Active galaxies can make axionic dark energy

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ABSTRACT

AGN jets carry helical magnetic fields, which can affect dark matter if the latter is axionic. This preliminary study shows that, in the presence of strong helical magnetic fields, the nature of the axionic condensate may change and become dark energy. Such dark energy may affect galaxy formation and galactic dynamics, so this possibility should not be ignored when considering axionic dark matter.

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1. Introduction

As supersymmetric particles have not been observed in the LHC yet, interest in axionic dark matter is increasing. Such dark matter has a loop-suppressed interaction with the electromagnetic field, which opens up observational possibilities that aim to exploit the photon-axion conversion in astrophysical magnetic fields. Many authors have considered the electromagnetic interaction of axion particles [1]. However, the effect of this interaction to the axionic condensate itself has been largely ignored, assuming that it is negligible. In this paper we investigate the effect of an helical magnetic field on an axionic condensate. We find that, if the magnetic field is strong enough, axionic dark matter is modified to lead to the violation of the strong energy condition and behave as dark energy.[1] Then we apply our findings to the helical magnetic fields in the jets of Active Galactic Nuclei (AGN). We find that the magnetic fields near the central supermassive black hole may be strong enough to make axionic dark energy and thereby affect galaxy formation and dynamics. We use natural units, for which $c = \hbar = 1$ and $m_P^{-2} = 8\pi G$, with $m_P = 2.4 \times 10^{18}$ GeV being the reduced Planck mass. For the signature of the metric we take $(+1, -1, -1, -1)$.

2. Electromagnetically dominated axionic condensate

The axion (or an axion like particle) field ϕ, at tree level has a coupling to a fermionic field ψ of the form $\phi\bar{\psi}\psi$. Therefore, the axion couples to the photon via a fermionic loop as

$$\mathcal{L}_{\phi\gamma} = -\frac{1}{4}g_{\phi\gamma}\phi F_{\mu\nu}\tilde{F}^{\mu\nu} = g_{\phi\gamma}\phi\, \boldsymbol{E} \cdot \boldsymbol{B}, \tag{1}$$

where $F_{\mu\nu}$ is the Faraday tensor, $\tilde{F}^{\mu\nu}$ is its dual and $\boldsymbol{E}$ and $\boldsymbol{B}$ are the electric and magnetic field respectively. In the above we have defined the dimensionful coupling $g_{\phi\gamma} = \alpha\,\mathcal{N}/2\pi f_a$, where the f_a is the Peccei-Quinn (PQ) scale, while $\alpha \simeq 1/137$. We will assume $\mathcal{N} = 1$. The Lagrangian density for the axion is

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}g_{\phi\gamma}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}, \tag{2}$$

with

$$V(\phi) = m^2 f_a^2\left[1 - \cos\left(\frac{\phi}{f_a}\right)\right], \tag{3}$$

where m is the axion mass.[2] If inflation occurred after the PQ transition, the axion field is homogenised.[3] For a homogeneous axion

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[1] Axionic dark energy has been proposed before, see for example Ref. [2].

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[2] Such a potential has been utilised to realise cosmic inflation, in Natural Inflation originally in Ref. [3] considering string axions. Much more recently, inflationary models considering also the axial, Chern-Simons term $\propto F\tilde{F}$, have been considered in Gauge-flation [4] and Chromo-natural Inflation [5]. The axial term has also been employed to generate steep inflation with parity-violating gravitational waves [6].

[3] Alternatively, we may have production of a network of cosmic strings, which however, we will not consider further since the scaling solution results in a few open strings per horizon volume anyway, so the field is aligned for distances ~ 100 Mpc, unless one is near a cosmic string.

in an expanding Universe, the Klein-Gordon equation of motion is

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = g_{\phi\gamma} \boldsymbol{E} \cdot \boldsymbol{B}, \tag{4}$$

where $H(t)$ is the Hubble parameter, the dot denotes derivative with respect to the cosmic time and the prime denotes derivative with respect to ϕ. In the early Universe the electromagnetic source term in the above may be neglected[4].

Originally, the axion mass is zero and the vacuum manifold is flat. However, at the quark confinement phase transition, QCD instantons tilt the vacuum manifold (for $\mathcal{N} = 1$) and generate a sinusoidal potential for the axion, as shown in Eq. (3). For sub-Planckian f_a, the axion begins coherently oscillating immediately after the phase transition, with original amplitude $\sim f_a$ [7]. However, very soon after the onset of the oscillations $\phi \ll f_a$ and the potential becomes quadratic $V \simeq \frac{1}{2}m^2\phi^2$. Soon the period of oscillations becomes exponentially smaller than the Hubble time, so that the expansion of the Universe can be ignored for timescales shorter than H^{-1}. Then, Eq. (4) becomes $\ddot{\phi} + m^2\phi \simeq 0$ with solution

$$\phi = \Phi \cos(mt + \beta), \tag{5}$$

with Φ being the oscillation amplitude and β being an initial phase. Over cosmological timescales the Hubble friction term in Eq. (4) cannot be neglected, so that $\Phi = \Phi(t)$. A homogeneous scalar field oscillating coherently with a quadratic potential can be regarded as pressureless matter whose density scales as [7]

$$\rho_\phi = \frac{1}{2}m^2\Phi^2 \propto a^{-3} \propto T^3 \Rightarrow \Phi \sim f_a \left(\frac{T}{\Lambda_{qcd}} \right)^{3/2}, \tag{6}$$

where the scale factor is $a \propto 1/T$ and the temperature at the onset of the oscillations is determined by the scale of the QCD transition Λ_{qcd}. We have also assumed that m remains constant but this is strictly speaking not true because $m = m(T)$ initially grows while the oscillations are not harmonic near their onset, so Eq. (6) is off by a few orders of magnitude. In fact, the axion mass is estimated as [7]

$$m \sim \frac{\Lambda_{qcd}^2}{f_a} \Rightarrow \frac{m}{10^{-5}\,\text{eV}} \sim \frac{10^{12}\,\text{GeV}}{f_a}, \tag{7}$$

where $\Lambda_{qcd} \sim 100\,\text{MeV}$. Since the axion is to be the observed dark matter, its present density is $\rho_{DM0} \simeq 0.3\rho_0 \sim 10^{-30}\,\text{g/cm}^3$. Because $\rho_{DM0} = \frac{1}{2}(m\Phi_0)^2$, we find $\Phi_0 \sim 10^{-24}\,\text{GeV}^2/m$, where Φ_0 is the average value of Φ today. Using, Eq. (7) we readily find

$$\Phi_0 \sim 10^{-1}\left(\frac{f_a}{10^{12}\,\text{GeV}}\right)\text{eV}. \tag{8}$$

Now, suppose that the electromagnetic source term in Eq. (4) cannot be neglected. In this case, the oscillating axion may have a spatial dependence as well, due to the helical magnetic field. Considering timescales much smaller than H^{-1} and assuming Minkowski spacetime for the moment, the equation of motion becomes

$$\ddot{\phi} - \nabla^2\phi + m^2\phi = g_{\phi\gamma} \boldsymbol{E} \cdot \boldsymbol{B}. \tag{9}$$

To keep things simple, let us consider that the electromagnetic source term has no time dependence. This implies that we consider the magnetic field of gravitationally bound structures (such as galaxies) which do not expand with the Universe expansion. Then, the solution to the above is of the form

$$\phi = \Phi \cos(mt + \beta) + Q, \tag{10}$$

where $\dot{Q} = 0$. Putting this in Eq. (9) we have

$$(m^2 - \nabla^2)Q = g_{\phi\gamma} \boldsymbol{E} \cdot \boldsymbol{B}. \tag{11}$$

Now, let's assume that the characteristic size of the helical magnetic field configuration is much larger than the axion Compton wavelength. Then $\nabla^2 Q \ll m^2 Q$, so we find

$$Q \simeq g_{\phi\gamma} \frac{\boldsymbol{E} \cdot \boldsymbol{B}}{m^2}. \tag{12}$$

For the energy-momentum tensor we have

$$T_{\mu\nu} = 2\frac{\partial \mathcal{L}}{\partial g^{\mu\nu}} - g_{\mu\nu}\mathcal{L} \tag{13}$$

The axion-photon coupling in Eq. (1) can be written as

$$\mathcal{L}_{\phi\gamma} = \frac{1}{2}g_{\phi\gamma}\phi \frac{\epsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}}F_{\mu\nu}F_{\rho\sigma} \tag{14}$$

where $g \equiv \det(g_{\mu\nu})$ and $\epsilon^{\mu\nu\rho\sigma}$ is the Levi-Civita symbol in flat spacetime ($= 0, \pm 1$). Because $\frac{\partial}{\partial g^{\mu\nu}}\left(\frac{1}{\sqrt{-g}}\right) = \frac{1}{2}\frac{g_{\mu\nu}}{\sqrt{-g}}$ it can be shown that $2\frac{\partial \mathcal{L}_{\phi\gamma}}{\partial g^{\mu\nu}} = g_{\mu\nu}\mathcal{L}_{\phi\gamma}$. In view of Eq. (13), this implies that $\mathcal{L}_{\phi\gamma}$ does not contribute to $T_{\mu\nu}$.[5] Naïvely, one might think that the electromagnetic field cannot influence the energy-momentum of the condensate, but this is incorrect because the effect of the helical magnetic field is included in the Q-dependence of ϕ in Eq. (10).

Considering $\mathcal{L}$ as given in Eq (2) but ignoring $\mathcal{L}_{\phi\gamma}$, we find for the energy density of the axion condensate

$$\rho_\phi = T_{00} = \frac{1}{2}[\dot{\phi}^2 + (\nabla\phi)^2] + \frac{1}{2}m^2\phi^2, \tag{15}$$

The pressure corresponds to the spatial components of $T_{\mu\nu}$. The off diagonal spatial components are simply $T_{ij} = \partial_i\phi\,\partial_j\phi$, where $i, j = 1, 2, 3$ and $i \neq j$. The diagonal components (the principal pressures) are given by $T_{ii} = (\partial_i\phi)^2 + p_\phi$ with

$$p_\phi = \mathcal{L} - \mathcal{L}_{\phi\gamma} = \frac{1}{2}[\dot{\phi}^2 - (\nabla\phi)^2] - \frac{1}{2}m^2\phi^2. \tag{16}$$

The spatial gradients of the axion configuration mirror the spatial gradients of the magnetic field. If the characteristic dimensions of the latter are much larger than the axion Compton wavelength then, on average, $|\partial_i\phi| \ll m|\phi|$. This means that the off diagonal components of $T_{\mu\nu}$ become negligible and the principal pressures all become approximately equal to p_ϕ in Eq. (16). We insert Eq. (10) in the above to find

$$\bar{\rho}_\phi \simeq \frac{1}{2}m^2\Phi^2 + \frac{1}{2}m^2Q^2 \quad \text{and} \quad \bar{p}_\phi \simeq -\frac{1}{2}m^2Q^2, \tag{17}$$

where $\bar{\rho}_\phi$ and $\bar{p}_\phi$ is the average density and pressure over many oscillations such that $\overline{\cos\omega} = 0$ and $\overline{\sin^2\omega} = \overline{\cos^2\omega} = \frac{1}{2}$ with $\omega = mt + \beta$ and we considered a magnetic field configuration with typical scale larger than m^{-1} so that $(\nabla Q)^2 \ll (mQ)^2$.

It is evident that, in the absence of a magnetic field, we have $\bar{p}_\phi = 0$ so the axionic condensate behaves as dark matter with $\rho_\phi = \frac{1}{2}m^2\Phi^2$. However, we will assume that a helical magnetic field is present. If the magnetic field contribution in the above is dominant (i.e. $\Phi^2 < 2Q^2$) we have $\bar{p}_\phi/\bar{\rho}_\phi < -\frac{1}{3}$ and the axionic condensate violates the strong energy condition and behaves as dark energy. In effect, when $Q^2 \gg \Phi^2$ the helical magnetic field halts in its track the oscillating condensate such that $\phi \simeq Q$. The condensate density becomes $\rho_\phi \simeq V(Q) \simeq \frac{1}{2}m^2Q^2 \simeq$ constant (in time). Thus, the axionic condensate becomes potentially dominated and acts similarly to the inflaton condensate during slow-roll inflation. Note however that, from Eq. (17) we have $\bar{\rho}_\phi + \bar{p}_\phi =$

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[4] Unless an extremely strong, helical primordial magnetic field is present. We will not consider this possibility here.

[5] This is why $\mathcal{L}_{\phi\gamma}$ is sometimes called "topological".

$\frac{1}{2} m^2 \Phi^2 > 0$, as with the case of axionic dark matter since the contribution of the magnetic field cancels out.[6] Thus, the condensate does not violate the null energy condition.

A non-oscillatory scalar field condensate does not have a particle interpretation. So, what happens to the axion particles, which the oscillatory condensate coresponds to, when an intense helical magnetic field is present? Eq. (10) suggests that, even when Q is sizable, the axion oscillations continue, with the same frequency but not arround zero; around Q instead. Because the effect of the helical magnetic field is additive in Eq. (10), we expect that the axion dark matter particles coexist with a smooth axion field component, like photons travelling inside a constant electromagnetic field. The smooth axionic condensate is gravitationally repulsive (as dark energy is) so we would expect axion dark matter particles to be driven away from the axionic dark energy.

Therefore, the conditions necessary for turning axionic dark matter into dark energy are

$$(\nabla Q)^2 \ll (m\,Q)^2 \qquad \text{and} \qquad 2Q^2 > \Phi^2, \tag{18}$$

where Q is determined by the helical magnetic field as shown in Eq. (12).

3. Axionic matter in AGNs

It so happens that the conditions in Eq. (18) may be satisfied in AGN jets. Observations suggest that AGN jets feature powerful helical magnetic fields [8]. Most spiral galaxies are assumed to go through the AGN phase when their central supermassive black hole is formed. The AGN jet can be huge in length (up to Mpc scales). Its spine, however, is narrow; about $d \sim (10^{-5} - 50)\,\text{pc}$, which, however, is typically much larger than the axion Compton wavelength m^{-1}. Therefore, $(\nabla Q)^2 \ll (m\,Q)^2$ is satisfied. Now, the AGN spine is electromagnetically dominated, so that $2Q^2 > \Phi^2$ seems reasonable. Let us estimate this.

The helicity of the magnetic field along the AGN jet is thought to be due to the rotation of the accretion disk [9]. It can be modelled as a longitudinal (poloidal) field $\boldsymbol{B}_\parallel$ and a transverse (toroidal) field $\boldsymbol{B}_\perp$, which is a Biot-Savart field due to the current along the jet, such that $\boldsymbol{J} = \nabla \times \boldsymbol{B}_\perp$, where $\boldsymbol{J} = \sigma\,\boldsymbol{E}$ is the current density and σ is the plasma conductivity. Thus, from Eq. (12) we have

$$Q = \frac{\alpha \eta}{2\pi f_a m^2}(\nabla \times \boldsymbol{B}_\perp) \cdot \boldsymbol{B} \Rightarrow |Q| \sim \frac{\alpha \eta}{f_a m^2}\frac{B_\perp B_\parallel}{d}, \tag{19}$$

where $\eta = 1/\sigma$ is the plasma resistivity. The above needs to be compared with Φ, the amplitude of the oscillating axionic condensate.

The average amplitude at present is given by Eq. (8). However, inside galaxies, the dark matter density is expected to be much larger than the average density of dark matter at present ρ_{DM0}. Indeed, a typical estimate for dark matter in the vicinity of the Earth is $\rho_{\text{DME}} \simeq 0.3\,\text{GeV/cm}^3$, which is about $10^5 \rho_{\text{DM0}}$. Depending on the dark matter halo model, the density of dark matter can be ~ 50 times bigger in the core. Because $\rho_{\text{DM}} \propto \Phi^2$ this means that the core value of Φ can be $\Phi_C \sim 10^3 \Phi_0$.

From Eqs. (7), (8) and (19), it is straightforward to show that

$$\left.\frac{|Q|}{\Phi}\right|_C \sim \frac{\eta}{d}\frac{(B_\perp B_\parallel)_{\text{core}}}{10^{14}\,\text{G}^2}, \tag{20}$$

where the dependence on the Peccei-Quinn scale f_a cancels out.

The onset of the jet is near the event horizon. To suppress relativistic corrections, we choose $r_0 \sim 10^2 r_S$ for the onset of the jet,

where $r_S = 2GM_{\text{BH}}$ is the Scwarzschild radius

$$r_S \sim 10^{-5}\left(\frac{M_{\text{BH}}}{10^8 M_\odot}\right)\text{pc}. \tag{21}$$

Now, flux conservation suggests that $B_\perp \propto 1/r$ and $B_\parallel \propto 1/r^2$, where r denotes the distance from the central supermassive black hole [9,10]. Therefore,

$$\frac{(B_\perp B_\parallel)_{\text{core}}}{(B_\perp B_\parallel)_{\text{lobe}}} \sim \left[\frac{1\,\text{Mpc}}{10^{-3}\,\text{pc}}\left(\frac{10^8 M_\odot}{M_{\text{BH}}}\right)\right]^3 = 10^{27}\left(\frac{10^8 M_\odot}{M_{\text{BH}}}\right)^3, \tag{22}$$

where we assumed that the jet is about 1 Mpc long. The *minimum* magnetic field at the lobes is of equipartition value $\sim \mu\text{G}$ [9]. So, $(B_\perp B_\parallel)_{\text{lobe}} \sim 10^{-12}\,\text{G}^2$. Therefore,

$$\left.\frac{|Q|}{\Phi}\right|_C \sim 10\,(\eta/d) \times \left(\frac{10^8 M_\odot}{M_{\text{BH}}}\right)^3. \tag{23}$$

The resistivity of the plasma is still an open issue, although it is implicitly estimated in MHD simulations. This is because the plasma in jets is far from thermal equilibrium, while it is still uncertain which particles are the primary charge carriers for the current. An estimate, based on numerical investigation, is provided by [11]

$$\frac{\eta}{10^{14}\,\text{m}^2\text{s}^{-1}} \sim \sqrt{\frac{M_{\text{BH}}}{M_\odot}\frac{r_0}{r_\odot}} \Rightarrow \eta \sim 10^{-4}\,\text{pc} \times \left(\frac{M_{\text{BH}}}{10^8 M_\odot}\right), \tag{24}$$

where we considered $r_0 \sim 10^2 r_S$. Assuming the width of the jet spine near the core is $d \sim r_0$, we have

$$\left.\frac{|Q|}{\Phi}\right|_C \sim \left(\frac{10^8 M_\odot}{M_{\text{BH}}}\right)^3. \tag{25}$$

Thus, we see that the axionic condensate can become dark energy near the AGN core if $M_{\text{BH}} \lesssim 10^8 M_\odot$, which is quite plausible. For example, the supermassive black hole in the centre of the Milky Way has mass $\sim 10^6 M_\odot$.[7]

Furthermore, from Eq. (7) we find

$$m^{-1} \sim 10^{-18}\left(\frac{f_a}{10^{12}\,\text{GeV}}\right)\text{pc} \tag{26}$$

Therefore, $m^{-1} \ll d$ for sub-Planckian f_a and the magnetic field configuration is safely much larger than the axionic Compton wavelength.

In general, we expect to be in Kerr spacetime. Jets are projected along the directions of the poles, which implies $\sin\theta \simeq 0$. Then, the Kerr metric reduces to

$$ds^2 = \frac{\Delta}{r^2}dt^2 - \frac{r^2}{\Delta}dr^2 - r^2 d\theta^2, \tag{27}$$

where $\frac{\Delta}{r^2} = 1 - \frac{r_S}{r} + \left(\frac{a}{r}\right)^2$, with $a = GJ/M_{\text{BH}}$; J being the angular momentum. Assuming that the black hole is not extremal means $a < r_S$. So, taking $d \sim r_0 \sim 10^2 r_S$ ensures that relativistic corrections are small.

4. Conclusions

We have investigated the behaviour of axionic dark matter in the presence of a helical magnetic field and found that, when the condensate becomes electromagnetically dominated, it ceases to be dark matter and becomes dark energy instead. We have applied our findings in AGNs and showed that the helical magnetic

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[6] This can be readily seen from Eqs. (15) and (16), which give $\rho_\phi + p_\phi = \dot\phi^2 = m^2\Phi^2 \sin^2\omega$.

[7] Surprisingly, the effect is intensified for smaller black holes because the horizon size is smaller, while the AGN jets are always taken to be of Mpc scales. This, might be augmented when more realistic AGN jets are considered.

field along the AGN jets near the AGN core can be strong enough to convert axionic dark matter into dark energy. Lacing the AGN black holes with dark energy may have profound implications for galaxy formation and galactic dynamics (e.g. rotation curves), because dark energy is gravitationally repulsive. For example, it might assist with the central cusp problem of dark matter halos. Note that, in contrast to the evenly distributed dark energy, which dominates the Universe today, the axionic dark energy in AGNs is localised in the core parts of the AGN jets. Our study demonstrates that the issue warrants deeper investigation. It is likely that relativistic corrections may influence the results.

Acknowledgements

References

[1] A. Mirizzi, G.G. Raffelt, P.D. Serpico, Phys. Rev. D 72 (2005) 023501; A. Mirizzi, J. Redondo, G. Sigl, JCAP 0908 (2009) 001; M. Fairbairn, T. Rashba, S.V. Troitsky, Phys. Rev. D 84 (2011) 125019; F. Tavecchio, M. Roncadelli, G. Galanti, Phys. Lett. B 744 (2015) 375.
[2] S. Alexander, R. Brandenberger, J. Froehlich, ArXiv:1601.00057 [hep-th].
[3] K. Freese, J.A. Frieman, A.V. Olinto, Phys. Rev. Lett. 65 (1990) 3233; F.C. Adams, J.R. Bond, K. Freese, J.A. Frieman, A.V. Olinto, Phys. Rev. D 47 (1993) 426.
[4] A. Maleknejad, M.M. Sheikh-Jabbari, Phys. Lett. B 723 (2013) 224, doi:10.1016/j.physletb.2013.05.001; A. Maleknejad, M.M. Sheikh-Jabbari, J. Soda, JCAP 1201 (2012) 016; R. Namba, E. Dimastrogiovanni, M. Peloso, JCAP 1311 (2013) 045; C.M. Nieto, Y. Rodriguez, Mod. Phys. Lett. A 31 (21) (2016) 1640005. [arXiv:1102.1513[hep-ph]].
[5] P. Adshead, M. Wyman, Phys. Rev. Lett. 108 (2012) 261302; E. Dimastrogiovanni, M. Peloso, Phys. Rev. D 87 (10) (2013) 103501; P. Adshead, E. Martinec, M. Wyman, Phys. Rev. D 88 (2) (2013) 021302. JHEP 1309; (2013) 087; P. Adshead, E. Martinec, E. I. Sfakianakis M. Wyman, arXiv:1609.04025 [hep-th]
[6] L. Sorbo, JCAP 1106 (2011) 003; J.L. Cook, L. Sorbo, Phys. Rev. D 85 (2012) 023534; M.M. Anber, L. Sorbo, Phys. Rev. D 85 (2012) 123537; R. Namba, M. Peloso, M. Shiraishi, L. Sorbo, C. Unal, JCAP 1601 (01) (2016) 041. Erratum: Phys. Rev. D 86;(2012) 069901, JCAP 1311; (2013) 047
[7] E.W. Kolb, M.S. Turner, The early universe, Front. Phys. 69 (1990) 1.
[8] K. Asada, M. Inoue, Y. Uchida, S. Kameno, K. Fujisawa, S. Iguchi, M. Mutoh, Publ. Astron. Soc. Jap. 54 (2002) L39; D.C. Gabuzda, E. Murray, P. Cronin, Mon. Not. Roy. Astron. Soc. 351 (2004) L89; J.-L. Gomez, A.P. Marscher, S.G. Jorstad, I. Agudo, M. Roca-Sogorb, Astrophys. J. 681 (2008) L69; D.C. Gabuzda, A. Reichstein, E.L. O'Neill, Mon. Not. Roy. Astron. Soc. 444 (1) (2014) 172.
[9] J.H. Krolik, Active Galactic Nuclei: From the Central Black Hole to the Galactic Environment, Princeton Series in Astrophysics, Princeton Univ. Pr., 1999.
[10] A. Ferrari, Ann. Rev. Astron. Astrophys. 36 (1998) 539.
[11] M. Cemeljic, J. Gracia, N. Vlahakis, K. Tsinganos, Mon. Not. Roy. Astron. Soc. 389 (2008) 1022.