Available online at www.sciencedirect.com

ScienceDirect

AKCE International Journal of Graphs and Combinatorics 13 (2016) 191–199

AKCE International Journal of Graphs and Combinatorics

www.elsevier.com/locate/akcej

On total labelings of graphs with prescribed weights

Muhammad Irfan[a], Andrea Semaničová-Feňovčíková[b,*]

[a] Abdus Salam School of Mathematical Sciences, GC University, Lahore, Pakistan

[b] Department of Applied Mathematics and Informatics, Technical University, Košice, Slovakia

Received 1 March 2016; received in revised form 30 May 2016; accepted 7 June 2016

Available online 29 June 2016

Abstract

Let $G = (V, E)$ be a finite, simple and undirected graph. The edge-magic total or vertex-magic total labeling of G is a bijection f from $V(G) \cup E(G)$ onto the set of consecutive integers $\{1, 2, \dots, |V(G)| + |E(G)|\}$, such that all the edge weights or vertex weights are equal to a constant, respectively. When all the edge weights or vertex weights are different then the labeling is called edge-antimagic or vertex-antimagic total, respectively.

In this paper we provide some classes of graphs that are simultaneously super edge-magic total and super vertex-antimagic total, that is, graphs admitting labeling that has both properties at the same time. We show several results for fans, sun graphs, caterpillars and prisms.

Keywords: Super edge-magic total labeling; Super vertex-antimagic total labeling; Total labeling

1. Introduction

We consider $G = (V, E)$ finite undirected graphs without loops and multiple edges with vertex set $V(G)$ and edge set $E(G)$, where $n = |V(G)|$, $m = |E(G)|$. A *labeling* of a graph G is any mapping that maps a certain set of graph elements to a certain set of positive integers. If the domain is the vertex (or edge) set, respectively, the labeling is called *vertex* (or *edge*) *labeling*, respectively. If the domain is both vertices and edges then the labeling is called a *total labeling*. The *edge weight* of an edge under the total labeling is the sum of the edge label and the labels of its end vertices. The *vertex weight* of a vertex under the total labeling is defined as sum of the vertex label itself and the labels of its incident edges.

A labeling is called *edge-magic total* (*vertex-magic total*) if the edge weights (vertex weights) are all the same. If the edge weights (vertex weights) are pairwise distinct then the total labeling is called *edge-antimagic total* (*vertex-antimagic total*). A graph that admits edge-magic total (edge-antimagic total) labeling or vertex-magic total (vertex-antimagic total) labeling is called an *edge-magic total* (*edge-antimagic total*) *graph* or *vertex-magic total* (*vertex-antimagic total*) *graph*, respectively.

Peer review under responsibility of Kalasalingam University.

* Corresponding author.

E-mail addresses: m.irfan.assms@gmail.com (M. Irfan), andrea.fenovcikova@tuke.sk (A. Semaničová-Feňovčíková).

http://dx.doi.org/10.1016/j.akcej.2016.06.001

0972-8600/

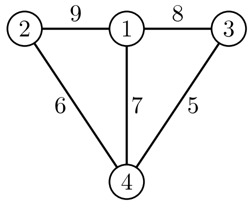

Fig. 1. A simultaneously super edge-magic total and super vertex-antimagic total labeling of the fan F_3.

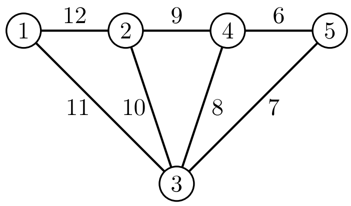

Fig. 2. A simultaneously super edge-magic total and super vertex-antimagic total labeling of the fan F_4.

The subject of edge-magic total graph has its origin in the work of Kotzig and Rosa [1]. Vertex-magic total graphs are introduced by MacDougall, Miller, Slamin and Wallis in [2], see also [3,4]. The notation of edge-antimagic total labeling was introduced by Simanjuntak, Bertault and Miller in [5] as a natural extension of magic valuation defined by Kotzig and Rosa in [1]. The vertex-antimagic total labeling of graphs was introduced in [6], see also [7]. If moreover the vertices are labeled with the smallest possible labels then, as is customary, the labeling is referred to as *super*. The concept of super-magic graphs was introduced by Stewart [8]. For further information on these types of labelings, one can see [9,7,10,11].

In [12] Bača, Miller, Phanalasy, Ryan, Semaničová-Feňovčíková and Sillasen proved that there exist some classes of stars, paths and cycle graphs which are simultaneously edge-magic total and vertex-antimagic total or simultaneously vertex-magic total and edge-antimagic total.

In this paper we will prove that some classes of fans, suns, caterpillars and prism graphs are simultaneously super edge-magic total and super vertex-antimagic total. For every of these graph we describe a total labeling that has both properties at the same time.

2. Fans, sun graphs, caterpillars and prisms

A *fan* F_n, $n \geq 2$, is a graph obtained by joining all vertices of path P_n on n vertices to another vertex, called center. The fan graph F_n contains $n+1$ vertices and $2n-1$ edges.

Theorem 1. *The fan F_n is simultaneously super edge-magic total and super vertex-antimagic total if and only if $3 \leq n \leq 6$.*

Proof. In [13], see also [7], is proved that the fan F_n has a super edge-magic total labeling if and only if $2 \leq n \leq 6$. The fan F_2 is isomorphic to the cycle C_3. In [12] is showed that the cycle C_3 is not simultaneously super edge-magic total and super vertex-antimagic total.

For $3 \leq n \leq 6$ are the required labelings depicted in Figs. 1 through 4.

Fig. 1 depicts a simultaneously super edge-magic total and super vertex-antimagic total labeling for F_3 with edge weights equal 12 and vertex weights 16, 17, 22, 25.

A super edge-magic total labeling of the fan F_4 with edge weights 15 is illustrated in Fig. 2. This total labeling is also super vertex-antimagic with vertex weights 18, 24, 27, 33, 39.

Fig. 3 shows a simultaneously super edge-magic total and super vertex-antimagic total labeling of the fan F_5 with edge weights 18 and with vertex weights 21, 29, 31, 38, 43, 57.

A total labeling of the fan F_6 with constant edge weights 21 is given in Fig. 4. This total labeling is also super vertex-antimagic with vertex weights 25, 34, 35, 39, 47, 52, 82. ■

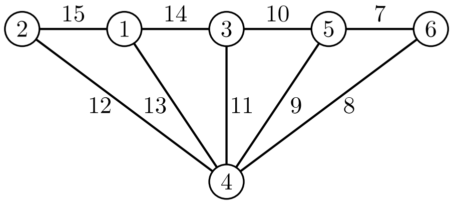

Fig. 3. A simultaneously super edge-magic total and super vertex-antimagic total labeling of the fan F_5.

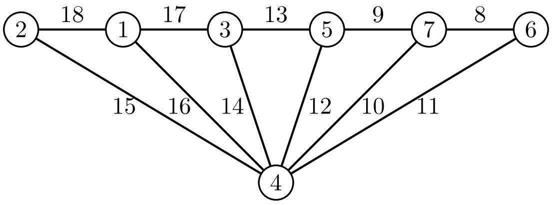

Fig. 4. A simultaneously super edge-magic total and super vertex-antimagic total labeling of the fan F_6.

In the following lemma we prove that number of pendant edges incident with a given vertex in a simultaneously super edge-magic total and super vertex-antimagic total graph is at most 1.

Lemma 1. *Let G be a simultaneously super edge-magic total and super vertex-antimagic total graph. Then every vertex of G can be incident with at most one pendant edge.*

Proof. Let G be a simultaneously super edge-magic total and super vertex-antimagic total graph and let f be the corresponding labeling of G. Let us assume that there is a vertex $v \in V(G)$ that is adjacent to more than one vertex of degree 1, say u_1, u_2 are two of them. As f is a super edge-magic total labeling then the weights of edges vu_1 and vu_2 are the same, i.e.,

$$wt_f(vu_1) = wt_f(vu_2)$$
$$f(v) + f(vu_1) + f(u_1) = f(v) + f(vu_2) + f(u_2)$$

and thus

$$f(vu_1) + f(u_1) = f(vu_2) + f(u_2)$$
$$wt_f(u_1) = wt_f(u_2)$$

what is a contradiction to the fact that f is also vertex-antimagic. ■

Now we will investigate graphs obtained by attaching n pendant vertices to every vertex of a given graph G. These graphs can be alternatively obtained as a corona product of a graph G and a totally disconnected graph on m vertices, denoted by $G \odot \overline{K}_m$. A cycle of order n with m pendant edges attached at each vertex, i.e., $C_n \odot mK_1$, is called an *m-crown* with cycle of order n. A 1-crown, or only a *crown* or a *sun graph*, is a cycle with exactly one pendant edge attached at each vertex of the cycle. We will use notation $Sun(n)$ for this graph. We denote the elements of the sun graph $Sun(n)$ such that its vertex set is

$$V(Sun(n)) = \{x_i, y_i : 1 \le i \le n\}$$

and its edge set is

$$E(Sun(n)) = \{x_i x_{i+1}, x_i y_i : 1 \le i \le n\},$$

where the indices are taken modulo n. Thus the sun $Sun(n)$ has $2n$ vertices and $2n$ edges, see Fig. 5.

According to Lemma 1 we immediately have the following result.

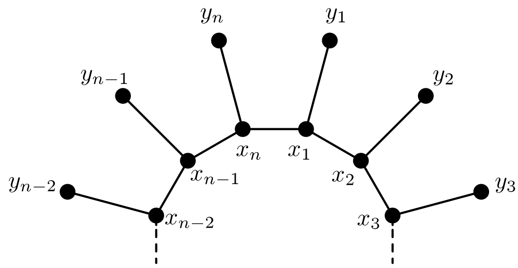

Fig. 5. Sun graph $Sun(n)$.

Observation 1. *If the m-crown with cycle of order n, i.e., $C_n \odot mK_1$, is a simultaneously super edge-magic total and super vertex-antimagic total graph then $m = 1$.*

Next we will deal with the existence of total labeling of the sun graph $Sun(n)$ which has simultaneously super edge-magic properties and super vertex-antimagic properties.

Theorem 2. *For every odd positive integer n, $n \geq 3$, the sun graph $Sun(n)$ is simultaneously super edge-magic total and super vertex-antimagic total.*

Proof. Define a total labeling $f : V(Sun(n)) \cup E(Sun(n)) \to \{1, 2, \ldots, 4n\}$ in the following way:

$$\begin{aligned}
f(x_i) &= \begin{cases} \frac{i+1}{2}, & \text{for } i \equiv 1 \pmod 2,\ 1 \leq i \leq n \\ \frac{n+i+1}{2}, & \text{for } i \equiv 0 \pmod 2,\ 2 \leq i \leq n-1 \end{cases} \\
f(y_i) &= 2n - i, \quad \text{for } 1 \leq i \leq n-1 \\
f(y_n) &= 2n, \\
f(x_i x_{i+1}) &= 4n - i, \quad \text{for } 1 \leq i \leq n-1 \\
f(x_n x_1) &= 4n, \\
f(x_i y_i) &= \begin{cases} \frac{5n+i+2}{2}, & \text{for } i \equiv 1 \pmod 2,\ 1 \leq i \leq n-2 \\ \frac{4n+i+2}{2}, & \text{for } i \equiv 0 \pmod 2,\ 2 \leq i \leq n-1 \end{cases} \\
f(x_n y_n) &= 2n + 1.
\end{aligned}$$

It is not difficult to see that the labeling f is a total labeling and moreover the vertex labels are the smallest possible labels. For edge weights we have

$$\begin{aligned}
wt_f(x_i x_{i+1}) &= f(x_i) + f(x_i x_{i+1}) + f(x_{i+1}) \\
&= \begin{cases} \frac{n+i+1}{2} + \big(4n - i\big) + \frac{i+2}{2} = \frac{9n+3}{2}, & \text{for } i \equiv 0 \pmod 2,\ 2 \leq i \leq n-1 \\ \big(\frac{i+1}{2}\big) + \big(4n - i\big) + \big(\frac{n+i+2}{2}\big) = \frac{9n+3}{2}, & \text{for } i \equiv 1 \pmod 2,\ 1 \leq i \leq n-2 \end{cases} \\
wt_f(x_n x_1) &= f(x_n) + f(x_n x_1) + f(x_1) = \frac{n+1}{2} + 4n + 1 = \frac{9n+3}{2}, \\
wt_f(x_i y_i) &= f(x_i) + f(x_i y_i) + f(y_i) \\
&= \begin{cases} \frac{n+i+1}{2} + \frac{4n+i+2}{2} + \big(2n - i\big) = \frac{9n+3}{2}, & \text{for } i \equiv 0 \pmod 2,\ 2 \leq i \leq n-1 \\ \frac{i+1}{2} + \frac{5n+i+2}{2} + \big(2n - i\big) = \frac{9n+3}{2}, & \text{for } i \equiv 1 \pmod 2,\ 1 \leq i \leq n-2 \end{cases} \\
wt_f(x_n y_n) &= f(x_n) + f(x_n y_n) + f(y_n) = \frac{n+1}{2} + \big(2n + 1\big) + 2n = \frac{9n+3}{2}.
\end{aligned}$$

Thus the total labeling f is a super edge-magic labeling of the sun graph $Sun(n)$ for every odd n, $n \geq 3$. Let us consider the vertex weights under the total labeling f.

$$wt_f(x_i) = f(x_{i-1}x_i) + f(x_i) + f(x_iy_i) + f(x_ix_{i+1})$$
$$= \begin{cases} \big(4n-(i-1)\big) + \frac{n+i+1}{2} + \frac{4n+i+2}{2} + \big(4n-i\big) = \frac{21n+5}{2} - i, & \text{for } i \equiv 0 \pmod 2,\ 2 \le i \le n-1 \\ \big(4n-(i-1)\big) + \frac{i+1}{2} + \frac{5n+i+2}{2} + \big(4n-i\big) = \frac{21n+5}{2} - i, & \text{for } i \equiv 1 \pmod 2,\ 1 \le i \le n-2 \end{cases}$$
$$wt_f(x_n) = f(x_{n-1}x_n) + f(x_n) + f(x_ny_n) + f(x_nx_1) = \big(4n-(n-1)\big) + \frac{n+1}{2} + \big(2n+1\big) + 4n = \frac{19n+5}{2},$$
$$wt_f(y_i) = f(y_i) + f(x_iy_i)$$
$$= \begin{cases} \big(2n-i\big) + \frac{4n+i+2}{2} = 4n+1-\frac{i}{2}, & \text{for } i \equiv 0 \pmod 2,\ 2 \le i \le n-1 \\ \big(2n-i\big) + \frac{5n+i+2}{2} = \frac{9n+2-i}{2}, & \text{for } i \equiv 1 \pmod 2,\ 1 \le i \le n-2 \end{cases}$$
$$wt_f(y_n) = f(y_n) + f(x_ny_n) = 2n + \big(2n+1\big) = 4n+1.$$

As for $1 \le i \le n$

$$wt_f(x_i) = \frac{21n+5}{2} - i, \tag{1}$$

we have that the weights of vertices x_i form an arithmetic progression with difference $d = 1$. Also the weights of vertices y_i constitute the set of consecutive integers

$$\{wt(y_i) : 1 \le i \le n\} = \left\{\frac{7n+3}{2}, \frac{7n+5}{2}, \dots, \frac{9n+1}{2}\right\}.$$

We can also easily see that $wt_f(x_i) \neq wt_f(y_i)$ for all $1 \le i \le n$. This shows that f is vertex-antimagic as well. It means that for every odd n, $n \ge 3$, the sun graph $Sun(n)$ is simultaneously super edge-magic total and super vertex-antimagic total. ■

If we remove one edge on the cycle of the sun graph $Sun(n)$ then the resulting graph is a tree and it is a special class of caterpillar. We denote this tree as $Cat(n)$.

Theorem 3. *For every odd positive integer n, $n \ge 3$, the tree $Cat(n)$ is simultaneously super edge-magic total and super vertex-antimagic total.*

Proof. Let us consider the tree $Cat(n)$ as the sun graph $Sun(n)$ with the edge x_1x_n deleted. By the labeling f defined in the proof of Theorem 2, we define a labeling g of $Cat(n)$ as follows:

$$g(x) = f(x)$$

for every $x \in V(Sun(n)) \cup E(Sun(n)) - \{x_1x_n\}$.

Since the label of removed edge x_1x_n in $Sun(n)$ under the total labeling f is the largest one then the labeling g is a total labeling of the tree $Cat(n)$ with labels from 1 up to $4n-1$. It is easy to see that the total labeling g preserves super edge-magic properties.

For proving that the total labeling g preserves also super vertex-antimagic properties it is sufficient to show that the vertex weights of vertices x_1 and x_n are distinct from other vertices of the tree $Cat(n)$. In fact, using (1), for $2 \le i \le n-1$ we get

$$wt_g(x_1) = wt_f(x_1) - f(x_nx_1) = \left(\frac{21n+5}{2} - 1\right) - 4n = \frac{13n+3}{2} < wt_f(x_i) = \frac{21n+5}{2} - i$$

and for $1 \le i \le n$

$$wt_g(x_1) > wt_f(y_i).$$

Also for $2 \le i \le n-1$

$$wt_g(x_n) = wt_f(x_n) - f(x_nx_1) = \left(\frac{21n+5}{2} - n\right) - 4n = \frac{11n+5}{2} < wt_f(x_i) = \frac{21n+5}{2} - i$$

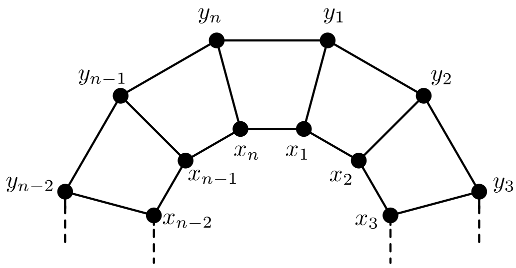

Fig. 6. The prism D_n.

and for $1 \leq i \leq n$

$$wt_g(x_n) > wt_f(y_i).$$

This proves that the tree $Cat(n)$ is simultaneously super edge-magic total and super vertex-antimagic total for every odd n, $n \geq 3$. ■

The prism D_n, $n \geq 3$, is a cubic graph which can be represented as a Cartesian product $P_2 \square C_n$ of a path on two vertices with a cycle on n vertices. Let

$$V(D_n) = \{x_i, y_i : 1 \leq i \leq n\}$$

be the vertex set and

$$E(D_n) = \{x_i x_{i+1}, y_i y_{i+1}, x_i y_i : 1 \leq i \leq n\},$$

where the indices are taken modulo n, be the edge set of the prism D_n, see Fig. 6. The prism D_n has $2n$ vertices and $3n$ edges.

Next theorem shows that there exists a total labeling of the prism D_n which has super edge-magic and also super vertex-antimagic properties.

Theorem 4. *For every odd positive integer n, $n \geq 3$ and $n \not\equiv 0 \pmod 5$ the prism D_n is simultaneously super edge-magic total and super vertex-antimagic total.*

Proof. We define a total labeling $h : V(D_n) \cup E(D_n) \to \{1, 2, \ldots, 5n\}$ as follows:

$$h(x_i) = \begin{cases} \frac{i+1}{2}, & \text{for } i \equiv 1 \pmod 2,\ 1 \leq i \leq n \\ \frac{n+i+1}{2}, & \text{for } i \equiv 0 \pmod 2,\ 2 \leq i \leq n-1 \end{cases}$$

$$h(y_i) = \begin{cases} \frac{3n+i}{2}, & \text{for } i \equiv 1 \pmod 2,\ 1 \leq i \leq n \\ \frac{2n+i}{2}, & \text{for } i \equiv 0 \pmod 2,\ 2 \leq i \leq n-1 \end{cases}$$

$$h(x_i x_{i+1}) = 5n - i, \quad \text{for } 1 \leq i \leq n-1$$

$$h(x_n x_1) = 5n,$$

$$h(y_i y_{i+1}) = 3n + 1 - i, \quad \text{for } 1 \leq i \leq n-1$$

$$h(y_n y_1) = 2n + 1,$$

$$h(x_i y_i) = 4n + 1 - i, \quad \text{for } 1 \leq i \leq n.$$

It is not difficult to check that h is a bijection and the vertices of prism D_n under the total labeling h receive labels from the set $\{1, 2, \ldots, 2n\}$. So, the total labeling h is super. For edge weights under the labeling h we have

$$wt_h(x_ix_{i+1}) = h(x_i) + h(x_ix_{i+1}) + h(x_{i+1})$$
$$= \begin{cases} \frac{n+i+1}{2} + (5n-i) + \frac{i+2}{2} = \frac{11n+3}{2}, & \text{for } i \equiv 0 \pmod 2,\ 2 \le i \le n-1 \\ \frac{i+1}{2} + (5n-i) + \frac{n+i+2}{2} = \frac{11n+3}{2}, & \text{for } i \equiv 1 \pmod 2,\ 1 \le i \le n-2 \end{cases}$$
$$wt_h(x_nx_1) = h(x_n) + h(x_nx_1) + h(x_1) = \frac{n+1}{2} + 5n + 1 = \frac{11n+3}{2},$$
$$wt_h(y_iy_{i+1}) = h(y_i) + h(y_iy_{i+1}) + h(y_{i+1})$$
$$= \begin{cases} \frac{2n+i}{2} + (3n+1-i) + \frac{3n+i+1}{2} = \frac{11n+3}{2}, & \text{for } i \equiv 0 \pmod 2,\ 2 \le i \le n-1 \\ \frac{3n+i}{2} + (3n+1-i) + \frac{2n+i+1}{2} = \frac{11n+3}{2}, & \text{for } i \equiv 1 \pmod 2,\ 1 \le i \le n-2 \end{cases}$$
$$wt_h(y_ny_1) = h(y_n) + h(y_ny_1) + h(y_1) = \frac{3n+n}{2} + (2n+1) + \frac{3n+1}{2} = \frac{11n+3}{2},$$
$$wt_h(x_iy_i) = h(x_i) + h(x_iy_i) + h(y_i)$$
$$= \begin{cases} \frac{n+i+1}{2} + (4n-i+1) + \frac{2n+i}{2} = \frac{11n+3}{2}, & \text{for } i \equiv 0 \pmod 2,\ 2 \le i \le n-1 \\ \frac{i+1}{2} + (4n-i+1) + \frac{3n+i}{2} = \frac{11n+3}{2}, & \text{for } i \equiv 1 \pmod 2,\ 1 \le i \le n. \end{cases}$$

Thus the labeling h is a super edge-magic total labeling.

Let us consider the vertex weights under the labeling h:

$$wt_h(x_n) = h(x_{n-1}x_n) + h(x_n) + h(x_ny_n) + h(x_nx_1)$$
$$= (5n-(n-1)) + \frac{n+1}{2} + (4n-n+1) + 5n = \frac{25n+5}{2},$$
$$wt_h(x_i) = h(x_{i-1}x_i) + h(x_i) + h(x_iy_i) + h(x_ix_{i+1})$$
$$= \begin{cases} (5n-(i-1)) + \frac{n+i+1}{2} + (4n-i+1) + (5n-i) = \frac{29n+5-5i}{2}, \\ \quad \text{for } i \equiv 0 \pmod 2,\ 2 \le i \le n-1 \\ (5n-(i-1)) + \frac{i+1}{2} + (4n-i+1) + (5n-i) = \frac{28n+5-5i}{2}, \\ \quad \text{for } i \equiv 1 \pmod 2,\ 1 \le i \le n-2 \end{cases}$$
$$wt_h(y_1) = h(y_ny_1) + h(y_1) + h(x_1y_1) + h(y_1y_2) = (2n+1) + \frac{3n+1}{2} + 4n + 3n = \frac{21n+3}{2},$$
$$wt_h(y_i) = h(y_{i-1}y_i) + h(y_i) + h(x_iy_i) + h(y_iy_{i+1})$$
$$= \begin{cases} (3n+1-(i-1)) + \frac{2n+i}{2} + (4n+1-i) + (3n+1-i) = \frac{22n+8-5i}{2}, \\ \quad \text{for } i \equiv 0 \pmod 2,\ 2 \le i \le n-1 \\ (3n+1-(i-1)) + \frac{3n+i}{2} + (4n+1-i) + (3n+1-i) = \frac{23n+8-5i}{2}, \\ \quad \text{for } i \equiv 1 \pmod 2,\ 3 \le i \le n. \end{cases}$$

Since $n \not\equiv 0 \pmod 5$ then $wt_h(x_i) = \frac{29n+5-5i}{2}$ for i even, $2 \le i \le n-1$, is distinct to $wt_h(x_i) = \frac{28n+5-5i}{2}$ for i odd, $1 \le i \le n-2$. Moreover, $wt_h(x_n) = \frac{25n+5}{2} \ne wt_h(x_i)$ for $1 \le i \le n-1$.

By the same reason $wt_h(y_i) = \frac{22n+8-5i}{2}$ for i even, $2 \le i \le n-1$, is distinct to $wt_h(y_i) = \frac{23n+8-5i}{2}$ for i odd, $3 \le i \le n$. It is easy to see that $wt_h(y_1) = \frac{21n+3}{2} \ne wt_h(y_i)$ for $2 \le i \le n$.

Hence h is a super vertex-antimagic total labeling.

Therefore the prism D_n is simultaneously super edge-magic total and super vertex-antimagic total for every odd n, $n \ge 3$, $n \not\equiv 0 \pmod 5$. This concludes the proof. ■

3. Conclusion

In the paper we dealt with the problem of finding total labelings of some classes of graphs that are simultaneously super vertex-magic and super edge-antimagic. We showed the existence of such labelings for certain classes of graphs, namely fans, sun graphs, one class of caterpillars and prisms. In [12] authors ask not only for finding classes of graphs that are simultaneously vertex-magic and edge-antimagic but also graphs that are simultaneously vertex-antimagic and edge-magic.

For the fan F_n we proved that it is simultaneously super edge-magic total and super vertex-antimagic total if and only if $3 \le n \le 6$. In [14] it is proved that the fan F_n is vertex-magic total if and only if $2 \le n \le 10$. Note that for

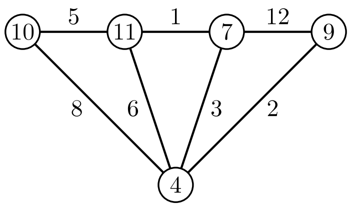

Fig. 7. A simultaneously vertex-magic total and edge-antimagic total labeling of the fan F_4.

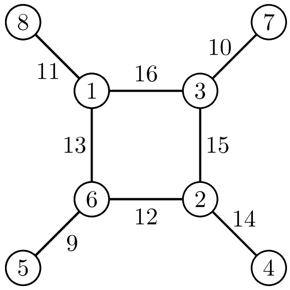

Fig. 8. A simultaneously super edge-magic total and super vertex-antimagic total labeling of the sun graph S_4.

$n = 3, 5$ and $n = 6$ the described labelings induce also distinct edge weights. In Fig. 7 we present a simultaneously vertex-magic total and edge-antimagic total labeling of the fan F_4.

Moreover, in [15], it is showed that the fan F_n is super vertex-magic total if and only if $n = 2$. However, in this case the considered edge weights are also the same, which means that no fan is simultaneously super vertex-magic total and super edge-antimagic total. So we pose the following open problem.

Open Problem 1. For the fan F_n, $7 \leq n \leq 10$, determine if there is a simultaneously vertex-magic total and edge-antimagic total labeling.

For sun graph we proved that for every n odd, $n \geq 3$, the sun graph $Sun(n)$ is simultaneously super edge-magic total and super vertex-antimagic total. Another interesting question is to settle the existence question for even orders. We found a simultaneously super edge-magic total and super vertex-antimagic total labeling for sun graph S_4, see Fig. 8. Although we were unable to find such labelings for bigger even numbers, we still believe that a computer aided search would help to find them.

Open Problem 2. For the sun graph $Sun(n)$, n even, $n \geq 6$, determine if there exists a total labeling which is simultaneously super edge-magic and super vertex-antimagic.

It is a simple observation that the minimum degree of a super vertex-magic total graph must be at least 2. Thus, immediately from this we have that no m-crown with cycle of order n, i.e., $C_n \odot mK_1$, and thus no $Sun(n)$, is simultaneously super vertex-magic total and super edge-antimagic total. On the other hand, it is evident that no simultaneously vertex-magic total and edge-antimagic total graph can contain $K_{1,2}$ as an induced subgraph. This means that if an m-crown with the cycle of order n, i.e., $C_n \odot mK_1$, is a simultaneously vertex-magic total and edge-antimagic total graph, then $m = 1$. Thus we state the following for further investigation.

Open Problem 3. For the sun graph $Sun(n)$, $n \geq 3$, determine if there exists a total labeling which is simultaneously vertex-magic total and edge-antimagic total.

For prism we showed that for every odd n, $n \geq 3$ and $n \not\equiv 0 \pmod 5$, the prism D_n is simultaneously super edge-magic total and super vertex-antimagic total. For prism D_n, when $n \equiv 0 \pmod 5$ and n is odd or n is even, we did not find any total labeling with required properties. Therefore we propose the following open problem.

Open Problem 4. For the prism D_n, $n \equiv 5 \pmod{10}$ or n even, find a total labeling which is simultaneously super edge-magic and super vertex-antimagic.

For further investigation we also state the following open problem.

Open Problem 5. For the prism D_n, $n \geq 3$, determine if there exists a total labeling which is simultaneously super vertex-magic and super edge-antimagic.

Acknowledgments

The research for this article was supported by APVV-15-0116 and by KEGA 072TUKE-4/2014.

References

[1] A. Kotzig, A. Rosa, Magic valuation of finite graphs, Canad. Math. Bull. 13 (1970) 451–461.
[2] J.A. MacDougall, M. Miller, Slamin, W.D. Wallis, Vertex-magic total labelings of graphs, Util. Math. 61 (2002) 3–21.
[3] A.M. Marr, W.D. Wallis, Magic Graphs, Birkhäuser, New York, 2013.
[4] W.D. Wallis, Magic Graphs, Birkhäuser, Boston, Basel, Berlin, 2001.
[5] R. Simanjuntak, F. Bertault, M. Miller, Two new (a, d)-antimagic graph labelings, in: Proc. Eleventh Australas. Workshop Combin. Alg., AWOCA, 2000, pp. 179–189.
[6] M. Bača, F. Bertault, J.A. MacDougall, M. Miller, R. Simanjuntak, Slamin, Vertex-antimagic total labelings of graphs, Discuss. Math. Graph Theory 23 (2003) 67–83.
[7] M. Bača, M. Miller, Super Edge-antimagic Graphs, Brown Walker Press, Boca Raton, Florida, USA, 2008.
[8] B.M. Stewart, Magic graphs, Canad. J. Math. 18 (1966) 1031–1056.
[9] M. Bača, Y. Lin, A. Semaničová-Feňovčíková, Note on super antimagicness of disconnected graphs, AKCE Int. J. Graphs Comb. 6 (1) (2009) 47–55.
[10] J.A. Gallian, A dynamic survey of graph labeling, Electron. J. Combin. 16 (2013) #DS6.
[11] M. Javaid, Super (a, d)-EAT labeling of subdivided stars, AKCE Int. J. Graphs Comb. 12 (1) (2015) 14–18.
[12] M. Bača, M. Miller, O. Phanalasy, J. Ryan, A. Semaničová-Feňovčíková, A.A. Sillasen, Total labelings of graphs with prescribed weights, J. Combin. Math. Combin. Comput. (2016) in press.
[13] M. Bača, Y. Lin, M. Miller, M.Z. Youssef, Edge-antimagic graphs, Discrete Math. 307 (2007) 1232–1244.
[14] J.A. MacDougall, M. Miller, W.D. Wallis, Vertex-magic total labelings of wheel and related graphs, Util. Math. 62 (2002) 175–183.
[15] M. Bača, P. Kovář, A. Semaničová-Feňovčíková, J. Zlámalová, Vertex-antimagic labelings of wheels and related graphs, submitted for publication.