Natural Gas Network Modeling for Power Systems Reliability Studies

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***Abstract*—The growth of power generation in Spain and several other countries is mainly based on the construction of combined-cycle power plants. As the number of this type of plants increases, the gas and electricity systems are linked together. Therefore, power system reliability studies should consider the gas supply reliability. As a consequence, the new generation of reliability models should take into account the joint operation of electrical and gas systems.**

This paper presents a model to compute the maximum amount of power that can be supplied by the combined-cycle power plants in a system. The gas network is modeled. The effect of compressors to enlarge the transmission capacity of the network is included. The developed model will be integrated into a higher level model that analyzes the joint reliability of the electrical and gas systems.

A case study based on the Belgian high-calorific gas network is analyzed.

Index Terms— Combined-cycle power plant, Natural gas, Networks, Optimization methods, Power generation, Power system reliability.

I. INTRODUCTION

The structure of the power industry has undergone important changes in many countries during the last years. The general trend is to move towards a greater competition which means larger risks for private companies. These changes have been driven by political, economic and technical reasons. Among the technical reasons, the outstanding feature is the development of combined-cycle power plants. These plants are efficient power plants that use natural gas to generate electricity. They present several advantages with respect to traditional thermal and nuclear power plants. Among others, they require lower investment costs and shorter depreciation periods. Therefore, the old economies of scale that were the reason for the existence of big regulated utilities either disappear or are greatly reduced. This new frame compels the restructuring of the power industry into a free electricity market.

Due to the benefits of combined-cycle power plants, the growth of generating power in Spain and several other countries is mainly based on the construction of this kind of plants. The increase of electrical generation by this technology has promoted the merge of the electrical system and the natural gas system into an only energy system.

Furthermore, gas systems have also been restructured from a regulated market to a free competitive market in the last few years. In Spain, gas companies are building combined-cycle power plants to get into the electricity market. Such a gas company could choose, according to electricity and gas market prices, between selling its gas as fuel in the gas market or selling its electrical energy in the electricity market. Therefore, the gas market price and the electricity market price are related and the coupling between these two sectors is stronger.

Electrical and gas systems are quite similar. Both systems are designed to carry energy from suppliers to customers. They can be structured into:

- Suppliers (electrical power plants or gas fields)
- Transmission (high voltage network or high pressure pressure network)
- Distribution (medium/low voltage network or medium/low pressure network)
- Customers (electricity customers or gas customers).

Nevertheless, there are some differences between these systems. Natural gas constitutes a primary form of energy that comes straight from gas fields, while electrical energy is a secondary form of energy which comes from the transformation, in a power plant, of a primary energy (fuel). Moreover, gas systems can store energy to be used in peak load periods while electrical energy cannot efficiently be stored.

The gas is carried from the gas fields (suppliers) to customers in Liquefied Natural Gas (LNG) ships and/or flows through pipelines. The pipeline network is a complex and expensive grid which feeds gas customers. Unfortunately, the transmission capacity of a gas pipeline is not unbounded. It depends on the pressure difference between the two ends of

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the pipeline. To increase the transmission capacity of the network, compressors, which enlarge the pressure difference between two nodes, may be added at certain locations. However, the amount of gas that can be supplied to customers is always limited.

Gas consumers can be classified into domestic and industrial customers. Combined-cycle power plants are natural gas industrial customers that use gas to generate electricity. In this paper, gas customers will be structured into electric generation consumers (combined-cycle power plants) and non-electrical consumers.

The priority of the natural gas network is to serve non-electric natural gas consumption. Combined-cycle power plants would produce electric power only if sufficient natural gas exists for all system non-electric customers. This is the case, at least for the moment, in Spain, where investments on the gas transmission network should be made. In mature electricity and gas markets, customers not supplied should be those not willing to pay the market price.

In the new deregulated environment, the electrical supply reliability studies are very important for the incurrent companies, the possible newcomers, the regulatory commission and the customer unions. Because of the link between the gas and the electricity supply (tighter as new combined-cycle power plants are built), power system reliability studies should consider the gas supply reliability. Therefore, the new generation of reliability models should take into account the joint operation of the electrical and the gas systems. These models have not yet been developed and should consider the different technical features of the electrical and the gas systems. Fig. 1 illustrates the joint operation of the electrical and the gas systems.

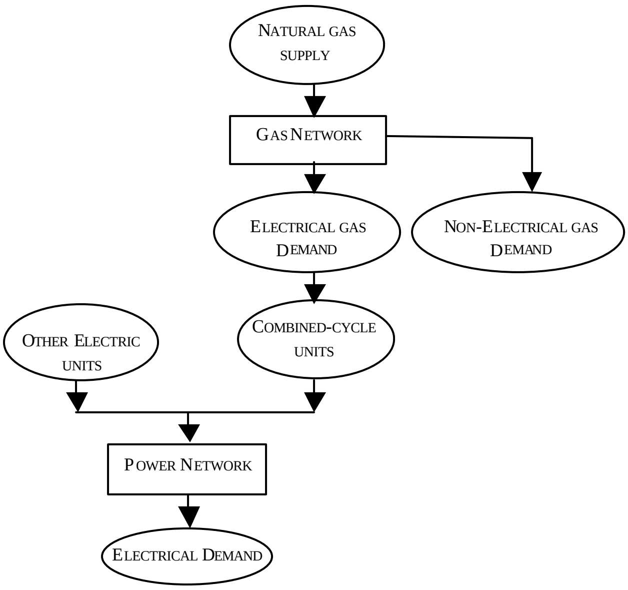

Fig. 1. Joint operation of the electric and natural gas systems.

Electric energy system reliability models have been developed since the 60´s, based mainly on the hierarchical levels structure proposed by Billinton and Allan [1].

Natural gas system reliability studies started to be developed from the late 80´s. The similarities between the electrical and the gas sectors suggest, for natural gas reliability models, a three hierarchical level structure, equivalent to the electrical one developed by Billinton and Allan. These hierarchical levels are based on three functional zones: gas production, transmission facilities and distribution facilities [2].

The scope of this paper is to develop a model of the natural gas transmission network to be included into a reliability model of the joint operation of electrical and gas systems. This reliability model is still under development.

To perform electrical energy reliability studies, the maximum amount of power that can be supplied by each generator at each time period is required. For traditional thermal plants, the maximum power is a constant known parameter. Nevertheless, the maximum amount of power that can be supplied by a combined-cycle power plant at each time period is a nonlinear function of the amount of available fuel at that time period. Therefore, it depends on the conditions of the gas network, on the available gas supply and on the gas required by other gas customers at each time period.

This paper presents a model to compute the maximum amount of energy that can be provided by each combined-cycle power plant of a gas system at each time period. The transmission network constraints are considered. Compressors to enlarge the transmission capacity of the network are included and modeled.

This optimization process will be inserted into a higher level model that analyzes the electrical supply reliability according to Fig. 1.

II. COMBINED-CYCLE POWER PLANTS

Combined-cycle power plants present several advantages with respect to traditional thermal and nuclear power plants. They have higher full-load efficiencies (58% Vs. 38%) and require lower investment costs and lower depreciation periods. Because of their high thermal efficiency, low initial cost, high reliability, relatively low gas prices and low air emissions, combined-cycle power plants have been the new resource of choice for bulk power generation for well over a decade.

The power generated by a combined-cycle power plant is a nonlinear function of the gas supply:

$$Pgcc = m(e) * LHV * e \qquad (1)$$

where *Pgcc* is the electrical power generated in MW, *e* is the power plant gas supply in m^3/s, *m* is the combined-cycle efficiency and *LHV* is the Low Heating Value for natural gas (35.07 $MW(m^3/s)$) [3]. Combined-cycle efficiency *m* can be described as a quadratic function of the gas supply to the power plant. As a consequence, the electrical power *Pgcc* generated in a combined-cycle power plant is a cubic function of the gas supply *e*. So, the above equation can be formulated as the following cubic function:

$$Pgcc = K_3 e^3 + K_2 e^2 + K_1 e \quad (2)$$

where the coefficients K_3, K_2 and K_1 depend on the combined-cycle power plant characteristics.

III. NATURAL GAS SYSTEM

Natural gas has made a strong comeback in the global energy balance since the mid-1970s as a direct response to the increase in crude oil prices that started in that period. This development was given further impetus from the late 1980s in the light of new concerns about a potential global warming and climate change. The low carbon intensity of natural gas (lowest among the fossil fuels) has made it the fuel of choice from an environmental point of view.

Pipeline transportation is more economical over short distances, while LNG shipping is more attractive over greater distances [4]. Once the natural gas is in the transmission network, it travels from suppliers to customers over long distances. The gas network includes supply nodes, demand nodes and intermediate nodes. The gas is injected into the system through the supply nodes and it flows out of the system through the demand nodes. Demand nodes are classified into electrical customers and non-electrical customers. Electrical customers are combined-cycle power plants which use the gas as fuel to produce electrical energy. Non-electrical customers are the remainder natural gas system customers.

The gas network consists of nodes and pipelines. A pipeline is represented by an arc linking two nodes. The network is defined as the pair (N,A), where N is the set of nodes and $A \subseteq N x N$ is the set of arcs (or pipelines) connecting these nodes. A gas network is illustrated in Fig. 2.

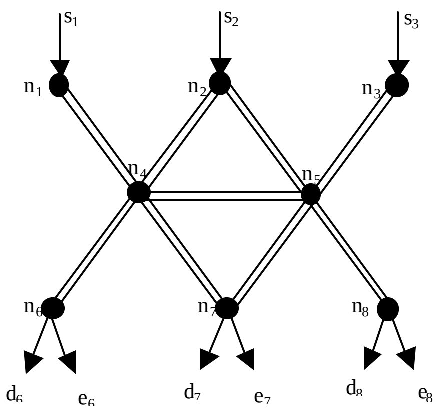

Fig. 2. Natural gas network.

Nodes n_1, n_2 and n_3 in Fig. 2 are supply nodes while nodes n_6, n_7 and n_8 are demand nodes. The gas supply into a node i is denoted as s_i . The non-electrical demand gas demand out of a node j is denoted d_j, while the combined-cycle gas demand out of a node j is denoted e_j.

Each node i is defined by its pressure p_i. A gas flow f_{ij} is associated to each pipeline (i,j). The gas flow through each pipeline depends on the pressures at the two end nodes of the pipeline. The behavior of the network is modeled as follows. The gas transmission company cannot take gas at pressures higher than the ones ensured by the suppliers. Conversely, at each exit point, the demand must be satisfied at a minimal pressure guaranteed to the customer. Therefore:

$$P_{i,\min} \le p_i \le P_{i,\max} \quad (3)$$

The flow conservation equation ensures the gas balance at node i (Fig. 3):

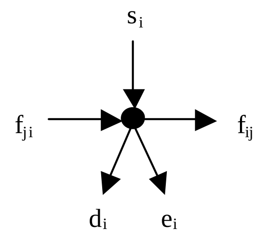

Fig. 3. Gas flow balance at node i.

Mathematically, the flow conservation can be expressed as:

$$s_i + \sum_j f_{ji} = \sum_j f_{ij} + d_i + e_i \quad (4)$$

There are two types of pipelines: passive pipelines and active pipelines. Passive pipelines correspond to regular pipelines. Active pipelines are regular pipelines with compressors. Compressors may be included at certain locations to enlarge the pressure difference between the two end nodes of a pipe. The introduction of compressors increases the transmission capacity of the network.

The gas flow through each passive pipeline f_{ij} is a quadratic function of the pressures at the end nodes:

$$sign(f_{ij}) f_{ij}^2 = C_{ij}^2 \left(p_i^2 - p_j^2 \right) \quad (5)$$

where C_{ij} is a constant that depends on the properties of the pipeline (length, diameter and the absolute rugosity) and on the gas composition. Note that the flow is unrestricted in sign. If $f_{ij}>0$, the gas flows from node i to node j and if $f_{ij}<0$, it flows from node j to node i.

The gas flow through an active pipeline is also a quadratic function of the pressures at the end nodes. In this case, the pressure at the incoming node i is lower than the pressure at the outcoming node j ($p_i < p_j$) and the gas flows from node i to node j ($f_{ij}>0$). Mathematically,

$$f_{ij}^2 \ge -C_{ij}^2 \left(p_i^2 - p_j^2 \right) \quad (6)$$

where the constant C_{ij} is a constant that depends on the properties of the pipeline (length, diameter and the absolute rugosity), on the gas composition and on the compressor characteristics.

The pressure at the exit of each compressor is bounded:

$$p_j \le P_{j\max} \quad (7)$$

IV. NOTATION

The notation used through the whole paper is explained next. Capital greek letters represent sets, capital letters represent parameters or data, and lowercase letters represent variables.

Sets:

Ω : gas network nodes

Ω_e : subset of electric demand nodes ($\Omega_e \subset \Omega$)

Ω_{ne} : subset of non-electric demand nodes ($\Omega_{ne} \subset \Omega$)

Ω_s : subset of supply nodes ($\Omega_s \subset \Omega$)

Φ : network pipelines

Φ^P : subset of passive pipelines ($\Phi^P \subset \Phi$)

Φ^A : subset of active pipelines ($\Phi^A \subset \Phi$)

Parameters:

$K_{1,i}$, $K_{2,i}$, $K_{3,i}$: Coefficients of the cubic approximation of the electrical power generated by combined-cycle power plant i, $i \in \Omega_e$

C_{ij} : Pipeline constant, $(i,j) \in \Phi$

$S_{i,max}$, $S_{i,min}$: supply bounds, $i \in \Omega_s$

$D_{i,max}$, $D_{i,min}$: non-electric demand bounds, $i \in \Omega_{ne}$

$E_{i,max}$, $E_{i,min}$: electric demand bounds, $i \in \Omega_e$

$P_{i,max}$, $P_{i,min}$: pressure bounds, $i \in \Omega$

$\Pi_{i,max}$, $\Pi_{i,min}$: squared pressure bounds, $i \in \Omega$

A_i^I, A_i^{II} : Coefficients of the linear approximation of the electrical power generated by combined-cycle power plant i, $i \in \Omega_e$

$\overline{X}_i^I$, $\overline{X}_i^{II}$: Bounds of the gas supply intervals used in the linear approximation of the electrical power generated by combined-cycle power plant i, $i \in \Omega_e$

$\overline{F}_{ij}^P$: Maximum positive flow, $(i,j) \in \Phi^P$

$\overline{F}_{ij}^N$: Maximum negative flow, $(i,j) \in \Phi^P$

F_{ij} : Max { F_{ij}^P, F_{ij}^N } , $(i,j) \in \Phi^P$

D_{ij}^0 : Parameter used for the linear approximation of the passive pipeline gas flow equations, $(i,j) \in \Phi^P$

$\underline{Y}_{ij}$: Flow direction parameter, $(i,j) \in \Phi^P$

$\overline{\Pi}_{ij}^P$: Maximum squared pressure difference for positive flow, $(i,j) \in \Phi^P$

$\overline{\Pi}_{ij}^N$: Maximum squared pressure difference for negative flow, $(i,j) \in \Phi^P$

$\overline{\Pi}_{ij}^0$: Max $\{D_{ij}^0 \overline{\Pi}_{ij}^P , D_{ij}^0 \overline{\Pi}_{ij}^N\}$, $(i,j) \in \Phi^P$

Variables:

s_i : natural gas supply at node i, $i \in \Omega_s$

d_i : non-electric natural gas demand at node i, $i \in \Omega_{ne}$

e_i : electric natural gas demand at node i, $i \in \Omega_e$

p_i : pressure at node i, $i \in \Omega$

π_i: squared pressure at node i, $i \in \Omega$

x_i^I, x_i^{II}: auxiliary variables for the linear approximation of the electric power generated by combined-cycle power plant i, $i \in \Omega_e$

w_i : binary variable for the linear approximation of the electric power generated by combined-cycle power plant i, $i \in \Omega_e$

f_{ij} : gas flow through pipeline (i,j), $(i,j) \in \Phi$

f_{ij}^P : positive gas flow through pipeline (i,j), $(i,j) \in \Phi^P$

f_{ij}^N : negative gas flow through pipeline (i,j), $(i,j) \in \Phi^P$

y_{ij}: binary variable associated to the direction of the gas flow through pipeline (i,j), $(i,j) \in \Phi^P$

V. PROBLEM FORMULATION

The scope of this paper is to develop a model to compute the maximum amount of power generated by each combined-cycle power plant in a gas system. For this purpose the gas network is modeled. The effect of compressors is included. At this stage, no storage nodes have been defined. The effect of gas storage will be considered in future works.

The optimization problem is formulated below. It will be called the Original Problem.

$$Max \sum_{i \in \Omega_e} K_{1,i} e_i + K_{2,i} e_i^2 + K_{3,i} e_i^3 \qquad (8)$$

subject to

$$s_i + \sum_{j/(j,i) \in \Phi} f_{ji} = \sum_{j/(i,j) \in \Phi} f_{ij} + d_i + e_i \qquad \forall i \in \Omega \qquad (9)$$

$$sign(f_{ij}) f_{ij}^2 = C_{ij}^2 \left(p_i^2 - p_j^2 \right) \qquad \forall (i,j) \in \Phi^P \qquad (10)$$

$$f_{ij}^2 \geq -C_{ij}^2 \left(p_i^2 - p_j^2 \right) \qquad \forall (i,j) \in \Phi^A \qquad (11)$$

$$D_{i,\min} \leq d_i \leq D_{i,\max} \qquad \forall i \in \Omega_{ne} \qquad (12)$$

$$E_{i,\min} \leq e_i \leq E_{i,\max} \qquad \forall i \in \Omega_e \qquad (13)$$

$$P_{i,\min} \leq p_i \leq P_{i,\max} \qquad \forall i \in \Omega \qquad (14)$$

$$S_{i,\min} \leq s_i \leq S_{i,\max} \qquad \forall i \in \Omega_s \qquad (15)$$

$$f_{ij} \geq 0 \qquad \forall (i,j) \in \Phi^A \qquad (16)$$

Equation (8) is the total electrical power produced by all the combined-cycle power plants of the system.

Equations (9) are the flow conservation equations at each node of the system.

Equations (10) define the gas flows through passive pipelines while equations (11) model the gas flows through active pipelines. Note that the direction of the flow through an active pipeline is fixed: from node *i* to node *j*, where *j* is the node with a higher pressure.

Equations (12) and (13) bound, respectively, the non-electrical gas demand and the combined-cycle gas demand. If the non-electrical gas demand is fixed, then $D_{i\ min} = D_{i\ max}$. On the other hand, gas demand in electric nodes is bounded between zero and full-load combined-cycle power plant gas supply.

Equations (14) bound the pressures in the network.

Equations (15) bound the gas supply.

Equations (16) define the active pipeline flows as nonnegative variables.

Note that because no storage nodes are included, no constraints linking time periods are defined. Therefore, one optimization problem can be formulated for each time period of the time horizon analyzed.

VI. Solution Procedure

Constraints (10) make the Original Problem (8)-(16) a non-convex optimization problem. However, once the flow directions through passive pipelines are known, the problem becomes convex.

This problem (8)-(16) is solved in two consecutive phases. In the first phase the flow directions through passive pipelines are computed. Furthermore, a good initial solution for the second phase is achieved. For this purpose, a Mixed Integer Linear Programming (MILP) optimization problem is formulated and solved. In the second phase, all the flow directions are known and a NonLinear Programming (NLP) optimization problem is solved to get the maximum amount of electrical power generated by each combined-cycle power plant in the system.

Phase 1

The scope of Phase 1 is to determine the gas flow directions through passive pipelines. To do so, a new optimization problem is formulated. In this problem (10)-(11) are not included. The constraints that model the feasible region of Phase 1 optimization problem will be detailed next. They are all linear equations.

Phase 1 optimization problem includes constraints (9), (12), (13) and (15) of the Original Problem. It also includes other constraints, which are explained below.

In Phase 1 optimization problem a new array of variables p_i ($i \in \Omega$) is introduced, where

$$p_i^2 = p_i \qquad \forall i \in \Omega \qquad (17)$$

Therefore, equations (14) of the Original Problem (8)-(16) become:

$$\Pi_{i,\min} \le p_i \le \Pi_{i,\max} \qquad \forall i \in \Omega \qquad (18)$$

where:

$$\Pi_{i,\max} = P_{i,\max}^2 \qquad \forall i \in \Omega \qquad (19)$$

$$\Pi_{i,\min} = P_{i,\min}^2 \qquad \forall i \in \Omega \qquad (20)$$

To model the gas flow directions through each passive pipeline f_{ij} ($(i,j) \in \Phi^P$), two nonnegative variables, f_{ij}^P and f_{ij}^N, are defined for each passive pipeline. The gas flow f_{ij} is defined by the following equations:

$$f_{ij} = f_{ij}^P - f_{ij}^N \qquad \forall (i,j) \in \Phi^P \quad (21)$$

If the gas flows from node i to node j, then $f_{ij}>0$, and:

$$f_{ij} = f_{ij}^P \quad and \quad f_{ij}^N = 0 \qquad \forall (i,j) \in \Phi^P \quad (22)$$

Conversely, if the gas flows from node j to node i, then $f_{ij}<0$, and:

$$f_{ij} = -f_{ij}^N \quad and \quad f_{ij}^P = 0 \qquad \forall (i,j) \in \Phi^P \quad (23)$$

To model the alternative constraints, (22) and (23), the following equations are included [5]:

$$0 \le f_{ij}^P \le \overline{F}_{ij}(1 - y_{ij}) \qquad \forall (i,j) \in \Phi^P \quad (24)$$

$$0 \le f_{ij}^N \le \overline{F}_{ij} y_{ij} \qquad \forall (i,j) \in \Phi^P \quad (25)$$

where

$$\overline{F}_{ij} = \max\left\{\overline{F}_{ij}^P, \overline{F}_{ij}^N\right\} \qquad \forall (i,j) \in \Phi^P \quad (26)$$

$$\overline{F}_{ij}^P = \sqrt{C_{ij}^2\left(P_{i,\max}^2 - P_{j,\min}^2\right)} \qquad \forall (i,j) \in \Phi^P \quad (27)$$

$$\overline{F}_{ij}^N = \sqrt{C_{ij}^2\left(P_{j,\max}^2 - P_{i,\min}^2\right)} \qquad \forall (i,j) \in \Phi^P \quad (28)$$

Equations (10) at the Original Problem could be expressed as a piecewise linear function. However, taking into account that in radial gas networks the flow directions of most passive pipelines (the scope of Phase 1) are determined by the balance equation (9), the equality constraints (10) may be linearized and expressed as upper bounds to the gas flows. These bounds are linear functions of the squared pressure differences, as shown below:

$$f_{ij}^P \le D_{ij}^0(p_i - p_j) + \overline{\Pi}_{ij}^0 y_{ij} \qquad (29)$$

$$f_{ij}^N \le D_{ij}^0(p_j - p_{ij}) + \overline{\Pi}_{ij}^0 (1 - y_{ij}) \qquad (30)$$

where

$$D_{ij}^0 > 0 \qquad (31)$$

$$\overline{\Pi}_{ij}^0 = \max\left\{D_{ij}^0 \overline{\Pi}_{ij}^P, D_{ij}^0 \overline{\Pi}_{ij}^N\right\} \qquad (32)$$

$$\overline{\Pi}_{ij}^P = P_{i,\max}^2 - P_{j,\min}^2 \qquad (33)$$

$$\overline{\Pi}_{ij}^N = P_{j,\max}^2 - P_{i,\min}^2 \qquad (34)$$

It should be noted that equations (24)-(25) and (29)-(30), enforce the relationship between flow and pressure in passive pipelines to be consistent:

- If $y_{ij} = 0 \Rightarrow f_{ij}^P \ge 0$, $f_{ij}^N = 0 \Rightarrow \pi_i \ge \pi_j$
- If $y_{ij} = 1 \Rightarrow f_{ij}^P = 0$, $f_{ij}^N \ge 0 \Rightarrow \pi_i \le \pi_j$

Occasionally, there are parallel pipelines in a gas network. In this case and because the pressure difference between the two end nodes of the parallel pipelines is the same, the gas is distributed through the pipelines according to pipeline constants, as shown in (10). To achieve a reasonable distribution of the gas through parallel pipelines the following equation should be added:

$$\frac{f_{i'j'}^P - f_{i'j'}^N}{f_{ij}^P - f_{ij}^N} = \frac{C_{i'j'}^2}{C_{ij}^2} \qquad \forall (i',j') = (i,j) \in \Phi \quad (35)$$

where (i',j') and (i,j) are two parallel pipelines joining nodes i and j. The variables $f_{i'j'}^P$ and f_{ij}^P are the positive flows through the pipelines (i',j') and (i,j); $f_{i'j'}^N$ and f_{ij}^N are the negative flows through the pipelines (i',j') and (i,j), and $C_{i'j'}$ and C_{ij} are the constants of the pipelines (i',j') and (i,j).

The objective function of Phase 1 optimization problem is to maximize the total electric power generated by combined-cycle power plants. It is a nonlinear function of the gas supply to the combined-cycle power plants in the system. In order to use a MILP solver, the objective function will be approximated by a piecewise linear approximation. A binary variable w_i ($i \in$

Ω_e) is defined to perform such a piecewise linear approximation. The power generated by the combined-cycle power plant i, $Pgcc_i$ ($i \in \Omega_e$) is computed as:

$$Pgcc_i = A_i^I x_i^I + A_i^{II} x_i^{II} \qquad (36)$$

$$e_i = x_i^I + x_i^{II} \qquad \forall i \in \Omega_e \qquad (37)$$

$$\overline{X}_i^I w_i \le x_i^I \le \overline{X}_i^{II} \qquad \forall i \in \Omega_e \qquad (38)$$

$$0 \le x_i^{II} \le \left(\overline{X}_i^{II} - \overline{X}_i^I\right) w_i \qquad \forall i \in \Omega_e \qquad (39)$$

In the above equations, e_i is the gas supply to combined-cycle power plant i; x_i^I and x_i^{II} are auxiliary variables; A_i^I and A_i^{II} are the slopes for the linear approximation of electrical power generated by combined-cycle power plant i and X_i^I and X_i^{II} define the gas supply linearization intervals.

Phase 1 optimization problem is formulated as follows:

$$Max \sum_{i \in \Omega} A_i^I x_i^I + A_i^{II} x_i^{II} \qquad (40)$$

subject to

$$e_i = x_i^I + x_i^{II} \qquad \forall i \in \Omega \qquad (41)$$

$$\overline{X}_i^I w_i \le x_i^I \le \overline{X}_i^{II} \qquad \forall i \in \Omega \qquad (42)$$

$$0 \le x_i^{II} \le \left(\overline{X}_i^{II} - \overline{X}_i^I\right) w_i \qquad \forall i \in \Omega \qquad (43)$$

$$s_i + \sum_{j/(i,j) \in \Phi^A} f_{ji} + \sum_{j/(j,i) \in \Phi^P} \left(f_{ji}^P - f_{ji}^N\right) = $$

$$= \sum_{j/(i,j) \in \Phi^A} f_{ij} + \sum_{j/(i,j) \in \Phi^P} \left(f_{ij}^P - f_{ij}^N\right) + d_i + e_i \quad \forall i \in \Omega \qquad (44)$$

$$f_{ij}^P \le \overline{F}_{ij}\left(1 - y_{ij}\right) \qquad \forall (i,j) \in \Phi^P \qquad (45)$$

$$f_{ij}^N \le \overline{F}_{ij} y_{ij} \qquad \forall (i,j) \in \Phi^P \qquad (46)$$

$$f_{ij}^P \le D_{ij}^0 (\pi_i - \pi_j) + \overline{\Pi}_{ij}^0 y_{ij} \qquad \forall (i,j) \in \Phi^P \qquad (47)$$

$$f_{ij}^N \le D_{ij}^0 (\pi_j - \pi_i) + \overline{\Pi}_{ij}^0 \left(1 - y_{ij}\right)$$

$$\forall (i,j) \in \Phi^P \qquad (48)$$

$$\frac{f_{i'j'}^P - f_{i'j'}^N}{f_{ij}^P - f_{ij}^N} = \frac{C_{i'j'}^2}{C_{ij}^2} \qquad \forall (i',j') = (i,j) \in \Phi \qquad (49)$$

$$S_{i,\min} \le s_i \le S_{i,\max} \qquad \forall i \in \Omega_s \qquad (50)$$

$$D_{i,\min} \le d_i \le D_{i,\max} \qquad \forall i \in \Omega_{ne} \qquad (51)$$

$$E_{i,\min} \le e_i \le E_{i,\max} \qquad \forall i \in \Omega_e \qquad (52)$$

$$\Pi_{i,\min} \le p_i \le \Pi_{i,\max} \qquad \forall i \in \Omega \qquad (53)$$

$$f_{ij}^P, f_{ij}^N \ge 0 \qquad \forall (i,j) \in \Phi^P \qquad (54)$$

$$f_{ij} \ge 0 \qquad \forall (i,j) \in \Phi^A \qquad (55)$$

$$w_i \in \{0,1\} \qquad \forall i \in \Omega_e \qquad (56)$$

$$y_{ij} \in \{0,1\} \qquad \forall (i,j) \in \Phi^P \qquad (57)$$

This problem is a Mixed Integer Linear programming problem which is easily solved using an appropiate solver. The solution of this problem is close to the final problem solution.

Phase 2

In Phase 2, once the flow directions through passive pipelines are known, a nonlinear optimization problem is solved. Phase 2 includes two stages. In the first stage, the flow directions through passive pipelines are computed. A parameter Y_{ij} is defined for each passive pipeline, as follows:

$$Y_{ij} = 1 - 2y^*_{ij} \qquad \forall (i,j) \in \Phi^P \qquad (58)$$

where y^*_{ij} ($(i,j) \in \Phi^P$) is the Phase 1 optimal value for variable y_{ij}. Note that:

- if $y^*_{ij} = 0 \Rightarrow Y_{ij} = 1$ (and the gas flows from node i to node j)
- and if $y^*_{ij} = 1 \Rightarrow Y_{ij} = -1$ (and the gas flows from node j to node i)

Phase 2 optimization problem is formulated as follows:

$$Max \sum_{i \in \Omega_e} K_{1,i} e_i + K_{2,i} e_i^2 + K_{3,i} e_i^3 \qquad (59)$$

subject to

$$s_i + \sum_{j/(j,i) \in \Phi} f_{ji} = \sum_{j/(i,j) \in \Phi} f_{ij} + d_i + e_i \qquad \forall i \in \Omega \qquad (60)$$

$$f_{ij}^2 = Y_{ij} C_{ij}^2 \left(\pi_i - \pi_j\right) \qquad \forall (i,j) \in \Phi^P \qquad (61)$$

$$f_{ij}^2 \ge -C_{ij}^2 \left(\pi_i - \pi_j\right) \qquad \forall (i,j) \in \Phi^A \qquad (62)$$

$$D_{i,\min} \le d_i \le D_{i,\max} \qquad \forall i \in \Omega \qquad (63)$$

$$E_{i,\min} \le e_i \le E_{i,\max} \qquad \forall i \in \Omega \qquad (64)$$

$$\Pi_{i,\min} \le p_i \le \Pi_{i,\max} \qquad \forall i \in \Omega \qquad (65)$$

$$S_{i,\min} \le s_i \le S_{i,\max} \qquad \forall i \in \Omega \qquad (66)$$

$$Y_{ij} f_{ij} \ge 0 \qquad \forall (i,j) \in \Phi^P \qquad (67)$$

$$f_{ij} \ge 0 \qquad \forall (i,j) \in \Phi^A \qquad (68)$$

Equation (59) is the objective function of the Original Problem (8)-(16).

Constraints (60), (62)-(64), (66) and (68) were also included in the Original Problem formulation.

Equations (61) compute gas flows through passive pipelines.

Equations (65) are equivalent to equations (14) (see equations (17)).

Equations (67) define each passive pipeline gas flow as a nonnegative or a nonpositive variable, depending on the flow direction through that pipeline.

The above problem (59)-(68) is solved starting from the solution of Phase 1 optimization problem. Because a good initial solution is used, the problem is solved efficiently.

VII. CASE STUDY

A case study is presented. It is based on the Belgian high-calorific gas network [6]. The network includes 20 nodes and 24 pipelines (3 active and 21 passive). The original demand and supply data have been modified in order to have a transmission capacity constrained network. Using the data from [6], the gas transmission network behaves as an infinite capacity network, and the electric power generated by both combined-cycle power plants is the full-load power generation.

Network data are presented in Appendix A. Table I shows the node data and Table II shows the pipeline data. For the purpose of this paper, two 400 MW combined-cycle power plants have been added to the Belgian gas network in nodes 12 and 20.

The priority of the gas network is to meet non-electrical gas demand. The purpose of the case study analyzed is to compute the maximum electrical power generated by the two combined-cycle power plants.

The model has been developed using GAMS [7]. The MILP problem has been solved using CPLEX, and the NLP problem using CONOPT.

The tests have been carried out on a 200 Mhz PC with 48 MB of RAM. Total Phase 1 + Phase 2 execution time is 2.030 seconds. The sofware version is GAMS IDE 2.0.13.0.

In Phase 1, the passive pipeline flow directions are determined. All the passive pipeline flows resulted positive, except for the pipeline number 8. Tables III and IV show the result of Phase 2.

Combined-cycle power plant located at node 12 produces 400 MW. Combined-cycle power plant located at node 12 produces 255 MW. The full-load electrical power generated by each power plant is 400 MW. Because the transmission capacity of the network is bounded, the combined-cycle power plant situated in node 20 cannot operate at full-load in the case study analized.

VIII. Conclusions

This paper presents an optimization model to compute the maximum power generated by the combined-cycle power plants in a system. The maximum power generated by each combined-cycle power plant at each time period depends on the gas supply to the power plant at that time period.

The paper models the gas network including the effect of compressors to enlarge the gas transmission capacity. It formulates an optimization problem and establishes a solution procedure.

The problem to be solved is a non-convex optimization problem. However, once the flow directions through passive pipelines are known, the problem becomes convex.

The problem is solved in two consecutive phases. In the first phase, the flow directions are determined by solving a MILP problem. In the second phase a NLP problem is solved.

This optimization process will be inserted into a higher level model that analyzes the electrical system supply reliability taking into account natural gas system features.

IX. Appendix A

TABLE I
NODE DATA

Node	Town	$s_{i\,min}$ (Mm^3/day)	$s_{i\,max}$ (Mm^3/day)	$d_{i\,min}$ (Mm^3/day)	$d_{i\,max}$ (Mm^3/day)	$e_{i\,min}$ (Mm^3/day)	$e_{i\,max}$ (Mm^3/day)	$p_{i\,min}$ (bar)	$p_{i\,max}$ (bar)
1	Zeebrugge	8.87	17.39	0	0	0	0	0	77
2	Dudzele	0	12.6	0	0	0	0	0	77
3	Brugge	0	0	5.88	5.88	0	0	30	80
4	Zomergem	0	0	0	0	0	0	0	80
5	Loenhout	0	7.2	0	0	0	0	0	77
6	Antwerpen	0	0	6.05	6.05	0	0	30	80
7	Gent	0	0	7.88	7.88	0	0	30	80
8	Voeren	20.34	33.02	0	0	0	0	50	66.2
9	Berneau	0	0	0	0	0	0	0	66.2
10	Liége	0	0	9.55	9.55	0	0	30	66.2
11	Warnand	0	0	0	0	0	0	0	66.2
12	Namur	0	0	0	0	0	1.71	0	66.2
13	Anderlues	0	1.8	0	0	0	0	0	66.2
14	Peronnes	0	1.44	0	0	0	0	0	66.2
15	Mons	0	0	10.27	10.27	0	0	0	66.2
16	Blaregnies	0	0	23.42	23.42	0	0	50	66.2
17	Wanze	0	0	0	0	0	0	0	66.2
18	Sinsin	0	0	0	0	0	0	0	63.0
19	Arlon	0	0	0.33	0.33	0	0	0	66.2
20	Petange	0	0	0	0	0	1.71	25	66.2

TABLE II
PIPELINE DATA

Pipeline	From	To	C_{ij}	Type of arc
1	Zeebrugge	Dudzele	3.012	Passive
2	Zeebrugge	Dudzele	3.012	Passive
3	Dudzele	Brugge	2.459	Passive
4	Dudzele	Brugge	2.459	Passive
5	Brugge	Zomergem	1.181	Passive
6	Loenhout	Antwerpen	0.317	Passive
7	Antwerpen	Gent	0.386	Passive
8	Gent	Zomergem	0.476	Passive
9	Zomergem	Peronnes	0.812	Passive
10	Voeren	Berneau	2.694	Active
11	Voeren	Berneau	0.329	Active
12	Berneau	Liége	1.347	Passive
13	Berneau	Liége	0.164	Passive
14	Liége	Warnand	1.204	Passive
15	Liége	Warnand	0.147	Passive
16	Warnand	Namur	0.929	Passive
17	Namur	Anderlues	0.952	Passive
18	Anderlues	Peronnes	2.694	Passive
19	Peronnes	Mons	1.905	Passive
20	Mons	Blaregnies	1.205	Passive
21	Warnand	Wanze	0.227	Passive
22	Wanze	Sinsin	0.080	Active
23	Sinsin	Arlon	0.041	Passive
24	Arlon	Petange	0.167	Passive

X. APPENDIX B

TABLE III
NODE RESULTS

Node	Town	s_i (Mm^3/day)	d_i (Mm^3/day)	e_i (Mm^3/day)	p_i (bar)
1	Zeebrugge	17.39	0	0	64.12
2	Dudzele	12.60	0	0	64.05
3	Brugge	0	5.88	0	63.76
4	Zomergem	0	0	0	60.41
5	Loenhout	7.2	0	0	63.05
6	Antwerpen	0	6.05	0	58.80
7	Gent	0	7.88	0	58.73
8	Voeren	25.83	0	0	64.60
9	Berneau	0	0	0	64.03
10	Liége	0	9.55	0	61.71
11	Warnand	0	0	0	60.52
12	Namur	0	0	1.71	58.40
13	Anderlues	1.8	0	0	56.76
14	Peronnes	1.44	0	0	56.49
15	Mons	0	10.27	0	53.65
16	Blaregnies	0	23.42	0	50.00
17	Wanze	0	0	0	60.16
18	Sinsin	0	0	0	63.00
19	Arlon	0	0.33	0	51.50
20	Petange	0	0	1.16	51.02

TABLE IV
PIPPELINE RESULTS

Pipeline	From	To	f_{ij} (Mm^3/day)
1	Zeebrugge	Dudzele	8.695
2	Zeebrugge	Dudzele	8.695
3	Dudzele	Brugge	14.995
4	Dudzele	Brugge	14.995
5	Brugge	Zomergem	24.114
6	Loenhout	Antwerpen	7.2
7	Antwerpen	Gent	1.149
8	Gent	Zomergem	- 6.735
9	Zomergem	Peronnes	17.379
10	Voeren	Berneau	23.023
11	Voeren	Berneau	2.809
12	Berneau	Liége	23.023
13	Berneau	Liége	2.809
14	Liége	Warnand	14.514
15	Liége	Warnand	1.771
16	Warnand	Namur	14.787
17	Namur	Anderlues	13.077
18	Anderlues	Peronnes	14.877
19	Peronnes	Mons	33.696
20	Mons	Blaregnies	23.424
21	Warnand	Wanze	1.498
22	Wanze	Sinsin	1.498
23	Sinsin	Arlon	1.498
24	Arlon	Petange	1.165

XI. REFERENCES

[1] R. Billinton and R. N. Allan, *Reliability evaluation of power systems*, 2nd ed., Plenum Press, 1996, pp.10-11.

[2] W. Mansell Research Ltd., “Methodology for evaluating natural gas transmission system reliability levels and alternatives,” for the Canadian Petroleum Association, Sept. 1991.

[3] R. H. Kehlhofer, J. Warner, H. Nielsen, R. Bachmann, *Combined-Cycle Gas&Steam Turbine Power Plants*. (2nd Edition). Tulsa: PennWell Publishing Company, 1999, p. 17.

[4] B. E. Okogu, "Issues in global gatural gas: a primer and analysis", International Monetary Fund, Middle Eastern Department, May 2002.

[5] Bradley. S. P., Hax A. C., Magnanti T.L., *Applied Mathematical Programming*. Reading, Massachussets: Addison-Weley Publishing Corporation, 1977. pp 374-375.

[6] D. de Wolf, Y. Smeers, “The Gas Transmission Problem Solved by an Extension of the Simplex Algorithm,” *Management Science*, vol 46, N. 11, pp. 1454-1465, 2000.

[7] A. Brooke, D. Kendrick, A. Meeraus, R. Raman. *GAMS: A User´s Guide*. GAMS Development Corporation, 1998. [Online]. Available: http://www.gams.com

XII. BIOGRAPHIES

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