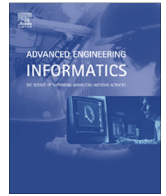




Contents lists available at ScienceDirect

Advanced Engineering Informatics

journal homepage: www.elsevier.com/locate/aei

An integrated AHP and VIKOR for design concept evaluation based on rough number

Guo-Niu Zhu, Jie Hu*, Jin Qi, Chao-Chen Gu, Ying-Hong Peng

State Key Laboratory of Mechanical System and Vibration, School of Mechanical Engineering, Shanghai Jiao Tong University, Shanghai 200240, PR China

ARTICLE INFO

Article history:

Received 22 May 2014

Received in revised form 10 December 2014

Accepted 26 January 2015

Available online xxx

Keywords:

Design concept evaluation

Vagueness and subjectivity

Rough number

VIKOR

AHP

ABSTRACT

Design concept evaluation at the early stage of product design has been widely recognized as one of the most critical phases in new product development as it determines the direction of the downstream design activities. However, the information at this stage is mainly subjective and imprecise which only depends on experts' judgments. How to handle the vagueness and subjectivity in design concept evaluation becomes a critical issue. This paper presents a systematic evaluation method by integrating rough number based analytic hierarchy process (AHP) and rough number based compromise ranking method (also known as VIKOR) to evaluate design concepts under subjective environment. In this study, rough number is introduced to aggregate individual judgments and preferences and deal with the vagueness in decision-making. A novel AHP based on rough number is presented to determine the weight of each evaluation criterion. Then an improved rough number based VIKOR is proposed to evaluate the design concept alternatives. Sensitivity analysis is conducted to measure the impact of the decision makers' risk to the final evaluation results. Finally, a practical example is put forward to validate the performance of the proposed method. The result shows that the proposed decision-making method can effectively enhance the objectivity in design concept evaluation under subjective environment.

© 2015 Elsevier Ltd. All rights reserved.

1. Introduction

New product development (NPD) has been considered as one of the corporate key functions and a critical factor to maintain competitiveness in a fast-growing global market under highly competitive environment [19,23]. The rapid advances in technology, fast changing customer needs, combined with increasing market competition force the companies to develop new product with shorter time, lower cost and higher quality [34]. However, it is a process involving uncertainty and risk. To resolve the stochastic problem and ensure the success of the product development, people should take into account these factors and constraints as early as possible and make accurate decisions. A bad selection of final design concept for a particular product may not only increase the development cost, but also cause additional modification or even endanger the success of the overall NPD. It is reported that up to 70–80% of the whole product development cost is determined at early design stage [29]. In addition, the deficiencies at this stage can hardly be compensated in the subsequent design process. Due to its important role on the consequent design activities, the

final design concept evaluation in the early product development is perhaps the most crucial stage among NPD.

Generally, design concept evaluation is a complex multi-criteria decision-making (MCDM) problem which concerns many factors ranging from customer needs to constraints and resources of the enterprise [47,14]. At the same time, it is also a group decision-making; many experts are invited to implement the final performance evaluation. The data and information in the decision-making process are usually subjective and imprecise, which can only rely on experts' subjective judgments. It ranges from individual judgments to methods that aggregate and rank these alternatives [38]. How to effectively and objectively aggregate individual judgments to evaluate the design concepts under such subjective environment at the early stage of NPD becomes a critical issue and a hot topic among researchers.

This paper aims to propose a systematic approach to manipulate the vagueness and subjectivity to enhance the objectivity in design concept evaluation by combining with rough number, analytic hierarchy process (AHP) and compromise ranking method (also known as VIKOR). Rough number is introduced to aggregate individual judgments and priorities and handle the vagueness in decision-making process. Then a rough number based AHP (rough AHP) is developed to calculate criteria weights and a rough number

* Corresponding author. Tel.: +86 21 34206552.

E-mail address: hujie@sjtu.edu.cn (J. Hu).

based VIKOR (rough VIKOR) is presented to evaluate design concept alternatives.

The remainder of this paper is organized as follows. Section 2 outlines the brief review and background. The rough number based approach is proposed in Section 3. Section 4 presents a case study. Section 5 draws the conclusion.

2. Brief review and background

2.1. Design concept evaluation

Due to its important and fundamental role in NPD, design concept evaluation is an attractive topic among researchers in decision-making. Various decision-making methods have conducted to select the best alternatives among design concepts. Calantone et al. [9] discussed the usefulness of the AHP in knowledge acquisition and management, and used it as a decision support tool to assist managers in new product ideas screening. To address the variety of interactions and dependencies in conceptual design alternative evaluation, Ayağ and Özdemir [5] presented an analytic network process (ANP) based methodology to select the best concept to satisfy the expectations of both company and customers. Meanwhile, many integrated methods were developed. Ayağ [4] integrated AHP and simulation analysis in design concept selection in a NPD environment. AHP was adopted to evaluate concept design alternatives while simulation analysis was used to carry out further economic analysis, and finally the best design concept alternatives were determined by a final benefit/cost analysis. Lin et al. [27] proposed an integrated AHP and TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) to assist the process of customer-driven product design. AHP was applied to determine the weights of customer needs and design characteristics while TOPSIS was adopted to execute competitive benchmarking in design alternative evaluation. Takai and Ishii [40] integrated target costing and perception-based concept evaluation approach for complex and large-scale systems. The method decomposed the QFD (Quality Function Deployment) matrices simultaneously for structure and their requirements, and evaluated concepts with regard to target requirements and cost. Besharati et al. [8] presented a generalized purchase model that accounts for market demand, designers' preferences and uncertainty associated with design attribute levels. Based on the customer-based utility metric, a decision support system was developed for product design selection.

However, most of the information in design concept evaluation comes from experts' subjective judgments, which are imprecise, vague or even inconsistent. The simple crisp pair-wise comparison with conventional decision-making approach is unable to capture the true perception of the decision maker effectively and precisely. In order to manipulate the vagueness and uncertainty in decision-making, some researchers introduced fuzzy logic into decision-making algorithm and developed various fuzzy sets based decision-making methods. Ayağ [3] integrated fuzzy AHP and simulation technique to assist concept selection. A fuzzy AHP was employed to reduce candidate concepts while simulation analysis was introduced to evaluate the rest ones. Finally, a further preference ratio analysis was conducted to determine the final design alternative. To compensate the deficiency of conventional AHP in addressing the variety of interactions and dependencies in decision-making, Ayağ and Özdemir [6] extended the pair-wise comparison in ANP with fuzzy sets and presented a fuzzy ANP model to evaluate conceptual design alternatives. Vanegas and Labib [42] suggested a novel approach to handle fuzzy sets and calculate desirability levels through a new fuzzy-weighted average in engineering design evaluation. Vanegas and Labib [43] presented a

survey of several fuzzy methods in engineering design evaluation. They posed two metrics for determining performance levels and suggested a new fuzzy goal and a novel method of measuring design candidates by computing an aggregate fuzzy set. Wang [44] developed an improved design concept evaluation model by combining with fuzzy sets and Pugh's concept selection approach, which can not only evaluate the candidate concepts but also aid designers for further promoting the concepts under evaluation. Kahraman et al. [23] proposed an integrated method with two phases to promote the quality of new product alternatives selection by combining with fuzzy sets, TOPSIS and heuristic multi-attribute utility approach. Malekly et al. [28] stated an integrated optimization-based approach for conceptual bridge design evaluation by combining QFD and TOPSIS under fuzzy environment. To strengthen the effectiveness of design alternative evaluation in complex product development, Zhang and Chu [50] attempted to integrate QFD, group decision-making and fuzzy sets, and developed a new model.

The intention of the fuzzy valued evaluation technique is to transform crisp numbers into fuzzy ones with the aid of membership function to deal with the vagueness in decision-making. However, the selection of membership function in fuzzy sets is mainly depending on subjective judgment. In addition, the boundary of fuzzy set is difficult to determine accurately.

Consider the deficiency of fuzzy sets in dealing with the vagueness in decision-making, some researchers introduced interval arithmetic and developed many hybrid methods. To optimize the decision-making process of NPD assessment under complex multi-attribute environment, Chin et al. [12] developed a novel evidential reasoning interval based method, which can effectively address uncertain and incomplete data, and information in various types. Guo et al. [16] further enhanced the evidential reasoning under both fuzzy and interval environment. Akay et al. [2] extended fuzzy information axiom with interval type-2 fuzzy sets and developed a novel concept selection approach known as interval-type-2 fuzzy information axiom. Sayadi et al. [37] integrated VIKOR with interval numbers for decision-making while Vahdani et al. [41] further introduced interval-valued fuzzy sets in VIKOR. Furthermore, Kuo and Liang [24] presented a new integrated interval-valued fuzzy sets and VIKOR. Park et al. [32] extended VIKOR with interval-valued intuitionistic fuzzy information for group decision-making. Chen and Tsao [11] combined with interval-valued fuzzy sets and TOPSIS while Park et al. [33] further extended TOPSIS with interval-valued intuitionistic fuzzy sets.

Likewise, the basic idea of interval based decision-making algorithm is to use interval number to represent decision information, many approaches may also integrate with fuzzy sets. However, the interval boundary is also difficult to determine and involving subjectivity.

Furthermore, as a famous method in dealing with uncertain information system, rough set and its variant models are also applied in decision-making. Xie et al. [45] combined with variable precision rough set and AHP for group decision-making. The integrated model was mainly used to calculate the weights of attributes. Guo and Zhang [15] integrated VIKOR and rough set theory in supplier selection. Rough set was employed to calculate relative importance and VIKOR was adopted to rank candidate suppliers. Aydoğan [7] presented a hybrid rough-AHP and TOPSIS methods for Turkish aviation firms' performance measurement under fuzzy environment. Rough set was adopted to reduce evaluation bias in pair-wise comparison process of AHP in criteria weighting. Li et al. [26] proposed an integrated model by combining with Kano's model, rough set and AHP to rank customer requirements' importance. Rough set was used to identify customer requirements and simplify the decision system. Besides, rough set was also used in the rule mining in decision-making.

Chang and Hung [10] and Zou et al. [51] introduced rough set in supplier selection by using them in rule mining. Li et al. [25] posed a grey-based rough decision-making method. Rough set was adopted in the knowledge discovery and data mining. In a word, the rough set in rough set based methods was mainly used in criteria weighting or rules mining to eliminate bias. The entire decision-making process was separated and only part of them was taken into consideration. Therefore, much vagueness in the whole process was ignored.

In addition, many other methods were introduced in decision-making and many new models were developed. Akay and Kulak [1] integrated information axiom, fuzzy sets and grey theory, and developed a grey-fuzzy information axiom to evaluate product concepts. Zhai et al. [47] combined with rough set and grey relation analysis and developed a rough-grey analysis model to optimize the design concept evaluation. Geng et al. [14] proposed a novel design concept evaluation method by integrating vague sets with TOPSIS. Huang et al. [17] presented a computational intelligence method to assist conceptual design. Fuzzy neural network was employed to implement concept evaluation. Moreover, several design selection methods were put forth by Huang et al. [18] to execute conceptual design evaluation under different conditions based on computational intelligence methods, such as physical programming, genetic algorithm, neural network and fuzzy logic. Song et al. [39] developed a novel rough group AHP and rough group TOPSIS to evaluate design concept under subjective environment. Jenab et al. [21] posed a multi-layer graph model to conduct conflict resolution among experts' opinions and developed a fuzzy graph-based model to evaluate design concepts.

In summary, the traditional fuzzy and interval based approaches cannot tackle the subjectivity in decision-making effectively. Most of the methods need to introduce much auxiliary information, such as predetermination of the membership function and the interval boundary, which are subjective as well. The rough set based algorithm can only address part of them, such as criteria weighting. Much of them were ignored. To resolve the dilemma, rough number is introduced to manipulate the subjectivity and vagueness in decision-making. It only relies on the data only, without any auxiliary numbers. Thus it can avoid the subjectivity at the most extent and measure the vagueness in various group decision-making areas. Not only can it be used in the determination of criteria weights, but also can it be adopted in the alternative ranking.

2.2. Rough number

Due to the subjectivity and group characteristics of the design concept evaluation, how to aggregate individual judgments and priorities from group experts and manage the subjectivity among them become urgent tasks. In this paper, rough number is introduced to handle these problems. Inspired by rough set theory, rough number is first proposed by Zhai et al. [46] with the purpose of handling subjective judgments of customers and determining the boundary intervals. A rough number usually contains lower limit, upper limit and the rough boundary interval, which only depends on the original data. Thus it does not require any auxiliary information and can better capture the experts' real perception and improve the objectivity of the decision-making.

Suppose U is the universe which contains all the objects, Y is an arbitrary object of U , R is a set of t classes (G_1, G_2, \dots, G_t) that cover all the objects in U , $R = \{G_1, G_2, \dots, G_t\}$. If these classes are ordered as $G_1 < G_2 < \dots < G_t$, then $\forall Y \in U, G_q \in R, 1 \leq q \leq t$, the lower approximation ($\underline{Apr}(G_q)$), upper approximation ($\overline{Apr}(G_q)$) and boundary region ($Bnd(G_q)$) of class G_q are defined as:

$$\underline{Apr}(G_q) = \bigcup \{Y \in U/R(Y) \leq G_q\} \tag{1}$$

$$\overline{Apr}(G_q) = \bigcup \{Y \in U/R(Y) \geq G_q\} \tag{2}$$

$$Bnd(G_q) = \bigcup \{Y \in U/R(Y) \neq G_q\} = \{Y \in U/R(Y) > G_q\} \bigcup \{Y \in U/R(Y) < G_q\} \tag{3}$$

Then G_q can be represented by a rough number ($RN(G_q)$), which is determined by its corresponding lower limit ($\underline{Lim}(G_q)$) and upper limit ($\overline{Lim}(G_q)$), where

$$\underline{Lim}(G_q) = \frac{1}{M_L} \sum R(Y) | Y \in \underline{Apr}(G_q) \tag{4}$$

$$\overline{Lim}(G_q) = \frac{1}{M_U} \sum R(Y) | Y \in \overline{Apr}(G_q) \tag{5}$$

$$RN(G_q) = [\underline{Lim}(G_q), \overline{Lim}(G_q)] \tag{6}$$

where M_L, M_U are the number of objects that contained in $\underline{Apr}(G_q)$ and $\overline{Apr}(G_q)$, respectively.

Obviously, the lower limit and upper limit denote the mean value of elements included in its corresponding lower approximation and upper approximation, respectively. Their difference is defined as rough boundary interval ($IRBnd(G_q)$):

$$IRBnd(G_q) = \overline{Lim}(G_q) - \underline{Lim}(G_q) \tag{7}$$

The rough boundary interval denotes the vagueness of G_q , where a larger one means more vague while a smaller one denotes a better precise. Then the subjective information can be denoted by rough number.

Take a data set $U = \{3, 5, 7, 3, 7\}$ for example, it has three classes and $R = \{G_1, G_2, G_3\} = \{3, 5, 7\}$. Take G_2 to explain the definition of the rough number, according to Eqs. (1)–(3):

$$\underline{Apr}(5) = \bigcup \{Y \in U/R(Y) \leq 5\} = \{3, 5, 3\}$$

$$\overline{Apr}(5) = \bigcup \{Y \in U/R(Y) \geq 5\} = \{5, 7, 7\}$$

$$Bnd(5) = \bigcup \{Y \in U/R(Y) \neq 5\} = \{3, 7, 3, 7\}$$

Therefore, the corresponding rough number of G_2 is calculated by Eqs. (4)–(6):

$$\underline{Lim}(5) = \frac{1}{M_L} \sum R(Y) | Y \in \underline{Apr}(5) = \frac{1}{3} (3 + 5 + 3) = 3.67$$

$$\overline{Lim}(5) = \frac{1}{M_U} \sum R(Y) | Y \in \overline{Apr}(5) = \frac{1}{3} (5 + 7 + 7) = 6.33$$

$$RN(5) = [\underline{Lim}(5), \overline{Lim}(5)] = [3.67, 6.33]$$

The rough boundary interval of G_2 is defined as:

$$IRBnd(5) = \overline{Lim}(5) - \underline{Lim}(5) = 2.66$$

Finally, the element '5' in U is represented by a rough number $RN(5) = [3.67, 6.33]$. Similarly, other elements in U are determined in the same way.

Because of the similarity with interval number, the arithmetic rules of interval number can also be used in rough number [48]. Suppose $RN(\alpha) = [\underline{Lim}(\alpha), \overline{Lim}(\alpha)]$ and $RN(\beta) = [\underline{Lim}(\beta), \overline{Lim}(\beta)]$ are two rough numbers, μ is a nonzero constant, then:

$$RN(\alpha) \times \mu = [\underline{Lim}(\alpha), \overline{Lim}(\alpha)] \times \mu = [\mu \times \underline{Lim}(\alpha), \mu \times \overline{Lim}(\alpha)] \tag{8}$$

$$RN(\alpha) + RN(\beta) = [\underline{Lim}(\alpha), \overline{Lim}(\alpha)] + [\underline{Lim}(\beta), \overline{Lim}(\beta)] = [\underline{Lim}(\alpha) + \underline{Lim}(\beta), \overline{Lim}(\alpha) + \overline{Lim}(\beta)] \tag{9}$$

$$RN(\alpha) \times RN(\beta) = [\underline{Lim}(\alpha), \overline{Lim}(\alpha)] \times [\underline{Lim}(\beta), \overline{Lim}(\beta)] = [\underline{Lim}(\alpha) \times \underline{Lim}(\beta), \overline{Lim}(\alpha) \times \overline{Lim}(\beta)] \tag{10}$$

2.3. VIKOR

VIKOR (Serbian name: ViseKriterijumska Optimizacija I Kompromisno Resenje), also known as compromise ranking method, is an effective tool in MCDM. It is developed from the L_p -metric in compromise programming:

$$L_{p,i} = \left\{ \sum_{j=1}^m [w_j (f_j^* - f_{ij}) / (f_j^* - f_j^-)]^p \right\}^{1/p}$$

$$1 \leq p \leq \infty; \quad i = 1, 2, \dots, n \tag{11}$$

where $L_{p,i}$ is considered as an aggregating function; f_{ij} is the evaluation value of criterion j for alternative i ; f_j^+, f_j^- are the best and worst value of criterion j , respectively; w_j is the weight of criterion j ; m is the number of criteria; n is the number of alternatives; p denotes the weight of the maximal deviation from the ideal solution.

In VIKOR, $L_{1,i}$ (expressed as S_i) and $L_{\infty,i}$ (expressed as R_i) are used to formulate ranking measure. The final compromise solution is the one with a maximum group utility (min S_i) of the majority, and a minimum of individual regret (min R_i) of the opponent. It is a feasible solution which is the closest to the ideal [34].

VIKOR is particularly powerful under such environment where the decision maker is unable, or does not know how to express his preference at the early stage of product development [30]. Furthermore, it has been combined with other methods including fuzzy sets, interval numbers and outranking methods to enhance its performance [41,31,24,49]. Due to its unique superiority, VIKOR has been widely used in various decision-making areas, such as material selection, robot selection and supplier selection [20,22,13,36].

Among various decision-making techniques, AHP is widely used in the determination of relative importance while VIKOR is a powerful alternative evaluation method. The rough number is a good choice to manipulate the subjectivity and aggregate individual judgments and priorities under group decision-making environment. Thus these three methods can be combined to integrate the merit of AHP in hierarchy evaluation, the superiority of rough number in manipulating vagueness and the virtue of VIKOR in modeling MCDM to improve the objectivity of decision-making in design concept evaluation.

3. Proposed method

3.1. Framework of the proposed method

In general, the design concept evaluation mainly contains two parts: the determination of relative importance of evaluation criterion and the alternative ranking. In order to eliminate the bias of the evaluation process, the two phases must be taken into consideration simultaneously. For the purpose of handling the vagueness and subjectivity in product design evaluation, this paper proposes an integrated approach by introducing rough number into AHP and VIKOR. Rough number is adopted and combined with AHP to calculate relative importance. Then the paper presents a rough VIKOR to evaluate design concept alternatives. By combining with rough AHP and rough VIKOR, both relative importance of each criterion and final alternative ranking are determined without any auxiliary information. Thus, the proposed method can effectively reflect the decision makers' true perception and strengthen the objectivity of design concept evaluation. The framework of the proposed method is depicted in Fig. 1.

3.2. Rough AHP for criteria weighting

As one of the most popular methods, AHP is widely used in various decision-making problems, especially in criteria weighting. It provides the ability to measure consistency of preferences, manipulate multiple decision makers, handle tangible and non-tangible criteria, and manage decision-making involving subjective judgments. Due to the subjectivity and vagueness in decision-making, this paper introduces rough number to combine with AHP to aggregate individual judgments and compute relative importance of each criterion. The procedure of the rough AHP is described as follows.

Step 1: Identify the evaluation objective, criteria and alternatives. Construct a hierarchical structure with the evaluation objective at the top layer, criteria at the middle and alternatives at the bottom.

Step 2: Conduct AHP survey and construct a group of pair-wise comparison matrices. The pair-wise comparison matrix of the e th expert is described as:

$$B_e = \begin{bmatrix} 1 & x_{12}^e & \dots & x_{1m}^e \\ x_{21}^e & 1 & \dots & x_{2m}^e \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1}^e & x_{m2}^e & \dots & 1 \end{bmatrix} \tag{12}$$

where $x_{gh}^e (1 \leq g \leq m, 1 \leq h \leq m, 1 \leq e \leq s)$ is the relative importance of criterion g on criterion h given by expert e , m is the number of criteria, s is the number of experts.

Calculate the maximum eigenvalue λ_{max}^e of B_e , then compute the consistency index $CI = (\lambda_{max}^e - m) / (m - 1)$.

Determine the random consistency index (RI) in Table 1 according to m . Compute the consistency ratio $CR = CI/RI$.

Conduct consistency test. If $CR < 0.1$, the comparison matrix is acceptable. Otherwise, experts' judgments should be adjusted until $CR < 0.1$.

Then the integrated comparison matrix \tilde{B} is built as:

$$\tilde{B} = \begin{bmatrix} 1 & \tilde{x}_{12} & \dots & \tilde{x}_{1m} \\ \tilde{x}_{21} & 1 & \dots & \tilde{x}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \dots & 1 \end{bmatrix} \tag{13}$$

where $\tilde{x}_{gh} = \{x_{gh}^1, x_{gh}^2, \dots, x_{gh}^s\}$, \tilde{x}_{gh} is the sequence of relative importances of criterion g on criterion h .

Step 3: Construct a rough comparison matrix.

Translate the element x_{gh}^e in \tilde{B} into rough number $RN(x_{gh}^e)$ using Eqs. (1)–(6):

$$RN(x_{gh}^e) = [x_{gh}^{eL}, x_{gh}^{eU}] \tag{14}$$

where x_{gh}^{eL} is the lower limit of $RN(x_{gh}^e)$ while x_{gh}^{eU} is the upper limit.

Then the rough sequence $RN(\tilde{x}_{gh})$ is represented as:

$$RN(\tilde{x}_{gh}) = \{ [x_{gh}^{1L}, x_{gh}^{1U}], [x_{gh}^{2L}, x_{gh}^{2U}], \dots, [x_{gh}^{sL}, x_{gh}^{sU}] \} \tag{15}$$

It is further translated into an average rough number $RN(x_{gh})$ by rough arithmetic Eqs. (8)–(10):

$$RN(x_{gh}) = [x_{gh}^L, x_{gh}^U] \tag{16}$$

$$x_{gh}^L = \frac{x_{gh}^{1L} + x_{gh}^{2L} + \dots + x_{gh}^{sL}}{s} \tag{17}$$

$$x_{gh}^U = \frac{x_{gh}^{1U} + x_{gh}^{2U} + \dots + x_{gh}^{sU}}{s} \tag{18}$$

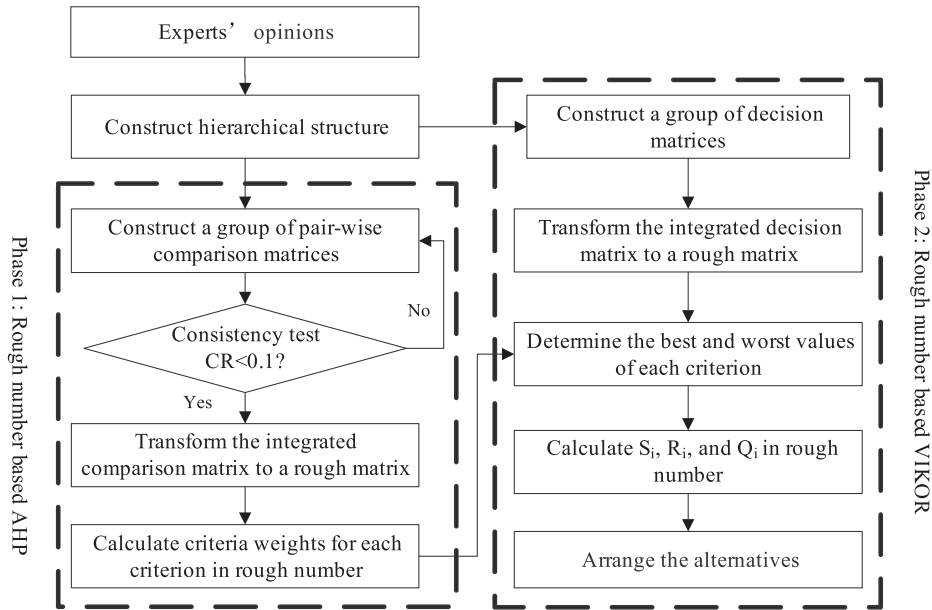


Fig. 1. Framework of the proposed design concept evaluation approach.

Table 1
Random consistency index (RI) [35].

<i>m</i>	3	4	5	6	7	8	9	10
<i>RI</i>	0.52	0.89	1.11	1.25	1.35	1.40	1.45	1.49

where x_{gh}^L is the lower limit of $RN(x_{gh})$ and x_{gh}^U is the upper limit. Then the rough comparison matrix *M* is formed as:

$$M = \begin{bmatrix} [1, 1] & [x_{12}^L, x_{12}^U] & \cdots & [x_{1m}^L, x_{1m}^U] \\ [x_{21}^L, x_{21}^U] & [1, 1] & \cdots & [x_{2m}^L, x_{2m}^U] \\ \vdots & \vdots & \ddots & \vdots \\ [x_{m1}^L, x_{m1}^U] & [x_{m2}^L, x_{m2}^U] & \cdots & [1, 1] \end{bmatrix} \quad (19)$$

Step 4: Calculate the rough weight w_g of each criterion:

$$w_g = \left[\sqrt[m]{\prod_{h=1}^m x_{gh}^L}, \sqrt[m]{\prod_{h=1}^m x_{gh}^U} \right] \quad (20)$$

$$w'_g = w_g / \max(w_g^U) \quad (21)$$

where w'_g is the normalization form.

Finally, the criteria weights are obtained.

3.3. Rough VIKOR for alternative evaluation

Based on the relative importance of each criterion calculated by rough AHP, rough VIKOR is proposed to aggregate individual priorities and evaluate design concept alternatives, which is conducted as follows.

Step 1: Construct a group of decision matrices and translate them into a rough decision matrix *D* according to Eqs. (1)–(10):

$$D = \begin{bmatrix} [f_{11}^L, f_{11}^U] & [f_{12}^L, f_{12}^U] & \cdots & [f_{1m}^L, f_{1m}^U] \\ [f_{21}^L, f_{21}^U] & [f_{22}^L, f_{22}^U] & \cdots & [f_{2m}^L, f_{2m}^U] \\ \vdots & \vdots & \ddots & \vdots \\ [f_{n1}^L, f_{n1}^U] & [f_{n2}^L, f_{n2}^U] & \cdots & [f_{nm}^L, f_{nm}^U] \end{bmatrix} \quad (22)$$

where $\tilde{f}_{ij} = \{f_{ij}^1, f_{ij}^2, \dots, f_{ij}^s\}, f_{ij}^e$ is the evaluation value of criterion *j* for alternative *i* given by expert *e*, the construction of matrix *D* is similar as *M* as stated in Section 3.2.

Step 2: Identify the best value f_j^* and the worst value f_j^- of each criterion in *D*. For the benefit criterion which belongs to the “larger-the-better” category: $f_j^* = \max_j f_{ij}^U, f_j^- = \min_j f_{ij}^L$; For the cost criterion which belongs to the “smaller-the-better” category: $f_j^* = \min_j f_{ij}^L, f_j^- = \max_j f_{ij}^U$; that is

$$f_j^* = \left\{ \left(\max_i f_{ij}^U \mid j \in B \right) \text{ or } \left(\min_i f_{ij}^L \mid j \in C \right) \right\} \quad (23)$$

$$f_j^- = \left\{ \left(\min_i f_{ij}^L \mid j \in B \right) \text{ or } \left(\max_i f_{ij}^U \mid j \in C \right) \right\} \quad (24)$$

where *B* is associated with the benefit criterion while *C* is associated with the cost criterion.

Step 3: Calculate the values $[S_i^L, S_i^U]$ and $[R_i^L, R_i^U]$:

$$S_i^L = \sum_{j \in B} w_j^L (f_j^* - f_{ij}^U) / (f_j^* - f_j^-) + \sum_{j \in C} w_j^L (f_{ij}^L - f_j^*) / (f_j^- - f_j^*) \quad (25)$$

$$S_i^U = \sum_{j \in B} w_j^U (f_j^* - f_{ij}^L) / (f_j^* - f_j^-) + \sum_{j \in C} w_j^U (f_{ij}^U - f_j^*) / (f_j^- - f_j^*) \quad (26)$$

$$R_i^L = \max_j \begin{cases} w_j^L (f_j^* - f_{ij}^U) / (f_j^* - f_j^-) \mid j \in B \\ w_j^L (f_{ij}^L - f_j^*) / (f_j^- - f_j^*) \mid j \in C \end{cases} \quad (27)$$

$$R_i^U = \max_j \begin{cases} w_j^U (f_j^* - f_{ij}^L) / (f_j^* - f_j^-) \mid j \in B \\ w_j^U (f_{ij}^U - f_j^*) / (f_j^- - f_j^*) \mid j \in C \end{cases} \quad (28)$$

where w_j^L is the lower limit and w_j^U is the upper limit of the weight of each criterion, which are calculated by rough AHP stated in Section 3.2.

Step 4: Calculate the values $[Q_i^L, Q_i^U]$:

$$Q_i^L = \nu (S_i^L - S^*) / (S^- - S^*) + (1 - \nu) (R_i^L - R^*) / (R^- - R^*) \quad (29)$$

$$Q_i^U = \nu (S_i^U - S^*) / (S^- - S^*) + (1 - \nu) (R_i^U - R^*) / (R^- - R^*) \quad (30)$$

where $S^* = \min_i S_i^L, S^- = \max_i S_i^U, R^* = \min_i R_i^L, R^- = \max_i R_i^U$; Q is an aggregating index; ν is the weight of the tactics of the majority of criteria, $\nu \in [0, 1]$; usually $\nu = 0.5$.

Step 5: Rank the alternatives in ascending order, on the basis of S, R, Q . Then three arrangements are obtained.

The ranking rule of interval numbers is described as follows [46]:

1. If the rough boundary interval of a rough number is not strictly bound by another:
 - (a) If $\underline{Lim}(\alpha) \geq \underline{Lim}(\beta)$ and $\overline{Lim}(\alpha) > \overline{Lim}(\beta)$, or $\underline{Lim}(\alpha) > \underline{Lim}(\beta)$ and $\overline{Lim}(\alpha) \geq \overline{Lim}(\beta)$, then $RN(\alpha) > RN(\beta)$.
 - (b) If $\underline{Lim}(\alpha) = \underline{Lim}(\beta)$ and $\overline{Lim}(\alpha) = \overline{Lim}(\beta)$, then $RN(\alpha) = RN(\beta)$.
2. If the rough boundary interval of a rough number is strictly bound by another, suppose $M(\alpha)$ and $M(\beta)$ are the middle values of $RN(\alpha)$ and $RN(\beta)$, respectively:
 - (a) If $\underline{Lim}(\beta) > \underline{Lim}(\alpha)$ and $\overline{Lim}(\beta) < \overline{Lim}(\alpha)$: if $M(\alpha) \leq M(\beta)$, then $RN(\alpha) < RN(\beta)$; if $M(\alpha) > M(\beta)$, then $RN(\alpha) > RN(\beta)$.
 - (b) If $\underline{Lim}(\alpha) > \underline{Lim}(\beta)$ and $\overline{Lim}(\alpha) < \overline{Lim}(\beta)$: if $M(\alpha) \leq M(\beta)$, then $RN(\alpha) < RN(\beta)$; if $M(\alpha) > M(\beta)$, then $RN(\alpha) > RN(\beta)$.

Step 6: Propose the alternative A_a as a compromise solution, which is the best ranked with respect to Q (minimum), if the following two conditions are satisfied [34]:

C1: Acceptable advantage:

$$\sqrt{\frac{1}{2} [(Q^U(A_b) - Q^U(A_a))^2 + (Q^L(A_b) - Q^L(A_a))^2]} \geq 1 / (n - 1) \quad (31)$$

where A_b is the second alternative ranked by Q ;

C2: Acceptable stability in decision-making:

A_a must also be the best ranked with respect to S or/and R . This compromise solution is stable in decision-making process. When $\nu > 0.5$, it could be the strategy of maximum group utility, or "with veto" ($\nu < 0.5$), or "by consensus" ($\nu \approx 0.5$).

When C1 or C2 is not satisfied, a series of compromise solutions are selected as follows:

- (a) A_a and A_b if only condition C2 is not satisfied, or
- (b) A_a, A_b, \dots, A_k if C1 is not satisfied. The maximum k in A_k is determined by $\sqrt{\frac{1}{2} [(Q^U(A_k) - Q^U(A_a))^2 + (Q^L(A_k) - Q^L(A_a))^2]} < 1 / (k - 1)$.

By combining with rough AHP and rough VIKOR, the design concept evaluation is conducted and the subjectivity is effectively addressed.

4. Case study

In this section, the proposed approach is used in the concept selection of a lithography tool to assist design concept evaluation. Generally, lithography is a critical technology in the manufacturing of the integrated circuit and the lithography tool is regarded as the most important equipment in the integrated circuit industry. Meanwhile, the lithography tool is a complex and expensive product due to its high precision. The evaluation of the conceptual design becomes very important. In a lithography tool manufacturing company, a total of six design conceptual alternatives have been generated in the conceptual design, namely, A_1, A_2, A_3, A_4, A_5 and A_6 . The properties of the alternatives are outlined in Table 2.

The objective of the design concept evaluation is to select the best alternative from the six design concepts for subsequent design

phases. In the early design stage, people are mainly concerning with the seven following criteria including: linewidth (C_1), field size (C_2), throughput (C_3), overlay (C_4), illumination uniformity (C_5), manufacturing cost (C_6) and power consumption (C_7). Among them, C_1, C_4, C_6 and C_7 are the cost criteria while C_2, C_3 and C_5 are the benefit ones. Five experts are invited as decision makers to give their own opinions and judgments in the evaluation of the lithography tool design concepts.

Fig. 2 shows the hierarchical structure of the design concept evaluation of the lithography tool. Typically, the evaluation can be divided into two phases: criteria weighting calculated by rough AHP and alternatives ranking determined by rough VIKOR.

4.1. Criteria weighting by rough AHP

In the lithography tool design concept evaluation, rough AHP is employed to aggregate individual judgments and calculate the weight of each criterion.

Step 1: Build a hierarchical structure for the lithography tool concept evaluation which is shown in Fig. 2.

Step 2: Collect individual judgments and construct a group of pair-wise comparison matrices. Take the consistency examination until all the comparison matrices can pass through. Integrate individual comparison matrices to generate an integrated comparison matrix. The individual pair-wise comparison matrices are as follows:

$$B_1 = \begin{bmatrix} 1 & 5 & 4 & 2 & 3 & 5 & 9 \\ 1/5 & 1 & 1/3 & 1/5 & 1/4 & 1/2 & 5 \\ 1/4 & 3 & 1 & 1/5 & 1/3 & 2 & 5 \\ 1/2 & 5 & 5 & 1 & 3 & 5 & 9 \\ 1/3 & 4 & 3 & 1/3 & 1 & 3 & 9 \\ 1/5 & 2 & 1/2 & 1/5 & 1/3 & 1 & 5 \\ 1/9 & 1/5 & 1/5 & 1/9 & 1/9 & 1/5 & 1 \end{bmatrix}, \quad CR_1 = 0.0585 < 0.1$$

$$B_2 = \begin{bmatrix} 1 & 5 & 3 & 1 & 1 & 4 & 9 \\ 1/5 & 1 & 1/2 & 1/5 & 1/5 & 1/3 & 5 \\ 1/3 & 2 & 1 & 1/5 & 1/2 & 3 & 7 \\ 1 & 5 & 5 & 1 & 2 & 4 & 9 \\ 1 & 5 & 2 & 1/2 & 1 & 3 & 7 \\ 1/4 & 3 & 1/3 & 1/4 & 1/3 & 1 & 3 \\ 1/9 & 1/5 & 1/7 & 1/9 & 1/7 & 1/3 & 1 \end{bmatrix}, \quad CR_2 = 0.0508 < 0.1$$

$$B_3 = \begin{bmatrix} 1 & 7 & 3 & 1 & 3 & 5 & 9 \\ 1/7 & 1 & 1/2 & 1/7 & 1/3 & 1/3 & 3 \\ 1/3 & 2 & 1 & 1/5 & 1/2 & 3 & 5 \\ 1 & 7 & 5 & 1 & 2 & 4 & 9 \\ 1/3 & 3 & 2 & 1/2 & 1 & 3 & 5 \\ 1/5 & 3 & 1/3 & 1/4 & 1/3 & 1 & 3 \\ 1/9 & 1/3 & 1/5 & 1/9 & 1/5 & 1/3 & 1 \end{bmatrix}, \quad CR_3 = 0.0388 < 0.1$$

$$B_4 = \begin{bmatrix} 1 & 7 & 5 & 2 & 3 & 7 & 9 \\ 1/7 & 1 & 1/3 & 1/6 & 1/5 & 1/2 & 3 \\ 1/5 & 3 & 1 & 1/3 & 1/3 & 2 & 5 \\ 1/2 & 6 & 3 & 1 & 3 & 5 & 7 \\ 1/3 & 5 & 3 & 1/3 & 1 & 4 & 7 \\ 1/7 & 2 & 1/2 & 1/5 & 1/4 & 1 & 4 \\ 1/9 & 1/3 & 1/5 & 1/7 & 1/7 & 1/4 & 1 \end{bmatrix}, \quad CR_4 = 0.0458 < 0.1$$

Table 2
Brief description of the conceptual alternatives.

Alternatives	Brief description
A ₁	A ₁ is a step-and-repeat lithographic system which adopts 4X reduction ArF projection lens with conventional illumination, a wavelength 193 nm ArF excimer laser, an ultra-precision screw rail stage with automatic focusing and leveling technology
A ₂	A ₂ is a step-and-repeat lithographic system which uses 4X reduction KrF projection lens with off-axis illumination, a wavelength 248 nm KrF excimer laser, an ultra-precision screw rail dual-stage with self-adaptive focusing and leveling technology
A ₃	A ₃ is a step-and-scan lithographic system which adopts 4X reduction KrF projection lens with off-axis illumination and resolution enhancement techniques, a wavelength 248 nm KrF excimer laser, an aerostatic bearing stage with self-adaptive focusing and leveling technology
A ₄	A ₄ is a step-and-scan lithographic system which uses 4X reduction ArF projection lens with optional off-axis illumination and resolution enhancement techniques, a wavelength 193 nm ArF excimer laser, an ultra-precision aerostatic bearing dual-stage with self-adaptive focusing and leveling technology
A ₅	A ₅ is a step-and-repeat lithographic system which adopts 4X reduction KrF projection lens with off-axis illumination and resolution enhancement techniques, a wavelength 248 nm KrF excimer laser, a magnetic levitation with automatic focusing and leveling technology
A ₆	A ₆ is a step-and-scan lithographic system which uses 4X reduction ArF projection lens with conventional illumination, a wavelength 193 nm ArF excimer laser, a magnetic levitation with self-adaptive focusing and leveling technology

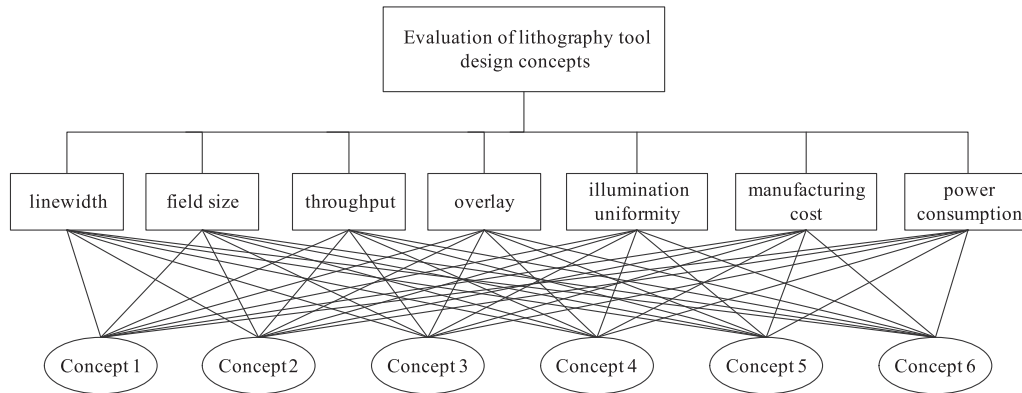


Fig. 2. The hierarchical structure of the lithography tool design concept evaluation.

$$B_5 = \begin{bmatrix} 1 & 7 & 5 & 1 & 2 & 5 & 7 \\ 1/7 & 1 & 1/3 & 1/5 & 1/4 & 1/2 & 3 \\ 1/5 & 3 & 1 & 1/3 & 1/3 & 2 & 5 \\ 1 & 5 & 3 & 1 & 2 & 5 & 7 \\ 1/2 & 4 & 3 & 1/2 & 1 & 3 & 5 \\ 1/5 & 2 & 1/2 & 1/5 & 1/3 & 1 & 2 \\ 1/7 & 1/3 & 1/5 & 1/7 & 1/5 & 1/2 & 1 \end{bmatrix}, \quad CR_5 = 0.0329 < 0.1$$

Obviously $CR_e < 0.1$ ($e = 1, 2, \dots, 5$), all the comparison matrices are acceptable. Then the integrated comparison matrix \tilde{B} is generated by combining with the above five individual comparison matrices.

$$\tilde{B} = \begin{bmatrix} 1, 1, 1, 1, 1 & 5, 5, 7, 7, 7 & \dots & 5, 4, 5, 7, 5 & 9, 9, 9, 9, 7 \\ \frac{1}{5}, \frac{1}{5}, \frac{1}{7}, \frac{1}{7}, \frac{1}{7} & 1, 1, 1, 1, 1 & \dots & \frac{1}{2}, \frac{1}{3}, \frac{1}{3}, \frac{1}{2}, \frac{1}{2} & 5, 5, 3, 3, 3 \\ \frac{1}{4}, \frac{1}{3}, \frac{1}{3}, \frac{1}{5}, \frac{1}{5} & 3, 2, 2, 3, 3 & \dots & 2, 3, 3, 2, 2 & 5, 7, 5, 5, 5 \\ \frac{1}{2}, 1, 1, \frac{1}{2}, 1 & 5, 5, 7, 6, 5 & \dots & 5, 4, 4, 5, 5 & 9, 9, 9, 7, 7 \\ \frac{1}{3}, 1, \frac{1}{3}, \frac{1}{3}, \frac{1}{2} & 4, 5, 3, 5, 4 & \dots & 3, 3, 3, 4, 3 & 9, 7, 5, 7, 5 \\ \frac{1}{5}, \frac{1}{4}, \frac{1}{5}, \frac{1}{7}, \frac{1}{5} & 2, 3, 3, 2, 2 & \dots & 1, 1, 1, 1, 1 & 5, 3, 3, 4, 2 \\ \frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{7}, \frac{1}{7} & \frac{1}{5}, \frac{1}{5}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3} & \dots & \frac{1}{5}, \frac{1}{3}, \frac{1}{4}, \frac{1}{2} & 1, 1, 1, 1, 1 \end{bmatrix}$$

Step 3: Translate the elements in \tilde{B} into rough numbers and correspondingly the original integrated comparison matrix \tilde{B} is converted into a rough comparison matrix.

Take $\tilde{x}_{16} = \{5, 4, 5, 7, 5\}$ as an example,

$$\begin{aligned} \underline{Lim}(4) &= 4, & \overline{Lim}(4) &= \frac{1}{5}(5 + 4 + 5 + 7 + 5) = 5.2 \\ \underline{Lim}(5) &= \frac{1}{4}(5 + 4 + 5 + 5) = 4.75, & \overline{Lim}(5) &= \frac{1}{4}(5 + 5 + 7 + 5) = 5.5 \\ \underline{Lim}(7) &= \frac{1}{5}(5 + 4 + 5 + 7 + 5) = 5.2, & \overline{Lim}(7) &= 7 \end{aligned}$$

Thus, x_{16}^e can be expressed in rough number:

$$\begin{aligned} RN(x_{16}^1) &= RN(x_{16}^3) = RN(x_{16}^5) = RN(5) = [4.75, 5.5] \\ RN(x_{16}^2) &= RN(4) = [4, 5.2] \\ RN(x_{16}^4) &= RN(7) = [5.2, 7] \end{aligned}$$

According to Eqs. (16)–(18):

$$\begin{aligned} x_{16}^L &= \frac{x_{16}^1 + x_{16}^2 + \dots + x_{16}^5}{s} = \frac{4.75 + 4 + 4.75 + 5.2 + 4.75}{5} = 4.69 \\ x_{16}^U &= \frac{x_{16}^1 + x_{16}^2 + \dots + x_{16}^5}{s} = \frac{5.5 + 5.2 + 5.5 + 7 + 5.5}{5} = 5.74 \end{aligned}$$

Thus the rough sequence \tilde{x}_{16} in \tilde{B} is transformed into a rough number $RN(x_{16}) = [4.69, 5.74]$. The transformation of other elements in \tilde{B} are implemented in the same way.

Then, the rough comparison matrix is obtained:

$$M = \begin{bmatrix} [1, 1] & [5.72, 6.68] & \dots & [4.69, 5.74] & [8.28, 8.92] \\ [0.15, 0.18] & [1, 1] & \dots & [0.39, 0.47] & [3.32, 4.28] \\ [0.23, 0.30] & [2.36, 2.84] & \dots & [2.16, 2.64] & [5.08, 5.72] \\ [0.68, 0.92] & [5.17, 6.06] & \dots & [4.36, 5.84] & [7.72, 8.68] \\ [0.38, 0.65] & [3.75, 4.64] & \dots & [3.04, 3.36] & [5.72, 7.51] \\ [0.18, 0.22] & [2.16, 2.64] & \dots & [1, 1] & [2.75, 4.08] \\ [0.11, 0.12] & [0.25, 0.31] & \dots & [0.26, 0.39] & [1, 1] \end{bmatrix}$$

Step 4: Calculate rough weights of the criteria using Eqs. (20) and (21):

$$\begin{aligned} w &= \{w_1, w_2, w_3, w_4, w_5, w_6, w_7\} \\ &= \{[2.902, 3.556], [0.428, 0.506], [0.898, 1.073], [2.663, 3.123], \\ &\quad [1.563, 1.935], [0.595, 0.703], [0.216, 0.254]\} \end{aligned}$$

Then its normalization form w' is obtained:

$$w' = \{w_1, w_2, w_3, w_4, w_5, w_6, w_7\}$$

$$= \{[0.816, 1], [0.120, 0.142], [0.253, 0.302], [0.749, 0.878], [0.440, 0.544], [0.167, 0.198], [0.061, 0.071]\}$$

4.2. Alternative evaluation by rough VIKOR

When the criteria weights are obtained, rough VIKOR is adopted to determine the final ranking. For quantitative criteria, each expert suggests an appropriate value according to his own perception and experience. For example, the five experts choose value of '95 nm', '99 nm', '98 nm', '97 nm' and '93 nm' for the linewidth of alternative 1 (A_1), respectively. For qualitative criteria, experts evaluate the design concept with scale of 1, 3, 5, 7, 9, which represents 'very low', 'low', 'medium', 'high' and 'very high', respectively. Then the performance of a design concept can be represented by a set of such values from experts' estimation. Table 3 shows the experts' evaluation values for design concepts.

Step 1: Change the original group decision data in Table 3 into a rough decision matrix according to Eqs. (1)–(10):

$$D = \begin{bmatrix} [94.83, 97.83] & [851.30, 854.88] & \dots & [4.49, 6.28] & [5.72, 6.68] \\ [90.63, 93.12] & [852.45, 855.89] & \dots & [5.72, 6.68] & [4.30, 5.70] \\ [88.12, 91.70] & [851.84, 855.76] & \dots & [7.08, 7.72] & [8.28, 8.92] \\ [85.78, 88.27] & [854.79, 856.52] & \dots & [7.32, 8.28] & [7.32, 8.28] \\ [89.13, 91.71] & [855.84, 858.75] & \dots & [5.32, 6.28] & [7.08, 7.72] \\ [89, 91] & [854.02, 855.54] & \dots & [7.72, 8.68] & [5.32, 6.28] \end{bmatrix}$$

Step 2: Identify the best value f_j^* and worst value f_j^- for each criterion as listed in Table 4.

Step 3: Calculate the values $[S_i^L, S_i^U]$ and $[R_i^L, R_i^U]$ as given in Tables 5 and 6.

Step 4: Calculate the values $[Q_i^L, Q_i^U]$, which are shown in Table 7.

Step 5: Rank the design concepts in ascending order based on S, R, Q . According to the ranking rules in Section 3.3, final ranking is obtained as follows: $A_4 < A_6 < A_3 < A_5 < A_2 < A_1$. Obviously, A_4 is the best design alternative.

4.3. Comparison and discussion

To measure the influence of experts' risks to the final product ranking, sensitivity analysis is conducted which is shown in Table 8. The results show that concept A_4 has a maximum priority at all the situations. When $v < 0.7$, final ranking is obtained as $A_4 < A_6 < A_3 < A_5 < A_2 < A_1$; otherwise, the final ranking is obtained as $A_4 < A_6 < A_5 < A_3 < A_2 < A_1$. That is, concept A_3 and concept A_5 are dependent of the risk preferences of decision makers.

To illustrate the proposed rough AHP and rough VIKOR methodology, a lot of experiments are executed for comparison on the basis of the data stated above. First of all, traditional AHP in crisp form (crisp AHP) and fuzzy AHP are introduced to compute criteria weights and compare with the proposed rough AHP. Symmetrical triangular membership function is employed in fuzzy AHP. Fig. 3 shows the comparison of criteria weights calculated by crisp AHP, fuzzy AHP and the proposed rough AHP. From Fig. 3, it can be concluded that the three methods generate the same sequence of the criteria weights ($C_1 > C_4 > C_5 > C_3 > C_6 > C_2 > C_7$), but differ in size. The crisp AHP method calculates weights by crisp number without considering subjectivity and vagueness while the

Table 3
Evaluation data for design concepts.

Alternatives	Experts	Evaluation criteria						
		C ₁	C ₂	C ₃	C ₄	C ₅	C ₆	C ₇
A ₁	1	95	852	208	12	97	5	7
	2	99	851	210	13	98	7	7
	3	98	850	206	11	98	5	7
	4	97	857	205	12	97	7	5
	5	93	855	207	13	99	3	7
A ₂	1	93	853	208	11	98	7	5
	2	95	857	209	11	98	5	7
	3	90	850	203	12	97	7	3
	4	91	857	206	12	98	5	5
	5	90	854	205	11	99	7	5
A ₃	1	88	852	201	10	98	7	9
	2	93	850	208	11	98	7	9
	3	92	852	205	10	98	7	9
	4	86	858	207	10	97	9	7
	5	91	857	202	11	98	7	9
A ₄	1	85	855	200	9	99	7	9
	2	90	856	205	8	98	9	7
	3	87	858	198	7	99	9	7
	4	85	854	204	8	98	7	7
	5	88	855	203	8	99	7	9
A ₅	1	89	855	204	11	98	5	7
	2	93	857	207	11	98	5	9
	3	90	855	206	12	98	7	7
	4	88	862	207	10	99	5	7
	5	92	857	203	11	98	7	7
A ₆	1	88	853	204	10	98	9	7
	2	92	856	203	9	99	9	5
	3	89	855	208	10	98	7	5
	4	90	854	207	11	99	7	5
	5	91	856	202	10	98	9	7

Table 4
The best value and worst value of each criterion.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
f_j^*	85.78	858.75	208.46	7.65	98.84	4.49	4.30
f_j^-	97.83	851.30	200.12	12.64	97.36	8.68	8.92

Table 5
 $[S_i^L, S_i^U]$ values.

	S_i^L	S_i^U	S_i	Rank
A_1	1.485	2.773	[1.485, 2.773]	6
A_2	1.120	2.131	[1.120, 2.131]	5
A_3	1.062	2.022	[1.062, 2.022]	4
A_4	0.333	1.124	[0.333, 1.124]	1
A_5	0.952	1.779	[0.952, 1.779]	3
A_6	0.833	1.665	[0.833, 1.665]	2

Table 6
 $[R_i^L, R_i^U]$ values.

	R_i^L	R_i^U	R_i	Rank
A_1	0.615	1	[0.615, 1]	6
A_2	0.527	0.702	[0.527, 0.702]	5
A_3	0.377	0.526	[0.377, 0.526]	3
A_4	0.144	0.302	[0.144, 0.302]	1
A_5	0.450	0.651	[0.450, 0.651]	4
A_6	0.300	0.475	[0.300, 0.475]	2

Table 7
 $[Q_i^L, Q_i^U]$ values.

	Q_i^L	Q_i^U	Q_i	Rank
A_1	0.511	1	[0.511, 1]	6
A_2	0.385	0.694	[0.385, 0.694]	5
A_3	0.285	0.569	[0.285, 0.569]	3
A_4	0	0.254	[0, 0.254]	1
A_5	0.306	0.593	[0.306, 0.593]	4
A_6	0.194	0.466	[0.194, 0.466]	2

fuzzy AHP and rough AHP represent weights by interval numbers. The different interval sizes mean different levels of vagueness due to different mechanisms in dealing with the subjectivity. Both the intervals reflect the vagueness containing in the criteria weighting process by the corresponding AHP model while the rough number adopts a flexible interval boundary, and the fuzzy set uses a fixed boundary in accordance with corresponding membership function. However, the predetermination of the membership function increases additional subjective information, which enlarges the vagueness as shown in the interval boundary. Thus, the proposed rough AHP can effectively measure the vagueness among the evaluation process and reflect the decision makers' true perception with more objectivity.

Based on the criteria weights calculated above, crisp AHP and VIKOR (crisp AHP–VIKOR), fuzzy AHP and VIKOR (fuzzy AHP–VIKOR) are carried out for comparison to validate the proposed rough AHP and rough VIKOR (rough AHP–VIKOR) which is shown in Fig. 4. In crisp and fuzzy approach, the ranking result is $A_4 < A_6 < A_5 < A_3 < A_2 < A_1$, while $A_4 < A_6 < A_3 < A_5 < A_2 < A_1$ in rough method. Actually, the difference between A_3 and A_5 in crisp method and rough method is subtle. In particular, the VIKOR in

Table 8
Sensitivity analysis.

	$v = 0$	Rank	$v = 0.1$	Rank	$v = 0.2$	Rank
A_1	[0.551, 1]	6	[0.543, 1]	6	[0.535, 1]	6
A_2	[0.447, 0.652]	5	[0.435, 0.660]	5	[0.422, 0.669]	5
A_3	[0.272, 0.446]	3	[0.274, 0.471]	3	[0.277, 0.495]	3
A_4	[0, 0.184]	1	[0, 0.198]	1	[0, 0.212]	1
A_5	[0.358, 0.592]	4	[0.347, 0.592]	4	[0.337, 0.592]	4
A_6	[0.182, 0.387]	2	[0.184, 0.402]	2	[0.187, 0.418]	2
<hr/>						
	$v = 0.3$		$v = 0.4$		$v = 0.5$	
A_1	[0.527, 1]	6	[0.519, 1]	6	[0.511, 1]	6
A_2	[0.410, 0.677]	5	[0.397, 0.686]	5	[0.385, 0.694]	5
A_3	[0.280, 0.520]	3	[0.282, 0.545]	3	[0.285, 0.569]	3
A_4	[0, 0.226]	1	[0, 0.240]	1	[0, 0.254]	1
A_5	[0.326, 0.592]	4	[0.316, 0.592]	4	[0.306, 0.593]	4
A_6	[0.189, 0.434]	2	[0.191, 0.450]	2	[0.194, 0.466]	2
<hr/>						
	$v = 0.6$		$v = 0.7$		$v = 0.8$	
A_1	[0.503, 1]	6	[0.496, 1]	6	[0.488, 1]	6
A_2	[0.372, 0.703]	5	[0.360, 0.711]	5	[0.348, 0.720]	5
A_3	[0.288, 0.594]	3	[0.291, 0.619]	4	[0.293, 0.643]	4
A_4	[0, 0.268]	1	[0, 0.282]	1	[0, 0.296]	1
A_5	[0.295, 0.593]	4	[0.285, 0.593]	3	[0.274, 0.593]	3
A_6	[0.196, 0.482]	2	[0.198, 0.498]	2	[0.200, 0.514]	2
<hr/>						
	$v = 0.9$		$v = 1.0$			
A_1	[0.480, 1]	6	[0.472, 1]	6		
A_2	[0.335, 0.728]	5	[0.323, 0.737]	5		
A_3	[0.296, 0.668]	4	[0.299, 0.692]	4		
A_4	[0, 0.310]	1	[0, 0.324]	1		
A_5	[0.264, 0.593]	3	[0.254, 0.593]	3		
A_6	[0.203, 0.530]	2	[0.205, 0.546]	2		

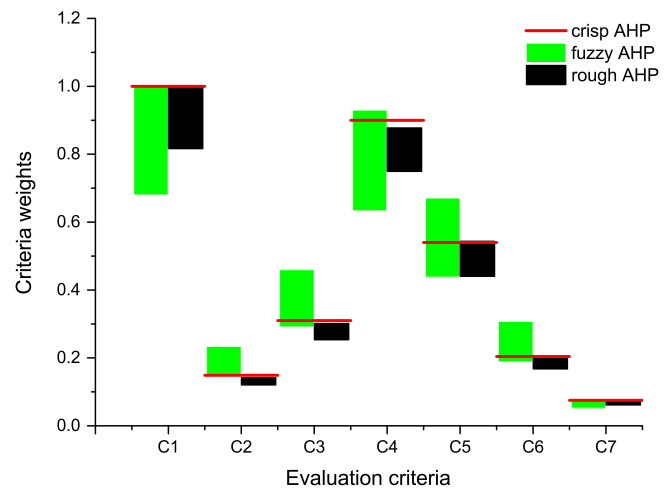


Fig. 3. Comparison of the criteria weighting.

fuzzy AHP–VIKOR takes crisp form; the interval boundary is a little smaller than rough AHP–VIKOR. However, if it adopts fuzzy value, it would be much larger than the rough AHP–VIKOR. Among the three methods, A_4 is the best design concept alternative among the whole candidate alternatives.

In addition, crisp AHP and crisp TOPSIS (crisp AHP–TOPSIS) are introduced to compare with the proposed rough AHP–VIKOR. Table 9 shows the final ranking of the crisp AHP–TOPSIS and rough AHP–VIKOR. The final ranking of both techniques is $A_4 < A_6 < A_3 < A_5 < A_2 < A_1$. Similarly, A_4 is the best one according to the two methods.

Moreover, the integrated rough AHP and rough VIKOR method can not only determine the criteria importance but also the alternatives ranking based on the original data, without any auxiliary information. The rough number can naturally aggregate and

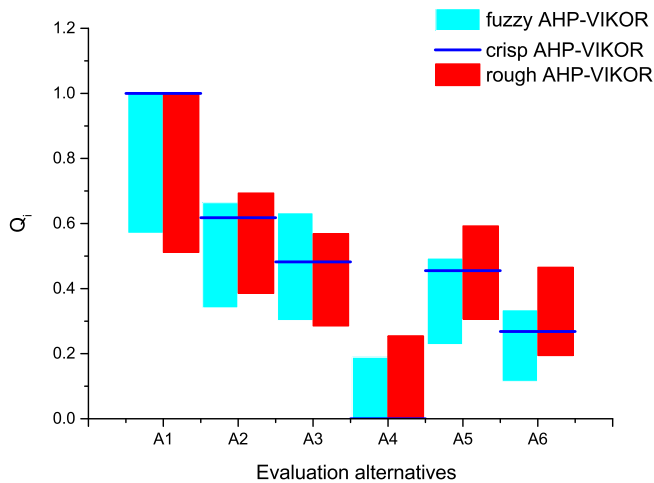


Fig. 4. Comparison of the alternative ranking.

Table 9
Comparison of the crisp AHP-TOPSIS and rough AHP-VIKOR.

	Crisp AHP-TOPSIS		Rough AHP-VIKOR	
	Closeness coefficient	Rank	$Q(v = 0.5)$	Rank
A ₁	0.190	6	[0.511, 1]	6
A ₂	0.273	5	[0.385, 0.694]	5
A ₃	0.438	3	[0.285, 0.569]	3
A ₄	0.827	1	[0, 0.254]	1
A ₅	0.353	4	[0.306, 0.593]	4
A ₆	0.512	2	[0.194, 0.466]	2

translate crisp pair-wise comparison numbers into interval numbers without any additional tools. It adopts flexible interval boundary instead of a fixed predefined one, which denotes the vagueness of the decision results, where a larger one means vaguer while a smaller one represents a better precise. Thus, the proposed method can effectively reflect the decision makers' true perception and enhance the objectivity of decision-making for design concept evaluation. Comparing with the crisp and fuzzy methods, the rough number based decision-making method does not need any auxiliary information except the original data collected from the decision maker. It provides a novel way to aggregate individual judgments and priorities in group decision-making. It avoids pre-determination of the membership function in fuzzy method while maintaining the ability of reflecting the vagueness in decision-making process.

5. Conclusion

To deal with the subjectivity in design concept evaluation, this paper proposes an integrated rough AHP and rough VIKOR methodology to handle the vagueness and subjectivity. In this study, rough number is introduced to aggregate individual judgments and priorities and measure vagueness. It is firstly used to integrate with AHP to calculate the relative importance for each criterion. Then it is employed to combine with VIKOR to arrange the alternatives. By combining with rough AHP and rough VIKOR, both relative importance of each criterion and final alternatives ranking are determined. Finally, the proposed approach is applied in a practical case study to assist design concept evaluation. To further evaluate the risk of decision makers' perception, sensitivity analysis is conducted to the case study. Then the proposed approach is used

to compare with crisp AHP-VIKOR, fuzzy AHP-VIKOR and crisp AHP-TOPSIS. By comparing with these methods, the proposed method can effectively reflect the true perception of the decision maker. The results are more objective and the vagueness is quantified and addressed properly.

The proposed rough number based approach is also applicable to many other group decision-making areas. In the future, many other methods such as ANP and their integrated models will be combined with rough number to extend the application areas and tackle vagueness under subjective environment in decision-making.

Acknowledgments

This research is supported by the National Natural Science Foundation of China (Nos. 51475288, 51275293, 51305260), National Key Scientific Instruments and Equipment Development Program of China (Nos. 2011YQ030114, 2013YQ03065105), Shanghai Committee of Science and Technology (No. 13111102800), "Shu Guang" project supported by Shanghai Municipal Education Commission and Shanghai Education Development Foundation (12SG14), Natural Science Foundation of Shanghai (No. 13ZR1421400).

References

- [1] D. Akay, O. Kulak, Evaluation of product design concepts using grey-fuzzy information axiom, *J. Grey Syst.* 19 (3) (2007) 221–234.
- [2] D. Akay, O. Kulak, B. Henson, Conceptual design evaluation using interval type-2 fuzzy information axiom, *Comput. Ind.* 62 (2) (2011) 138–146.
- [3] Z. Ayağ, A fuzzy AHP-based simulation approach to concept evaluation in a NPD environment, *IIE Trans.* 37 (9) (2005) 827–842.
- [4] Z. Ayağ, An integrated approach to evaluating conceptual design alternatives in a new product development environment, *Int. J. Prod. Res.* 43 (4) (2005) 687–713.
- [5] Z. Ayağ, R. Özdemir, An analytic network process-based approach to concept evaluation in a new product development environment, *J. Eng. Des.* 18 (3) (2007) 209–226.
- [6] Z. Ayağ, R.G. Özdemir, A hybrid approach to concept selection through fuzzy analytic network process, *Comput. Ind. Eng.* 56 (1) (2009) 368–379.
- [7] E.K. Aydoğan, Performance measurement model for Turkish aviation firms using the rough-AHP and TOPSIS methods under fuzzy environment, *Expert Syst. Appl.* 38 (4) (2011) 3992–3998.
- [8] B. Besharati, S. Azarm, P. Kannan, A decision support system for product design selection: a generalized purchase modeling approach, *Decis. Support Syst.* 42 (1) (2006) 333–350.
- [9] R.J. Calantone, C.A. Benedetto, J.B. Schmidt, Using the analytic hierarchy process in new product screening, *J. Prod. Innov. Manage.* 16 (1) (1999) 65–76.
- [10] B. Chang, H.-F. Hung, A study of using RST to create the supplier selection model and decision-making rules, *Expert Syst. Appl.* 37 (12) (2010) 8284–8295.
- [11] T.-Y. Chen, C.-Y. Tsao, The interval-valued fuzzy TOPSIS method and experimental analysis, *Fuzzy Sets Syst.* 159 (11) (2008) 1410–1428.
- [12] K.-S. Chin, J.-B. Yang, M. Guo, J.-K. Lam, An evidential-reasoning-interval-based method for new product design assessment, *IEEE Trans. Eng. Manage.* 56 (1) (2009) 142–156.
- [13] K. Devi, Extension of VIKOR method in intuitionistic fuzzy environment for robot selection, *Expert Syst. Appl.* 38 (11) (2011) 14163–14168.
- [14] X. Geng, X. Chu, Z. Zhang, A new integrated design concept evaluation approach based on vague sets, *Expert Syst. Appl.* 37 (9) (2010) 6629–6638.
- [15] J. Guo, W. Zhang, Selection of suppliers based on rough set theory and VIKOR algorithm, in: *International Symposium on Intelligent Information Technology Application Workshops, 2008, HITAW'08, IEEE, 2008*, pp. 49–52.
- [16] M. Guo, J.-B. Yang, K.-S. Chin, H.-W. Wang, X.-B. Liu, Evidential reasoning approach for multiattribute decision analysis under both fuzzy and interval uncertainty, *IEEE Trans. Fuzzy Syst.* 17 (3) (2009) 683–697.
- [17] H.-Z. Huang, R. Bo, W. Chen, An integrated computational intelligence approach to product concept generation and evaluation, *Mech. Mach. Theory* 41 (5) (2006) 567–583.
- [18] H.-Z. Huang, Y. Liu, Y. Li, L. Xue, Z. Wang, New evaluation methods for conceptual design selection using computational intelligence techniques, *J. Mech. Sci. Technol.* 27 (3) (2013) 733–746.
- [19] X. Huang, G.N. Soutar, A. Brown, Measuring new product success: an empirical investigation of Australian SMEs, *Ind. Mark. Manage.* 33 (2) (2004) 117–123.
- [20] A. Jahan, F. Mustapha, M.Y. Ismail, S. Sapuan, M. Bahraminasab, A comprehensive VIKOR method for material selection, *Mater. Des.* 32 (3) (2011) 1215–1221.

- [21] K. Jenab, A. Sarfaraz, M.T. Ameli, A conceptual design selection model considering conflict resolution, *J. Eng. Des.* 24 (4) (2013) 293–304.
- [22] R. Jeya Girubha, R. Vinodh, Application of fuzzy VIKOR and environmental impact analysis for material selection of an automotive component, *Mater. Des.* 37 (2012) 478–486.
- [23] C. Kahraman, G. Büyükoçkan, N.Y. Ateş, A two phase multi-attribute decision-making approach for new product introduction, *Inform. Sci.* 177 (7) (2007) 1567–1582.
- [24] M.-S. Kuo, G.-S. Liang, A soft computing method of performance evaluation with MCDM based on interval-valued fuzzy numbers, *Appl. Soft Comput.* 12 (1) (2012) 476–485.
- [25] G.-D. Li, D. Yamaguchi, M. Nagai, A grey-based rough decision-making approach to supplier selection, *Int. J. Adv. Manuf. Technol.* 36 (9–10) (2008) 1032–1040.
- [26] Y. Li, J. Tang, X. Luo, J. Xu, An integrated method of rough set, Kano's model and AHP for rating customer requirements final importance, *Expert Syst. Appl.* 36 (3) (2009) 7045–7053.
- [27] M.-C. Lin, C.-C. Wang, M.-S. Chen, C.A. Chang, Using AHP and TOPSIS approaches in customer-driven product design process, *Comput. Ind.* 59 (1) (2008) 17–31.
- [28] H. Malekly, S. Meysam Mousavi, H. Hashemi, A fuzzy integrated methodology for evaluating conceptual bridge design, *Expert Syst. Appl.* 37 (7) (2010) 4910–4920.
- [29] J.L. Nevins, D.E. Whitney, *Concurrent Design of Products and Processes: A Strategy for the Next Generation in Manufacturing*, McGraw-Hill Companies, 1989.
- [30] S. Opricovic, G.-H. Tzeng, Compromise solution by MCDM methods: a comparative analysis of VIKOR and TOPSIS, *Eur. J. Oper. Res.* 156 (2) (2004) 445–455.
- [31] S. Opricovic, G.-H. Tzeng, Extended VIKOR method in comparison with outranking methods, *Eur. J. Oper. Res.* 178 (2) (2007) 514–529.
- [32] J.H. Park, H.J. Cho, Y.C. Kwun, Extension of the VIKOR method for group decision making with interval-valued intuitionistic fuzzy information, *Fuzzy Optimiz. Decis. Mak.* 10 (3) (2011) 233–253.
- [33] J.H. Park, I.Y. Park, Y.C. Kwun, X. Tan, Extension of the TOPSIS method for decision making problems under interval-valued intuitionistic fuzzy environment, *Appl. Math. Model.* 35 (5) (2011) 2544–2556.
- [34] R.V. Rao, *Decision Making in Manufacturing Environment Using Graph Theory and Fuzzy Multiple Attribute Decision Making Methods*, vol. 2, Springer, 2012.
- [35] T.L. Saaty, L.G. Vargas, *Models, Methods, Concepts & Applications of the Analytic Hierarchy Process*, vol. 175, Springer, 2012.
- [36] A. Sanayei, S. Farid Mousavi, A. Yazdankhah, Group decision making process for supplier selection with VIKOR under fuzzy environment, *Expert Syst. Appl.* 37 (1) (2010) 24–30.
- [37] M.K. Sayadi, M. Heydari, K. Shahanaghi, Extension of VIKOR method for decision making problem with interval numbers, *Appl. Math. Model.* 33 (5) (2009) 2257–2262.
- [38] M.J. Scott, Quantifying uncertainty in multicriteria concept selection methods, *Res. Eng. Des.* 17 (4) (2007) 175–187.
- [39] W. Song, X. Ming, Z. Wu, An integrated rough number-based approach to design concept evaluation under subjective environments, *J. Eng. Des.* 24 (5) (2013) 320–341.
- [40] S. Takai, K. Ishii, Integrating target costing into perception-based concept evaluation of complex and large-scale systems using simultaneously decomposed QFD, *J. Mech. Des.* 128 (6) (2006) 1186–1195.
- [41] B. Vahdani, H. Hadipour, J.S. Sadaghiani, M. Amiri, Extension of VIKOR method based on interval-valued fuzzy sets, *Int. J. Adv. Manuf. Technol.* 47 (9–12) (2010) 1231–1239.
- [42] L. Vanegas, A. Labib, Application of new fuzzy-weighted average (NFWA) method to engineering design evaluation, *Int. J. Prod. Res.* 39 (6) (2001) 1147–1162.
- [43] L.V. Vanegas, A.W. Labib, Fuzzy approaches to evaluation in engineering design, *J. Mech. Des.* 127 (1) (2005) 24–33.
- [44] J. Wang, Improved engineering design concept selection using fuzzy sets, *Int. J. Comput. Integr. Manuf.* 15 (1) (2002) 18–27.
- [45] G. Xie, J. Zhang, K.K. Lai, L. Yu, Variable precision rough set for group decision-making: an application, *Int. J. Approx. Reason.* 49 (2) (2008) 331–343.
- [46] L.-Y. Zhai, L.-P. Khoo, Z.-W. Zhong, A rough set enhanced fuzzy approach to quality function deployment, *Int. J. Adv. Manuf. Technol.* 37 (5–6) (2008) 613–624.
- [47] L.-Y. Zhai, L.-P. Khoo, Z.-W. Zhong, Design concept evaluation in product development using rough sets and grey relation analysis, *Expert Syst. Appl.* 36 (3) (2009) 7072–7079.
- [48] L.-Y. Zhai, L.-P. Khoo, Z.-W. Zhong, A rough set based QFD approach to the management of imprecise design information in product development, *Adv. Eng. Inform.* 23 (2) (2009) 222–228.
- [49] N. Zhang, G. Wei, Extension of VIKOR method for decision making problem based on hesitant fuzzy set, *Appl. Math. Model.* 37 (7) (2013) 4938–4947.
- [50] Z. Zhang, X. Chu, A new integrated decision-making approach for design alternative selection for supporting complex product development, *Int. J. Comput. Integr. Manuf.* 22 (3) (2009) 179–198.
- [51] Z. Zou, T.-L.B. Tseng, H. Sohn, G. Song, R. Gutierrez, A rough set based approach to distributor selection in supply chain management, *Expert Syst. Appl.* 38 (1) (2011) 106–115.