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Helicopter blade reliability: Statistical data analysis and modeling

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ABSTRACT

The concept of reliability has been attracting attentions in mechanical engineering following the developments of the aerospace industries. Limited failure data and statistical analyses of helicopter components reliability exist in the technical literature. For filling this gap, a nonparametric analysis is conducted on the performance of the 338 blades of some Iranian helicopters, which were in service between 1974 and 2012. These blades have 41 different failure modes. In this paper, statistical reliability analysis is conducted based on two strategies: In strategy I, general failure is defined as scrapping or retirement of the blade. In strategy II, the blade is assumed to be subjected to different modes of failure and the cumulative mode-specific functions are derived for each failure modes using Nelson–Aalen estimator. The Kaplan–Meier estimator is used for calculating the nonparametric reliability functions. Confidence intervals are derived for the reliability results and parametric fits are conducted using the maximum likelihood estimation. An important result from parametric analysis is that the blade reliability has a 3-parameter Weibull distribution and so the blades exhibit an increasing failure rate. Finally, considering the mode-specific hazard functions, the failure mode 1, i.e., excessive vibration is observed to have major contribution to the blade failures.

1. Introduction

The growing trends of the application of the concept of reliability in mechanical engineering design owe to the statistical nature of the various failure modes of the mechanical components. It is essential that the mechanical equipment operate reliably under all the conditions in which it is used; however, the requirement for reliability is different for each application. Reliability is the probability that a component, equipment, or system will perform a required function under the operating conditions encountered for a stated period of time [1]. There are many different operational requirements and various environments, thus reliability is quantified in many different ways. One of them is the statistical analysis that is referred to as lifetime, survival time, or failure time data. Some methods of dealing with lifetime data are quite old, but starting at about 1970 the field expanded rapidly with respect to methodology, theory, and fields of applications [2].

Limited failure data and statistical analyses of helicopter components reliability exist in the technical literature and few statistical studies were made to model the reliability of these parts. Bell Helicopters Company documented the reliability for some OH-58D components using the strength/load interaction formulation and established a life versus reliability relationship [3]. Sometimes there is a single lifetime for each individual, but failure may be of different modes of types. Often the modes refer to cause of failure, in which case the term “competing risks” or “multiple modes of failure” is sometimes used [2]. There have been many efforts to consider failure mechanisms (competing risks) for each component to analyze reliability [4–6]. Kaplan and Meier [7] presented the Kaplan–Meier estimator for calculating the nonparametric reliability function. In the nonparametric analysis, the confidence intervals are derived to inform about the dispersion around the nonparametric reliability function [8,9]. Nelson and Aalen presented the Nelson–Aalen estimator for calculating the cumulative mode-specific functions for each failure modes [10–13].

In this paper, failure data are collected for 338 helicopter blades. These blades have 41 different failure modes. Two different strategies are adopted. In the first one, general failure is defined as scrapping of the blade, which results in retirement of the blade. A nonparametric analysis of blade reliability is conducted for 338 blades, which were in service between 1974 and 2012. Because in this case the dataset is censored due to the fact that some of the blades are still operational at the end of gathering data, the Kaplan–Meier estimator is used for calculating the blade reliability function. In addition, confidence intervals are derived for the nonparametric reliability result. In the second strategy, the blade is

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1270-9638/

Table 1
Data collection template and sample data for statistical analysis of blade reliability.

Failure mode	Failure date	Mode of failure	Flight times (hours)
1	8/10/2002	Excessive Vibration	683
2	27/2/1985	Corroded	746
3	15/12/1982	Chipped	1204
4	30/12/1975	Cracked	603
5	12/8/1985	Bent/Dented	719
6	23/8/1998	Scrapping	1108
7	7/9/1999	Hard Landing	90
8	28/7/1976	Sudden Stop	1413
10	4/5/1982	Worn Excessively	1501
17	11/7/1978	Delaminated	1155

assumed to be subjected to different modes of failure. In the other words, causes other than scrapping such as fatigue, excessive vibration, corrosion, … can also make the blade to fail. So, the blade is faced with multiple modes of failure in this strategy. In this case, the dataset is unsensored because the first time failures of the blades are available. A nonparametric analysis of blade reliability is conducted using the Kaplan–Meier estimator [7] and the confidence intervals are derived for the nonparametric reliability result. In addition, the cumulative mode-specific functions are derived for each failure modes by using Nelson–Aalen estimator. In both cases, parametric fits are conducted using the maximum likelihood estimation (MLE) and graphical approach. Moreover, the goodness of fit tests are used to justify the choice of a 3-parameter Weibull distribution for modeling blade reliability and then with the maximum likelihood estimation, the parameters of the 3-parameter Weibull distribution are calculated.

Applying the MLE procedure, the values of the shape parameter ($\beta = 1.793$) and the scale parameter ($\theta = 9390.97$ hours) are derived for the first strategy, and the values of the shape parameter ($\beta = 2.187$) and the scale parameter ($\theta = 1234.9$ hours) are derived for the second one. It is seen that the shape parameter β is greater than one in both strategies. Consequently, the helicopter blades in both cases (strategies I and II) suffer increasing failure rate or wear-out failures and as a result their failure probability of occurrence increases over time. Hence, with regarding the bathtub hazard rate curve their expected average life in wear-out period of life can be much smaller than their mean life in the period of their useful life.

2. Dataset description

For the purpose of this study, the data taken from the Iranian helicopter industry (PANHA) are used. This dataset provides extensive data on helicopter blade failures, as well as flight and in service histories since 1974.

For each blade in dataset, these data are collected from the dataset: (1) its flight times; (2) its failure date, if failure occurred; (3) the failure mode according to each flight time; and (4) the "censored time," if no failure occurred. This last point is further explained in the following section where data censoring and the Kaplan–Meier estimator are discussed. The data collection template and sample data for this analysis for the most important failure modes are shown in Table 1. In the following section, the data are collected to conduct a nonparametric reliability analysis of all the blades identified previously.

3. Nonparametric reliability analysis of helicopter blade

Two different reliability analyses are accomplished:

Strategy I: General failure is defined as scrapping or retirement of the blade.

Strategy II: Failure of the blade occurs as a result of different modes of failure.

In the second strategy, failure does not mean the retirement of the blade. In fact, the distinction of the two strategies is that the first one accounts for the repairments done on the blades and as a result the repaired blade can be treated as a new one. The retirement of the blades in this strategy occurs when the repairments can no longer be helpful and so, the blade has to be scrapped. However, the second strategy considers that the blade stops to work properly when each of the failure modes occur.

3.1. Censored data sample and Kaplan–Meier estimator as applied in the context of the first strategy

Censoring occurs when life data for statistical analysis of a set of items is incomplete, as in the case the dataset corresponding to the first strategy. More specifically, right censoring occurs. This means the following: (1) the blades in the present dataset are activated at different points in time but all these activation times in this dataset are known, (2) failure dates and censoring are stochastic, and (3) censoring occurs because the blade is still operational at the end of gathering data. In this work, the powerful Kaplan–Meier estimator [7] is adopted, which is best suited for handling the type of censoring in the present dataset. The Kaplan–Meier estimator of the reliability function with censored data is given by:

$$\hat{R}(t) = \prod_{\textit{all } i \textit{ such that } t_{(i)} \le t} \frac{n_i - d_i}{n_i} \tag{1}$$

where:

$$\begin{aligned}
&t_{(i)}\text{: time to the i'th failure (arranged in ascending order)}\\
&n_i = \text{number of operational units right before } t_{(i)}\\
&\quad = n - [\text{number of censored units right before } t_{(i)}]\\
&\qquad - [\text{number of failed units right before } t_{(i)}]\\
&d_i = \text{number of failure units at } t_{(i)}
\end{aligned} \tag{2}$$

3.2. Confidence interval analysis

The Kaplan–Meier estimator (Eq. (1)) provides a maximum likelihood estimate of reliability but does not inform about the dispersion around $\hat{R}(t_i)$. This dispersion is captured by the variance or standard deviation of the estimator, which is then used to derive the upper and lower bounds for say a 95% confidence interval (that is, a 95% likelihood that the actual reliability will fall between the two calculated bounds, with the Kaplan–Meier analysis providing us with the most likely estimate). The variance of the estimator is provided by Greenwood's formula [8] and [9]:

$$\mathrm{var}\left[R(t_i)\right] \equiv \sigma^2(t_i) = \left[\hat{R}(t_i)\right]^2 \sum_{j \le i} \frac{d_j}{n_j(n_j - d_j)} \tag{3}$$

Moreover, the 95% confidence interval is determined by:

$$R_{95\%} = \hat{R}(t_i) \pm 1.96\sigma(t_i) \tag{4}$$

More details about these equations can be found in [8] and [9].

3.3. Kaplan plot of blade reliability in presence of censored data

With the brief overview of censoring, of the Kaplan–Meier estimator, and of confidence intervals, the blade reliability can be analyzed from the present dataset. According to the first strategy for the 338 blades analyzed, 310 censored and 28 failure times are obtained. The data is treated with the Kaplan–Meier estimator (Eq. (1)), and the Kaplan–Meier plot of the reliability of helicopter

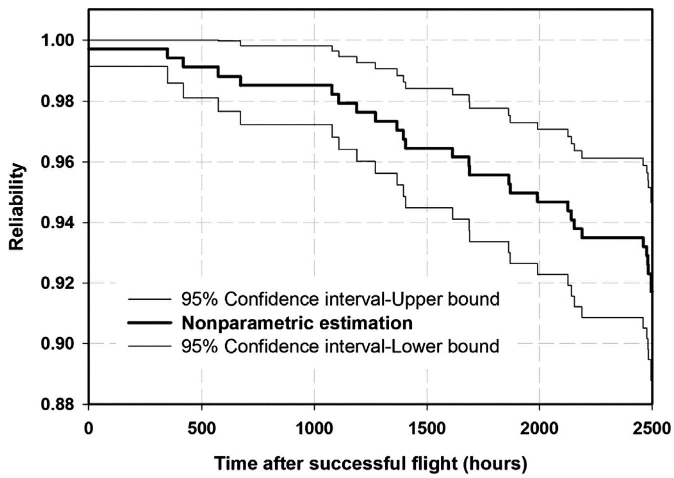


Fig. 1. Blade reliability with 95% confidence intervals based on strategy I.

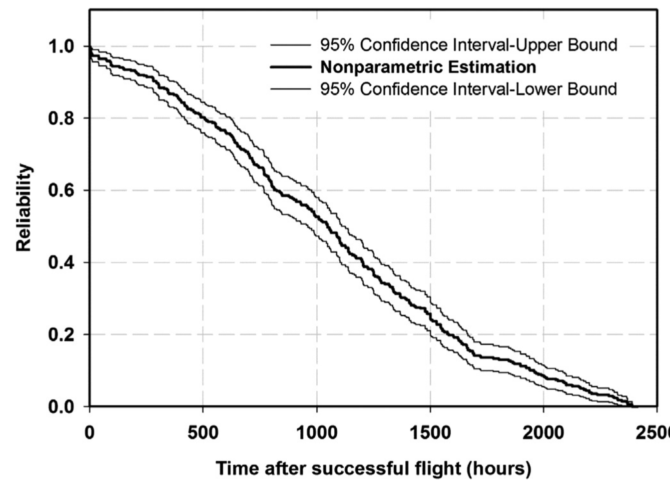


Fig. 2. Blade reliability with 95% confidence intervals based on strategy II.

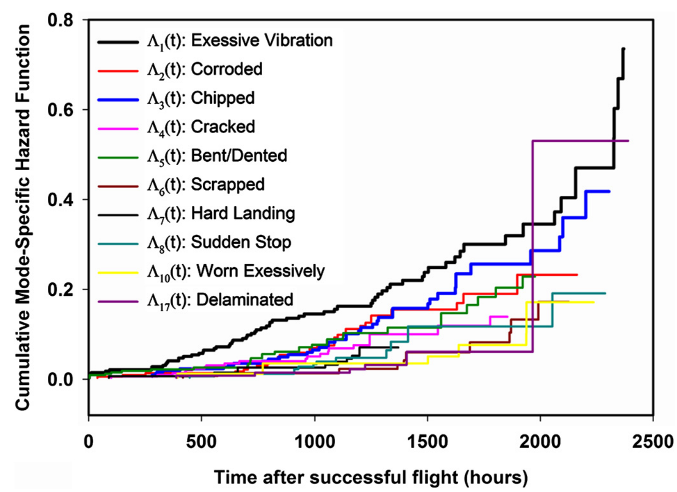


Fig. 3. NA estimates of cumulative mode-specific hazard functions based on strategy II.

blade is obtained. In addition, when Eqs. (3) and (4) are applied to the data, the 95% confidence interval curves are obtained. These are shown in Fig. 1.

Fig. 1 shows that the reliability of the blade decreases to 91.71% after 2504 flight hours. The complete tabular data corresponding to Fig. 1 is provided in Appendix A, Table A.1.

3.4. *Uncensored data sample: the second strategy of reliability analysis*

Considering all of the failure modes, the dataset is uncensored and the second strategy holds true. In this case, the blades can fail in different ways, i.e., multiple modes of failure exist. These modes may refer to the cause of failures, in which case they are often termed as competing risks. In the present dataset, the blades have 41 different failure modes.

With considering the first time failure of the blades, the non-parametric reliability function ($\hat{R}(t)$) is estimated by ignoring the associated types of the failure modes and using the Kaplan–Meier estimator based on the data that is given by Eq. (1) [2]. It should be noted that the Kaplan–Meier estimator formula could be used in both cases: censored data and uncensored data [2]. When there is no censoring, $n_1 = n$ and $n_i = n_{i-1} - d_{i-1}$ $(i = 2, 3, \dots, k)$ and Eq. (1) for each t_i reduces to:

$$\hat{R}(t_i) = \frac{\textit{Number of observations} \geq t_i}{n}; t_i \geq 0 \tag{5}$$

In both censored and uncensored cases $\hat{R}(t)$ is a left-continuous step function which is equal to 1 at $t = 0$ and drops in a stepwise manner by a factor $(n_i - d_i)/n_i$ immediately after each life time $t_{(i)}$. To estimate the confidence intervals, Eqs. (3) and (4) should be used.

In this case, in addition, the Nelson–Aalen (NA) estimator is adopted to estimate the cumulative mode-specific hazard functions corresponding to each failure modes [2]. The derivation of the Nelson–Aalen estimator formula can be found in [10–13]. The Nelson–Aalen estimator of the cumulative mode-specific hazard function with multiple failure modes for failure mode j is given by:

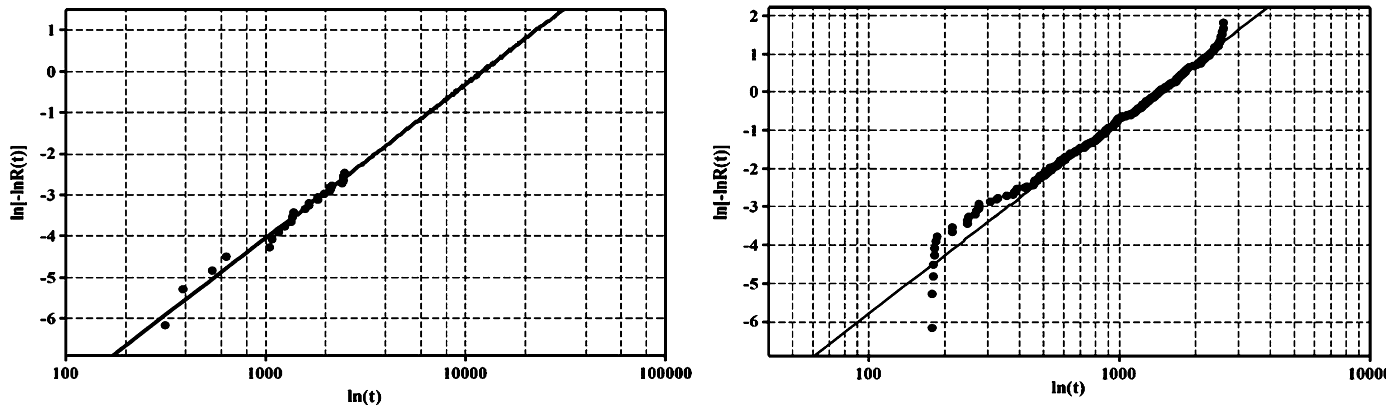
$$\hat{\Lambda}_j(t) = \sum_{\textit{all } i \textit{ such that } t_{(i)} \leq t} \frac{\delta_{ij}}{n_i}; \quad j = 1, \dots, k \tag{6}$$

where:

$t_{(i)}$: *time to i'th failure*
k: *number of failure* mod *es* (7)
n_i: *number of operational units right before* $t_{(i)}$

With the brief overview of uncensored data, of the Kaplan–Meier estimator in case of multiple failure modes, and of cumulative mode-specific hazard function, the blade reliability can be analyzed from the present uncensored dataset. Based on the second strategy, for the 338 blades analyzed, the first time failures of the blades are available. The Kaplan plot of blade reliability along with the 95% confidence intervals is shown in Fig. 2. This figure shows that the reliability of the blade decreases to 84.31% after 418 flight hours.

The cumulative mode-specific hazard functions corresponding to each failure modes are derived. The plots of the $\hat{\Lambda}_j(t)$'s are given in Fig. 3 for the most important failure modes that are shown in Table 1. Failure mode 1 is excessive vibration. Excessive quivering or tumbling vibration can cause structural stress on the helicopter during flight or ground running. Failure mode 2 is corrosion. Corrosion is a natural chemical process that gradually destroys most metals by a chemical reaction due to the effect of environment. Corrosion of rotor blades is common and requires a vigilant effort to control. Because of that pitting can occur on a surface. Pitting of a surface is breakdown of a material due to chemical or electrochemical attack by atmosphere, moisture or other agent. Failure mode 3 is chipping. Chipping is the actual breaking out of some small pieces of metal usually caused by heavy impact. Failure mode 4 is cracking. The rotating machines such as helicopter rotor blades are subjected to mixed high level loading and



(a) (b)

Fig. 4. Weibull plots of the blade reliability based on: (a) strategy I, (b) strategy II.

initiation of a crack can lead to fracture under the effect of these loads. Failure mode 5 is denting. When surface is dented, surface indent with rounded bottom that usually caused by impact of a foreign object and parent material is displaced but not separated. Failure mode 6 is scrapping. The retirement of the blades occurs when the repairments can no longer be helpful and so, the blade has to be scrapped. Failure mode 7 is hard landing. A hard landing is any accident or incident in which impact of the A/C caused severe pitching of main rotor allowing hard contact of hub with mast or results in cracking the aft. Failure mode 8 is sudden stop. Sudden stop is defined as any rapid deceleration of the drive system, whether by internal seizure of the transmission or by main or tail rotor blades striking something which causes rapid deceleration or enough tail rotor damage or require replacement. Failure mode 10 is wear. Excessive deterioration of a surface occurs due to material removal caused by relative motion between it and another part. Failure mode 17 is delamination. Delamination occurs when a part is being separated into constituent layers due to heat, pressure, stress or expired adhesive.

The cumulative mode-specific hazard functions ($\hat{\Lambda}_j(t)$) show that the failure mode 1 that is the "excessive vibration" is the major reason for the blade failures. After "excessive vibration" the failure mode 3, i.e., "the chipped" is the second rank cause of failure of the blades. The slopes of the plots in Fig. 3 provide rough estimation of the hazard functions $\lambda_j(t)$.

4. Parametric reliability of the helicopter blade

Nonparametric analysis provides powerful results since the reliability calculation is unconstrained to fit any particular pre-defined lifetime distribution. However, this flexibility makes nonparametric results neither easy nor convenient to use for various purposes often encountered in engineering design (e.g., reliability-based design optimization). Several possible methods are available to fit a parametric distribution to the nonparametric estimated reliability function (as provided by the Kaplan–Meier estimator), such as graphical procedures and inference procedures. In the following, two such methods namely the probability plots and the maximum likelihood estimation are reviewed briefly. The goodness of fit tests are used to justify the choice of a 3-parameter Weibull distribution for modeling the blade reliability and then with the maximum likelihood estimation, the parameters of each 3-parameter Weibull distribution are calculated.

4.1. Probability plots or graphical estimation

The review of fitting techniques are begun with the easy-to-use and visually appealing graphical technique known as probability plotting (or plotting positions). This technique is used to demonstrate that the 3-parameter Weibull distribution is an appropriate choice in both strategies for capturing the failure behavior of blades. Probability plots constitute a simple graphical procedure for fitting a parametric distribution to nonparametric data. This procedure is based on the fact that some parametric models, such as the exponential or Weibull distribution for example, can have their reliability function that are linearized by using a particular mathematical transformation. Considering the 2-parameter Weibull distribution, we have:

$$R(t) = \exp\left[-\left(\frac{t}{\theta}\right)^{\beta}\right] \quad (8)$$

Taking the natural logarithm of both sides of Eq. (8), yields:

$$\ln\big[R(t)\big] = -\left(\frac{t}{\theta}\right)^{\beta} \quad (9)$$

Taking again the natural logarithm of both sides of this equality, results in:

$$\ln\big[-\ln R(t)\big] = \beta\ln(t) - \beta\ln(\theta) \quad (10)$$

If $\ln[-\ln\hat{R}(t_i)]$ is plotted as a function of $\ln(t_i)$ and data points are obtained that are aligned in the $(\ln(t); \ln[-\hat{R}(t)])$ space, the resulting graph is termed the Weibull plot. In addition, shape parameter β of the Weibull distribution is provided with the slope of the line that fits the data point with least square method. Moreover, the scale parameter θ can be evaluated for example from the value of the intersection of the line with the y-axis.

Examining different line fitting techniques according to the 3-parameter Weibull, 2-parameter Weibull, exponential, normal and lognormal distribution probability plots, it is concluded that a 3-parameter Weibull distribution leads to the best approximation. Therefore, it can be stated that blade reliability can be properly approximated by 3-parameter Weibull distribution in each of the two strategies. Fig. 4 shows the Weibull plots for the blade reliability based on the two strategies.

Probability plots or graphical methods for parametric fit have a powerful advantage in their simplicity: they are easy to set up, they do not require involved calculations, and they give immediate (visual) information about the validity of the assumed parametric distribution. In addition, the parameters of the assumed distribution can be calculated by simple least-square linear fit of the data on the probability plots. However, probability plots have some disadvantages when used to calculate the actual parameters of the distribution. For example,

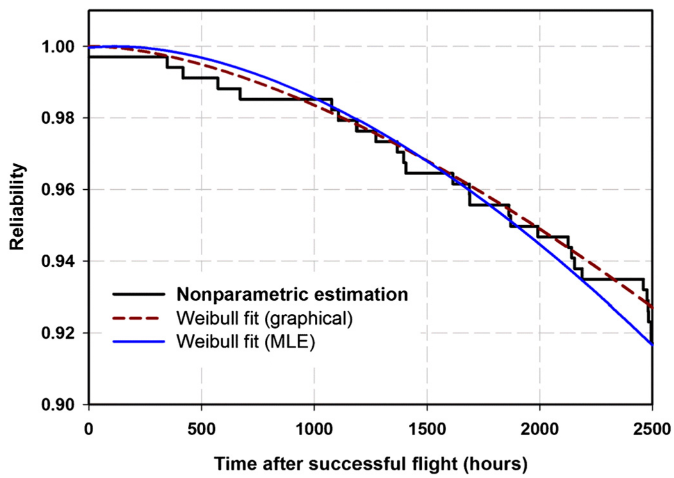


Fig. 5. Nonparametric reliability and 3-parameter Weibull fits for the strategy I.

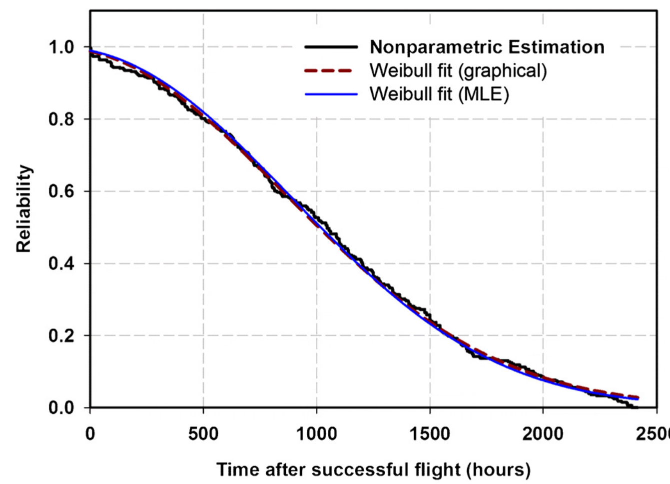


Fig. 6. Nonparametric reliability and 3-parameter Weibull fits for the strategy II.

(i) With distributions requiring logarithm time transformations (e.g., Weibull or lognormal distribution), excessive weight is given to the early failure times, and consequently, the resulting parametric fit is biased (towards more precision for early failures).

(ii) As a consequence of (i), the least-square fit on the probability plot does not result in minimum variance estimate of the actual distribution.

(iii) The estimation of the parameters may be poor if the failure times are not scattered properly across the data range.

If the purpose or objectives of conducting the reliability study do not require "precise" results, then probability plots or graphical estimations are adequate for conducting parametric fits. Otherwise, one can revert to the more precise, but analytically involved, maximum likelihood estimation method, discussed later. For this purpose, probability plots are used to justify the choice of a 3-parameter Weibull distribution for modeling the blade reliability, and then MLE is used to calculate the parameters of the 3-parameter Weibull distribution.

4.2. Maximum likelihood estimation

Maximum likelihood estimation (MLE) addresses all the limitations of probability plots and provides more precise parametric fits than graphical estimation. While conceptually simple, the MLE method analytically requires:

(1) Determining the right formulation of a function (known as the likelihood function) depending on several parameters (e.g., censoring type, chosen parametric distribution), and (2) Searching for an optimum of this function, which can prove analytically tedious by calculating a set of partial derivatives of the logarithm of the likelihood function, and/or numerically intense by non-linear optimization techniques. In the following, a brief overview of this method is provided.

Conceptually, MLE is based on the following: given a set of observed data, and assuming a parametric life distribution with unknown parameters (e.g., three parameters for the 3-parameter Weibull distribution), likelihood function is defined as the probability of obtaining the observed data from the chosen parametric distribution. When an exhaustive search is conducted over the unknown parameters of the distribution, the values of these parameters that maximize the likelihood function are termed as the maximum likelihood estimates and the method is known as the MLE.

Fig. 5 shows the nonparametric reliability curve for the strategy I, as well as the best 3-parameter Weibull fit extracted using MLE and graphical parameters. Fig. 5 provides a visual verification

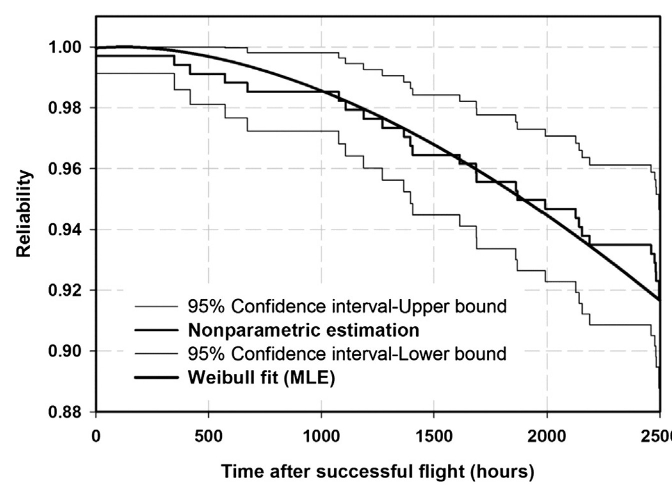


Fig. 7. Nonparametric reliability and 3-parameter Weibull fit for the strategy I.

for the 3-parameter Weibull distribution of the nonparametric reliability computed based on the first strategy. Similar results for the failure data obtained following the second strategy are shown in Fig. 6.

The MLE procedure is applied to determine the parameters of 3-parameter Weibull distribution for the helicopter blade reliability. Considering the strategy I, its nonparametric reliability is best approximated by the following 3-parameter Weibull distribution:

$$R(t) = \exp\left[-\left(\frac{t - 125.697}{9265.273}\right)^{1.793}\right] \tag{11}$$

The values of the shape parameter ($\beta = 1.793$) and the scale parameter ($\theta = 9390.97$) are the maximum likelihood estimates. Now considering the strategy II, its nonparametric reliability is best approximated by the following 3-parameter Weibull distribution:

$$R(t) = \exp\left[-\left(\frac{t + 174.830}{1409.73}\right)^{2.187}\right] \tag{12}$$

The values of the shape parameter ($\beta = 2.187$) and the scale parameter ($\theta = 1234.9$) are the maximum likelihood estimates. Fig. 7 and Fig. 8 show the nonparametric reliability curve (with 95% confidence intervals) for the helicopter blades, as well as the best 3-parameter Weibull fit (with MLE parameters).

The maximum error of the goodness-of-fit for the 3-parameter Weibull distribution is 4.24% for strategy I and 0.408% for strat-

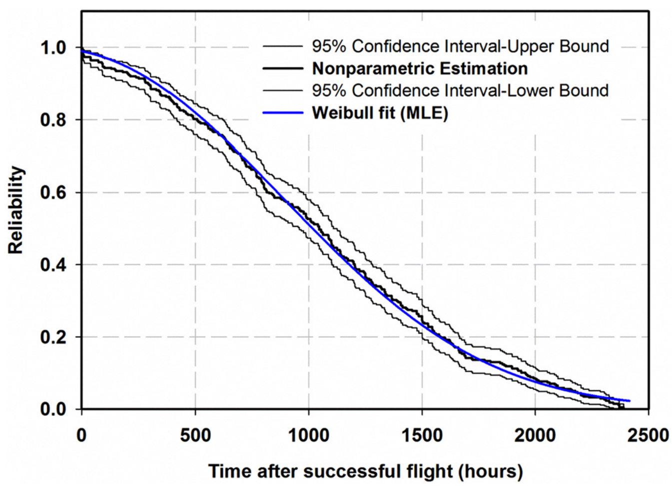


Fig. 8. Nonparametric reliability and 3-parameter Weibull fits for the strategy II.

egy II. This represents a remarkable accuracy for a 3-parameter Weibull distribution.

The important result is that the helicopter blades in both cases (strategies I and II) undergo increasing failure rate or wear-out failures (shape parameter $\beta > 1$). Hence, with regarding the bathtub hazard rate curve their expected average life in wear-out period of life can be much smaller than their mean life in the period of their useful life.

5. Conclusion

Limited failure data and statistical analyses of helicopter components reliability exist in the technical literature and few statistical studies were made to model the reliability of these parts. This issue motivates the development of the present statistical analysis of helicopter blade reliability. In this work, this gap is filled by conducting a nonparametric statistical analysis of helicopter blade reliability. It is demonstrated that the 3-parameter Weibull distribution is a good fit for helicopter blade reliability. The Weibull parameters are calculated using the maximum likelihood estimation technique. One important result from the parametric analysis is that the blade reliability function versus life has 3-parameter Weibull distribution. Regarding the Kaplan plots in both strategies, they show that in the first strategy the reliability of the blade decreases to 91.71% after 2504 flight hours. Nevertheless, in the second strategy the reliability of the blade decreases to 84.31% after 418 flight hours. Further, the blades exhibit an increasing failure rate or wear-out. In other words, their failure probability of occurrence increases over time and with regarding the bathtub hazard rate curve their expected average life in wear-out period of life can be much smaller than their mean life in the period of their useful life. This finding has important implications for the helicopter industry and should prompt serious consideration for wear-out procedures. Finally, considering the cumulative mode-specific hazard functions it was observed that the failure mode 1, i.e., the "excessive vibration" is the major reason for the blade failure. After "excessive vibration" the failure mode 3, i.e., "the chipped" is the second major reason for failure of the blades. The slopes of the cumulative mode-specific functions provide rough estimates of the hazard functions $\lambda_j(t)$.

Conflict of interest statement

None declared.

Acknowledgement

The authors would like to appreciate PANHA Company for supporting the project of reliability analysis of helicopter blades.

Appendix A. Nonparametric blade reliability and tabular data according to the first strategy

Table A.1
Tabular data for the Kaplan–Meier plot of blade reliability in Fig. 1.

Failure time t_i (hours)	$\hat{R}(t_i)$	95% Confidence interval—lower bound	95% Confidence interval—upper bound
348	0.997041	0.991251	1.00000
418	0.994083	0.985907	1.00000
573	0.991124	0.981125	1.00000
672	0.988166	0.976637	0.99969
1078	0.985207	0.972337	0.99808
1108	0.982249	0.968171	0.99633
1188	0.979290	0.964108	0.99447
1271	0.976331	0.960125	0.99254
1367	0.973373	0.956210	0.99054
1395	0.970414	0.952350	0.98848
1405	0.967456	0.948539	0.98637
1614	0.964497	0.944770	0.98422
1687	0.961538	0.941037	0.98204
1689	0.958580	0.937337	0.97982
1863	0.955621	0.933667	0.97758
1870	0.952663	0.930024	0.97530
1991	0.949704	0.926404	0.97300
2126	0.946746	0.922808	0.97068
2141	0.943787	0.919232	0.96834
2154	0.940828	0.915675	0.96598
2188	0.937870	0.912136	0.96360
2460	0.934911	0.908613	0.96121
2476	0.931953	0.905106	0.95880
2481	0.928994	0.901613	0.95637
2483	0.926036	0.898135	0.95394
2495	0.923077	0.894669	0.95148
2504	0.917160	0.887774	0.94655

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