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Product market segmentation and output collusion within substitute products

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ABSTRACT

We extend the differentiated product model, first developed by Bowley (1924), by relaxing the assumption that each firm produces only one differentiated product. By doing so, we are able to analyze the potential for collusive market segmentation in a two-stage decision framework, first in product space and second in output. We find that when firms cannot coordinate on output, the required discount factor that supports collusive market segmentation is strictly decreasing in product substitutability and is greater than partial output and full collusion. Overall we find that output collusion alone is easier to sustain than collusive product market segmentation.

1. Introduction

Product differentiation and market segmentation have long been recognized as important strategic choices by firms (Smith, 1956). Firms may strategically differentiate their product(s) by brand and/or quality attributes to uniquely position their product(s) with consumers. Market segmentation is the strategy of choosing which products to produce from a finite set of existing or potential product(s).

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United States and European courts and antitrust authorities have long recognized the potential reduction in competition from tacit or overt horizontal agreements to allocate consumers, products and/or geographic territories (Belleflamme & Bloch, 2004; Sullivan & Grimes, 2000). Welfare reducing collusion in regards to product space entails an agreement that 'you produce product A while I produce product B' when independent and competitive decisions would dictate both firms produce both products.

The ability of firms to tacitly collude in restricting output or raising prices in repeated games is significantly impacted by the differentiability of the firms' product(s) (e.g. Chang, 1991; Häckner, 1994; Ross, 1992; Singh & Vives, 1984). To date, the differentiated products literature has largely focused on collusive output/pricing decisions rather than also addressing collusive multiproduct (conglomerate) decisions. There is a limited literature that addresses the collusive potential among multiproduct (conglomerate) firms, each tackling the problem from different directions (i.e. Bernheim & Winston, 1990; Symeonidis, 2002).[2] An even smaller amount of literature has considered firm decisions as a two-stage game, first in product space and second in price (Dobson & Waterson, 1996; Shaked & Sutton, 1990) or second in quantity (Fraja, 1992), but this line of literature has not addressed collusion at either stage.

The objective of our research is to analyze the potential for tacit collusion at both the product choice and quantity decision(s) stages. In the first stage, firms make strategic decisions over which of the available differentiated product(s) they will produce and in the second stage make their respective output decisions. To accomplish our objective, we first extend a commonly used differentiated products model developed by Bowley (1924) by relaxing the long running assumption that a finite set of differentiated products are uniquely produced by each firm (i.e. Dixit, 1979; Häckner, 2000; Singh & Vives, 1984; Symeonidis, 2002). By doing so, the market segmentation decisions of firms can be endogenized and allow firms to produce perfect overlapping products.

We further consider instances in which the firms are able to only partially or fully collude across both decision stages. When firms are unable to collude across both product space and output, we find the dominant strategy between symmetric firms during product selection, regardless of product substitutability, is for both firms to conglomerate and produce multiple products in contrast to the findings of earlier work by Shaked and Sutton (1990). Furthermore, the required discount factor that supports collusive market segmentation is strictly decreasing in product substitutability.

Interestingly, we find under partial collusion that the required minimum discount factor that supports output collusion alone given ex ante non-cooperative multiproduct (conglomeration) is strictly less than that required for collusive market segmentation alone. Additionally, the required minimum discount factor is constant; a result contrary to both horizontally and vertically differentiated product modeling thus far. We also find the required minimum discount factor that supports output collusion under non-cooperative market segmentation is monotonically increasing; a result that is consistent with price collusion in Chang's (1991) and Ross's (1992) horizontally differentiated product models, as well as the Cournot setting of Deneckere (1983). However, this result is in contrast to a Bertrand setting where Deneckere (1983) and Häckner (1994) found a non-monotonic and monotonically decreasing result, respectively.

Finally, when firms are able to consider full collusion across both decision stages, we find that the required minimum discount factor that supports both collusive market segmentation and output is monotonically decreasing as products become closer substitutes. The minimum required discount factor in this setting is less than that required for collusive market segmentation alone but greater than that required for output collusion alone. Therefore, if firms are found to collusively segment the market, output collusion is a logical progression of the firms' decision making.

The remainder of the paper is organized as follows: Section 2 provides a review of the literature. Section 3 describes the economic model, the solutions of Nash and sub-game perfect equilibria. Finally, in Section 4 we present our conclusions.

[2] Bernheim and Winston (1990) analyze the incentive constraints of collusive pricing strategies of firms experiencing multimarket contact holding product-firm space, product differentiation and geographic locations constant. Symeonidis (2002) analyzes the impact of exogenous changes in the number of firms, number of products produced by both firms and product substitutability on the likelihood of collusion via comparative statics.

2. Literature

Though the product differentiation literature is voluminous, we discuss only that which we believe is most relevant to our analysis. There are several popular linear models of aggregate demand for horizontally differentiated products. An intensively used model was developed by Hotelling (1929). It is often assumed that there are two firms, each firm produces only one product at constant marginal cost, demand is inelastic, and the differentiated commodity is uniformly distributed.

Using this spatial competition framework with respect to pricing strategies, Chang (1991) examined the relationship between the degree of product substitutability and the required discount factor to sustain collusion. Chang (1991) concluded the relationship is monotonically increasing; as products become closer substitutes, the required discount factor which sustains price collusion increases.

Based on a vertical differentiation model, Häckner (1994) contradicts the results of Chang (1991) and found a negative relationship between the degree of product differentiation and sustainability of price collusion. The argument is that when products are remote substitutes, the firm producing the high quality product is quite well off even without collusion and is less likely to collude. Therefore, the discount factor required to support collusion is relatively small since the low quality firm would agree to collude at a low discount factor. For the horizontally differentiated products, the author argued the opposite result can be obtained and concluded that theory cannot predict when products contain both attributes.

Häckner (1995) extends Chang's (1991) paper by considering the endogenous choice of Bertrand duopoly competitors in selecting horizontally differentiated products as a means to facilitate collusion in an infinitely repeated game. The author finds that the firms' ability to maintain collusive pricing increases by choosing an optimal degree of differentiation given varying required discount factors. For sufficiently high discount factors, firms chose intermediate degree of product substitutability. For sufficiently low discount factors, firms increase the differentiation of their products to maintain collusion.

The Bowley (1924) model has been a popular linear aggregate demand model, where the utility function is assumed to be quadratic and strictly concave (i.e. Häckner, 2000; Mukherjee, 2005; Singh & Vives, 1984; Symeonidis, 2002). Singh and Vives (1984) analyze the duality of Bertrand and Cournot competition in a differentiated symmetric duopoly. The authors concluded that if firms can precommit to either Cournot or Bertrand competition, the dominant strategy is for firms to choose Cournot when products are substitute goods and Bertrand for complements. However, Bertrand competition yielded higher total welfare in equilibrium, regardless of whether the goods are substitutes or complements.

Häckner (2000) extended Singh and Vives (1984) to include $n > 2$ heterogeneous firms. The heterogeneity is in regards to vertical product differentiation on quality and substitutability/complementarity of products. The author found that the dichotomy between Bertrand and Cournot competition is sensitive to the duopoly assumption and the results of Singh and Vives (1984) cannot be generalized to the n-firm specification if quality differences between firms are large and products are complements.

More recently, Mukherjee (2005) compared both Cournot and Bertrand competition over substitute goods when there is free entry. The author demonstrates that welfare is higher under Cournot competition for sufficiently differentiated products, but it is higher yet under Bertrand if the products are close substitutes, a result on par with Singh and Vives (1984). The reason being is that increases in Cournot competition result in a larger market size which overtakes the generally more competitive Bertrand competition.

An early infinitely repeated game 'grim' strategy collusion analysis by Deneckere (1983) found that the stability of collusion for differentiated substitute goods is monotonically decreasing with product homogeneity in Cournot setting but non-monotonic in Bertrand setting. Following the work of Singh and Vives (1984), Ross (1992) analyzed the relationship between vertical and horizontal differentiation and the sustainability of collusion between symmetric firms under Bertrand competition based on the quadratic utility model. Assuming firms' marginal costs are identical and set equal to zero, a non-monotonic relationship between the degree of substitution and collusion stability can obtain, a result consistent with that of Deneckere (1983) under price competition. The reason being is greater homogeneity/heterogeneity can reduce cartel stability by increasing the incentive to defect, but when the products are moderate substitute's collusion becomes easier. Under horizontal differentiation, however, Ross (1992) found the stability of price collusion monotonically decreases with product homogeneity.

Symeonidis (2002) analyzes cartel stability in an infinitely repeated game among multiproduct firms in a horizontally differentiated market under both Cournot and Bertrand competition. In the model, it is assumed that the number of firms in the market and the number of varieties that each firm produces are both exogenous. Via comparative statics, the author generally finds that an increase in the number of varieties produced by each firm makes collusion more difficult to sustain, except for the case where the number of firms is small and the products are close substitutes.

Shaked and Sutton (1990) study a two-stage game where Bertrand duopolists consider product expansion within firm, the competition effects from the expansion and entry deterrence in horizontally (Hotelling) and vertically differentiated (Bowley) products. The authors generally find in a simultaneous game as two products become closer substitutes, the value of product expansion decreases in relation to the increase in competition for more similar products. The analysis demonstrates a multitude of potential Nash equilibria, but none include the firms producing both products.

Fraja (1992) extends Shaked and Sutton's (1990) work by explicitly considering economies of scope and its impacts on which of two products are produced by two firms. Furthermore, the author utilizes a variant of the Bowley model that allows for changing intercepts and slopes of the respective demands as a function of the quantities produced of each product. The author finds that Cournot competition in the second stage results in a full range of product combination (market structure) possibilities. However, Bertrand competition results in only a pure monopoly of both products or a pure monopoly by each firm over each product. The main finding is that a decrease in economies of scope and an increase in product substitutability induce a shortening of the product line.

Dobson and Waterson (1996) expand the vertical differentiated Bowley demand model of Shaked and Sutton (1990) and Fraja (1992) to include consumer's intrinsic value of consumption from a specific firm, entry costs, economies of scope, as well as Cournot competitors. They generally find that the added dimensions result in an array of Nash equilibria in a simultaneous product choice game that depend on the parameters of the model. Among other results, they find that for intense intra-product rivalry (close substitutes) firms produce only one of the two products and for close complements firms produce both products. They suggest that when two firms optimally produce both products, collusion is beneficial. Asymmetric duopoly equilibria (one firm produces both products while the rival produces one) are not observed under a symmetric cost assumption. Economies of scope increase the incentive (likelihood) of observing both firms producing both products.

Doraszelski and Gaganska (2006) study the determinants of firms' market segmentation strategies under spatial competition model assuming firms compete in prices. A two-stage game is considered. In the first stage, firms simultaneous decide on their produce offerings; in the second stage, price competition takes place. In this framework, they considered both the utility increases for some consumers (due to increased fit) and the utility decreases for others (due to increased misfit) as a result of the firms' offering a targeted product instead of general purpose products. Their results suggest that in addition to the degree of fit and misfit, the intensity of competition and the fixed cost of offering an additional product determine firms' market segmentation strategies.

3. Economic model

We first identify two stages of firm decision making. In the first stage, two firms consider which of two differentiated products to produce in the market, A and/or B. In stage two, firms consider their output choices for each product produced.

As noted in Bernheim and Winston (1990), the definition of the markets may be identified by product and/or geographic delineations. For simplicity, we describe the markets unidimensionally as products. Our modeling framework is most applicable for analyzing a finite set of substitutable commodities, such as meats or sweeteners. For example, consumers view differentiated products such as beef, pork, chicken and fish as imperfect substitutes of a larger class of meat protein (Kinnucan, Xiao, Hsia, & Jackson, 1997).

We utilize the quadratic consumer utility function developed by Bowley (1924). One of the strengths of the Bowley is that we need not distinguish between horizontal and vertical differentiation, since the model can be used in both situations (Martin, 2002). One weakness of the model, however, is that the addition of products necessarily increases the market size, a phenomena not always present in real markets. However, when the product mix is fixed and has long been supplied by at least one seller, as in commodities, the weakness is not as apparent.

In this section, we first provide three scenarios resulting in partial collusion at only the first stage product space or only at the second stage output decision. A potential cause of partial collusion is that though firms are long lived, different levels of management and their goals may change over time. The final scenario we consider is that of complete collusion from backward induction across both decision stages within a trading period. We finish this section with a comparison and discussion of the required minimum discount factors across product substitutability to maintain each type of collusion, partial and complete.

3.1. Scenario 1 – partial collusion, product space only

The first scenario we consider is when firm product line decisions are made by long lived upper management who delegate output decisions to lower level management with short time horizons. In such a case, upper management of each firm considers product space collusion, each believing their rival's lower level management will competitively choose output. In this scenario, firms only consider taking three courses of actions: produce product A only, produce B only, or produce both products A and B. Rivals are assumed able to perfectly observe the product choices of rival firms at no cost.

To begin, the Bowley quadratic and strictly concave utility function of the representative consumer for the two primary products is $U(q_A, q_B) = a(q_A + q_B) - (1/2)b(q_A^2 + 2\theta q_A q_B + q_B^2) + m$. The quantities demanded of the two differentiated products in the market are q_A and q_B, m represents all other goods with price normalized to 1, and θ represents the degree of substitution of the two products. We consider only substitutes goods as we rely on the findings of Singh and Vives (1984) and others that the dominant strategy for symmetric quality duopoly is to choose quantity when products are substitutes, as such, $0 \leq \theta \leq 1$. As $\theta \to 0$, the two products are increasingly independent indicating multiproduct firms in this spectrum of products are more akin to conglomerates. As $\theta \to 1$, the two products become closer substitutes indicating the multiproduct firms experience increasing intra-firm (or intra-product) rivalry. Finally, although the utility function can be generalized by allowing the positive parameters a and b to vary across products, for clarity we do not pursue the generalization.

From the quadratic utility, the corresponding inverse demand functions for each product are $p_A = a - b(q_A + \theta q_B)$ and $p_B = a - b(\theta q_A + q_B)$. A basic assumption of the model is that firms are unable to identify consumer groups per se, thus reducing the strategic choices of the firms to product space. We extend Bowley's (1924) model by assuming each firm has the option of producing any combination of the products within the available product space. That is to say, each firm can produce product A, product B, or both products, and the quantity produced of each product is based on the resulting competition and substitutability of the products. The aggregate quantities of A and B produced are $q_A = q_{A,1} + q_{A,2} > 0$ and $q_B = q_{B,1} + q_{B,2} > 0$, where $q_{A,i}$ and $q_{B,i}$ are the quantities of product A and B produced by firm $i = 1,2$. The prices paid by consumers for products A and B are p_A and p_B.

We assume entry costs and economies of scope are symmetric across firms and products, and without a loss in generality we set them equal to 0. As will become apparent, our model does not require significant economies of scope to result in competitive multiproduct (conglomerate) market structure as was required in Fraja (1992) and Dobson and Waterson (1996). Variable costs are generated from fixed proportion technology for products A and B, are constant and for simplicity assumed symmetric. Therefore, $c_A = c_B = c$, where the costs of production $c < a$ for positive production. Admittedly, it cannot generally be assumed that production costs of differentiated products are symmetric and leave this as a future extension.

We consider four general cases which map to the nine simultaneous game payoff profiles summarized in Table 1. The payoffs depicted in Table 1 are static product choice conditional on firms

Table 1
First stage product choice conditional on Cournot output competition.

Firm 1	Firm 2		
	A	B	A&B
A	$\frac{(a-c)^2}{9b}, \frac{(a-c)^2}{9b}$	$\frac{(a-c)^2}{b(2+\theta)^2}, \frac{(a-c)^2}{b(2+\theta)^2}$	$\frac{(a-c)^2}{9b}, \frac{(13-5\theta)(a-c)^2}{36b(1+\theta)}$
B	$\frac{(a-c)^2}{b(2+\theta)^2}, \frac{(a-c)^2}{b(2+\theta)^2}$	$\frac{(a-c)^2}{9b}, \frac{(a-c)^2}{9b}$	$\frac{(a-c)^2}{9b}, \frac{(13-5\theta)(a-c)^2}{36b(1+\theta)}$
A&B	$\frac{(13-5\theta)(a-c)^2}{36b(1+\theta)}, \frac{(a-c)^2}{9b}$	$\frac{(13-5\theta)(a-c)^2}{36b(1+\theta)}, \frac{(a-c)^2}{9b}$	$\frac{2(a-c)^2}{9b(1+\theta)}, \frac{2(a-c)^2}{9b(1+\theta)}$

symmetrically choosing Cournot output into the unforeseeable future. Detailed solutions to the payoffs depicted in Table 1 and Nash equilibrium are provided in Appendix 1.

In the first case, let each firm consider producing only product A. When both firms produce only A $q_B = 0$, the game degenerates to a classic Cournot duopoly where the representative firm's objective function is $\underset{q_{A,i}|q_{A,2}}{\text{Max}}\ \pi_{A,j} = [a - b(q_{A,i} + q_{A,j}) - c]q_{A,i},\ i \neq j$. The resulting payoffs are depicted in the top left and middle center cells of Table 1.

In the second case, each firm considers producing each product separately, thus the game degenerates to the cases analyzed in past research that uses a Bowley model. Let Firm 1 consider producing only A and firm 2 producing only B. In this case, the objective functions for the two firms are $\underset{q_{A,1}|q_{B,2}}{\text{Max}}\ \pi_{A,1} = [a - b(q_{A,1} + \theta q_{B,2}) - c]q_{A,1}$ and $\underset{q_{B,2}|q_{A,1}}{\text{Max}}\ \pi_{B,2} = [a - b(\theta q_{A,1} + q_{B,2}) - c]q_{B,2}$, where each firm's payoffs are depicted in the middle left and middle top cells of Table 1.

In the third case, we begin extending the assumption that firms are uniquely identified by their product. Let Firm 1 consider producing both A and B, while firm 2 considers producing only A. The firms objective functions are now $\underset{q_{A,1},q_{B,1}|q_{A,2}}{\text{Max}}\ \pi_{A\&B,1} = [a - b(q_{A,1} + q_{A,2} + \theta q_{B,1}) - c]q_{A,1} + [a - b(\theta(q_{A,1} + q_{A,2}) + q_{B,1}) - c]q_{B,1}$ and $\underset{q_{A,2}|q_{A,1},q_{B,1}}{\text{Max}}\ \pi_{A,2} = [a - b(q_{A,1} + q_{A,2} + \theta q_{B,1}) - c]q_{A,2}$ resulting in the respective firm payoffs depicted in the bottom left, top right, middle left and right cells of Table 1.

In the fourth case, let each firm consider producing both products. The representative firm's objective function $i = 1,2$ is therefore $\underset{q_{A,i},q_{B,i}|q_{A,j},q_{B,j}}{\text{Max}}\ \pi_{A\&B,i} = [a - b(q_{A,i} + q_{A,j} + \theta(q_{B,i} + q_{B,j})) - c]q_{A,i} + [a - b(\theta(q_{A,i} + q_{A,j}) + q_{B,i} + q_{B,j}) - c]q_{B,i},\ i \neq j$, where each firm's total payoffs are depicted in the bottom right cell of Table 1.

Given the matrix of Cournot outcomes depicted in Table 1, we prove in Appendix 1 that producing both products is a unilateral strictly dominant strategy. Therefore, the unique pure strategy Nash equilibrium {A&B | Cournot, A&B | Cournot}. Interestingly, these results hold for multiproduct firm structure producing close substitutes or, as classically defined, conglomeration across weak and independent products.

We now solve for the conditions for upper management to maintain tacitly collusive market segmentation as a pure strategy subgame perfect Nash equilibria (SPNE), conditional on both firms maintaining Cournot output. We assume in an infinitely repeated game firms follow the standard 'grim' or 'trigger' strategy. Applicable to our setting, such a strategy dictates that each firm chooses to abstain from producing the same product as their rival in any given trading period. If either firm enters their product market in one period, the other firm punishes by permanently entering their rival's market.

For simplicity we will denote π^c as the collusive market segmentation payoff, π^d as the defection payoff and π^N is the Nash equilibrium payoff. Under a 'grim' strategy the following condition must be satisfied for sustainable tacitly collusive market segmentation (MS) is $\delta \geq \delta^{MS} = ((\pi^d - \pi^c)/(\pi^d - \pi^N))$, where δ and δ^{MS} is the required and minimum discount factor to maintain tacitly collusive market segmentation. Because of the symmetric outcomes of the model we ignore firm identifiers. By substituting the corresponding payoffs from Table 1 into the collusion condition we have $\delta \geq \delta^{MS} = ((5\theta^2 + 12\theta + 16)/(5(2+\theta)^2))$. Notice that the minimum discount factor is a function of the substitutability parameter θ and, due to firm and product symmetries, is independent of

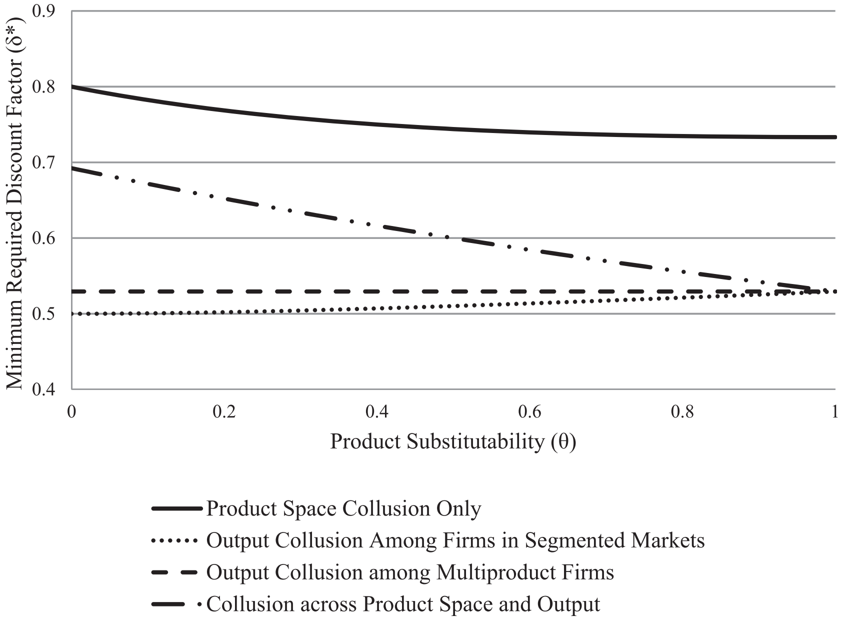

Fig. 1. Comparison of minimum required discount factor to maintain collusion as a function of product substitutability.

cost and the slope (elasticity) of demand. Because there are two possible market segmentations, {A, B} and {B, A}, we have the two pure strategy subgame perfect Nash equilibria $\text{SPNE}_1 = \{\text{A}, \text{B}\,;\, \delta \geq ((5\theta^2 + 12\theta + 16)/(5(2+\theta)^2))\}$ and $\text{SPNE}_2 = \{\text{B}, \text{A}\,;\, \delta \geq ((5\theta^2 + 12\theta + 16)/(5(2+\theta)^2))\}$. Given $0 \leq \theta < 1$ in the relevant game, δ^{MS} lies in the interval $\delta^{\text{MS}} \in (11/15, 4/5]$. These results are graphically depicted in Fig. 1.

Given there are two SPNE, coordination at the initial stage of market segmentation could realistically be a "noisy" process. For simplicity, we will assume firms' are able to coordinate by making ex ante public statements and firm 1 always chooses product A first. In pure strategies, $(\partial\delta^{\text{MS}}/\partial\theta) = ((8(\theta-1))/(5(2+\theta)^3)) < 0\ \forall\ 0 \leq \theta < 1$ and is equal to zero otherwise. This result indicates the minimum discount factor that supports market segmentation is monotonically decreasing as the degree of substitution of the two goods increases. That is to say, as the goods become closer substitutes, it is easier for the firms to collude and segment the market. Alternatively, as goods become increasingly independent, multiproduct production (conglomeration) is more likely.

Finally, it is quite apparent that each firm would like to produce A&B while the other produces only A or B. The payoff $(((13-5\theta)(a-c)^2)/(36b(1+\theta)))$ in the top right and bottom left cells of Table 1 are the highest possible payoff in the game. Therefore, if a first mover $i = 1, 2$ were able to create an entry cost $\varepsilon_i \geq \pi|\{\text{A\&B, B}\}_j - \pi|\{\text{A\&B, A\&B}\}_j\ i \neq j$, as in Fraja (1992) and Dobson and Waterson (1996), the first mover could preempt entry rendering collusive market segmentation a moot point. In such a game, other antitrust issues of how firms erect the barriers to entry become focal.

3.2. *Partial collusion, output only*

The second and third scenarios we consider are when making product line adjustments or entering new markets is a long process so that the primary focus of upper (and hence lower) management is on output competition. In such a case, firms/management may only consider output collusion. Though there are nine potential product combinations between the two firms, we will focus on the Nash (multiproduct production (conglomeration)) and collusive (market segmentation) equilibria to provide the first stage initial conditions for our analysis of output collusion.

Table 2
Second stage output choice conditional on prior market segmentation.

Firm 1 produces product A	Firm 2 produces product B	
	Monopoly output	Cournot output
Monopoly output	$\frac{(a-c)^2}{4b(1+\theta)}, \frac{(a-c)^2}{4b(1+\theta)}$	$\frac{(a-c)^2(2-\theta(\theta-2))}{8b(1+\theta)^2}, \frac{(a-c)^2(2+\theta)^2}{16b(1+\theta)^2}$
Cournot output	$\frac{(a-c)^2(2+\theta)^2}{16b(1+\theta)^2}, \frac{(a-c)^2(2-\theta(\theta-2))}{8b(1+\theta)^2}$	$\frac{(a-c)^2}{b(2+\theta)^2}, \frac{(a-c)^2}{b(2+\theta)^2}$

3.2.1. Scenario 2 – ex ante market segmentation

In scenario two, we assume firms are producing separate products with no plans of altering their product mix. This state may arise due to (i) an existing competitive advantage, (ii) a locational constraint, or (iii) the firm's ability to tacitly collude over product space prior to foreseeing their capabilities of output collusion. Assuming firms are producing only one product, the payoffs from the collusive equilibrium {A, B} from Table 1 is the starting point for second stage tacit output collusion. Table 2 summarizes the payoff matrices from output decisions. Detailed solutions to the payoffs depicted in Table 2 and Nash equilibrium are provided in Appendix 2.

Firms now consider their collusive joint profit maximization objective function

$\underset{q_{A,1},q_{B,2}}{\text{Max}} \ \pi_{A,1} + \pi_{B,2} = (a - b(q_{A,1} + \theta q_{B,2}) - c)q_{A,1} + (a - b(\theta q_{A,1} + q_{B,2}) - c)q_{B,2}$. This is the discriminating monopolist's objective function. The resulting collusive outcomes are provided in the upper left cell and the defection payoffs are provided in the upper left and lower right cells in Table 2. We subsequently prove in Appendix 2 that there is a unilateral incentive to deviate from the collusive output resulting in the unique pure strategy Nash equilibrium of {Cournot, Cournot}.

Given the relevant payoffs, the required minimum discount factor for output collusion under ex ante market segmentation (EMS) following a 'grim' strategy requires $\delta \geq \delta^{EMS} = ((2+\theta)^2/(8+\theta(8+\theta)))$. Therefore, the subgame perfect Nash equilibrium is $SPNE_{EMS} = \{q_{A,1} = ((a-c)/(2b(1+\theta))) = q_{B,2}\, ; \delta \geq ((2+\theta)^2/(8+\theta(8+\theta)))\}$. Given $0<\theta<1$, δ^{EMS} lies in the interval $\delta^{EMS} \in (1/2, 9/17)$. Comparing collusive market segmentation and non-cooperative output to non-cooperative market segmentation and output collusion we find the following relationship, $\delta^{EMS} < \delta^{MS}\ \forall\, 0<\theta<1$. Thus, collusion is easier to obtain when markets are initially segmented. These relationships are graphically depicted in Fig. 1.

In pure strategies, the first order condition, $(\partial\delta^{EMS}/\partial\theta) = ((4\theta(2+\theta))/(8+\theta(8+\theta))^2) > 0\ \forall 0<\theta\leq 1$ and zero otherwise, reveals increasing product substitutability monotonically increases the required discount factor to maintain collusion. This result is in stark contrast to our previous results regarding collusion in product space alone.

This scenario most closely aligns with past collusion literature. Our result using the Bowley model is consistent with price collusion in horizontally differentiated product models where consumer tastes are heterogeneous and demand is piecewise linear (Chang, 1991; Ross, 1992). Additionally, we find similar results under quantity collusion assuming a piecewise demand function (Deneckere, 1983). However, our result is in contrast to the *non-monotonic* relationship found when assuming price collusion and a quadratic utility function (Ross, 1992). Furthermore, our result is in contrast to Häckner (1994) who found that increasing product substitutability monotonically *decreases* the required discount factor to maintain collusion. An important reason for the difference between our result and Häckner (1994) lies in the fact that Häckner's model explicitly includes a continuous quality variable absent from the Bowley model. Häckner also considers cost asymmetries resulting from quality differentiation, while we assume cost is symmetric across products and firms.

3.2.2. Scenario 3 – ex ante multiproduct production (conglomerate)

In the third scenario, we assume that conglomerate firms are producing both products. This state may arise due to (i) a lack of a competitive advantage, (ii) no locational constraint, or (iii) falling prey to the competitive pressures to produce both products before foreseeing their capabilities of output collusion. When firms are producing both products, the payoffs from the Nash equilibrium {A&B,

Table 3
Second stage output choice conditional on prior competitive multiproduct (conglomeration).

Firm 1 produces A&B	Firm 2 produces A&B	
	Monopoly output	Cournot output
Monopoly output	$\frac{(a-c)^2}{4b(1+\theta)}, \frac{(a-c)^2}{4b(1+\theta)}$	$\frac{3(a-c)^2}{16b(1+\theta)}, \frac{9(a-c)^2}{32b(1+\theta)}$
Cournot output	$\frac{9(a-c)^2}{32b(1+\theta)}, \frac{3(a-c)^2}{16b(1+\theta)}$	$\frac{2(a-c)^2}{9b(1+\theta)}, \frac{2(a-c)^2}{9b(1+\theta)}$

A&B} in Table 1 is the starting point for output collusion. Table 3 summarizes the payoff matrices from output decisions. Detailed solutions to the payoffs depicted in Table 3 are provided in Appendix 3.

In this scenario, firms now consider their collusive joint profit maximization objective function across both products $\underset{q_{A,1},q_{B,1},q_{A,2},q_{B,2}}{\text{Max}} \pi_{A\&B,1} + \pi_{A\&B,2} = [a-c-b(q_{A,1}+q_{A,2}+\theta(q_{B,1}+q_{B,2}))]q_{A,1} + [a-c-b(q_{B,1}+q_{B,2}+\theta(q_{A,1}+q_{A,2}))]q_{B,1} + [a-c-b(q_{A,1}+q_{A,2}+\theta(q_{B,1}+q_{B,2}))]q_{A,2} + [a-c-b(q_{B,1}+q_{B,2}+\theta(q_{A,1}+q_{A,2}))]q_{B,2}$. The collusive payoffs are provided in the upper left cell and the defection payoffs are provided in the upper left and lower right cells in Table 2. We prove in Appendix 3 that there is a unilateral incentive to deviate from joint collusive output resulting in the unique pure strategy Nash equilibrium of {Cournot, Cournot}.

The required discount factor for output collusion under ex ante multiproduct (conglomeration) (EMP) following a 'grim' strategy is constant $\delta \geq \delta^{EMP} = 9/17$. Because there is only one Nash equilibrium from the first stage game, there is only one pure strategy subgame perfect Nash equilibrium in the subsequent second stage game of $\text{SPNE}_{EMP} = \{A\&B, A\&B; \delta \geq 9/17\}$. Comparing market segmentation alone and output collusion under either ex ante multiproduct (conglomeration) or market segmentation we find that $\delta^{EMS} < \delta^{EMP} < \delta^{MS}\ \forall\ 0 < \theta < 1$. Thus, output collusion alone is easier to obtain regardless of whether markets are initially segmented. These relationships are graphically depicted in Fig. 1.

In pure strategies notice that minimum required discount factor is independent of θ, a seemingly odd result. The reasons the degree of the substitutability of the products no longer impact the minimum required discount factor to maintain collusion are (1) consumers are entirely trapped and cannot flee higher prices, and (2) given firms are already producing both products, they need not be concerned about the value of defection (product expansion) relative to changes in competition for more similar/dissimilar products. Interestingly, there is no distinction between conglomerate firms competing for near independent goods and multiproduct firms producing close substitutes ability to collude.

3.3. Scenario 4 – complete collusion across product space and output

The previous partial collusive equilibria were based on the assumptions that product space and output decisions may be carried out by various levels of management, each with varying time horizons. If however, management decisions of the firm are fully integrated and long lived, then the firm is able to fully foresee the benefits from collusion across both decision stages, further assuming there are no constraints as to competitive advantage or location.

In regards to observability of defection, we consider the case where firms make product space decision in stage one, but either do not make them publically known until placed in the market, or even if observable, the rival waits to verify product placement in one period before defection is subsequently punished. In such a setting, the two-stage decision process reduces to a single game of paired choices (product space and quantity) played once each period.

We discuss three possible defection cases. The first is where one firm is observed to defect only on the output dimension, which we have already addressed in scenario two above. The second is when one firm defects only on the product dimension. However, given we assume the rival will not punish defection on product space without committed production leaves us with the third case, defection in paired choices (product space and quantity). Therefore, verifiable defection in product space cannot occur without defection in output, especially in the market entered. To maintain collusion across both

Table 4
Joint product space and output choices.

Firm 1	Firm 2	
	Produce product B at monopoly output	Produce products {A&B} at Cournot output
Produce product a at monopoly output	$\frac{(a-c)^2}{4b(1+\theta)}, \frac{(a-c)^2}{4b(1+\theta)}$	$\frac{(a-c)^2(1+2\theta)}{8b(1+\theta)^2}, \frac{(a-c)^2(5+4\theta)}{16b(1+\theta)^2}$
Produce products {A&B} at Cournot output	$\frac{(a-c)^2(5+4\theta)}{16b(1+\theta)^2}, \frac{(a-c)^2(1+2\theta)}{8b(1+\theta)^2}$	$\frac{2(a-c)^2}{9b(1+\theta)}, \frac{2(a-c)^2}{9b(1+\theta)}$

product space and output, the within period backward induction process across decision stages by firms must consider credible punishments, and optimal defection strategies.

In regards to defection, we find that the defector has two strategies available contingent upon the range of product substitutability. The first defection strategy takes place only at the output decision stage resulting in a defection payoff of $(((a-c)^2(2+\theta)^2)/(16b(1+\theta)^2))$, (payoff located in the upper/lower right/left cells in Table 2, scenario 2). The second defection strategy is across both decision stages. Appendix 4 provides the derivation of defection per period payoffs for Table 4. We find when the colluding firm produces the monopoly output within its believed segmented market, results in a set of defection and collusion payoffs of $\pi_{\text{defection},i} = (((a-c)^2(5+4\theta))/(16b(1+\theta)^2))i \neq j$. If the defector is rational, the firm will compare these two payoffs and choose an optimal one time defection.

We now compare the two defection payoffs and note that θ is the primary variable determining the relative size of the defection payoffs. We find that $(((a-c)^2(5+4\theta))/(16b(1+\theta)^2)) > (((a-c)^2(2+\theta)^2)/(16b(1+\theta)^2))\,\forall\, 0 \leq \theta < 1$. Therefore, for the exception of perfect substitutes we find the optimal defection includes both entering the rival's previously segmented market and producing Cournot output. In Table 4 we provide the respective defection payoffs in the lower right and upper left cells.

Next we consider credible punishment. If a firm is observed to cheat in only the second stage, it was the case in scenario 2 that the optimal punishment by the rival in the next period is to produce its Cournot output in only its segmented product market. However, under full collusion the second stage subgame Nash equilibrium does not represent a credible punishment, but rather the paired strategies {A&B, Cournot; A&B, Cournot}. Therefore, the only credible punishment for verifiable defection in product space and output results in symmetric payoffs of $((2(a-c)^2)/(9b(1+\theta)))$ (bottom right payoffs in Table 1, scenario 1).

Table 4 summarizes the relevant payoffs of the game played each period. The collusive payoffs from scenario 2 constitute full collusion payoffs in the upper left cell. The credible punishment just discussed constitutes full defection in the bottom right cell. We prove in Appendix 4 that there is a unilateral incentive to deviate from maintaining market segmentation and monopoly output resulting in the unique pure strategy Nash equilibrium {A&B, Cournot; A&B, Cournot}.

Following a 'grim' strategy, the required discount factor for both market segmentation and output collusion requires $\delta \geq \delta^{\text{Full}} = (9/(13+4\theta))$. Therefore, the subgame perfect Nash equilibrium is SPNE = {A, B, $q_{A,1} = ((a-c)/(2b(1+\theta))) = q_{B,2}$; $\delta \geq (9/(13+4\theta))$}. Given $0 \leq \theta < 1$, δ^{Full} lies in the interval $\left[9/13, 9/17\right)$.

By comparing all partial collusion scenarios to full collusion we find that $\delta^{\text{EMS}} < \delta^{\text{EMP}} < \delta^{\text{Full}} < \delta^{\text{MS}}\,\forall\, 0 < \theta < 1$. Thus, output collusion alone is easier to obtain than coordinating over product space. Taking the first order condition, $(\partial\delta^{\text{Full}}/\partial\theta) = -(36/(13+4\theta)^2) < 0\,\forall\, 0 \leq \theta < 1$, reveals that increasing product substitutability monotonically decreases the required discount factor to maintain complete collusion. These results are graphically represented in Fig. 1.

3.4. Summary of the minimum required discount factors for partial and complete collusion

Given our modeling framework, it appears that product space collusion is more difficult to attain than output collusion. Overall, as long as product space is major consideration of collusion, closer substitutes stabilizes collusion. Alternatively, as long as output is the focal point of collusion, closer substitutes weakly destabilizes collusion. However, output, rather than market segmentation, collusion is generally easier to obtain and more so as product markets become increasingly weaker substitutes.

It is important to note that output collusion among multiproduct (conglomerate) firms results in the same payoffs as output collusion after market segmentation. To achieve the same level of collusion, and hence profit, firms/management that first segment the market then later consider restricting quantity have two hurdles of coordination to overcome in the long run. However, if firms/management have a high ‘enough’ discount factor to collusively segment the market in the first place, they would necessarily be able to collude over output at a later date. On the other hand, if firms/management have low ‘enough’ discount factor and cannot initially segment the market, the resulting multiproduct (conglomerate) firms would experience nearly the same level of difficulty to collude in output as segmented firms.

4. Conclusions

By accounting for a two-stage decision process, we were able to evaluate the impact of the firms’ ability to collusively segment a differentiated product market, an important issue for antitrust agencies. Our first important findings is that under partial collusion, i) collusive product market segmentation is unlikely to occur due to the value of entry, and ii) multiproduct competition (conglomeration) and output collusion is more likely to occur than collusive product market segmentation. Secondly, when firms are able to consider collusion over both product space and output, maintaining market segmentation is more likely to occur as products become closer substitutes. Lastly, if firms are found to have collusively segmented the product market, but not output, the impacts are not as severe as with output collusion. However, output collusion is a natural progression if firms are able to coordinate over product space.

Given the output collusive payoffs are the same post market segmentation and multiproduct (conglomeration) market structure, even firms with sufficiently high discount factors to initially segment the market may want to consider the overall ease of collusion. For instance, segmented industries must monitor entry and output competition, as well as consumer perceptions of the differentiability of their products. Though we find output collusion is more likely after market segmentation than not, under more realistic informational assumptions our results suggest collusion may be more likely among multiproduct (conglomerate) firms. For instance, firms need not be concerned about entry, as their rival is already in the market, and need not be concerned about adequately estimating the consumers’ view of the differentiability of their products. Therefore, monitoring costs would more likely be lower in the case of a conglomerate industry. Consequently, periodic distortions in the market, changes in consumer preferences and/or imperfections in monitoring that may lead to competitive disagreements that are more likely overcame if the firms are able to focus on one rather than two product dimensions.

In relation to the literature, we find that producing both products can be a symmetric and unique Nash equilibrium without requiring significant entry costs and economies of scope (Dobson & Waterson, 1996; Fraja, 1992), and for the exception of perfect substitutes, is independent of the degree of substitutability. These results are in stark contrast with earlier two-stage Bertrand models by Shaked and Sutton (1990) where multiple product mix equilibria are found but firms never produce both products for the exception of complements (Shaked & Sutton, 1990). The discrepancy is due to the Cournot assumption in our framework. Under Bertrand–Nash competition, the prices would fall to marginal cost where firms will not produce both products.

Our results under ex ante market segmentation are consistent with the findings of horizontally differentiated products and Bertrand competitors analyzed by Chang (1991) and Ross (1992). However, this result is inconsistent with Ross’s (1992) non-monotonic result based on the quadratic utility model and Häckner’s (1994) analysis on a vertical differentiation model. One reason being is that firms must first overcome the competitive pressure to product both products in the first stage. More interestingly, if firms are unable to segment the market in the first stage, the results indicate that the stability of collusion is independent of product substitutability for conglomerates. The reason being is that product substitutability is only an intra-firm competition concern leaving only aggregate output as the target for collusion.

Finally, our modeling approach is easily extended in obvious and much needed ways. For instance, the assumption of symmetric production costs for each product should be relaxed. For example in the meat processing industry, beef is significantly more expensive to process than chicken, thus altering the incentives of the high cost firm to refrain from entering the low cost industry. Interestingly, if one or the other product is more profitable due to asymmetries in the size of each market, demand elasticity's and/or production costs, our intuition is that for a firm to be willing to continue producing only the less profitable product would require further assumptions about entry costs, product expertise or overt (collusive) compensation. Furthermore, an extension of our modeling framework to analyze multiproduct mergers could consider both economies of scope and increases in the inelasticity of residual aggregate demand when firms produce a wider range of complimentary/substitute products (Bailey & Friedlaender, 1982; Hausman, Leonard, & Zona, 1994).

Appendix 1. Scenario 1 – product space collusion with ex post Cournot output

Case 1. Let each firm consider producing only product A. When both firms produce only A $q_B = 0$, the game degenerates to a classic duopoly where the representative firm's objective function is $\underset{q_{A,i}|q_{A,2}}{\text{Max}}\ \pi_{A,j} = [a - b(q_{A,i} + q_{A,j}) - c]q_{A,i},\ i \neq j$. By doing so, the system of reaction functions of each firm are $q_{A,1} = ((a-c)/2b) - (q_{A,2}/2)$ and $q_{A,2} = ((a-c)/2b) - (q_{A,1}/2)$, where the optimal quantity for each firm is $q^*_{A,1} = ((a-c)/3b) = q^*_{A,2}$. Therefore, under Cournot competition, each firm earns $\pi_{A,1} = ((a-c)^2/9b) = \pi_{A,2}$. Similarly, when both firm produce only B, $q_A = 0$ and $q^*_{B,1} = ((a-c)/3b) = q^*_{B,2}$, resulting in $\pi_{B,1} = ((a-c)^2/9b) = \pi_{B,2}$. These payoffs are depicted in the top left and middle center cells of Table 1.[3]

Case 2. Let each firm considers producing each product separately, thus the game degenerates to the cases analyzed in past research that uses a Bowley model. Let firm 1 consider producing only A and firm 2 producing only B. In this case, the objective functions for the two firms are $\underset{q_{A,1}|q_{B,2}}{\text{Max}}\ \pi_{A,1} = [a - b(q_{A,1} + \theta q_{B,2}) - c]q_{A,1}$ and $\underset{q_{B,2}|q_{A,1}}{\text{Max}}\ \pi_{B,2} = [a - b(\theta q_{A,1} + q_{B,2}) - c]q_{B,2}$ resulting in the inter-firm reaction functions, $q_{A,1} = ((a-c)/2b) - (\theta q_{B,2}/2)$ and $q_{B,2} = ((a-c)/2b) - (\theta q_{A,1}/2)$. The optimal quantities for firm 1 and 2 of product A and B is $q^*_{A,1} = ((a-c)/(b(2+\theta))) = q^*_{B,2}$. Therefore, under Cournot competition, each firm earns $\pi_{A,1} = ((a-c)^2/(b(2+\theta)^2)) = \pi_{B,2}$ and conversely, $\pi_{B,1} = ((a-c)^2/(b(2+\theta)^2)) = \pi_{A,2}$. These payoffs are depicted in the middle left and middle top cells of Table 1.[4]

Case 3. We begin extending the assumption that firms are uniquely identified by their product. Let Firm 1 consider producing both A and B, while firm 2 considers producing only A. The firms objective functions are now $\underset{q_{A,1},q_{B,1}|q_{A,2}}{\text{Max}}\ \pi_{A\&B,1} = [a - b(q_{A,1} + q_{A,2} + \theta q_{B,1}) - c]q_{A,1} + [a - b(\theta(q_{A,1} + q_{A,2}) + q_{B,1}) - c]q_{B,1}$ and $\underset{q_{A,2}|q_{A,1},q_{B,1}}{\text{Max}}\ \pi_{A,2} = [a - b(q_{A,1} + q_{A,2} + \theta q_{B,1}) - c]q_{A,2}$, which result in the system of intra- and inter-firm reaction functions $q_{A,1} = ((a-c)/2b) - ((bq_{A,2} + 2b\theta q_{B,1})/2b)$, $q_{B,1} = ((a-c)/2b) - ((2b\theta q_{A,1} + b\theta q_{A,2})/2b)$ and $q_{A,2} = ((a-c)/2b) - ((bq_{A,1} + b\theta q_{B,1})/2b)$. The optimal quantities of each product for firm 1 are $q^*_{A,1} = (((2-\theta)(a-c))/(6b(1+\theta)))$ and $q^*_{B,1} = ((a-c)/(2b(1+\theta)))$, while for firm 2 $q^*_{A,2} = ((a-c)/3b)$. Therefore, under Cournot competition the total payoff for firm 1 is $\pi_{A\&B,1} = (((13-5\theta)(a-c)^2)/(36b(1+\theta)))$ and for firm 2 is $\pi_{A,2} = ((a-c)^2/9b)$, the same holding true

[3] The second order derivatives for the profit functions are both $-2b$, which is negative since b is positive. Therefore, the second order conditions are satisfied.

[4] Again, the second order conditions are satisfied since the second order derivatives for the profit functions of both firms are $-2b$, which is negative.

for various similar relative product positions. These payoffs are depicted in the bottom left, top right, middle left and right cells of Table 1.[5]

Case 4. Let each firm consider producing both products. The representative firm's objective function $i=1,2$ is therefore $\underset{q_{A,i},q_{B,i}|q_{A,j},q_{B,j}}{\text{Max}}\ \pi_{A\&B,i} = [a-b(q_{A,i}+q_{A,j}+\theta(q_{B,i}+q_{B,j}))-c]q_{A,i}+[a-b(\theta(q_{A,i}+q_{A,j})+q_{B,i}+q_{B,j})-c]q_{B,i}\ i \neq j$, which results in the system of intra- and inter-firm reaction functions $q_{A,1}=((a-c)/2b)-((bq_{A,2}+2b\theta q_{B,1}+b\theta q_{B,2})/2b)$, $q_{B,1}=((a-c)/2b)-((bq_{B,2}+2b\theta q_{A,1}+b\theta q_{A,2})/2b)$, $q_{A,2}=((a-c)/2b)-((abq_{A,1}+b\theta q_{B,1}+2b\theta q_{B,2})/2b)$ and $q_{B,2}=((a-c)/2b)-((bq_{B,1}+b\theta q_{A,1}+2b\theta q_{A,2})/2b)$. The optimal quantities for both firms are $q^*_{A,i}=q^*_{B,i}=((a-c)/(3b(1+\theta)))=q^*_{A,j}=q^*_{B,j}$. Therefore, under Cournot competition, each firm's total payoff is $\pi_{A\&B,i}=((2(a-c)^2)/(9b(1+\theta)))=\pi_{A\&B,j}$. These payoffs are depicted in the bottom right cell of Table 1.[6]

Equilibrium: Based on the matrix of Cournot outcomes across product profiles depicted in Table 1, several features of the market segmentation game can now be discussed. First, when A and B are perfect substitutes ($\theta=1$), all payoffs are equal and the problem degenerates to the classic Cournot game between two firms producing a homogenous good. Secondly, denoting by the conditional choice payoffs in Table 1 as π|{choice firm 1, choice firm 2}, we find from firm 2's perspective that $\pi|\{A, A\}<\pi|\{A, B\}<\pi|\{A, A\&B\}, \pi|\{B, B\}<\pi|\{B, A\}<\pi|\{B, A\&B\}$, and $\pi|\{A\&B, A\}=\pi|\{A\&B, B\}<\pi|\{A\&B, A\&B\}\ \forall 0\leq\theta<1$. Firm 1 has same incentive to defect to multiproduct production. Therefore producing both products is a unilateral strictly dominant strategy, resulting in the unique pure strategy Nash equilibrium {A&B | Cournot, A&B | Cournot}.

Appendix 2. Scenario 2 – output collusion under ex ante market segmentation

Firms now consider their collusive joint profit maximization objective function $\underset{q_{A,1},q_{B,2}}{\text{Max}}\ \pi_{A,1}+\pi_{B,2}=(a-b(q_{A,1}+\theta q_{B,2})-c)q_{A,1}+(a-b(\theta q_{A,1}+q_{B,2})-c)q_{B,2}$. This is the discriminating monopolist's objective function. First order conditions result in the system of inter-firm reaction functions $q_{A,1}=((a-c)/2b)-\theta q_{B,2}$ and $q_{B,2}=((a-c)/2b)-\theta q_{A,1}$. The optimal level of output by Firm 1 and 2 is $q^*_{A,1}=((a-c)/(2b(1+\theta)))=q^*_{B,2}$. By substitution of the optimal levels of output into the objective function results in total joint profits of $\pi_{A,1}+\pi_{B,2}=((a-c)^2/(2b(1+\theta)))$. Therefore, due to firm and demand symmetry results in equal market shares and each firm's collusive earnings are $\pi_{\text{collusion},1}=((a-c)^2/(4b(1+\theta)))=\pi_{\text{collusion},2}$. These payoffs are provided in the upper left cell in Table 2.

By substitution of either firm's collusive output quantity into the rival's reaction function results in the defector's output of $q^*_{\text{defection},i}=(((a-c)(2+\theta))/(4b(1+\theta)))\ \forall i=1,2$. Substituting the defector and colluder's outputs into each firms objective function results in the defector and colluder earnings of $\pi_{\text{defection},i}=(((a-c)^2(2+\theta)^2)/(16b(1+\theta)^2))>(((a-c)^2(2-\theta(\theta-2)))/(8b(1+\theta)^2))=\pi_{\text{collusion},j}\ i\neq j$ and $\forall 0<\theta<1$. These relative payoffs are provided in the upper right and lower left cells in Table 2. The lower right payoffs are the Cournot outcomes of a segmented market from Table 1.

Equilibrium: Comparing the payoffs in Table 2 between the colluder's payoff when their rival defects illustrates that $\pi_{\text{collusion},i}<\pi_{\text{Cournot},j}\ \forall 0\leq\theta<1$. Because $\pi_{\text{Cournot},i}>\pi_{\text{collusion},i}\ \forall 0\leq\theta<1$ when j defects, defection in the static game is symmetrically preferred. Therefore, producing Cournot output is a unilateral strictly dominant strategy resulting in the unique pure strategy Nash equilibrium under ex ante market segmentation of {Cournot, Cournot}.

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[5] The Hessian matrix for the profit function of the firm that produces both products is $\begin{bmatrix} -2b & -2b\theta \\ -2b\theta & -2b \end{bmatrix}$, which is negative definite since $|H_1|=-2b<0$ and $|H_2|=4b^2(1-\theta^2)>0$ given that $b>0$ and $0\leq\theta<1$, and the second order derivative for profit function of the firm that produce a single product is -2b. Therefore, the second order conditions are satisfied. $|H_2|=0$ when $\theta=1$, which is a special case that we discuss later.

[6] Again, the Hessian matrices for both firms' profit functions are $\begin{bmatrix} -2b & -2b\theta \\ -2b\theta & -2b \end{bmatrix}$, which is negative definite, therefore, the second order conditions are satisfied.

Appendix 3. Scenario 3 – output collusion under ex ante multiproduct (conglomerate) production

By taking the first order condition of the joint profit maximization objective function $\underset{q_{A,1},q_{B,1},q_{A,2},q_{B,2}}{\text{Max}}$ $\pi_{A\&B,1}+\pi_{A\&B,2}=[a-c-b(q_{A,1}+q_{A,2}+\theta(q_{B,1}+q_{B,2}))]q_{A,1}+[a-c-b(q_{B,1}+q_{B,2}+\theta(q_{A,1}+q_{A,2}))]q_{B,1}+[a-c-b(q_{A,1}+q_{A,2}+\theta(q_{B,1}+q_{B,2}))]q_{A,2}+[a-c-b(q_{B,1}+q_{B,2}+\theta(q_{A,1}+q_{A,2}))]q_{B,2}$ results in the system of inter- and intra-firm reaction functions $q_{A,1}=((a-c)/2b)-(q_{A,2}+\theta(q_{B,1}+q_{B,2}))$, $q_{B,1}=((a-c)/2b)-(q_{B,2}+\theta(q_{A,1}+q_{A,2}))$, $q_{A,2}=((a-c)/2b)-(q_{A,1}+\theta(q_{B,1}+q_{B,2}))$ and $q_{B,2}=((a-c)/2b)-(q_{B,1}+\theta(q_{A,1}+q_{A,2}))$. Because of firm symmetry resulting in equal market shares the optimal collusive quantities for both firms and each product are $q^*_{A,1}=q^*_{B,1}=((a-c)/(4b(1+\theta)))=q^*_{A,2}=q^*_{B,2}$. Substitution of the optimal levels of output into the objective function results in total joint profits $\pi_{A\&B,1}+\pi_{A\&B,2}=((a-c)^2/(2b(1+\theta)))$. Therefore, each firm's equal market share of the collusive earnings are $\pi_{\text{collusion},1}=((a-c)^2/(4b(1+\theta)))=\pi_{\text{collusion},2}$. These payoffs are located in the upper left cell of Table 3.

By substitution of either firm's collusive output quantity into their rivals Cournot intra- and inter-firm reaction functions from the first stage results in the defectors output of $q^*_{\text{defection},i}=((3(a-c))/(8b(1+\theta)))$ $i\neq j$. By substitution of collusion and defection outputs into each firms objective function results in the defector/colluder earnings of $\pi_{\text{defection},i}=((9(a-c)^2)/(32b(1+\theta)))>((3(a-c)^2)/(16b(1+\theta)))=\pi_{\text{collusion},j} i\neq j$. These relative payoffs are provided in the upper right and lower left cells in Table 3. The lower right payoffs are the Cournot outcomes of a multiproduct market from Table 1.

Equilibrium: Comparing the payoffs in Table 3 between symmetric collusion and Cournot we have $\pi_{\text{collusion},i}>\pi_{\text{Cournot},i}\ \forall\, i=1,2$. Again, output collusion in both products Pareto dominates multiproduct (conglomerate) Cournot. Comparison of the colluder's payoff when their rival defects illustrates that $\pi_{\text{collusion},i}<\pi_{\text{Cournot},j}\ \forall 0\leq\theta<1$. Because $\pi_{\text{Cournot},i}>\pi_{\text{collusion},i}\ \forall\, 0\leq\theta<1$ when j defects, defection in the static game is symmetrically preferred. Therefore, producing Cournot output is a unilateral strictly dominant strategy resulting in the unique pure strategy Nash equilibrium under ex ante multiproduct (conglomerate) Nash equilibrium of {Cournot, Cournot}.

Appendix 4. Scenario 4 – collusion across both product space and output

Following scenario 1, case 3, let Firm 1 consider producing both A and B, while firm 2 considers producing only B. The firms objective functions would be $\underset{q_{A,1},q_{B,1}|q_{B,2}}{\text{Max}}\ \pi_{A\&B,1}=[a-b(q_{A,1}+\theta(q_{B,1}+q_{B,2}))-c]q_{A,1}+[a-b(\theta q_{A,1}+q_{B,1}+q_{B,2})-c]q_{B,1}$ and $\underset{q_{B,2}|q_{A,1},q_{B,1}}{\text{Max}}\ \pi_{B,2}=[a-b(\theta q_{A,1}+q_{B,1}+q_{B,2})-c]q_{B,2}$, which would result in the system of intra- and inter-firm reaction functions $q_{A,1}=((a-c)/2b)-((2b\theta q_{B,1}+b\theta q_{B,2})/2b)$, $q_{B,1}=((a-c)/2b)-((2b\theta q_{A,1}+bq_{B,2})/2b)$ and $q_{B,2}=((a-c)/2b)-((b\theta q_{A,1}+bq_{B,1})/2b)$. However, under complete collusion firm 2 continues producing monopoly output of product B, $q^*_{B,2}=((a-c)/4b(1+\theta))$, under the belief that firm 1 is producing only product A at the joint profit maximizing output (Appendix 2, scenery 2). If firm 1 defects and produces product B while firm 2 continues to produce the monopoly output of B, firm 1's optimal quantities of each product are $q^*_{A,1}=((a-c)/(2b(1+\theta)))$ and $q^*_{B,1}=((a-c)/(4b(1+\theta)))$. Substituting the defector and colluder's outputs into each firms objective function results in defector and colluder earnings of $\pi_{\text{defection},1}=((a-c)^2(5+4\theta)/(16b(1+\theta)^2))>(((a-c)^2(1+2\theta))/8b(1+\theta)^2)=\pi_{\text{collusion},2}\ \forall\, 0\leq\theta<1$. The same relative payoffs hold true from defection by firm 2 if firm 1 maintains collusion. These payoffs are depicted in the top right and bottom left cells of Table 4.

Equilibrium: Comparing the payoffs in Table 4 the colluder's payoff when their rival defects illustrates that $\pi_{\text{collusion},i}<\pi_{\text{defection},j}\ \forall\, 0\leq\theta<1$. Because $\pi_{\text{Cournot},i}>\pi_{\text{collusion},i}\ \forall\, 0\leq\theta<1$ when j defects, defection in the static game is symmetrically preferred. Therefore, producing Cournot output and entering the rivals market is a unilateral strictly dominant strategy resulting in the unique Nash equilibrium of {A&B, Cournot; A&B, Cournot}.

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