}\left(1-\mu_j^{(k)}\right)^{\lambda_k}, \prod_{k=1}^{l}\left(\upsilon_j^{(k)}\right)^{\lambda_k}, \prod_{k=1}^{l}\left(1-\mu_j^{(k)}\right)^{\lambda_k} - \prod_{k=1}^{l}\left(\upsilon_j^{(k)}\right)^{\lambda_k}\right] \\ W &= \left[w_1, w_2, w_3, \ldots, w_j\right] \\ w_j &= \left(\mu_j, \upsilon_j, \pi_j\right) \ (j = 1, 2, \ldots, n) \end{aligned} \tag{12}$$

Step 4. Constructing the weighted aggregate intuitionistic fuzzy decision matrix based on decision-makers' opinions:

After determining the weight of each risk factor (W) and aggregate intuitionistic fuzzy decision matrix, weighted aggregate intuitionistic fuzzy decision matrix can be obtained by the following equation [4]:

$$R \otimes W = \left\{\langle c, \mu_{FM_i}(c).\mu_W(c), \upsilon_{FM_i}(c).\upsilon_W(c) - \upsilon_{FM_i}(c).\upsilon_W(c) \,\middle|\, x \in X\rangle\right\} \tag{13}$$

$$\pi_{FM_iW}(c) = 1 - \upsilon_{FM_i}(c) - \upsilon_W(c) - \mu_{FM_i}(c).\mu_W(c) + \upsilon_{FM_i}(c).\upsilon_W(c) \tag{14}$$

Then weighted aggregate intuitionistic fuzzy decision matrix will be as follows:

$$R = \begin{bmatrix} \left(\mu_{FM_1W}(O), \upsilon_{FM_1W}(O), \pi_{FM_1W}(O)\right) & \left(\mu_{FM_1W}(S), \upsilon_{FM_1W}(S), \pi_{FM_1W}(S)\right) & \left(\mu_{FM_1W}(D), \upsilon_{FM_1W}(D), \pi_{FM_1W}(D)\right) \\ (\mu_{FM2W}(O), \upsilon_{FM2W}(O), \pi_{FM2W}(O)) & (\mu_{FM2W}(S), \upsilon_{FM2W}(S), \pi_{FM2W}(S)) & (\mu_{FM2W}(D), \upsilon_{FM2W}(D), \pi_{FM2W}(D)) \\ (\mu_{FM3W}(O), \upsilon_{FM3W}(O), \pi_{FM3W}(O)) & (\mu_{FM3W}(S), \upsilon_{FM3W}(S), \pi_{FM3W}(S)) & (\mu_{FM3W}(D), \upsilon_{FM3W}(D), \pi_{FM3W}(D)) \\ \vdots & \ddots & \vdots \\ (\mu_{FM_nW}(O), \upsilon_{FM_nW}(O), \pi_{FM_nW}(O)) & (\mu_{FM_nW}(S), \upsilon_{FM_nW}(S), \pi_{FM_nW}(S)) & (\mu_{FM_nW}(D), \upsilon_{FM_nW}(D), \pi_{FM_nW}(D)) \end{bmatrix} \rightarrow \begin{bmatrix} r'_{1O} & r'_{1S} & r'_{1D} \\ r'_{2O} & r'_{2S} & r'_{2D} \\ r'_{3O} & r'_{3S} & r'_{3D} \\ \vdots & \ddots & \vdots \\ r'_{nO} & r'_{nS} & r'_{nD} \end{bmatrix}$$

where: $r'_{ij} = \left(\mu'_{ij}, \upsilon'_{ij}, \pi'_{ij}\right) = \left(\mu_{FMW}(c), \upsilon_{FMW}(c), \pi_{FMW}(c)\right)$ are the elements of weighted aggregate intuitionistic fuzzy decision matrix.

Step 5. determining the intuitionistic fuzzy positive and negative ideal values

Suppose that J_1 and J_2 are the profit and cost criteria respectively. Intuitionistic fuzzy positive ideal solution FM^+ and intuitionistic fuzzy negative ideal solution FM^- can be determined as below:

$$\begin{aligned} FM^+ &= \left(\mu_{FM^+W}\left(c_j\right), \upsilon_{FM^+W}\left(c_j\right)\right) \\ FM^- &= \left(\mu_{FM^-W}\left(c_j\right), \upsilon_{FM^-W}\left(c_j\right)\right) \end{aligned} \tag{15}$$

where:

$$\mu_{FM^+W}\left(c_j\right) = \left(\langle \max_i \mu_{FM_i.W}\left(c_j\right) \,\middle|\, j \in J_1\rangle, \langle \min_i \mu_{FM_i.W}\left(c_j\right) \,\middle|\, j \in J_2\rangle\right) \tag{16}$$

$$\upsilon_{FM^+W}\left(c_j\right) = \left(\langle \min_i \upsilon_{FM_i.W}\left(c_j\right) \,\middle|\, j \in J_1\rangle, \langle \max_i \upsilon_{FM_i.W}\left(c_j\right) \,\middle|\, j \in J_2\rangle\right) \tag{17}$$

$$\mu_{FM^-W}\left(c_j\right) = \left(\langle \min_i \mu_{FM_i.W}\left(c_j\right) \,\middle|\, j \in J_1\rangle, \langle \max_i \mu_{FM_i.W}\left(c_j\right) \,\middle|\, j \in J_2\rangle\right) \tag{18}$$

$$\upsilon_{FM^-W}\left(c_j\right) = \left(\langle \max_i \upsilon_{FM_i.W}\left(c_j\right) \,\middle|\, j \in J_1\rangle, \langle \min_i \upsilon_{FM_i.W}\left(c_j\right) \,\middle|\, j \in J_2\rangle\right) \tag{19}$$

Step 6. Calculating the distance of failure modes from the positive and negative ideals:

Atanassov [6], Szmidt and Kacprzyk [50] and Grzegorzewski [36] have proposed different methods to calculate the distance between two intuitionistic fuzzy sets including Hamming distance, Euclidean distance and their normalized distances. Yang and Chiclana [58] in their article clearly showed that the distance between two fuzzy sets is exactly the same distance between two intuitionistic fuzzy sets and The only difference is in the degree of their membership. So that in the intuitionistic fuzzy sets, the membership, non-membership degrees and the degree of hesitancy are involved but in fuzzy sets, only the membership degree is involved. This means that if we move both sets from 3D space to 2D space with the same changes in membership, non-membership and hesitancy degrees, then we obtain exactly the same distance between the two fuzzy sets. Accordingly, because human perception is not necessarily always linear in some cases, this linear feature of the previous distances may not be adequate. Therefor it is better to be used spherical distance and normalized spherical distance between two intuitionistic fuzzy sets.

Table 3
Failure modes for the quality of internet banking services.

	Factors
A_1	Low speed connection in the online banking program
A_2	Ignoring the accuracy of data sent from users in the online banking program
A_3	Failure to protect data submitted by users through the online banking application security measures
A_4	Failure to provide easy navigation in the online banking program
A_5	Slow loading of the online banking program
A_6	Possible changes in the data submitted by users through the online banking application security measures
A_7	Failure to provide an easy understanding of the content for the online banking program
A_8	Failure to easily handle the processes in the online banking program
A_9	Failure to provide easy online transaction processes in the online banking program
A_{10}	Failure to properly transfer data sent by users through the online banking application security measures
A_{11}	Lack of design attractiveness in the online banking program
A_{12}	Failure to provide useful information content by online banking program
A_{13}	Failure to provide related information content by online banking program
A_{14}	Failure to respond to customers in the online banking program
A_{15}	Failure to address customer dissatisfaction and problems in the online banking program
A_{16}	Failure to provide proper commercial content in the online banking program
A_{17}	Lack of elegant and ordered design in the online banking program
A_{18}	Failure to provide ease of use in the online banking program
A_{19}	Low speed of feedback to problems in the online banking program
A_{20}	Low speed of transactional processes in the online banking program

Thus in this study, normalized spherical distance [1] is used to calculate the distance of i-th failure mode from positive and negative ideals that are shown by S^+ and S^- respectively. So we have:

$$S^+ = \frac{2}{3.14 \times n} \sum_{j=1}^{n} \arccos \left(\sqrt{\mu_{FM_i}.w(c_j)\mu_{FM^+}w(c_j)} + \sqrt{\upsilon_{FM_i}.w(c_j)\upsilon_{FM^+}w(c_j)} + \sqrt{\pi_{FM_i}.w(c_j)\pi_{FM^+}w(c_j)} \right) \quad (20)$$

$$S^- = \frac{2}{3.14 \times n} \sum_{j=1}^{n} \arccos \left(\sqrt{\mu_{FM_i}.w(c_j)\mu_{FM^-}w(c_j)} + \sqrt{\upsilon_{FM_i}.w(c_j)\upsilon_{FM^-}w(c_j)} + \sqrt{\pi_{FM_i}.w(c_j)\pi_{FM^-}w(c_j)} \right) \quad (21)$$

Step 7. Calculating the relative closeness coefficient with respect to intuitionistic ideal:

The relative closeness coefficient for failure mode FM_i with respect to intuitionistic fuzzy positive ideal solution FM^+ is defined as follows:

$$C_i^+ = {S_i^-}/{S_i^+ + S_i^-}, \quad 0 \le C_i^+ \le 1 \quad (22)$$

Step 8 Ranking the failure modes

After determining the relative closeness coefficient for each failure mode, failure modes can be ranked in a descending order based on C_i^+ values. In other words, failure modes with larger C have higher priority.

4. Numerical example

In this section, the proposed model is used to evaluate the failure modes for the quality of internet banking services. The model was implemented for Refah Bank, Yazd Province, Iran. Based on earlier studies Zeithaml et al. [60]; Büyüközkan and Çifçi [18], Hamadi [37], Loiacono et al. [41], Beheshti Zavareh et al. [10], Hsu et al. [38], Ariff et al. [2], Einasto [31], Bhatti et al. [12], Wan [53], Valarie et al. [52] and surveys obtained from banking and academic experts, a total number of 20 failure modes across six dimensions were determined for the quality of internet banking services (Table 3).

After determining the modes of failure for the quality of internet banking services, a questionnaire was designed and distributed among experts. Table 4 illustrates the results of failure modes evaluations for the case study of Refah Bank in Yazd Province, based on the opinions obtained from 5 experts. These results are based on converting linguistic terms listed in Table 1 to their corresponding intuitionistic fuzzy numbers.

Step 1. Determining the weight of decision-makers:

The importance of each decision maker based on linguistic terms shown in Table 2 is presented in the table below.

By converting the listed linguistic terms to intuitionistic fuzzy numbers of Table 2 and using Eq. (10), weight values (λ_k) for each decision maker were determined as presented below.

$$\lambda_k = \left| \begin{matrix} K_1 & K_2 & K_3 & K_4 \\ 0.299 & 0.175 & 0.262 & 0.262 \end{matrix} \right|$$

Step 2. Constructing the aggregate intuitionistic fuzzy decision matrix based on decision-makers' opinions:

Based on the weights obtained for each of the decision makers and Eq. (11), the aggregate decision matrix (r_{ij}) was determined as below (Tables 5 and 6).

Step 3. Determining the weight of risk factors

The results of risk factors assessment based on the linguistic terms of Table 2 are shown in Table 7.

Table 4
Evaluation of failure modes by intuitionistic fuzzy numbers.

Items	D_1			Items	D_2		
	O	S	D		O	S	D
A1	(0.9,0.1,0)	(0.8,0.1,0.1)	(0.1,0.75,0.15)	A1	(0.9,0.1,0)	(0.6,0.3,0.1)	(0.6,0.3,0.1)
A2	(0.9,0.1,0)	(1,0,0)	(0.1,0.9,0)	A2	(0.5,0.45,0.05)	(0.6,0.3,0.1)	(0.6,0.3,0.1)
A3	(0.75,0.2,0.05)	(0.9,0.1,0)	(0.7,0.2,0.1)	A3	(0.35,0.6,0.05)	(0.6,0.3,0.1)	(0.8,0.1,0.1)
A4	(0.5,0.45,0.05)	(1,0,0)	(0.1,0.9,0)	A4	(0.5,0.45,0.05)	(0.1,0.9,0)	(0.6,0.3,0.1)
A5	(0.75,0.2,0.05)	(0.9,0.1,0)	(0.1,0.9,0)	A5	(0.9,0.1,0)	(0.6,0.3,0.1)	(0.1,0.75,0.15)
A6	(0.9,0.1,0)	(0.8,0.1,0.1)	(0.5,0.4,0.1)	A6	(0.35,0.6,0.05)	(0.7,0.2,0.1)	(0.6,0.3,0.1)
A7	(0.5,0.45,0.05)	(0.8,0.1,0.1)	(0.6,0.3,0.1)	A7	(0.75,0.2,0.05)	(0.9,0.1,0)	(0.8,0.1,0.1)
A8	(0.35,0.6,0.05)	(0.5,0.4,0.1)	(0.9,0.1,0)	A8	(0.9,0.1,0)	(0.1,0.9,0)	(0.1,0.75,0.15)
A9	(0.5,0.45,0.05)	(0.8,0.1,0.1)	(0.4,0.5,0.1)	A9	(0.5,0.45,0.05)	(0.1,0.75,0.15)	(0.1,0.9,0)
A10	(0.9,0.1,0)	(0.6,0.3,0.1)	(0.9,0.1,0)	A10	(0.1,0.9,0)	(0.6,0.3,0.1)	(0.1,0.75,0.15)
A11	(0.35,0.6,0.05)	(0.7,0.2,0.1)	(0.4,0.5,0.1)	A11	(0.9,0.1,0)	(0.5,0.4,0.1)	(0.1,0.75,0.15)
A12	(0.75,0.2,0.05)	(0.8,0.1,0.1)	(0.1,0.9,0)	A12	(0.9,0.1,0)	(0.4,0.5,0.1)	(0.8,0.1,0.1)
A13	(0.9,0.1,0)	(0.7,0.2,0.1)	(0.4,0.5,0.1)	A13	(0.75,0.2,0.05)	(1,0,0)	(0.25,0.6,0.15)
A14	(0.5,0.45,0.05)	(0.25,0.6,0.15)	(0.25,0.6,0.15)	A14	(0.5,0.45,0.05)	(0.9,0.1,0)	(0.25,0.6,0.15)
A15	(0.1,0.9,0)	(0.4,0.5,0.1)	(0.4,0.5,0.1)	A15	(0.75,0.2,0.05)	(0.25,0.6,0.15)	(0.1,0.75,0.15)
A16	(0.9,0.1,0)	(0.8,0.1,0.1)	(0.9,0.1,0)	A16	(0.9,0.1,0)	(0.25,0.6,0.15)	(0.5,0.4,0.1)
A17	(0.1,0.9,0)	(0.1,0.75,0.15)	(0.4,0.5,0.1)	A17	(0.1,0.9,0)	(0.6,0.3,0.1)	(0.9,0.1,0)
A18	(0.75,0.2,0.05)	(0.8,0.1,0.1)	(0.25,0.6,0.15)	A18	(0.75,0.2,0.05)	(0.8,0.1,0.1)	(1,0,0)
A19	(0.75,0.2,0.05)	(0.25,0.6,0.15)	(0.25,0.6,0.15)	A19	(0.75,0.2,0.05)	(0.8,0.1,0.1)	(0.1,0.75,0.15)
A20	(0.1,0.9,0)	(0.8,0.1,0.1)	(0.9,0.1,0)	A20	(0.1,0.9,0)	(0.8,0.1,0.1)	(0.6,0.3,0.1)
Items	D_3			Items	D_4		
	O	S	D		O	S	D
A1	(0.9,0.1,0)	(0.6,0.3,0.1)	(1,0,0)	A1	(0.9,0.1,0)	(0.1,0.75,0.15)	(0.1,0.9,0)
A2	(0.1,0.9,0)	(0.1,0.9,0)	(0.1,0.75,0.15)	A2	(0.1,0.9,0)	(0.25,0.6,0.15)	(0.4,0.5,0.1)
A3	(0.75,0.2,0.05)	(0.6,0.3,0.1)	(0.1,0.75,0.15)	A3	(0.75,0.2,0.05)	(0.5,0.4,0.1)	(0.1,0.75,0.15)
A4	(0.75,0.2,0.05)	(0.8,0.1,0.1)	(0.5,0.4,0.1)	A4	(0.9,0.1,0)	(0.4,0.5,0.1)	(0.4,0.5,0.1)
A5	(0.1,0.9,0)	(0.1,0.75,0.15)	(0.9,0.1,0)	A5	(0.1,0.9,0)	(0.6,0.3,0.1)	(0.25,0.6,0.15)
A6	(0.9,0.1,0)	(0.8,0.1,0.1)	(0.1,0.75,0.15)	A6	(0.75,0.2,0.05)	(0.1,0.9,0)	(0.5,0.4,0.1)
A7	(0.9,0.1,0)	(0.6,0.3,0.1)	(0.1,0.9,0)	A7	(0.1,0.9,0)	(0.1,0.75,0.15)	(0.4,0.5,0.1)
A8	(0.75,0.2,0.05)	(0.1,0.75,0.15)	(0.6,0.3,0.1)	A8	(0.9,0.1,0)	(0.9,0.1,0)	(0.6,0.3,0.1)
A9	(0.5,0.45,0.05)	(0.9,0.1,0)	(0.6,0.3,0.1)	A9	(0.1,0.9,0)	(0.8,0.1,0.1)	(0.6,0.3,0.1)
A10	(0.75,0.2,0.05)	(0.25,0.6,0.15)	(0.8,0.1,0.1)	A10	(0.5,0.45,0.05)	(0.8,0.1,0.1)	(0.6,0.3,0.1)
A11	(0.9,0.1,0)	(0.1,0.75,0.15)	(0.6,0.3,0.1)	A11	(0.35,0.6,0.05)	(1,0,0)	(0.9,0.1,0)
A12	(0.9,0.1,0)	(0.7,0.2,0.1)	(0.7,0.2,0.1)	A12	(0.5,0.45,0.05)	(0.7,0.2,0.1)	(0.1,0.75,0.15)
A13	(0.5,0.45,0.05)	(1,0,0)	(0.8,0.1,0.1)	A13	(0.9,0.1,0)	(0.1,0.75,0.15)	(0.8,0.1,0.1)
A14	(0.35,0.6,0.05)	(0.4,0.5,0.1)	(0.8,0.1,0.1)	A14	(0.5,0.45,0.05)	(0.25,0.6,0.15)	(0.6,0.3,0.1)
A15	(0.5,0.45,0.05)	(0.25,0.6,0.15)	(0.4,0.5,0.1)	A15	(0.1,0.9,0)	(0.1,0.75,0.15)	(0.5,0.4,0.1)
A16	(0.35,0.6,0.05)	(0.25,0.6,0.15)	(0.9,0.1,0)	A16	(0.9,0.1,0)	(0.25,0.6,0.15)	(0.7,0.2,0.1)
A17	(0.75,0.2,0.05)	(0.4,0.5,0.1)	(0.4,0.5,0.1)	A17	(0.5,0.45,0.05)	(0.25,0.6,0.15)	(0.8,0.1,0.1)
A18	(0.9,0.1,0)	(0.8,0.1,0.1)	(0.4,0.5,0.1)	A18	(0.35,0.6,0.05)	(0.25,0.6,0.15)	(0.1,0.75,0.15)
A19	(0.5,0.45,0.05)	(0.6,0.3,0.1)	(0.4,0.5,0.1)	A19	(0.5,0.45,0.05)	(0.4,0.5,0.1)	(0.8,0.1,0.1)
A20	(0.1,0.9,0)	(0.4,0.5,0.1)	(0.9,0.1,0)	A20	(0.9,0.1,0)	(0.9,0.1,0)	(1,0,0)

Table 5
The importance of each decision maker.

	K_1	K_2	K_3	K_4
Linguistic terms	Very Important	Medium	Important	Important

By converting the linguistic terms listed in Table 7 to intuitionistic fuzzy numbers and weight of each decision maker calculated in step (1), weight of each risk factor was determined as presented below.

$$W_j = \left| \begin{matrix} O & S & D \\ (0.821, 0.168, 0.012) & (0.778, 0.192, 0.03) & (0.794, 0.188, 0.018) \end{matrix} \right|$$

The results suggest that the weight of factor O is higher than that of the other two factors.

Step 4. Constructing the weighted aggregate intuitionistic fuzzy decision matrix:

By multiplying the weight vector of risk factors by matrix r_{ij}, based on the Eqs. (13) and (14), weighted aggregate intuitionistic fuzzy decision matrix for the evaluation of failure modes related to the quality of internet banking service $R \otimes W$, was obtained as follows.

Step 5. Determining the intuitionistic fuzzy positive and negative ideal values

Based on the results of Table 8 and the Eqs. (15)–(19), the positive and negative ideal values were determined as the following.

$$\begin{matrix} FM^+ = \\ FM^- = \end{matrix} \left| \begin{matrix} O & S & D \\ (0.739, 0.251, 0.01) & (0.778, 0.192, 0.03) & (0.794, 0.188, 0.0183) \\ (0.315, 0.647, 0.038) & (0.205, 0.679, 0.116) & (0.237, 0.681, 0.083) \end{matrix} \right|$$

Step 6. Calculating the distance of failure modes from the positive and negative ideals and obtaining the relative closeness coefficient with respect to the intuitionistic ideals:

Table 6
Aggregate decision matrix (r_{ij}).

Items	O	S	D
A1	(0.9,0.1,0)	(0.598,0.275,0.128)	(1,0,0)
A2	(0.58,0.413,0.008)	(1,0,0)	(0.298,0.606,0.095)
A3	(0.704,0.242,0.053)	(0.72,0.233,0.047)	(0.502,0.355,0.143)
A4	(0.727,0.245,0.028)	(1,0,0)	(0.398,0.514,0.087)
A5	(0.583,0.39,0.027)	(0.673,0.275,0.052)	(0.518,0.44,0.042)
A6	(0.823,0.164,0.012)	(0.681,0.201,0.118)	(0.439,0.449,0.112)
A7	(0.661,0.316,0.023)	(0.685,0.227,0.089)	(0.512,0.378,0.11)
A8	(0.777,0.205,0.018)	(0.576,0.378,0.046)	(0.696,0.253,0.051)
A9	(0.417,0.54,0.044)	(0.783,0.142,0.075)	(0.479,0.424,0.097)
A10	(0.715,0.262,0.024)	(0.607,0.27,0.124)	(0.746,0.19,0.064)
A11	(0.714,0.274,0.013)	(1,0,0)	(0.638,0.308,0.054)
A12	(0.799,0.183,0.018)	(0.7,0.191,0.109)	(0.482,0.393,0.125)
A13	(0.821,0.168,0.012)	(1,0,0)	(0.65,0.222,0.129)
A14	(0.464,0.485,0.05)	(0.503,0.418,0.079)	(0.551,0.312,0.137)
A15	(0.384,0.576,0.04)	(0.264,0.602,0.134)	(0.386,0.506,0.108)
A16	(0.836,0.16,0.003)	(0.495,0.351,0.154)	(0.823,0.153,0.024)
A17	(0.449,0.505,0.046)	(0.331,0.542,0.128)	(0.672,0.247,0.081)
A18	(0.747,0.222,0.03)	(0.717,0.16,0.123)	(1,0,0)
A19	(0.64,0.306,0.054)	(0.524,0.348,0.127)	(0.484,0.371,0.145)
A20	(0.495,0.505,0)	(0.778,0.153,0.07)	(1,0,0)

Table 7
Assessment of risk factors.

Criteria	K_1	K_2	K_3	K_4
O	*VI*	*I*	*VI*	*M*
S	*I*	*M*	*I*	*VI*
D	*M*	*VI*	*VI*	*I*

Table 8
Weighted aggregate intuitionistic fuzzy decision matrix for the evaluation of failure modes: $R \otimes W$.

Items	O	S	D
A1	(0.739,0.251,0.01)	(0.465,0.414,0.121)	(0.794,0.188,0.018)
A2	(0.476,0.511,0.013)	(0.778,0.192,0.03)	(0.237,0.681,0.083)
A3	(0.578,0.369,0.052)	(0.56,0.38,0.06)	(0.399,0.476,0.125)
A4	(0.597,0.372,0.032)	(0.778,0.192,0.03)	(0.316,0.606,0.078)
A5	(0.478,0.493,0.029)	(0.524,0.414,0.062)	(0.412,0.545,0.043)
A6	(0.676,0.304,0.02)	(0.53,0.355,0.115)	(0.348,0.552,0.099)
A7	(0.543,0.43,0.027)	(0.533,0.375,0.092)	(0.407,0.495,0.098)
A8	(0.638,0.338,0.024)	(0.448,0.497,0.054)	(0.552,0.394,0.054)
A9	(0.342,0.617,0.041)	(0.609,0.307,0.084)	(0.381,0.532,0.087)
A10	(0.587,0.385,0.028)	(0.472,0.41,0.118)	(0.593,0.342,0.065)
A11	(0.586,0.396,0.019)	(0.778,0.192,0.03)	(0.507,0.438,0.055)
A12	(0.656,0.32,0.024)	(0.545,0.346,0.109)	(0.383,0.508,0.11)
A13	(0.674,0.307,0.019)	(0.778,0.192,0.03)	(0.516,0.368,0.116)
A14	(0.381,0.572,0.047)	(0.391,0.53,0.079)	(0.437,0.442,0.121)
A15	(0.315,0.647,0.038)	(0.205,0.679,0.116)	(0.306,0.599,0.094)
A16	(0.687,0.301,0.013)	(0.385,0.476,0.139)	(0.654,0.312,0.034)
A17	(0.369,0.588,0.043)	(0.257,0.63,0.113)	(0.533,0.389,0.078)
A18	(0.613,0.353,0.034)	(0.558,0.321,0.121)	(0.794,0.188,0.018)
A19	(0.525,0.423,0.052)	(0.408,0.474,0.119)	(0.384,0.49,0.126)
A20	(0.406,0.588,0.006)	(0.605,0.315,0.08)	(0.794,0.188,0.018)

Based on Eqs. (20) and (21), distance of each failure mode from the positive and negative ideals and the relative closeness coefficient (Eq. (22)) were obtained as presented in Table 9.

5. Conclusion

Among the various methods for risk assessment, FMEA is one of the most effective approaches, which is capable of detecting and assessing risks [46]. History of multi-criteria decision-making methods denotes that such methods have been used either separately or with other methods for the assessment of risks in different cases. Fuzzy sets are vague sets with imprecise boundaries, which were first introduced by Zadeh in an article in 1965 [59], which aimed to create a simpler model for complex systems. Following the development of fuzzy logic, intuitionistic fuzzy logic and fuzzy sets were introduced by Atanassov in 1983, as an extension to fuzzy logic [4]. Apart from a degree of membership, intuitionistic fuzzy sets also have a degree of non-membership. This leads to a decision matrix with a more accurate and reliable assessment and subsequently a more efficient and effective decision-making capability. Theory of intuitionistic fuzzy set does not rule out the theory of fuzzy set and does not diminish its capabilities. Instead, it provides a more effective and efficient tool for dealing with uncertainty by using the extended form of fuzzy sets. On this basis, the current paper used the theory of intuitionistic fuzzy sets for the analysis of failure mode and effects. The proposed model was tested for prioritizing the failure modes for quality of internet banking

Table 9
Distance of each failure mode from positive and negative ideals and the relative closeness coefficient.

	Factors	S_i^+	S_i^-	C_i	Rank
A1	Low speed connection in the bank's online banking program	0.072	0.281	0.797	1
A2	Ignoring the accuracy of data sent from the user in the bank's online banking program	0.184	0.167	0.476	13
A3	Failure to protect data submitted by the user by the bank's online banking application security measures	0.182	0.184	0.503	12
A4	Failure to provide easy navigation in the bank's online banking program	0.141	0.21	0.599	7
A5	Slow loading of the bank's online banking program	0.201	0.149	0.426	15
A6	Possible changes in the data submitted by the user by the bank's online banking application security measures	0.175	0.183	0.512	11
A7	Failure to provide an easy understanding of the bank's online banking program content	0.189	0.166	0.468	14
A8	Failure to easily handle the processes in the bank's online banking program	0.154	0.197	0.562	9
A9	Failure to provide easy online transaction processes in the bank's online banking program	0.221	0.133	0.374	17
A10	Failure to properly transfer data sent by the user by the bank's online banking application security measures	0.154	0.202	0.567	8
A11	The lack of attractiveness of the bank's online banking program design	0.1	0.248	0.713	4
A12	Failure to provide useful information content by the bank's online banking program	0.17	0.191	0.529	10
A13	Failure to provide related information content by the bank's online banking program	0.084	0.276	0.766	2
A14	Failure to respond to customers in the bank's online banking program	0.248	0.112	0.312	18
A15	Failure to address customer dissatisfaction and problems in the bank's online banking program	0.332	0.018	0.052	20
A16	Failure to provide proper commercial content in the bank's online banking program	0.135	0.219	0.62	6
A17	Lack of elegant and ordered design in the bank's online banking program	0.259	0.094	0.265	19
A18	Failure to provide theease of use in the bank's online banking program	0.085	0.274	0.763	3
A19	Low speed of feedback to the problems in the bank's online banking program	0.228	0.139	0.379	16
A20	Low speed of transactional processes in the bank's online banking program	0.116	0.247	0.681	5

services by Refah Bank (Yazd Province, Iran) based on experts' opinions. First, decision-makers weight and risk factors were calculated based on linguistic terms and intuitionistic fuzzy numbers of Table 2. Aggregate decision matrix was then calculated based on the obtained weights and principles of intuitionistic fuzzy numbers. Finally, the intuitionistic fuzzy TOPSIS technique [32] was used to prioritize the failure modes for the quality of internet banking services. Study results showed that the following failure modes had a high priority in the failure of internet banking programs:

- Low speed connection in the bank's online banking program
- Failure to provide related information content by the bank's online banking program
- Failure to provide theease of use in the bank's online banking program

These priorities are arranged in the order of damages inflicted by failing to meet customers' needs. Study findings can be used by the bank to improve the quality of its internet banking services. Prioritizing the failure modes can be a stepping stone for devising strategies and plans for continuous improvement programs. Results of the model analysis provides fresh insights to managers and decision makers. The proposed model of the current study resolves the problems pertaining the ambiguities of FMEA techniques in assessing qualitative factors. Moreover, the use of intuitionistic fuzzy TOPSIS technique provides a strong basis for dealing with uncertain and ambiguous factors which is common in business environments. This approach facilitates more accurate and realistic decisions making. It can also be used to assess failure mode and effects both in service and manufacturing sector.

References

[1] M. Abramowitz, I.A. Stegun (Eds)., Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables, Dover, New York, USA (1972).
[2] M.S.M. Ariff, L. Ooi Yun, N. Zakuan, A. Jusoh, Examining dimensions of electronic service quality for internet banking services, Soc. Behav. Sci. 65 (2012) 854–859.
[3] K.T. Atanassov, Intuitionistic fuzzy sets, in: VII ITKR's Session, Sofia, Bulgarian, 1983.
[4] K.T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets Syst. 20 (1986) 87–96.
[5] K.T. Atanassov, More on intuitionistic fuzzy sets? Fuzzy Sets Syst. 33 (1) (1989) 37–46.
[6] K.T. Atanassov, Intuitionistic Fuzzy Sets, Physica-Verlag, Heidelberg, New York, 1999.
[7] K.T. Atanassov, Two theorems for Intuitionistic fuzzy sets, Fuzzy Sets Syst. 110 (2000) 267–269.
[8] K.T. Atanassov, G. Gargov, Interval-valued intuitionistic fuzzy sets, Fuzzy Sets Syst. 31 (3) (1989) 343–349.
[9] A.I. Ban, Nearest Interval Approximation of an Intuitionistic Fuzzy Number, Computational Intelligence. Theory and Applications, Springer-Verlag, Berlin, Heidelberg, 2006, pp. 229–240.
[10] F. Beheshti Zavareh, M.S.M. Ariff, A. Jusoh, N. Zakuan, A. Zaidi Bahari, M. Ashourian, E-service quality dimensions and their effects on E-customer satisfaction in internet banking services, Soc. Behavi. Sci. 40 (2012) 441–445.
[11] M. Ben-Daya, A. Raouf, A revised failure mode and effects analysis model, Int. J. Qual. Reliab. Manag. 13 (1) (1996) 43–47.
[12] N. Bhatti, A. Bouch, A. Kuchinsky, Integrating user- perceived quality into web server design, in: Proceedings of the 9th Intrnational World Wide Web Conference (WWW9), Amsterdam, The Netherlands, 2000.
[13] J.B. Bowles, An assessment of PRN prioritization in a failure modes effects and criticality analysis, J. IEST 47 (2004) 51–56.
[14] J.B. Bowles, C.E. Peláez, Fuzzy logic prioritization of failures in a system failure mode, effects and criticality analysis, Reliab. Eng. Syst. Saf. 50 (1995) 203–213.
[15] M. Braglia, M. Frosolini, R. Montanari, Fuzzy criticality assessment model for failure modes and effects analysis, Int. J. Qual. Reliab. Manag. 20 (4) (2003) 503–524.
[16] M. Braglia, M. Frosolini, R. Montanari, Fuzzy TOPSIS approach for failure mode, effects and criticality analysis, Int. J. Qual. Reliab. Eng. 19 (2003) 425–443.
[17] T. Buhaesku, Some observations on intuitionistic fuzzy relations, itinerant seminar of functional equations, Approx. Convexity (1989) 111–118.
[18] G. Büyüközkan, G. Çifçi, A combined fuzzy AHP and fuzzy TOPSIS based strategic analysis of electronic service quality in healthcare industry, Expert Syst. Appl. 39 (3) (2012) 2341–2354.
[19] N. Chanamool, T. Naenna, Fuzzy FMEA application to improve decision making process in an emergency department, Appl. Soft Comput. (2016), http://dx.doi.org/10.1016/j.asoc.2016.01.007.
[20] C.L. Chang, C.C. Wei, Y.H. Lee, Failure mode and effects analysis using fuzzy method and grey theory, Integr. Manuf. Syst. 12 (3) (2001) 211–216.
[22] C.L. Chang, C.C. Wei, Y.H. Lee, Failure mode and effects analysis using fuzzy method and grey theory, Kybernetes 28 (1999) 1072–1080.
[23] L.H. Chen, W.C. Ko, Fuzzy linear programming models for new product design using QFD with FMEA, Appl. Math. Modell. (2007), http://dx.doi.org/10.1016/j.apm.2007.11.029.
[24] T.Y. Chen, C.H. Li, Objective weights with intuitionistic fuzzy entropy measures and computational experiment analysis, Appl. Soft Comput. 11 (2011) 5411–5423.
[25] P.D.T. Connor, Practical Reliability Engineerin, Heyden, London, 2002.
[26] C. Corporation, F.M. Company, G.M. Corporation, Potential failure mode and effect analysis (FMEA), Ref. Man. (1995).
[27] R. Daniela, O. Dosoinescu, The Adoption Electronic Banking Services in Developing Countries., Department of Business Information Systems.Bd Carol Jnr. (2004) 22–35.

[28] G. Deschrijver, E.E. Kerre, On the relationship between intuitionistic fuzzy sets and some other extensions of fuzzy set theory, J. Fuzzy Math. 10 (3) (2002) 711–724.
[29] G. Deschrijver, C. Cornelis, E.E. Kerre, On the representation of intuitionistic fuzzy t-norms and t-conorms, IEEE Trans. Fuzzy Syst. 12 (2004) 45–61.
[30] C. Ebeling, An Introduction to Reliability and Maintainability Engineering, Tata McGraw-Hill, New York, NY, 2001.
[31] O. Einasto, E-service quality criteria in university library: a focus group study, Soc. Behav. Sci. (2014) 561–566.
[32] F. EmreBoran, S. Genç, M. Kurt, D. Akay, A multi-criteria intuitionistic fuzzy group decision making for supplier selection with TOPSIS method, Expert Syst. Appl. 36 (2009) 11363–11368.
[33] P.A.A. Garcia, R. Schirru, P.F.F. E Melo, A fuzzy data envelopment analysis approach for FMEA, Prog. Nucl. Energy 46 (3–4) (2005) 359–373.
[34] W.L. Gau, D.J. &Buehrer, Vague sets, IEEE Trans. Syst. Man Cybern. 23 (1993) 610–614.
[35] W. Gilchrist, Modelling failure modes and effects analysis, Int. J. Qual. Reliab. Manag. 10 (5) (1993) 16–23.
[36] P. Grzegorzewski, Distances between intuitionistic fuzzy sets and/or interval-valued fuzzy sets based on the hausdorffmetric, Fuzzy Sets Syst. 148 (2004) 319–328.
[37] C. Hamadi, The impact of quality of online banking on customer commitment, Commun. IBIMA (2010) 1–8.
[38] T.H. Hsu, L.C. Hung, J.W. Tang, A hybrid ANP evaluation model for electronic service quality, Appl. Soft Comput. 12 (2012) 72–81.
[39] G. Ireson, W. Coombs, F. Clyde, Y.M. Richard, Handbook of Reliability Engineering and Management, 2nd ed., McGraw-Hill Professional, New York, N. Y, 1995.
[40] M. Kumru, P. YıldızKumru, Fuzzy FMEA application to improve purchasing process in a public hospital, Appl. Soft Comput. (2013) 13721–13733.
[41] Eleanor T. Loiacono, T. Richard Watson, L. Dale Goodhue, WebQual™: a measure of web site quality, American Marketing Association, Winter 2002 (2002).
[42] R. McDermott, R. Mikulak, M. Beauregard, The basics of FMEA. USA: Productivity (1996).
[43] J.H. Park, H.J. Cho, Y.C. Kwun, Extension of the VIKOR method to dynamic intuitionistic fuzzy multiple attribute decision making, Comput. Math. Appl. 65 (2013) 731–744.
[44] J.H. Park, I.Y. Park, Y.C. Kwun, X. Tan, Extension of the TOPSIS method for decision making problems under interval-valued intuitionistic fuzzy environment, Appl. Math. Modell. 37 (2011) 2544–2556.
[45] A. Pillay, J. Wang, Modified failure mode and effects analysis using approximate reasoning, Reliab. Eng. Syst. Saf. 79 (2003) 69–85.
[46] SAE, Potential failure mode and effects analysis in design (design FMEA), potential failure mode and effects in manufacturing and assembly processes (process FMEA), and potential failure mode and effects analysis for machinery (machinery FMEA); SAE (2002) J1746.
[47] N.R. Sankar, B.S. Prabhu, Modified approach for prioritization of failures in a system failure mode and effects analysis, Int. J. Qual. Reliab. Manag. 18 (3) (2001) 324–335.
[48] A. Segismundo, P. Augusto, C. Miguel, Failure mode and effects analysis(FMEA) in the context of riskmanagement in new product development, a case study in an automotive company, Int. J. Qual. Reliab. Manag. 25 (9) (2008) 899–912.
[49] D. Stoyanova, More on Cartesian product over intuitionistic fuzzy sets, BUSEFAL 54 (1993) 9–13.
[50] E. Szmidt, J. Kacprzyk, Distances between intuitionistic fuzzy sets, Fuzzy Sets Syst. 114 (3) (2000) 505–518.
[51] E. Szmidt, J. Kacprzyk, Entropy for intuitionistic fuzzy sets, Fuzzy Sets Syst. 118 (2001) 467–477.
[52] A. Valarie, L. Zeithaml Leonard, A. Berry Parasurman, Communication and control processes in delivery of service quality, J. Mark. (1998) 35–38.
[53] H.I. Wan, Opportunities to inhance a commercial web site, Inf. Manag. 38 (1) (2000) 15–21.
[54] Y.M. Wang, K.S. Chin, K.K.G. Poon, J.B. Yang, Risk evaluation in failure mode and effects analysis using fuzzy weighted geometric mean, Expert Syst. Appl. 36 (2) (2009) 1195–1207.
[55] J.Z. Wu, Q. Zhang, Multicriteria decision making method based on intuitionistic fuzzy weighted entropy, Expert Syst. Appl. 38 (2011) 916–922.
[56] Z.S. Xu, Intuitionistic fuzzy aggregation operators, IEE Trans. Fuzzy Syst. 15 (6) (2007) 1179–1187.
[58] Y. Yang, F. Chiclana, Intuitionistic fuzzy sets: spherical representation and distances, Int. J. Intell. Syst. 24 (4) (2009) 399–420.
[59] L.A. Zadeh, Fuzzy sets, Inform. Control 8 (1965) 338–353.
[60] V.A. Zeithaml, A. Parasuraman, L.L. Berry, Delivering Quality Service: Balanceing Customer Perceptions and Expectations, Free Press, New York, 1990.
[61] Q.S. Zhang, S. Jiang, B. Jia, S. Luo, Some information measures for interval-valued intuitionistic fuzzy sets, Inf. Sci. 180 (2010) 5130–5145.
[62] X. Zhao, G. Wei, Some intuitionistic fuzzy Einstein hybrid aggregation operators and their application to multiple attribute decision making, Knowl. Based Syst. 37 (2013) 472–479.
[63] M. Rafie, F. Samimi Namin, Prediction of subsidence risk by FMEA using artificial neural network and fuzzy inference system, Int. J. Min. Sci Technol. 25 (4) (2015) 655–663.
[64] Mehmet Ekmekçioğlu, Ahmet Can Kutlu, A fuzzy hybrid approach for fuzzy process FMEA: an application to a spindle manufacturing process, Int. J. Comput. Intell. Syst. 5 (4) (2016) 611–626.
[65] Hu-Chen Liu, Long Liu, Nan Liu, Ling-Xiang Mao, Risk evaluation in failure mode and effects analysis with extended VIKOR method under fuzzy environment, Expert Syst. Appl. 39 (2012) 12926–12934.
[66] S.K.R. De Biswas, A.R. Roy, Some operations on intuitionistic fuzzy sets, Fuzzy Sets Syst. 114 (2000) 477–484.

Hossein Sayyadi Tooranloo I received Ph.D. degree in "Operational Research" in 2014 from Tarbiat Modares University. Now I am Assistant Professor of Management Faculty of Vali-e-Asr University. I am currently writing a book titled intuitionistic fuzzy group decision-making. I have also fuzzy several articles in national and international scientific journals.

Arezoo sadat Ayatollah I received Master Science in information technology management from University of Science and Arts, Yazd, Iran, in 2014. Now I am doing several researches using fuzzy logic and intuitionistic fuzzy. I am currently writing a book titled intuitionistic fuzzy group decision-making.