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Optimization of pump efficiencies with different pumps characteristics working in parallel mode

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ABSTRACT

This paper concentrates on an algorithm for the prediction of steady running variable speed pumps (VSPs) working in parallel to keep them running close to the best efficiency point (BEP) provided by the pump manufacturer. Special focus is on the pumps that have different efficiency and performance characteristics. The complex optimization task to maximize the total efficiency of the pump system and thereby minimize energy consumption was solved with the customized optimization software using the Levenberg–Marquardt algorithm (LMA). Three different theoretical scenarios were analyzed: with working pumps having identical, slightly and largely different characteristics. The transition curves are proposed on the basis of optimization that indicate water discharge and pressure head when an additional pump should be switched on to ensure the highest total efficiency of the pump system.

1. Introduction

This paper shows that energy consumed by a Water Distribution System (WDS) may be reduced by increasing the efficiency of the pumps. Improved efficiency of the pumping systems also contributes to the reduction of greenhouse gas emissions and reduction of environmental impact. The task is essential in WDSs with no elevated storage tanks where pressure in the system is regulated only by pumps. The WDS of Tallinn Water is an example of such a system [1].

First methods to solve complex WDS optimization tasks appeared along with the first digital computers already in the 1960s. Since then the development of different optimization algorithms has increased rapidly. For example, Coulbeck and Orr [2] used the dynamic programming (DP) technique to optimize the basic pump scheduling procedure and pump combination selection. Also, Zessler and Shamir [3] carried out optimization to find the optimal decisions for pumps and valves for a 24-hour period using the progressive optimality (PO) technique based on dynamic programming (DP). Ormsbee, et al. [4] focused on finding an optimal pump usage policy to minimize the pump operational cost in a water distribution system according to variable electricity rate schedules and system demand schedules. As a result, an optimal water level trajectory in the tank and pump operating policy was developed to achieve higher efficiency of a pump system. Yu, Powell and Sterling [5] used an algorithm based on nonlinear programming to determine optimal pump schedules. But the complexity of WDSs limits the use of conventional optimization methods such as linear and nonlinear programming. A need to optimize more than one objective or goal led to the development of more complex multi-objective stochastic optimization methods such as simulated annealing, evolutionary algorithms and neural network.

To achieve maximum efficiency of the pump system, optimization of pump work schedules and pump control settings has been a high priority research theme for many years. Pump's efficiency usually degrades during normal operation due to wear by as much as 10–25% before it is replaced. Efficiencies of 50 to 60% or lower are quite common in older pumping stations, as described in [6]. It is estimated in [7] that ca 75% of the pumping systems today are oversized. A pump is considered as generally oversized when operated at lower than 20% of its best efficiency point (BEP) [8], although it is normally considered acceptable if the duty point of the pump falls within 20% range of the BEP flow rate. At the same time, pumps can also become undersized due to the WDS increased. A pump operating beyond its BEP will also experience increased noise and vibration. Running a pump continuously at such extreme regime will reduce pump's life time considerably. In practical applications, operating a pump continuously at its BEP is uncommon, because pumping systems must adapt to the constantly changing flow rate and the system head. To achieve better results, different mathematical models and techniques are used in the simulation and optimization of pump stations [3,4,5,9,10]. Comprehensive and practical methods and tools were required to estimate and optimize the energy efficiency. Many best practice guides were prepared, for example in [6] to help

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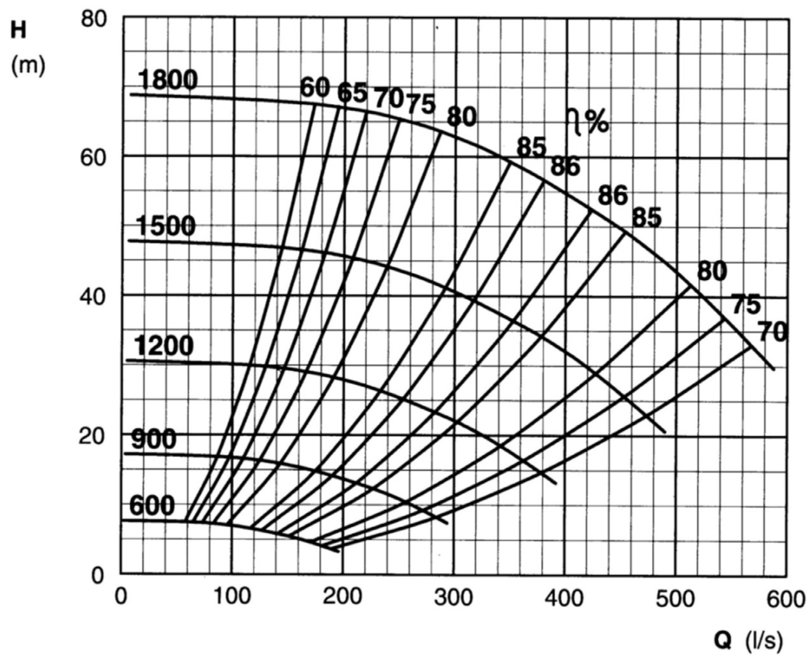

Fig. 1. Universal characteristics of the new pump.

Table 1
Water discharge at the highest combined efficiency calculated for a different number of pumps.

Number of pumps	Pump 1 flow (l/s)	Pump 2 flow (l/s)	Pump 3 flow (l/s)	Pump 4 flow (l/s)	Pump 5 flow (l/s)	Total flow, (l/s)	Total efficiency, (%)
3	366.67	366.67	366.67	–	–	1100	81.9
4	275	275	275	275	–	1100	86.7
5	220	220	220	220	220	1100	84.6

industries to start improving their pumping systems step by step. Describing the pumps working in the oil industry, Crease [11] states that identical pumps work with the highest efficiency (running in parallel) if they operate at the same conditions. Tianyi, et al. [12] also showed that pumps running in parallel and controlled by variable frequency drives are most efficient when all pumps work at the same speed. Both of these investigations were carried out for pumps working outside WDSs (in the oil industry and in air-conditioning systems). Analysis of pumps for a WDS is more complex, as a WDS usually contains elevated storage tanks and therefore joint action of pumps and elevated storage tanks must be analyzed. The task may be very complex. Some steps to solve the problem are proposed in [13], including simplification of the WDS model. Efficiency of a variable speed pump (VSP) working under both varied system demand and pump head was investigated in [14,15]. In this regard, the Tallinn WDS differs from other systems because it contains no elevated storage tanks and the analyses of pumps covered in [11,12,14,15] may be useful in this case. Two common issues to be overcome in the operation are unnecessary demands in the network and oversized pumps. Focus of this paper is on the creation of a control algorithm that enables maximization of the total efficiency of a pump group and recommends when to start/stop extra pumps based on the required flow. Pump (Q, H) performance and efficiency characteristics are usually provided by manufacturers. The performance of a single new pump is typically described by a graph plotting the pressure head generated by the pump against the needed flow rate. The performance curves for a typical centrifugal pump are shown in Fig. 1. A single pump is often unable to consistently operate close to its BEP because of a wide variation in the WDS requirements. Therefore, pump batteries consisting of several smaller pumps running in parallel are often used to serve the pumping requirements of a WDS, particularly those with large differences between the flow rate required during the normal system operation and that required during the maximum system flow conditions. Koor, et al. [16] carried out practical research to find an optimal number of working pumps for identical pumps working in parallel. Optimal pump count areas with pump switching points were calculated based on the required flow and the pump head.

The aim of this paper is to analyze further possibilities to optimize the work of a pump group that consists from non-identical pumps. The paper is based on the previous research of Koor, et al. [1], but significant improvements and extensions have been made. The equation for the calculation of the total efficiency of the pump group was changed, another method of calculation of the efficiency at different pump heads was proposed. To solve the task, the optimization software developed by Argonne National Laboratory (University of Chicago) was used. Minpack package, which uses the LMA algorithm, has been rewritten from FORTRAN into Visual Basic and into Visual C++. Software was slightly modified to enable its usage with the current optimization task. The result was tested on the basis of three scenarios - pumps are identical, characteristics of pumps are slightly different and differences between the characteristics of pumps are quite large.

2. Analysis

2.1. Description of the tasks

Let us assume that the number of pumps in the pumping station is n; all the pumps can be switched on and off and their revolutions per minute (rpm) are adjustable. In such cases, a complex optimization

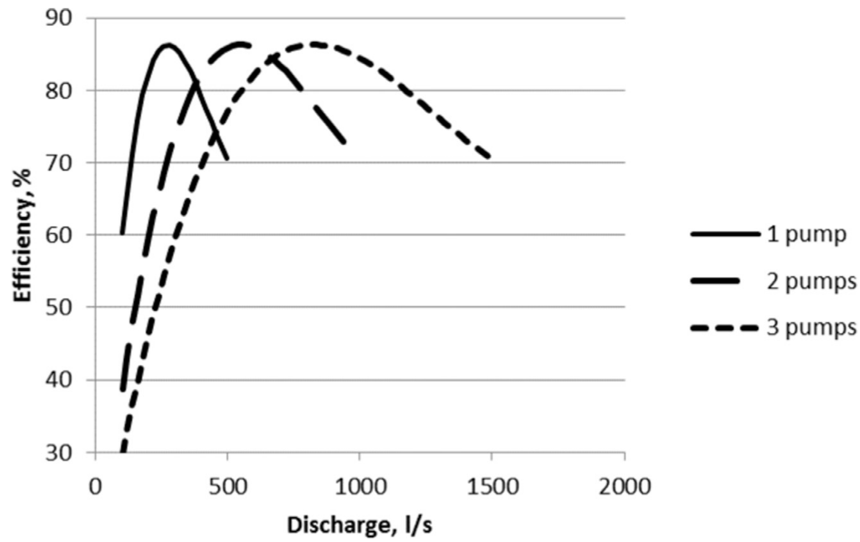

Fig. 2. Dependence of the pump efficiency on the discharge (at H = 25 m) for a different number of pumps.

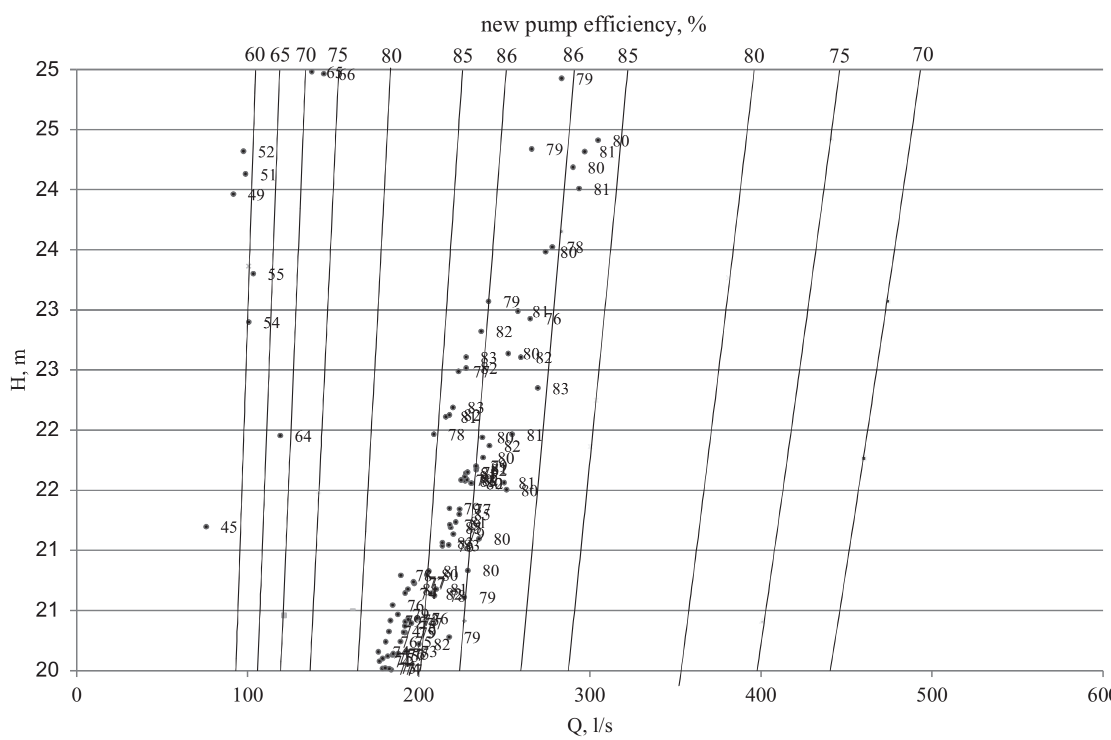

Fig. 3. Efficiency (η) of the new pump (lines on the chart) and the old pump (points with efficiency indicated next to points).

Table 2
Water discharge at the best combined efficiency calculated for a different number of pumps.

Number of pumps	*Pump 1 flow (l/s)*	*Pump 2 flow (l/s)*	*Pump 3 flow (l/s)*	*Pump 4 flow (l/s)*	*Pump 5 flow (l/s)*	*Total flow, (l/s)*	*Total efficiency, (%)*
Alternative 1							
3	383	364	353			1100	78.0
4	283	272	261	284		1100	83.4
5	230	218	204	230	218	1100	81.2
Alternative 2							
3	366	344	390			1100	81.9
4	275	260	289	276		1100	86.7
5	222	212	231	222	213	1100	84.9

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Table 3
Water discharge at the best combined efficiency calculated for a different number of pumps.

Number of pumps	Pump 1 flow (l/s)	Pump 2 flow (l/s)	Pump 3 flow (l/s)	Pump 4 flow (l/s)	Pump 5 flow (l/s)	Total flow, (l/s)	Total efficiency, (%)
3	362	288	450			1100	77.9
4	271	191	367	271		1100	82.2
5	234	153	325	234	154	1100	80.7

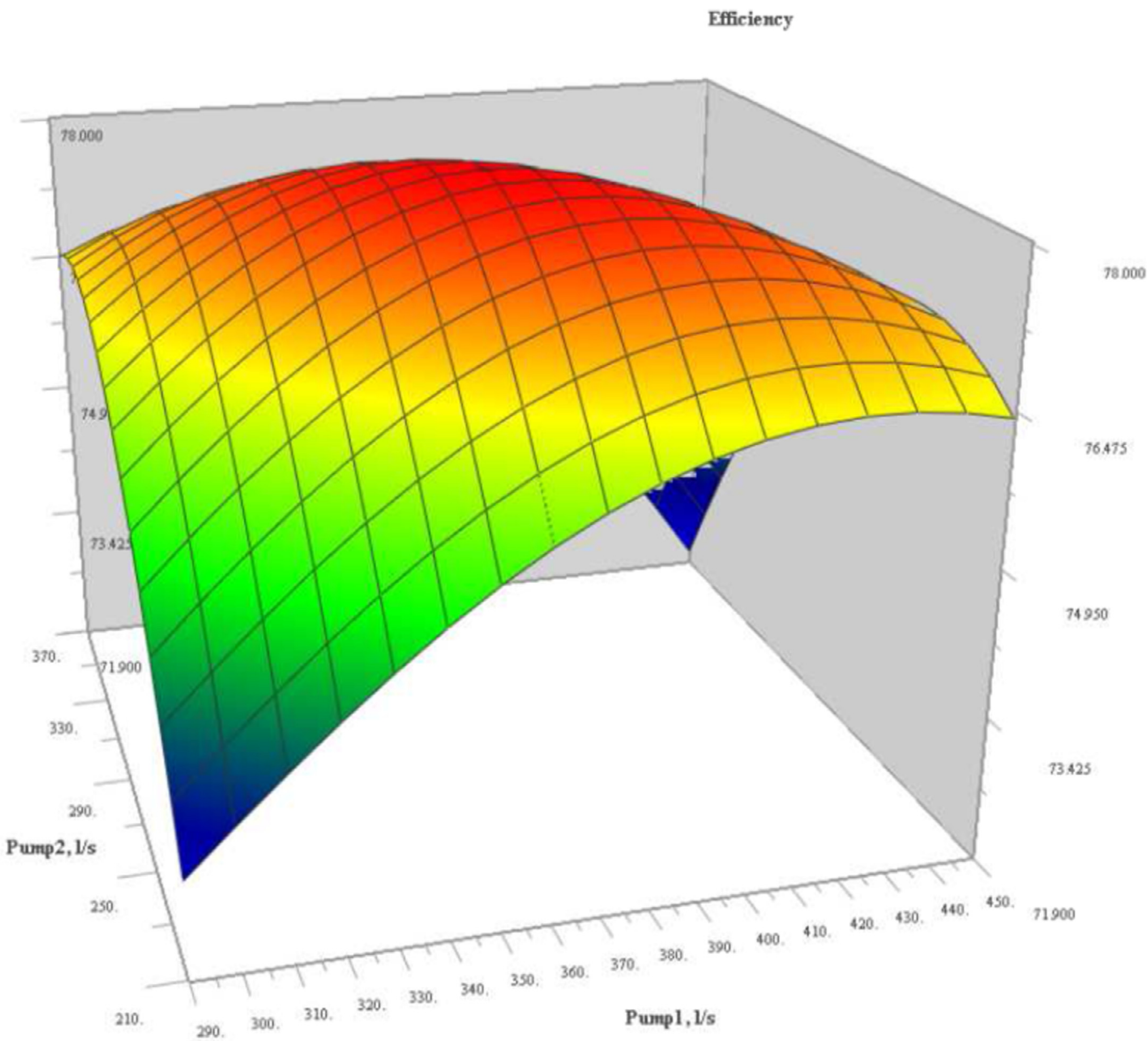

Fig. 4. Surface of combined efficiency in the case of three pumps with quite large differences.

task should be resolved with the objective function of maximizing the total efficiency of the pump system.

- Initially, the usage of several pumps with the same technical characteristics was considered.
- Then, several pumps with slightly different characteristics were examined. A similar situation can happen if some pumps from the initial set of identical pumps worked more than the other ones. Pumps working more often show faster degrading in terms of their efficiency because of wear. Also, a theoretical case when one or more pumps were replaced with a pump that has similar characteristics was investigated. For example, an old broken pump in the pump buttery will be usually replaced with a new pump that has similar but not identical characteristics.
- Finally, a case with quite large differences in the pump parameters was analyzed.

2.2. Scenarios

2.2.1. Identical pumps

The scenario with identical pumps was used mostly to test the software. Let us consider the pump with the characteristics presented in Fig. 1.

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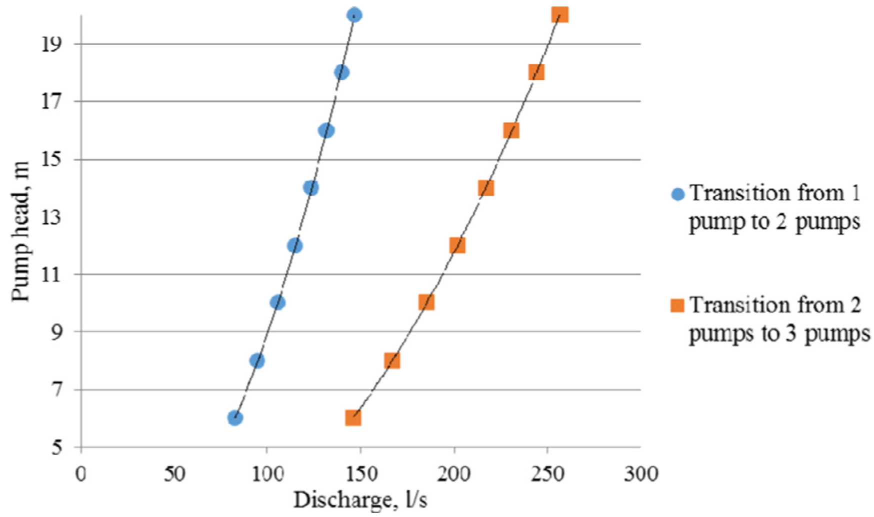

Fig. 5. Transition curves from 1 pump to 2 pumps and from 2 pumps to 3 pumps.

For example, a WDS needs a flow $Q = 1100$ l/s at the pump head $H = 25$ m. Our task is to find the combination of discharges at which the pumps work with maximal total efficiency. In the case of n pumps, the total efficiency can be formulated as the following Eq. (1):

$$\eta_{tot} = \frac{\rho g H Q}{\rho g H\left(\sum_{i=1}^{n} \frac{Q_i}{\eta_i}\right)} \tag{1}$$

where

Q_i – discharge from the ith pump; Q – sum of discharges from all pumps; n – number of pumps; η_i– efficiency of the ith pump (%); ρ – density; g – gravitational acceleration. Taking into account that ρ, g, H, Q remain unchanged, the task may be formulated as a search of a minimum of the part of the denominator $\sum_{i=1}^{n} Q_i/\eta_i$.

Efficiency of the pump at different discharges and at the same head H=25 m may be obtained from Fig. 1 and represented by the polynomial function as given in (2):

$$\eta = 1,758E - 0,6^{*}Q^3 - 0,00213^{*}Q^2 + 0,7691^{*}Q \tag{2}$$

Free term in the approximation (2) equals zero (efficiency equals zero if the discharge equals zero). Table 1 below presents the results of the optimization for a different number of identical pumps. The software calculates the individual discharge Q_i for each working pump to maximize the total efficiency of the pump system. Individual flow of every pump can be different during the optimization.

Table 1 shows that in all cases the optimization program proposes a situation when all the pumps produce the same discharge flow. These results coincide with the findings that identical pumps work at the best efficiency if they operate at the same rotation speed (the same discharge) [11,12,16]. Thus, it is easy to find the discharge flow and the corresponding rotation speed for each pump by dividing the total flow by the number of pumps. The question of how many pumps should work to deliver the arbitrary total flow at the needed pump head is more complex. The fact that all identical pumps must work at the same rotation speed makes it easier to find optimal pump switching points when the number of pumps should be changed to maintain maximal total efficiency. Since all pumps are identical, it is easy to generate dependencies of efficiency on the flow for a different number of pumps using Eq. (3):

$$\eta_{np} = c_0 + c_1\frac{Q}{np} + c_2\frac{Q^2}{np^2} + c_3\frac{Q^3}{np^3} \tag{3}$$

where np – number of pumps; coefficients c0 = 0; c1 = 0.7691; c2 = −0.00213; c3 = 1.758E−06 (Eq. 2). Dependencies of efficiency on the discharge for a different number of pumps are presented in Fig. 2.

As Fig. 2 shows, one pump works with the highest efficiency at 280 l/s and then the efficiency drops down. Two identical pumps work with the highest total efficiency at 560 l/s. Discharge value at the cross point of the curves for 1 and 2 pumps in Fig. 2 is the flow when to switch on the second pump because the total efficiency of two pumps is higher when the flow increases. One way to find this flow is by solving the following Eq. (4):

$$c_0 + c_1\frac{Q}{np-1} + c_2\frac{Q^2}{(np-1)^2} + c_3\frac{Q^3}{(np-1)^3} = c_0 + c_1\frac{Q}{np} + c_2\frac{Q^2}{np^2} + c_3\frac{Q^3}{np^3} \tag{4}$$

where np – is the number of pumps ($np = 2$ in this case).

After removing c0 that is the same on the left and right side of the equation and dividing the left and the right side by the discharge Q, the equation can be simplified as Eq. (5), which can be solved easily:

$$c_1\frac{1}{np-1} + c_2\frac{Q}{(np-1)^2} + c_3\frac{Q^2}{(np-1)^3} = c_1\frac{1}{np} + c_2\frac{Q}{np^2} + c_3\frac{Q^2}{np^3} \tag{5}$$

Another way is to find the flow with minimal difference between η in the interval from the maximal efficiency of one and two pumps by optimization, which is more preferable since it allows a different type of approximation of the dependence of the efficiency on the discharge.

2.2.2. Pumps with slightly different characteristics

As was mentioned above, pumps can be degraded during operation due to wear. Fig. 3 presents efficiencies of a new pump and of the same pump after 16 years of operation in the WDS of Tallinn City. Operational parameters were constantly measured and stored in the SCADA system that gives an opportunity to analyze pump parameter changes in time. It is commonly impossible to plan and carry out measurements to cover the whole operational area of the chart by the measured points. The set of data depends on the required discharge and the pump head that change constantly according to the customers' requirements. Therefore, the chart contains two groups of points that correspond to the night- and daytime. As can be seen, the efficiency of the pump is slightly decreased (by 5–7%). It means that the pump used in active service for several years is not identical to the new one and the statement that pumps must work at the same speed and discharge is incorrect in this case.

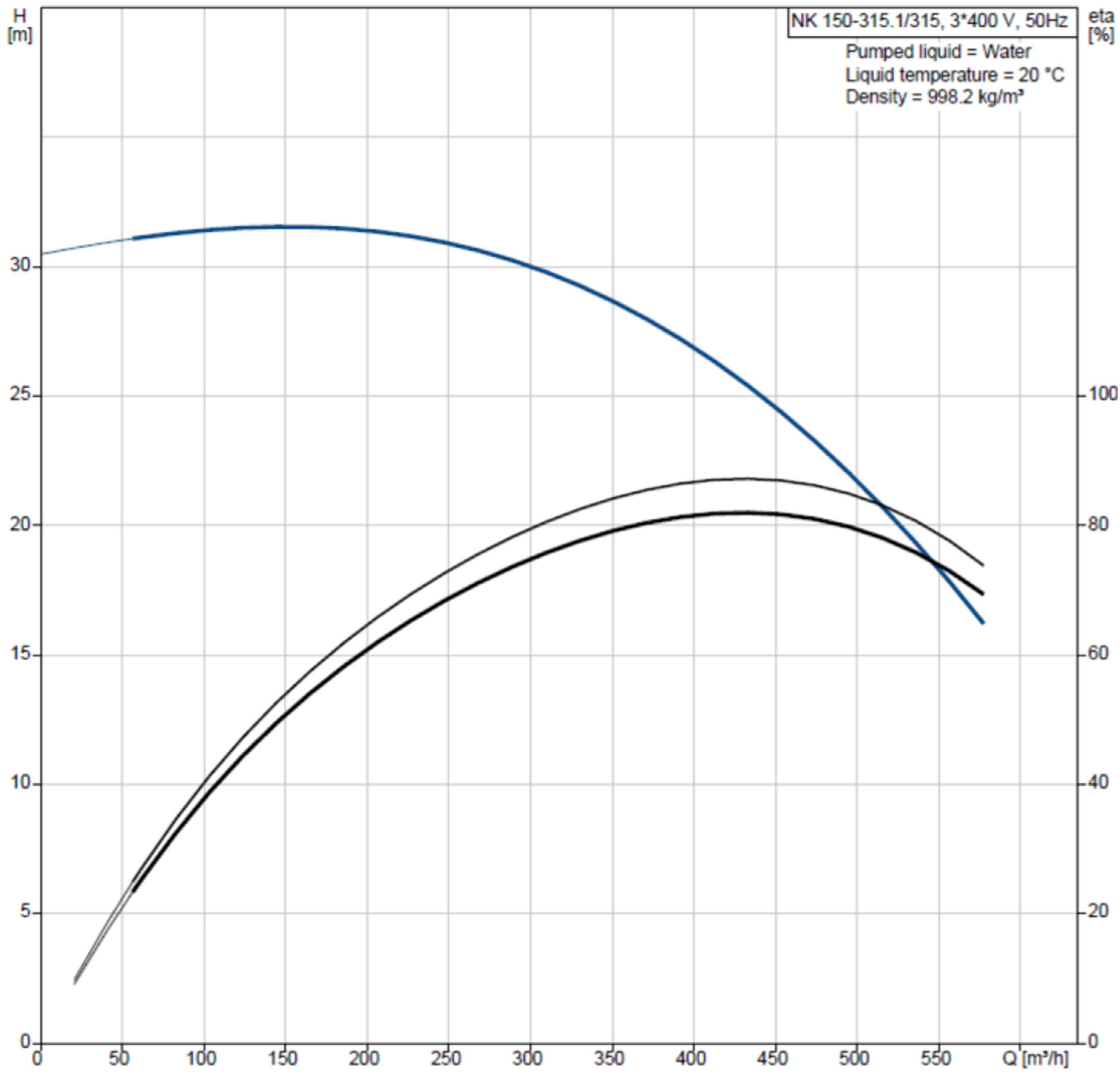

Fig. 6. Pump characteristics.

Therefore, two alternatives of the calculations were made in this section for the pump groups consisting of 3, 4 and 5 pumps to test the software. In the first scenario, one of the pumps was considered new and the efficiency of the second and the third pump was respectively decreased by 5% and 10%. The efficiency of pumps 4 and 5 is taken the same as the efficiency of pumps 1 and 2.

In the second alternative, the discharge for pumps 2 and 3 corresponding to the maximal efficiency was decreased/increased by 5% and 10%, respectively (curves $\eta=f(Q)$ were shifted to left and to right). The efficiency of pumps 4 and 5 was taken the same as the efficiency of pumps 1 and 2. The dependencies of individual pump efficiency $\eta=f(Q)$ were represented by six different polynoms (three of them for the first alternative and the next three for the second one).

Alternative 1

(1) $\eta = 1.758E - 06 * Q^3 - 0.00213 * Q^2 + 0.7691 * Q$
(2) $\eta = 1.670E - 06 * Q^3 - 0.00203 * Q^2 + 0.7306 * Q$
(3) $\eta = 1.583E - 06 * Q^3 - 0.00192 * Q^2 + 0.6922 * Q$

Alternative 2

(1) $\eta = 1.758E - 06 * Q^3 - 0.00213 * Q^2 + 0.7691 * Q$
(2) $\eta = 2.412E - 06 * Q^3 - 0.00263 * Q^2 + 0.8545 * Q$
(3) $\eta = 1.3210E - 06 * Q^3 - 0.00176 * Q^2 + 0.6992 * Q$

Table 2 presents the calculation results. The discharges at the best combined efficiency differ in this case (not the same as for identical pumps).

As expected, optimal water discharge is higher for pumps that work with higher efficiency. The calculations showed that software works successfully also with slightly different pumps.

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2.2.3. Pumps with quite large differences

The pumps with quite large differences were used in this case. Curves η=*f(Q)* were represented by the following polynoms for pumps 1–3:

(1) $\eta = 1.758E - 06 * Q^3 - 0.00213 * Q^2 + 0.7691 * Q$
(2) $\eta = 5.797E - 06 * Q^3 - 0.00447 * Q^2 + 1.072 * Q$
(3) $\eta = 2.426E - 07 * Q^3 - 0.000687 * Q^2 + 0.428 * Q$

Optimization was done for a different number of pumps and as in the previous case, three different curves used for pumps 1-3 and curves for pumps 4 and 5 repeat the curves for pumps 1 and 2, respectively. Table 3 presents the results of the optimization.

Discharges differ significantly because of high differences in the characteristics of the pumps.

For better visualization, the surface of total efficiency was calculated as an example for the case with three pumps (Fig. 4).

Calculations were made for different combinations of discharges. Water discharge was changed from 290 to 450 l/s for pump 1 and from 210–370 l/s for pump 2 in this case. Water discharge for pump 3 was estimated as the difference between 1100 l/s and the sum of discharges for pumps 1 and 2. Results of the calculations in Fig. 4 show that even with significantly different pumps, the combined efficiency surface is quite simple and allows using the LMA algorithm for optimization.

2.3. Discussion

The calculations above showed that it is possible to find maximal total efficiency for a different number of pumps even if they differ considerably. These calculations were made for a fixed *H* (pump head) but the pressure *H* may be changed during the day and night in an operational WDS in order to decrease energy consumption. It means that curves η=*f(Q)* will change with *H* and it is necessary to solve this task for the whole possible interval of pump heads. The approximation of Fig. 1 in the whole pressure head range by some polynomial may be used and an equation may be found to estimate a number of pumps that work with the highest efficiency for different flow and pump heads [16]. But the solution proposed in [16] is valid for identical pumps only. Secondly, sometimes the polynomial type approximation proposed in [16] is incapable of describing all possible variants even if identical pumps were used. Therefore, an alternative solution was considered.

First of all, it is necessary to find dependencies of the efficiency on the water flow for each possible pump head (and for each pump if they differ). Affinity Laws (6) described in [17] for centrifugal pumps can be used to approximate the influence of the change of speed of centrifugal pumps on the performance of the pump:

$$\frac{Q_n}{Q_x} = \frac{n_n}{n_x};\ \frac{H_n}{H_x} = \left(\frac{n_n}{n_x}\right)^2;\ \left(\frac{N_1}{N_2}\right)^3 = \frac{P_1}{P_2};\ \frac{\eta_n}{\eta_x} = 1 \quad (6)$$

Affinity Laws (6) imply that the pump efficiency is the same at the rotation speed n_n and n_x but in practice a reduction of the speed results in a slight reduction in efficiency [14,15]. Efficiency at a reduced speed (n_x) can be estimated by the following Sárbu and Borza's [18] formula (7), which is valid for speed reduction down to 50% of the maximum speed:

$$\eta_x = 1 - (1 - \eta_n) \cdot \left(\frac{n_n}{n_x}\right)^{0,1} \quad (7)$$

Eqs. (6) and (7) allow obtaining the dependencies of the pump efficiency on the water flow at different fixed pump heads that in turn may be approximated by some polynomial functions. Parallel processing can be used to estimate these functions for large sets of pump heads to reduce calculation time. Farming algorithm described in [19] was used in the current case. The next step (estimation of a water flow at the highest efficiency) was described in the previous section.

Dependencies of the best number of pumps at the required Q and *H* can be estimated by the calculations for different water discharges and heads to simplify operational work. Taking into account that the characteristics of pumps and the range of changes in *H* and Q are known, transition curves should be calculated only once and subsequently may be used in the form of a table or functions. Fig. 5 presents an example of such curves for pumps with characteristics presented in Fig. 6. Unlike identical pumps, estimation of conditions to switch on an additional pump is more difficult in the case of non-identical pumps. As opposed to identical pumps, in this case it is necessary to estimate proportions of discharges for each pump, which ensures maximal efficiency. Those estimations should cover all sets of total discharges and pump heads. As was mentioned above, this task may be solved by the optimization procedure for each set of Q and H.

Let us consider the pumps with quite large differences. First of all, it is necessary to define the order or priorities of pumps – the first pump that works at the beginning, the second pump that will be added in the first place and so on. It depends in a large measure on the dynamics of demands. After the order of pumps is defined, it is necessary to estimate curves $\eta = f(Q)$ for different *H*. The next step is to estimate the proportions of discharges between the pumps that ensure maximal efficiency for different total discharge and different *H*. That computer resource consuming process may be solved using parallel processing [19].

3. Conclusions

Our investigation demonstrated that

- Optimization software can be used to estimate the best combined efficiency of the pumps working in parallel.
- Identical pumps work at the best combined efficiency if the discharge is identical for all pumps, as reported in several investigations.
- Non-identical pumps work at the best combined efficiency if the water discharges are different.
- The optimal number of working pumps can also be estimated using the optimization software.

Acknowledgments

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