Journal of Family Business Strategy xxx (2014) xxx–xxx

Contents lists available at ScienceDirect

Journal of Family Business Strategy

journal homepage: www.elsevier.com/locate/jfbs

Bayesian methods in family business research

Joern H. Block [a,*], Danny Miller [b,1], Dominik Wagner [a,2]

[a] Universität Trier, Universitätsring 15, DM Gebäude, 54296 Trier, Germany
[b] HEC Montreal and University of Alberta, 3000 Cote Ste. Catherine Road, Montreal, Canada H3T 2A7

ARTICLE INFO

Article history:
Received 17 October 2013
Received in revised form 17 December 2013
Accepted 22 December 2013

Keywords:
Bayesian methods
Null hypothesis significance testing
Family business research methods
Regression models

ABSTRACT

Bayesian methods constitute an alternative to null hypothesis significance testing (NHST). This article briefly reviews the concept of Bayesian methods, describes their differences from NHST, and discusses the potential of Bayesian methods to advance family business research and practice. We argue that Bayesian methods are well suited to account for the significant heterogeneity that exists in the population of family firms. The article closes with a short guide to using Bayesian methods and reporting their results in the context of research on family businesses. The article's focus is on regression models.

1. Introduction

Bayesian analysis is a commonly encountered and well-accepted statistical method that is employed in academic disciplines including medicine (Armitag et al., 2009; Ashby, 2006; Berry, 2006), psychology (Edwards, Lindman, & Savage, 1963), physics (Cousins, 1995), genetics (Shoemaker, Painter, & Weir, 1999), and biology (Huelsenbeck, 2001). With the exception of marketing research (Rossi & Allenby, 2003) and decision analysis (Grover, 2013), however, Bayesian methods are rarely utilised in business or management research (Kruschke, Aguinis, & Joo, 2012; Zyphur & Oswald, 2013).[3] A number of recent studies lament this situation and suggest that Bayesian methods may be a useful alternative to null hypothesis significance testing (NHST) (Hahn & Doh, 2006; Hansen, Perry, & Reese, 2004; Kruschke, Aguinis, & Joo, 2012; The Economist, 2006), which has long been the dominant mode of statistical analysis in management research (Schwab, Abrahamson, Starbuck, & Fidler, 2011).[4] This article extends this small but growing literature on the Bayesian approach and examines the potential applicability of Bayesian methods to family business research and practice. The focus of the article is on regression models.

We proceed as follows: Section 2 describes NHST and introduces Bayesian analysis. Section 3 presents the most cogent criticisms of NHST, and Section 4 enumerates the differences between Bayesian analysis and NHST. Section 5 discusses the potential contributions of Bayesian analysis to family business research, and Section 6 provides a short illustrative example of the difference between Bayesian analysis and NHST. Section 7 contains a short guide for how to use Bayesian methods and report results in the context of research on family businesses. Section 8 concludes.

2. NHST and Bayesian analysis

The NHST approach defines a population (e.g., all students in Germany) and draws a sample from this population (e.g., students at the University of Trier) to learn about the value of a particular parameter in the population (e.g., mean age). The assumption is that a parameter varies across the population. One sample taken from the population will yield a particular parameter value, whereas a different sample will yield another value, and the difference between the samples is referred to as sampling variation. The statistician's task is to arrive at the 'true' parameters of the population using the evidence provided by the sample. To accomplish this objective, a sample estimator and an accompanying test statistic are selected. Thus, the NHST approach is tied to the notion of a sample and a population; this approach uses sample estimators and test statistics to learn something about the 'true' parameters in the population. The Bayesian approach is different because it is *not* tied to the notion of a sample and a population.

* Corresponding author. Tel.: +49 6512013032; fax: +49 6512013029.
E-mail addresses: block@uni-trier.de (J.H. Block), danny.miller@hec.ca (D. Miller), dominik.wagner@uni-trier.de (D. Wagner).
[1] Tel.: +1 514 340 6501; fax: +1 514 340 6382.
[2] Tel.: +49 6512013031; fax: +49 6512013029.
[3] The situation in economics is comparable: with the exception of macroeconomics (e.g., Smets & Wouters, 2007), most empirical research in economics is based on NHST.
[4] We found only two articles researching family business using Bayesian methods (Block et al., 2011; Block & Wagner, 2013).

1877-8585/$ – see front matter
http://dx.doi.org/10.1016/j.jfbs.2013.12.003

Bayesian methods rely on Bayes' theorem in probability theory (Bayes, 1763), which is given by:

$$Pr(\theta|y) = \frac{Pr(y|\theta)Pr(\theta)}{Pr(y)} \quad (1)$$

where θ represents the set of unknown parameters and y represents the data. $\Pr(\theta)$ is the prior distribution of the parameter set θ,[5] which may be derived from theory, expert opinion, or other external sources. $Pr(y|\theta)$ is the likelihood function, which is the probability of the data y given the unknown parameter set θ. $\Pr(y)$ is the marginal distribution of the data y; finally, $Pr(y|\theta)$ represents the posterior distribution,[6] which is the probability of the parameter set θ given the data y. Eq. (1) may also be written as:

$$Pr(\theta|y) \propto Pr(y|\theta)Pr(\theta), \quad (2)$$

where $\propto$ indicates 'proportional to'. The posterior distribution is proportional to the likelihood function multiplied by the prior distribution.[7] In Bayesian analysis, inference comes from the posterior distribution, which states the likelihood of a particular parameter value.

When testing a hypothesised relationship between two variables, Bayesian analysis proceeds in the following three steps. First, a priori beliefs (from theory, prior empirical research or an interview) about the relationship of interest are formulated (the prior distribution, $\Pr(\theta)$). Next, a probability of occurrence of the data given these a priori beliefs is assumed (the likelihood function, $Pr(y|\theta)$). In the third step, data are used to update these beliefs. The result is the posterior distribution, $Pr(\theta|y)$). This posterior distribution gives a density function of the parameter of interest (i.e., the coefficient that describes the relationship between the two variables). The posterior distribution allows for statements in terms of likely and unlikely parameter values.

3. Criticism of NHST

Since its introduction by Fisher (1925), NHST has been criticised for myriad reasons (e.g., Cohen, 1994; Schmidt, 1996; Starbuck, 2006).[8] Fisher himself was aware of the problems associated with NHST and recommended its use primarily when researchers have little prior knowledge about the object of their research (Gigerenzer, Krauss, & Vitouch, 2004).

One of the main problems of NHST is argued to be the statistical significance level required for publication (in most cases 5%), which is arbitrary and has no mathematical basis; this statistical significance level is found simply as the result of long tradition (Gigerenzer, Krauss, & Vitouch, 2004). Frequently, researchers applying NHST ignore the fact that the obtained significance level is tied to the test's statistical power and the sample size. In addition, NHST interprets the result of an empirical analysis as dichotomous, i.e., either an empirical result is statistically significant or not. Small differences in data (e.g., a p-value that drops from $p = 0.051$ to $p = 0.048$) can therefore lead to major differences in inference (Schwab et al., 2011) and interpretation of empirical results.[9] A more pedagogical criticism addresses the fact that overstressing statistical significance draws attention from the size of an effect (economic significance) (Combs, 2010). NHST does not distinguish between economically and practically important versus unimportant effects; this judgement is simply left to the researcher. Additionally, NHST is sensitive to the size of the sample, and a statistically significant result can almost always be found if a large enough sample is analysed (Berkson, 1938; Combs, 2010).[10] The outcome of a research project therefore is critically dependent on a researcher' ability to obtain sufficient data (Sawyer & Ball, 1981). Another criticism concerns the interpretation of a result that does not allow the rejection of the null hypothesis (e.g., $p > 0.05$ with a 5% significance level). A non-significant result can result from a small sample size, a violation of assumptions of the specific estimators and statistical tests used, or a non-existing relationship. Finally, scientific journals almost never publish statistically non-significant results and thereby present a biased picture of reality. Meta-analyses, in particular, can suffer from this publication bias (Stanley, 2005).

4. Differences between NHST and Bayesian analysis

The Bayesian approach is fundamentally different from NHST. The main differences are the following:

- *Posterior distribution* instead of a point estimate: As explained in Section 2, the result of Bayesian analysis is a posterior distribution of the parameter of interest, which differs from NHST in that the outcome of the estimation is not a point estimate (i.e., whether a value is either statistically significant or not) but an entire distribution function. Thus, Bayesian analysis allows for statements such as "the probability of a positive effect of A on B is 70%", which is not possible with NHST. NHST only permits a statement such as "the effect of A on B is positive. The probability of making an error with this statement is below 5% (10%)."
- *Notion of sample and population*: Bayesian analysis is *not* tied to the notion of a sample and a population. Its results are statements about the particular data that are used in the analysis. Thus, there is no statement about a 'true' parameter in an underlying distribution. When Bayesians refer to their data as a sample, it is simply out of convention.
- *Prior*: A Bayesian researcher must formulate an assumption about the distribution function of the coefficient of interest. This so-called prior (probability) can be either subjective and informative or objective and minimally informative. Assume that a researcher believes that variable A has a positive influence on variable B. A subjective (informative) prior would refer to a probability distribution function with a positive mean and few or no values smaller than zero; an objective (minimally informative) prior would refer to a flat distribution function or a Gaussian distribution with a mean of zero.[11] In most cases, a Bayesian researcher should investigate the sensitivity of the results that are obtained to the specification of the prior.
- *Likelihood function*: In addition to specifying the prior, a Bayesian[12] researcher must attach probabilities to the values of the data observed. As explained in Section 2, attaching the

[5] This distribution is often referred to as the *prior*.
[6] This distribution is often referred to as the *posterior*.
[7] The denominator $\Pr(y)$ can be neglected in Bayesian estimation; $\Pr(y)$ is constant and often unknown and it is also independent from the parameters of the model.
[8] The criticism has been most intense in the field of psychology. The American Psychological Association (APA) reacted and enhanced their publication guidelines by reducing the relative importance of NHST. Reporting confidence intervals for effect sizes has become standard.
[9] It is notable that this overstressing of a drop in the p-value is more of a problem regarding the application of NHST (and the interpretation of the p-value) than of the statistic itself (Hubbard & Bajarri, 2003).
[10] By definition, one in 10 studies will produce a significant result ($p < 10\%$, two-sided test). If this study is then published and the other studies are not published, the uninformed reader will conclude that there is a significant relationship in the population.
[11] The researcher can also formulate a prior opposite to her expectations. This type of prior formulation is frequently used as a robustness check. An empirical result is considered particularly robust if the evidence (the data) is able to "correct" a prior formulated against the researcher's beliefs.
[12] People who follow Bayesian methods are sometimes called Bayesians. People who follow NHST are sometimes referred to as 'frequentists'.

Please cite this article in press as: Block, J. H., et al. Bayesian methods in family business research. *Journal of Family Business Strategy* (2014), http://dx.doi.org/10.1016/j.jfbs.2013.12.003

Table 1
NHST versus Bayesian analysis.

NHST	Bayesian analysis
Sample and population, sample characteristics	
A sample is taken from a population to learn about the parameters in the population.	There is no notion of a sample and a population. The results of the estimation apply only to the specific data used.
Small samples can include so much uncertainty that the null hypothesis cannot be rejected.	Bayesian analysis is able to make use of information contained in small samples.
Large-sample approximations may not hold for skewed samples.	Distributional results are exact under skewed samples. There is no inference made regarding a pre-defined population.
Assumptions about parameters, test statistics, and significance levels	
NHST does not formulate priors. No prior assumptions about the distribution of the parameters and their likelihood are required.	Prior assumptions about the distribution of parameters and their likelihood are necessary.
An assumption about the significance level is required, typically 10%, 5%, or 1%.	No assumption regarding significance level is required.
Violations of asymptotic properties of estimators or test statistics (e.g., due to multicollinearity or heteroscedasticity) can lead to biased estimates.	Bayesian analysis does not rely on asymptotic theory. Violations of asymptotic properties of test statistics or estimators do not lead to biased estimates.
Results and interpretation	
The result of NHST is often a fixed parameter (point estimate) that is either statistically significant or not.	The result of Bayesian analysis is a probability distribution of the parameter of interest, i.e., the result is not a point estimate but an entire distribution function.
Interpretation of the 95% confidence interval: if the empirical analysis is repeated multiple times, the confidence interval includes the true population parameter value 95% of the time.	Interpretation of the 95% credibility interval: the interval, which includes 95% of the parameter values.
NHST does not allow a researcher to measure evidence that there is no relationship between two variables (the null hypothesis of $\beta = 0$ cannot be accepted).	The result of Bayesian analysis (the posterior distribution) can provide evidence that there is no relation between two variables.

probabilities is typically performed in three stages. First, the Bayesian researcher assumes a prior distribution $\Pr(\theta)$ (based on theory or other sources). Then, she assumes a likelihood function $Pr(y|\theta)$, the probability of the data given the prior beliefs. These two distributions are then utilised to calculate the posterior distribution $Pr(\theta|y)$, i.e., the result of the Bayesian analysis. The Bayesian approach is thus more demanding than NHST in the pre-estimation phase because specific assumptions regarding the likelihood function and the prior distribution are required. However, this investment is rewarded because the posterior distribution permits inferences that are 'exact'.

- *Asymptotic properties* of estimators or test statistics: Contrary to NHST, Bayesian analysis requires no assumptions regarding asymptotic properties of estimators or test statistics. Consequently, any violations of asymptotic properties of estimators (e.g., problems of multicollinearity or heteroscedasticity) are less problematic than under NHST (Leamer, 1973).

Table 1 provides an overview of the main differences between Bayesian methods and NHST.

5. The potential utility of Bayesian methods for family business research

This section discusses the potential for Bayesian methods in family business research. Some arguments apply to management research in general, whereas others are tailored specifically to family business research. Our main argument is that Bayesian methods are well suited to account for the significant heterogeneity that exists within the population of family firms (Westhead & Howorth, 2007). The focus of this study is on Bayesian versus classical (i.e., NHST) regression methods.

5.1. Bayesian methods and small sample sizes

As discussed above, Bayesian analysis fully exploits the information provided in small samples and is able to investigate relationships between variables using small and skewed samples. As opposed to NHST, sample size does not influence the Bayesian method's ability to test whether a particular relationship is "true" or not (i.e., statistically significant). Bayesian analysis is therefore well suited to analyse research questions in relation to small but important sub-populations of family firms such as large multi-generation family firms (Pieper, 2007) or family firms listed in small countries with unique corporate governance systems. By means of Bayesian analysis, it is possible to analyse rare family-firm-specific events (e.g., succession, sale of the family firm, etc.) with a quantitative approach. Thus, until recently, qualitative research in the form of case studies has primarily been utilised to address these particular types of research questions.

To summarise, Bayesian methods do not "force" the researcher to pool several samples into one sample to increase the power of statistical tests (Schwab et al., 2011). Instead, Bayesian analysis is able to account for the considerable heterogeneity that exists within the group of family firms (e.g., (Sharma & Nordqist, 2007; Villalonga & Amit, 2006; Westhead & Howorth, 2007) using small and skewed samples as data underlying an empirical investigation.

5.2. Bayesian methods and multicollinearity

Empirical family firm research using NHST frequently confronts severe multicollinearity problems. Multicollinearity arises when two or more independent variables are strongly correlated with one another; thus, multicollinearity reduces the efficiency of estimations, inflates standard errors, and ultimately influences the level of statistical significance obtained. In severe cases, multicollinearity can lead coefficients in regression models to switch signs and lead to biased results.

The problem of multicollinearity restricts the research possibilities of the family business researcher using NHST. For example, it is difficult to assess and compare the effects of different family firm dimensions (e.g., management, ownership, and control) regarding an outcome variable of interest because these firm dimensions are typically highly correlated, and the researcher thus cannot include these different dimensions in the same regression (e.g., Miller, Le Breton-Miller, Lester, & Cannela, 2007; Villalonga & Amit, 2006). This problem is exacerbated when the sample is small. With NHST, the researcher faces a dilemma: she can either increase the sample size (which is frequently not possible due to data restrictions) or leave out one of the correlated variables, which thereby increases the risk of omitted variables bias. Another option would be to group several family firm dimensions into one variable (that is, using a family firm dummy instead of separate variables for family firm management or ownership). This grouping, however, does not account for the full heterogeneity that exists within the group of family firms, which can substantially restrict the research possibilities of the family business researcher.

As discussed above, Bayesian methods do not rely on asymptotic theory and therefore multicollinearity poses less of a problem. The researcher can include several correlated variables as independent variables in one regression. To illustrate this advantage, imagine a situation in which a researcher is interested in the effects of family management and family ownership on firm development. Furthermore, imagine an extreme situation in which family ownership and family management are perfectly correlated. In this example, neither NHST nor Bayesian analysis can disentangle their individual effects on firm development (Bayesian analysis would simply return the prior). Bayesian methods, however, are advantageous when the correlation is less than perfect, but still high enough to pose problems for NHST (e.g., the correlation between family ownership and family management is $r = 0.7$ and the sample size is "only" 50). Using NHST, the researcher faces a dilemma. Due to problems involving multicollinearity, she cannot include family ownership and family management in the same regression. Instead, she must run two separate regressions to learn about the respective effects of family management and family ownership on firm development. However, she then faces potential problems with omitted variables bias. Thus, NHST has brought forward a variety of approaches for handling multicollinearity issues, albeit often imperfect, such as partial least squares (PLS) regression or PLS structural equation modeling (Abdi, 2010; Hair, Hult, Ringle, & Sarstedt, 2014).

The situation is different with respect to Bayesian methods: with a less than perfect correlation between family ownership and family management (e.g., $r = 0.7$), the posterior distribution (the results of Bayesian analysis) will differ from the prior distribution, which allows the researcher to draw conclusions about the distinct effects of family management and family ownership. This characteristic of Bayesian methods allows the researcher to separate the effects of family ownership and family management on firm development without risking omitted variables bias.

5.3. Bayesian methods to test and correct for endogeneity

Family firm research is regularly confronted with endogeneity issues of various types. For example, research on the performance of family versus non-family firms frequently faces the problem of reverse causality; for example, does family ownership lead to higher firm performance or does low firm performance lead family owners to sell their shares. To correct for endogeneity, instrumental variables (IV) are frequently considered an option (Angrist, Imbens, & Rubin, 1996). The objective of IV regressions is to locate an instrument that is correlated with the independent variable of interest (e.g., family ownership) but is not directly correlated with the dependent variable (e.g., family firm performance).

Bayesian methods allow assessing to what degree violations of the strict validity assumptions of instruments affect the estimation results of IV regressions (Hoogerheide, Block, & Thurik, 2012). To this end, the Bayesian researcher may assume a tight prior near zero for the instrument's direct effect on the dependent variable and subsequently considers priors that allow for an increasing direct effect.

Bayesian analysis also helps the researcher to assess the severity of endogeneity problems (Block, Hoogerheide, & Thurik, 2012). Thus, the researcher is able to learn whether endogeneity is actually present and to what degree it influences the estimation results. Instead of showing the results of a statistical test for endogeneity (as is done in NHST), Bayesian methods provide a full distribution of the correlation between the suspected endogenous variable and the error term. To assess the quantitative effect of endogeneity regarding estimation results, a Bayesian researcher compares the distribution of the parameter of interest obtained from a simple regression against the distribution obtained from an IV model. If the differences are large, endogeneity not only is present but also has a substantial effect on the results.

5.4. Hierarchical Bayes models

Family firm research frequently involves different levels of analysis (e.g., the firm, the family and its members, and the owners) (e.g., Gersick, Davis, & Lansberg, 1997). Using group level instead of individual level data can lead to aggregation bias. To overcome this bias, utilising multi-level or hierarchical analysis is suggested. Bayesian analysis has strong properties regarding multi-level or hierarchical analysis. Hierarchical Bayes models (so-called HB models), which are hierarchical models analysed using Bayesian methods, allow a researcher to account for data uncertainty at different levels of analysis. Because of its strong small sample properties and its ability to address data uncertainty, hierarchical Bayes models are well suited to calculate within-unit effects and overcome aggregation bias. By means of Bayesian methods, the researcher can thus calculate the effects of particular administrative decisions for specific family firms and account for industry- and country-level differences.

5.5. Predictive statements

Bayesian analysis has strong predictive properties. It uses an entire distribution function to make predictive statements and states an exact probability of occurrence for each event; these characteristics distinguish it from NHST, which uses point estimates together with confidence intervals to make predictions. With Bayesian estimates, decision-makers can make probability statements about decisions and account for uncertainty. This feature can have practical relevance for family firms to quantify the consequences of different scenarios (e.g., different types of firm succession, failure of different types of family firms under different conditions, the addition of new generations of family owners or board members). Moreover, Bayesian analysis allows researchers to compare the relative effects on these outcomes of closely related family firm governance factors.

5.6. Testing for non-existing relationships

As discussed above, NHST can never conclude that there is no relationship between two variables. Particularly with small sample sizes, the NHST researcher cannot rule out that a non-significant result is the result of the sample size being too small or because of multicollinearity problems. In a strict sense, the same is also true for Bayesian analysis. However, Bayesian analysis enables the researcher (at a minimum) to conclude that a given variable has little or no effect on an outcome variable in the *particular* data that are analysed. In that case, the probability of a positive (vs. negative) effect of a variable on an outcome variable would be approximately 50%. For example, it might be established that family firm performance or innovation outcomes are not influenced by generational changes beyond the second generation.

This characteristic of Bayesian methods also can be used to compare the characteristics of family firms with those of other firms. For example, it might be discovered that there is no difference in the acquisition strategies of family firms and non-family firms.

5.7. Bayesian meta-analyses

There are two meta-analyses regarding the performance of family firms versus other firms (O'Boyle, Pollack, & Rutherford, 2012; Carney, van Essen, Gedajlovic, & Heugens, 2013). These meta-analyses, however, do not provide a clear answer about whether family firms exhibit superior performance over other firms; typically, such meta-analyses suffer from small sample sizes (i.e., a small number of primary studies) and inconsistencies regarding family firm definitions and performance measurements. A Bayesian meta-regression technique (van Houwelingen, Arends, & Stijnen, 2002; Thompson & Higgins, 2002) can be used to overcome these method-related problems and may be able to determine whether and when family firms exhibit superior or inferior performance. At a minimum, Bayesian methods allow researchers to take into account the heterogeneity of primary studies on family business performance and conclude that there is little or no relationship between family firms and firm performance (see Section 5.6).

5.8. Bayesian methods as robustness checks

Bayesian analysis can be used in combination with NHST and thereby serve as a useful robustness check when there may be problems affecting the efficiency of estimators such as heteroscedasticity or multicollinearity. Bayesian analysis can also be used to establish the robustness of results across different definitions of family firms. This form of robustness check has become commonplace in finance research into the financial performance of family firms (see, for example, Miller, Le Breton-Miller, Lester, & Cannela, 2007). Unlike NHST, Bayesian analysis is not limited to statistical significance tests in making statements about the robustness and replication of results across different definitions of family firms.

5.9. Bayesian methods and replication studies

Unlike in the medical or natural sciences, there are few replication studies in management research, and family firm research is no exception. Replication studies using NHST face the problem of finding criteria to decide whether a certain result has been replicated or not (e.g., regarding level of significance, effect size, etc.) and whether the test for replication can be considered "fair". Bayesian methods offer a different perspective on replication. The Bayesian researcher does not need to define criteria of whether a result has been replicated or not. Instead, she includes the results of prior empirical research in the formulation of the prior and uses data to update the prior. Thus, the researcher is not required to state whether a finding from prior research has been replicated or not. Instead, she can present an updated view of the cumulative knowledge in the field.

6. Illustrative example comparing NHST and Bayesian analysis

We will use an example taken from Block, Miller, & Jaskiewicz (2011) to compare NHST and Bayesian analysis. That article investigates the specific effects of family ownership and family management on firm performance using Bayesian random-effects regressions. Figs. 1 and 2 display the results of these analyses. Fig. 1 shows the posterior distribution of the effect of family ownership. The figure shows that family ownership exerts a positive effect on firm performance (measured as the log of market-to-book value); the probability of a positive effect is 96%, and the median effect is $\beta = 0.43$.

Fig. 2 represents the posterior distribution of the effect of family management and shows that family management has a neutral effect of firm performance; that the probability of a positive effect is 42%; and that the median effect is $\beta = -0.01$. Running the same regression model with NHST (see Table A1 in Block et al., 2011), we obtain an effect of $\beta = 0.33$ ($p = 0.056$, two-sided test) for family ownership and $\beta = -0.032$ ($p = 0.445$, two-sided test) for family management. Thus, following conventional NHST reasoning and employing a significance level of 5%, we would not be able to make a statement about the specific effects of family management and family ownership on firm performance. Importantly, using the Bayesian analysis allowed us to make such a statement.

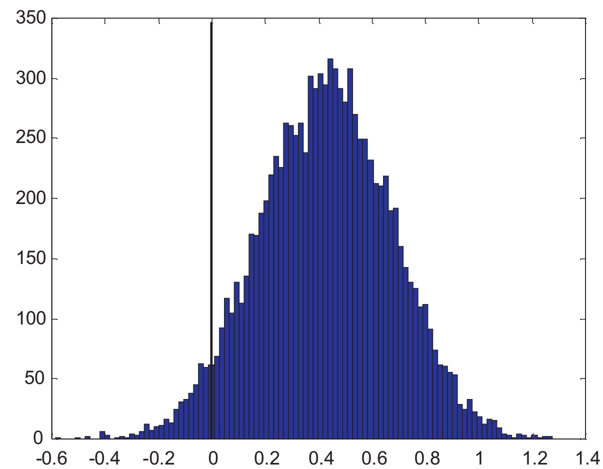

Fig. 1. The performance effect of *ownership by family*.
Note: The figure shows the (posterior) distribution of the variable *ownership by family* and is based on the random-effects regression shown in Table 4 of Block et al. (2011). The probability that the variable exerts a positive effect is 96%. The median effect is $\beta = 0.43$.

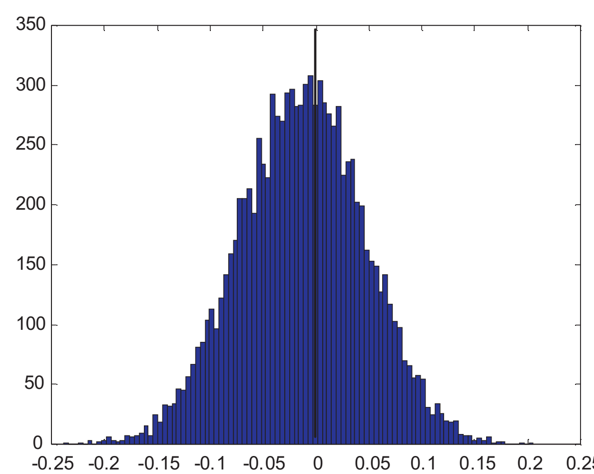

Fig. 2. The performance effect of *management by family*.
Note: The figure shows the (posterior) distribution of the variable *management by family* and is based on the random-effects regression shown in Table 4 of Block et al. (2011). The probability that the variable exerts a positive effect is 42%. The median effect is $\beta = -0.01$.

7. Short guide for how to use Bayesian methods in family business research

This section describes the basic steps for performing a Bayesian analysis in the context of family business research. In addition, we outline how to report the results of Bayesian methods and briefly comment on the software packages that are available to conduct Bayesian analysis.

1. *Need and justification for Bayesian methods*: To begin, the researcher must assess whether Bayesian methods are actually necessary and can provide a benefit compared with NHST. Often, the researcher should justify why s/he uses Bayesian methods, particularly because many editors and reviewers are not

Please cite this article in press as: Block, J. H., et al. Bayesian methods in family business research. *Journal of Family Business Strategy* (2014), http://dx.doi.org/10.1016/j.jfbs.2013.12.003

particularly familiar with Bayesian methods. The following points and questions can help the researcher justify the use of Bayesian methods: Is the sample skewed or small? Is multicollinearity or autocorrelation present? Are research questions severely restricted by such issues? Does the researcher seek to compare empirical results across different definitions of family firms? To what degree would the research benefit from obtaining a full distribution of effects instead of point estimates only? Does the empirical model suffer from serious endogeneity problems?

2. *Empirical model*: After selecting a Bayesian approach, the researcher must formulate an empirical model. This formulation should be guided by theory and data availability. The choice is similar to that under NHST. The following questions should be addressed: Is the relationship between the variables of interest linear or non-linear? Are the data cross-sectional, longitudinal or both (i.e., panel data)? Which dependent variable, independent variables, and control variables should be utilised?
3. *Assumptions about the prior*: This step is unique and crucial for Bayesian methods and has no counterpart in NHST. The choice of the prior can have a strong influence on the results of Bayesian analysis and should therefore be made carefully (Van Dongen, 2006). The researcher must make explicit assumptions about the parameter that is investigated (e.g., the effect of variable A on variable B). This assumption is called the prior and is updated with data gathering in the course of the Bayesian analysis. The researcher can choose between an informative or non-informative prior. When using an informative prior, the family business researcher should attempt to incorporate family business theory or prior empirical family business research findings to justify a particular distribution of the parameter that is being investigated. Another possibility might be to include results from qualitative interviews or case studies. Typically, the researcher assumes a mean and a standard deviation of a prior distribution. If the researcher does not want to incorporate prior research or subjective beliefs about the parameter of interest into the model, s/he can choose a non-informative prior such as a flat distribution function that weighs every possible outcome equally. Thus, the influence of the prior distribution on the posterior distribution will be minimal. Using a non-informative prior versus an informative resembles NHST in the sense that the researcher's prior beliefs about the parameter do not influence the results (Gill, 2008).
4. *Assumption about likelihood function*: The likelihood function reflects the probability of the data given the unknown parameters. The likelihood function is used together with the data to update the prior. The result is the posterior distribution, which is what Bayesians typically report in a paper. In Bayesian analysis, the researcher must assume a particular likelihood function for each parameter of interest, i.e., she assumes a family of probability distributions, one for each parameter. The choice of the "correct" likelihood function is more art than science. The choice should reflect the economic model that is under investigation and must allow the researcher to calculate a posterior distribution. Fig. 3 shows how the prior and the likelihood function lead to the posterior.
5. *Bayes theorem and posterior distribution*: Bayes theorem (see formulas (1) and (2) above) is used in calculating the posterior distribution. The resulting posterior distribution is frequently multidimensional, i.e., it encompasses several parameters in one distribution. The researcher, however, wants to learn about the specific distribution of one particular parameter. Because these specific distributions cannot be deduced analytically by using, for example, methods of numerical integration, a simulation approach is typically employed. Generally, this simulation is conducted by Markov Chain Monte Carlo Techniques (MCMC) (Gilks, Richardson, & Spiegelhalter, 1996) in combination with a Gibbs Sampler to arrive at the corresponding univariate distributions of the coefficients. The number of draws used to initialise the MCMC sampler is referred to as the burn-in draws. These burn-in draws frequently oscillate a substantial amount and are therefore not included in the simulation of the posterior distribution. The convention is to assume that a sampler has converged at approximately 1000 draws.

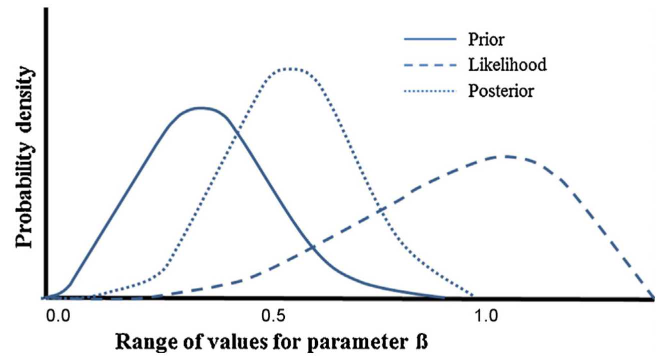

Fig. 3. Prior, posterior distribution influenced by the likelihood (data), adapted from Zyphur and Oswald, 2013, p. 7.

6. *Robustness tests*: It is common to check the sensitivity of the results to the specific assumptions about the prior, i.e., a researcher should assume different priors and compare the results across the different prior specifications. A robust result is obtained when the posterior distribution is not influenced by the choice of the prior.
7. *Reporting the results of Bayesian analysis* (i.e., *the posterior distribution*): In addition to displaying the specific posterior distributions of the parameters of interest graphically, the researcher should describe the posterior distribution with descriptive statistics. Typically, a Bayesian researcher reports the mean, the median, and the 5th and 95th quantiles of the posterior distribution. Some Bayesians also report the percentage of the posterior distribution that is positive. Probabilities near 100% indicate probable positive effects of the independent variable on the dependent variable; those near 0% indicate probable negative effects (see, e.g., Hansen et al., 2004). Researchers could also report a highest density interval (HDI) (Kruschke, 2011, pp. 40–41), which includes 90%, 95% or 99% of the posterior distribution,[13] which allows the researcher to be 90%, 95%, or 99% certain that the parameter of interest lies within this interval. The researcher can also evaluate whether the null value is within a particular HDI or not, which resembles the conclusion from NHST (Zyphur & Oswald, 2013). Fig. 4 displays different posterior distributions and different HDIs. Panel a shows a posterior function with a positive mean effect, in which the null value lies within the 95% HDI (weak positive effect); panel b shows a posterior function with a positive mean effect, where the null value lies outside the 95% HDI (strong positive effect); panel c shows a posterior function with a negative mean effect in which the null value lies outside the 95% HDI (strong negative effect).
8. *The limitations of Bayesian analysis*: Of course, Bayesian analysis has limitations, which the Bayesian researcher should discuss whenever applicable. A primary limitation is that the results of Bayesian analysis apply only to the *specific* sample used. Unlike NHST, Bayesian analysis is not tied to the notion of sample and population, which reduces the generalisability of the results. Other limitations concern the difficulty of choosing the prior, the

[13] Other researchers use the term highest probability density, Chen and Shao (1999), highest posterior density interval, Koop (2010), or Bayesian confidence interval Levy (2012).

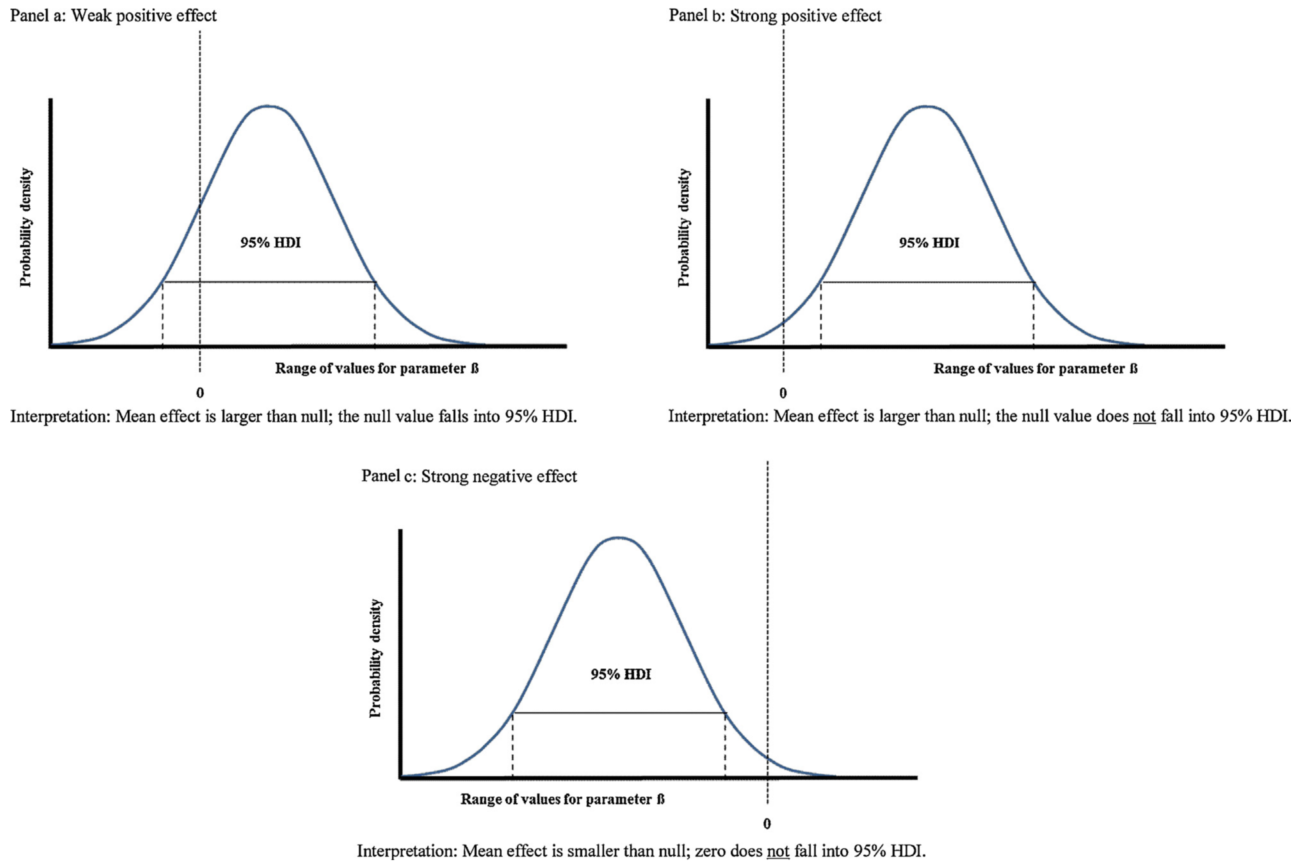

Fig. 4. Example posterior functions.
Panel a: Weak positive effect. Interpretation: Mean effect is larger than null; the null value falls into 95% HDI.
Panel b: Strong positive effect. Interpretation: Mean effect is larger than null; the null value does *not* fall into 95% HDI. Panel c: Strong negative effect. Interpretation: Mean effect is smaller than null; null does *not* fall into 95% HDI.

sensitivity of the posterior distribution to the choice of the prior, and the computational demands of the algorithm.

Several software packages can be used to conduct Bayesian analysis. The most widely used is the freely available BUGS family (WINBUGS, OPENBUGS, JAGS). [14] BUGS can be used in connection with the R software (see Kruschke, 2011, and Lancaster, 2008 for Bayesian tutorials with R and BUGS). In addition to BUGS and R, researchers can also use MatLab, MPlus, Sawtooth (particularly for hierarchical Bayes models), and SAS to conduct Bayesian analysis. However, these software packages are not free. Koop (2010) includes a MatLab tutorial (with MatLab code); the use of MPlus to conduct Bayesian analysis is described in Muthén (2010), Orme (2000) and Johnson (2000) refer to Sawtooth.

8. Conclusion

Bayesian methods are well suited to account for the heterogeneity that exists among family firms (Westhead & Howorth, 2007) due to its applicability in small samples, its ability to address multicollinearity problems, and the possibility to test the sensitivity of the results of an empirical analysis with the specific definition of the firm that is utilised. Thus, Bayesian methods can help family firm researchers investigate alternative definitions and outcomes associated with family firms. For these and other reasons discussed above, we believe that the time has come to make far more use of Bayesian methods in family business research.

[14] The BUGS project is described at http://www.mrc-bsu.cam.ac.uk/bugs (accessed 16.10.13).

Acknowledgements

We thank Christian Fisch for comments regarding earlier versions of the manuscript. We also thank the reviewers for their constructive and helpful comments.

References

Abdi, H. (2010). Partial least squares regression and projection on latent structure regression (PLS Regression). *Wiley Interdisciplinary Reviews: Computational Statistics, 2*(1), 97–106.

Angrist, J. D., Imbens, G. W., & Rubin, D. B. (1996). Identification of causal effects using instrumental variables. *Journal of the American Statistical Association, 91*(434), 444–455.

Armitag, P., Berry, G., Matthews, J., Armitage, P., Berry, G., & Matthews, J. N. S. (2009). *Statistical methods in medical research* (4th ed., reprinted 7). Malden, MA: Blackwell Science.

Ashby, D. (2006). Bayesian statistics in medicine: A 25 year review. *Statistics in Medicine, 25*(21), 3589–3631.

Bayes, T. (1763). An essay towards solving a problem in the doctrine of chances. *Philosophical Transactions of the Royal Society of London, 53*, 370–418.

Berkson, J. (1938). Some difficulties of interpretation encountered in the application of the chi-square test. *Journal of the Statistical Association, 33*(203), 526–536.

Berry, D. A. (2006). Bayesian clinical trials. *Nature Reviews Drug Discovery, 5*(1), 27–36.

Block, J., Hoogerheide, L., & Thurik, R. (2012). Are education and entrepreneurial income endogenous? A Bayesian analysis. *Entrepreneurship Research Journal, 2*(3), 1–27.

Block, J., Miller, D., & Jaskiewicz, P. (2011). Ownership versus management effects on performance in family and founder companies: A Bayesian reconciliation. *Journal of Family Business Strategy, 2*(4), 232–245.

Block, J., & Wagner, M. (2013). The effect of family ownership on different dimensions of corporate social responsibility: Evidence from large US firms. *Business Strategy and the Environment* http://dx.doi.org/10.1002/bse.1798.

Carney, M., van Essen, M., Gedajlovic, E. R., & Heugens, P. P. (2013). What do we know about private family firms? A meta-analytical review. *Entrepreneurship Theory and Practice* http://dx.doi.org/10.1111/etap.12054.

Chen, M.-H., & Shao, Q.-M. (1999). Monte Carlo estimation of Bayesian credible and HPD intervals. *Journal of Computational and Graphical Statistics, 8*(1), 69–92.
Cohen, J. (1994). The earth is round ($p < .05$). *American Psychologist, 49*(12), 997–1003.
Combs, J. (2010). Big samples and small effects: Let's not trade relevance and rigor for power. *Academy of Management Journal, 53*(1), 9–13.
Cousins, R. D. (1995). Why isn't every physicist a Bayesian? *American Journal of Physics, 63*(5), 398–410.
Edwards, W., Lindman, H., & Savage, L. (1963). Bayesian statistical inference or psychological research. *Psychological Review, 70*(3), 193–242.
Fisher, R. A. (1925). Applications of "Student's" distribution. *Metron, 5*, 90–104.
Gersick, K., Davis, J., & Lansberg, I. (1997). *Generation to generation: Life cycles of the family business*. Boston, MA: Harvard Business School.
Gigerenzer, G., Krauss, S., & Vitouch, O. (2004). The null ritual: What you always wanted to know about significance testing but were afraid to ask. *The Sage handbook of quantitative methodology for the social sciences*, Thousand Oaks (CA): Sage, 391–408.
Gill, J. (2008). Bayesian methods: A social and behavioral sciences approach *Statistics in the social and behavioral sciences series*, (2nd ed.). Boca Raton, FL: Chapman & Hall.
Gilks, W. R., Richardson, S., & Spiegelhalter, D. J. (1996). *Markov chain Monte Carlo in practice*. London: Chapman & Hall.
Grover, J. (2013). *Strategic economic decision-making: Using Bayesian belief networks to solve complex problems*. *Springer briefs in statistics* (Vol. 9). New York, NY: Springer.
Hahn, E. D., & Doh, J. P. (2006). Using Bayesian methods in strategy research: An extension of Hansen et al.. *Strategic Management Journal, 27*(8), 783–798.
Hair, J. F., Hult, G. T. M., Ringle, Ch.M., & Sarstedt, M. (2014). *A primer on partial least squares structural equation modeling*. Thousand Oaks, CA: Sage.
Hansen, M. H., Perry, L. T., & Reese, C. S. (2004). A Bayesian operationalization of the resource-based view. *Strategic Management Journal, 25*(13), 1279–1295.
Hoogerheide, L., Block, J., & Thurik, R. (2012). Family background variables as instruments for education in income regressions: A Bayesian analysis. *Economics of Education Review, 31*(5), 515–523.
van Houwelingen, H., Arends, L., & Stijnen, T. (2002). Advanced methods in meta-analysis: Multivariate approach and meta-regression. *Statistics in Medicine, 21*, 589–624.
Hubbard, R., & Bajarri, M. J. (2003). Confusion over measure of evidence (*p*'s) versus errors (alphas) in classical statistics testing. *American Statistician, 57*(3), 171–178.
Huelsenbeck, J. P. (2001). Bayesian inference of phylogeny and its impact on evolutionary biology. *Science, 294*(5550), 2310–2314.
Johnson, R. M. (2000). *Understanding HB: An intuitive approach*. Sequim, WA: Sawtooth Software Inc.
Koop, G. (2010). *Bayesian econometrics*. Chichester: Wiley.
Kruschke, J. K. (2011). *Doing Bayesian data analysis. A tutorial with R and BUGS*. Oxford: Elsevier.
Kruschke, J. K., Aguinis, H., & Joo, H. (2012). The time has come: Bayesian methods for data analysis in the organizational sciences. *Organizational Research Methods, 15*(4), 722–752.
Lancaster, T. (2008). *An introduction to modern Bayesian econometrics ([reprint.])*. Malden, MA: Blackwell.
Leamer, E. E. (1973). Multicollinearity: A Bayesian interpretation. *Review of Economics and Statistics, LV*(3), 371–380.
Levy, R. (2012). *Probabilistic models in the study of language*.
Miller, D., Le Breton-Miller, I., Lester, R. H., & Cannela, A. A. (2007). Are family firms really superior performers? *Journal of Corporate Finance, 13*(5), 829–858.
Muthén, B. (2010). *Bayesian Analysis in Mplus: A brief introduction*.
O'Boyle, E. H., Pollack, J. M., & Rutherford, M. W. (2012). Exploring the relation between family involvement and firms' financial performance: A meta-analysis of main and moderator effects. *Journal of Business Venturing, 27*(1), 1–18.
Orme, B. (2000). Hierarchical Bayes: Why all the attention? Quirk's Marketing Research Review. *14*(3), 16–63.
Pieper, T. (2007). *Mechanisms to assure long-term family business survival*. Frankfurt: Peter Lang.
Rossi, P., & Allenby, G. (2003). Bayesian statistics and marketing. *Marketing Science, 22*(3), 304–328.
Sawyer, A. G., & Ball, A. D. (1981). Statistical power and effect size in marketing research. *Journal of Marketing Research, 18*(3), 275–290.
Schmidt, F. (1996). Statistical significance testing and cumulative knowledge in psychology. *Psychological Methods, 1*(2), 115–129.
Schwab, A., Abrahamson, E., Starbuck, W. H., & Fidler, F. (2011). Researchers should make thoughtful assessments instead of null-hypothesis significance tests. *Organization Science, 22*(4), 1105–1120.
Sharma, P., & Nordqist, M. (2007). A typology for capturing the heterogeneity of family firms. *Academy of Management Annual Meeting Proceedings, 8*(1), 1–6.
Smets, F., & Wouters, R. (2007). Shocks and frictions in US business cycles: A Bayesian DSGE approach. *American Economic Review, 97*(3), 586–606.
Shoemaker, J. S., Painter, I. S., & Weir, B. S. (1999). Bayesian statistics in genetics: A guide for the uninitiated. *Trends in Genetics, 15*(9), 354–358.
Stanley, T. D. (2005). Beyond publication bias. *Journal of Economic Surveys, 19*(3), 309–345.
Starbuck, W. H. (2006). *The production of knowledge. The challenge of social science research*. New York: Oxford University Press.
The Economist. (2006). *Bayes rules: A once-neglected statistical technique may help to explain how the mind works. Print edition (Jan 5th)*.
Thompson, S., & Higgins, J. (2002). How should meta-regression analyses be undertaken and interpreted? *Statistics in Medicine, 21*, 1559–1573.
Villalonga, B., & Amit, R. (2006). How do family ownership, control and management affect firm value? *Journal of Financial Economics, 80*(2), 385–417.
Van Dongen, S. (2006). Prior specification in Bayesian statistics: Three cautionary tales. *Journal of Theoretical Biology, 242*(1), 90–100.
Westhead, P., & Howorth, C. (2007). Types of private family firms: An exploratory conceptual and empirical analysis. *Entrepreneurship & Regional Development: An International Journal, 19*(5), 405–431.
Zyphur, M. J., & Oswald, F. L. (2013). Bayesian estimation and inference: A user's guide. *Journal of Management* http://dx.doi.org/10.1177/0149206313501200.

Please cite this article in press as: Block, J. H., et al. Bayesian methods in family business research. *Journal of Family Business Strategy* (2014), http://dx.doi.org/10.1016/j.jfbs.2013.12.003