

Capturing the Evolution of Customer–Firm Relationships: How Customers Become More (or Less) Valuable Over Time[☆]

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Abstract

Few studies have examined the influence of marketing activities while accounting for customer dynamics over time. The authors contribute to this growing literature by extending the hurdle model to capture customer dynamics using a hidden Markov chain. We find our dynamic model performs better than static and latent class models. Our results suggest the customer base can be segmented into four segments: Deal-prone, Dependable, Active, and Event-driven. Each segment reacts differentially to marketing activities. Although catalogs influence both purchase incidence and the number of orders, this marketing activity has the largest impact on purchase incidence across all four segments. In contrast, retail promotions are more likely to influence the number of orders a customer will make for all of the segments except for the Deal-prone segment. For this segment, retail promotions have the strongest impact on purchase incidence.

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The Chief Marketing Officer at Sobeys, the second largest Canadian grocery retailer commented, “We try to talk about being meaningfully relevant [to consumers], which will drive more goodwill and more desire to shop in your store.” (Shaw 2010). Sobeys promotions focus on targeting the right customer at the right time. Despite industry coupon redemption rates around two percent, Sobeys enjoys a double-digit redemption rate due largely to their targeted promotions. How do retailers determine if a promotion is relevant to a consumer? What are the most effective segmentation approaches to guide the creation of targeted promotions? Does a customer remain in a segment over the duration of the relationship or is she transient? Fundamental

to these questions is an understanding of how marketing promotions influence a consumer or segment to become loyal, an important question to both retailers and academics (Grewal and Levy 2007). We begin to answer these questions by investigating the impact of marketing activities on buying behaviors while accounting for the evolution of customer–firm relationships.

Research on customer relationship management has evolved from developing individual-level customer profitability models (Fader, Hardie, and Lee 2005; Mulhern 1997) to models that aggregate these calculations to determine the overall value of the customer base (Johnson and Selnes 2004). These models can then be used as a proxy to determine the market value of the firm (Gupta, Lehmann, and Stuart 2004) or as a mechanism for evaluating marketing investments (Kumar, Shah, and Venkatesan 2006; Rust, Lemon, and Zeithaml 2004). More recently, academics have expanded these models to incorporate customer dynamics. Specifically, researchers have investigated customer dynamics as they relate to choice modeling (Netzer, Lattin, and Srinivasan 2008), behavioral changes over time (Rust and Verhoef 2005), retention rates (Fader and Hardie 2010), and customer portfolio management (Homburg, Steiner, and Totzek 2009). A consistent finding across studies has been that ignoring

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customer dynamics underestimates the value of a firm's customer base (Fader and Hardie 2010).

From a marketing perspective, however, few studies have examined the influence of marketing activities on customer dynamics (one exception is Montoya, Netzer, and Jedidi 2010). As such, our first objective of this article is to investigate customer dynamics in a retail context and second, to assess the impact of marketing activities on customer buying behaviors while accounting for the evolution of customer–firm relationships. Formally, our research questions are:

- (1) How do relationships between customers and retailers evolve over time?
- (2) How do marketing activities differentially influence purchasing behaviors across segments while accounting for customer dynamics over time?

To answer these research questions, we adapt, extend, and empirically validate a customer dynamics framework. First, we adapt a customer dynamics framework to a retail context using a hidden Markov model (HMM). This enables us to contribute to the growing literature on customer dynamics as we are first to apply a dynamic segmentation approach using a HMM to the retail context. This type of approach has been applied in the pharmaceutical context (Montoya, Netzer, and Jedidi 2010); however, we argue that the retail environment differs from the pharmaceutical environment for three reasons: (1) retail marketing activities vary from those employed by pharmaceutical sales representatives (e.g., detailing versus coupons), (2) retail marketing activities are aimed at influencing the end consumer to buy a product rather than aimed at the physician and his prescription behavior, and (3) retailers must account for both habitual behavior and longer inter-purchase times in their models. To address these concerns, we develop a dynamic hurdle model by incorporating a Markov chain. The unobservable latent states influence a customer's propensity-to-buy (i.e., buy/no buy decision) as well as the number of orders. Moreover, customer dynamics can be analyzed by estimating a customer's membership to a latent state at each observation period. In addition, we empirically validate our model using data from a North American retailer and compare our customer dynamics model to several other models to assess its performance. We find our dynamic model better fit the data when compared to other models, including a latent class model. For this retailer, we find the static models fail to capture the power of catalogs to trigger a purchase event and as such underestimate the influence of marketing activities. Furthermore, the retailer would make erroneous marketing decisions by assuming a static model when identifying segments and investigating the impact of marketing activities on customer buying behaviors.²

For this retailer, we identify four segments where the first state is the least valuable to the firm and the fourth state is the most valuable: (1) Deal-prone, (2) Dependable, (3) Active, and (4) Event-driven. From a customer dynamics perspective, we

find most customers migrate from a less valuable state to a more valuable state over time. Initially, the Active segment consists of five percent of the customer base, but doubles in size by the second year and continues to grow to twenty-two percent of the customer base by the end of the observation period. Although some customers make downward transitions (e.g., from a more valuable state to a less valuable state), for this retailer, only four percent of customers transition from the Dependable or Active segments to the Deal-prone state.

We also find differential impacts of marketing activities on buying behaviors for each of these states. Our results suggest that customers belonging to the Deal-prone state make few purchases. For these customers, retail promotions are more likely to encourage a purchase incidence whereas catalogs will influence both purchase incidence as well as the number of orders these customers will make. The Dependable segment responds more favorably to retail promotions relative to the other states. For this segment, retail promotions are more likely to influence the amount of orders these customers will make in any month. The active segment buys consistently over the nine years and is the only segment that responds well to both catalogs and retail promotions. Catalogs and retail promotions work equally well at influencing a purchase incidence. In addition, both of these activities have a significant impact on the number of orders these customers place, with retail promotions having a slightly stronger effect. Finally, we refer to the fourth segment as the event-driven segment since these customers tend to make purchases of large monetary value after receiving catalog promotions, which often happens during holidays. These customers will make a purchase with almost certainty when they receive a catalog; however, catalogs do not influence how much they will buy. This marketing activity tends to serve as a reminder to these customers that an event is approaching which immediately triggers a purchase activity.

The remainder of this paper is organized as follows. We begin with a review of the literature on customer management and the impact of marketing mix variables on customer dynamics. This section is followed by a description of our customer dynamics model and the data used to empirically validate our model. The third section presents the empirical results of our analysis. We conclude our article with a discussion section, limitations, and suggestions for future research.

Theoretical background

Customer management

Research in customer equity literature has moved in a direction in favor of viewing individual customers as part of a portfolio of customers (Johnson and Selnes 2004; Tarasi et al. 2011). Customer portfolio management offers a lens through which to segment customers based on different types of customer–firm relationships. Johnson and Selnes (2004) adopt the terminology of acquaintances, friends, and partners to characterize relationship types. When customers are considered at the aggregate level, they argue, customers progress from acquaintances, to friends, to partners. From the firm's

² We thank an anonymous reviewer for this suggestion.

perspective, building a competency in converting customers to closer relationship types (i.e., partners) and retaining these valuable customers has several benefits. First, relationships built on trust and commitment are less likely to dissolve, resulting in lower switching probabilities (Morgan and Hunt 1994). Lower switching probabilities are important because it is less costly to convert existing customers than it is to acquire new customers (Reichheld and Teal 1996). Second, customers with closer relationship types are more willing to pay a price premium and accept new products through cross-buying and up-selling initiatives (Reichheld and Teal 1996). Thus, in this research we assume different relationships exist among customers and retailers, and relationships evolve over time.

Frameworks linking marketing investments to a firm's profitability

Marketing researchers have linked marketing activities to a firm's profitability both conceptually and empirically. One of the earliest frameworks linking marketing activities to firm value is the service-profit chain (SPC). Conceptually, the SPC posits direct relationships among employee productivity, value of goods and services, customer satisfaction, loyalty, and profitability (Heskett et al. 2008). Kamakura et al. (2002) corroborate the links in the SPC with a model that includes both strategic and operational details. At the strategic level, the model links attitudes and behaviors to profitability, whereas the operational level translates components of the strategic model into measurable behaviors that lead to superior customer satisfaction ratings.

Rust, Lemon, and Zeithaml (2004) extend this framework by developing a model to link various marketing investments to a firm's profitability. Their return-on-marketing framework quantifies customers' purchase intentions through a Lifetime Value model thereby establishing a direct link from marketing investments to purchase intentions. Several conceptual models have also been proposed to link marketing investments to a firm's profitability. For example, Berger and his co-authors (Berger et al. 2002) propose a customer asset management framework that provides a roadmap for utilizing available transactional and point of contact data so as to maximize marketing productivity.

Similarly, Bolton, Lemon, and Verhoef (2004) propose a customer asset management of services framework which posits that customer perceptions moderate the relationship between marketing activities and customer behaviors. They argue that marketing activities influence customers' perceptions of a firm's service offerings. These perceptions, in turn, influence customers' decisions to buy more, to buy more frequently, and whether or not to continue the relationship with the service provider. These behaviors directly influence the costs and revenues associated with serving each customer. By reducing the costs necessary to serve a customer and encouraging greater depth and breadth of purchases, firms will increase the value of their customer base.

The central premise of these conceptual frameworks is that firms which offer value to customers will be rewarded by favorable customer behaviors, thus impacting a firm's overall profitability. While the earlier frameworks provide an overview

of the impact of marketing activities on a firm's overall profitability, later frameworks and empirical investigations offer guidance for leveraging internal resources in order to link marketing activities directly to individual-level behaviors. Moreover, the abundance of conceptual frameworks suggests that there are many opportunities for future research to test links between marketing activities and customer behaviors. Thus, in this paper, we begin to address this need by developing and empirically validating a dynamic segmentation model that incorporates segment-level responses to marketing activities, and provides insight into the value of each segment to the retailer.

Customer dynamics and behavior

Customer portfolio management moves existing relationship marketing models into the realm of customer dynamics by incorporating conversion and switching probabilities. Conversion probabilities refer to the progression of customers from one type of relationship to another whereas switching probabilities refer to customers leaving the firm for a competitor's product. Johnson and Selnes' (2004) findings suggest that even marginal increases in a firm's conversion probabilities and corresponding reduction in switching probabilities will result in a significant increase in the value of a firm's customer portfolio. A limitation of their research is that they make assumptions about conversion and switching probabilities rather than estimating these behaviors using historical data. We argue that retailers require a rigorous approach to estimate conversion probabilities among low, medium, and high value customers. Specifically, there is a need to capture how relationships evolve between retailers and customers and how segments, using relationship-type as a basis, react to marketing activities so as to contribute to the growing literature linking observable metrics to financial performance (Gupta and Zeithaml 2006). We begin to address this gap in the literature by applying a customer dynamics framework in the retail context. Table 1 provides a summary of customer dynamics research.

Modeling customer dynamics and the marketing mix

Recent research in customer relationship management emphasizes the importance of simultaneously capturing the heterogeneity of customers and the heterogeneity in responses to the marketing mix (Rust and Verhoef 2005). A variety of dynamic models have been applied in marketing research to estimate interpurchase times (Allenby, Leone, and Lichung 1999), change in gross profit so as to maximize customer profitability (Rust and Verhoef 2005), consumer choice models (Netzer, Lattin, and Srinivasan 2008) and, more recently, to model a physician's prescription behavior (Montoya, Netzer, and Jedidi 2010). Subsequently, we elaborate on the most relevant dynamic models for our research.

Netzer, Lattin, and Srinivasan (2008) extend discrete choice models to account for customer dynamics. Using alumni data, they argue future donations are driven by latent relationship states. Latent relationship states are defined by current period donating behavior and are modeled using a HMM. The authors

Table 1
Summary of customer dynamics research.

	Lewis (2005)	Rust and Verhoef (2005)	Netzer, Lattin, and Srinivasan (2008)	Homburg, Steiner, and Totzek (2009)	Khan, Lewis, and Singh(2009)	Montoya, Netzer, and Jedidi (2010)	This research
Context	Consumer products: Internet Retailer	Consumer product: Insurance	Alumni Relations with University	Banking, telecommunications, pharmaceutical, and chemicals	Online retailer: grocery and drugstore	Pharmaceutical	Consumer products: Retailer
Analysis	Dynamic programming and simulation	RFM, Finite Mixture Model, and Hierarchical Bayesian Model	Discrete choice model with hidden Markov chain	Observed customer switching between segments	Discrete choice model with time varying covariates	Hidden Markov model; optimization	Dynamic hurdle model with hidden Markov chain
Data	Individual transaction histories	Individual transaction histories	Individual donation histories	Individual transaction histories	Individual transaction histories	Physician's prescription behavior	Individual transaction histories
Length of observation period	Thirteen months	Two years	25 years	Ranged from four to five periods	Fourteen months	24 months	Nine years
Dependent variable	Individual sequential choice	Change in gross profit	Alumni donations	Present customer value	Purchase incidence and expenditures	Prescribing new drug	Purchase incidence and number of orders
Price promotions	Yes	No	No	No	Yes	No	No
Direct marketing campaign	No	No	No	No	No	No	Yes
Coupons	No	No	No	No	Yes	No	Yes
Reward program	Yes	Yes	No	No	Yes	No	No

find significant improvement in alumni responses when marketing campaigns are targeted at individuals based on their latent relationship state. A limitation of their research is the restrictive assumptions regarding donors' migration among states. Specifically, they assume a one-step transition from one state to an adjacent state. In addition, their model does not provide insight into the value of relationship states, an important marketing metric (Bolton, Lemon, and Verhoef 2004; Kumar, Shah, and Venkatesan 2006).

Similarly, Homburg, Steiner, and Totzek (2009) investigate customer dynamics using a multi-step modeling procedure that observes customer switching between segments. They argue for strategic use of offensive and defensive management of customer relationships. Offensive management refers to developing existing relationships and acquiring new customers; whereas, defensive management refers to reducing the likelihood of customers migrating to lower valued segments as well as maintaining stable relationships (no migration). Using a simulation study, the authors found that marketing tactics aimed at promoting deeper relationships are more effective for customers belonging to the least profitable segment. Customers belonging to more profitable segments, however, should be targeted with marketing programs that encourage these customers to maintain existing relationships.

More recently, Montoya, Netzer, and Jedidi (2010) apply a HMM to estimate a physician's likelihood to prescribe a new drug. In their study, the latent states represent physicians' prescription behaviors. They find detailing is better at acquiring new physicians while sampling works best as a retention tool. Montoya et al.'s model works well in a context where there is a constant stream of behavior (e.g., prescribing). However, the retail industry, especially in the apparel category, there are many periods where no purchases occur, not because of a change in latent state, but rather due to different or varying inter-purchase cycles. Therefore, in this research, we develop a dynamic model to account for the unique buying characteristics present in a retail context.

In Montoya et al.'s customer dynamics model (2010), the authors assume the latent states represent physician's prescription behavior whereby their model allows the transition from one state to another to be directly influenced by pharmaceutical samples and detailing. We consider an alternative approach to modeling customer dynamics based on the concept of habit persistence (Haaijer and Wedel 2001; Roy, Chintagunta, and Haldar 1996). Habit persistence is the "effect of prior propensities to select a brand on current selection probabilities." (cf. Heckman 1981 cited in Roy, Chintagunta, and Haldar 1996, p. 281). Early dynamic brand choice models emphasized the importance of capturing habit persistence and heterogeneity in order to improve the accuracy of parameter estimates. As such, we investigate the habitual aspects of consumer behavior by assuming the latent states represent a customer's propensity-to-buy. Once we account for a customer's prior propensities using a hidden Markov chain specification, we can better understand how responses to marketing activities vary by a customer's propensity-to-buy type (e.g., state). Our customer dynamics framework is flexible in that the Markov model enables us to

estimate how many "propensity-to-buy" states exist in the customer base, and the likelihood that an individual will transition from one state to another. Moreover, a customer's transition from one state to another provides insight into the evolution of the relationship between the customer and the retailer.

To recapitulate, the objectives of our research are (1) to empirically investigate customer dynamics and the different types of relational patterns between customers and retailers; and (2) to assess the impact of marketing mix variables on buying behaviors while accounting for customer dynamics. We accomplish these objectives by extending the hurdle model to incorporate a hidden Markov chain thereby capturing customer dynamics over time. Failure to adequately capture customer dynamics results in marketing analytics that might under or over estimate the influence of marketing activities on building and maintaining customer-firm relationships.

Customer dynamics hurdle model

We develop a customer dynamics model based on the evolution of customer-firm relationships, and assess the impact of marketing variables on a customer's propensity to buy from a retailer. To begin this section, we describe limitations with existing Poisson models when modeling longitudinal data. We then present our model, which extends the Poisson hurdle model by including a latent Markov chain. Finally, we discuss the data used to empirically validate our customer dynamics hurdle model.

The model described in this section deals with the analysis of a longitudinal dataset of customer buying behavior. A transaction database captures a rich set of customer information over several time periods. Such data structure shows a number of characteristics that are referred to as the dependence of a target variable on covariates, serial dependence, and heterogeneity among customers. An appealing approach to account for longitudinal data features is to use HMMs (Netzer, Lattin, and Srinivasan 2008). The application of HMMs is justified by their versatility and mathematical tractability; availability of all moments; the likelihood computation is linear in the number of observations; the marginal distributions are easy to determine and missing observations can be handled with minor effort; the conditional distributions are available; outliers identification is possible; and forecast distributions can be calculated.

In a basic HMM for longitudinal data, the existence of two processes is assumed: an unobservable finite-state first-order Markov chain, S_{it} , $i = 1, \dots, n$, $t = 1, \dots, T$, with state space $S = \{1, \dots, m\}$ and an observed process, Y_{it} , where Y_{it} denotes the response variable for individual i at time t . The distribution of Y_{it} depends only on S_{it} , specifically the Y_{it} are conditionally independent given the S_{it} . However, without this conditioning the Y_{it} are not independent in time. Thus, the unknown parameters in a HMM involve both the parameters of the Markov chain and the state-dependent distributions of the random variables Y_{it} . In particular, the parameters of the Markov chain are the transition probabilities $Q = \{q_{ijk}\}$ where $q_{ijk} = \Pr(S_{it} = k | S_{i,t-1} = j)$, $j, k \in S$ is the probability that individual i visit state k at time t given that at time $t - 1$ she was in state j and the initial probabilities $\delta_{ij} = \Pr(S_{i1} = j)$, that is, the probability of being in state j

a non-informative prior have been carried out. Unfortunately, the Hessian matrix, which is obtained as by-product from the quasi-Newton algorithm, does not provide numerically stable results for long time series. Therefore, standard errors of the parameter estimated have to be computed by a parametric bootstrap approach (Bulla and Berzel 2008; Visser, Raijmakers, and Molenaar 2002).

A by-product of the estimation procedure are the state classification or smoothing probabilities, which can be interpreted as the a posteriori probability of a customer i belonging to a state j at time t given all observations y_{it} , $t = 1, \dots, T$. By means of these probabilities, the hidden state sequence can be estimated for every customer. A local decoding approach has been adopted, that is, for each customer we determine the most probable state at each time, given the observations. Of course different approaches, like global decoding, can be pursued.

Data

The data for this research are from a major North American retailer that sells both apparel and household goods. For confidentiality reasons, the retailer has requested to remain anonymous. To empirically validate our dynamic hurdle model, data from a cohort of 9,487 customers were collected over a nine-year period, beginning in January 2001. Each customer in this cohort made her first purchase from the retailer in the first year of observation (i.e., January 2001 to December 2001). The data include daily transaction records for customers aggregated monthly resulting in a dataset of 1,024,596 observations. In this dataset, we find 61.90 percent of the observations are zeros. Moreover, our dependent variable (i.e., the number of orders) exhibits substantial overdispersion: We test for equidispersion and for the appropriateness of the simple Poisson model (i.e., $H_0: Y \sim \text{Poisson}(\lambda)$ against $H_1: \text{var}(Y) > \lambda$). We employed two different statistics, the first suggested by Böhning (1994) and the second one derived by (Baksh, Böhning, and Lerdsuwansri 2011). Both tests reject the hypothesis of equidispersion in favor of overdispersion due to zero-inflation (p -value = 0). This casts doubts on the unit variance-to-mean ratio implied by the Poisson model. Thus, we investigate alternative approaches for modeling longitudinal data with zero-inflation as described above.

As we are interested in the evolution of customer–firm relationships, we took a random sample of customers from the retailer’s database who made at least three purchases in at least one year over the nine years. That is, we developed our customer dynamics model using active customers, which is consistent with modeling dynamics in the customer dynamics literature (Ansari, Mela, and Neslin 2008).

We aggregated the data to a monthly level to capture the effects of marketing variables on buying behaviors. Our sample consists of mostly females (81 percent) and 72 percent of them are married. Over the nine-year period, these customers made an average number of orders of 67.06 with a mean total order value of \$6,000.11. For confidentiality reasons, the data are multiplied by a factor so as to disguise actual values. The marketing promotions employed by the retailer included mailing catalogs to customers and retail promotions (e.g., coupons). On average,

the retailer mailed 487 catalogs to each customer and distributed 6.18 retail promotions over the nine years. Fig. 1 shows average values on a monthly basis of the number of purchases and the number of catalogs and retail promotions received, respectively.

Results

Model selection

We compared our dynamic hurdle model to several other models allowing for zero-inflation to assess the performance of our model. First, we estimated a simple hurdle model without marketing covariates to establish a base line comparison. The second model is a hurdle model including the effect of marketing covariates, which can also be interpreted as a dynamic Hurdle model with one state. The third model is a latent class model where customers do not transition among the states. The fourth model is a hurdle model with a hidden Markov chain, thereby capturing heterogeneity and dynamics in the customer base (see Table 2 for details). When estimating different models, the appropriate model and, in particular, the number of states must be selected. We estimated different models with two to five states. Taking the BIC as formal model selection criterion and good parameter interpretability as an additional condition, the four-state dynamic hurdle model was finally selected.

HMM parameter estimation results

In this section, we describe the hidden Markov chain parameters. The initial probabilities indicate that the large majority of customers (75 percent) belong to the second state at the beginning of the observation period, followed by the first state (18 percent), third state (6 percent) and fourth state (1 percent). The transition probability matrix reveals a high persistence of the first three states, while the fourth state is transient. This is not unusual from a statistical point of view as the first three diagonal entries of the transition probability matrix are close to one. In other words, customers mainly do not transition backward and forward from one state to another; rather, when a customer migrates to a new state, the change is mostly persistent. For example, if a customer transitions from state 2 to state 3, the customer will very probably remain in state 3 for the duration of the relationship with the retailer. However, it also suggests that if a customer transitions to a lesser value state (e.g., state 2 to state 1), the customer will most likely remain in state 1 for the remainder of the relationship. Persistent states have also been found in the environmental literature (Bulla et al. 2012) and in the statistical literature (Bartolucci and Farcomeni 2009; Bulla 2011). Interestingly, customers from state 1 do not transit into the fourth state. See Table 3 for the parameter estimates of the hidden Markov chain.

State profiles

To profile each of the four states, we refer to the intercepts of the model and the buying characteristics of customers conditional on being active in a state. In the first step of the model

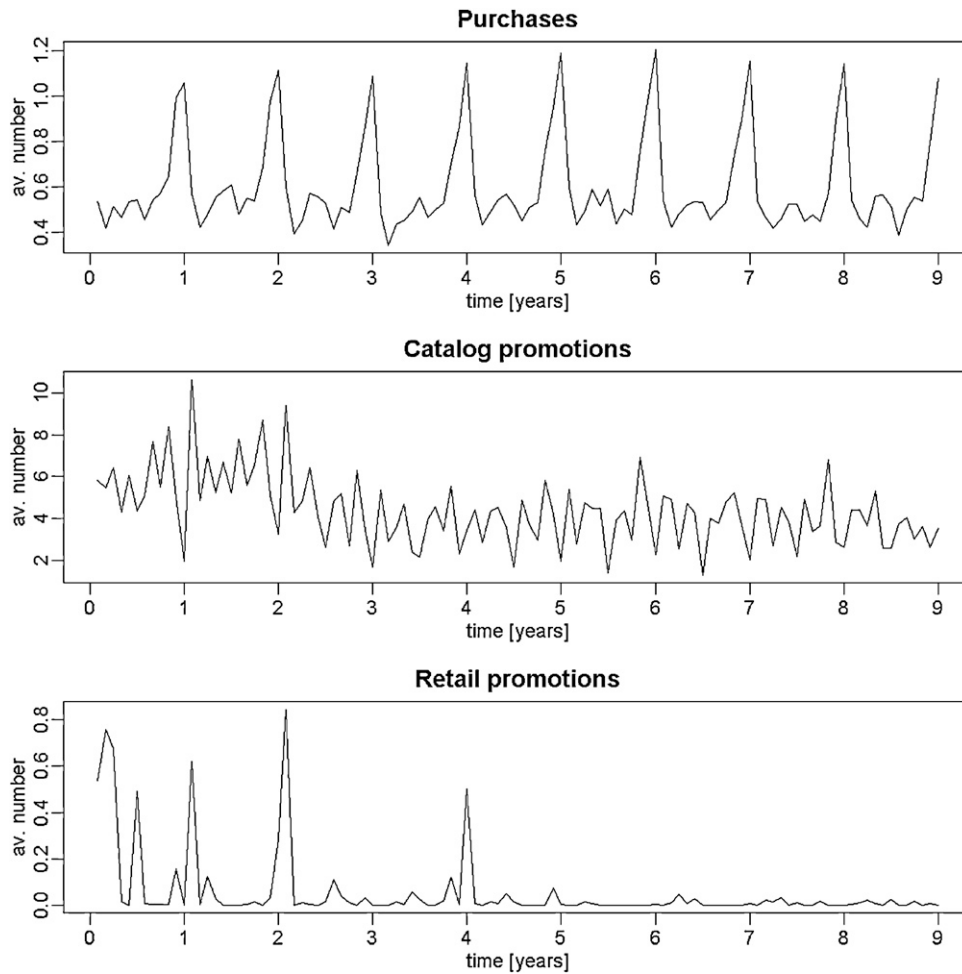


Fig. 1. Time trends of purchases and marketing activities.

($y = 0$), the first state exhibits the highest intercept and thus the lowest probability of a purchase activity, followed by the second, the third, and the fourth states. The intercept of the second part of the model ($y > 0$), modeling the number of orders conditional on making a purchase, shows that consumers belonging to the fourth state have the highest purchasing amounts, followed by states three, two, and one.

We empirically investigate the estimated state sequences by assigning each customer to the most likely state at each time via a local decoding procedure. In other words, as a by-product of the estimation procedure, we compute the posterior probabilities of belonging to a state for each time point. Then, we assign a customer to the state with the highest posterior probability for each month. Thus, according to this procedure, we obtain a

Table 2
Log-likelihood and Bayesian information criteria (BIC) for models.

	Simple hurdle	Latent class model with varying number of states			
		2 states	3 states	4 states	5 states
Log-likelihood	-1,099,966	-1,050,726.09	-1,034,468.11	-1,030,242.46	-1,029,786.67
Number of parameters	2	25	38	51	64
BIC	2,199,960	2,101,798	2,069,462	2,061,191	2,060,459
	Hurdle with covariates	Dynamic hurdle model with varying number of states			
		2 states	3 states	4 states	5 states
Log-likelihood	-1,083,364	-1,044,987.03	-1,028,065.19	-1,022,433.11	-1,022,327.54
Number of parameters	12	27	44	63	84
BIC	2,166,894	2,090,348	2,056,739	2,045,738	2,045,818

Table 3
Hidden Markov model parameters.

	Transition matrix				Initial probabilities	
	State 1	State 2	State 3	State 4		
State 1	<i>1.0000</i> (0.0001)	0.0000 (0.0001)	0.0000 (0.0000)	0.0000 (0.0000)	State 1	<i>0.1810</i> (0.0048)
State 2	0.0009 (0.0001)	<i>0.9927</i> (0.0002)	<i>0.0037</i> (0.0004)	<i>0.0027</i> (0.0004)	State 2	<i>0.7486</i> (0.0057)
State 3	0.0000 (0.0000)	<i>0.0025</i> (0.0007)	<i>0.9403</i> (0.0043)	<i>0.0572</i> (0.0047)	State 3	<i>0.0598</i> (0.0041)
State 4	0.0002 (0.0001)	<i>0.0530</i> (0.0084)	<i>0.5439</i> (0.0146)	<i>0.4029</i> (0.0188)	State 4	<i>0.0107</i> (0.0023)

^aEntries in bold and italic script are significantly different to zero with $p = .05$. Standard errors in parenthesis below each estimate were determined by non-parametric bootstrap.

^bStandard errors are presented below coefficients in parentheses.

sequence of most probable states for each customer. Once this sequence of states has been determined, a state can be profiled by calculating the average buying characteristics for each state conditional on a customer being active in the state.

We find customers classified to the first state show the least amount of buying activity. More specifically, customers in the first state buy only in 12.5 percent of the months with a conditional average number of orders of 1.2; whereas, customers in state 2 show corresponding values of 39.0 percent and 1.28, respectively and average monthly values of \$132.47 and \$155.67 for states 1 and 2, respectively. Customers classified to the third state buy rather frequently and have large number of purchases over the observation period. For these customers, a buying activity is recorded for 60.1 percent of the months. Additionally, conditional on a purchase, the average number of orders for this segment is 1.87 with an average monthly value of \$169.84. Customers belonging to state 4 tend to make 5.36 orders on average and have an average monthly value of \$487.34. Therefore, state 1 is the least valuable to the retailer followed by state 2; whereas, state 4 is the most valuable to the retailer followed by state 3. We subsequently interpret the covariates of the dynamic hurdle model to further derive meaning to each state.

Interpretation of covariates

In this article, our dynamic hurdle model allows the effects of covariates to change with the state membership. That is, our model is a random coefficient model rather than only a random intercept model (see Alfó and Maruotti 2010). Consequently, the estimated parameters must be interpreted. We begin with an examination of the covariates explaining the probability of (not) making a purchase ($y = 0$). See Table 4 for a description of the independent variables and Tables 5 and 6 for the results of both parameter vectors. We find that catalogs increase the likelihood of purchase across all four states, having the largest impact on purchase incidence for state 4 customers (state 1: $\phi_1 = -0.1606, p < .05$; state 2: $\phi_2 = -0.0580, p < .05$; state 3: $\phi_3 = -0.0834, p < .05$; state 4: $\phi_4 = -6.9864, p < .05$). One may note that the effect of catalogs in state 4 is so strong that it results in a propensity to buy very close to 100 percent, independent of all other covariates. Retail promotions increase the likelihood of

making a purchase for both the state 1 and state 3 customers, but are not significant for the state 2 customers. Furthermore, we observe a negative correlation between retail promotions and the propensity of buying for customers clustered in state 4 (state 1: $\phi_1 = -0.2448, p < .05$; state 2: $\phi_2 = 0.0204$; state 3: $\phi_3 = -0.1435, p < .05$; state 4: $\phi_4 = 0.2889, p < .05$). However, this may simply result from multicollinearity effects of catalog and retail promotions in this state. Customers who are married have significantly higher likelihood of purchase in three of the four states (state 1: $\phi_1 = -0.0349$; state 2: $\phi_2 = -0.0371, p < .05$; state 3: $\phi_3 = -0.0959, p < .05$; state 4: $\phi_4 = -0.7816, p < .05$). A similar result is obtained by looking at gender effects. For states 2–4, females are more likely to make a purchase, while gender is not significant for customers belonging to state 1 (state 1: $\phi_1 = 0.0038$; state 2: $\phi_2 = 0.0672, p < .05$; state 3: $\phi_3 = 0.1930, p < .05$; state 4: $\phi_4 = 0.3599, p < .05$). Finally, a seasonal effect is captured by our model specification. The dummy variable introduced to capture the effects of Christmas holidays is significant and strongly affects the probability of purchasing for all states (state 1: $\phi_1 = -1.6240, p < .05$; state 2: $\phi_2 = -0.9519, p < .05$; state 3: $\phi_3 = -0.9926, p < .05$; state 4: $\phi_4 = -1.0263, p < .05$).

The second component of the dynamic hurdle model is the number of orders ($y > 0$) (see Table 6 for the results). As expected, marketing covariates have a positive impact on the number of orders. State 2 customers respond slightly more favorably to retail promotions than customers belonging to states 3 and 4, but this effect is not significant for state 1 customers (state 1: $\beta_1 = 0.1241$; state 2: $\beta_2 = 0.1178, p < .05$; state 3: $\beta_3 = 0.0960, p < .05$; state 4: $\beta_4 = 0.1069, p < .05$). Similarly, catalogs have a positive impact on the number of orders for state 1, 2, and 3 customers, even with different magnitude, but have no significant effect for state 4 customers (state 1: $\beta_1 = 0.0437, p < .05$; state 2: $\beta_2 = 0.0041, p < .05$; state 3: $\beta_3 = 0.0299, p < .05$; state 4: $\beta_4 = 0.0041$). We find marital status to influence the number of orders in a similar way as described for the probability of purchasing (state 1: $\beta_1 = 0.0387$; state 2: $\beta_2 = 0.0533, p < .05$; state 3: $\beta_3 = 0.0554, p < .05$; state 4: $\beta_4 = 0.0769, p < .05$). Additionally, females are more likely to place a higher number of orders than males across states 3 and 4, but gender is not significant for states 1 and 2 (state 1: $\beta_1 = -0.0142$; state 2: $\beta_2 = -0.0275$; state 3: $\beta_3 = -0.0756, p < .05$; state 4: $\beta_4 = -0.0781, p < .05$). As

Table 4
Variables and descriptives.

Variables	Operationalization	Percentage/mean	Standard deviation
Number of orders	The total number of orders placed during the observation period.	67.06	40.95
Total order value	The total order value of purchases made during the observation period.	\$6,001.11	\$3,782.55
Retail promotions	Total number of retail promotions.	6.18	4.06
Number of catalogs	Total number of catalogs mailed to each customer.	486.98	126.31
Marital status	1 if married, 0 if not married.	71.63 percent married; 28.37 percent not married;	–
Gender	1 if female, 0 otherwise.	81.11 percent female 18.89 percent male	–
Holidays	1 in November and December, 0 otherwise.	–	–

Table 5
Binary model ($y = 0$).

Parameter	State 1 ϕ_1	State 2 ϕ_2	State 3 ϕ_3	State 4 ϕ_4
Intercept	2.5267 (0.0168)	0.7642 (0.0140)	0.1486 (0.0393)	–2.4244 (0.0019)
Married	–0.0349 (0.0192)	–0.0371 (0.0145)	–0.0959 (0.0354)	–0.7816 (0.0011)
Gender	0.0038 (0.0173)	0.0672 (0.0138)	0.1930 (0.0429)	0.3599 (0.0019)
Retail promotions	–0.2448 (0.0433)	0.0204 (0.0107)	–0.1435 (0.0292)	0.2889 (0.0007)
Catalog promotions	–0.1606 (0.0044)	–0.0580 (0.0016)	–0.0834 (0.0026)	–6.9864 (0.0019)
Holidays	–1.6240 (0.0392)	–0.9519 (0.0195)	–0.9926 (0.0770)	–1.0263 (0.0001)

^aEntries in bold and italic script are significantly different to zero with $p = .05$. Standard errors in parenthesis below each estimate were determined by non-parametric bootstrap.

^bStandard errors are presented below coefficients in parentheses.

^cWhen interpreting the sign of a coefficient, recall that the binary part of the model estimates the probability of no purchase ($y = 0$).

expected, during the Christmas holidays the amount of purchases tends to increase significantly for all states (state 1: $\beta_1 = 0.4949$, $p < .05$; state 2: $\beta_2 = 0.4932$; state 3: $\beta_3 = 0.4947$, $p < .05$; state 4: $\beta_4 = 0.3061$, $p < .05$).

Following the analysis of the model parameters and the inferred state sequences, the four states can be interpreted as Deal-prone (state 1), Dependable (state 2), Active (state 3) and Event-driven (state 4).

Customer dynamics

Empirical inquiry into the estimated state sequences also provides insight into the evolution of the relationships between the customers and retailer. Fig. 2 displays the proportion of customers classified in states 1, 2, 3, and 4, respectively, at the aggregate level. Table 7 displays the inferred transitions among the states while Table 8

Table 6
Truncated Poisson model ($y > 0$).

Parameter	State 1 β_1	State 2 β_2	State 3 β_3	State 4 β_4
Intercept	–1.1715 (0.0462)	–0.4152 (0.0190)	–0.1007 (0.0355)	1.1997 (0.0422)
Married	0.0387 (0.0384)	0.0533 (0.0179)	0.0554 (0.0243)	0.0769 (0.0343)
Gender	–0.0142 (0.0428)	–0.0275 (0.0164)	–0.0756 (0.0286)	–0.0781 (0.0375)
Retail promotions	0.1241 (0.0964)	0.1178 (0.0134)	0.0960 (0.0195)	0.1069 (0.0246)
Catalog promotions	0.0437 (0.0090)	0.0041 (0.0020)	0.0299 (0.0022)	0.0041 (0.0030)
Holidays	0.4949 (0.0597)	0.4932 (0.0145)	0.4947 (0.0371)	0.3061 (0.0457)

^aEntries in bold and italic script are significantly different to zero with $p = .05$. Standard errors in parenthesis below each estimate were determined by non-parametric bootstrap.

^bStandard errors are presented below coefficients in parentheses.

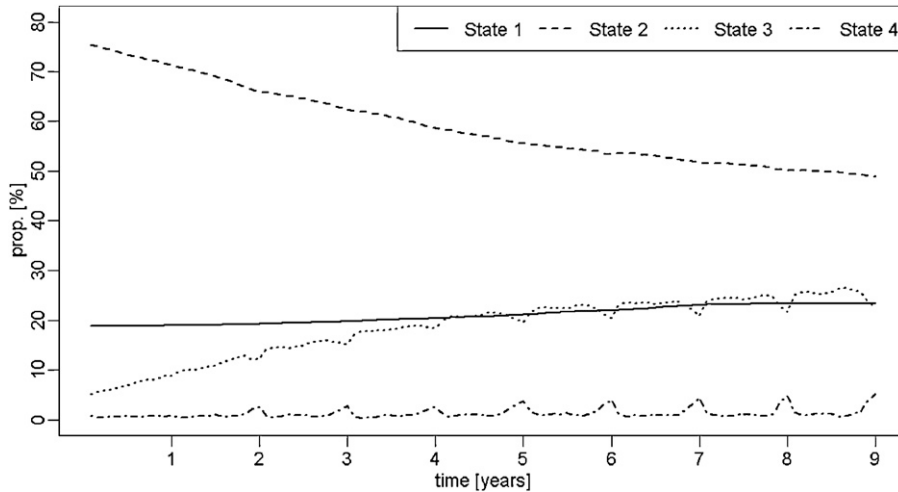


Fig. 2. Proportion of customers in states 1–3.

Table 7
Inferred transitions.

States	States			
	1	2	3	4
1	213,506	1	0	0
2	427	597,258	2,516	744
3	0	737	180,707	6,824
4	1	427	6,705	5,256

summarizes the total number of inferred transitions in our sample.

We find that the estimated proportion of customers in state 1 increases relatively slowly over the observation period, from 18.8 percent to 23.3 percent. The estimated transitions (via a maximum a posteriori analysis of the posterior probabilities) underline that once customers enter state 1, they basically remain in state 1. As for state 2, the estimated proportion of customers in this state diminishes significantly over the observation period. The initial state probabilities attribute 75.5 percent of our sample to this state; however, state membership decreases to 48.9 percent by year nine. According to Fig. 2, the trajectory representing membership to state 2 is downward sloping and has

the largest number of customers migrating to another state of all four states. In contrast, state 3 gains the largest number of customers over time. Initially, this state has a smaller proportion of customers, namely 5.0 percent, and grows to 22.5 percent of the customer base by the ninth year. Finally, we find state 4’s trajectory is highly seasonal with peaks mostly during the holiday seasons. This state begins with an annual average of 0.63 percent of the customers in the first year and almost triples to an annual average of 1.67 percent in the last year. Furthermore, while this state is visited by only 0.89 percent of the customers in the first December observed, the respective figure in December rises almost by a factor of six to 5.25 percent in the final year.

This empirical inquiry further enables the retrieval of the different behavioral patterns. While 59.6 percent of the customers remain in one state for the entire observation period, 9.7 percent make one transition, 11.1 percent make two to three transitions, and 4.8 percent of the customers reach ten and more transitions (see Table 8). Furthermore, we find much of the dynamics occurs among the more valuable states. That is, we find the largest number of inferred transitions occurs between states 3 and 4, followed by transitions between states 2 and 3. Our empirical findings suggest that when customers make a transition, they are more likely to transition to more valuable states.

Table 8
Customer dynamics.

Number of inferred transitions	Proportion of customers (percent)	Cumulative proportion of customers (percent)
0	59.64	59.64
1	9.73	69.37
2	4.34	73.71
3	6.72	80.44
4	3.49	83.93
5	3.91	87.84
6	2.37	90.21
7	2.25	92.45
8	1.51	93.96
9	1.28	95.24
10+	4.76	100.00

Discussion

Academics and practitioners have long assumed that customer–firm relationships strengthen over time (Reichheld 1996; Rust, Lemon, and Zeithaml 2004) but little is known about why or how these relationships evolve over time. Does every relationship strengthen at the same rate over time? Are there different evolutionary patterns for the most profitable customers? More importantly from a retailer’s perspective, how can marketers tailor their activities to accommodate for these differences? Our customer dynamics framework begins to address these questions.

Our empirical analysis indicates that, at least for this retailer, there is evidence of customer dynamics. Specifically, the Active

segment (state 3) gains the largest number of customers over time. Initially, this state has an estimated smaller proportion of customers, namely 5.0 percent, and grows to 22.5 percent of the customer base by the tenth year. This behavior is most favorable for the retailer as it suggests that the retailer is successfully converting customers from “friends” to “partners” (Johnson and Selnes 2004). Moreover, state 3 customers are valuable to the retailer and therefore, with increasing numbers of “Active” customers, the value of the customer portfolio will increase. More importantly, once customers migrate to state 3, the likelihood of them transitioning to a less profitable state is very low. From a marketing perspective, these customers react well to receiving catalogs as it increases the likelihood of them making a purchase. These customers are also positively influenced by retail promotions, and they are more likely to buy more when they receive a retail promotion.

As for state 2, the Dependable segment, the proportion of customers in this state diminishes significantly over the observation period. According to Fig. 2, the trajectory representing state membership is downward sloping and has the largest number of customers migrating to another state of all four states. Fortunately for the retailer, a relevant number of these transitions are to states 3 and 4. State 4 is transient and attracts a small amount of customers overall, most coming exclusively from states 2 and 3. Customers visiting state 4 represent the most valuable to the retailer because they buy more frequently and the monetary value of their orders is the largest of all the segments. In addition, these customers react to certain catalogs with almost 100 percent certainty. Catalogs seem to serve as a reminder for this segment that an event (e.g., holidays) is soon arriving and are thus an effective tool to encourage customers to make a purchase.

However, some of the customers are classified to or migrate into state 1, the least valuable state. We find very little dynamics for customers belonging to state 1, the Deal-prone segment. These customers do not transition to a more valuable state. This finding suggests that despite the preference of these customers to not strengthen their relationship with the retailer (i.e., transition to a more valuable state), they demonstrate their commitment to the relationship by retaining the retailer’s products in their consideration set as evidenced by the total number of purchases over the observation period. However, it is possible that some of the customers in state 1 have also decided to terminate their relationship with the retailer. This phenomenon is not directly captured in our model as we make the assumption that the customers are always-a-share (Dwyer 1997; Rust, Kumar, and Venkatesan 2011). An always-a-share customer is defined as a customer that has several companies in her consideration set and thus allocates a share of purchases to each of them over time. Unlike contractual relationships where attrition can be captured, it is more difficult to capture the moment where a customer decides to terminate her relationship with the retailer in the retail context. More research is needed to establish frameworks that account for customer attrition or incorporate lost-for-good customers into customer dynamics research. It might be beneficial for the retailer to investigate whether it is possible to influence favorable migration patterns for these

customers and decrease the likelihood of a downward migration pattern.

Finally, state 2 customers can be characterized as the Dependable segment as these customers make steady purchases from the retailer throughout the observation period. They have a higher propensity to buy than the Deal-prone customers, but less than the Active customers. These customers’ propensity to buy can be slightly increased by catalog promotions; however, a retail promotion does not have a significant impact on triggering a purchase occasion. Rather, for these customers, retail promotions increase the number of orders they will make. Thus, the retailer may continuously invest in marketing activities to increase the overall value of these customers to the retailer.

Theoretical and methodological contributions

This research contributes to our understanding of customer management as follows. First, we provide insight into the evolution of customer–firm relationships. Relationship marketing theory suggests that relationships evolve monotonically over time (Reichheld and Teal 1996); however, our empirical inquiry is more consistent with Fournier’s (1998) brand-relationship theory where different relationship patterns emerge among customers and retailers. We find 40.4 percent of customers make at least one transition among the four states during the observation period in which almost two thirds of the transitions are upward. In other words, for this retailer, customers who continue their relationships with the firm at some level are more likely to transition from a less valuable state to a more valuable state (e.g., states 2 to state 3). Although this finding is consistent with Reichheld and Teal’s research, we also find that several customers have unique trajectories that share more similarities with the various patterns in brand relationship theory (Fournier 1998). For example, at the individual-level, we find that one customer has seven transitions over the duration of the relationship suggesting an “Approach-Avoidance” relationship pattern while another customer has four transitions, which is more similar to the “Cyclical Resurgence” pattern described by Fournier (1998, p. 364).

Second, we contribute to the growing customer dynamics literature as we are the first to develop a dynamic hurdle model to account for unique buying characteristics in a retail context, such as habitual behavior and longer inter-purchase times. Indeed, we find our dynamic segmentation model outperforms other models that do not capture these dynamics. As a result, our estimates of customer responses to marketing activities are more precise and therefore better inform management when making resource allocation decisions.

Our third contribution to the customer management literature is the finding that marketing resources might be more effective when targeted at the “middle tier” rather than the “top tier” of a customer pyramid. Zeithaml, Rust, and Lemon (2001) developed a segmentation approach to classify customers into various tiers based on their relative profitability to the firm. The premise behind their segmentation approach is that companies should allocate resources to the most profitable customers because these customers will reward firms by having favorable behavioral

outcomes (e.g., positive word of mouth, repeat purchases, less costly to serve, etc.). However, the customer pyramid approach assumes stability over time (Rust, Kumar, and Venkatesan 2011), whereas, in our research, we account for dynamics by modeling the transition among segments over time. Segmentation models should account for these dynamics since ignoring them may result in under-valuing customers belonging to other segments who have potential to become more valuable to the firm over time.

Finally, this article contributes to the customer dynamics literature by extending the hurdle model to incorporate a HMM. This methodological contribution is important to this research stream as we are the first to account for time varying association structure and zero-inflation, which are common findings in customer transaction databases. Our dynamic model accounts for these unique longitudinal data features thereby making model estimates more reliable.

Implications for retailers

Our results suggest that retailers would benefit from a segmentation model that incorporates customer dynamics. The model proposed and tested here will enable marketers to better understand the impact of marketing variables on buying behavior. It is important for retailers to identify how their customer base is changing over time, especially to understand whether the number of customers in more valuable states is growing or shrinking over time and the manner in which customers are transitioning from one state to another. Armed with results from such a model, retailers can take steps to retain and grow the value of their best customers while also developing and implementing strategies to reduce the probability that customers transition to less valuable states over time.

In addition, our research contributes to the growing body of literature emphasizing the importance of dynamic models as these models provide retailers with consistent and reliable estimates. Increased reliability gives retailers more confidence in the results and enables retailers to improve their marketing decisions. In comparison, retailers, in assuming a static model, might make suboptimal marketing decisions because of erroneous inferences.

As retailers gain access to increasingly more data, it is imperative that methodologies are tailored to and best fit the type of data (i.e., longitudinal) they collect. Our dynamic model provides more reliable estimates than prior approaches by accounting for the unique features of longitudinal data and thus enables the retailer to identify segments that previously might not be noticed by earlier modeling approaches. For example, our Event-driven segment, the most valuable segment, would not have been identified by prior approaches. Although this segment is transient, customers transiting into this state respond very well to catalog mailings and have an average monthly value of \$487.34. Awareness of the effectiveness of catalogs at triggering a purchase event for these valuable customers is a key advantage of our model. The retailer might want to design catalogs around meaningful events throughout the year so as to influence additional purchases from these Event-driven customers. The ability

to identify such new, profitable segments is a new capability for retail marketing management.

Limitations and directions for future research

One limitation of our research is the inability to evaluate the effectiveness of a targeted marketing campaign. We believe that customer dynamics research would benefit from a field study that compares a retailer's existing marketing campaign to one that is customized based on our customer dynamics model. Specifically, one could create a marketing campaign for each of the segments identified in a customer base using a customer's propensity to buy as a basis. Once identified and profiled, a targeted marketing campaign could be created for each segment. Ideally, a field experiment could be conducted to assess the effectiveness of both campaigns.

Consistent with other research in this domain, our empirical analysis only used data from repeat customers in our sample. The use of longitudinal data from repeat customers limits our ability to investigate customer attrition. However, it would be valuable for retailers to develop a model that predicts when a customer transitions into an inactive state and which marketing activities are most effective at reducing the likelihood of customers transitioning into this unprofitable state. In addition, early detection of inactive customers can enable a retailer to terminate additional investment in building and maintaining relationships with these customers.

A third limitation to our research is the use of a longitudinal dataset from a single retailer. Although it is difficult to gain access to a longitudinal dataset that captures customers' choices across a variety of retailers, customer dynamics research would benefit from a model that includes this type of data. One possible solution would be to adopt the survey methodology employed by Rust, Lemon, and Zeithaml (2004) to develop a customer dynamics model that incorporates competition. Customer dynamics research would also benefit from a model that incorporates macro-level data so as to better understand variations in customer or segment level profiles over time.

Other aspects also deserve further investigation. It would be ideal to distinguish *true* heterogeneity among customers that arises when subpopulations are present in the data, and behavioral dynamics over time. With this aim, a natural extension of our approach can be pursued in the mixed HMM class (Maruotti 2011). Similarly, we may encounter endogeneity issues in providing a statistical analysis. Several proposals exist in the literature to deal with the latter issue. An interesting approach is provided in Alfó, Maruotti, and Trovato (2011) where a multivariate model is specified including equations for endogenous variables.

Conclusion

In this research, we develop a dynamic segmentation model to assess the impact of marketing activities on purchase incidence as well as the number of orders. We believe that our model provides new insights into customer–firm relationship dynamics

and adds to the growing body of research on customer management. Overall, our model enables retailers to understand how marketing activities vary by segment and suggests how to tailor activities for each segment so as to increase the effectiveness of and improve the return on their investments.

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