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Active vibration suppression in flexible spacecraft with optical measurement

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ABSTRACT

With the development of the space station technology in China, the active vibration suppression of the lager flexible antennas and solar panels that mounted on the space station is demanded. The active vibration suppression is hard to achieve because these appendages are not allowed to mount actuators on it. Further, the practical goal is usually unachievable by the controller based on the unreliability or impracticality observed vibration parameters. In this paper, the active vibration suppression during the spacecraft attitude maneuver by the optical camera monitoring the dynamic behaviors of the flexible appendages is introduced under the restriction of the freedom of the actuators. The modal parameters of the flexible appendage used in the controller are obtained by the optical monitoring approach. A referenced angular velocity is set as the virtual input by the back-stepping control and Lyapunov method, and the control law is designed to track this virtual input. The constraint of the coefficient in the controller is investigated to guarantee the asymptotic convergence of the system considering angular velocity tracking error. The control manage coefficient is defined to describe the distribution coefficient of the vibration suppression in the design of control law. The relationship between the attitude maneuver accuracy and active vibration suppression is introduced. The appropriate control manage coefficient is obtained numerically. To guarantee the reliability or practicality of the designate flexible spacecraft control system, the optical measurements are used to measure the dynamic behaviors of the large flexible structures. The absence of displacement velocity sensors is compensated by the presence of appropriate dynamics in the controller. The results of simulation validate the feasibility of the proposed control strategy.

1. Introduction

The active suppression of flexible vibration during spacecraft attitude maneuvers is an intriguing problem that has recently stimulated research activities. Both small satellites with flexible booms, and large space stations composed of light deformable structures will get benefits from the development of vibration damping control. The dynamic behaviors of the large flexible structures are difficult to be predicted analytically, not to mention the unreliability or impracticality of structural tests on Earth, the performance of controller designed on the basis of the perfect model is deteriorated. These difficulties will lead to the derivation between the on-orbit behaviors of spacecraft and the preflight ground test measures or analytic predictions [1]. To overcome these problems, the adaptive control schemes, belonging to the called centralized active vibration control approach, are proposed estimating unavailable states by the observer. An alternative to this approach is the use of structures with distributed actuators and sensors, which is also called the distributed active vibration control. With the development of space station instrumented with large antennas and solar panels in China, further investigation on the practical method accomplishing the active vibration suppression for these appendages, which are not allowed to mount actuators, is expected.

The active vibration control is consisting of the centralized active vibration control and the distributed active vibration control. The former is designed for the appendages that are not collocated with distributed actuators. Researchers derived a class of centralized active vibration controllers for the flexible spacecraft [2–6]. A control approach that integrates the command input shaping and the technique of dynamic variable structure output feedback control was put forward for the vibration control of flexible spacecraft during attitude maneuver [2]. An L1 adaptive controller was developed for the pitch angle control of an orbiting flexible spacecraft with a moment producing device located on the central rigid

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body. A state predictor was added up to the controller generating the assessments of the unknown parameters for feedback [3]. Treating the vibration mode of flexible appendages as the inherent perturbation, a robust control strategy was developed for the flexible spacecraft during large-angle attitude maneuver [4]. At the terminal of the attitude maneuver, a state feedback controller was employed to damp the residual vibration of appendages. A robust fuzzy controller for attitude stabilization of a rigid platform with a flexible appendage was introduced by proposing a fuzzy observer to estimate the unavailable states [5]. Haibo Du [6] studied on a distributed attitude cooperative control strategy to solve the problem of attitude synchronization for a group of flexible spacecraft during formation maneuvers. Based on the back-stepping design, a distributed attitude cooperative control law was explicitly constructed by a modal observer. However, the dynamic behaviors of the large flexible structures are difficult to be predicted analytically, which cause that the requirements of control in the practical mission are hardly to be satisfied by the traditional vibration suppression method without measurements of the dynamics behaviors. For the distributed active vibration control, the number of materials of actuators and sensors had been investigated and fabricated over the years; some of them were shape memory alloys, piezoelectric materials, optical fibers, electro-rheological fluids, magneto-strictive materials [7]. Among all materials, piezoelectric materials were widely used as sensor and actuator because of its numerous advantages like low cost, quick dynamic response, low power consumption, excellent electromechanical coupling, large operating range, light weight and ease in bonding on structure [8–12]. Nevertheless, these actuators and sensors have changed the structure property of the flexible appendage by mechanical interfere, which is not allowed by most large light flexible structure. It implies the vibration suppression utilizing the piezoelectric sensors is not appropriate from the engineering viewpoint. Flexible appendages cannot be allocated with actuators and the structural tests on Earth are neither unreliable nor impractical. To improve such group of flexible appendages, it is significance to find a middle ground between the centralized and decentralized active vibration control. To obtain the vibration parameters, the full-bridge strain gauges and a camera [13] and a laser displacement sensor as well as a laser vibrometer [14] were employed to sense the necessary data for vibration control. The feasibility of modal modification technology in the flight was researched by V. Wickramasinghe [15]. The methods to get the modal parameters of flexible appendages through monitoring and identification approaches were proposed [16–20]. Taking advantages of the aforementioned control schemes, the centralized active vibration suppression with the optical camera monitoring the dynamic behaviors of the flexible appendages is investigated as the first contribution of this paper. The modal parameters of the flexible appendage used in the controller are obtained by the optical monitoring approach.

Despite the displacement information on the vibration can be obtained by traditional piezoelectric sensors and optical measurements, it is hard for these sensors to obtain the velocity of vibration. Although the micro-electromechanical systems (MEMS) may be used to measure the velocity of vibration in the near future, they are unacceptable for most of large light flexible appendage, because of their mechanical interface and complexity. Therefore, the alternative way is to compensate the absence of velocity vibration by the dynamics property. The controller is constructed by the back-steeping control and Lyapunov control methods for the flexible spacecraft with an actuator collocated at the spacecraft body. The constraint of the control coefficients is introduced to guarantee the asymptotical stability of the whole system, which is ignored in the previous literatures. Because the challenge of the vibration suppression during the attitude maneuver lies in the restriction of the freedom of the actuators, the relationship between the attitude maneuver and the vibration suppression should be investigated. Therefore, as the second contribution of this paper, a new variable named control manage coefficient is defined to describe the relationship between the attitude maneuver and active vibration suppression. The appropriate control manage coefficient is obtained numerically, which not only effectively damps the vibration but also greatly improves the accuracy of the attitude maneuver and the performance of the control torque. The remainder of this paper is outlined as follows. In section 2, the mathematical model of a spacecraft with flexible appendages and the optical measurement is introduced. In section 3, the referenced angular velocity is constructed by back-steeping control and Lyapunov methods, the control law is derived and the relationship between the attitude maneuver and active vibration suppression is introduced. In Section 4, the controller is tested and the appropriate control manage coefficient is obtained, followed by analyses and conclusions.

2. Mathematical models

The mathematical models of a flexible spacecraft are consisting of the kinematic and the dynamic model. The mathematical models proposed in this paper were introduced in the Ref. [8]. The dynamic behaviors of the large flexible structures are difficult to be predicted analytically, not to mention the unreliability or impracticality of structural, and the optical measurements are used to measure the displacement of the flexible appendages for the distributed sensors are not allowed to mount on the appendages. By setting the position of these optical signs, the displacements of the whole appendages can be monitored.

2.1. Mathematical equations

The flexible spacecraft is composed of a rigid main body and flexible appendages. The kinematics equations are defined by the attitude motion of the main body and expressed by the unit quaternion as

$$\begin{bmatrix} \dot{q}_0 \\ \dot{\boldsymbol{q}} \end{bmatrix} = \frac{1}{2} \boldsymbol{Q}_0(q_0, \boldsymbol{q}) \boldsymbol{\omega} \tag{1}$$

where $\boldsymbol{\omega}$ is the angular velocity of the spacecraft in the body fixed frame, $[q_0 \ \boldsymbol{q}^T]^T$ the unit quaternion vector, $\boldsymbol{q}$ the vector components of unit quaternion, and

$$\boldsymbol{Q}_0(q_0, \boldsymbol{q}) = \begin{bmatrix} -\boldsymbol{q}^T \\ q_0 \boldsymbol{E}_3 + \tilde{\boldsymbol{q}} \end{bmatrix}$$

with $\tilde{\boldsymbol{q}}$ as the cross product as

$$\tilde{\boldsymbol{q}} = \begin{bmatrix} 0 & -q_z & q_y \\ q_z & 0 & -q_x \\ -q_y & q_x & 0 \end{bmatrix}$$

and $\boldsymbol{E}_3$ the identity matrix.

By Lagrangian approach, the dynamical equations of the flexible spacecraft with the piezoelectric actuators were introduced in Ref. [1]. This paper considers the situation without the piezoelectric actuators. With $\boldsymbol{q}_{fi}$ as the modal coordinate vector, the dynamical equations of the spacecraft with flexible appendages are expressed as

$$\begin{cases} \boldsymbol{I}_{bt}\dot{\boldsymbol{\omega}} + \boldsymbol{H}\ddot{\boldsymbol{q}}_{fi} + \tilde{\boldsymbol{\omega}}\boldsymbol{I}_{bt}\boldsymbol{\omega} + \tilde{\boldsymbol{\omega}}\boldsymbol{H}\dot{\boldsymbol{q}}_{fi} = \boldsymbol{T}_b \\ \ddot{\boldsymbol{q}}_{fi} + \boldsymbol{C}_{fi}\dot{\boldsymbol{q}}_{fi} + \boldsymbol{K}_{fi}\boldsymbol{q}_{fi} = -\boldsymbol{H}^T\dot{\boldsymbol{\omega}} \end{cases} \tag{2}$$

where $\boldsymbol{I}_{bt}$ is the symmetric inertia matrix of the whole structure, $\boldsymbol{H}$ the coupling matrix between the rigid body and the second derivative of the modal coordinates, $\boldsymbol{T}_b$ the control torque, $\boldsymbol{C}_{fi}$ the damping matrices, and $\boldsymbol{K}_{fi}$ the stiffness matrices.

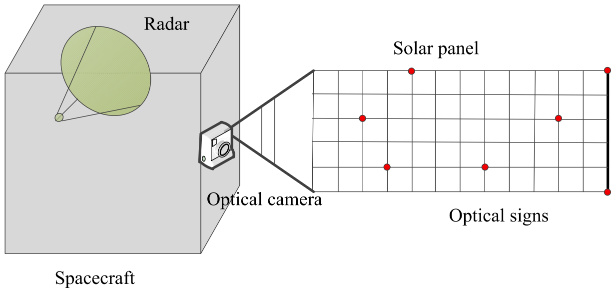

Fig. 1. Configuration of the flexible spacecraft.

To compensate for the absence of displacement velocity sensors, suppose $\boldsymbol{\psi} = \boldsymbol{H}^T\boldsymbol{\omega} + \dot{\boldsymbol{q}}_{fi}$, hence the mathematical models of the flexible spacecraft are given as

$$\begin{cases} \dot{\boldsymbol{q}} = \dfrac{1}{2}\boldsymbol{E}(\boldsymbol{q})\boldsymbol{\omega} \\ \dot{\boldsymbol{\psi}} = -\boldsymbol{C}_{fi}\boldsymbol{\psi} - \boldsymbol{K}_{fi}\boldsymbol{q}_{fi} + \boldsymbol{C}_{fi}\boldsymbol{H}^T\boldsymbol{\omega} \\ (\boldsymbol{I}_{bt} - \boldsymbol{H}\boldsymbol{H}^T)\dot{\boldsymbol{\omega}} + (\tilde{\boldsymbol{\omega}}\boldsymbol{H} - \boldsymbol{H}\boldsymbol{C}_{fi})(\boldsymbol{\psi} - \boldsymbol{H}^T\boldsymbol{\omega}) \\ \quad - \boldsymbol{H}\boldsymbol{K}_{fi}\boldsymbol{q}_{fi} + \tilde{\boldsymbol{\omega}}\boldsymbol{I}_{bt}\boldsymbol{\omega} = \boldsymbol{T}_b \end{cases} \tag{3}$$

The absence of velocity sensor makes $\dot{\boldsymbol{q}}_{fi}$ immeasurable. In equation (3), $\dot{\boldsymbol{q}}_{fi}$ is replaced by $\boldsymbol{\psi}$ which is yielded by a recursive method. The convergence in obtaining $\boldsymbol{\psi}$ is proven in the Refs. [6, 8].

2.2. Optical measurement

The appendages instrumented on the spacecraft, e.g. antennas, solar arrays and solar panels, are not allowed to mount piezoelectric actuators, thus a optical camera is utilized to measure the displacements of the structure and the centralized active vibration suppression strategy is developed during the attitude maneuver. The configuration of the whole spacecraft composed of a main body, a flexible appendage and a optical camera, is presented in Fig. 1.

To measure the displacements of the vibration, several optical reflective signs are first stuck on the surface to the flexible appendage. Then, three-dimensional position of the optical signs is measured by optical measurements using the current optical measurement theory, which is called as stereo imaging [21].

Because the vibration displacement components of the optical signs orthogonal to the surface of the flexible appendage is large than the other two directions, the vibration displacements and mode along this direction are employed to calculate the modal coordinates. The relationship between the vibration displacements, vibration mode and the modal coordinate components along this direction are defined as [16]

$$\boldsymbol{S} = \boldsymbol{N}\boldsymbol{q}_{fi} \tag{4}$$

where $\boldsymbol{S} = [S_1^T,\ S_2^T,\ \ldots,\ S_n^T]^T, \boldsymbol{N} = [N_1^T,\ N_2^T,\ \ldots,\ N_n^T]^T$. S_i $(i = 1, 2, \ldots, n)$ is the displacements of the ith optical sign, N_i $(i = 1, 2, \ldots, n)$ the vibration mode of the ith optical sign, and n the number of the optical signs.

The modal of the appendage is modified by the displacements of the appendage measured by the optical camera. Through the monitoring and identification approaches given by Refs. [16–20], the modal coordinates describing the vibration of the appendages can be expressed as

$$\boldsymbol{q}_{fi} = \left[(\boldsymbol{N}^m)^T\boldsymbol{N}^m\right]^{-1}(\boldsymbol{N}^m)^T\boldsymbol{S} \tag{5}$$

where $\boldsymbol{N}^m$ is the modified modal of the appendages. In equation (5), the number of the candidate optical sign n and the order of the vibration mode s satisfy that $n \geq s$, and the location of the optical sign ensures $(\boldsymbol{N}^m)^T\boldsymbol{N}^m$ is a invertible matrix. The modal coordinates defined by equation (5) are utilized in the control strategy. We assume the modal is modified in the following sections.

3. Control strategy

The control strategies for the rest-to-rest maneuver presented in the literature can be divided into groups based on their functions: to generate a closed-loop feedback controller or a robust controller and to track an optimal slew trajectory. In the case of the fast large-angle attitude maneuvers with limited energy and control capability, the third control strategy is more applicable from the viewpoint of engineering. Without the loss of generality, the target attitude position discussed in this paper is supposed to be zero, and the other rest-to-rest maneuvers can be similarly produced by replacing the unit quaternion with its error to the target quaternion. The modal coordinates used in the control strategy are obtained by the vibration displacement measured by the optical measurements.

3.1. Referenced angular velocity design

The slew trajectory is designed by the back-stepping control and Lyapunov approaches. The referenced angular velocity is set as virtual control input. The attitude synchronization and the vibration suppression can be achieved asymptotically by tracking this virtue control input. Since the modal coordinate $\boldsymbol{q}_{fi}$ and the modal variable $\boldsymbol{\psi}$ of the controller are obtained by the optical measurements and the designate observer, respectively, the control strategy is investigated in the following part.

Lemma 1. *For the system (3), if the referenced angular velocity is designed as*

$$\omega_d = -\boldsymbol{k}_\beta\left[\boldsymbol{q} + \boldsymbol{H}\boldsymbol{k}_1(\boldsymbol{C}_{fi}\boldsymbol{\psi} - \boldsymbol{K}_{fi}\boldsymbol{q}_{fi})\right] \tag{6}$$

where $\boldsymbol{k}_\beta > \boldsymbol{0}$, and $\boldsymbol{k}_1 > \boldsymbol{0}$ is defined as the control manage coefficient, then the attitude synchronization and the vibration suppression is achieved asymptotically.

Proof. To prove Lemma 1, considering the candidate Lyapunov function

$$V_1 = 2(1 - q_0) + \frac{1}{2}\boldsymbol{\psi}^T\boldsymbol{k}_1\boldsymbol{\psi} + \frac{1}{2}\boldsymbol{q}_{fi}^T\boldsymbol{k}_1\boldsymbol{K}_{fi}\boldsymbol{q}_{fi} \tag{7}$$

The derivative of equation (7) along system (3) is

$$\dot{V}_1 = -\boldsymbol{\psi}^T\boldsymbol{k}_1\boldsymbol{C}_{fi}\boldsymbol{\psi} + \left[\boldsymbol{q}^T + (\boldsymbol{\psi}^T\boldsymbol{C}_{fi} - \boldsymbol{q}_{fi}^T\boldsymbol{K}_{fi})\boldsymbol{k}_1\boldsymbol{H}^T\right]\boldsymbol{\omega} \tag{8}$$

Denote $\boldsymbol{\beta} = [\boldsymbol{q} + \boldsymbol{H}\boldsymbol{k}_1(\boldsymbol{C}_{fi}\boldsymbol{\psi} - \boldsymbol{K}_{fi}\boldsymbol{q}_{fi})]$, which implies that $\boldsymbol{\omega}_d = -\boldsymbol{k}_\beta\boldsymbol{\beta}$. Substituting the virtual control law (6) into (8), the derivative of the candidate Lyapunov function yields

$$\dot{V}_1 = -\boldsymbol{\psi}^T\boldsymbol{C}_{fi}\boldsymbol{\psi} - \boldsymbol{\beta}^T\boldsymbol{k}_\beta\boldsymbol{\beta} \leq 0 \tag{9}$$

Noticing that $\boldsymbol{k}_\beta > \boldsymbol{0}$, it is concluded that $V_1 \to 0$ as $t \to \infty$ by the LaSalle's invariant principle, which implies that $\{\boldsymbol{\psi}, \boldsymbol{q}_{fi}, \boldsymbol{\beta}\} \to \boldsymbol{0}$ and $q_0 \to 1$ as $t \to \infty$. By the definition of quaternions and modal variables, the attitude synchronization and the vibration suppression can be achieved asymptotically.

In the above references, the modal variables $\boldsymbol{q}_{fi}$ and $\boldsymbol{\psi}$ were not measured, the states $(\boldsymbol{q}_{fi}, \boldsymbol{\psi})$ of the controller is predicted by an observer. Due to the complicated space environments and the unreliability or impracticality of structural, the dynamic behaviors of the large flexible structures are difficult to be predicted analytically, and the controller based on the observer is improper for in-orbit services. Considering the aforementioned facts, it is better to take place of the observer by the optical measurement obtaining the vibration state of the appendage, and $\boldsymbol{\psi}$ used in the controller is yielded by a convergence observer. Because the vibration suppression is highly effected by $\boldsymbol{q}_{fi}$ not $\boldsymbol{\psi}$, the control strategy proposed in this paper is capable of improving the reliability or practicality of the controller. □

3.2. Control law design

A control law for $\boldsymbol{T}_b$ is designed to achieve attitude consensus and vibration suppression.

Theorem 1. *For the flexible spacecraft system (3), if the control is designed as*

$$\begin{aligned}\boldsymbol{T}_b &= (\tilde{\boldsymbol{\omega}}\boldsymbol{H} - \boldsymbol{H}\boldsymbol{C}_{fi})(\boldsymbol{\psi} - \boldsymbol{H}^T\boldsymbol{\omega}) - \boldsymbol{H}\boldsymbol{K}_{fi}\boldsymbol{q}_{fi} + \tilde{\boldsymbol{\omega}}\boldsymbol{I}_{bt}\boldsymbol{\omega} \\ &\quad + (\boldsymbol{I}_{bt} - \boldsymbol{H}\boldsymbol{H}^T)\left[\dot{\boldsymbol{\omega}}_d - \boldsymbol{k}_\omega(\boldsymbol{\omega} - \boldsymbol{\omega}_d)\right]\end{aligned} \tag{10}$$

where $\boldsymbol{k}_\beta, \boldsymbol{k}_\omega > \mathbf{0}$, the attitude synchronization and the vibration suppression is achieved with

$$\begin{aligned}\dot{\boldsymbol{\omega}}_d &= -\frac{1}{2}\boldsymbol{k}_\beta\boldsymbol{E}(\boldsymbol{q})\boldsymbol{\omega} + \boldsymbol{k}_\beta\boldsymbol{H}\boldsymbol{k}_1\left(\boldsymbol{C}_{fi}^2 + \boldsymbol{K}_{fi}\right)(\boldsymbol{\psi} - \boldsymbol{H}^T\boldsymbol{\omega}) \\ &\quad + \boldsymbol{k}_\beta\boldsymbol{H}\boldsymbol{k}_1\boldsymbol{C}_{fi}\boldsymbol{K}_{fi}\boldsymbol{q}_{fi}\end{aligned} \tag{11}$$

Proof. Define $\boldsymbol{e}_{\boldsymbol{\omega}} = \boldsymbol{\omega} - \boldsymbol{\omega}_d$ the tracking error of angular velocity, substituting the control law (10) into system (3), the angular tracking error is obtained as

$$\dot{\boldsymbol{e}}_{\boldsymbol{\omega}} = -\boldsymbol{k}_\omega\boldsymbol{e}_{\boldsymbol{\omega}} \tag{12}$$

In conclusion, the closed-loop system yields

$$\begin{cases}\dot{\boldsymbol{q}} = \frac{1}{2}\boldsymbol{E}(\boldsymbol{q})\boldsymbol{\omega}_d + \frac{1}{2}\boldsymbol{E}(\boldsymbol{q})\boldsymbol{e}_{\boldsymbol{\omega}} \\ \dot{\boldsymbol{\psi}} = -\boldsymbol{C}_{fi}\boldsymbol{\psi} - \boldsymbol{K}_{fi}\boldsymbol{q}_{fi} + \boldsymbol{C}_{fi}\boldsymbol{H}^T\boldsymbol{\omega}_d + \boldsymbol{C}_{fi}\boldsymbol{H}^T\boldsymbol{e}_{\boldsymbol{\omega}} \\ \boldsymbol{\omega}_d = -\boldsymbol{k}_\beta\left[\boldsymbol{q} + \boldsymbol{H}\boldsymbol{k}_1(\boldsymbol{C}_{fi}\boldsymbol{\psi} - \boldsymbol{K}_{fi}\boldsymbol{q}_{fi})\right] \\ \dot{\boldsymbol{e}}_{\boldsymbol{\omega}} = -\boldsymbol{k}_\omega\boldsymbol{e}_{\boldsymbol{\omega}}, \boldsymbol{q}_{fi} = \left[(\boldsymbol{N}^m)^T\boldsymbol{N}^m\right]^{-1}(\boldsymbol{N}^m)^T\boldsymbol{S}\end{cases} \tag{13}$$

Since $\boldsymbol{k}_\omega$ is definite positive, the subsystem $\dot{\boldsymbol{e}}_{\boldsymbol{\omega}} = -\boldsymbol{k}_\omega\boldsymbol{e}_{\boldsymbol{\omega}}$ is globally asymptotically stable. When, $\boldsymbol{e}_{\boldsymbol{\omega}}$ converges to zero, it follows from the Lemma 1 that the attitude synchronization and the vibration suppression can be achieved asymptotically. At the beginning of the attitude maneuver, the tracking error of angular velocity $\boldsymbol{e}_{\boldsymbol{\omega}}$ may be large, so the influence of the tracking error to the closed-loop system should be considered in the control law. □

Theorem 2. *For the closed-loop system (13), if the gain matrix $\boldsymbol{k}_\beta, \boldsymbol{k}_\omega$ in the referenced angular velocity and control law satisfy that $\boldsymbol{k}_\beta\boldsymbol{k}_\omega \geq 0.25\boldsymbol{E}_3$ ($\boldsymbol{k}_\beta, \boldsymbol{k}_\omega > \mathbf{0}$), the attitude synchronization and the vibration suppression are achieved asymptotically.*

Proof. To strictly prove the stabilization of the closed-loop system, the Lyapunov function is constructed as

$$V_4 = 2(1 - q_0) + \frac{1}{2}\boldsymbol{\psi}^T\boldsymbol{k}_1\boldsymbol{\psi} + \frac{1}{2}\boldsymbol{q}_{fi}^T\boldsymbol{k}_1\boldsymbol{K}_{fi}\boldsymbol{q}_{fi} + \frac{1}{2}\boldsymbol{e}_{\boldsymbol{\omega}}^T\boldsymbol{e}_{\boldsymbol{\omega}} \tag{14}$$

The derivative of equation (14) along system (13) is

$$\dot{V}_4 = -\boldsymbol{\psi}^T\boldsymbol{k}_1\boldsymbol{C}_{fi}\boldsymbol{\psi} - \left(\boldsymbol{\beta}^T\boldsymbol{k}_\beta\boldsymbol{\beta} - \boldsymbol{\beta}^T\boldsymbol{e}_{\boldsymbol{\omega}} + \boldsymbol{e}_{\boldsymbol{\omega}}^T\boldsymbol{k}_\omega\boldsymbol{e}_{\boldsymbol{\omega}}\right) \tag{15}$$

Decompose these two gain matrix as

$$\begin{cases}\boldsymbol{k}_\beta = \boldsymbol{k}'_\beta + \boldsymbol{k}''_\beta \\ \boldsymbol{k}_\omega = \boldsymbol{k}'_\omega + \boldsymbol{k}''_\omega\end{cases}$$
$$\left\{\boldsymbol{k}'_\beta, \boldsymbol{k}'_\omega, > \mathbf{0}\ \boldsymbol{k}''_\beta, \boldsymbol{k}''_\omega \geq \mathbf{0} \quad \text{and} \quad \boldsymbol{k}'_\beta\boldsymbol{k}'_\omega = 0.25\boldsymbol{E}_3\right\} \tag{16}$$

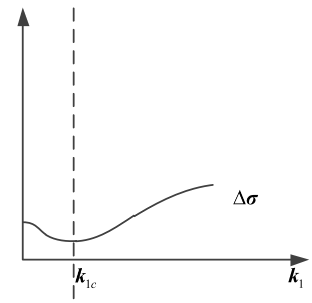

Fig. 2. Attitude control accuracy about $\boldsymbol{k}_1$.

Substituting equation (16) into equation (15), yields

$$\begin{aligned}\dot{V}_4 &= -\boldsymbol{\psi}^T\boldsymbol{C}_{fi}\boldsymbol{\psi} - \left(\sqrt{\boldsymbol{k}'_\beta}\boldsymbol{\beta} - \sqrt{\boldsymbol{k}'_\omega}\boldsymbol{e}_{\boldsymbol{\omega}}\right)^T\left(\sqrt{\boldsymbol{k}'_\beta}\boldsymbol{\beta} - \sqrt{\boldsymbol{k}'_\omega}\boldsymbol{e}_{\boldsymbol{\omega}}\right) \\ &\quad - \boldsymbol{\beta}^T\boldsymbol{k}''_\beta\boldsymbol{\beta} - \boldsymbol{e}_{\boldsymbol{\omega}}^T\boldsymbol{k}''_\omega\boldsymbol{e}_{\boldsymbol{\omega}} \leq 0\end{aligned} \tag{17}$$

Noting that if $\boldsymbol{k}_\beta\boldsymbol{k}_\omega \geq 0.25\boldsymbol{E}_3$ ($\boldsymbol{k}_\beta, \boldsymbol{k}_\omega > \mathbf{0}$), the Lyapunov function of the whole system satisfies $V_4 \geq 0$ and $\dot{V}_4 \leq 0$, $\dot{V}_4 = 0$ only when $V_4 = 0$. According to the LaSalle's invariant principle, it is obtained that if the gain matrixes ($\boldsymbol{k}_\beta, \boldsymbol{k}_\omega$) satisfy the constraint introduced in Theorem 2, the attitude synchronization and the vibration suppression is asymptotic converge under the tracking error of angular velocity. In the controller, the coefficient $\boldsymbol{k}_\omega$ should be large enough to guarantee the convergence rate of the closed-loop system by reducing the tracking error and tracking time. □

3.3. Control manage coefficient

In the centralized vibration suppression during the attitude maneuver, the three degrees of freedom control torques provided by the spacecraft actuator are used to control the three degrees of freedom attitude and the vibration of the flexible appendages. The whole system is a controlled under-actuated system. The distribution of the control torque is introduced in this section by defining the control manage coefficient. In the controller, $\boldsymbol{k}_1$ in the referenced angular velocity design expressed by equation (6) is defined as the control manage coefficient, which represents the distribution coefficient of the vibration suppression in the design of control law. The larger the control manage coefficient is, the higher the distribution proportion of the vibration suppression in the controller become. The relationship of the control manage coefficient $\boldsymbol{k}_1$ with the attitude maneuver accuracy is illustrated in Fig. 2.

The left side of the dotted line in Fig. 2 reveals that the vibration suppression of the flexible appendage in the controller improves the accuracy of the attitude maneuver in a certain period of time, and the attitude maneuver accuracy is increased by magnifying the control manage coefficient. In the controller, increasing the control manage coefficient results in the reduction of the distribution of the attitude control. The right side of the dotted line in Fig. 2 shows that the attitude maneuver accuracy is lowered in a certain period of time by increasing the control manage coefficient because of the lower attitude convergence rate. On the purpose of increasing the attitude accuracy, the appropriate control manage coefficient $\boldsymbol{k}_{1c}$ is set to guarantee the higher accuracy of the attitude maneuver in a certain period of time. In this paper, the control manage coefficient $\boldsymbol{k}_{1c}$ is obtained numerically.

Due to the nonlinearity of the dynamics of system and the property of back-stepping controller, the relationship between the attitude and the modal coordinates is unachievable theoretically. Hence, a method to obtain the appropriate control manage coefficient numerically is put forward in this paper, which is applicable in engineering. The procedure to obtain the appropriate control manage coefficient consists of two steps: preliminary selection and

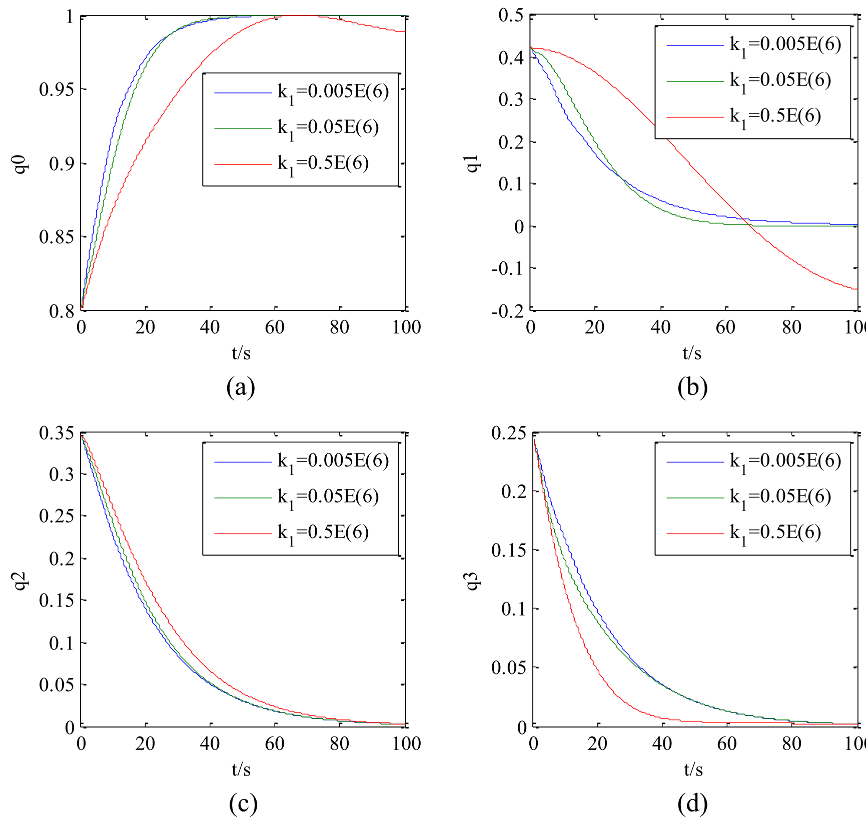

Fig. 3. Attitude of the flexible spacecraft.

accurate determination. The former gives the rough range of the appropriate control manage coefficient while the latter provides accurate values, which satisfy the engineering requirements.

4. Simulation results

The simulation scenario in this paper is a spacecraft body with a flexible appendage mounted along the yaw axis. The attitude of the roll axis suffers most from the vibration of the flexible appendage. The control strategy proposed in this paper is simulated in this section.

4.1. Simulations

The flexible spacecraft introduced in this paper consists of the main body and a solar panel. The parameters of the flexible spacecraft are set as follows: the mass of the main body is $m_b = 0.746 \times 10^3$ kg, the mounting location of the solar panel is $[0\ 0.86\ 0]^T$, the size of the solar panel is 2 m square and 8 m long, the flexible parameters are given as $\omega_{f1} = 0.0908$, $\omega_{f2} = 0.6420$, $\omega_{f3} = 0.8852$, $\omega_{f4} = 1.8166$, $\omega_{f5} = 2.6876$, $\omega_{f6} = 3.7665$, the moment of the spacecraft body and flexible appendage,

$$\boldsymbol{I}_{bt} = \begin{bmatrix} 1655.1 & 3.94 & -2.32 \\ 3.94 & 530.4 & -2.79 \\ -2.32 & -2.79 & 1685.5 \end{bmatrix} \text{kg}\,\text{m}^2, \quad \text{and}$$

$$\boldsymbol{H} = \begin{bmatrix} -32.52 & 7.53 & 0.06 & 3.43 & 0.14 & -2.39 \\ 0.016 & -0.079 & 2.67 & -0.027 & 0.90 & 0.0021 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

The simulation examples are performed to verify the feasibility of the vibration suppression method and the control torque manage proposed in this paper, and the parameters are $\boldsymbol{k}_\beta = \operatorname{diag}\left(0.1\ \ 0.1\ \ 0.1\right)$ and $\boldsymbol{k}_\omega = \operatorname{diag}\left(2\ \ 2\ \ 2\right)$. To roughly describe the effects of manage coefficients on the performance of this controller, the simulation are performed with the control manage coefficient $\boldsymbol{k}_1 = 0.005\boldsymbol{E}(6)$, $\boldsymbol{k}_1 = 0.05\boldsymbol{E}(6)$ and $\boldsymbol{k}_1 = 0.5\boldsymbol{E}(6)$. The simulation results of controller with these three control manage coefficients are shown as follows.

4.2. Results and analyses

Such conclusion can be easily deduced from the simulation results that the active vibration in flexible spacecraft with the optical measurement is feasible in engineering. Fig. 3 indicates the tendency of the attitude control accuracy as the control manage coefficient increases, and it is concluded that the controller constructed in this paper guarantees the convergence of the attitude control. It also shows that the attitude maneuver accuracy is affected by the control manage coefficient, and the controller with appropriate control manage coefficient is able to accomplish the vibration suppression and improve the attitude maneuver accuracy. However, the quite larger control manage coefficient in the controller decreases the attitude control accuracy because decreasing the distribution proportion of attitude control makes the controller insensitive to the attitude error.

It can be concluded from Fig. 4 that the convergence rate of vibration suppression of flexible appendage is easily affected by the incensement of the control manage coefficient. From the value of the coupling matrix $\boldsymbol{H}$, it indicates that the roll angle is mostly coupled with the first and second order modal vibration, the pitch angle is mostly coupled with the third modal vibration and the yaw attitude is not apparently affected by the modal vibration. It is deduced from Fig. 4(a) and Fig. 4(b) that the appropriate control manage coefficient are capable of improving the accuracy of the first- and second-order modal vibration suppression. Nevertheless, the large control manage coefficient decreases the accuracy because it is coupled with the attitude motion. The results demon-

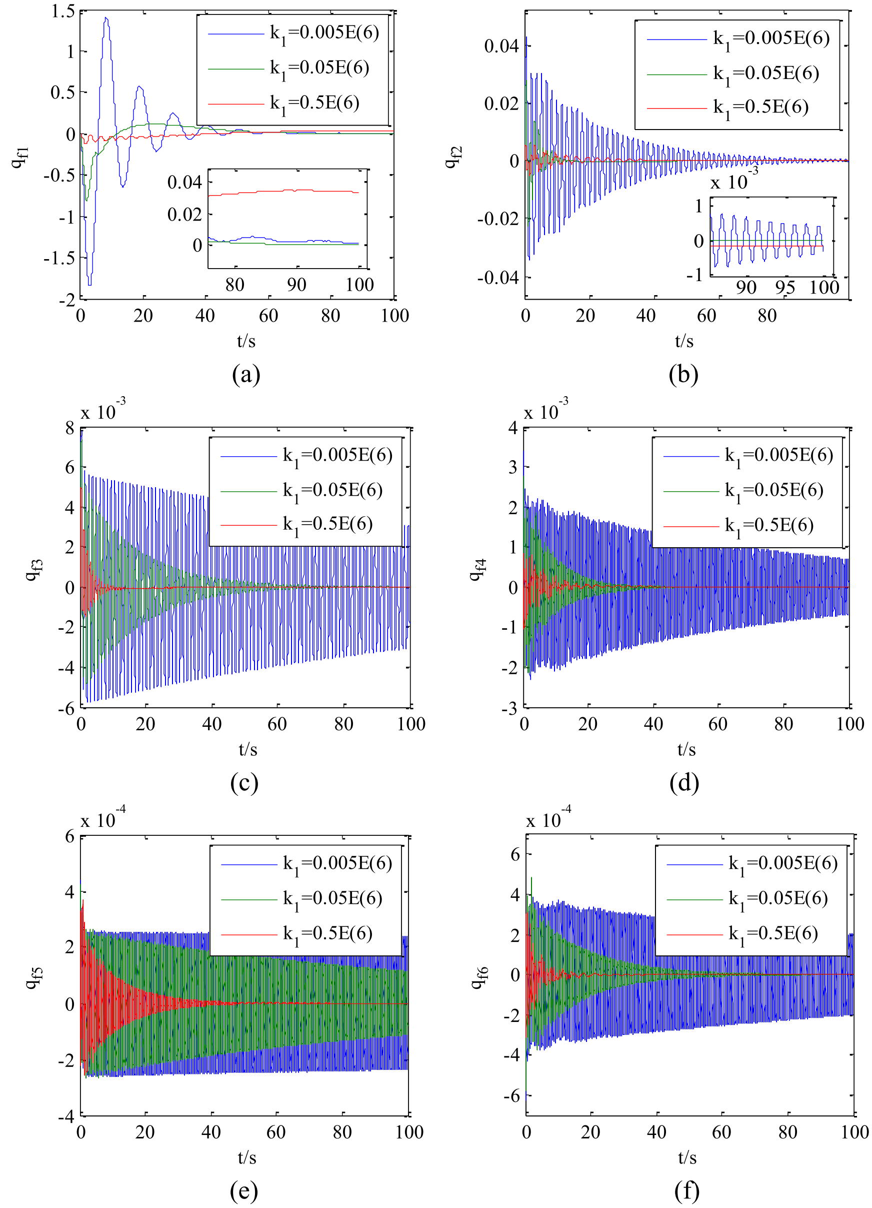

Fig. 4. Modal coordinates of the flexible appendages.

strated in Fig. 5 also indicate that the appropriate control manage coefficient has greatly improved the performance of the control torque.

The simulation results show that the control method with the optical measurements is feasible from engineering viewpoint. The appropriate control manage coefficient not only effectively damps the vibration, but also improves the accuracy of the attitude maneuver and the performance of the control torque of the flexible spacecraft. The appropriate control manage coefficient proposed in this paper is obtained numerically, shown in the following section.

4.3. Appropriate control manage coefficient

In obtaining the appropriate control manage coefficient, the effect of high orders is ignored because they are not effectively coupled with the attitude motion, and the control manager coefficient is given as $\boldsymbol{k}_1(4,4) = \boldsymbol{k}_1(5,5) = \boldsymbol{k}_1(6,6) = 0.05$.

The preliminary selection to obtain the appropriate control manage coefficient is first introduced. This progress is based on the relationship between the attitude maneuver accuracy $\Delta\boldsymbol{q}$ and the changing control manage coefficient, shown in Table 1.

Table 1
Preliminary progress of obtaining $\boldsymbol{k}_{1c}$.

$\boldsymbol{k}_1(1,1)$	$\boldsymbol{k}_1(2,2)$	$\boldsymbol{k}_1(2,2)$	Δq_1	Δq_2
–	–	–	3.23×10^{-3}	2.67×10^{-3}
0.005	0.005	0.005	2.52×10^{-3}	2.35×10^{-3}
0.01	0.01	0.01	2.146×10^{-3}	2.35×10^{-3}
0.02	0.02	0.02	1.4835×10^{-3}	2.35×10^{-3}
0.03	0.03	0.03	8.534×10^{-4}	2.354×10^{-3}
0.04	0.04	0.04	2.73×10^{-4}	2.358×10^{-3}
0.05	0.05	0.05	-2.40×10^{-4}	2.362×10^{-3}
0.05	0.05	0.5	-2.428×10^{-4}	2.19×10^{-3}
0.05	0.05	1	-2.453×10^{-4}	2.00×10^{-3}
0.05	0.05	2	-2.50×10^{-4}	1.625×10^{-3}
0.05	0.05	4	-2.586×10^{-4}	9.01×10^{-4}
0.05	0.05	6	-2.644×10^{-4}	2.345×10^{-4}
0.05	0.05	8	-2.67×10^{-4}	-3.45×10^{-4}

The first row in Table 1 demonstrates the results without vibration suppression in the controller. The control manage coef-

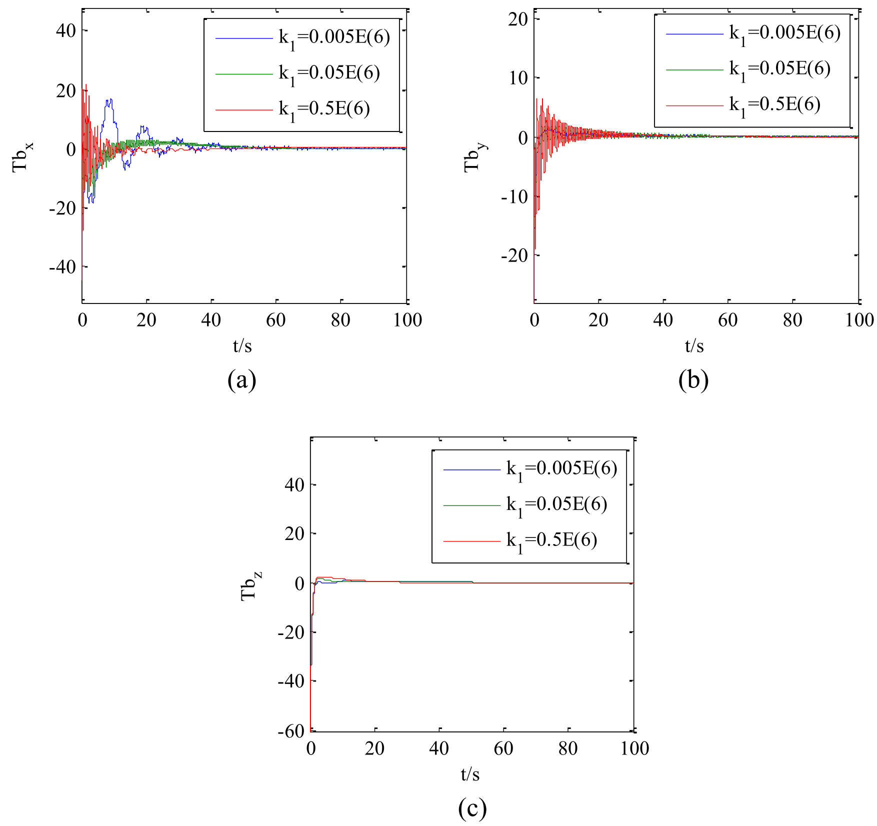

Fig. 5. Control torque of the flexible spacecraft.

Table 2
Accurate progress of obtaining $\boldsymbol{k}_{1c}$.

$\boldsymbol{k}_1(1,1)$	$\boldsymbol{k}_1(2,2)$	$\boldsymbol{k}_1(2,2)$	Δq_1	Δq_2
0.05	0.05	7	-2.66×10^{-4}	-6.675×10^{-5}
0.05	0.05	6.5	-2.653×10^{-4}	8.10×10^{-5}
0.05	0.05	6.75	-2.657×10^{-4}	6.39×10^{-6}
0.045	0.045	6.75	-1.36×10^{-4}	7.79×10^{-6}
0.0425	0.0425	6.75	1.19×10^{-4}	8.445×10^{-6}
0.04375	0.04375	6.875	5.2×10^{-5}	-2.85×10^{-5}
0.044375	0.044375	6.8125	2.0×10^{-5}	-1.05×10^{-5}

ficient in Table 1 is gradually increasing. From Table 1, it can be concluded that the attitude maneuver accuracy $\Delta\boldsymbol{q}$ increases as the control manage coefficient increases within the range of $\boldsymbol{k}_1(1,1), \boldsymbol{k}_1(2,2) < 0.04$, and $\boldsymbol{k}_1(3,3) < 6$. However, the attitude maneuver accuracy $|\Delta\boldsymbol{q}|$ decreases when $\boldsymbol{k}_1(1,1), \boldsymbol{k}_1(2,2) = 0.05$, and $\boldsymbol{k}_1(3,3) = 8$. Then, it shows that the range of the appropriate control manage coefficient is $0.04 < \boldsymbol{k}_1(1,1), \boldsymbol{k}_1(2,2) < 0.05$, and $6 < \boldsymbol{k}_1(3,3) < 8$. The accurate determination to refine the appropriate control manage coefficient is shown in Table 2.

From the accurate determination obtaining $\boldsymbol{k}_{1c}$ shown in Table 2, it can be obtained numerically that the appropriate control manage coefficients satisfying $\Delta q_1, \Delta q_2 \le 2 \times 10^{-5}$ are $\boldsymbol{k}_1(1,1) = \boldsymbol{k}_1(2,2) = 0.044375$ and $\boldsymbol{k}_1(3,3) = 6.8125$.

5. Conclusion

In this paper, the active vibration suppression of the flexible appendages, which are not allowed to be allocated with actuators, is introduced. The optical measurements, instead of an observer, are used to measure the dynamic behaviors of the large flexible structures in order to guarantee the reliability and practicality of the designate control system in the actual on-orbit service. The active vibration controller during the spacecraft attitude maneuver is proposed under the restriction of the freedom of the actuators. The simulation results show the feasibility of the newly proposed active vibration suppression method. The control manage coefficient is defined to describe the distribution coefficient of the vibration suppression in the controller. The relationship between the control manage coefficient, the attitude control accuracy and the vibration suppression is discussed. The controller with appropriate control manage coefficient is not only able to effectively damp the vibration, but also to improve the accuracy of the attitude maneuver and the performance of the control torque of the flexible spacecraft. The numerical method to obtain the appropriate control manage coefficient is introduced. The control manage coefficient defined in this paper can also be determined by the requirements of actual control missions. The control manage coefficient $\boldsymbol{k}_{1c}$ is of first priority when the mission is concentrated on the attitude control accuracy. Numerical simulations validate the proposed active vibration suppression algorithm. Furthermore, the control manage presented in this paper is considered as an instruction for the actual control mission.

The problem discussed in this paper derives from the requirements of actual control missions and extends the existing control methods. The method and results put forward in this paper is supposed to be utilized in the actual control mission of the space station.

Conflict of interest statement

The authors declared that they have no conflicts of interest to this work. We declare that we do not have any commercial or associative interest that represents a conflict of interest in connection with the work submitted.

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