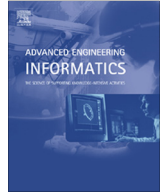




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Schedule design for sustainable container supply chain networks with port time windows [☆]

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ABSTRACT

This paper studies a practical liner shipping schedule design problem with port time windows for container supply chain networks. A mixed-integer nonlinear non-convex model that incorporates the availability of ports is proposed to minimize the sum of ship cost and fuel cost (and thereby pollutant emission). In view of the structure of the problem, we reformulate it as an integer linear optimization model and propose an iterative optimization approach. The proposed solution method is applied to two liner networks operated by a global shipping line.

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1. Introduction

As reported by UNCTAD [15], the world gross domestic product in 2012 grew by 2.2% with the reorientation in global production. In the meantime, the international trade grew by 1.8%, reaching 18.3 trillion USD in merchandise export. Maritime transportation is the backbone of international trade. For several decades, containerized trade has been the fastest growing shipping segment. It accounted for over 16% of global seaborne trade by volume in 2012 and more than half by value in 2007. In 2012, containerized trade volume expanded to 155 million twenty-foot equivalent units (TEUs) [15].

Containers are transported by global liner companies on regularly scheduled ship routes. A large variety of general cargoes are containerized, such as manufactured products, food, and garment. Liner shipping services have fixed sequences of ports of call and fixed schedules, i.e., arrival and departure times at each port of call. Liner services are announced in advance to attract potential customers. For example, Fig. 1 shows a liner service – Singapore West Asia Express (SWX) – operated by American President Lines (APL) [1]. The ports of call and schedule are announced in the website of APL. Customers can arrange the delivery of their cargo based on the available date of the cargo at the origin port and the expected

arrival date at the destination port [14]. Therefore, container liner shipping is of significant importance to the global supply chain network.

A container liner shipping network consists of many ship routes, and a shipping line has to determine the schedule for each ship route. Schedule design for a liner ship route is a tactical-level planning decision that is made every three to six months. To design the schedule of a ship route, the first factor to consider is the service availability of the ports. Since a port has limited berths and needs to provide services for a number of liner shipping companies and a number of ships, it cannot guarantee the availability of services whenever a ship arrives. We use the term “port time window” to refer to the time in a week that berths at the port can provide services to ships. A schedule designed without considering the availability of ports may be infeasible in reality.

Different schedules mean different sailing times between ports, which dictate different sailing speeds. It is known in the shipping industry that the daily fuel consumption of ships increases approximately proportional to the sailing speed cubed. Therefore, schedule design affects the bunker fuel consumption and thereby air pollutant emission. Reducing the fuel consumption will also improve the sustainability of the global container transportation network.

Container shipping lines provide weekly services for transporting containers, which means that the rotation time in terms of weeks for visiting all ports of call on a ship route is equal to the number of ships deployed. As a consequence, each port of call has a ship departure on the same day every week. When the speed

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of ships is higher, the rotation time is shorter, and hence fewer ships are required to maintain the weekly frequency.

The objective of this paper is to design the schedule for each ship route in a container liner shipping network. The aim is to minimize the sum of ship cost and fuel cost, while ensuring that ports are available to serve the ships on the planned days. The main contribution of the paper to the state-of-the-art literature and practice is that it takes the initiative to address a practical liner shipping network schedule design problem with port time windows (NSDPTW). The designed schedules are preferable as the sum of ship cost and fuel cost is minimized. The schedules are feasible because the availability of berths at each port on each day is explicitly considered in the model. The results from the model need no or very little modification before put to use. Hence, this study provides a useful decision-support tool for liner shipping companies to plan their services.

The remainder of the paper is organized as follows. Section 2 is the literature review. Section 3 describes the problem and formulates a mathematical model. Section 4 proposes a tailored solution approach to address the problem. Section 5 reports case studies based on a network consisting of three service routes. Section 6 concludes and points out future research directions.

2. Literature review

There is not much research on schedule design for container liner shipping networks, see e.g. Christiansen et al. [5], Meng et al.

[10] for reviews. The first category of relevant works is on schedule design at the tactical planning level. Wang and Meng [18] investigated the schedule design and container routing problem in a general liner shipping network with many ports and many ship routes with fixed sailing speed. Qi and Song [11] designed an optimal schedule for a liner ship route to minimize the total expected fuel cost. The port time is random, and a certain level of service, which is the probability that the containership would arrive at a port no later than the announced arrival time, has to be maintained in the model. Wang and Meng [20] designed a robust schedule for a liner ship route in which uncertainties in port operations and schedule recovery by fast steaming were captured endogenously. They assumed that ships are able to catch up the delayed schedule after a long leg that transverses an ocean. Wang and Meng [19] extended the work of Wang and Meng [18] by incorporating the optimization of speed, the uncertainty at port and the uncertainty at sea. None of the above four studies have considered the port time windows in schedule design. Wang et al. [17] developed a dynamic programming approach to design a schedule for a single ship route with port time windows. However, they assumed that each port on the ship route can only be visited once, whereas in reality many ship routes have ports that are visited twice. Wang et al. [16] extended the previous work by allowing a port to be visited twice on a ship route. Still, they focus on a single ship route, rather than a liner shipping network with many ship routes.

For the operational-level schedule adjustment, Yan et al. [21] developed a container routing model from the perspective of a

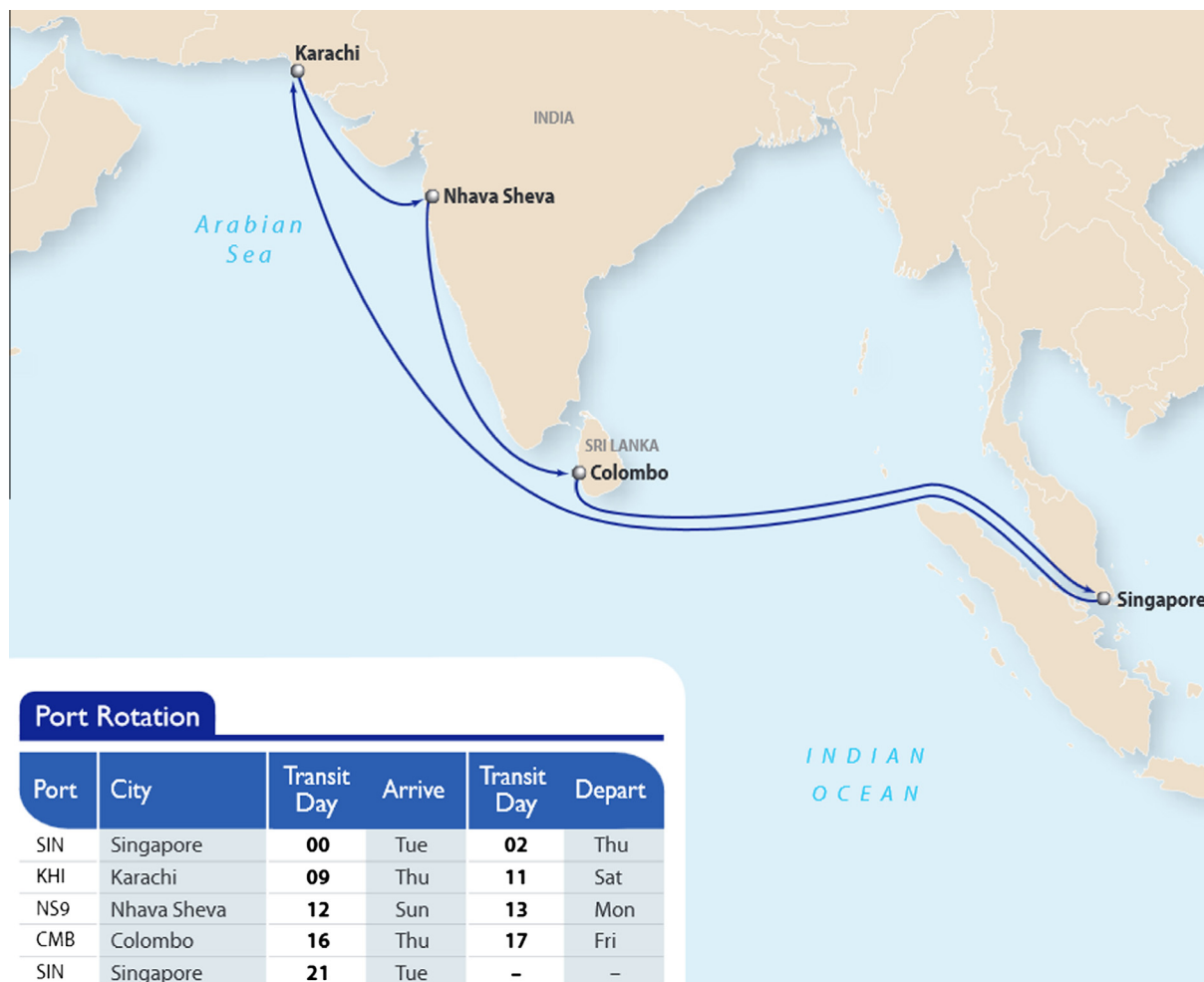


Fig. 1. SWX service operated by APL [1].

liner shipping company with the objective of maximizing the operating profit. Brouer et al. [2] examined a vessel schedule recovery problem, by evaluating a given disruption scenario and selecting a recovery action balancing the tradeoff between increased bunker consumption, the impact on cargo in the remaining network and the customer service level. None of the above two studies have taken into consideration the port time windows in schedule adjustment or schedule recovery.

Another category of relevant studies is focused on port operations, e.g., Chang et al. [3,4], He et al. [7], Du et al. [6], Yan et al. [22], Sun et al. [13], Yin et al. [23], Salido et al. [12], He et al. [8,9]. Both quay-side operations including berth allocation and quay crane assignment and yard-side operations such as yard template planning and yard truck scheduling have been investigated. These models are developed from the viewpoint of port operators: given the liner shipping schedules, they aim to better plan port operations to serve the ships more efficiently. For example, to evaluate the operational capability and efficiency of different designs of seaport container terminals, Sun et al. [13] introduced MicroPort which is a general simulation framework to provide an integrated and flexible modeling system. Yin et al. [23] proposed a distributed agent system for dynamic port planning and scheduling and they examined the system based on a case study. Salido et al. [12] developed a planning technique and a set of optimized allocation algorithms to solve the container stacking problem and the berth allocation problem. He et al. [9] proposed a strategy to resolve the problem of sharing internal trucks among multiple container terminals and proposed a simulation optimization method to obtain near-optimal solutions. He et al. [8] proposed a simulation-based heuristic method to solve the problem of multi-echelon container supply chain network.

The above literature review shows that schedule design for a container liner shipping network with port time windows has not been addressed in the existing literature, whereas this problem has practical significance for the profitability of liner shipping companies and the facilitation of global supply chains.

3. Problem description

Consider a liner container shipping company that operates a number of ship routes, denoted by the set \mathcal{R} , regularly serving a group of ports denoted by the set \mathcal{P} . The port rotation of a ship route $r \in \mathcal{R}$ can be expressed as:

$$p_{r1} \rightarrow p_{r2} \rightarrow \dots \rightarrow p_{rN_r} \rightarrow p_{r1} \quad (1)$$

where N_r is the number of ports of call on the ship route and $p_{ri} \in \mathcal{P}$ is the physical port corresponding to i th port of call. Let I_r be the set of ports of call of ship route $r \in \mathcal{R}$, i.e., $I_r = \{1, 2, \dots, N_r\}$. For brevity, we define $\langle r, i \rangle$ as the port of call i on ship route r . Defining

$p_{r,N_r+1} = p_{r1}$, the voyage from p_{ri} to $p_{r,i+1}$ is called *leg i* , $i \in I_r$. Fig. 2 shows a liner shipping network with three ship routes operated by APL [1], where ship route 1 is SWX in Fig. 1, ship route 2 is Surabaya Feeder Service (SUR) and ship route 3 is Semarang Feeder Service (SEM):

$$r = 1, N_r = 4: p_{r1}(SG) \rightarrow p_{r2}(KR) \rightarrow p_{r3}(NS) \rightarrow p_{r4}(CB) \rightarrow p_{r1}(SG)$$

$$r = 2, N_r = 2: p_{r1}(SG) \rightarrow p_{r2}(SB) \rightarrow p_{r1}(SG)$$

$$r = 3, N_r = 2: p_{r1}(SG) \rightarrow p_{r2}(SR) \rightarrow p_{r1}(SG) \rightarrow p_{r1}(CB)$$

We further define \mathcal{R}_p as the set of ship routes that visit port $p \in \mathcal{P}$, and define I_p as the set of ports of call on ship route $r \in \mathcal{R}_p$ that correspond to port p . In the above example, $\mathcal{R}_{SG} = \{1, 2, 3\}$, $I_{1,CB} = \{4\}$, $I_{2,SG} = \{1\}$, and $I_{3,CB} = \emptyset$.

3.1. Weekly service and schedules

A string of homogeneous ships are deployed on each ship route $r \in \mathcal{R}$ to maintain a weekly service frequency. Let L_{ri} (n mile) be the voyage distance of the i th leg of route $r \in \mathcal{R}$, $t_{ri}^{\text{port}} \in \{1, 2\}$ be the fixed time (day) a ship spends at port of call i on ship route $r \in \mathcal{R}$ for container handling, m_r be the number of ships deployed on ship route $r \in \mathcal{R}$, and t_{ri} be the sailing time (day) on the i th leg of ship route $r \in \mathcal{R}$. We then have the relation:

$$\sum_{i \in I_r} (t_{ri} + t_{ri}^{\text{port}}) = 7m_r, \quad \forall r \in \mathcal{R}$$

where 7 is the number of days in a week. Represent by C_r^{ship} the fixed cost (USD/week) associated with a ship on route $r \in \mathcal{R}$. C_r^{ship} includes the capital cost and the operating cost. The fixed cost associated with all the ships is:

$$\sum_{r \in \mathcal{R}} C_r^{\text{ship}} m_r$$

Define the beginning of a particular Sunday as day 0, and let t_{ri}^{arr} be the arrival time (day) at port of call i on ship route r . For instance, $t_{ri}^{\text{arr}} = 13$ means that ships arrive at $\langle r, i \rangle$ at the beginning of next Saturday. The time components of a ship route have the relation

$$t_{r,i+1}^{\text{arr}} = t_{ri}^{\text{arr}} + t_{ri}^{\text{port}} + t_{ri}, \quad r \in \mathcal{R}, i \in I_r$$

We assume that the sailing time on a leg has a minimum required value, denoted by t_{ri}^{min} . Hence,

$$t_{ri} \geq t_{ri}^{\text{min}}, \quad r \in \mathcal{R}, i \in I_r$$

Note that t_{ri}^{min} can be used to incorporate potential buffer time. For instance, if a leg is short, and the container handling time at port of call i is unreliable, then t_{ri}^{min} should be much larger than $\frac{L_{ri}}{24V_r^{\text{max}}}$ so as to hedge against potential delay by fast steaming, where V_r^{max} is the maximum speed (knots) of ships deployed on ship route r .

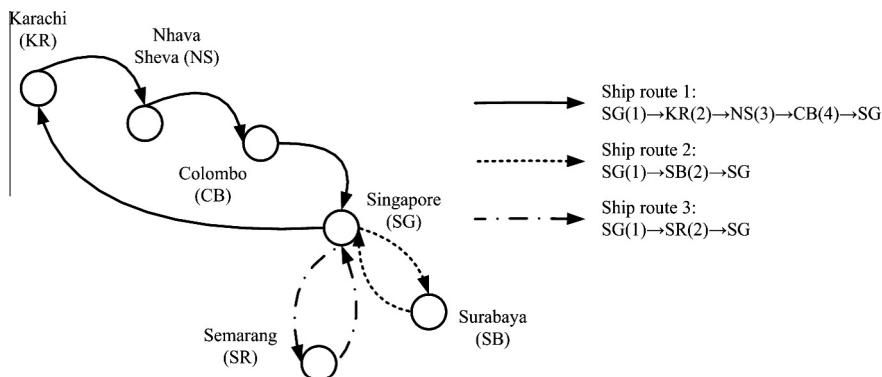


Fig. 2. A liner shipping network with three ship routes.

Because liner ship routes provide weekly services, to simplify the notation, we define W to be a set of all days in a week, that is, $W := \{0, 1, 2, 3, 4, 5, 6\}$

where 0 represents Sunday, 1 represents Monday, etc. Without loss of generality, we require

$$t_{r1}^{arr} \in W, \quad r \in \mathcal{R}$$

In fact, there is no difference between $t_{r1}^{arr} = 2$ and $t_{r1}^{arr} = 9$ due to the weekly frequency of liner services. Hence, the schedule of ship routes in the liner shipping network can be defined by the following vector

$$(t_{ri}^{arr}, r \in \mathcal{R}, i \in I_r; m_r, r \in \mathcal{R}) \quad (2)$$

The number of ships m_r is essential in the above vector, because without it the sailing time on leg N_r cannot be determined. The weekly service implies that

$$t_{r,N_r+1}^{arr} = t_{r1}^{arr} + 7m_r, \quad r \in \mathcal{R}$$

Therefore, we can also define the schedule as

$$(t_{ri}^{arr}, r \in \mathcal{R}, i \in I_r \cup \{N_r + 1\}) \quad (3)$$

which is equivalent to the schedule defined by (2).

The arrival time t_{ri}^{arr} at a port of call corresponds to day $t_{ri}^{arr} \bmod 7$ of a week. To be more specific, we let z_{ri}^w be a binary variable which equals 1 if and only if ships arrive on day $w \in W$ of a week at (r, i) . Mathematically

$$t_{ri}^{arr} \bmod 7 = w' \iff \begin{cases} z_{ri}^{w'} = 1 \\ z_{ri}^w = 0, \quad w \in W \setminus \{w'\} \end{cases}$$

3.2. Port time windows

A port may have more than one berth. Hence, we let B_p be the set of berths at the port $p \in \mathcal{P}$. A berth may not always be available in a week because a port needs to serve more than one liner shipping company. Therefore, we define the parameter δ_b^w which equals 1 if berth $b \in \cup_{p \in \mathcal{P}} B_p$ is free on day $w \in W$ and 0 otherwise.

A ship uses exactly one berth when it visits a port of call and a berth cannot serve more than one ship at the same time. To formulate this constraint, let $z_{ri}^{b,w}$ be a binary variable which equals 1 if and only if ships use berth $b \in B_{p_{ri}}$ when visiting (r, i) on day w . For notational convenience, we define $z_{ri}^{b,-1} := z_{ri}^{b,6}$. We have

$$\sum_{r \in \mathcal{R}} \sum_{p_i \in I_{rp}, t_{ri}^{port} = 1} z_{ri}^{b,w} + \sum_{r \in \mathcal{R}} \sum_{p_i \in I_{rp}, t_{ri}^{port} = 2} (z_{ri}^{b,w-1} + z_{ri}^{b,w}) \leq \delta_b^w, \quad p \in \mathcal{P}, b \in B_p, w \in W \quad (4)$$

The first term on the left-hand side means that a ship uses the berth on day w if $t_{ri}^{port} = 1$ and the arrival day is w . The second term on the left-hand side means that a ship uses the berth on day w if $t_{ri}^{port} = 2$ and the arrival day is $w - 1$ or w . The overall constraint indicates that an available berth cannot serve more than one ship on the same day.

We assume further that each port has a premium berth \bar{b} , which is always available and can accommodate any number of ships. However, the liner shipping company needs to pay a high penalty cost $C_{\bar{b}}$ each time the berth is used.

3.3. Bunker consumption

Represent by $g_{ri}(v)$ the bunker consumption (tons/n mile) function with regard to the speed v on leg i of ship route r . Based on the

results in existing studies, we assume that $g_{ri}(v)$ is a power function of the form:

$$g_{ri}(v) = a_{ri} v^{b_{ri}}, \quad a_{ri} > 0, \quad b_{ri} > 2 \quad (5)$$

Hence, we let $Q_{ri}(t_{ri})$ be the bunker consumption on leg i of ship route r , and it can be calculated by:

$$Q_{ri}(t_{ri}) = L_{ri} g_{ri}(L_{ri}/t_{ri})$$

3.4. Model

Before presenting the model, we list the notation below.

Variables

- m_r Number of ships deployed on the ship route $r \in \mathcal{R}$
- t_{ri}^{arr} Arrival time (day) at port of call i on ship route r
- t_{r,N_r+1}^{arr} The time (day) when the ship returns to the 1st port of call on ship route r
- t_{ri} Sailing time (day) on the i th leg of ship route $r \in \mathcal{R}$
- k_{ri} An integer that is associated with the arrival time at the port of call i on ship route $r, r \in \mathcal{R}, i \in I_r$
- z_{ri}^b A binary variable which equals 1 if and only if ships use berth b when visiting the port of call i on ship route r including the premium berth \bar{b}
- $z_{ri}^{\bar{b}}$ A binary variable which equals 1 if and only if ships use a premium berth \bar{b} when visiting the port of call i on ship route r
- z_{ri}^w A binary variable that equals 1 if and only if ships arrive on day $w \in W$ of a week at the port of call i on ship route r
- $z_{ri}^{b,w}$ A binary variable that equals 1 if and only if ships arrive on day $w \in W$ of a week at the berth b at the port of call i on ship route r

Parameters

- α The bunker fuel price (USD/ton)
- δ_b^w A parameter that equals 1 if berth $b \in \cup_{p \in \mathcal{P}} B_p$ is free on day $w \in W$ and 0 otherwise
- a_{ri} A coefficient calibrated from operating data and satisfy $a_{ri} > 0$
- b_{ri} A coefficient calibrated from operating data and satisfy $b_{ri} > 1$
- B_p The set of berths at the port $p \in \mathcal{P}$
- C_r^{ship} The weekly operating cost of a ship deployed on ship route r
- $C_{\bar{b}}$ The penalty cost of using a premium berth \bar{b}
- I_r The set of ports of call on ship route r
- L_{ri} Oceanic distance (n mile) of the i th leg of route r
- N_r Number of ports on the ship route r
- p_{ri} The physical port that corresponds to the i th port of call on the ship route r
- t_{ri}^{port} Time (day) a ship spends at port of call i on the ship route r
- t_{ri}^{min} A minimum required value of the sailing time on the i th leg of route r
- \bar{V}_{ri} Number of containers (TEUs) on leg i on the ship route r
- V_r^{max} Maximum speed of the ships on the ship route r
- m_r^{max} Maximum number of ships deployed on the ship route r
- \mathbb{Z}^+ Set of nonnegative integers

The NSDPTW can be formulated as an optimization model below. The objective function is:

[NSDPTW]

$$\min \sum_{r \in \mathcal{R}} C_r^{\text{ship}} m_r + \alpha \sum_{r \in \mathcal{R}} \sum_{i \in I_r} L_{ri} a_{ri} (L_{ri} / (24t_{ri}))^{b_i} + C_b \sum_{r \in \mathcal{R}} \sum_{i \in I_r} z_{ri}^b \quad (6)$$

which aims to minimize the sum of ship cost and fuel cost while penalizing the violation of berth time windows.

The NSDPTW is subject to a number of constraints. The first set is the basic logical constraints in the schedule:

$$0 \leq t_{ri}^{\text{arr}} \leq 6, \quad r \in \mathcal{R} \quad (7)$$

$$t_{ri} \geq \left\{ t_{ri}^{\text{min}}, \left\lceil \frac{L_{ri}}{24V_r^{\text{max}}} \right\rceil \right\}, \quad r \in \mathcal{R}, i \in I_r \quad (8)$$

$$t_{r,i+1}^{\text{arr}} = t_{ri}^{\text{arr}} + t_{ri}^{\text{port}} + t_{ri}, \quad r \in \mathcal{R}, i \in I_r \quad (9)$$

$$t_{r,N_r+1}^{\text{arr}} = t_{r1}^{\text{arr}} + 7m_r, \quad r \in \mathcal{R} \quad (10)$$

$$m_r \in \{1, 2, 3, \dots, m_r^{\text{max}}\}, \quad r \in \mathcal{R} \quad (11)$$

$$t_{ri}^{\text{arr}} \in \mathbb{Z}^+, \quad r \in \mathcal{R}, i \in I_r \quad (12)$$

The objective function (6) minimizes the sum of ship cost, bunker cost and penalty cost. Constraint (7) eliminates symmetric solutions. Constraint (8) confirms that the sailing time on a leg is not less than a minimum required value and ships cannot sail at a speed that exceeds V_r^{max} . Constraint (9) defines the relation of different time components in a round-trip journey. Constraint (10) defines the time when the ship returns to the 1st port of call after one round-trip. Constraint (11) indicates that the number of ships is a positive integer that does not exceed a pre-specified maximum value. Constraint (12) indicates that the arrival time at each port of call is a nonnegative integer.

The second set of constraints formulates the day of a week for arrival at each port of call in the network:

$$\sum_{w \in W} w z_{ri}^w = t_{ri}^{\text{arr}} - 7k_{ri}, \quad r \in \mathcal{R}, i \in I_r \quad (13)$$

$$\sum_{w \in W} z_{ri}^w = 1, \quad r \in \mathcal{R}, i \in I_r \quad (14)$$

$$z_{ri}^w \in \{0, 1\}, \quad r \in \mathcal{R}, i \in I_r, w \in W \quad (15)$$

$$k_{ri} \in \{0, 1, 2, \dots, m_r - 1\}, \quad r \in \mathcal{R}, i \in I_r \quad (16)$$

Constraint (13) defines the arrival day of a week at each port call on the route. Constraint (14) requires that a ship arrives exactly once a week at each port of call. Constraint (15) defines z_{ri}^w as a binary variable. Constraint (16) defines the auxiliary variable k_{ri} as a nonnegative integer.

The third set of constraints considers the availability of berths:

$$\sum_{r \in \mathcal{R}} \sum_{p_i \in I_r, t_{ri}^{\text{port}}=1} z_{ri}^{bw} + \sum_{r \in \mathcal{R}} \sum_{p_i \in I_r, t_{ri}^{\text{port}}=2} (z_{ri}^{b,w-1} + z_{ri}^{bw}) \leq \delta_b^w, \quad p \in \mathcal{P}, b \in B_p, w \in W \quad (17)$$

$$\sum_{b \in B_{p_i} \cup \{\bar{b}\}} z_{ri}^b = 1, \quad r \in \mathcal{R}, i \in I_r \quad (18)$$

$$z_{ri}^{bw} \leq z_{ri}^b, \quad r \in \mathcal{R}, i \in I_r, b \in B_{p_i} \cup \{\bar{b}\}, w \in W \quad (19)$$

$$z_{ri}^{bw} \leq z_{ri}^w, \quad r \in \mathcal{R}, i \in I_r, b \in B_{p_i} \cup \{\bar{b}\}, w \in W \quad (20)$$

$$z_{ri}^{bw} \geq z_{ri}^b + z_{ri}^w - 1, \quad r \in \mathcal{R}, i \in I_r, b \in B_{p_i} \cup \{\bar{b}\}, w \in W \quad (21)$$

$$z_{ri}^{bw} \in \{0, 1\}, \quad r \in \mathcal{R}, i \in I_r, b \in B_{p_i} \cup \{\bar{b}\}, w \in W \quad (22)$$

$$z_{ri}^b \in \{0, 1\}, \quad r \in \mathcal{R}, i \in I_r, b \in B_{p_i} \cup \{\bar{b}\} \quad (23)$$

Constraint (17) indicates that an available berth cannot serve more than one ship on the same day. Constraint (18) requires that a ship uses exactly one berth each time it visits a port including the premium berth \bar{b} . Constraints (19)–(21) impose that a ship uses a berth

at a port of call once a week. Constraints (22) and (23) define z_{ri}^{bw} and z_{ri}^b as binary variables.

4. Solution method

The model [NSDPTW] is a mixed-integer nonlinear non-convex optimization problem. It is difficult to solve because (i) it has a large number of discrete variables and (ii) it has nonlinear objective function. After carefully examining the properties of the problem, we develop a tailored solution method that overcomes these difficulties.

4.1. Linearization of the objective function

The nonlinear term $t_{ri}^{-b_i}$ in the objective function (6) can be linearized due to the following property.

Proposition 4.1. *The optimal t_{ri} can be determined by the optimal z_{ri}^w and $z_{r,i+1}^w$, $w \in W$.*

Proof. Given z_{ri}^w and $z_{r,i+1}^w$, $w \in W$, t_{ri} can only take values as follows: $\sum_{w \in W} w z_{r,i+1}^w - \sum_{w \in W} w z_{ri}^w - t_{ri}^{\text{port}}$, $\sum_{w \in W} w z_{r,i+1}^w - \sum_{w \in W} w z_{ri}^w - t_{ri}^{\text{port}} + 7$, $\sum_{w \in W} w z_{r,i+1}^w - \sum_{w \in W} w z_{ri}^w - t_{ri}^{\text{port}} + 14$, etc., while satisfying constraint (8). The possible values of t_{ri} differ from each other by an integer number of weeks. As a result, different t_{ri} impact the bunker cost on the leg and the number of ships to deploy on the ship route, and do not affect other components of the model. Hence, given $\sum_{w \in W} w z_{r,i+1}^w - \sum_{w \in W} w z_{ri}^w$, we can find the best t_{ri} . □

We use an example to demonstrate the proposition. Suppose that $z_{ri}^2 = 1$ (arrival on Tuesday), $z_{r,i+1}^3 = 1$ (arrival on Wednesday), $t_{ri}^{\text{port}} = 2$, $\alpha = 500$, $L_{ri} = 12,000$, $a_{ri} = 0.001$, $b_i = 2$, $C_r^{\text{ship}} = 500,000$.

Table 1
Impact of different t_{ri} on the costs (million USD/week).

t_{ri}	20	27	34	41	48	55	62
Bunker cost	3.75	2.06	1.30	0.89	0.65	0.50	0.39
Ship cost	0.00	0.50	1.00	1.50	2.00	2.50	3.00
Total cost	3.75	2.56	2.30	2.39	2.65	3.00	3.39

Table 2
Ship fleet.

Ship type (TEUs)	1200	2600	6500
Weekly cost (USD)	94,829	159,621	267,393
Max speed (knot)	18.3	20.9	20.9
Bunker consumption (ton/day)	0.000287 v^2	0.000358 v^2	0.000559 v^2

Table 3
Parameters of the three ship routes.

Route	Port of call	Port	Port ID	Port time	Distance
1	1	Singapore	1	2	2895
	2	Karachi	2	2	509
	3	Nhava Sheva	3	1	891
	4	Colombo	4	1	1575
2	1	Singapore	1	1	767
	1	Surabaya	5	2	767
3	1	Singapore	1	1	668
	2	Semarang	6	1	668

Suppose further that the minimum value of t_{ri} determined by constraint (8) is 15. Then t_{ri} can take the value of 20, 27, 34, etc. Table 1 reports the bunker cost on the leg, the additional ship cost compared with the minimum possible $t_{ri} = 20$, and the total cost. Hence, the optimal value of t_{ri} is 34.

Based on Proposition 4.1, we can linearize the objective function (6). We define binary variables y_{ri}^w to be 1 if and only if the difference of the arrival times at $\langle r, i + 1 \rangle$ and $\langle r, i \rangle$ is w days, $w \in W$. We represent by t_{ri}^{w*} the optimal value of t_{ri} when the difference of the arrival times at $\langle r, i + 1 \rangle$ and $\langle r, i \rangle$ is w days. We stress here that t_{ri}^{w*} can be computed a priori and is not a decision variable. We further define auxiliary binary variables \bar{k}_{ri} . The model [NSDPTW] can be transformed to the following integer linear programming (ILP) model:

[NSDPTW-ILP]

$$\min \sum_{r \in \mathcal{R}} C_r^{\text{ship}} m_r + \alpha \sum_{r \in \mathcal{R}} \sum_{i \in I_r} L_{ri} a_{ri} (L_{ri}/24)^{b_i} \sum_{w \in W} (t_{ri}^{w*})^{-b_i} y_{ri}^w + C_b \sum_{r \in \mathcal{R}} \sum_{i \in I_r} z_{ri}^b \quad (24)$$

with constraints

$$t_{ri} = \sum_{w \in W} t_{ri}^{w*} y_{ri}^w, \quad r \in \mathcal{R}, i \in I_r \quad (25)$$

$$\sum_{w \in W} w y_{ri}^w = \sum_{w \in W} w z_{ri}^w - \sum_{w \in W} w z_{ri}^{\text{port}} - t_{ri}^{\text{port}} + 7 \bar{k}_{ri}, \quad r \in \mathcal{R}, i \in I_r \quad (26)$$

$$\sum_{w \in W} y_{ri}^w = 1, \quad r \in \mathcal{R}, i \in I_r \quad (27)$$

$$y_{ri}^w \in \{0, 1\}, \quad r \in \mathcal{R}, i \in I_r, w \in W \quad (28)$$

$$\bar{k}_{ri} \in \{0, 1, 2\}, \quad r \in \mathcal{R}, i \in I_r \quad (29)$$

and constraints from (7)–(23).

Constraint (25) defines t_{ri}^{w*} which is the optimal value of t_{ri} when the difference of the arrival times at $\langle r, i + 1 \rangle$ and $\langle r, i \rangle$ is w days. Constraint (26) defines the difference of the arrival times between ports. Constraint (27) requires that the difference of the arrival times is a fix number between 0 and 6 at each port of call. Constraint (28) defines y_{ri}^w as a binary variable. Constraint (29) defines the auxiliary variable \bar{k}_{ri} as a nonnegative integer.

4.2. Iterative optimization approach

Model [NSDPTW-ILP] is an integer linear programming formulation. Small-scale instances can be solved by off-the-shelf solvers. To solve large-scale instances, we propose an iterative optimization approach below:

Algorithm 1. Iterative optimization approach

Step 0. (Initialization): We define vector $(m_r = m_r^*, r \in \mathcal{R}; t_{ri}^{\text{arr}} = t_{ri}^{\text{arr}*}, r \in \mathcal{R}, i \in I_r; z_{ri}^{bw} = z_{ri}^{bw*}, r \in \mathcal{R}, i \in I_r, b \in B_{p_{ri}} \cup \{\bar{b}\}, w \in W)$ as the best solution obtained (for brevity, we use $(m_r^*, t_{ri}^{\text{arr}*}, z_{ri}^{bw*})$ to represent the vector). Note that we do not need to record the values of the variables $t_{ri}, z_{ri}^w, k_{ri}, z_{ri}^b, y_{ri}^w$, or \bar{k}_{ri} , because they can be derived from $(m_r^*, t_{ri}^{\text{arr}*}, z_{ri}^{bw*})$. Find a feasible $(m_r^*, t_{ri}^{\text{arr}*}, z_{ri}^{bw*})$. Since there is a premium berth at each port, such a feasible schedule always exists. The total cost can be calculated and is represented by C^* .

Step 1. (Ship route schedule optimization): Define $C_0 = C^*$.

Step 1.0 Set $\bar{r} = 1$. Define $C_1 = C^*$.

Step 1.1 (Optimize the schedule for ship route \bar{r}) Fix the schedule of all ship routes $r \in \mathcal{R} \setminus \{\bar{r}\}$ and optimize schedule for ship route \bar{r} . That is, we solve model [NSDPTW-ILP] with the following constraints:

$$m_r = m_r^*, \quad r \in \mathcal{R} \setminus \{\bar{r}\} \quad (30)$$

$$t_{ri}^{\text{arr}} = t_{ri}^{\text{arr}*}, \quad r \in \mathcal{R} \setminus \{\bar{r}\}, i \in I_r \quad (31)$$

$$z_{ri}^{bw} = z_{ri}^{bw*}, \quad r \in \mathcal{R} \setminus \{\bar{r}\}, i \in I_r, b \in B_{p_{ri}} \cup \{\bar{b}\}, w \in W \quad (32)$$

Update $(m_{\bar{r}}^*, t_{\bar{r}i}^{\text{arr}*}, z_{\bar{r}i}^{bw*})$ using the corresponding optimal solution obtained. The resulting total cost is denoted by \hat{C} . Note that $\hat{C} \leq C^*$. Set $C^* = \hat{C}$.

Step 1.2 If $\bar{r} < |\mathcal{R}|$, set $\bar{r} = \bar{r} + 1$ and go to Step 1.1.

Step 1.3 If $C_1 > C^*$, go to Step 1.0.

Step 1.4 Go to Step 2.0.

Step 2 (Port arrival time optimization):

Step 2.0 Set $\bar{p} = 1$. Define $C_2 = C^*$.

Step 2.1 (Optimize the arrival times at port \bar{p}) Fix the schedule of all ports $p \in \mathcal{P} \setminus \{\bar{p}\}$ and optimize schedule at port \bar{p} . That is, we solve model [NSDPTW-ILP] with the following constraints:

$$m_r = m_r^*, \quad r \in \mathcal{R} \quad (33)$$

$$t_{ri}^{\text{arr}} = t_{ri}^{\text{arr}*}, \quad r \in \mathcal{R}, i \in I_r \setminus I_{r\bar{p}} \quad (34)$$

$$z_{ri}^{bw} = z_{ri}^{bw*}, \quad r \in \mathcal{R}, i \in I_r \setminus I_{r\bar{p}}, b \in B_{p_{ri}} \cup \{\bar{b}\}, w \in W \quad (35)$$

Update $(t_{\bar{p}i}^{\text{arr}*}, z_{\bar{p}i}^{bw*}, r \in \mathcal{R}_p, i \in I_{r\bar{p}})$ using the corresponding optimal solution obtained. The resulting total cost is denoted by \hat{C} . Note that $\hat{C} \leq C^*$. Set $C^* = \hat{C}$.

Step 2.2 If $\bar{p} < |\mathcal{P}|$, set $\bar{p} = \bar{p} + 1$ and go to Step 2.1.

Step 2.3 If $C_2 > C^*$, go to Step 2.0.

Step 2.4 If $C_0 > C^*$, go to Step 1.

Step 2.5 Stop (It means we cannot improve the solution by optimizing the schedule for one route or optimizing the arrival times at one port). □

Table 4 Available time at each port.

Port ID	Port	Berth	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
1	Singapore	1	Free	Free		Free	Free		
		2	Free	Free					
2	Karachi	1	Free	Free	Free	Free	Free	Free	Free
		2			Free				
3	Nhava Sheva	1		Free		Free	Free		
		2	Free	Free		Free	Free		
4	Colombo	1	Free	Free			Free	Free	
5	Surabaya	1	Free	Free		Free	Free		
6	Semarang	1	Free	Free				Free	Free
		2			Free				

Table 5
Different cases of available time at Singapore.

Case	Berth	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
1	1					Free		
	2	Free	Free			Free		
2	1					Free		Free
	2	Free	Free			Free		
3	1			Free		Free		Free
	2	Free	Free			Free		
4	1			Free		Free		Free
	2	Free	Free			Free	Free	
5	1		Free	Free		Free		Free
	2	Free	Free			Free	Free	
6	1		Free	Free		Free		Free
	2	Free	Free	Free		Free	Free	
7	1		Free	Free		Free	Free	Free
	2	Free	Free	Free		Free	Free	

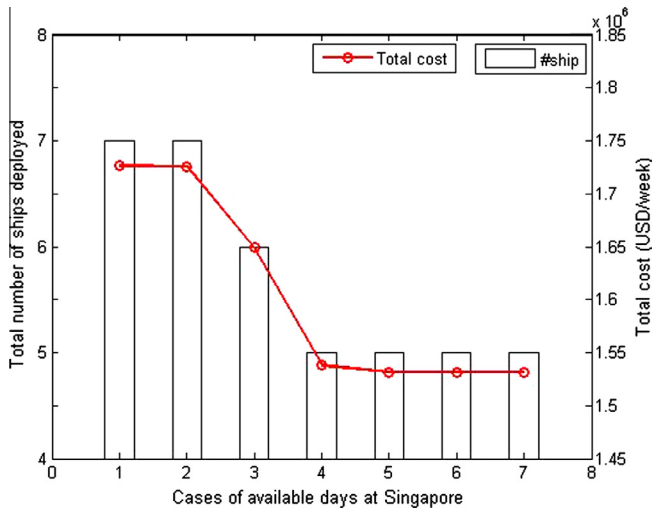


Fig. 3. Impact of port time windows.

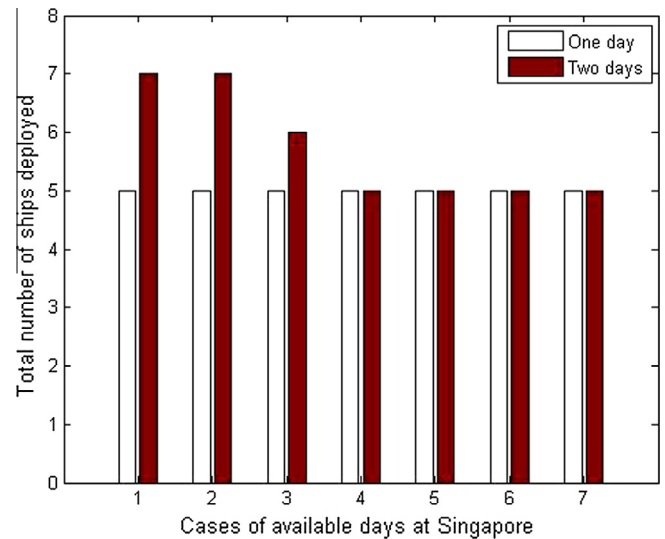


Fig. 4. Impact of port time at Singapore on ship route 1 on the number of ships deployed.

5. Case studies

5.1. A six-port case study

We conduct a case study based on the ocean carrier APL to evaluate the applicability of the proposed models and methods. The network has a total of 6 ports, as shown in Fig. 2. There are 3 types of ship and 3 ship routes, as shown in Table 2, which also shows the daily bunker consumption functions related to the sailing speed v (knot). The port time at each port of call (day) and distance on each leg (n mile) are shown in Table 3. We assume that these 6 ports have a total of 10 berths, whose available times are shown in Table 4 and the bunker price $\alpha = 400$ USD/ton.

5.1.1. Impact of port time windows

Firstly, we examine the effect of port time windows on the total cost and the optimal schedule. We consider the example of the port of Singapore, which is visited three times a week on all the ship routes in the network. Both the operator of the port of Singapore and the liner shipping company that operates the network are interested in looking at the result if more available berth time is provided at Singapore. We hence examine 7 berth availability cases of Singapore, as shown in Table 5. The two berths at Singapore have more and more available days from case 1 to case 7.

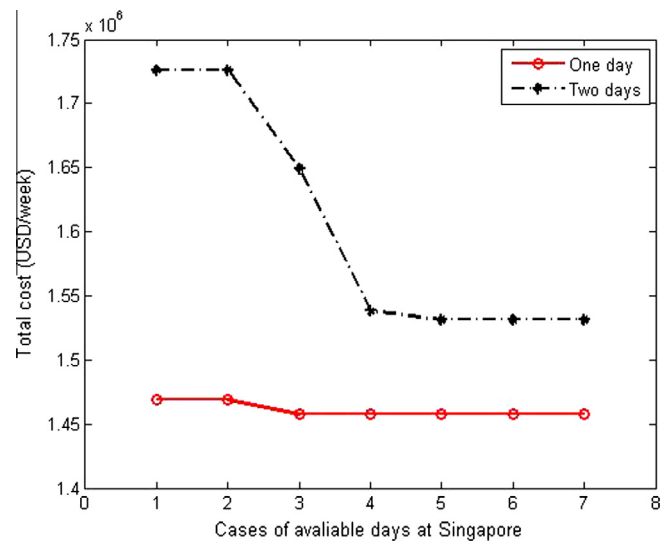


Fig. 5. Impact of port time at Singapore on ship route 1 on the total cost.

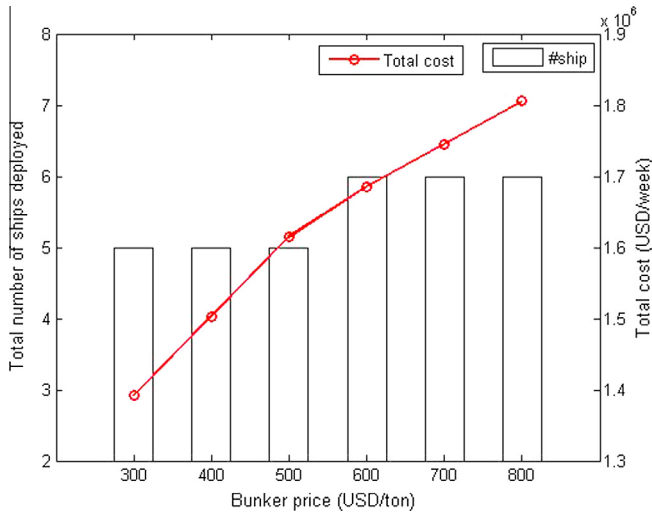


Fig. 6. Result of bunker prices on the total cost and the number of ships deployed.

The results of the 7 berth availability cases are shown in Fig. 3. It can be seen that more available days at Singapore leads to a lower total cost: from case 1 to case 7, the total cost is reduced by 194,969 USD/week due to the increase of the number of available days at berth. Fig. 3 also demonstrates that the number of available days at a port may affect the optimal number of ships deployed. In particular, when there are not many available days for berthing, more ships have to be deployed to satisfy the stringent port time window constraints.

The optimal ship route schedules for the 7 berth availability cases are shown in Table 6, which reports the day of arrival at each port of call. We observe that the time windows at the port of Singapore affect the optimal arrival times at all the ports of call in the liner shipping network.

5.1.2. Consequence of port efficiency

It is important for port operators to improve the container handling efficiency, i.e., reducing t_i^{port} . To investigate the effect of port handling efficiency, we compare the default parameter settings with the situation of reducing the port time at Singapore on ship route 1 from two days to one day. We find that the optimal number of ships and the total cost increase with the time spent at Singapore, as shown in Figs. 4 and 5, in all the 7 cases of available days at Singapore. These results demonstrate that it is of significant importance for port operators to improve the container handling efficiency.

5.1.3. Result of bunker prices

In this section, we study the impact of the bunker price on the total cost and the number of ships deployed in the liner shipping network. We increase the bunker price from 300, 400, 500, 600, 700, to 800 and the other parameters are the same as Tables 2–4. The results are shown in Fig. 6. We observe that a higher bunker price always leads to a higher total cost for liner shipping companies and there is a rise in the number of ships used when the bunker price becomes higher. This is because when the bunker price is higher, the sailing speed is reduced by deploying more ships to make the bunker consumption lower. Overall, the total cost increases concavely with the increase of bunker price.

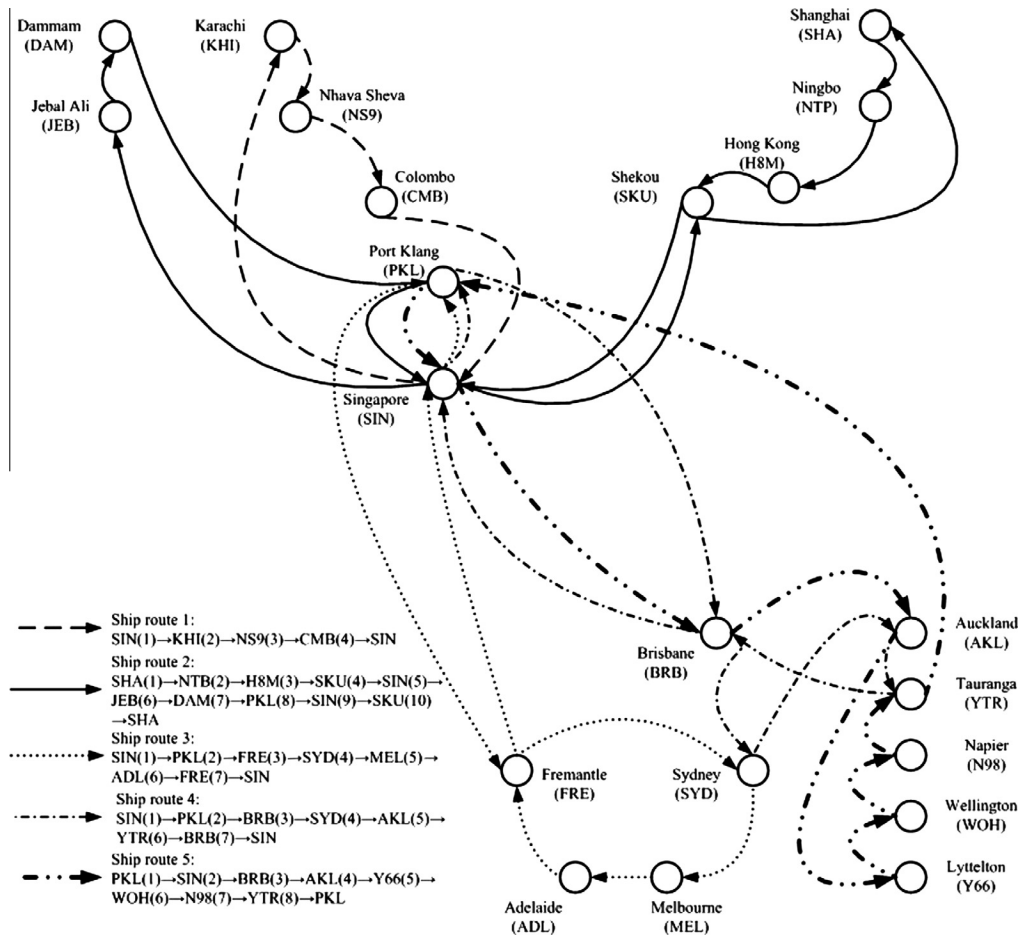


Fig. 7. A liner shipping network with five ship routes.

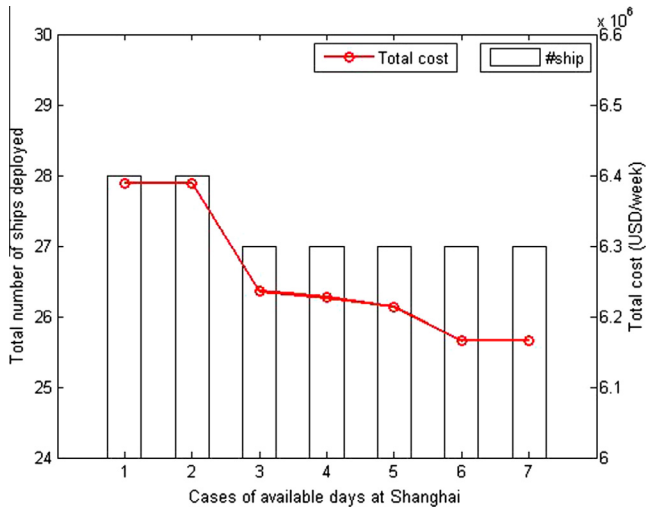


Fig. 8. Impact of port time windows.

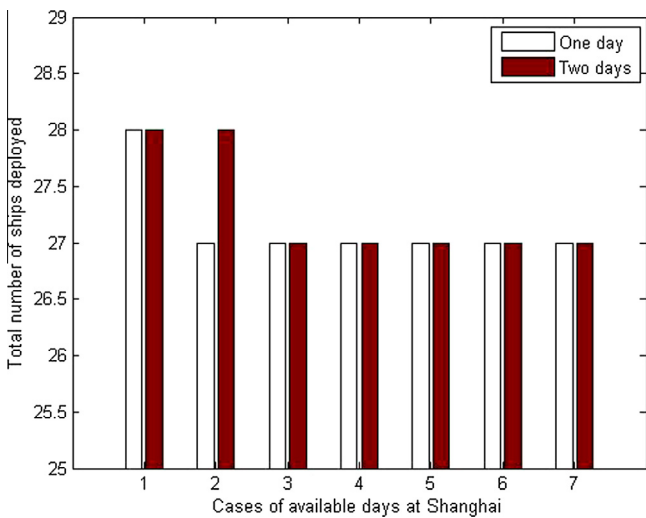


Fig. 9. Impact of port time at Shanghai on ship route 2 on the number of ships deployed.

5.2. A 21-port case study

We examine another a case study based on the ocean carrier APL to evaluate the applicability of the proposed models and methods. The network has a total of 21 ports, as shown in Fig. 7. The other parameters are assumed based on practical operations.

5.2.1. Impact of port time windows

Firstly, we examine the effect of port time windows on the total cost and the optimal schedule. We consider the example of the port of Shanghai, which is visited one time a week on in this network. We hence examine 7 berth availability cases of Shanghai. The berths at Shanghai have more and more available days from case 1 to case 7.

The results of the 7 berth availability cases are shown in Fig. 8. It can be seen that more available days at Shanghai leads to a lower total cost: from case 1 to case 7, the total cost is reduced by 222,675 USD/week due to the increase of the number of available days at berth. Fig. 8 also demonstrates that the number of available days at a port may affect the optimal number of ships deployed. In particular, when there are not many available days for berthing, more ships have to be deployed to satisfy the stringent port time window constraints.

5.2.2. Consequence of port efficiency

It is important for port operators to improve the container handling efficiency, i.e., reducing t_i^{port} . To investigate the effect of port handling efficiency, we compare the default parameter settings with the situation of reducing the port time at Shanghai on ship route 1 from two days to one day. We find that the optimal number of ships and the total cost increase with the time spent at Shanghai, as shown in Figs. 9 and 10, in all the 7 cases of available days at Shanghai. These results demonstrate that it is of significant importance for port operators to improve the container handling efficiency.

5.2.3. Result of bunker prices

In this section, we study the impact of the bunker price on the total cost and the number of ships deployed in the liner shipping network. We increase the bunker price from 300, 400, 500, 600, 700, to 800. The results are shown in Fig. 11. We observe that a higher bunker price always leads to a higher total cost for liner shipping companies and there is a rise in the number of ships used when the bunker price becomes higher. This is because when the bunker price is higher, the sailing speed is reduced by deploying more ships to make the bunker consumption lower. Overall, the total cost increases concavely with the increase of bunker price.

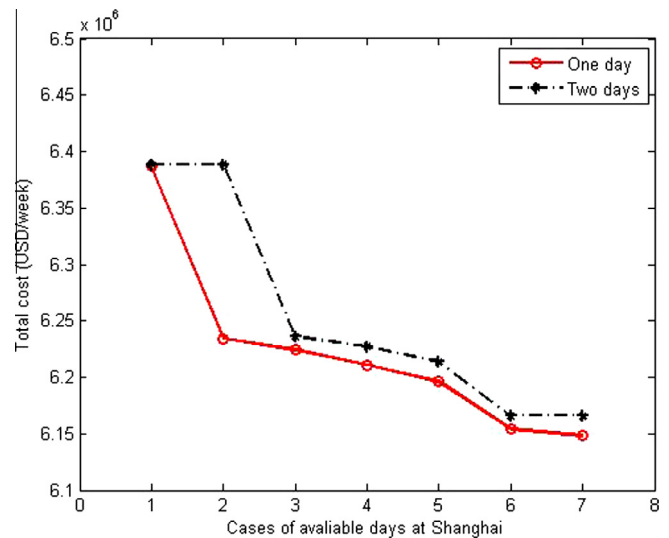


Fig. 10. Impact of port time at Shanghai on ship route 2 on the total cost.

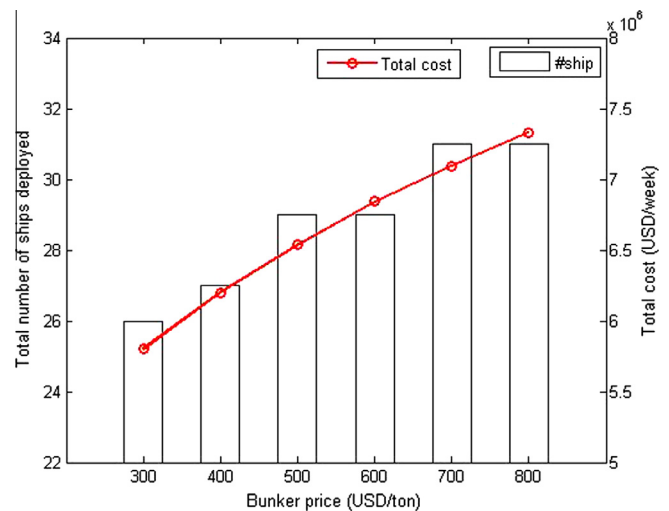


Fig. 11. Result of bunker prices on the total cost and the number of ships deployed.

Table 6
Impact of port time window on the optimal schedule.

Route	Port of call	Port	Cases 1	Case 2	Cases 3	Case 4	Case 5	Cases 6	Case 7
1	1	Singapore	0	0	0	4	1	1	1
	2	Karachi	8	8	8	12	11	11	11
	3	Nhava Sheva	11	11	11	15	15	15	15
	4	Colombo	15	15	15	19	18	18	18
2	1	Singapore	4	4	4	0	0	0	0
	5	Surabaya	10	10	10	3	3	3	3
3	1	Singapore	4	6	2	2	1	1	2
	6	Semarang	12	13	6	5	5	5	5

6. Conclusions and future work

This paper has studied the practical liner shipping network schedule design problem with port time windows. This is a significant tactical planning decision problem because it considers the availability of ports when planning liner shipping services. As a result, the designed schedule can be applied in practice without or with only minimum revisions. This problem is formulated as a mixed-integer nonlinear non-convex optimization model. In view of the problem structure, we reformulated the problem as an integer linear optimization model and proposed an iterative optimization approach.

The proposed solution method was applied to two networks, consisting of six ports and 21 ports, operated by APL. The results demonstrate that the port time windows, port handling efficiency, and bunker price all affect the total cost, the optimal number of ships to deploy, and the optimal schedule. Higher availability at ports, shorter port time, and lower bunker price result in a lower total cost and a smaller number of ships to deploy. Therefore, port operators can apply the proposed method to quantify whether the benefits to liner shipping companies are worthwhile compared to the cost of expanding the ports' capacity and improving their efficiency. Liner shipping companies may need to charter in more ships if they predict that the future bunker price will increase.

In future we will investigate the schedule design problem with port time windows while considering the inventory cost. Moreover, we will also investigate how the schedule design and container routing can be jointly planned.

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