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The co-occurrence test for non-monotonic inference

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ABSTRACT

According to the co-occurrence test, q is (non-monotonically) inferrible from p if and only if q holds in all the reasonably plausible belief change outcomes in which p holds. A formal model is introduced that contains representations of both the co-occurrence test (for non-monotonic inference) and the Ramsey test (for conditionals). In this model, (non-nested) conditionals and non-monotonic inference satisfy the same logical principles. However, in spite of this similarity the two notions do not coincide. They should be carefully distinguished from each other.

1. Introduction

It is often assumed that the logic of non-monotonic inference coincides with that of "the flat (i.e. nonnested) fragment of a conditional logic" [17, p. 171]. Under the equally common assumption that conditionals are related to belief revision via the Ramsey test this means that the Ramsey test will be applicable to non-monotonic inference as well. This was also suggested by Makinson and Gärdenfors [19], according to whom "q follows non-monotonically from p" means that q holds if an "arbitrary but fixed background theory" (p. 189) is revised by p. Denoting that theory by K, belief revision by $*$, and non-monotonic consequence by $\mid\!\sim$ we obtain:

The Ramsey test for non-monotonic inference
$p \mid\!\sim q$ holds if and only if $q \in K * p$.

This proposal has been further investigated by Gärdenfors and Makinson [7], Wobcke [24], del Val [4], Rott [23, pp. 111–119], and many others. However conditionality and inferribility are distinct albeit related concepts. Even if both of them are connected with belief revision they need not be connected in the same way. It is the purpose of the present contribution to investigate what happens if we replace

Ramsey's criterion
If the agent revises her beliefs by p, then she will believe that q.

by the following alternative criterion:

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The co-occurrence criterion
If the agent comes to believe that p, then she will believe that q.

The two criteria differ since an agent can come to believe in p not only as the result of revising her beliefs by p but also as the result of revising them by some other input. According to the co-occurrence criterion q has to be an element not only of $K * p$ but also of other belief sets containing p. The criterion concerns whether we will *in general* (given our present epistemic commitments) believe in q if we come to believe in p, not only whether we will do so in one singular case. This seems to make it better aligned with the notion of inferribility (in contradistinction to conditionality) than what Ramsey's criterion is.

The co-occurrence test needs to be specified with respect to which of the belief sets containing p we should include in the analysis. A simple answer would be to include all potential belief change outcomes that contain p, i.e. all belief sets $K * r$ such that $p \in K * r$. However, such an approach would be inadequate since it does not reflect the essential feature of non-monotonic reasoning that comparatively remote possibilities are left out of consideration. When you conclude from "Tweety is a bird" that "Tweety can fly" that is precisely because you do not take remote possibilities into account. The degree of remoteness referred to here is relative to the antecedent. Some of the possibilities that are too remote to be taken into account when considering "Tweety is a bird" would be quite close at hand when considering "Tweety is a bird who was born in Antarctica".

To sum this up, when evaluating non-monotonic inferences with p as the antecedent we have to consider not only $K * p$ but also other, reasonably plausible, p-containing belief sets. This amounts to the following test of inferribility:

The co-occurrence test for non-monotonic inference
$p \mid\!\sim q$ holds if and only if q holds in all the p-satisfying belief change outcomes that are reasonably plausible as compared to other p-satisfying belief change outcomes.

It should be noted that this criterion does not exclude the existence of some belief change outcome in which $p\&\neg q$ holds. There can be some sentence r, less plausible than p, such that $p\&\neg q$ holds in some or all of the r-satisfying belief change outcomes. (For an example, let p denote that Bitsy is a female mammal, q that Bitsy gives birth to live young, and r that Bitsy is a platypus.)

One immediate difference between the Ramsey test and the co-occurrence test concerns the following property of conditionals: (The symbol $\rightarrowtail$ will be used to denote "standard" Ramsey test conditionals.)

Property CS
If p and q both hold then so does $p \rightarrowtail q$.

It is usually assumed that if $p \in K$, then $K * p = K$. From this it follows directly that a relation $\rightarrowtail$ that is based on $*$ via the Ramsey test will satisfy CS. However, an operation $\mid\!\sim$ based on the co-occurrence test will not in general do so. Even if $K * p = K$ and $q \in K$ there may be other reasonably plausible p-containing belief sets that do not contain q, and then $p \mid\!\sim q$ will not hold.

This difference between the two tests seems to correspond fairly well to a difference between conditionality and inferribility. To see that, let p denote that it rains in London today and q that Real Madrid wins the match they are playing tonight in Berlin. If I become convinced that both p and q are true, then I may arguably conclude that "if p then q".[1] However, it would be absurd to also conclude that "from p it can be inferred that q".[2] More generally speaking, inferribility seems to imply conditionality, but not the other way around.

In Section 2 a framework for belief revision will be presented that is useful for our present purposes since it contains a straightforward representation of the differences in plausibility between belief sets. In Section 3, representations of conditionality ($\rightarrowtail$) according to the Ramsey test and non-monotonic inference ($\mid\!\sim$) according to the co-occurrence test will be introduced into that framework. In Section 4 a theorem is presented, showing that they have (nearly) the same logic. Finally, in Section 5 the interpretation of that theorem is discussed and it is argued that the two concepts should nevertheless be distinguished from each other.

2. The formal framework

The analysis will be based on descriptor revision, a model of belief change that was introduced in [11]. Like most other formal models of belief change it employs a selection mechanism, but that mechanism does not operate on possible worlds

[1] CS holds in many systems of conditional logic, see for instance [18, pp. 26–31], [22, p. 249], and [21]. However, it has also been criticized, for instance by Bennett [2, pp. 386–388], and Nozick [20, p. 176].

[2] One way to explain the difference is that when determining whether or not q can be inferred from p we consider whether we would believe in q after some revision that made us believe in p, whereas when determining whether "if p then q" holds we only consider whether revision by p would have that effect.

or remainders but on the belief sets that are potential outcomes of change. Several such selection mechanisms are available [11–13]. Here the following simple construction will be used.

A *centrolinear model* is a pair $\langle \mathbb{X}, \leqq \rangle$, where $\mathbb{X}$ is a set of logically closed sets (belief sets) in the object language ($\mathcal{L}$), and $\leqq$ (with the strict part $<$) is an ordering on $\mathbb{X}$, such that for any sentence p, there is a unique $\leqq$-minimal element of $\mathbb{X}$ that contains p.[3]

A centrolinear model is *exhaustive* if and only if $\bigcup \mathbb{X} = \mathcal{L}$.

Intuitively, $\mathbb{X}$ consists of all the belief sets that are stable, coherent, or plausible enough to be outcomes of a belief change. $\leqq$ represents degrees of plausibility or closeness.

The use of a relation to guide choices among epistemic objects has a long tradition in formal epistemology. In his account of counterfactuals, David Lewis [18, p. 13] employed a notion of degrees of similarity to the actual world. The position of a possible world in Adam Grove's sphere system for belief change is said to represent its closeness to the set of worlds that are compatible with the agent's present beliefs [9, p. 159]. Peter Gärdenfors has proposed a criterion of informational economy, based on the idea that "when we, for some reason or other, change our beliefs we want to retain as much as possible of our old beliefs", in particular the more important of them [6, p. 136]. These and many other approaches in formal epistemology are based on comparisons of epistemic objects in terms of how much they differ from the agent's current beliefs in the respects deemed important. In formal languages, such comparisons are expressed with a relation. As was noted in the AGM article, such a relation will have to be the same for all inputs since otherwise all choices among the epistemic objects would be trivially relational [1, p. 518].

Many if not most approaches to belief change employ a relation to select among epistemic objects. However, there is a wide variation in the types of epistemic objects that these relations are applied to: possible worlds, (inclusion-maximal) remainders, sentences to remove etc. [15]. It is a weakness of many of these constructions that the relation operates on epistemic objects that are cognitively inaccessible, and in some cases the relation between the objects of selection and the actual outcome is far from transpicuous. To avoid these problems, this investigation will be performed in a framework where the relation operates directly on the potential belief outcomes themselves, rather than on intermediate objects to be used in a construction of the outcome. (For additional justification of this approach, see [11].) There are various ways in which such a relation can be interpreted. From a formal point of view, the crucial aspects of the construction are that the belief change is obtained by a direct choice among the potential belief sets, that this choice is relational, and that the relation is the same for all inputs. Informally, this relation ($\leqq$) will be referred to as a representation of the plausibility of the potential outputs. If an agent chooses the belief set X rather than the belief set Y as an outcome, then this can be interpreted as meaning that X is a more plausible belief set than Y. However, nothing hinges on this intuitive interpretation.

The $\leqq$-minimal element of $\mathbb{X}$ is assumed to be the original belief set. Following convention in the belief revision literature it will be denoted K. (Thus, $K \leqq X$ for all $X \in \mathbb{X}$.) For simplicity we will assume that the strict part of $\leqq$ is a wellordering with an order type that is either finite or ω.[4]

Belief revision has an obvious interpretation in this framework. When we revise by p we choose the $\leqq$-minimal potential outcome:

Belief revision in an exhaustive centrolinear model
$K * p$ is the $\leqq$-minimal element of $\mathbb{X}$ that contains p.

Three comments are in place before we proceed to introduce conditionality and inferribility into this framework.

First, a centrolinear model has to be exhaustive in order for the success postulate for revision ($p \in K * p$) to hold. Although good arguments can be given for relaxing that criterion [10], we will only consider exhaustive models here. In this way we avoid the rather uninteresting limiting cases of conditionality and inference with non-satisfiable antecedents ($p \rightarrowtail q$ and $p \mid\!\sim q$ when the epistemic agent cannot be brought to believe that p is true).

Secondly, the belief revision model presented here is a generalization of the "gold standard" in the belief revision literature [1], namely transitively relational partial meet revision (revision satisfying all the basic and supplementary AGM postulates). In other words, if $*$ is a transitively relational partial meet revision on the belief set K, then there is a centrolinear model $\langle \mathbb{X}, \leqq \rangle$ such that K is the $\leqq$-minimal element of $\mathbb{X}$ and that $*$ coincides with the belief revision derived from $\langle \mathbb{X}, \leqq \rangle$ according to the simple principle just referred to [14].

Thirdly, the model proposed here is a fragment of descriptor revision, a framework that provides a unified approach to a wide variety of belief change operations. In descriptor revision the inputs are not sentences in the object language but instead metalinguistic sentences constructed with a belief predicate $\mathfrak{B}$. For any sentence p in the object language, $\mathfrak{B}p$ signifies that p is believed. A belief descriptor is a set of molecular combinations of such $\mathfrak{B}$-sentences, for instance $\{\neg\mathfrak{B}p, \neg\mathfrak{B}\neg p\}$ means that neither p nor its negation is believed and $\{\mathfrak{B}p \vee \mathfrak{B}q\}$ that either p or q is believed. With this

[3] This means that $\leqq$ has to satisfy a well-foundedness condition, see [11, p. 959].

[4] This means that the strict part $<$ of $\leqq$ is either isomorphic with a finite string $\langle 0, 1, \ldots n \rangle$ of natural numbers or with the full infinite series $\langle 0, 1, 2 \ldots \rangle$ of natural numbers.

notation, a wide variety of belief change operations can be subsumed under a unified operation $\circ$. Hence $K \circ \{\neg\mathfrak{B}p\}$ is the $\leqq$-minimal element of $\mathbb{X}$ in which $\neg\mathfrak{B}p$ is satisfied (corresponding to contraction by p) and $K \circ \{\neg\mathfrak{B}p, \mathfrak{B}q\}$ is the $\leqq$-minimal element of $\mathbb{X}$ in which $\{\neg\mathfrak{B}p, \mathfrak{B}q\}$ is satisfied (corresponding to replacement of p by q). In this more general framework, $K * p$ is of course an abbreviation of $K \circ \{\mathfrak{B}p\}$.[5]

3. Introducing the two tests

We are now ready to introduce the two tests into the formal framework. The Ramsey test can be applied in the usual way:

The Ramsey test in an exhaustive centrolinear model
$p \rightarrowtail q$ holds if and only if $q \in K * p$.

For the co-occurrence test we need to identify, for each sentence p, a set of reasonably plausible belief sets containing p. We can assume that this set satisfies the following continuity property: If Z is a reasonably plausible p-containing set, and Y is a p-containing set such that $Y \leqq Z$, then Y is a reasonably plausible p-containing set. Furthermore, we can assume that $K * p$ is the $\leqq$-minimal p-containing set and that it is reasonably plausible.[6] It then remains to identify, for each p, the $\leqq$-outermost reasonably plausible p-containing set. We will assume for simplicity, that if $K * p_1 = K * p_2$ then the sets of plausible belief sets containing p_1 respectively p_2 have the same outer limit. This provides us with the following definition:

The triple $\langle \mathbb{X}, \leqq, \ell \rangle$ is a *dilated centrolinear model* if and only if $\langle \mathbb{X}, \leqq \rangle$ is a centrolinear model and ℓ (the *delimiter*) is a function from and to $\mathbb{X}$ such that $X \leqq \ell(X)$ for all $X \in \mathbb{X}$.[7]

A belief set $X \in \mathbb{X}$ is *self-limited* according to ℓ if and only if $X = \ell(X)$.

(Another plausible property of ℓ is: If $X \leqq Y$ then $\ell(X) \leqq \ell(Y)$. It will not be needed here.)

We can now express the co-occurrence test in a fully precise manner:

The co-occurrence test in an exhaustive and dilated centrolinear model
$p \mid\!\sim q$ holds if and only if it holds for all $Y \in \mathbb{X}$ that if $K * p \leqq Y \leqq \ell(K * p)$ and $p \in Y$, then $q \in Y$.

In the limiting case when all elements of $\mathbb{X}$ are self-limited according to ℓ, the co-occurrence test coincides with the Ramsey test (and thus $\mid\!\sim$ coincides with $\rightarrowtail$).

4. The theorem

Theorem. *Let $\langle \mathbb{X}, \leqq, \ell \rangle$ be an exhaustive and dilated centrolinear model such that the strict part of $\leqq$ is a wellordering with an order type that is either finite or ω. Furthermore, let $\mid\!\sim$ be the non-monotonic inference relation that is based on $\langle \mathbb{X}, \leqq, \ell \rangle$ via the co-occurrence test. Then there is a centrolinear model $\langle \mathbb{X}', \leqq' \rangle$ such that $\mid\!\sim$ coincides with the conditional $\rightarrowtail$ that is based on $\langle \mathbb{X}', \leqq' \rangle$ via the Ramsey test.*

Furthermore, if K is the $\leqq$-minimal element of $\mathbb{X}$ and K' is the $\leqq'$-minimal element of $\mathbb{X}'$, then $K' \subseteq K$.

Proof. *General structure of the proof:* We will use the following alternative notation for the centrolinear model $\langle \mathbb{X}, \leqq \rangle$:

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[5] As was shown in [13], the framework of descriptor revision allows for a generalization of standard (sentential) conditionals to *Ramsey test descriptor conditionals*, denoted $\Psi \Rightarrow \Xi$ where Ψ and Ξ are descriptors. The interpretation is that if the agent's belief state is changed to satisfy Ψ, then it will also satisfy Ξ. Hence, $\mathfrak{B}p \vee \mathfrak{B}\neg p \Rightarrow \mathfrak{B}q \vee \mathfrak{B}\neg q$ signifies that if the agent makes up her mind about p then she will also have an opinion on whether q is true or not. The extensibility of the descriptor framework to this more general class of conditional sentences is one reason why the descriptor framework was chosen for this investigation. Another reason is the easy accessibility in this framework of the notion of a class of reasonably plausible belief sets satisfying a specific condition, see footnote 7.

[6] $\leqq$ is assumed to contain enough information to provide us with the outcome of the operation. It follows that ties are excluded, i.e. if X and Y are both among the potential outcomes of revision by some input, then $\leqq$ cannot rank them both highest among those potential outcomes. This is the price paid for the determinateness of the operation. (If $\leqq$ has ties, then we will have an indeterministic operation of belief change, i.e. one in which change by a particular input can have more than one outcome.) In contrast, the AGM model uses selection mechanisms that provide us with a set of equally ranked epistemic objects, and the outcome is obtained as the intersection of these objects. The latter approach has several disadvantages: (1) The epistemic objects on which the selection mechanism operates are entities such as possible worlds and remainders that are not themselves plausible potential outcomes of the operation. This makes the choice indirect and different to interpret. (2) If each element of a collection of belief sets is maximally plausible in some sense, then this does not necessarily tell us much about the plausibility of their intersection. (3) Some success conditions (such as $\mathfrak{B}p \vee \mathfrak{B}\neg p$) do not hold in general for the intersection of a set all of whose elements satisfy it. These problems with the standard, select-and-intersect method were among the reasons for introducing descriptor revision with its direct selection of an outcome. For details see [11].

[7] The same construction can be used in a more general approach employing Ramsey test descriptor conditionals. (See footnote 5.) In the AGM framework there is no equally straightforward way to delimit the set of reasonably plausible p-containing sets. The best option would probably be to refer to a sphere in a Grove system such that a p-containing belief set satisfies the plausibility requirement if and only if the possible worlds containing it are all elements of that sphere. However, this construction cannot be generalized to Ramsey test descriptor conditionals.

$$\mathcal{X}_0 = X_1, X_2, X_3, X_4, \ldots$$

where X_1 is the $\leqq$-minimal element of $\mathbb{X}$, X_2 the $\leqq$-minimal element of $\mathbb{X} \setminus \{X_1\}$, etc. Clearly, $\mathcal{X}_0$ contains the same information as $\langle \mathbb{X}, \leqq \rangle$, and therefore we can use $\langle \mathcal{X}_0, \ell \rangle$ as an alternative notation for $\langle \mathbb{X}, \leqq, \ell \rangle$. (This alternative notation will also be used for other centrolinear models to be constructed in the proof.)

We will use K as an alternative notation for the original belief set, i.e. $K = X_1$.

The proof will proceed by mathematical induction. In the base case we need to show that there is a series

$$\mathcal{X}_1 = Z_1^1, \ldots, Z_{m_1}^1, X_2, X_3, X_4, \ldots$$

and a delimiter ℓ_1 for the set consisting of its elements, such that each of $Z_1, \ldots, Z_m$ is self-limited according to ℓ and that $\langle \mathcal{X}_1, \ell_1 \rangle$ generates the same inference relation as $\langle \mathcal{X}_0, \ell \rangle$ (and thus the same as $\langle \mathbb{X}, \leqq, \ell \rangle$). We also need to show that $Z_1^1 \subseteq X_1$.

In the inductive step we will assume that we have a series

$$\mathcal{X}_{n-1} = Y_1, \ldots, Y_k, X_n, X_{n+1}, X_{n+2}, \ldots$$

(where $X_n, X_{n+1}, X_{n+2}, \ldots$ is the remaining part of $\mathcal{X}_0$ that has not been affected by the previous steps) and a delimiter ℓ_{n-1} according to which each of $Y_1, \ldots, Y_k$ is self-limited. We need to show that there is a series

$$\mathcal{X}_n = Y_1, \ldots, Y_k, Z_1^n, \ldots, Z_{m_n}^n, X_{n+1}, X_{n+2}, \ldots$$

and a delimiter ℓ_n for the set consisting of its elements, such that each of $Y_1, \ldots, Y_k, Z_1^n, \ldots, Z_{m_n}^n$ is self-limited according to ℓ_n and that $\langle \mathcal{X}_n, \ell_n \rangle$ generates the same inference relation as $\langle \mathcal{X}_{n-1}, \ell_{n-1} \rangle$.

Based on this, the whole of $\mathcal{X}_0$ can be replaced by the series

$$\mathcal{X}_\omega = Z_1^1, \ldots, Z_{m_1}^1, Z_1^2, \ldots, Z_{m_2}^2, Z_1^3, \ldots, Z_{m_3}^3 \ldots$$

with a delimiter ℓ_ω that is simply the identify function, i.e. $\ell_\omega(Z) = Z$ for all elements Z of the series. Then $\langle \mathcal{X}_\omega, \ell_\omega \rangle$ will yield the same inference relation as $\langle \mathcal{X}_0, \ell \rangle$. Furthermore, the inference relation obtainable from $\langle \mathcal{X}_\omega, \ell_\omega \rangle$ is the same as the conditional obtainable from $\mathcal{X}_\omega$ via the Ramsey test. This is what we had to prove.

The proofs of the base case and the inductive step are so similar that only the latter will be given in detail.

The inductive step: construction: We start with a series

$$\mathcal{X}_{n-1} = Y_1, \ldots, Y_k, X_n, X_{n+1}, X_{n+2}, \ldots$$

and a delimiter ℓ_{n-1} according to which each of $Y_1, \ldots, Y_k$ is self-limited. Let $\mathbb{Z}$ be the set of subsets of $\{X_n, X_{n+1}, X_{n+2}, \ldots\}$ such that (i) $X_n \in \mathbb{Z}$ and (ii) there is some sentence p such that $K * p = X_n$ and $\mathbb{Z} = \{Z \mid X_n \leqq Z \leqq \ell(X_n) \text{ and } p \in Z\}$.

Case i, $\mathbb{Z} = \varnothing$:

Let $\mathcal{X}_n = Y_1, \ldots, Y_k, X_{n+1}, X_{n+2}, \ldots$ and let ℓ_n be the restriction of ℓ_{n-1} to the elements of the new series.

Case ii, $\mathbb{Z} \neq \varnothing$: Let $\hat{\mathbb{Z}} = \{\bigcap V \mid V \in \mathbb{Z}\}$ and let $Z_1^n, Z_2^n, \ldots Z_{m_n}^n$ be a list on which each element of $\hat{\mathbb{Z}}$ appears exactly once, and such that if $Z, Z' \in \hat{\mathbb{Z}}$ and $Z' \subset Z$ then Z' comes before Z on the list. (The existence of such a series follows from the order extension principle that follows from the axiom of choice, see [16, p. 19].)

Now let:

$$\mathcal{X}_n = Y_1, \ldots, Y_k, Z_1^n \ldots, Z_{m_n}^n, X_{n+1}, X_{n+2}, \ldots$$

and let ℓ_n be such that (i) $\ell_n(X) = X$ for all $X \in \{Y_1, \ldots, Y_k, Z_1^n \ldots, Z_{m_n}^n\}$, and (ii) $\ell_n(X) = \ell_{n-1}(X)$ for all $X \in \{X_{n+1}, X_{n+2}, \ldots\}$.

The inductive step: verification: The verification is straightforward in Case i of the construction. In Case ii, let $\mid\!\sim_{n-1}$ be the inference relation derivable from $\langle \mathcal{X}_{n-1}, \ell_{n-1} \rangle$, and let $\mid\!\sim_n$ be the inference relation derivable from $\langle \mathcal{X}_n, \ell_n \rangle$. We are going to show that for all p and q: $p \mid\!\sim_n q$ if and only if $p \mid\!\sim_{n-1} q$. There are three cases.

Case 1, $p \in \bigcup\{Y_1, \ldots, Y_k\}$: The desired result follows directly since ℓ_n and ℓ_{n-1} coincide in this part of the series.

Case 2, $p \notin \bigcup\{Y_1, \ldots, Y_k\}$ and $p \in X_n$: Then $p \mid\!\sim_{n-1} q$ holds if and only if q holds in all elements of $\{X \mid X_n \leqq X \leqq \ell_{n-1}(X_n) \text{ and } p \in X\}$, i.e. if and only if $q \in \bigcap\{X \mid X_n \leqq X \leqq \ell_{n-1}(X_n) \text{ and } p \in X\}$. Now, $\bigcap\{X \mid X_n \leqq X \leqq \ell_{n-1}(X_n) \text{ and } p \in X\}$ is an element of $\widehat{\mathbb{Z}}$, and moreover it is the leftmost element of $\widehat{\mathbb{Z}}$ that contains p. (This is because it is a proper subset of all other elements of $\widehat{\mathbb{Z}}$ that contain p.) It follows that $p \mid\!\sim_n q$ holds if and only if $q \in \bigcap\{X \mid X_n \leqq X \leqq \ell_{n-1}(X_n) \text{ and } p \in X\}$, thus $p \mid\!\sim_n q$ holds if and only if $p \mid\!\sim_{n-1} q$ holds.

Case 3, $p \notin \bigcup\{Y_1, \ldots, Y_k, X_n\}$: The desired result follows directly since ℓ_n and ℓ_{n-1} coincide in the subseries $X_{n+1}, X_{n+2}, \ldots$. □

5. Discussion

The theorem shows that conditionality ($\rightarrowtail$) and non-monotonic inferribility ($|\!\sim$) have the same logic. But there are three important caveats.

First, the theorem was proved in one specific framework. There may be other frameworks in which both the Ramsey test and the co-occurrence test can be represented. The corresponding theorem may not be obtainable in all such frameworks.

Secondly, although the theorem provides us with a reconstruction of any co-occurrence test as a Ramsey test, this derived Ramsey test is based on another initial belief set and another operation of belief revision than those referred to in the co-occurrence test that we started with. Therefore, the logical properties that connect $\rightarrowtail$ to K or to $*$ need not hold if we replace $\rightarrowtail$ by $|\!\sim$, and vice versa. (The property CS mentioned in Section 2 is an example of this.)

Thirdly and most importantly, according to the theorem conditionality and non-monotonic inferribility *obey the same logical principles but do not coincide*. This is not an unusual situation. Logical necessity and physical necessity may both have the same (S5) logic, but that is no reason to conflate them [8, pp. 104–105], cf. [3] and [5]. In social choice theory, we usually assume that different persons' preferences satisfy the same logical rules, but in all non-trivial cases they differ in substance. Similarly, the analysis provided here gives us reason to treat conditionality and (non-monotonic) inferribility as distinct notions that satisfy the same logical principles. Possibly, the similarity of their logics has blinded us to the differences between the two concepts.

References

[1] Carlos Alchourrón, Peter Gärdenfors, David Makinson, On the logic of theory change: partial meet contraction and revision functions, J. Symb. Log. 50 (1985) 510–530.
[2] Jonathan Bennett, Counterfactuals and possible worlds, Can. J. Philos. 4 (1974) 381–402.
[3] John P. Burgess, Which modal logic is the right one?, Notre Dame J. Form. Log. 40 (1999) 81–93.
[4] Alvaro del Val, Non monotonic reasoning and belief revision: syntactic, semantic, foundational and coherence approaches, J. Appl. Non-Class. Log. 7 (1997) 213–240.
[5] Peter Fritz, What is the correct logic of necessity, actuality and apriority?, Rev. Symb. Log. 7 (2014) 385–414.
[6] Peter Gärdenfors, Epistemic importance and minimal changes of belief, Australas. J. Philos 62 (1984) 136–157.
[7] Peter Gärdenfors, David Makinson, Nonmonotonic inference based on expectations, Artif. Intell. 65 (1994) 197–245.
[8] James W. Garson, Modal Logic for Philosophers, second edition, Cambridge University Press, New York, 2013.
[9] Adam Grove, Two modellings for theory change, J. Philos. Log. 17 (1988) 157–170.
[10] Sven Ove Hansson, A survey of non-prioritized belief revision, Erkenntnis 50 (1999) 413–427.
[11] Sven Ove Hansson, Descriptor revision, Stud. Log. 102 (2014) 955–980.
[12] Sven Ove Hansson, Relations of epistemic proximity for belief change, Artif. Intell. 217 (2014) 76–91.
[13] Sven Ove Hansson, Iterated descriptor revision and the logic of Ramsey test conditionals, J. Philos. Log. (2016), in press.
[14] Sven Ove Hansson, A monoselective presentation of AGM revision, Stud. Log. (2016), in press.
[15] Sven Ove Hansson, Choice and change in an epistemic context, South Amer. J. Log. (2016), in press.
[16] Thomas J. Jech, The Axiom of Choice, Dover Publications, Mineola, N.Y., 2008.
[17] Sarit Kraus, Daniel Lehmann, Menachem Magidor, Nonmonotonic reasoning, preferential models and cumulative logics, Artif. Intell. 44 (1990) 167–207.
[18] David Lewis, Counterfactuals, Blackwell, Oxford, 1973.
[19] David Makinson, Peter Gäardenfors, Relations between the logic of theory change and nonmonotonic logic, in: André Fuhrmann, Michael Morreau (Eds.), The Logic of Theory Change, Springer, Berlin, 1991, pp. 185–205.
[20] Robert Nozick, Philosophical Explanations, Clarendon, Oxford, 1981.
[21] Donald Nute, Conditional logic, in: Dov M. Gabbay, Franz Guenthner (Eds.), Handbook of Philosophical Logic, vol. II, Reidel, Dordrecht, 1984, pp. 387–439.
[22] John L. Pollock, A refined theory of counterfactuals, J. Philos. Log. 10 (1981) 239–266.
[23] Hans Rott, Change, Choice and Inference: A Study of Belief Revision and Nonmonotonic Reasoning, Clarendon Press, Oxford, 2001.
[24] Wayne Wobcke, Belief revision, conditional logic and nonmonotonic reasoning, Notre Dame J. Form. Log. 36 (1995) 55–102.