Applied Soft Computing 49 (2016) 13–26

Contents lists available at ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

A multi-criteria route planning model based on fuzzy preference degrees of stops

Efendi Nasibov[a,c,*], Ahmet Can Diker[a], Elvin Nasibov[b]

[a] Department of Computer Science, Dokuz Eylul University, Buca, 35160 Izmir, Turkey
[b] Department of Mathematics, Ege University, Bornova, 35100 Izmir, Turkey,
[c] Institute of Control Systems, Azerbaijan National Academy of Sciences, AZ-1141 Baku, Azerbaijan,

ARTICLE INFO

Article history:
Received 1 March 2015
Received in revised form 20 June 2016
Accepted 31 July 2016
Available online 5 August 2016

Keywords:
Fuzzy modeling
Route planning
Preference degree
Heuristic algorithm

ABSTRACT

Integrated utilization of new technologies such as smart phones, tablet devices, and satellite maps has entered our daily lives recently. Nevertheless, many new applications are being developed mostly based on these technologies. The optimal route planning, which makes use of the public transport network structure between any selected origin and destination points, is one of the interesting applications among them. Route planning applications used today mostly focus on the aspects such that passengers use nearest stops around origin and destination geographical points, or use set of stops around these points within some walking radius. In these applications, which work on the classical (crisp) logic base, all stops on the walking distance have the same preference degree. However, in this study a novel fuzzy model is proposed which also takes into account preferences such as the stop's activity, and count of transit lines passing through the stop besides the walking distance. Using all these three preferences, aggregated fuzzy preference degrees of stops are calculated. The "optimum" routes between any origin and destination pair are constructed using feasible transfer points, which are chosen among the alternatives having the highest preference degrees overall. Fuzzy neighborhood relations such as "stop-stop", "stop-line", and "line-line" are introduced in order to employ in preference degree evaluations.

Apart from the aggregated degree of the preferences mentioned above, we also consider to minimize the total number of transit stops travelled on any route for establishing optimal routes. This additional preference can be described the time duration spent on transport vehicles, such as buses, trains, subways or ferries. Therefore, we propose a two-criteria route-planning problem in this study, where we try to maximize the aggregated preference degree of a route and to minimize the number of stops used on a route. Fuzzy optimal solutions for this problem are constructed via γ-level solutions of the fuzzy problem and a heuristic algorithm providing these solutions is proposed. This model and its algorithm can be considered as an optimal route search engine for mobile applications that could be used by urban public transport passengers.

1. Introduction

Increasing people's awareness to use public transportation more often has been regarded as one of the key factors for solving urban traffic problems over the last decade [1]. Most people believe that service quality of the public transportation is not adequate and eventually, the tendency to use private car among the society increases. This has caused the fact that route planning problem has become much more significant where it is required to obtain feasible route from origin to destination on any public transportation network. This problem has been considered as a subject of the Advanced Traveler Information Systems (ATIS) which provide information related with all traffic and weather conditions and it also helps to commuters or drivers by using this information before a trip [2,3]. Furthermore, with the advances of informatics and communication technologies, traveler information has become a valuable resource that it can implicitly contribute to achievement in service quality on public transportation [1,4]. There have been some previous studies that approaches to trip planning as an ATIS problem [1,4,5]. In these studies, it was shown that the people's route alternatives might change during their travel according to different conditions. Grotenhuis et al., discussed travel plan in three phases: pre-trip, wayside and on-board and they observed how

* Corresponding author at: Department of Computer Science, Dokuz Eylul University, Buca, 35160 Izmir, Turkey.
E-mail address: efendi.nasibov@deu.edu.tr (E. Nasibov).

http://dx.doi.org/10.1016/j.asoc.2016.07.052
1568-4946/

people's preferences changed depending on different phases and age groups [1]. Kramers suggested a similar three phase for the next generation such as pre-trip, on-trip and post trip but in his study, however, these phases are considered to vary according to the post actions on trip, then different travel information services with respect to these phases were compared [4]. Although there are other planning phases, pre-trip planning is the most frequently studied one in many systems. Different decision criteria such as shortest path, earliest arrival, at least price, or minimum number of transfer, etc. are taken into account for pre-trip planning objectives, as well as in multi-modal transportation and transport network modeling processes.

The route planning problem focuses on the issue that; how journey might be planned optimally for the routes between specified origin and destination under some criteria such as distance, time, traffic, etc. and various types of graphs is usually used in different kinds of systems. In most of the cases, the solution to this problem is given by shortest path algorithms on these graphs and bidirectional searches for a road network [6] that are generally accepted as the basis of the solution techniques [7–9]. Many of these algorithms, which are based on Dijkstra, have been modified to achieve satisfactory results when large road network is concerned because of their slow response time [10–14]. Although these algorithms are running successfully in the road network, they cannot be applied to public transportation network directly [15]. Solution for the route planning problem on the public transportation networks is known to be harder than road networks since transition between different modes could also be made possible and bus, metro or ferry have certain routes with timetables. Various methods were proposed in literature that they are called speed-up techniques based on narrowing the search space although shortest path algorithms on public transportation network have already been modified and utilized on graphs [12,14,16,17]. Hierarchical methods [12,14], goal directed search [16], reach-based search [17] are widely used with preprocessing as a common property of these techniques but where it done, memory cost is increased considerably despite the decrease in processing time. As it has been mentioned before, there are various criteria in real life problems where people prefer to decide upon routes according to their conditions and it is called as a multi-objective shortest path problem that usually arises in transportation problems. Solution techniques including labeling methods, ranking methods, constraint methods and parametric methods have been proposed to provide different effectiveness from each other [18]. It has been shown that, obtaining a solution that gives the best results for all criteria may not be possible since route decision of the people depending on their current circumstances could vary according to different scenarios.

Lozano and Storchi achieved pareto-optimal solutions by using shortest path algorithm in their study and they introduced a model that enables people to make decision depending on their demands [19]. In another study, where both time and number of transfer constraints were included, bi-criteria optimization problem was defined according to time-dependent and time-expanded models by obtaining pareto-optimal solution [20]. Delling et al. introduced Round Based Public Transportation Router (RAPTOR) with a different approach but they focused on the bi-criteria problem similarly by minimizing arrival time and the number of transfers from all pareto-optimal solutions [21]. In addition to these methods, some other approaches such as k-shortest path algorithms [22–24] and genetic algorithms [25,26] were implemented. Since in most of these algorithms, preprocessing increases memory cost, heuristic approaches were proposed in some other studies as an alternative methodology [15,27–29]. There are some other studies where the models were developed by excluding graphs, such as Liu et al. In these studies, algorithms aim to search common stops for different lines, although the model was constructed with graph in most of researches [29]. Stops from which those lines are passing were defined as hubs to narrow the search space. Chang et al. proposed a different approach on graphs that was constructed with nodes defining lines instead of stops [27]. In order to speed up this technique, they introduced criteria based on the features of stop preferences. It was assumed that the number of lines passing through on any stop is not a sufficient criterion, but there also has to be a short time interval between consecutive journeys on the same lines based on finding the fast connections. Bast et al. has suggested routes using transfer pattern on a graph that was modeled for a large transportation network [15].

There are few studies considering a fuzzy approach in public transportation networks in the literature [30,31]. Both Golnarkar et al. and Verga et al. discussed finding the best routes between origin and destination on multi-modal transportation network by taking arcs of graph as fuzzy numbers. Ghatee et al. proposed two algorithms for solving minimal cost flow problem as a multi-objective optimization problem, which concerns the transportation of hazardous materials from starting to end point with minimal cost [32]. The objective in their algorithms was to minimize fuzzy costs according to different criteria, such as giving a priority using lexicographic ordering. In those studies, time constraint has been pre-emptively discussed and representing arcs of graph with fuzzy numbers was one of the most common approaches. However, our approach in this study consists of neighborhood relation between stops and lines, which has not been addressed in the literature yet. In addition, a priority is given to stops according to a preference degree, which is explained in Section 2.2. This preference degree is a vital point of our study, from which we will benefit when solving our multi-criteria fuzzy route planning problem that is defined in Section 3. Note that, in most of the crisp route planning models in the literature, only stops within some fixed walking distance are evaluated ignoring the diversities among them or they could be handled only with respect to the ordering of their distances, where the nearest stop is considered first. Usually, there are no suggestions for the calculation of preference measure of stops, lines, or routes. However, our proposed model naturally allows the stops, lines and routes to be arranged in an order, using a technique based on fuzzy logic theory for determining their preference degrees. This approach seems more practical than its classical version, because it provides an ability to assign a membership degree in fuzzy sets, which may in turn allow the decision makers more flexibility.

In the rest of this paper, some mathematical definitions related with calculation of fuzzy preference degrees are given. Fuzzy neighborhoods among the stops, among the lines and between stops and lines are introduced in Section 2. In this section, preference degree of the stops for various criteria is presented as well. We describe our multi-criteria fuzzy route planning problem in Section 3. After an algorithm for solving this problem is proposed in Section 4 and we analyze an application to express running mechanism of the algorithm for this problem in Section 5. Finally, we conclude with future remarks in Section 6.

2. Fuzzy neighborhoods and preference degrees of stops

Heuristic approaches are required to speed-up algorithms, although classical optimization techniques are favored in solving route planning problems especially when problems are modeled by graphs. Determination of selection priority for the stops contributes to the search process and it is described as one of the heuristic approaches. In literature, a model without graph expression is shown to assign priority to stops according to number of the lines passing through [29]. Chang et al. proposed another model with graph representation, even though it seemed to have a similar approach with Liu et al., but the main difference of their model was the addition of a new criterion about short interval headway, i.e. passing frequency of any stops to have more chances when making transfer [27]. We assume that, prioritization of stops with respect to different decision criteria by using fuzzy logic would be a model that is closer to human reasoning.

Search process is usually performed between the stops that are nearest to both origin and destination points in route planning problem. This process limits the possible search domain among these stops, where algorithms may not find any solution or might be obtaining a solution with many unnecessary transfers. The main problem in this approach is the search process being only carried out between two fixed stops, based on the premise that people choose to walk to the nearest stop. This might be impractical to adapt for real life situations, since passengers would prefer to have more alternative route options depending on changing conditions or different criteria. Therefore, besides the optimal solutions achieved by conventional methods, evaluations of Pareto-optimal solutions have been proposed with alternative route recommendations [21,20]. In this paper, three decision criteria will be considered first for preference of stops: walking distance to the stop, activity of the stop and count of lines passing through the stop (level of being a hub).

Some definitions used in calculation of fuzzy preference degrees of stops and routes are given in the rest of this section.

2.1. Fuzzy neighborhood relations of stops and lines

It is observed in most of the related literature that the closest stops to origin and destination points are selected, and only the possible routes between these two stops are considered. As we mentioned previously in this paper, considering only the nearest two stops for the solution may cause some problems and therefore, all stops are evaluated whether they are within a "reasonable" walking distance with respect to the specified points. The level of reasonability is determined by using fuzzy logic approach. It is specified in [33–35] that the robustness of methods (robustness of values against variability) and representativeness of models increase while neighborhood relation between points is implemented as fuzzy. Therefore, in this study, we prefer to use a fuzzy relation with degrees instead of using a crisp relation.

Distances between stops and specified points on the map are computed with respect to spherical geometry, because all points are defined as geographical latitude-longitude coordinates in the related mobile applications. For this purpose, Haversine formula [36] is used to compute distance in meters between any two geographical points X and Y, as follows:

$$d(X,Y) = 2R\sin^{-1}\left(\left[\sin^2\left(\frac{\phi_1-\phi_2}{2}\right) + \cos\phi_1\cos\phi_2\sin^2\left(\frac{\lambda_1-\lambda_2}{2}\right)\right]^{0.5}\right), \quad (2.1)$$

where R is radius of the Earth ($R = 6367450$m); ϕ is latitude, λ longitude; X is given$X = (\phi_1, \lambda_1)$, and Y is given $Y = (\phi_2, \lambda_2)$. As more detailed distance data are available for urban network structure, such as Manhattan distance, or even more precise data considering all pedestrian utilization regarding underground passages, overhead crossings, stairs, street lights, etc., these could easily be implemented in our model as the primary distance metric.

Let us start by giving definitions of some of the terms used in formulation of the fuzzy problem. Let s be any stop in set of the stops $S\,(s \in S)$ and r be any line in set of the lines $R\,(r \in R)$. Any line can be defined as an ordered set of the certain stops. Stop-Line relation is denoted by $RS(r,s)$. The sequence number of a stop s on the line r is given as $RS(r,s) \geq 1$. Hence, $RS(r,s) = 0$ denotes that the stop s does not belong to the line r. So, the RS relation is formulated as follows:

$$RS(r,s) = \begin{cases} \geq 1, & \text{sequence number of the stop s on the line r,} \\ 0, & \text{the stop s is not on the line r.} \end{cases} \quad (2.2)$$

Let $S1(r)$ denote the set of stops on the line r passing through, i.e.

$$S1(r) = \left\{ s \in S : RS(r,s) \geq 1 \right\}.$$

Moreover, we denote PR_s as a set of lines that pass through the stop s and we define it as follows:

$$PR_s = \left\{ r : s\varepsilon S1(r) \right\}. \quad (2.3)$$

We use the following definitions for determining a preference degree of a stop according to the walking distance between stops:

Definition 1. (a neighborhood relation between stops): Let $s_i, s_j \in S$ be any two stops. Then a fuzzy neighborhood relation between stops is defined as below:

$$\mu^d\left(s_i, s_j\right) = max\left\{0, 1 - \frac{d\left(s_i, s_j\right)}{d_{walk}}\right\}, \quad (2.4)$$

where the parameter d_{walk} specifies the maximum reasonable walking distance, $d\left(s_i, s_j\right)$ is the distance between the stops s_i and s_j. Generally, the maximum reasonable walking distance d_{walk} is smaller than the maximum distance among the stops in the city, so we use *max* operator in the definition to guarantee the non-negativity of a membership degree. Note that the fuzzy relation $\mu^d : S \times S \to [0, 1]$ is both asymmetric and reflexive.

Definition 2. (a fuzzy neighbor stops set of a stop): The set of fuzzy neighbor stops of any given stop s is defined as

$$N_s = \left\{ \left(\mu^d\left(s, s'\right) / s' \right) | s' \in S \right\}. \quad (2.5)$$

Note that the membership degree of an element s' to the fuzzy set N_s is denoted as $N_s(s')$, and similar notation is used in the rest of the paper. Moreover, we usually use a description like $\left(degree/element\right)$ for a fuzzy set.

For specified γ level, N_s^γ represents γ -level set of the fuzzy set N_s and it is described as:

$$N_s^\gamma = \left\{ s' \in S : N_s\left(s'\right) \geq \gamma \right\}. \quad (2.6)$$

We will also call the N_s^γ set as a γ-neighbor set of the stop s. We also relate the degree of the fuzzy level with neighborhood radius as follows. For each γ –level there exists a radius ε such that following equation is satisfied:

$$\left\{ s' \in S : N_s\left(s'\right) \geq \gamma \right\} \equiv \left\{ s\prime \in S : d\left(s, s\prime\right) \leq \varepsilon \right\}. \quad (2.7)$$

Possibility of transferring between lines is determined by the presence of the exactly common stops of lines in some studies [29]. In this study, we consider the concept of fuzzy neighbor stops to ensure transition between lines that have no common stops among each other.

We introduce the following definitions to determine transfer possibilities between lines. In order to specify a fuzzy neighborhood relation between stops and lines, a set of fuzzy neighbor stops of a given line is defined as follows.

Definition 3. (a fuzzy neighbor stops set of a line): The fuzzy neighbor stops set of the line $r \in R$ is:

$$\mathrm{NRS_r} = \mathrm{V_{s\epsilon S1(r)}N_s} \quad (2.8)$$

Note that operator is used instead of the *max* operator, and $\wedge$ is used instead of the *min* operator henceforth in the rest of the paper.

Let us define:

$$PR_s(r) = \begin{cases} 1, if r \in PR_s \\ 0, otherwise. \end{cases} \quad (2.9)$$

The following definition determines that, any other line passing through any fuzzy neighbor stop of the line

will be a fuzzy neighbor line to the line r. We use this definition in the formulation of the fuzzy problem and in constructing the heuristic search algorithm.

Definition 4. (a fuzzy neighbor lines set of a line): A fuzzy neighbor lines set N_r of a line r is defined as the following composition:

$$N_r = NRS_r \circ PR_s, \quad (2.10)$$

in other form,

$$N_r(r\prime) = \vee_{s \in S} \left[NRS_r(s) \wedge PR_s\left(r'\right)\right], \forall r\prime \in R \quad (2.11)$$

The fuzzy relation N_r implies a fuzzy relation matrix $NR : R \times R \to [0, 1]$ among lines:

$$NR(i, j) = N_{r^i}\left(r^j\right), \forall r^i, r^j \in \mathrm{R} \quad (2.12)$$

This NR matrix is used in constructing the γ-connective routes. It is obvious that the relation NR is both symmetric and reflective. The γ −level set of the relation NR can be given as a matrix:

$$NR^\gamma(i, j) = \begin{cases} 1, if NR(i, j) \geq \gamma, \\ 0, otherwise. \end{cases} \quad (2.13)$$

This matrix also can be obtained as a matrix multiplication given below:

$$M = (RS)\left(SS^\gamma\right)(RS)^T, \quad (2.14)$$

Hence,

$$NR^\gamma(i, j) = \begin{cases} 1, if M(i, j) > 0, \\ 0, otherwise, \end{cases} \quad (2.15)$$

where $SS(i, j)$ is a matrix form of the fuzzy neighborhood relation between stops, i.e.

$$SS(i, j) = \mu^d\left(s_i, s_j\right), \quad (2.16)$$

and SS^γ is the γ-level set of the fuzzy relation SS:

$$SS^\gamma(i, j) = \begin{cases} 1, \mu^d\left(s_i, s_j\right) \geq \gamma, \\ 0, otherwise. \end{cases} \quad (2.17)$$

Note that any other aggregated membership function (see (2.21)) can be used instead of $\mu^d\left(s_i, s_j\right)$ (i.e. considering also activity and hub degree of stops given in the next subsection) to formulate fuzzy neighborhood (or fuzzy preference) degrees of stops/lines.

In fact, the lines which are actually not linked each other, become connected by using concept of the γ-neighborhood of a line, so a traveler can reach to destination with some transfers in related situation (Fig. 1).

According to Fig. 1, we can note that $\mathrm{S1}(\mathrm{r}_1) = \{\mathrm{s}_1, \mathrm{s}_2, \mathrm{s}_3\}$ and $\mathrm{NRS}^\gamma_{\mathrm{r}_1} = \{\mathrm{s}_1, \mathrm{s}_2, \mathrm{s}_3, \mathrm{s}_4, \mathrm{s}_5\}$. Although there is no exactly common stops between the lines r_1 and r_3, r_3 is a γ-neighbor of the line r_1 (i.e. $\mathrm{N}^\gamma_{\mathrm{r}_1} = \{\mathrm{r}_1, \mathrm{r}_2, \mathrm{r}_3\}$), and it can be used as a continuation line for the liner$_1$.

2.2. Fuzzy preference degree of a stop

Three criteria are used to determine fuzzy preferences of stops. First of them is the walking distance which is a distance between the stop and a chosen geographical point. Distances from stops to any points are restricted by some boundaries in crisp mathematics based on crisp 0/1 logic and it is generally about the walking distance limitation of an average person. In such cases, searching process is done by handling stops within the walking distance without any preference differentiation among stops. Instead of strictly including stops inside of the given neighborhood boundary, we may consider it according to its fuzzy neighborhood degree.

Suppose that the maximum feasible walking distance is denoted with the d_{walk}. While the distance is considered according to any selected geographical point X, a fuzzy membership function indicating the fuzzy neighborhood degree of any stop s, can be defined as

$$\mu^d_X(s) = max\left\{0, 1 - \frac{d(X, s)}{d_{walk}}\right\}. \quad (2.18)$$

As seen from this definition, the $\mu^d_X(s)$ value is the fuzzy neighborhood degree between the point X and the stop s according to Definition 1.

Second criterion to determine the preference degree of a stop is the stop activity. It is proposed for incorporating the passenger choices to use stops that are more active. In route planning, the mobility capacity of the stops might be recognized as a more important feature in cases where many people generally follow others' movements. It then becomes possible to measure stops' activity by smart card data acquired by vehicles on public transit [37]. Information about number of boarding will be required to find activity degree that is given to stops as a static value, since we will propose time independent (static) model. Instead of instantaneous value for boarding count, $B(s)$ value representing the average of total daily boarding of a stop s will be employed. On the other hand, activity degree of stops is a general term and independent of the chosen point$X$. We assume that activity degree is related with the count of boarding passengers, thus more boarding corresponds a higher degree of preference. The activity degree of a stop s is defined as below:

$$\mu^A(s) = \frac{B(s)}{\max_{s\varepsilon S} B(s)} \quad (2.19)$$

Last criterion for determining the preference degree of a stop is about its being a hub stop. Different lines passing through a stop can provide more alternatives to reach at more destinations. Hubs were used as transfer stations in the literature with classical approach [15,27–29]. However, there is a certain degree in fuzzy logic that depends on number of lines passing through a stop if the stop is represented as a hub.

Suppose that PR_s is given as the set of lines passing through stop s and let us denote the total number of these lines as $|PR_s|$. Then we define the hub degree of any stop s as below:

$$\mu^H(s) = \frac{|PR_s|}{\max_{s\varepsilon S}|PR_s|}. \quad (2.20)$$

Several aggregation methods might be used to determine the overall preference degree for the stop, called stop preference degree [38,39]. Minimum Aggregation method, which is one of the most common methods in fuzzy logic theory, can also be used in here to combine three criteria as we mentioned above.

$$\mu_X(s) = min\left\{\mu^d_X(s), \mu^A(s), \mu^H(s)\right\}. \quad (2.21)$$

So, $\mu_X(s)$ denotes a preference degree of the stop s according to the geographical point X. Hereby, for any geographical point X, a fuzzy set of preferred stops could be organized with respect to their preference degrees as follows:

$$S2(X) = \{\left(\mu_X(s)/s\right)|s \in S\}. \quad (2.22)$$

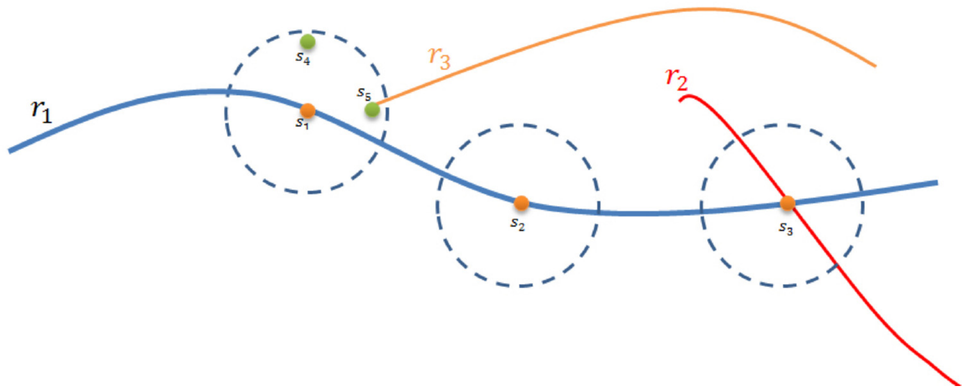

Fig. 1. The demonstration of a line r_3 that is a γ-neighbor line to the line r_1.

Note that also any stop with its geographical location can be used instead of the referenced point X.

The γ-level set $S2(X)^{\gamma}$ of the fuzzy set $S2(X)$ at a fixed level $\gamma \in (0, 1]$ is described as

$$S2(X)^{\gamma} = \left\{ s \in S : \mu_X(s) \geq \gamma \right\}. \tag{2.23}$$

It is obvious that the total number of stops within the $S2(X)^{\gamma}$ will decrease if γ –level value increases. Preference degree of the stop can present a more realistic solution than solving the problem only by choosing the nearest stop, since it does not only take into account distances, but also it includes degrees of both stop activity and being hub stop aggregated with fuzzy logic. This γ-level approach demonstrates the expectation degree of the travelers when making certain decisions and it therefore gives possibility to update their mobility demands and allow them to plan a route more flexibly.

3. A multi-criteria fuzzy route planning model

As it is known in the fuzzy set theory, general solution of the fuzzy problem can be given as a union of the γ –level solution sets. So, we will describe the problem via its $\gamma \in (0, 1]$ fuzzy level set decomposition.

Let us first introduce connection as 3-tuples (s_1, r, s_2) that is expressed as a travel from any stop $s_1 \varepsilon S$ to $s_2 \varepsilon S$ traveling via a line$r \varepsilon R$. Before introducing the problem, we will present the definitions of both possible connections and the route as below. The following definition determines possibility of travel only for one line:

Definition 5. (a possible connection): A 3-tuple (s_1, r, s_2) is a possible connection if it satisfies following conditions:

$$s_1 \in S1(r); \tag{3.1}$$

$$s_2 \in S1(r); \tag{3.2}$$

$$RS(r, s_1) < RS(r, s_2). \tag{3.3}$$

It is obvious that a 3-tuple (s_1, r, s_2) also is a possible connection if it satisfies the following conditions:

$$RS(r, s_1) \geq 1 \text{ and} RS(r, s_1) < RS(r, s_2). \tag{3.4}$$

Definition 6. (a fuzzy route): A sequence of m possible connections going from stop s_X to stop s_Y is a fuzzy route with membership degree $\mu\left(\pi_{s_X s_Y}\right)$ denoted as

$$\pi_{s_X s_Y} \triangleq \mu\left(\pi_{s_X s_Y}\right) / \left(s_{i_1}, r_{j_1}, s_{k_1}\right), \left(s_{i_2}, r_{j_2}, s_{k_2}\right), \ldots, \left(s_{im}, r_{jm}, s_{km}\right), \tag{3.5}$$

if it satisfies following conditions:

$$s_{i_1} = s_X; \tag{3.6}$$

$$s_{k_m} = s_Y; \tag{3.7}$$

$$\left(s_{i_t}, r_{j_t}, s_{k_t}\right)\ t = 1, \ldots, m, \qquad \text{are possible connections.} \tag{3.8}$$

The connectivity degree of the fuzzy route $\pi_{s_X s_Y}$ is determined as

$$\mu\left(\pi_{s_X s_Y}\right) = \wedge_{t=2,\ldots,m} \mu_{s_{k_{t-1}}}\left(s_{i_t}\right). \tag{3.9}$$

Note that the notation $\mu_{s_{k_{t-1}}}\left(s_{i_t}\right), t = 2, \ldots, m$, is a connectivity degree of a fuzzy transfer node and it illustrates a preference degree of the next boarding stop s_{i_t} according to the previous alighting stop $s_{k_{t-1}}$. Constraint (3.6) represents that beginning stop is equal to stop s_X, similarly last stop is taken as equal to stop s_Y according to the constraint (3.7). Traveler directly gets on the next vehicle on stops where she gets off from the previous vehicle without walking, if $s_{i_t} = s_{k_{t-1}}$ is satisfied; otherwise if $s_{i_t} \neq s_{k_{t-1}}$ is satisfied, she walks from alighting stop $s_{k_{t-1}}$ of the previous boarding to boarding stop s_{i_t} of the next line. It is clear that the number of transfers on route $\pi_{s_X s_Y}$ which consists of m lines is equal to $m - 1$.

It is known that a γ –level set decomposition approach to solve fuzzy optimization problems is widely used in fuzzy sets theory. So we will give a definition of γ –connectivity for a fuzzy route, and then formulate a γ-decomposition version of the fuzzy route planning problem.

Definition 7. (a fuzzy γ-connective route): A fuzzy γ –connective route $\pi^{\gamma}_{s_X s_Y}$ is a route having its connectivity degree greater than or equal to γ.

It is obvious that a γ –connective route is a sequence of m possible connections going from stop s_X to stop s_Y which satisfies the following conditions:

$$s_{i_1} = s_X; \tag{3.10}$$

$$s_{k_m} = s_Y; \tag{3.11}$$

$$\mu_{s_{k_{t-1}}}\left(s_{i_t}\right) \geq \gamma, t = 2, \ldots, m; \tag{3.12}$$

$$\left(s_{i_t}, r_{j_t}, s_{k_t}\right) \qquad \text{, are possible connections.} \tag{3.13}$$

Another decision criterion in this study considering optimality of the route is the minimization of number of stops passing through. Moreover, number of transfers is taken as a fixed parameter.

Let $|(s_i, r, s_k)|$ denote the total number of stops on the line $r \varepsilon R$ between the stops s_i and . Then the total number of stops on the route $\pi_{s_X s_Y}$ going from the stop s_X to the stop s_Y can be calculated as follows:

$$|\pi_{s_X s_Y}| = \sum_{t=1}^{m} |\left(s_{i_t}, r_{j_t}, s_{k_t}\right)|, \tag{3.14}$$

where $s_{i_1} = s_X$ and $s_{k_m} = s_Y$.

Thereby, a fuzzy multi criteria route planning problem in its γ −level decomposition form for any specified geographical points X and Y, can be presented as follows:

$$\max : \gamma \tag{3.15}$$

$$\min : |\pi_{s_X s_Y}| = \Sigma_{t=1}^{m} | \left(s_{i_t}, r_{j_t}, s_{k_t} \right) |, \tag{3.16}$$

s.t.:

$$RS\left(r_{j_t}, s_{i_t}\right) \geq 1 \; t = 1, \ldots, m, \tag{3.17}$$

$$RS(r_{j_t}, s_{k_t}) > RS(r_{j_t}, s_{i_t})t = 1, \ldots, m, \tag{3.18}$$

$$\mu_X(s_X) \geq \gamma, \tag{3.19}$$

$$\mu_Y(s_Y) \geq \gamma, \tag{3.20}$$

$$\mu_{s_{k_{t-1}}}\left(s_{i_t}\right) \geq \gamma, t = 2, \ldots, m; \tag{3.21}$$

$$s_{i_1} = s_X, \tag{3.22}$$

$$s_{k_m} = s_Y, \tag{3.23}$$

$$\gamma \in (0, 1], \tag{3.24}$$

where (3.15) states making choice from among possible routes as higher preference degree as possible, (3.16) indicates selecting shorter possible routes in terms of number of stops. Constraints (3.17)-(3.18) indicate possibility of each connection on the route. Constraints (3.19) and (3.20) are constraints to ensure that the first stop on the route should be fuzzy γ-preferable according to the geographical point X, similarly last stop on the route should be γ-preferable according to the geographical point Y. Constraints (3.21)–(3.24) enforce the fuzzy γ-connectivity of a route. Solution of the problem given in (3.15)–(3.24), provides choosing possible shorter routes among as more as high preferable fuzzy routes connecting the chosen origin and destination points X and Y. Moreover, the preference degree of a solution of the problem (3.15)–(3.24) will be equal to

$$\gamma^* = \mu_X(s_X) \wedge \mu_Y(s_Y) \wedge \mu\left(\pi_{s_X s_Y}\right). \tag{3.25}$$

In practical situations, the proposed solution results should be limited with sorted in descending order top q alternative routes set Π_{XY}^q in order to increase usability:

$$\Pi_{XY}^q = \left\{ \pi_{s_X s_Y}^{(1)}, \ldots, \pi_{s_X s_Y}^{(q)} | \pi_{s_X s_Y}^{(i)} \succ \pi_{s_X s_Y}^{(i+1)}, i = 1..q-1 \right\}, \tag{3.26}$$

where $\pi_{s_X s_Y}^{(i)} \succ \pi_{s_X s_Y}^{(i+1)}$ indicates preference of the route $\pi_{s_X s_Y}^{(i)}$ to the route $\pi_{s_X s_Y}^{(i+1)}$. The measure of preference may be constructed according to the multiple criteria (3.15) and (3.16). This set of non-dominant solutions is in pareto-optimal solution space of the problem. There exist various approaches to multiple criteria problem, such as lexicographical ordering of the criteria, weighted aggregation of the criteria etc. It should also be mentioned that there is even a more important fact in route suggestions, where the users usually prefer the stops with higher level of satisfactions regarding their socio-psychological expectations.

4. Algorithm

In this section, we propose a heuristic algorithm for obtaining the pseudo-optimal solutions of the problem handled in (3.15)–(3.24). This algorithm differs from Liu et al. due to including constitutively new fuzzy definitions and applying a new strategy accordingly. Although, our approach is still similar with respect to their search route, which is also based on selecting lines separately (Liu et al.). There are some different approaches in our algorithm, such as fuzzy neighborhood of stops and lines to provide transition between stops and lines; and fuzzy preference degree of the stop that allows for selecting the route with high availability, instead of trying all possible paths. Principally, the algorithm will be implemented in two phases depending on these definitions: First, lines passing through stops which are located around the specified points such as O and D, are prioritized by using fuzzy preference degrees; and second, we seek whether these lines are fuzzy neighbor to each other or not.

In order to find γ-optimal solution, departure and arrival stops on the route have be found first, then the number of transfers as a parameter (direct connection, single transfer and double transfer) must be applied respectively unless there are no results for the current case. Fuzzy preference degrees of the stops are searched around origin and destination points that are represented as O and D respectively, by satisfying the following condition: stops are incorporated if their preference degrees are greater or equal to the γ. Then, k_O count of departure stops around O and k_D count of arrival stops around D are found. The searching process is separately employed between these departure and arrival stops on each stage for $k_O \times k_D$ pairs. Initially, direct connection is searched and thus we examine whether an intersection among any line between departure and arrival stops exists or not. In case of a direct link absence, searching process continues with transfer case where the case of a single transfer case is considered first. It could be said that, there exists a single transfer if there is a fuzzy neighborhood relation between any lines departed from one of the $s_o \in S2(O)^\gamma$ stops and any other line arrived to one of the $s_d \in S2(D)^\gamma$ stops. To satisfy this necessity, it is required to introduce a transfer location that has ingoing line (s) from departure stops and that has also outgoing line (s) to arrival stops, and these lines must be fuzzy neighbors, thus the transfer point is able to connect origin to destination. There must be alighting (s_{c_1}) and boarding (s_{c_2}) stops in transfer area to ensure a possible connection exactly. We note that if alighting stop of previous trip is same with boarding stop of the next trip, passengers ride the line where they get off from previous line when transfer, otherwise they should walk between these stops before boarding the line. The route is determined then as two consecutive possible connections, which are $\left(s_o, r_1, s_{c_1}\right)$ and $\left(s_{c_2}, r_2, s_d\right)$ respectively (Fig. 2): the first one begins with departure stop s_o by boarding r_1 and it ends in alighting stop s_{c_1} by getting off r_1; and the second connection continues by boarding r_2 from s_{c_2} and the route ends in s_d stop by getting off from r_2 line.

According to Fig. 2, it can be written: $s_o^{(1)}, s_o^{(2)} \in S2(O)^\gamma$ and $s_d^{(1)}, s_d^{(2)}, s_d^{(3)} \in S2(D)^\gamma$; $s_o^{(2)}, s_{c_1} \in S1(r_2)$ and $RS\left(r_2, s_o^{(2)}\right) < RS\left(r_2, s_{c_1}\right)$; $s_d^{(3)}, s_{c_2} \in S1(r_5)$ and $RS\left(r_5, s_{c_2}\right) < RS\left(r_5, s_d^{(3)}\right)$; $\pi_{s_o^{(2)} s_d^{(3)}} = \langle \left(s_o^{(2)}, r_2, s_{c_1}\right), \left(s_{c_2}, r_5, s_d^{(3)}\right) \rangle$

Double transfers should be sought if there is no single transfer. Similarly, a route could be defined as a double transfer if there are three lines that have fuzzy neighborhood respectively and if they satisfy three specific conditions. These three conditions are as follows; first line of the route passes from origin to first transfer location, second line links between double transfer location directed towards a second transfer location and finally third line connects second transfer location to destination. Each transfer location is represented as a pair that consists of alighting and boarding stops respectively. Alighting stop $\mathbf{s_{c_1}}$ of the first transfer location links outgoing line $\mathbf{r_o}$ from $\mathbf{s_o}$ to itself that the stop will be on same line. Boarding stop $\mathbf{s_{c_2}}$ of the first transfer location is around $\mathbf{s_{c_1}}$ that it will satisfy condition (9) in Definition 7. It connects itself to alighting stop $\mathbf{s_{c_3}}$, which is first parameter of the second transfer location, with $\mathbf{r}$ line going towards $\mathbf{s_{c_3}}$. Boarding stop $\mathbf{s_{c_4}}$ of the second transfer links itself to $\mathbf{s_d}$ with $\mathbf{r_D}$ that they will satisfy conditions of possible connection. Similar to the procedure described in single transfer, three possible connections will be required $\left(\mathbf{s_o}, \mathbf{r_O}, \mathbf{s_{c_1}}\right)$,

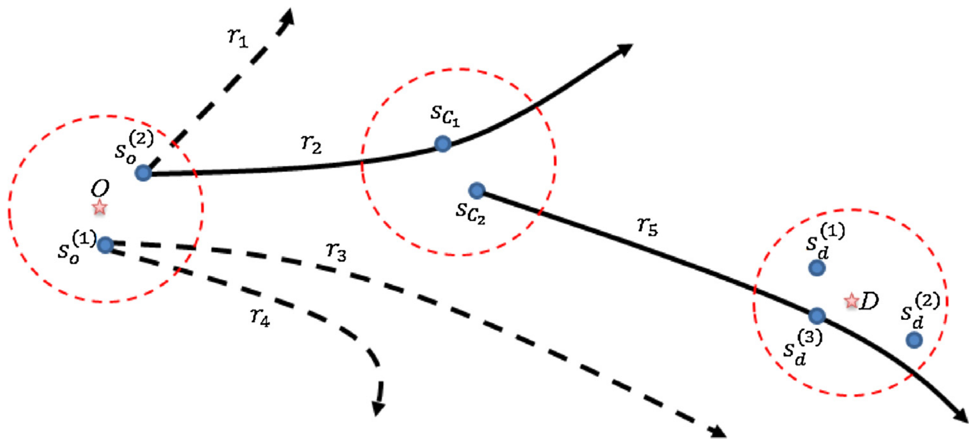

Fig. 2. Example of a route $\pi_{s_o^{(2)}s_d^{(3)}} = \left(s_o^{(2)}, r_2, s_{c_1}\right), \left(s_{c_2}, r_5, s_d^{(3)}\right)$ with a single transfer.

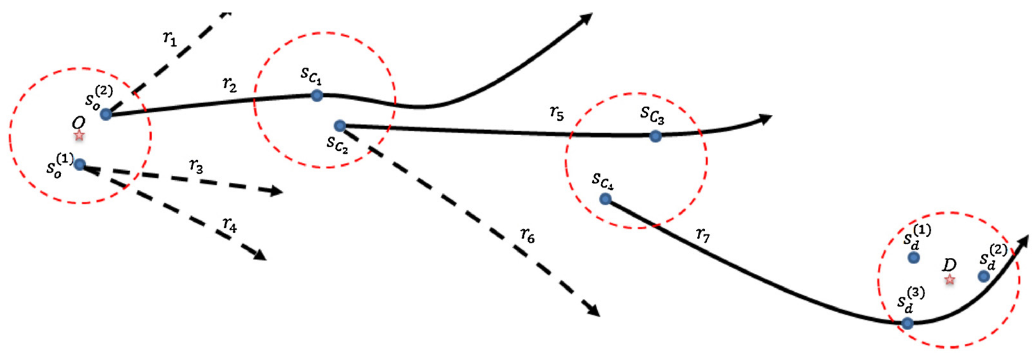

Fig. 3. Example of a route $\boldsymbol{\pi}_{s_o^{(2)}s_d^{(3)}} = \left(\mathbf{s_o^{(2)}}, \mathbf{r}_2, \mathbf{s}_{c_1}\right), \left(\mathbf{s}_{c_2}, \mathbf{r}_5, \mathbf{s}_{c_3}\right), \left(\mathbf{s}_{c_4}, \mathbf{r}_7, \mathbf{s_d^{(3)}}\right)$ with double transfers.

$\left(\mathbf{s}_{\mathbf{c}_2}, \mathbf{r}, \mathbf{s}_{\mathbf{c}_3}\right)$ and $\left(\mathbf{s}_{\mathbf{c}_4}, \mathbf{r_D}, \mathbf{s_d}\right)$ respectively and they are denoted in Fig. 3.

According to Fig. 3 it can be written: $\boldsymbol{s_o^{(1)}}, \boldsymbol{s_o^{(2)}} \in \boldsymbol{S}2(\boldsymbol{O})^{\boldsymbol{\gamma}}$ and $S_d^{(1)}, S_d^{(2)}, S_d^{(3)} \in S2(D)^{\gamma}$; $S_d^{(2)}, S_{c_1} \in S1(r_2)$ and $RS\left(r_2, S_0^{(2)}\right) < RS\left(r_2, S_{c_1}\right)$; $S_{c_2}, S_{c_3} \in S1(r_5)$ and $RS\left(r_5, s_{c_2}\right) < RS\left(r_5, s_{c_3}\right)$; $\boldsymbol{s_d^{(3)}}, \boldsymbol{s}_{c_4} \in \boldsymbol{S}1(\boldsymbol{r}_7)$ and $RS\left(r_7, s_{c_4}\right) < RS\left(r_7, s_d^{(3)}\right)$; $\pi_{s_o^{(2)}s_d^{(3)}} = \left(\boldsymbol{s_o^{(2)}}, \boldsymbol{r}_2, \boldsymbol{s}_{c_1}\right), \left(\boldsymbol{s}_{c_2}, \boldsymbol{r}_5, \boldsymbol{s}_{c_3}\right), \left(\boldsymbol{s}_{c_4}, \boldsymbol{r}_7, \boldsymbol{s_d^{(3)}}\right)$.

Maximum number of transfers that can be done for a city is taken as two (double transfers) in the proposed algorithm. Therefore, this algorithm will be terminated if there isn't any route available after completing the search for double transfers. Simplified flowchart of the proposed algorithm is presented in Figs. 4–7. Note that stops are processed every time with respect to their preference degrees sorted in descending order. More detailed pseudo-code of this algorithm is given as Appendix.

Let s be the count of all stops, l the count of lines, and c the count of stops on the longest line. We should use relations among stops, among lines and between stops and lines. So, space complexity of our algorithm can be evaluated as $\boldsymbol{O}\left(\boldsymbol{s}^2 + \boldsymbol{l}^2 + \boldsymbol{sl}\right)$.

Moreover, the time complexity of the algorithm can be calculated roughly as follows. Let an operation with single stop or line (i.e. access a preference degree, distance etc.) be handled roughly as a unit operation. Then the time complexity of the algorithm according to the flowchart is calculated roughly as follows. For the procedure *DirectConnection*:

1 Calculation of both $\boldsymbol{S}2(\boldsymbol{O})^{\boldsymbol{\gamma}}$ and $\boldsymbol{S}2(\boldsymbol{D})^{\boldsymbol{\gamma}}$ has complexity $\boldsymbol{O}(\boldsymbol{s})$;
2 Calculation of both $\overline{\boldsymbol{PR}}(\boldsymbol{O})$ and $\overline{\boldsymbol{PR}}(\boldsymbol{D})$ has complexity $\boldsymbol{O}(\boldsymbol{ls})$;
3 Calculation of $\boldsymbol{\Pi_{OD}}$ has complexity $\boldsymbol{O}\left(\boldsymbol{ls}^2\right)$;

So, total complexity of *DirectConnection* is $\boldsymbol{O}\left(\boldsymbol{ls}^2\right)$. Similarly, the complexities of the procedures *SingleTransfer* and *DoubleTransfer* are $\boldsymbol{O}\left(\boldsymbol{s}^2\boldsymbol{l}^2\boldsymbol{c}^2\right)$ and $\boldsymbol{O}\left(\boldsymbol{s}^2\boldsymbol{l}^3\boldsymbol{c}^4\right)$ respectively. Finally, the total time complexity of the algorithm is $\boldsymbol{O}\left(\boldsymbol{s}^2\boldsymbol{l}^3\boldsymbol{c}^4\right)$.

5. Computational example

In this section, a computational example is given that uses some relevant information about some stops such as geographical coordinates, stops activity, passing lines, etc. obtained from the public transportation network of Izmir, which is the third biggest metropolitan city in Turkey. There are about 6800 stops located on about 600 bus, metro or ferry transport lines in the Izmir [37]. Experiments with PC i7, 2.39 GHz, 8 GB RAM gave results around 2–4 s for constructing 2–3 alternative routes between ordinary chosen origin and destination points on the city map. However, since our main goal is mainly the presentation of a novel optimal route planning model based on fuzzy logic, more detailed performance analysis of the solution algorithm and its comparison to the other algorithms are left to future studies. A computational example explaining steps of the proposed algorithm is given below.

Let us apply the algorithm by taking $\boldsymbol{\gamma} = 0.005$ based on data from public transportation in Izmir with $\max_{s\varepsilon S}\boldsymbol{B}(\boldsymbol{s}) = 5999$, $\max_{s\varepsilon S}|\boldsymbol{PR_s}| = 28$. Moreover, the maximum walking distance is supposed to be $\boldsymbol{d_{walk}} = 1000$. Steps of the algorithm are processed as follows:

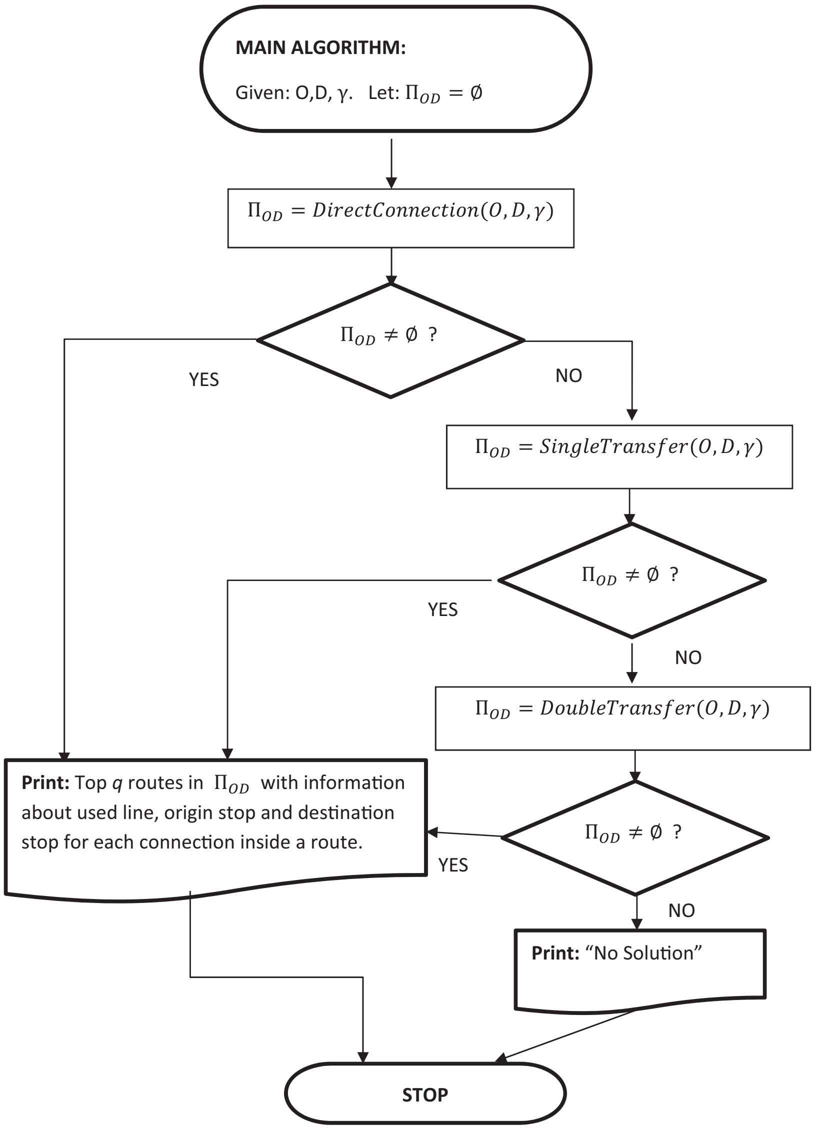

Fig. 4. Flowchart of the Main Algorithm.

Table 1
Walking distance, stop activity and hub values of stops and their degrees regarding to stops around the origin point O.

Stop ID (s)	$d(O, s)$	$\mu_O^d(s)$	$B(s)$	$\mu^A(s)$	$\lvert PR_S \rvert$	$\mu^H(s)$	$\mu_O(s)$
20243	50.3177678	0.94968223	34	0.00566761	1	0.03571428	0.00566761
20244	54.9615194	0.94503848	1	0.00016669	1	0.03571428	0.00016669
20233	155.202353	0.84479764	1	0.00016669	1	0.03571428	0.00016669
20066	169.671379	0.8303286	1207	0.20120020	4	0.14285714	0.14285714

Step 0: Two sets $\boldsymbol{S}2(\boldsymbol{O})^{0.005}$ and $\boldsymbol{S}2(\boldsymbol{D})^{0.005}$ are found by taking into account preference degrees of the stops around the points O and D, respectively. Search process will begin with the stop $\boldsymbol{s}_{20066}$ due to having maximum value among the preference degrees (Table 1).

Although $\boldsymbol{s}_{20243}$ is closer to origin point than $\boldsymbol{s}_{20066}$, it remains behind $\boldsymbol{s}_{20066}$ in ranking since its degrees of both stop activity and being hub stop is less than $\boldsymbol{s}_{20066}$. All these degrees are evaluated together as we mentioned in Section 2.2. Set of the stops around the point O is written by ordering as $\boldsymbol{S}2(\boldsymbol{O})^{0.005} = \left\{\boldsymbol{s}_{20066}^{(1)}, \boldsymbol{s}_{20243}^{(2)}\right\}$. Similarly, the preference degrees of the stops around D are found and given in Table 2.

As seen in Table 2, stops around D are prioritized by ordering with respect to the preference degree which is greater than 0.005 and these stops are found as $\boldsymbol{S}2(\boldsymbol{D})^{0.005} = \left\{\boldsymbol{s}_{30130}^{(1)}, \boldsymbol{s}_{30129}^{(2)}, \boldsymbol{s}_{30149}^{(3)}\right\}$. Searching process is continued by taking an element separately from sets both $\boldsymbol{S}2(\boldsymbol{O})^{0.005}$ ($\boldsymbol{k_O} = 2$) and $\boldsymbol{S}2(\boldsymbol{D})^{0.005}$ ($\boldsymbol{k_D} = 3$), respectively.

Step 1: $(\boldsymbol{S}2(\boldsymbol{O})^{0.005} \cap \boldsymbol{S}2(\boldsymbol{D})^{0.005}) = \phi$ equality is checked whether it is satisfied or not. Since the equality is satisfied, we proceed to Step 2.

Step 2: Procedure of *DirectConnection(O, D, 0.005)* is executed: we examine whether any line connects two stops selected in order from $\boldsymbol{S}2(\boldsymbol{O})^{0.005}$ and $\boldsymbol{S}2(\boldsymbol{D})^{0.005}$ or not. Therefore, we first take $\boldsymbol{\Pi_{OD}} = \phi$, then we investigate if there is an appropriate line

Table 2
Walking distance, stop activity and hub values of stops and their degrees regarding to stops around the destination point D.

Stop ID (s)	$d(D,s)$	$\mu_D^d(\boldsymbol{s})$	$\boldsymbol{B}(\boldsymbol{s})$	$\mu^A(\boldsymbol{s})$	$\lvert \boldsymbol{PR_S} \rvert$	$\mu^H(\boldsymbol{s})$	$\mu_D(\boldsymbol{s})$
30130	62.9699932	0.93703001	126	0.02100350	4	0.142857143	0.02100350
30129	78.8656185	0.92113438	125	0.02083681	2	0.071428571	0.02083681
30149	89.9143086	0.91008569	124	0.02067011	3	0.107142857	0.02067011

Table 3
Searching process for direct connection from departure $\boldsymbol{s_O}\varepsilon\boldsymbol{S}2(\boldsymbol{O})^{0.005}$ to arrival stops $\boldsymbol{s_D}\varepsilon\boldsymbol{S}2(\boldsymbol{D})^{0.005}$.

$\boldsymbol{s}_{\boldsymbol{O}}^{(\boldsymbol{i})}\varepsilon\boldsymbol{S}2(\boldsymbol{O})^{0.005}$	$\boldsymbol{PR}_{s_O^{(i)}}$	$\boldsymbol{s}_{\boldsymbol{D}}^{(\boldsymbol{j})}\varepsilon\boldsymbol{S}2(\boldsymbol{D})^{0.005}$	$\boldsymbol{PR}_{s_D^{(j)}}$	$\boldsymbol{PR}_{s_O^{(i)}} \cap \boldsymbol{PR}_{s_D^{(j)}}$
$\boldsymbol{s}_{20066}^{(1)}$	$\{\boldsymbol{r}_{77}, \boldsymbol{r}_{140}, \boldsymbol{r}_{147}, \boldsymbol{r}_{148}\}$	$\boldsymbol{s}_{30130}^{(1)}$	$\{\boldsymbol{r}_{53}, \boldsymbol{r}_{214}, \boldsymbol{r}_{348}, \boldsymbol{r}_{662}\}$	ϕ
		$\boldsymbol{s}_{30129}^{(2)}$	$\{\boldsymbol{r}_{54}, \boldsymbol{r}_{249}\}$	ϕ
		$\boldsymbol{s}_{30149}^{(3)}$	$\{\boldsymbol{r}_{59}, \boldsymbol{r}_{543}, \boldsymbol{r}_{564}\}$	ϕ
$\boldsymbol{s}_{20243}^{(2)}$	$\{\boldsymbol{r}_{361}\}$	$\boldsymbol{s}_{30130}^{(1)}$	$\{\boldsymbol{r}_{53}, \boldsymbol{r}_{214}, \boldsymbol{r}_{348}, \boldsymbol{r}_{662}\}$	ϕ
		$\boldsymbol{s}_{30129}^{(2)}$	$\{\boldsymbol{r}_{54}, \boldsymbol{r}_{249}\}$	ϕ
		$\boldsymbol{s}_{30149}^{(3)}$	$\{\boldsymbol{r}_{59}, \boldsymbol{r}_{543}, \boldsymbol{r}_{564}\}$	ϕ

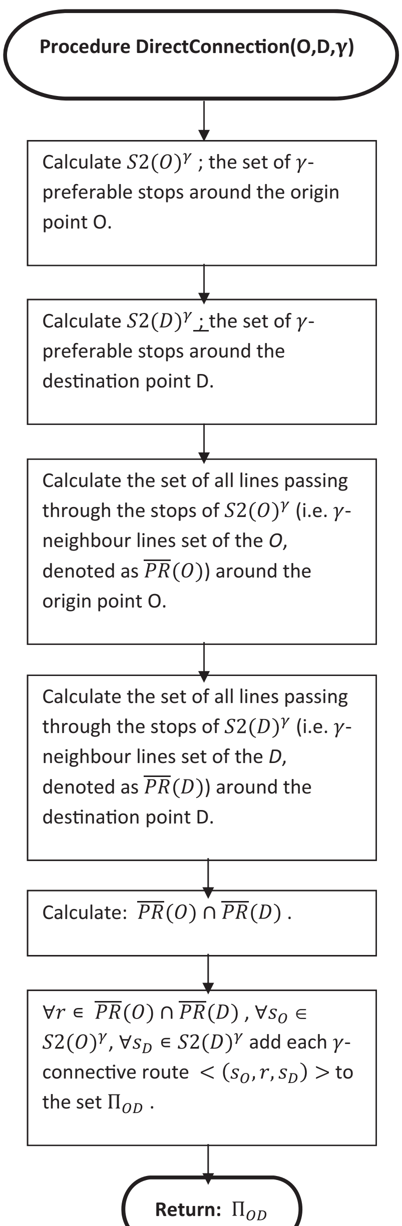

Fig. 5. Flowchart of the DirectConnection.

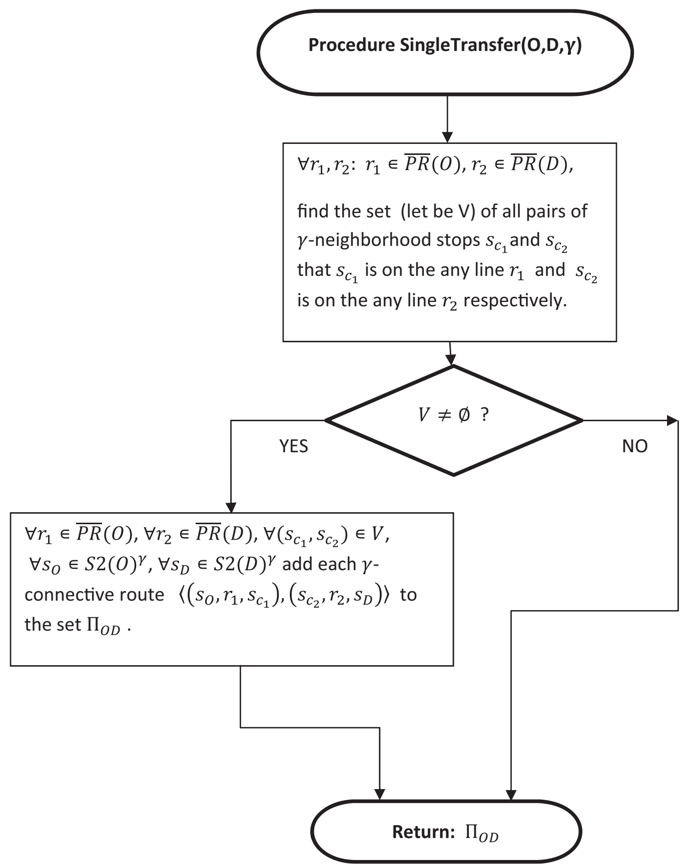

Fig. 6. Flowchart of the SingleTransfer.

passing from departure to arrival stops (in Table 3). As including $\{\boldsymbol{PR_{s_O}} \cap \boldsymbol{PR_{s_D}} : \boldsymbol{RS}(\boldsymbol{r}, \boldsymbol{s_O}) < \boldsymbol{RS}(\boldsymbol{r}, \boldsymbol{s_D})\} = \phi$ for $\forall \boldsymbol{s_O}\varepsilon\boldsymbol{S}2(\boldsymbol{O})^{0.005}$ and $\forall \boldsymbol{s_D}\varepsilon\boldsymbol{S}2(\boldsymbol{D})^{0.005}$ provides no direct connecting line, the *SingleTransfer(O, D,* 0.005) procedure will be operated in Step 3.

Step 3: Procedure of *SingleTransfer(O, D,* 0.005) is executed for finding the lines that pass from departure to transfer stops.

Fuzzy neighbors of stops on line $\boldsymbol{r}_1$ were marked as bold in Table 4. It is required to find the intersection between the stops $\boldsymbol{s_C}$ and $\boldsymbol{s_D}$ that provides the transfer from $\boldsymbol{s_C}$ to $\boldsymbol{s_D}$. If any intersection is found, then the solution set shall include two consecutive possible connections that comprise of these stops and lines. Departure stops in $\boldsymbol{s_O}$ ($\boldsymbol{s}_{20066}$ and $\boldsymbol{s}_{20243}$) that are given in Table 4 will be

Table 4
Passing lines from $\boldsymbol{s_O^{(i)}} \varepsilon \boldsymbol{S}2(\boldsymbol{O})^{0.005}$ with departure stops $\boldsymbol{s_c}$.

$\boldsymbol{s_O^{(i)}} \varepsilon \boldsymbol{S}2(\boldsymbol{O})^{0.005}$	$\boldsymbol{r}_1 = \boldsymbol{PR}_{\boldsymbol{s_O^{(i)}}}$	$\boldsymbol{s_c} \varepsilon \boldsymbol{NRS}_{\boldsymbol{r}_1}^{0.005}$
$\boldsymbol{s}_{20066}^{(1)}$	$\boldsymbol{r}_{77}$	$\{s_{20700}, s_{20066}, \boldsymbol{s_{20243}}, s_{20056}, \boldsymbol{s_{20054}}, s_{10492}\}$
	$\boldsymbol{r}_{140}$	$\{s_{20657}, s_{20628}, s_{20066}, \boldsymbol{s_{20243}}, s_{20054}, s_{10492}\}$
	$\boldsymbol{r}_{147}$	$\{s_{20580}, s_{20066}, \boldsymbol{s_{20243}}, s_{10492}\}$
	$\boldsymbol{r}_{148}$	$\{s_{20611}, s_{20066}, \boldsymbol{s_{20243}}, s_{20042}\}$
$\boldsymbol{s}_{20243}^{(2)}$	$\boldsymbol{r}_{361}$	$\{s_{20238}, s_{20243}, \boldsymbol{s_{20066}}, s_{20200}, \boldsymbol{s_{20166}}\}$

Table 5
Searching process on stops and lines for single transfer.

s_c	PR_{s_c}	$s_D^{(j)}$	$PR_{s_D^{(j)}}$	$PR_{s_O^{(i)}} \cap PR_{s_D^{(j)}}$
s_{20700}	$\{r_{77}\}$	$s_{30130}^{(1)}$	$\{r_{53}, r_{214}, r_{348}, r_{662}\}$	ϕ
		$s_{30129}^{(2)}$	$\{r_{54}, r_{249}\}$	ϕ
		$s_{30149}^{(3)}$	$\left\{r_{59}, r_{543}, r_{564}\right\}$	ϕ
s_{20056}	$\{r_{77}, r_{662}\}$	$s_{30130}^{(1)}$	$\{r_{53}, r_{214}, r_{348}, r_{662}\}$	$\{r_{662}\}$
		$s_{30129}^{(2)}$	$\{r_{54}, r_{249}\}$	ϕ
		$s_{30149}^{(3)}$	$\left\{r_{59}, r_{543}, r_{564}\right\}$	ϕ
s_{20054}	$\{\boldsymbol{r_{77}}, \boldsymbol{r_{140}}, r_{147}, r_{148}, r_{662}\}$	$s_{30130}^{(1)}$	$\{r_{53}, r_{214}, r_{348}, r_{662}\}$	$\{r_{662}\}$
		$s_{30129}^{(2)}$	$\{r_{54}, r_{249}\}$	ϕ
		$s_{30149}^{(3)}$	$\left\{r_{59}, r_{543}, r_{564}\right\}$	ϕ
s_{10492}	$\{r_{77}, r_{140}, r_{147}, r_{148}\}$	$s_{30130}^{(1)}$	$\{r_{53}, r_{214}, r_{348}, r_{662}\}$	ϕ
		$s_{30129}^{(2)}$	$\{r_{54}, r_{249}\}$	ϕ
		$s_{30149}^{(3)}$	$\left\{r_{59}, r_{543}, r_{564}\right\}$	ϕ
s_{20580}	$\{r_{147}\}$	$s_{30130}^{(1)}$	$\{r_{53}, r_{214}, r_{348}, r_{662}\}$	ϕ
		$s_{30129}^{(2)}$	$\{r_{54}, r_{249}\}$	ϕ
		$s_{30149}^{(3)}$	$\left\{r_{59}, r_{543}, r_{564}\right\}$	ϕ
s_{20042}	$\{r_{148}\}$	$s_{30130}^{(1)}$	$\{r_{53}, r_{214}, r_{348}, r_{662}\}$	ϕ
		$s_{30129}^{(2)}$	$\{r_{54}, r_{249}\}$	ϕ
		$s_{30149}^{(3)}$	$\left\{r_{59}, r_{543}, r_{564}\right\}$	ϕ
s_{20238}	$\{r_{361}\}$	$s_{30130}^{(1)}$	$\{r_{53}, r_{214}, r_{348}, r_{662}\}$	ϕ
		$s_{30129}^{(2)}$	$\{r_{54}, r_{249}\}$	ϕ
		$s_{30149}^{(3)}$	$\left\{r_{59}, r_{543}, r_{564}\right\}$	ϕ
s_{20166}	$\{r_{361}, r_{200}, r_{543}, r_{777}\}$	$s_{30130}^{(1)}$	$\{r_{53}, r_{214}, r_{348}, r_{662}\}$	ϕ
		$s_{30129}^{(2)}$	$\{r_{54}, r_{249}\}$	ϕ
		$s_{30149}^{(3)}$	$\left\{r_{59}, r_{543}, r_{564}\right\}$	$\{r_{543}\}$
s_{20200}	$\{r_{361}, \boldsymbol{r_{200}}, \boldsymbol{r_{543}}, \boldsymbol{r_{777}}\}$	$s_{30130}^{(1)}$	$\{r_{53}, r_{214}, r_{348}, r_{662}\}$	ϕ
		$s_{30129}^{(2)}$	$\{r_{54}, r_{249}\}$	ϕ
		$s_{30149}^{(3)}$	$\left\{r_{59}, r_{543}, r_{564}\right\}$	$\{r_{543}\}$

ignored in Table 5, since they are the initial stops on the route, but not the transfer stops.

Lines passing from fuzzy stop γ-neighbors are marked as bold in Table 5 and walking is necessary to reach s_D from s_C before getting on these lines from s_D in transfer case. As seen in Table 5, following comments can be made:

- After getting off at the stop s_{20056}, there can be a transfer to the line r_{662} from the same stop without walking.
- After getting off at the stop s_{20054}, there can be a transfer to the line r_{662} by walking to the stop s_{20056}.
- After getting off at the stop s_{20200}, there can be a transfer to the line r_{543} by walking to the stop s_{20166}.
- After getting off at the stop s_{20166}, there can be a transfer to the line r_{543} from the same stop without walking.

The routes in Table 6 are obtained by combining the results given in Tables 4 and 5.

It is finally required to compute the total number of the stops on these routes $|\pi_{s_O^{(i)} s_D^{(j)}}|$ for each $\pi_{s_O^{(i)} s_D^{(j)}}$. As it was previously mentioned, only representative stops were taken into consideration instead of all the stops due to the difficulty in representing all of the stops (Table 7). However, the total number of stops is calculated by using actual data with real values of stops' activity.

Alternative routes will be presented to the user in ascending order w.r.t. $|\pi_{s_O s_D}|$ values given below:

$$\Pi_{OD} = \left\{ \pi_{s_O s_D}^{(1)}, \pi_{s_O s_D}^{(2)}, \pi_{s_O s_D}^{(3)} \right\},$$

where

$$\pi_{s_O s_D}^{(1)} = \left(s_{20066}^{(1)}, r_{77}, s_{20056} \right), \left(s_{20054}, r_{662}, s_{30130}^{(1)} \right), \text{with total 8 stops;}$$

$$\pi_{s_O s_D}^{(2)} = \left(s_{20243}^{(2)}, r_{361}, s_{20200} \right), \left(s_{20166}, r_{543}, s_{30149}^{(1)} \right), \text{with total 11 stops;}$$

$$\pi_{s_O s_D}^{(3)} = \left(s_{20066}^{(1)}, r_{140}, s_{20054} \right), \left(s_{20054}, r_{662}, s_{30130}^{(1)} \right), \text{with total 14 stops.}$$

6. Conclusion

One of the basic concepts used in the literature concerning route planning is the walking distance of a person. Walking distance is known to be an important criterion in trip planning problem, but in real life situations it is not sufficient to determine which stop is the most suitable one. In this paper, we introduce two new criteria such as stop activity and being hub stop besides this criterion,

Table 6
Solutions to the problem with alternate routes.

$s_O^{(i)}$	r_{p_1}	s_{c_1}	s_{c_2}	r_{p_2}	$s_D^{(j)}$	$\pi_{s_O^{(i)} s_D^{(j)}}$
$s_{20066}^{(1)}$	r_{77}	s_{20056}	s_{20054}	r_{662}	$s_{30130}^{(1)}$	$\langle \left(s_{20066}^{(1)}, r_{77}, s_{20056}\right), \left(s_{20054}, r_{662}, s_{30130}^{(1)}\right) \rangle$
	r_{140}	s_{20054}	s_{20054}	r_{662}	$s_{30130}^{(1)}$	$\langle \left(s_{20066}^{(1)}, r_{140}, s_{20054}\right), \left(s_{20054}, r_{662}, s_{30130}^{(1)}\right) \rangle$
$s_{20243}^{(2)}$	r_{361}	s_{20200}	s_{20166}	r_{543}	$s_{30149}^{(3)}$	$\langle \left(s_{20243}^{(2)}, r_{361}, s_{20200}\right), \left(s_{20166}, r_{543}, s_{30130}^{(1)}\right) \rangle$

Table 7
The total number of stops within the routes.

$s_O^{(i)}$	r_1	s_{c_1}	$\lvert \left(s_O^{(i)}, r_1, s_{c_1}\right) \rvert$	s_{c_2}	r_2	$s_D^{(j)}$	$\lvert \left(s_{c_2}, r_2, s_D^{(j)}\right) \rvert$	$\lvert \pi_{s_O^{(i)} s_D^{(j)}} \rvert$
$s_{20066}^{(1)}$	r_{77}	s_{20056}	4	s_{20054}	r_{662}	$s_{30130}^{(1)}$	4	8
$s_{20066}^{(1)}$	r_{140}	s_{20054}	6	s_{20054}	r_{662}	$s_{30130}^{(1)}$	8	14
$s_{20243}^{(2)}$	r_{361}	s_{20200}	5	s_{20166}	r_{543}	$s_{30149}^{(3)}$	6	11

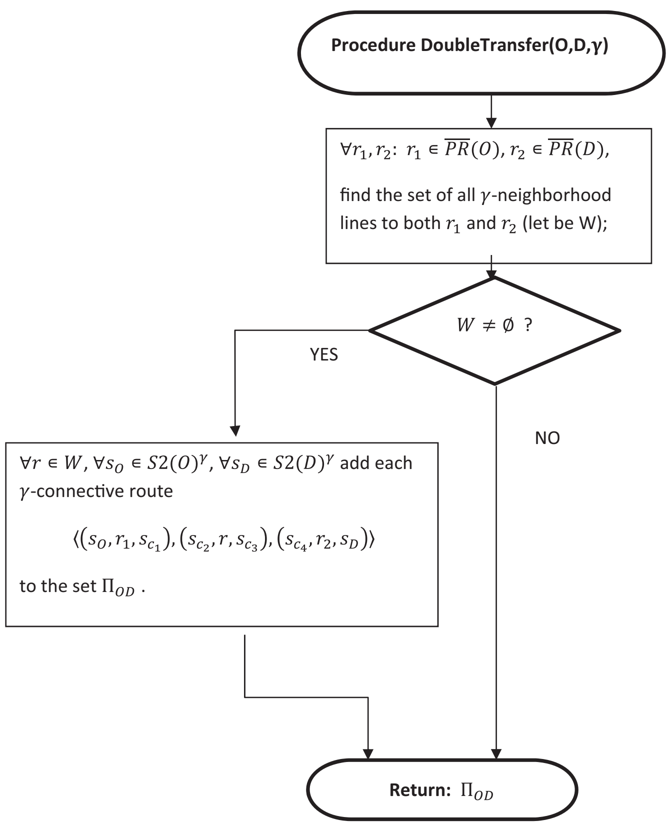

Fig. 7. Flowchart of the DoubleTransfer.

and they are evaluated together by using fuzzy logic, where these three criteria constitute the fuzzy preference degree of any stop. It should be noted that, in order to determine fuzzy preference degree of a stop in other real life applications, more criteria will also have to be incorporated in our problem. For instance, railway and ferry transportation might be more preferable than bus trips, since these transportation types and their routes are not directly affected by road traffic. Although people sometimes need to walk a long distance for reaching at a metro station or a pier, they might still prefer these transportation modes rather than walking to a bus station within a shorter distance. For this reason, the type of the stop/station such as bus stop, metro station, railway station and pier can be added as a new criterion for future studies. Other possible criteria regarding our problem might be considered as well, however, since we focus on demonstrating a fuzzy multi-criteria optimization approach in this study, this is left as another study topic for the future.

Finally, we can conclude that the fuzzy multi-criteria route planning model proposed in this study, which handles preference degrees of the stops, is a new approach to route planning problems based on fuzzy logic. The preference degrees are built on fuzzy neighborhood relations among stops, lines and between lines and stops. The proposed model can be employed to a search engine in optimal route planning applications for public transport users, to handle fuzzy human preferences more adequately.

Acknowledgements

This study is supported by the Scientific and Technological Research Council of Turkey (TUBITAK, grant No.113E535). Authors also are grateful to the anonymous referees for their valuable comments, which help to improve quality of the paper.

Appendix A.

<u>Algorithm (Solution of the Fuzzy Multi-Criteria Route Planning Problem):</u>

Input: O, D, γ;

Output: Π_{OD};

Step 0: It is computed $S2(O)^{\gamma}$ and $S2(D)^{\gamma}$ for O and D relatively. Then stops in $S2(O)^{\gamma}$ and $S2(D)^{\gamma}$ are sorted separately in descending order w.r.t preference degrees;

Set: $\Pi_{OD} = \emptyset$;

Step 1: *if* $(S2(O)^{\gamma} \cap S2(D)^{\gamma}) \neq \emptyset$ *then*

print "Destination is within walking distance"; **End;**

end if

Step 2: *DirectConnection*(O, D, γ);

Step 3: *if* $\Pi_{OD} = \emptyset$ *then SingleTransfer*(O, D, γ); *end if*

Step 4: *if* $\Pi_{OD} = \emptyset$ *then DoubleTransfer*(O, D, γ); *end if*

Step 5: *if* $\Pi_{OD} \neq \emptyset$ *then print* Π_{OD};

else

print "no solution";

end if

End.

Procedure DirectConnection(O, D, γ):

foreach $s_O \epsilon S2(O)^{\gamma}$

foreach $s_D \epsilon S2(D)^{\gamma}$

foreach r in $\{PR_{s_O} \cap PR_{s_D}\}$

if (s_O, r, s_D) is a possible connection *then*

$\Pi_{OD} = \Pi_{OD} \cup \langle (s_O, r, s_D) \rangle$;

end for r

end for s_D

end for s_O

return Π_{OD};

End;

Procedure SingleTransfer(O, D, γ):

foreach $s_O \epsilon S2(O)^\gamma$

foreach $s_D \epsilon S2(D)^\gamma$

foreach r_1 *in* PR_{s_O}

$S = \left\{\langle s_{c_1}, s_{c_2}\rangle : \left((s_O, r_1, s_{c_1})\ is\ possible\ connection\right) \wedge (s_{c_2} \in S2(s_{c_1})^\gamma)\right\}$;

foreach $\langle s_{c_1}, s_{c_2}\rangle$ *in* S // (s_{c_2} are sorted in descending order w.r.t preference degrees)

foreach r_2 *in* $(PR_{s_{c_2}} \cap PR_{s_D})\}$

if (s_{c_2}, r_2, s_D) is possible connection *then*

$\Pi_{OD} = \Pi_{OD} \cup \langle (s_O, r_1, s_{c_1}), (s_{c_2}, r_2, s_D)\rangle$;

end for r_2

end for $\langle s_{c_1}, s_{c_2}\rangle$

end for r_1

end for s_D

end for s_O

return Π_{OD};

End;

Procedure DoubleTransfer (O, D, γ):

foreach $s_O \epsilon S2(O)^\gamma$

foreach $s_D \epsilon S2(D)^\gamma$

foreach r_O *in* PR_{s_O}

$S_1 = \left\{\langle s_{c_1}, s_{c_2}\rangle : \left((s_O, r_O, s_{c_1})\ is\ possible\ connection\right) \wedge \left(s_{c_2} \in S2(s_{c_1})^\gamma\right)\right\}$

foreach r_D *in* PR_{s_D}

$S_2 = \left\{\langle s_{c_3}, s_{c_4}\rangle : \left((s_{c_4}, r_D, s_D)\ is\ possible\ connection\right) \wedge \left(s_{c_3} \in S2(s_{c_4})^\gamma\right)\right\}$

foreach r *in* R

// (s_{c_2} and s_{c_3} are sorted in descending order w.r.t. preference degrees)

if $\langle s_{c_1}, s_{c_2}\rangle \in S_1 \wedge \langle s_{c_3}, s_{c_4}\rangle \in S_2 \wedge (s_{c_2}, r, s_{c_3})$ is possible connection) *then*

$\Pi_{OD} = \Pi_{OD} \cup \langle (s_O, r_O, s_{c_1}), (s_{c_2}, r, s_{c_3}), (s_{c_4}, r_D, s_D)\rangle$

end for r

end for r_D

end for r_O

end for s_D

end for s_O

return Π_{OD};

End;

References

[1] J.-W. Grotenhuis, B.W. Wiegmans, P. Rietveld, The desired quality of integrated multimodal travel information in public transport: customer needs for time and effort savings, Trans. Policy 14 (1) (2007) 27–38.
[2] S. Ghosh, T.S. Lee, Intelligent Transportation Systems: Smart and Green Infrastructure Design, 2nd ed., CRC Press, 2010.
[3] J.S. Sussman, Perspectives on Intelligent Transportation Systems, Springer, 2005.
[4] A. Kramers, Designing next generation multimodal traveler information systems to support sustainability-oriented decisions, Environ. Modell. Softw. 56 (2014) 83–93.
[5] C.G. Chorus, E.J.E. Molin, W.B. Van, Use and effects of advanced traveller information services (ATIS): a review of the literature, Trans. Rev.: Trans. Transdiscip. J. 26 (2) (2006) 127–149.
[6] G.B. Dantzig, Linear Programming and Extensions, Princeton University Press, 1962.
[7] S. Derrible, C. Kennedy, Applications of graph theory and network science to transit network design, Trans. Rev.: Transnatl. Transdiscip. J. 31 (4) (2011) 495–519.
[8] C. von Ferber, T. Holovatch, Y. Holovatch, Modeling metropolis public transport, in: C. Appert-Rolland, F. Chevoir, P. Gondret, S. Lassarre, J.-P. Lebacque, M. Schrecenberg (Eds.), Traffic and Granular Flow '07, 2009, pp. 709–719.
[9] C. von Ferber, T. Holovatch, Y. Holovatch, V. Palchykov, Public transport networks: empirical analysis and modeling, Eur. Phys. J. B 8 (2) (2009) 261–275.
[10] D. Delling, M. Holzer, M. Kirill, F. Schulz, D. Wagner, High-Performance multi-Level graphs, 9TH DIMACS Implementation Challenge (2006) 52–65.
[11] D. Delling, T. Pajor, D. Wagner, Accelerating multi-modal route planning by access-Nodes, in: A. Fiat, P. Sanders (Eds.), Proc. Algorithms (Lecture Notes in Computer Science)—ESA 2009 (17th Annual European Symposium, Copenhagen, Denmark) (2009) 587–598.
[12] R. Geisberger, P. Sanders, D. Schultes, D. Delling, Engineering route planning algorithms, in: J. Lerner, D. Wagner, K.A. Zweig (Eds.), Algorithmics of Large and Complex Networks, Springer Berlin Heidelberg, 2008, pp. 117–139.
[13] M. Holzer, F. Schulz, D. Wagner, Engineering multilevel overlay graphs for shortest-path queries, J. Exp. Algorithm. 13 (2009).
[14] M. Thorup, Compact oracles for reachability and approximate distances in planar digraphs, J. ACM 51 (6) (2004) 993–1024.
[15] H. Bast, E. Carlsson, A. Eigenwillig, R. Geisberger, C. Harrelson, V. Raychev, F. Viger, Fast routing in very large public transportation networks using transfer patterns, in: M. Berg, U. Meyer (Eds.), Proc. ESA'10 Proceedings of the 18th Annual European Conference on Algorithms: Part I (2010) 290–301.
[16] H. Goldberg, Reach for A*: efficient point-to-Point shortest path algorithms, Workshop On Algorithm Engineering & Experiments (2006) 129–143.
[17] R. Gutman, Reach-Based routing: a new approach to shortest path algorithms optimized for road networks, Workshop on Algorithm Engineering and Experiments (ALENEX) (2004) 100–111.
[18] C. Xie, S.T. Waller, Parametric search and problem decomposition for approximating Pareto-optimal paths, Transp. Res. B: Methodol. 46 (8) (2012) 1043–1067.
[19] A. Lozano, G. Storchi, Shortest viable path algorithm in multimodal networks, Transp. Res. A: Policy Pract. 35 (3) (2001) 225–241.
[20] E. Pyrga, F. Schulz, D. Wagner, C. Zaroliagis, Efficient models for timetable information in public transportation systems, J. Exp. Algorithm. (2008) 12.
[21] D. Delling, T. Pajor, R.F. Werneck, Round-based public transit routing, Proc. 14th Meeting on Algorithm Engineering and Experiments (ALENEX'12) (2012) 130–140.
[22] M. Niksirat, M. Ghatee, S.M. Hashemi, Multimodal K-shortest viable path problem in Tehran public transportation network and its solution applying ant colony and simulated annealing algorithms, Appl. Math. Modell. 36 (11) (2012) 5709–5726.
[23] W. Xu, S. He, R. Song, S.S. Chaudhry, Finding the K shortest paths in a schedule-based transit network, Comput. Operat. Res. 39 (8) (2012) 1812–1826.
[24] L. Zhao, Y. Xiong, H. Sun, The K shortest transit paths choosing algorithm in stochastic transit network, in: G. Wang, T. Li, J.W. Grzymala-Busse, D. Miao, A. Skowron, Y. Yao (Eds.), Rough Sets and Knowledge Technology, Springer Berlin Heidelberg, 2008, pp. 747–754.
[25] J. Koszelew, An evolutionary algorithm for the urban public transportation, in: P. Jędrzejowicz, N.T. Nguyen, K. Hoang (Eds.), Computational Collective Intelligence Technologies and Applications, Lecture Notes in Computer Science, Springer Berlin Heidelberg, Berlin, 2011, pp. 234–243.
[26] A.J.S. Kumar, J.M.V. Arunadevi, Intelligent transport route planning using genetic algorithms in path computation algorithms, Eur. J. Sci. Res. 25 (3) (2009) 463–468.
[27] H. Chang, Y. Tai, J.Y. Hsu, Public transportation routing using route graph, Proc. 12th Conference on Artificial Intelligence & Applications (TAAI 2007) (2007).
[28] C. Liu, Best-path planning for public transportation systems, in: Proc. The IEEE 5th International Conference on Intelligent Transportation Systems, IEEE, 2002, pp. 834–839.
[29] C. Liu, T. Pai, C. Chang, C. Hsieh, Path-planning algorithms for public transportation systems, Proc. IEEE Intelligent Transportation Systems (ITSC 2001) (2001) 1061–1066.
[30] A. Golnarkar, A.A. Alesheikh, M.R. Malek, Solving best path problem on multimodal transportation networks with fuzzy costs, Iran. J. Fuzzy Syst. 7 (3) (2010) 1–13.
[31] J. Verga, R.C. Silva, A. Yamakami, W.V.I. Shirabayashi, Using the fuzzy sets theory in the multimodal transport network problem, Proc. IFSA World Congress and NAFIPS Annual Meeting (IFSA/NAFIPS) (2013) 801–806.
[32] M. Ghatee, H.S. Mehdi, M. Zarepisheh, E. Khorram, Preemptive priority-based algorithms for fuzzy minimal cost flow problem: an application in hazardous materials transportation, Comput. Ind. Eng. 57 (1) (2009) 341–354.
[33] E.N. Nasibov, G. Ulutagay, A new unsupervised approach for fuzzy clustering, Fuzzy Sets Syst. 158 (19) (2007) 2118–2133.
[34] E.N. Nasibov, G. Ulutagay, Robustness of density-based clustering methods with various neighborhood relations, Fuzzy Sets Syst. 160 (24) (2009) 3601–3615.
[35] E.N. Nasibov, A robust algorithm for solution of the fuzzy clustering problem on the basis of the fuzzy joint points method, Cybernet. Syst. Anal. 44 (1) (2008) 7–17.
[36] R.W. Sinnott, Virtues of the haversine, Sky Telesc. 68 (2) (1984) 158.
[37] E. Nasiboglu, U. Kuvvetli, M. Ozkilcik, U. Eliiyi, Origin-destination matrix generation using smart card data: case study for Izmir, Proc. 2012 IV International Conference Problems of Cybernetics and Informatics (PCI 2012) (2012) 1–4.
[38] E. Nasibov, Aggregation of fuzzy values in linear programming problems, Autom. Control Comput. Sci. 37 (2) (2003) 1–11.
[39] R.A. Nasibova, E.N. Nasibov, Linear aggregation with weighted ranking, Autom. Control Comput. Sci. 44 (2) (2010) 96–102.