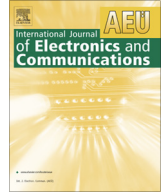




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Regular paper

## Performance of a bi-directional relaying system with one full duplex relay



Nagendrababu Kothapalli, Prabhat Sharma, Vinay Kumar\*

Department of Electronics and Communication Engineering, Visvesvaraya National Institute of Technology Nagpur, 440010, India

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### ABSTRACT

Full duplex communication system has the potential to double the spectral efficiency compared to traditional half duplex communication system. However self interference in full duplex relay is considered as the major concern to the performance of system. In this paper, performance of two way relaying communication system is studied with a full duplex relay. Specifically, the probability density function (PDF) of the signal-to-interference-plus-noise ratio (SINR) is derived. By using the derived PDF, closed form expressions for average bit error rate (BER) is derived for various binary modulation schemes such as, coherent binary phase shift keying (BPSK), differential BPSK, coherent binary frequency shift keying (BFSK), and non-coherent BFSK. Moreover, the ergodic capacity of the considered system is also analyzed.

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### 1. Introduction

The relay assisted multi-hop wireless communication system offers larger coverage and improved throughput [1,2]. The relay in multi-hop communication systems can operate in two modes (i) full duplex and (ii) half duplex. The half duplex mode does not allow the simultaneous transmission and reception in time and frequency, while full duplex mode does allow. Thus full duplex relaying based communication offers the advantage of better resource utilization and improved data rates. However, the full duplex relaying did not attract the researchers in the past due to strong self-interference (SI) in transmitting and receiving antennas of the relay node. The SI between transmitter and receiver exists because of imperfect SI suppression or cancellation methods. However, with the growing advancements in the SI cancellation techniques in recent past [3,4], the full duplex communication has emerged as a potential wireless technology in the recent literature [5,6].

In relay-assisted communication systems, the relay processes the incoming signal, and processed signal is then forwarded ahead to next node. Based on the processing at the relay node, several relaying protocols are used in the literature such as decode-and-forward (DF), amplify-and-forward (AF) etc. In DF, the relay forwards the decoded version of the received signal, whereas in AF, the relay first amplifies the received signal and then forwards it. The AF and DF protocols hence requires two time slots and thus under-utilize the available spectral resources. Recently, the two-

way relaying (TWR) technique (also known as bi-directional relaying) has emerged as a spectrally efficient means for the two user communication systems, and has been used widely in the recent literature of radio-frequency wireless communication and optical wireless communication [7–10]. In TWR based communication system, the relay assists two users to communicate in two time slots, and thus to achieve the spectrally efficient transmission.

#### 1.1. Motivation

The main idea behind this work is to find out the probability density function (PDF) for instantaneous SINR to analyse the considered system performance. The performances of full duplex and half duplex wireless communication systems are compared in several works. It is observed that in full duplex relaying, the saving of one time slot and thus betterment in the spectral efficiency is achieved at the cost of system performance. The full duplex relaying performs better than the half duplex relaying in the low signal-to-noise-ratio (SNR) regime. The authors in [11] derived the value of SI between transmitting and receiving antennas of relay below which full duplex relaying performs better than half duplex relaying in terms of system capacity. Riihonen et al. in [12] analysed the capacity trade-off between half duplex and full duplex relaying modes, and obtained the average end-to-end capacity for both full duplex and half duplex modes. The outage probability of AF cooperation with full duplex relay (FDR) over Rayleigh fading is presented in [13]. The selective decode-and-forward protocol in full duplex relaying was analysed in [14] over Rayleigh-fading channels.

One way relaying based DF systems are considered by the authors in [15–17], and the performance in terms of outage

\* Corresponding author.

E-mail addresses: [knagendra.kothapalli@gmail.com](mailto:knagendra.kothapalli@gmail.com) (N. Kothapalli), [prabhatsharma@ece.vnit.ac.in](mailto:prabhatsharma@ece.vnit.ac.in) (P. Sharma), [vinayrel01@gmail.com](mailto:vinayrel01@gmail.com) (V. Kumar).

probability, average symbol error rate were derived with and without co-channel interferers at the relay and destination nodes. The performance of a half duplex bi-directional relaying system was compared with that of one-way full duplex relaying in [18] in terms of outage probability and system throughput. It was concluded in [18] that full duplex one way relaying can outperform two way relaying half duplex communication system in the complex Gaussian channels. Dongwook et. al. in [19] analysed the full duplex two way relaying system, and considered that both the sources and relay are full duplex in nature. The achievable rate in quasi-static Rayleigh fading environment was found. The authors in [20] proposed a relay selection scheme for the full duplex bi-directional relaying, and obtained the outage probability, bit error rate, and ergodic capacity. The two way relaying based communication system considered in [20] consists the full duplex sources and multiple full duplex relays. For this considered system, the performance was analysed using cumulative distribution function (CDF) of the instantaneous SINR. We in this paper consider a two way relaying system with one full duplex relay, and we derive the system performance using probability density function (PDF) based approach.

### 1.2. Contributions

In summary, the main contributions of this paper are, The PDF of the instantaneous SINR is derived in presence of SI at the relay node. By using the derived PDF, average BER of the system is obtained for different modulation schemes such as coherent binary phase shift keying (BPSK), differential BPSK, coherent binary frequency shift keying (BFSK), and non-coherent BFSK. Moreover, the ergodic capacity of the considered system is derived.

Rest of the paper is organized as follows, system model is presented in Section 2, Section 3 describes the mathematical analysis, performance analysis is presented in Section 3.2. The simulation results are discussed in Section 4, and the paper is concluded in Section 5.

## 2. System model

The system model consists of two sources  $S_1$  and  $S_2$  and relay  $R$ . Relay  $R$  is assumed to be full duplex, thus we consider the effect of SI that exists between transmitting and receiving antennas of the relay. There is no direct link between the both the sources and we consider sources are far apart due to block channel fading effect [20].

The source  $S_1$  broadcasts information  $x_s(t)$  at an instant  $t$  with power  $P_s$  to the relay  $R$ . Similarly Source  $S_2$  also sends information  $x_r(t)$  with power  $P_r$  at the same instant. The signal received at the relay can be written as

$$y_r(t) = \sqrt{P_s}h_{11}x_s(t) + \sqrt{P_r}h_{21}x_r(t) + n_r(t) \quad (1)$$

where  $h_{11}$  and  $h_{21}$  are the channel coefficients from the sources  $S_1$  and  $S_2$  to the relay  $R$ , respectively. The relay node processes the information received by it and sends back to the sources. We consider here the decode-xor-forward (DxF) strategy in the relay node.

All the channels are assumed to be independent and identically distributed as complex gaussian distributions with zero mean and normalized variance  $\sigma$ . Thus the magnitude of the channel coefficients follows the Rayleigh distribution [23]. In general, a channel coefficient in Rayleigh fading can be characterized as,

$$f_X(x) = \frac{\chi e^{-\frac{x^2}{\sigma^2}}}{\sigma^2} \quad x \geq 0 \quad (2)$$

where  $\sigma$  is the variance of the Rayleigh distribution which indicates the power in the channel.

We consider a two-way (or Bi-Directional) relaying system, where there are two users which communicate to each other with

the help of a relay system. The relay first receives the signals from both users in first time slot, combines them through some kind of coding (in our paper we assume that the relay XORs the received signals), then in second time slot relay forwards the combined signals to both the users. The users can extract the information sent by the other user using its own message (in our case, any user can extract the message of other user from combined signal by simply XORing its own message with the combined signal). Hence it is not the case that user sends messages and receives it back after relaying. For this considered system we derived the unified expression for various binary modulation schemes, which, to the best of authors knowledge has not been explored in the literature.

## 3. Mathematical analysis

### 3.1. Signal-to-Noise-plus-Interference Ratio (SINR)

The SINR of the considered bi-directional relaying system can be given as in [20] as,

$$\gamma = \frac{\varphi_s \cdot \varphi_r |h_{11}|^2 |h_{21}|^2}{(\varphi_s + \varphi_r * \sigma_{e,r}^2 + 1) * |h_{11}|^2 + (\varphi_s * \sigma_{e,r}^2 + 1) * |h_{21}|^2 + c} \quad (3)$$

where  $c = (\sigma_{e,r}^2 + 1)$ ,  $\varphi_s = P_s/\sigma_n^2$ ,  $\varphi_r = P_r/\sigma_n^2$ ,  $\sigma_{e,r}^2$  represents variance of residual self-interference at the relay. Further,  $\varphi_s$  is the normalized source power, and  $\varphi_r$  is the normalized relay power.

**Lemma 1.** The PDF of instantaneous SINR  $\gamma$  in (3) can be given as

$$f_\gamma(\gamma) = \frac{2\sqrt{2a}}{\sigma\sqrt{\gamma}} e^{\left(\frac{-c}{2\sigma^2 a}\right)} e^{\left(\frac{-a\gamma^2}{4\sigma^2}\right)} W_{-1.5,0}\left(\frac{a\gamma^2}{2\sigma^2}\right) \quad (4)$$

where,  $pa$   $W_{a,b}(z)$  is the Whittaker function [22].

**Proof.** Detailed proof is given in Appendix A.  $\square$

In the next section the PDF of SINR is utilized to derive the performance metrics i.e. average bit error rate and ergodic capacity of the system.

### 3.2. Average BER

Average bit error rate(BER) can be written as

$$P_e = \int_0^\infty P(e|\gamma) f_\gamma(\gamma) d\gamma \quad (5)$$

where  $P(e|\gamma)$  is conditional bit error rate and  $f_\gamma(\gamma)$  is the PDF of SINR. The conditional error probability  $P(e|\gamma)$  is defined as in [21] as,

$$P(e|\gamma) = \frac{\Gamma(\mu_1, \mu_2 \gamma)}{2\Gamma(\mu_1)}, \quad (6)$$

where  $\mu_1, \mu_2$  are modulation dependent parameters such that  $\mu_1 = \mu_2 = 0.5$  characterizes the coherent BPSK,  $\mu_1 = 0.5, \mu_2 = 1$  represents coherent BFSK,  $\mu_1 = 1, \mu_2 = 0.5$  is used for non-coherent BFSK, and  $\mu_1 = \mu_2 = 1$  denotes the differential BPSK modulation.

**Lemma 2.** The average BER for the considered full duplex relay assisted bi-directional system can be given as,

$$P_e = \Phi \left( \Xi + \frac{c_{1p} F_q \left[ \{c_2, c_2, c_3\} \left\{ 1 + c_3, \frac{1}{2}, 2.75 + c_3 \right\}, \frac{\mu_2^2}{8n} \right]}{\Gamma[1 - c_3] \Gamma[2.75 + c_3]} - \frac{c_{4p} F_q \left[ \{c_5, c_5, \frac{1+2c_3}{2}\} \left\{ \frac{3+2c_3}{2}, 1.5, 3.25 + c_3 \right\}, \frac{\mu_2^2}{8n} \right]}{\Gamma[\frac{1}{2} - c_3] \Gamma[3.25 + c_3]} \right) \quad (7)$$

**Proof.** See Appendix B.  $\square$

3.3. Ergodic capacity

The ergodic capacity of a system can be given in terms of statistics of instantaneous SINR in the destination as,

$$C = \int_0^\infty \log_2(1 + \gamma) f_\gamma(\gamma) d\gamma \tag{8}$$

where  $f_\gamma(\gamma)$  is the probability density function of SINR.

**Lemma 3.** The ergodic capacity for the bi-directional relaying system considering the SI in the relay can be found to be,

$$C = \frac{2\sqrt{2a}}{\pi \log(16)\sigma} e^{\frac{(-\kappa)}{2\sigma^2 a}} G_{4,0}^{5,2} \left( 2\kappa \left| \frac{-1}{4}, \frac{1}{4}, \frac{3}{4}, 2.5 \frac{1}{2}, \frac{1}{2}, \frac{-1}{4}, \frac{-1}{4}, \frac{1}{4} \right. \right) \tag{9}$$

where,

$G_{p,q}^{m,n}(\cdot)$  is the Meijers-G function [24], and  $\kappa = \frac{a}{4\sigma^2}$ ,  $\sigma$  is the variance of the probability density function of the SINR.  $\pi$ ,  $\log(16)$  are the constants,  $\sigma$  is the standard deviation of the PDF of SINR.

**Proof.** See Appendix C. □

Hence, in this section we derived a new closed form expression for the ergodic capacity and average BER for various modulation schemes. (see Fig. 1).

4. Simulation results

In this section we provide the numerical results corresponding to the analytical expressions derived in this paper.

In Fig. 2, the average BER is plotted with average SNR(dB) for different modulation schemes such as BPSK, differential BPSK, coherent BFSK, and non-coherent BFSK. From the results, it can be observed that, among all modulation techniques considered in this work, coherent BPSK provides the better performance for the bi-directional DxF relaying system in presence of SI at the relay node. On the other hand, the performance of coherent BFSK is the worst among all the considered modulation schemes. Additionally, it can also be seen from the figure that the performance of differential BPSK is better than that of non-coherent BFSK at low SNR regime, however at higher SNRs, performance of both differential BPSK and non-coherent BFSK schemes are same.

Fig. 3 shows the error performance of the system for varying SI strengths. For this analysis plot, we consider the coherent BPSK

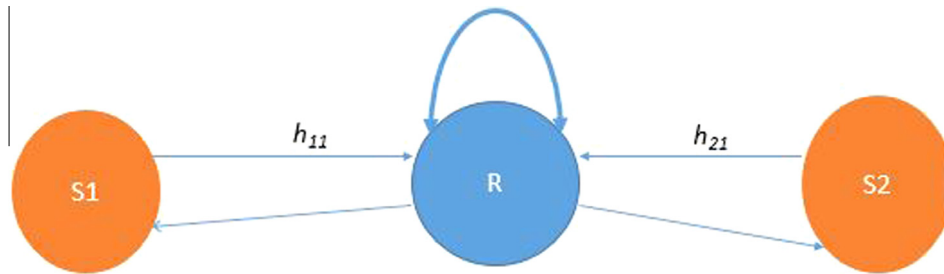


Fig. 1. Bi-directional communication with a full duplex relay.

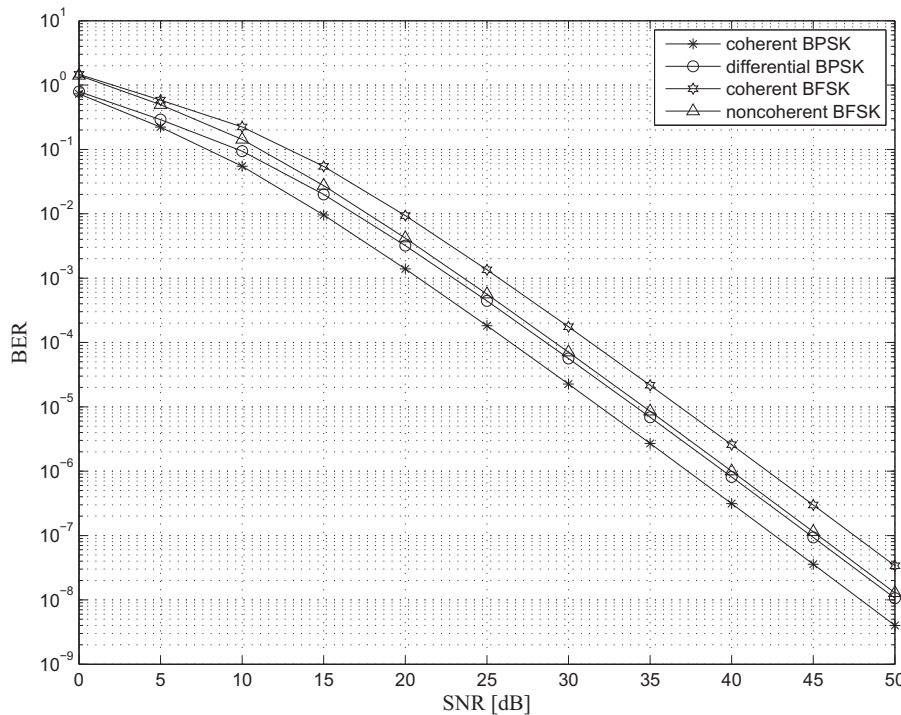


Fig. 2. Average BER for different binary modulation schemes.

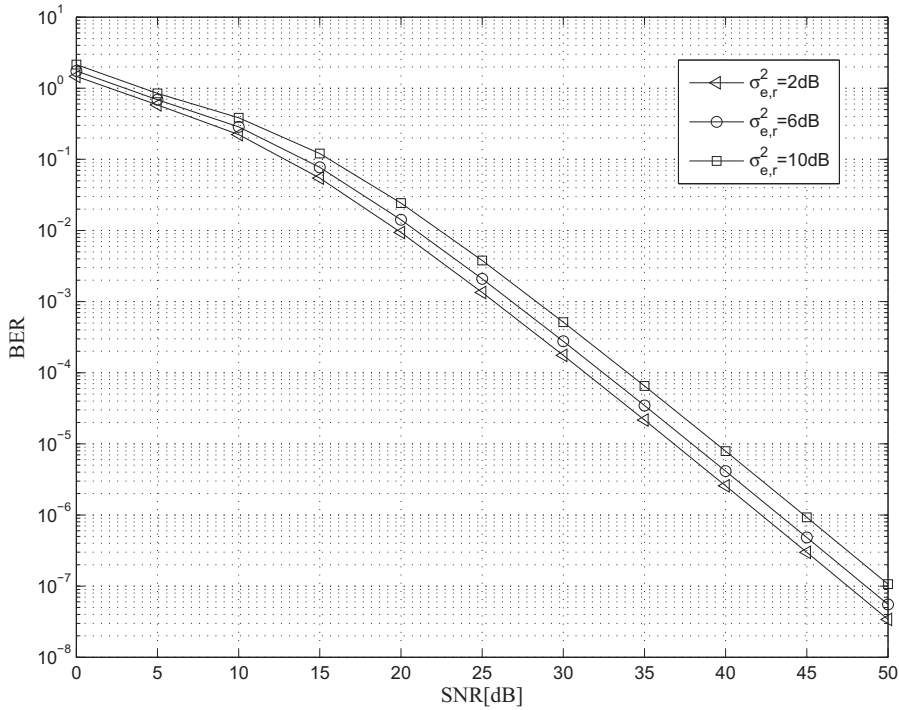


Fig. 3. Average BER for various SI powers.

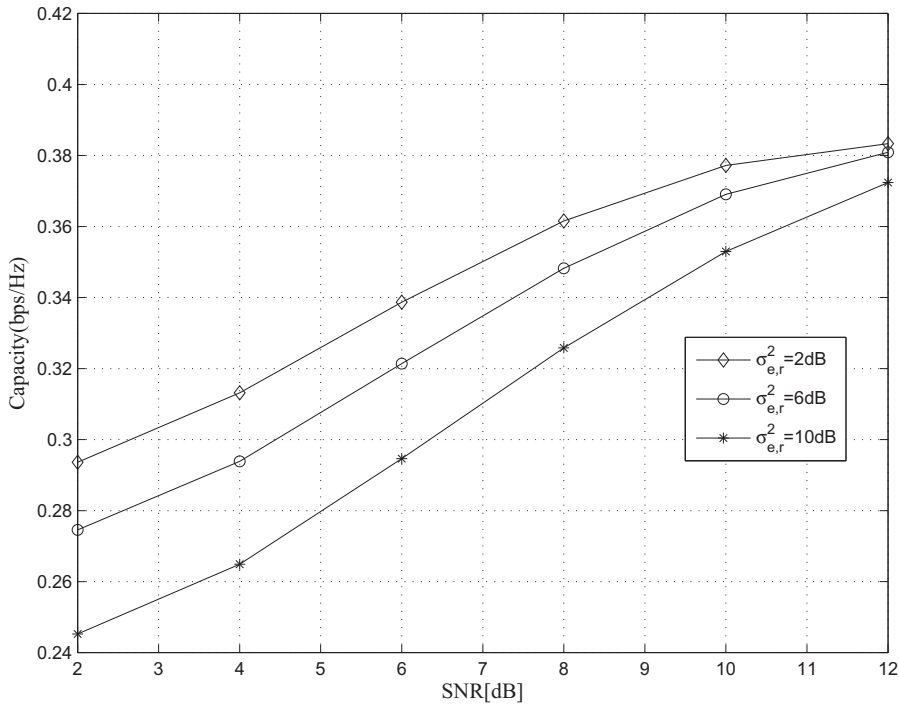


Fig. 4. Effect of self interference on ergodic capacity.

modulation scheme. We consider three different values of  $\sigma_{e,r}^2$  i.e.  $\sigma_{e,r}^2 = 2\text{ dB}$ ,  $\sigma_{e,r}^2 = 6\text{ dB}$ , and  $\sigma_{e,r}^2 = 10\text{ dB}$ . It can be seen from the result that as the strength of SI increases, the average BER of the system increases and system performance degrades. This result shows the detrimental effect of SI caused by the echo-interference between transmit and receive antennas of the full duplex relay.

The ergodic capacity analysis is illustrated in Fig. 4, where the effect of SI over the ergodic capacity can be observed in full duplex relaying systems. Here, we also assume three different values of parameter  $\sigma_{e,r}^2$  i.e.  $\sigma_{e,r}^2 = 2\text{ dB}$ ,  $\sigma_{e,r}^2 = 6\text{ dB}$ , and  $\sigma_{e,r}^2 = 10\text{ dB}$ . Similar to average BER results, the capacity performance (in bits per second (bps) per Hertz) degrades with the increase in the severity of self interfering power in the full duplex relay node. It can also be

stated, thus, that the SI in the relay node affects the performance of a bi-directional relaying system.

The ergodic capacity analysis is illustrated in Fig. 5, where the effect of SI for non-identical source powers over the ergodic capacity can be observed in full duplex relaying systems. Here we assume two different values of parameter  $\sigma_{e,r}^2$  i.e.  $\sigma_{e,r}^2 = 12$  dB,  $\sigma_{e,r}^2 = 5$  dB. Similar to average BER results, the capacity performance (in bits per second (bps) per Hertz) degrades with the increase in the severity of self-interfering power in the full duplex relay node.

It can be stated, thus, that the SI in the relay node adversely affects the performance of a bi-directional relaying system.

Fig. 6 shows the performance comparison between BPSK and BFSK for two different SI strengths. We assume two different values of parameter  $\sigma_{e,r}^2$  i.e.  $\sigma_{e,r}^2 = 3$  dB,  $\sigma_{e,r}^2 = 10$  dB. It can be shown that SI affects differently for different modulation techniques. As the SI strength increases BER performance degrades. Out of the two modulation techniques, BPSK provides better system performance for a given SI strength.

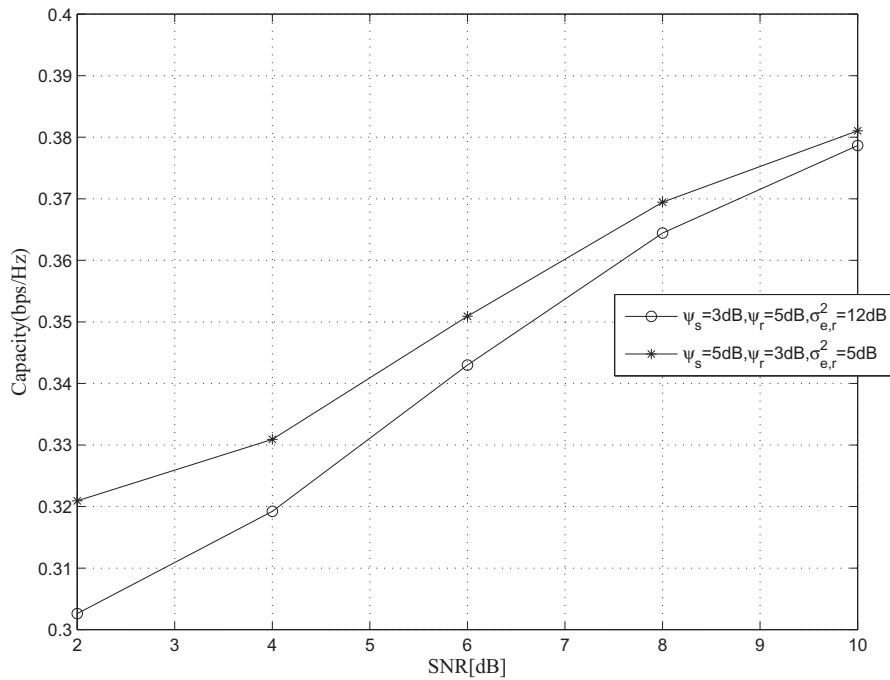


Fig. 5. Effect of self interference on ergodic capacity for different source powers.

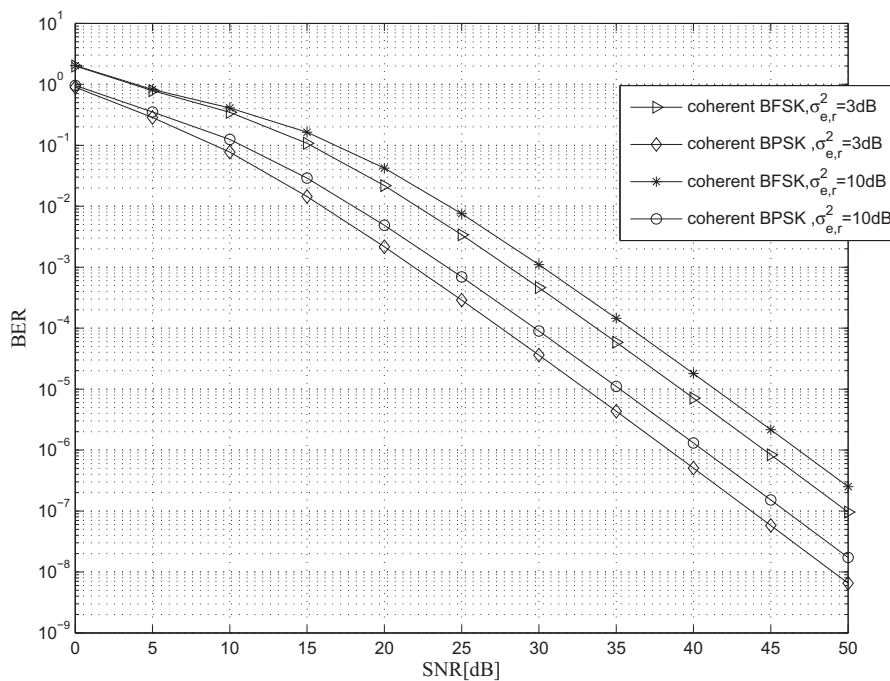


Fig. 6. Effect of self interference on modulation technique.

It is also found that as the strength of SI increases by interchanging the source powers, the average BER of the system increases and system performance degrades. This shows the detrimental effect of SI caused by the echo-interference between transmit and receive antennas of the full duplex relay.

**5. Conclusion and future work**

In this paper a bi-directional relaying system with DxF strategy is considered. In Rayleigh fading environment, the PDF of the instantaneous received SINR in the destination is derived. Using the PDF based approach the average BER is derived for various binary modulation schemes like binary phase shift keying (BPSK), differential BPSK, coherent binary frequency shift keying (BFSK), and non-coherent BFSK. Additionally a closed form expression for ergodic capacity of the considered system is also derived. It can be concluded that the SI in the full duplex relay node significantly affects the ergodic capacity and average BER performance of the considered bi-directional relaying system.

Consideration of imperfect state channel state information in the derivation will effect the overall system. Complexity will increase in the calculation of closed form expressions for BER and capacity. This scenario is currently being investigated as the part of future work.

**Appendix A. Proof of lemma 1**

X,Y are the two independent random variables follows normal distributions then  $|h_{11}|$  follows rayleigh distribution [23],  $H$  is the random variable of channel coefficient  $|h|$  the probability density function of  $|h|$  can be written as,

$$f_H(h) = \frac{he^{-\frac{h^2}{2\sigma^2}}}{\sigma^2}, \quad h \geq 0 \tag{10}$$

$K$  is the random variable of channel square coefficient, using the transformation of random variable technique probability density function for the square of the channel coefficient  $|h_{11}|^2$  can be written as,

$$f_K(k) = \frac{e^{-\frac{k}{2\sigma^2}}}{2\sigma^2}, \quad k \geq 0 \tag{11}$$

since the two channels are identical and independent, the density function for the square of the channel coefficient  $|h_{21}|^2$  can be written as

$$f_R(r) = \frac{e^{-\frac{r}{2\sigma^2}}}{2\sigma^2}, \quad r \geq 0 \tag{12}$$

product of the square of the channel coefficients  $|h_{11}|^2$  and  $|h_{21}|^2$  Eq. (3) is the another random variable  $P$ , can be written as,  $|h_{11}|^2 \cdot |h_{21}|^2 = P$  probability density function of the random variable  $P$  can be written as,

$$f_P(p) = \int_0^\infty f_K(k)f_R(p/k) \frac{1}{|k|} dk \tag{13}$$

$$= \frac{1}{4\sigma^4} \int_0^\infty \frac{1}{k} e^{-\frac{k}{2\sigma^2}} e^{-\frac{p}{2k\sigma^2}} dk \tag{14}$$

$$= \frac{1}{2\sigma^4} K_0\left(\frac{\sqrt{p}}{\sigma^2}\right) \tag{15}$$

where  $K_0\left(\frac{\sqrt{p}}{\sigma^2}\right)$  is the second order modified bessel function [24]

Noise power can be written as,

$$N = [\varphi_s + \varphi_r(\sigma_{e,r}^2 + 1)]|h_{11}|^2 + |h_{21}|^2 + [\sigma_{e,r}^2 + 1] \tag{16}$$

where  $\sigma_{e,r}^2$  is variance of the the self interference at the relay.  $a = [\varphi_s + \varphi_r(\sigma_{e,r}^2 + 1)]$ ,  $C = (\sigma_{e,r}^2 + 1) |h_{11}|^2$  is considered as the another random variable  $K$ ,  $|h_{21}|^2$  is the random variable  $R$ , by transforming into a single random variable  $N$ ,

$$N = aK + R + C \tag{17}$$

$$f_Y(y) = \frac{1}{a} f_K\left(\frac{y}{a}\right) \tag{18}$$

$N = Z + C$ ,  $Z$  is the random variable, the probability density function for the random variable  $N$  is given as,

$$f_N(n) = \frac{1}{2\sigma^2 a} e^{\left(\frac{-(n-c)}{2\sigma^2 a}\right)} \tag{19}$$

transforming total signal power into a random variable  $P$ , and noise power into a random variable  $N$ , the SINR is considered as a random variable  $X$ , the probability density function of the SINR can be given as follows,

$$SINR = \frac{P}{N} = \gamma \tag{20}$$

$$f_\gamma(\gamma) = \int_0^\infty n f_P(n\gamma) f_N(n) dn \tag{21}$$

$$= \int_0^\infty n \frac{1}{4\sigma^6 a} K_0\left(\frac{\sqrt{n\gamma}}{\sigma^2}\right) dn \tag{22}$$

solving the integral Eq. (21) using [22, Eq.(6.643.3)], the probability density function for SINR can be written as

$$f_\gamma(\gamma) = \frac{2\sqrt{2a}}{\sigma\sqrt{\gamma}} e^{\left(\frac{-c}{2\sigma^2 a}\right)} e^{\left(\frac{-\sigma^2}{4\sigma^2}\right)} W_{-1.5,0}\left(\frac{a\gamma^2}{2\sigma^2}\right) \tag{23}$$

where,  $W_{a,b}(z)$  is the Whittaker function [24],  $a = -1.5, b = 0, z = \left(\frac{a\gamma^2}{2\sigma^2}\right)$

**Appendix B. Proof of lemma 2**

$$P_e = \int_0^\infty P(e/\gamma) f_\gamma(\gamma) d\gamma \tag{24}$$

$P(e/\gamma)$  is the conditional error probability can be given as,

$$P(e/\gamma) = \frac{\Gamma(\mu_1, \mu_2 \gamma)}{2\Gamma(\mu_1)} \tag{25}$$

Substituting  $f_\gamma(\gamma)$  and Conditional probability density function  $P(e/\gamma)$ , average BER  $P_e$  can be written as

$$P_e = \int_0^\infty \frac{\Gamma(\mu_1, \mu_2 \gamma)}{2\Gamma(\mu_1)} \frac{2\sqrt{2a}}{\sigma\sqrt{\gamma}} e^{\left(\frac{-c}{2\sigma^2 a}\right)} e^{\left(\frac{-\sigma^2}{4\sigma^2}\right)} W_{-1.5,0}\left(\frac{a\gamma^2}{2\sigma^2}\right) d\gamma \tag{26}$$

By using the mathematical techniques the average bit error can be calculated as

$$P_e = \Phi \left( \Xi + \frac{c_{1p} F_q \left[ \{c_2, c_2, c_3\} \left\{ 1 + c_3, \frac{1}{2}, 2.75 + c_3 \right\}, \frac{\mu_2^2}{8n} \right]}{\Gamma[1 - c_3] \Gamma[2.75 + c_3]} - \frac{c_{4p} F_q \left[ \{c_5, c_5, \frac{1+2c_3}{2}\}, \left\{ \frac{3+2c_3}{2}, 1.5, 3.25 + c_3 \right\}, \frac{\mu_2^2}{8n} \right]}{\Gamma[\frac{1}{2} - c_3] \Gamma[3.25 + c_3]} \right) \tag{27}$$

where,  ${}^pF_q$  is the hypergeometric function,

$$\Xi = \frac{(0.110737) 2^{\mu_1} \Gamma(\frac{\mu_1}{2}) \Gamma(\frac{1+\mu_1}{2})}{-n^{\frac{1}{4}}}, \quad n = \frac{a}{4\sigma^2}$$

$$\Phi = \frac{\sqrt{2a}}{\sigma \Gamma(\mu_1)} e^{\left(\frac{-n}{2a\sigma^2}\right)}$$

$c_1 = 2^{(-\frac{3}{4}-\frac{\mu_1}{2})} n^{(-\frac{1}{4}-\frac{\mu_1}{2})} \mu_2^{\mu_1} \Gamma(\frac{3}{4} + \frac{\mu_1}{2})^2 \Gamma(-\frac{\mu_1}{2})$ ,  $c_2 = \frac{3+2\mu_1}{4}$ ,  $c_3 = \frac{\mu_1}{2}$ ,  $c_4 = 2^{(-\frac{1}{4}-\frac{\mu_1}{2})} n^{(-\frac{3}{4}-\frac{\mu_1}{2})} \mu_2^{(1+\mu_1)} \Gamma[-\frac{1}{2}-\frac{\mu_1}{2}] \Gamma[\frac{5}{4} + \frac{\mu_1}{2}]^2$ ,  $c_5 = \frac{1}{4}(5+2\mu_1)$ ,  $\mu_1, \mu_2$  are the modulation related parameters such that  $\mu_1 = 0.5, \mu_2 = 0.5$  for coherent BPSK,  $\mu_1 = 0.5, \mu_2 = 1$  coherent BFSK,  $\mu_1 = 1, \mu_2 = 0.5$  non-coherent BFSK,  $\mu_1 = 1, \mu_2 = 1$  differential BPSK. The average BER can be calculated for different modulation schemes by substituting the values of  $\mu_1, \mu_2$  accordingly.

### Appendix C. Proof of lemma 3

The ergodic capacity  $c_a$  can be expressed using the PDF of SINR as,

$$C = \int_0^\infty \log(1 + \gamma) \frac{2\sqrt{2a}}{\sigma\sqrt{\gamma}} e^{\left(\frac{-\gamma}{2\sigma^2 a}\right)} e^{\left(\frac{-a\gamma^2}{4\sigma^2}\right)} W_{-1.5,0}\left(\frac{a\gamma^2}{2\sigma^2}\right) d\gamma$$

$$= \frac{2\sqrt{2a}}{\sigma} e^{\left(\frac{-\gamma}{2\sigma^2 a}\right)} \int_0^\infty \log_2(1 + \gamma) \frac{1}{\sqrt{\gamma}} e^{\left(\frac{-a\gamma^2}{4\sigma^2}\right)} W_{-1.5,0}\left(\frac{a\gamma^2}{2\sigma^2}\right) d\gamma \quad (28)$$

The integral can in (28) can be solved using Mathematica [24], and the ergodic capacity of the system can be expressed as,

$$C = \frac{2\sqrt{2a}}{\pi \log \sin \sigma} e^{\left(\frac{-\gamma}{2\sigma^2 a}\right)} G_{p,q}^{m,n} \left( 2\kappa \left| \frac{-1}{4}, \frac{1}{4}, \frac{3}{4}, 2.5 \frac{1}{2}, \frac{1}{2}, \frac{-1}{4}, \frac{-1}{4}, \frac{1}{4} \right. \right) \quad (29)$$

where,

$G_{p,q}^{m,n}(\cdot)$  is the Meijers-G function [24], and  $\kappa = \frac{a}{4\sigma^2}$ .

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