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Z-number DEA: A new possibilistic DEA in the context of Z-numbers

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ABSTRACT

Data envelopment analysis (DEA) is a methodology that uses multiple inputs and outputs for measuring the efficiencies of a set of decision making units (DMUs). When data are crisp, conventional DEA models are used. However the values of inputs and outputs in many cases are imprecise and vague. In addition, most of these data are expert-based. Thus, taking into account expert's reliability is quite important. In this paper we propose a Z-number version of the CCR (named after Charnes, Cooper, and Rhodes) and BCC (named after Banker, Charnes and Coopers) DEA models. The proposed method can be converted into the fuzzy DEA model when experts are confident about their opinions. Also, it can be converted into the conventional DEA models when the inputs and outputs are crisp numbers. In this study, the Z-number DEA model is transformed into possible linear programming and then by applying an alternative α-cut approach, a crisp linear programming model is obtained. Furthermore, the proposed model is applied to a portfolio selection problem in IS/IT (Information Systems/Information Technology) project to tackle uncertainties, interactions between projects and reliabilities. To the best of our knowledge, this is the first study that presents a unique Z-number DEA model.

1. Introduction

DEA is a very important method in decision making that incorporates multiple inputs and outputs. It is an applicable methodology for selecting the best DMUs in real problems such as customer satisfaction, gas consumption, and investment selection in most engineering environments. But in most real world problems the data are represented by experts. Each expert often gives a linguistic expression for decision making data in real problems. It is obvious that expert has a different point of view about the same variables. Moreover, the reliabilities of experts in decision making problem is of high importance. Thus, in this paper a Z-number approach is developed based on DEA model to handle such situations. This is the first study proposing a Z-number DEA approach for real world problems.

DEA developed by Charnes et al. [1] is known as a performance measurement technique for computing the efficiency values of decision making units (DMUs) of a given system through comparing the values of outputs and inputs. Charnes et al. [1] generalized the definition of efficiency (single-output to single-input ratio) to multiple inputs and outputs. In Charnes, Cooper and Rhodes (CCR) model, it is claimed that the efficiency of a DMU can be created as a maximum ratio of weighted outputs to weighted inputs, with a restriction condition that the same ratio for all DMUs must be less than or equal to one. The DEA model must be run several times, once for each DMU to obtain the relative efficiency of all DMUs. Banker et al. [2] developed the BCC model to predict the efficiency of DMUs by referring to the efficient boundary [2]. It also recognizes whether a DMU is operating in increasing, decreasing or constant returning to scale [3].

DEA, like other ranking methods has some limitations. The most important limitation of this approach is its sensitivity to data. There are usually missing data, imperfect data or lack of data in forecasting problems. Therefore, the nature of data in real environments such as gas consumption, employee performance, machine performance, customer satisfaction and other engineering optimization problems is vague and linguistic. This means that data could not be collected in a deterministic fashion and new models for tackling such situations are required in this area. Since DEA focuses on frontiers or boundaries, any deviation from real data can cause a variation in the obtained efficiencies by DEA. Therefore, the input or output data must be exact for prosperous application of DEA and this method is very sensitive to data [4]. Moreover, in some real world problems, the data for evaluation of DMUs are often expert-based and in linguistic forms (such as good, medium or bad). Moreover, each expert has a different point of view about the linguistic variables. In other words, "judgmental data", as

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inputs and/or outputs, impose certain types of uncertainty and fuzziness to the problem. In such cases, fuzzy data envelopment analysis (FDEA), as a good alternative for DEA in uncertain environments can be used to cope with the uncertainty/fuzziness pertaining to the qualitative and judgmental data. This, in turn, provides a more realistic framework to the decision makers due to utilizing knowledge and judgment of the experts of the system. Sengupta [5] was the first person that introduced the fuzziness merging into the DEA model by defining tolerance levels. Several FDEA models are developed and used for many applications. Some of these models are mentioned in the next section.

It is obvious that data obtained from experts are vague and fuzzy, because in most data gathering situations experts most like to give the information in the base of mixed quantitative and qualitative numbers for modeling their sureness. Moreover, it is important to consider variation in viewpoint values and reliability of experts. Also, the concept of possibility is better than probability for future variables and predictions. This is because estimating the exact distribution of variables is not possible and occurrence of possible situations in real problems may be considered. This paper proposes a new integrated fuzzy and possibilistic DEA model that incorporates the reliability concept for expert judgment. In this model the fuzziness in the variables and possible situation in the future are combined to the formal DEA model. To achieve the objective of this study the context of Z-numbers introduced by Zadeh is used in the modeling process [6].

The concept of Z-number is intended to provide a basis for computation with numbers which are not totally reliable. A Z-number has two components denoted as Z = (A, B) for estimating the variable X. The first number (A) is the limitation on the values which X can take. The second number (B) is a restriction on the degree of reliability (certainty) that X is A [6]. It is assumed that the input and output variables of our DEA model are Z-numbers. Hence, the proposed model is referred to as a Z-number DEA model. This model is very practical in the real problems such as portfolio selection, energy consumption, and customer satisfaction.

The remainder of this paper is structured and discussed in the following sections. In Section 2, related literature has been investigated and discussed. In Section 3 overview of data envelopment analysis (DEA), fuzzy DEA and Z-number are discussed. Then, the proposed model and its features are discussed. Section 4 explains the case study and its features. Experimental results are presented in Section 5. Finally, summary and conclusions of our research are described in Section 6.

2. Literature review

2.1. Crisp, deterministic DEA models

There are two types of DEA models. Constant returns-to-scale (CRS) or CCR model that has been introduced by Charnes et al. [1] and a variable returns-to-scale (VRS) or BCC model that has been proposed by Banker et al. [7]. DEA applications are numerous in health care services, manufacturing, gas consumption, employee satisfaction, supplying chain management, etc. For example, in banking Emrouznejad et al. [8] recommended a new structure assessing the bank branches by using DEA method. For a recent bibliography of DEA see Manandhar and Tang [9].

2.2. Stochastic DEA models

Olesen and Petersen proposed a chance constrained DEA model which uses a piecewise linear envelopment of trust areas for observed stochastic multiple inputs and multiple outputs. Cooper et al. [10] combined the "satisficing concepts" into DEA and developed a new DEA model. Li [11] generalized two formal DEA models by incorporating random errors into input and output data. Sueyoshi [12] presented a stochastic DEA model and reformulated it as "DEA future analysis". He developed input-oriented CCR model with stochastic parameters and normal assumptions. He reformulated DEA future model and like previous scholars used probability bound for constraints which include stochastic parameters or outputs. His model is useful for DMUs that have stochastic outputs and crisp inputs.

2.3. Fuzzy DEA models

As mentioned before Karsak [13] was the first person that used the concept of fuzzy in the DEA model. The implementation of fuzzy theory in DEA is categorized in four groups: The tolerance approach, the α-level based approach, the fuzzy ranking approach and the possibility approach [13]. In the tolerance approach Kahraman and Tolga [14] incorporated the uncertainty into DEA models by defining tolerance levels on constraint outrages.

The α-level based approach is the most popular model in FDEA models. In this approach FDEA model is converted into the pair of parametric programs in order to find the efficiency scores of lower bound and upper bound. Girod [15] used this approach to formulate FDEA model. In this paper, the inputs and outputs could fluctuate between risk and infeasible bounds. Meada et al. [16] used this approach to gain the fuzzy interval efficiency of DMUs. Kao and Lio [17] used the idea of transforming the FDEA model to formal DEA models. They found membership functions of efficiency measures by using the Zadeh's extension principle and α-cut approach [18]. They proposed a couple of two-level mathematical model to calculate the lower bounds and upper bounds of efficiencies. Azadeh and Alem [19] used the simulation analysis to find a flexible Deterministic, Stochastic and Fuzzy DEA approach for supplying chain risk and vendor selection problem. In FDEA model they used α-cut method in five levels for α, to convert FDEA into interval programming. Saati et al. [20] introduced a Fuzzy CCR model as a possibilistic programming problem and converted it into interval programming problem using α-cut approach. The results that obtained from interval programming problem could be solved as a crisp LP model for a given α. Their model is derived for a particular case where the inputs and outputs are triangular fuzzy numbers. Azadeh et al. [21] proposed an integrated model of simulation FDEA to construct some scenarios with simulation model and determined optimum operator's allocation in cellular manufacturing systems with FDEA model. They used Saati et al.'s [20] method to rank DMUs in their FDEA model. Also, they applied clustering method for ranking the DMUs by Fuzzy C-Means method to show the desirability of operator allocation. Liu [22] developed an FDEA model that embedded with trust region concept. He applied α-cut and Zadeh's extension approach to transform their model into the pair of parametric mathematical programming and obtained the lower bounds and upper bounds of DMUs. Jahanshahloo et al. [23] proposed a fuzzy $l_1 - norm$ model that its variables are trapezoidal membership function. They used a Jiménez [24] ranking fuzzy numbers method and obtained a crisp α-parametric model.

The fuzzy ranking approach is another approach in the FDEA literature. The goal of this work is to detect the efficiency scores of DMUs using fuzzy linear programming which requires ranking fuzzy sets [25]. Moreover, a fuzzy CCR model is proposed in which fuzzy constraints are transformed into crisp constraints by defining the possibility levels. It is assumed that the variables of their model are triangular fuzzy numbers. Lertworasirikul et al. [26] proposed two FDEA models depending on ranking methods. The first model uses the Ramík and Řimánek [27] ranking method to obtain crisp efficiency scores of DMUs. In the second model they used the Leon et al. [28] ranking method to compute the efficiency scores for each possibility level. Jahanshahloo et al. [29] introduced the model that uses fuzzy ranking method for solving slack-based measure (SBM) model in DEA when the variables are triangular fuzzy numbers.

The possibility approach was first incorporated into FDEA model by Guo et al. [30]. Lertworasirikul et al. [31] developed two methods for solving decision making problem in FDEA models called "possibility and credibility approach". They proposed a possibility approach from optimistic and pessimistic views by assuming the fuzziness in objective function and constraints with possibility measures. In their second model that contains credibility approach, FDEA model converted into Credibility Programming-DEA model and fuzzy variables were replaced by "expected credits". Lertworasirikul et al. [4,32] proposed a possibility approach that is treated with the fuzzy constraints as fuzzy events. They solved a fuzzy CCR model with their approach. They assumed that the data are trapezoidal fuzzy numbers and converted the FDEA model into the possibility LP approach problem. Lertworasirikul et al. [33] used possibility and credibility approaches for solving the Fuzzy BCC models. They used the concept of chance-constrained programming and possibility of fuzzy constraints to obtain possibility BCC models. Khodabakhshi et al. [34] formulated two fuzzy and stochastic additive models to determine returns to scale in DEA. To learn more about fuzzy DEA models (see [35]).

2.4. Portfolio selection

Portfolio selection is a method for selecting investments from list of candidate investments in order to maximize profitability and other objectives without violating resource and other restrictions [36]. Portfolio problems are divided in two main categories according to [37]: Static and dynamic problems. Bard et al. [38] proposed that dynamic problems deal with two projects: active and candidate projects. Projects that are in progress are called active projects and projects that have not been started yet, are called candidate projects. In this category, there are decision points that conclude active and candidate projects in different budget levels. In static portfolio selection problems we deal only with candidate projects and limited resources for assigning to projects and activating them [39]. In this paper and in the case study section we focus on the static portfolio selection problem.

Early attempts on portfolio selection models usually used constrained optimization problems to maximize their considered objectives and prevent violating the constraints [36,40,41]. However, the stated models are not suitable in most real cases [42]. Loch et al. [43] described the limitations of these models in real situations. Consequently, portfolio selection models have been proposed to minimize the gaps in constrained optimization problems. Henriksen and Traynor [44] proposed a model based on decision trees and probability of success. The scoring method based on project merits and project costs was proposed by Beaujon et al. [45]. They proposed a model inspired from multi-dimensional knapsack problem by considering the input limitations. It should be noted that quantitative models could not cover all the portfolio selection situations.

Project interdependencies are known as important drawbacks in portfolio selection models [40]. In Schmidt [46] method, interactions between projects in outcome values and resources are considered. He used nonlinear integer programming method to allocate resources and then applied branch and bound algorithm to solve his model. Verma and Sinha [47] studied many cases of projects in manufacturing firms for understanding relations between projects in R&D portfolio selection environments. Bardhan et al. [48] studied IT investments portfolios and used discounted cash flow and nested real options to consider interdependencies between projects. Eilat et al. [49] proposed a method based on DEA for selecting portfolios in R&D environments with interactions between projects. The stated studies are not able to cope with uncertainties. Also, there are studies that cover uncertain situations but do not consider project interdependencies. For example, Huang et al. [50] applied fuzzy analytic hierarchy process (FAHP) for selecting portfolios in R&D environments. Tiryaki and Ahlatcioglu [51] used FAHP method in stock exchange market. Chen and Cheng [52] applied fuzzy multi-criteria decision making (FMCDM) for selecting portfolios in IS environments. Huang et al. [50] proposed a fuzzy portfolio selection based on credibility measures instead of possibility measures. His work is based on selecting high expected value of portfolios instead of selecting minimum variances of portfolios through a hybrid intelligent method. Ghapanchi et al. [53] proposed an FDEA model that considered interdependencies between projects and uncertain values. This work answered the questions on IS/IT portfolio selection problems, but the reliability issue was not considered and modeled. Moreover, it is the reliability of fuzzy values that experts assign to projects. We cover this situation with proposed Z-number DEA model and concurrently consider and model project uncertainties, interactions and reliabilities.

3. The proposed new fuzzy possibilistic DEA model

In this section we introduce our proposed model and its assumptions. To do this, the context of Z-numbers and its structures are explained. At the end of this section our model is presented based on Z-numbers inputs and outputs.

3.1. An overview on Z-numbers

Zadeh [6] introduced the concept of Z-numbers related with real-valued uncertain variable X. Certain Z-numbers concepts are described in the next sections. Z-number is referred to the measure of reliability of information and has two parts Z = (A, B). A is a fuzzy subset of the domain of the variable and B is a measure of reliability (certainty) of the A. Also, B can be related to sureness, confidence, strength of belief, probability, possibility, etc. Usually A and B are represented in a linguistic variable such as (High, Sure). The set of A, playing the role of fuzzy restriction R(x) on the values which X can take, with A, playing the role of the possibility set of X. In addition,

$$R(X):\ X\ is\ A \rightarrow Poss(X = u) = \mu_A(u) \tag{1}$$

where μ_A is the membership function of A and u is a generic value of X. μ_A may be viewed as a constraint which is associated with R(X). A Z-number is used to give information about the uncertain variable X, the ordered triple (X, A, B) is equivalent to the statement, X is (A, B) is referred to as a Z-valuation [54]. A Z-valuation is indicating that X takes the value of A with reliability of B. For example, some of these Z-valuations are as follows:

(Trust to the supplier: high, likely)
(Demand for product: low, sure)
(Anticipated budget increase: About 5 million, not sure)

A Z-number is closely related to natural language and Z-valuations prepares incomplete information about the associated variable [55]. Problems arise regarding these issues such as how Z-valuations could be used to manipulate information and combine multiple pieces of information. Answers are related upon the nature of the underlying uncertainty associated with the variable X. Zadeh [6] focuses on the situations of the uncertainty associated with variable is probabilistic and X is a random variable. Yager [54] gave some examples and assumed that probability area of variable is given and computes the combination of Z-numbers. Kang et al. [56] have proven a theorem that converts the Z-number with range [0, 1] to the usual fuzzy sets. Zadeh [57] shows that if X is a random variable, then X plays the role of fuzzy event in R. The probability of this event p is presented by Expression (2).

$$p = \int_R \mu_A(u) p_X(u) du \tag{2}$$

where p_X is the hidden probability density of X. Hence the Z-valuation (X, A, B) can be as a generalized constraint on the values X. The hidden probability distribution p_X, is not known. What is known is a restriction on p_X. Thus the membership function μ_B plays the role of certainty of the X. B is a constraint on the probability measure of A rather than on the exact distribution of A.

$$\int_R \mu_A(u) p_X(u) du \text{ is } B \tag{3}$$

Zadeh [6] introduced a concept of Z^+-number, which is closely related to concept of Z-number. Z^+-number is ordered pair of two numbers, $Z^+ = (A, R)$, which A plays the same role in Z-numbers, but R is a probability distribution of random number. Z^+-number may be expressed as (μ_A, p_X), where μ_A is the membership distribution of A. Moreover, p_X is unknown in the Z-number. However, the probability measure of A is known in the Z-number. Expression (4) shows the relation between Z-numbers and Z^+-numbers. Thus, Expression (14) presents that the probability measure of A and B is a possibilistic restriction on $\mu_A \cdot p_X$. This expression explains that in Z-number we don't know about probability measure of reliability value, what we know is that B is a restriction and in the type of linguistic variables or possibilities.

$$\begin{aligned} &\mu_A \cdot p_X = P_A = \int_R \mu_A(u) p_X(u) d(u) \\ &Z(A,B) = Z^+(A, \mu_A \cdot p_X \text{ is } B) \end{aligned} \tag{4}$$

3.2. The proposed integrated model

Suppose there are n DMUs with m inputs and s outputs. Each input and output is supposed to be a Z-number that consists of pairs of fuzzy numbers related to possibility of its reliability values. $\widetilde{Zx}_{ji} = (\widetilde{Ax}_{ji}, \widetilde{Bx}_{ji})$, (j = 1, 2, ..., m) and $\widetilde{Zy}_{ri} = (\widetilde{Ay}_{ri}, \widetilde{By}_{ri})$, (r = 1, 2, ..., s) represent the Z-number input and output for the DMU_i, (i = 1, 2, ..., n). $\widetilde{Ax}_{ji}$ is the fuzzy values which the j^{th} input of DMU_i can take with triangular fuzzy number. $\widetilde{Bx}_{ji}$ is the restriction of certainty on the $\widetilde{Ax}_{ji}$ and is a triangular fuzzy number. $\widetilde{Ay}_{ri}$ is the fuzzy values which the r^{th} output of DMU_i can take with triangular fuzzy number. $\widetilde{By}_{ri}$ is the restriction of certainty on the $\widetilde{Ay}_{ri}$. Moreover, it is a triangular fuzzy number. In Expressions (5) and (6) the primal and dual Z-number CCR mathematical models of input-oriented data envelopment analysis problem is presented. Eqs. (5) and (6) are the structures of the primal and dual of CCR model. They have already been verified and validated. Moreover, our method is introduced based on the stated primal and dual methods Linear programming toolbox in MatLab software is used for coding the proposed model. In addition, the proposed models of Eqs. (6) and (7) are common DEA models and have already been validated. Moreover, we changed the format of numbers in the stated models to Z-numbers. Second, the nature of data does not influence on correctness of the DEA mathematical models.

Indices	
i	Indices of DMUs
j	Indices of inputs
r	Indices of outputs
n	Number of DMUs
m	Number of inputs
s	Number of outputs
DMU i	The ith DMU
DMU 0	The target DMU (i = 0)
Parameters	
$\widetilde{Zx}_{ji}$	Z-number value of input j related to DMU i
$\widetilde{Ax}_{ji}$	Fuzzy value of input j related to DMU i
$\widetilde{Bx}_{ji}$	Fuzzy reliability value of input j related to DMU i
$\widetilde{Zy}_{ri}$	Z-number value of output r related to DMU i
Variables	
λ_i	Weight variables in the proposed model for obtaining the efficiencies of DMUs
θ_0	Objective value (efficiency) of the DEA model

$$\begin{aligned} &Min \quad \theta_0 \\ &\text{s.t.} \quad \sum_{i=1}^{n} \lambda_i \widetilde{Zx}_{ji} \leqslant \theta_0 \widetilde{Zx}_{j0} \quad j = 1, \ldots, m \\ &\qquad \sum_{i=1}^{n} \lambda_i \widetilde{Zy}_{ri} \geqslant \widetilde{Zy}_{r0} \quad r = 1, \ldots, s \\ &\qquad \lambda_i \geqslant 0 \quad i = 1, \ldots, n \end{aligned} \tag{5}$$

$$\begin{aligned} &MAX \quad \theta_0 = \sum_{r=1}^{s} u_r \widetilde{Zy}_{r0} \\ &\text{s.t.} \quad \sum_{j=1}^{m} v_j \widetilde{Zx}_{j0} = 1 \\ &\qquad \sum_{r=1}^{s} u_r \widetilde{Zy}_{ri} - \sum_{j=1}^{m} v_j \widetilde{Zx}_{ji} \leqslant 0, \quad i = 1, 2, \ldots, n \\ &\qquad u_r, v_j \geqslant 0, \quad r = 1, 2, \ldots, s, \quad j = 1, \ldots, m \end{aligned} \tag{6}$$

Models of (5) and (6) are not linear. To linearize these models, first we try to add the second part of each Z-number to its first number and convert the Z-number models to weighted fuzzy data envelopment models. Then, the weighted fuzzy numbers are transformed to regular fuzzy numbers with preserving the properties of reliabilities.

The second part of Z-number (reliability) is converted to crisp number with the defuzzification expression that is shown in Eq. (7). Let $\widetilde{B} = \{(x, \mu_{\widetilde{B}}(x)) | x \in [0, 1]\}$ denotes the reliability of Z-number and $\mu_{\widetilde{B}}(x)$ denotes membership function. Then, the crisp equivalent by center of gravity method is obtained as shown by (7).

$$\alpha = \frac{\int x \mu_{\widetilde{B}}(x) dx}{\int \mu_{\widetilde{B}}(x) dx} \tag{7}$$

If $\widetilde{B} \sim TFN(a, b, c)$ then, the center of gravity defuzzification of this set is $\frac{a+b+c}{3}$ [58]. This method has been chosen because of its simplicity in computation.

Definition 1. Let a fuzzy set $\widetilde{A}$ be defined in the universe X, Then $\widetilde{A} = \{(x, \mu_A(x)) | x \in X\}$. $\mu_A(x)$ is the membership function and is the degree of belongingness of $x \in X$. The fuzzy expectation of fuzzy set is denoted as shown by Expression (8) [56]:

$$E_A(x) = \int_X x\mu_A(x)dx \tag{8}$$

α is added to the first part of Z-number as probability measure of the first part. This is done to obtain the weighted Z-number for each input and output. If the Z-number is the pair of two triangular fuzzy numbers $\widetilde{Z} = (\widetilde{A}, \widetilde{B})$ and α is the crisp value of $\widetilde{B}$, the weighted Z-number can be denoted as $\widetilde{Z}^{\alpha} = \{(x, \mu^{\alpha}_{\widetilde{A}}(x))|x \in X\}$. Eq. (9) represents the relationship between a Z-number and its weighted fuzzy number.

$$\begin{aligned} &E_{\widetilde{A}^{\alpha}}(x) = \alpha E_{\widetilde{A}}(x), \quad x \in X \\ &\text{s.t.}\ \mu^{\alpha}_{\widetilde{A}}(x) = \alpha\mu_{\widetilde{A}}(x), \quad x \in X \end{aligned} \tag{9}$$

Proof.

$$E_{\widetilde{A}^{\alpha}}(x) = \int x\mu^{\alpha}_{\widetilde{A}}(x)dx = \int \alpha\mu_{\widetilde{A}}(x)dx = \alpha\int \mu_{\widetilde{A}}(x)dx = \alpha E_{\widetilde{A}}(x)$$

Fig. 1 shows the procedure of converting the triangular fuzzy set of first part of Z-number to the related weighted Z-number. □

Inputs and outputs of DMUs are transformed into weighted triangular fuzzy numbers according to their reliability values (Expression (9)). If the reliability of the number assigned by the expert is high then, the accuracy of fuzzy values that expert assigns to this number is high. On the contrary, if the reliability is low then, the accuracy of real characteristics of fuzzy number that the expert assigns to the input or output becomes low. Then, the weighted Z-numbers related to each input or output of DMUs is converted according to their reliabilities into normal fuzzy numbers. It is assumed that if the reliability of assigned Z-number to each input or output is low then, the domain of real fuzzy number is greater than the domain that expert assigned and vice versa. In Fig. 2, the weighted Z-number with height α is shown and the schematic conversion of normal fuzzy number is presented. If the membership function of weighted Z-number is triangular, it is assumed that relevant normal fuzzy number is triangular. Furthermore, the slope of its lines is equal to the weighted Z-number. It is obvious that the ratio of α impacts on the characteristics of first part of Z-numbers and this effect is relatively high if the amount of α is low. The impact has a direct relationship with slope of lines in weighted Z-numbers. Also, the slope of lines has a direct relationship with the value of α meaning that the impact has a direct relationship with value of α. These assumptions are used to find the characteristics of the relevant normal fuzzy numbers.

If the weighted Z-number has a triangular membership function with $\widetilde{Z}^{\alpha} \sim TFN(a,b,c)$ then its relevant normal fuzzy number has triangular membership function with $\widetilde{N} \sim TFN(a',b',c')$. It is assumed that $b = b'$ and the slope of lines in the two sets are equal. For finding the value of a' the left side slope of weighted Z-number is used. It equals to $\frac{\alpha}{b-a}$ and then the left side linear equation of normal fuzzy number is $\mu_{\widetilde{N}}(x) = \frac{\alpha}{b-a}x + h,\ x \leqslant b$. For finding the value of h, the point of $(b,1)$ is inserted in this equation. Thus:

$$\begin{aligned} &1 = \frac{\alpha}{b-a}b + h \rightarrow h = 1 - \frac{\alpha b}{b-a} \\ &\mu_{\widetilde{N}}(x) = \frac{\alpha}{b-a}x + 1 - \frac{\alpha b}{b-a}, \quad x \leqslant b \end{aligned} \tag{10}$$

If $\mu_{\widetilde{N}}(x) = 0$ in Expression (10), then the value of a' is identified by Expression (11).

$$\begin{aligned} &0 = \frac{\alpha}{b-a}a' + 1 - \frac{\alpha b}{b-a} \rightarrow \frac{\alpha}{b-a}a' = \frac{\alpha b - b + a}{b-a} \\ &a' = \frac{\alpha b - b + a}{\alpha} \end{aligned} \tag{11}$$

The same steps are performed for finding the value of c'. The right side slope of weighted Z-number and its relevant normal fuzzy number is $\frac{\alpha}{b-c}$. Then, the equation of right hand side of normal fuzzy

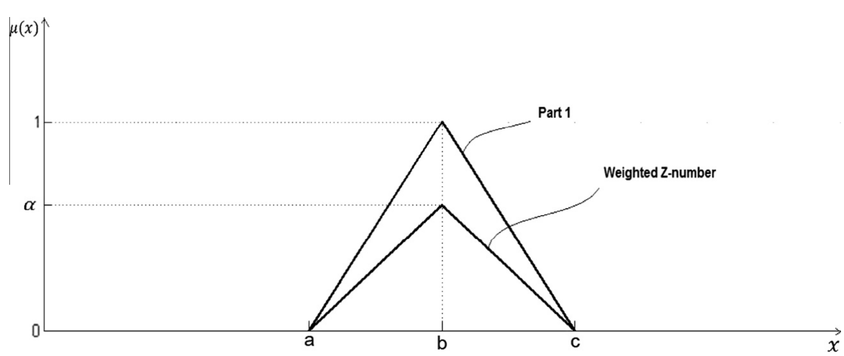

Fig. 1. Z-number after multiplying the reliability value.

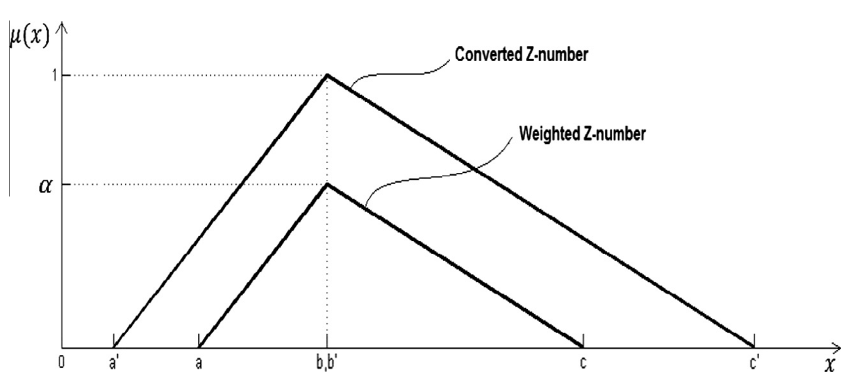

Fig. 2. Convert weighted Z-numbers to normal fuzzy numbers.

number is $\mu_{\widetilde{N}}(x) = \frac{\alpha}{b-c}x + h,\ x \geqslant b$. The values of h and then c' are consequently identified.

$$\begin{aligned}
&1 = \frac{\alpha}{b-c}b + h \rightarrow h = 1 - \frac{\alpha b}{b-c} \\
&\mu_{\widetilde{N}}(x) = \frac{\alpha}{b-c}x + 1 - \frac{\alpha b}{b-c}, \quad x \geqslant b \\
&\mu_{\widetilde{N}}(x) = 0 \\
&0 = \frac{\alpha}{b-c}c' + 1 - \frac{\alpha b}{b-c} \rightarrow \frac{\alpha}{b-c}c' = \frac{\alpha b - b + c}{b-c} \\
&c' = \frac{\alpha b - b + c}{\alpha}
\end{aligned} \tag{12}$$

In Z-number DEA model the expert gives the Z-number values for the inputs and outputs of the DMU_i. For example, he gives the pair of $(\widetilde{Ax}_{ji}, \widetilde{Bx}_{ji})$ for the j^{th} input of DMU_i. $\widetilde{Ax}_{ji} \sim TFN(a_{ji}, b_{ji}, c_{ji})$ and $\widetilde{Bx}_{ji} \sim TFN(d_{ji}, e_{ji}, f_{ji})$. $\widetilde{Bx}_{ji}$ is defuzzified with center of gravity method and the crisp reliability value α_{ji} is obtained and added to first part, $\widetilde{Ax}_{ji}$. Then, Expressions (11) and (12) are used to transform the weighted Z-number to normal fuzzy number, $\widetilde{Nx}_{ji} \sim TFN(a'_{ji}, b'_{ji}, c'_{ji})$.

$$\begin{aligned}
&\alpha_{ji} = \frac{d_{ji} + e_{ji} + f_{ji}}{3} \\
&b'_{ji} = b_{ji} \\
&a'_{ji} = \frac{\alpha_{ji} b_{ji} - b_{ji} + a_{ji}}{\alpha_{ji}} \\
&c'_{ji} = \frac{\alpha_{ji} b_{ji} - b_{ji} + c_{ji}}{\alpha_{ji}}
\end{aligned} \tag{13}$$

The possibilistic linear programming of Z-number DEA model is obtained with the stated conversion method. It is assumed that the inputs and outputs of model are ordered pair of triangular fuzzy numbers. Thus $(\widetilde{Ax}_{ji}, \widetilde{Bx}_{ji})$ is related to the input j of DMU_i. $\widetilde{Ax}_{ji} \sim TFN(ax^l_{ji}, ax^m_{ji}, ax^u_{ji})$ and $\widetilde{Bx}_{ji} \sim TFN(bx^l_{ji}, bx^m_{ji}, bx^u_{ji})$. Also, $(\widetilde{Ay}_{ri}, \widetilde{By}_{ri})$ is related to the input r of DMU_i. $\widetilde{Ay}_{ri} \sim TFN(ay^l_{ri}, ay^m_{ri}, ay^u_{ri})$ and $\widetilde{By}_{ji} \sim TFN(by^l_{ri}, by^m_{ri}, by^u_{ri})$. Expression (13) is used to convert the inputs and outputs of model to regular fuzzy numbers to obtain the possibilistic linear programming model. Expression (14) shows how to convert the inputs to the normal fuzzy numbers.

$$\begin{aligned}
&\beta x_{ji} = \frac{bx^l_{ji} + bx^m_{ji} + bx^u_{ji}}{3} \\
&x^m_{ji} = ax^m_{ji} \\
&x^l_{ji} = \frac{\beta x_{ji} ax^m_{ji} - ax^m_{ji} + ax^l_{ji}}{\beta x_{ji}} \\
&x^u_{ji} = \frac{\beta x_{ji} ax^m_{ji} - ax^m_{ji} + ax^u_{ji}}{\beta x_{ji}}
\end{aligned} \tag{14}$$

$\tilde{x}_{ji} \sim TFN(x^l_{ji}, x^m_{ji}, x^u_{ji})$ is the normal fuzzy converted number of j^{th} input of DMU_i. Expression (15) shows the equations of converting the outputs to the normal fuzzy numbers.

$$\begin{aligned}
&\beta y_{ri} = \frac{by^l_{ri} + by^m_{ri} + by^u_{ri}}{3} \\
&y^m_{ri} = ay^m_{ri} \\
&y^l_{ri} = \frac{\beta y_{ri} ay^m_{ri} - ay^m_{ri} + ay^l_{ri}}{\beta y_{ri}} \\
&y^u_{ri} = \frac{\beta y_{ri} ay^m_{ri} - ay^m_{ri} + ay^u_{ri}}{\beta y_{ri}}
\end{aligned} \tag{15}$$

Also $\tilde{y}_{ri} \sim TFN(y^l_{ri}, y^m_{ri}, y^u_{ri})$ is the normal fuzzy converted number of r^{th} output of DMU_i. Then the fuzzy programming of Z-number CCR model are presented in the Expression (16).

$$\begin{aligned}
MAX \quad & \theta_p = \sum_{r=1}^{s} u_r(y^l_{rp}, y^m_{rp}, y^u_{rp}) \\
\text{s.t.} \quad & \sum_{j=1}^{m} v_j(x^l_{jp}, x^m_{jp}, x^u_{jp}) = (1^l, 1, 1^u) \\
& \sum_{r=1}^{s} u_r(y^l_{ri}, y^m_{ri}, y^u_{ri}) - \sum_{j=1}^{m} v_j(x^l_{ji}, x^m_{ji}, x^u_{ji}) \leqslant 0, \quad i = 1, 2, \ldots, n \\
& u_r, v_j \geqslant 0, \quad r = 1, 2, \ldots, s, \quad j = 1, \ldots, m
\end{aligned} \tag{16}$$

where $1^l \leqslant 1$ and $1^u \geqslant 1$ are real numbers. There are several methods for solving the fuzzy linear models. In the most of these methods the researchers used the α-cut method. Saati et al. [20] proposed a fuzzy version of CCR using TFNs and offered a methodology based on α-cuts in the CCR fuzzy issue changed into determined intervals and chose a point in the intervals variable to assure the constraints and, at the same time, optimized the

Table 1
Fuzzy values assigned to projects criteria.

Project number	Cost of the project ($ million) Input	Number of potential subsequent investments Output 1	Contribution to the workflow improvement Output 2	Percentage of contribution to electronic readiness Output 3
1	(412, 435, 458)	(128, 132, 136)	(0.73, 0.865, 0.95)	(42, 46, 50)
2	(174, 178, 182)	(69, 75, 81)	(0.05, 0.16, 0.29)	(6, 9, 12)
3	(225, 242, 259)	(27, 28, 29)	(0.68, 0.74, 0.91)	(36, 41, 46)
4	(308, 323, 338)	(85, 90, 95)	(0.55, 0.7, 0.85)	(87, 90, 93)
5	(175, 189, 203)	(73, 75, 77)	(0.37, 0.55, 0.68)	(71, 75, 79)
6	(84, 93, 102)	(66, 70, 74)	(0.07, 0.17, 0.31)	(45, 47, 49)
7	(349, 370, 391)	(123, 130, 137)	(0.95, 0.99, 0.99)	(39, 44, 49)
8	(245, 271, 297)	(41, 43, 45)	(0.31, 0.45, 0.59)	(32, 37, 42)
9	(151, 154, 157)	(58, 60, 62)	(0.35, 0.45, 0.65)	(25, 27, 29)
10	(265, 281, 297)	(49, 52, 55)	(0.68, 0.79, 0.94)	(37, 41, 45)
11	(345, 362, 379)	(21, 24, 27)	(0.15, 0.18, 0.21)	(54, 58, 62)
12	(215, 222, 229)	(4, 6, 8)	(0.19, 0.2, 0.21)	(56, 59, 62)
13	(385, 391, 397)	(6, 8, 10)	(0.33, 0.34, 0.35)	(34, 36, 38)
14	(454, 474, 494)	(7, 9, 11)	(0.44, 0.47, 0.5)	(11, 13, 15)
15	(384, 390, 396)	(7, 8, 9)	(0.2, 0.22, 0.24)	(48, 51, 54)
16	(384, 391, 398)	(9, 11, 13)	(0.16, 0.18, 0.2)	(52, 54, 56)

objective function. We use this method to transform the fuzzy programming to crisp parametric linear programming model. Also we used this model to solve a DEA model based on envelopment formulation. The variables in the intervals are defined such that they satisfy the limitations and also optimize the objective function. Assume that α is a parameter depending to the interval [0, 1]. At last the parametric linear programming of dual Z-number CCR-ID model is represented in the expression (17).

$$\begin{aligned} MAX \quad & \theta_p = \sum_{r=1}^{s} \bar{y}_{rp} \\ \text{s.t.} \quad & \sum_{j=1}^{m} \bar{x}_{jp} = 1 \\ & \sum_{r=1}^{s} \bar{y}_{ri} - \sum_{j=1}^{m} \bar{x}_{ji} \leqslant 0, \quad i = 1, 2, \ldots, n \\ & v_j(\alpha x_{ji}^m + (1-\alpha) x_{ji}^l) \leqslant \bar{x}_{ji} \leqslant v_j(\alpha x_{ji}^m + (1-\alpha) x_{ji}^u, \\ & \quad i = 1, \ldots, n, \quad j = 1, \ldots, m \\ & u_r(\alpha y_{ri}^m + (1-\alpha) y_{ri}^l) \leqslant \bar{y}_{ri} \leqslant u_r(\alpha y_{ri}^m + (1-\alpha) y_{ri}^u, \\ & \quad i = 1, \ldots, n, \quad r = 1, \ldots, s \\ & u_r, v_j \geqslant 0, \quad r = 1, \ldots, s, \quad j = 1, \ldots, m \end{aligned} \tag{17}$$

Expression (17) is a crisp parametric linear programming model based on multiplier formulation. The objective function of this model is the same as CCR-ID model. It reflects the value of efficiency of the DMU under consideration. In CCR-ID model the efficiency is evaluated by the summation of all outputs multiplied by respective weights. It is obvious that if inputs and outputs of the proposed model are crisp then the crisp values of reliabilities take the value 1. Moreover, optimistic, pessimistic, and most likely values of inputs and outputs remain the same. Consequently, Model (17) is equivalent to conventional CCR-DEA model. In the rest of this paper we used an envelopment formulation to obtain our Z-number DEA model.

For linearization of the primal Z-number CCR-DEA model, it is assumed that the inputs and outputs are the same as the primal CCR model with triangular fuzzy numbers. Expressions (14) and (15) are used for conversion of inputs and outputs. The fuzzy programming is obtained by Expression (18). The Saati et al. [20] method is used to obtain the parametric linear programming of primal CCR Z-number DEA model. This model is represented by Expression (19).

$$\begin{aligned} Min \quad & \theta_p \\ \text{s.t.} \quad & \sum_{i=1}^{n} \lambda_i (x_{ji}^l, x_{ji}^m, x_{ji}^u) \leqslant \theta_p (x_{jp}^l, x_{jp}^m, x_{jp}^u) \quad j = 1, \ldots, m \\ & \sum_{i=1}^{n} \lambda_i (y_{ri}^l, y_{ri}^m, y_{ri}^u) \geqslant (y_{rp}^l, y_{rp}^m, y_{rp}^u) \quad r = 1, \ldots, s \\ & \lambda_i \geqslant 0 \quad i = 1, \ldots, n \end{aligned} \tag{18}$$

$$\begin{aligned} Min \quad & \theta_p \\ \text{s.t.} \quad & \theta_p(\alpha x_{jp}^m + (1-\alpha) x_{jp}^l) \geqslant \sum_{i=1}^{n} \lambda_i (\alpha x_{ji}^m + (1-\alpha) x_{ji}^u), \quad j = 1, \ldots, m \\ & \alpha y_{rp}^m + (1-\alpha) y_{rp}^u \leqslant \sum_{i=1}^{n} \lambda_i (\alpha y_{ri}^m + (1-\alpha) y_{ri}^l), \quad r = 1, \ldots, s \\ & \lambda_i \geqslant 0, \quad i = 1, \ldots, n \end{aligned} \tag{19}$$

The proposed model is tantamount to a parametric programming model while $\alpha \in [0, 1]$ is a parameter. If the constraint $\sum_{i=1}^{n} \lambda_i = 1$ is added to the model (17) then the BCC model is obtained. The only difference between the two is on embodiment of the convexity restrictions in the BCC model. It is noted that for each α we have a special optimal solution. Thus we can provide the solution table with different α in [0, 1] to decision makers.

4. The Case study: Methods and material

In this paper actual IS/IT projects are used as case studies. There are interactions between these projects due to usage of same resources, uncertainties and result of outcomes. Planning on IS/IT has two impacts. Right plans on IS/IT investments can have a good influence on organizations to reach their business missions. Poor plan on IS/IT projects can have a negative impact on organizational performance [59]. IS/IT projects are in the category of R&D and deployment of these projects need more time, cost and technology in comparison to other types of R&D projects. Consequently they are more risky in implementation phase [60]. Investments on IS/IT projects must be assigned to projects quickly. This is because rapid changes in organizations create uncertainties in such projects [61]. The existence of certainties and interactions in IS/IT projects cause complexity in IT/IS investment prioritization. Hence, in this paper we utilize the proposed model for selecting the best portfolios in IS/IT investments to cope with the complexity. This is achieved by comparing the reliability values. Also, we used Eilat et al. [49] method for considering the interactions in IS/IT projects. By customizing the reliability values related to each project, we applied the numerical example of Ghapanchi et al. [53] in a national governmental organization. Current study is only a research that considers the fuzziness, interactions and reliabilities together in portfolio selection for IS/IT investment. With the purpose of applying the proposed model for solving the case problem, we followed the following 4 steps:

(1) Modeling the problem;
(2) Selecting efficient projects with Z-number DEA model;
(3) Generating portfolios for selected projects;
(4) Selecting efficient portfolio with Z-number DEA model.

4.1. Modeling the problem

In the first step each project is considered as a DMU and the input and output values related to projects are estimated by experts. Each input and output is considered as a fuzzy triangular variable. Projects inputs and outputs and interactions between projects are taken from Ghapanchi et al. [53]. Moreover, for each value the reliabilities that experts are set to these values are used.

Table 2
Fuzzy reliability values assigned to projects.

Project number	Reliability of input	Reliability of output 1	Reliability of output 2	Reliability of output 3
1	Likely	Likely	Likely	Likely
2	Likely	Sure	Sure	Likely
3	Usually	Sure	Likely	Usually
4	Sure	Sure	Usually	Sure
5	Sure	Sure	Usually	Usually
6	Likely	Sure	Sure	Usually
7	Sure	Usually	Sure	Likely
8	Sure	Usually	Likely	Sure
9	Usually	Likely	Usually	Likely
10	Sure	Sure	Usually	Usually
11	Likely	Likely	Usually	Sure
12	Usually	Likely	Sure	Sure
13	Sure	Likely	Sure	Sure
14	Sure	Likely	Likely	Usually
15	Sure	Sure	Sure	Likely
16	Usually	Sure	Usually	Likely

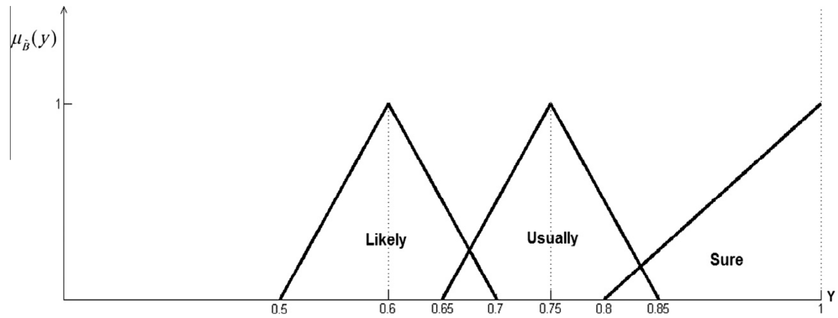

Fig. 3. Fuzzy sets of linguistic reliability values.

Table 3
Classification of reliability values given by experts to the projects criteria.

Z = (A, B)		Membership functions parameters
B	Sure	[0.8, 1, 1]
	Usually	[0.65, 0.75, 0.85]
	Likely	[0.5, 0.6, 0.7]

Reliabilities are classified in three categories which are likely, usually and sure according to Azadeh et al. [62,63].

This problem has one input indicator and three output indicators. Input indicator is cost of the project in millions of dollars. On the other hand, output 1 is the number of potential subsequent investments, output 2 is the contribution to the workflow improvement and output 3 is the percentage of contribution to electronic readiness respectively based on Ghapanchi et al. [53]. In addition, there is restriction on the assigned budget for the projects which is about $600,000,000. Table 1 shows the inputs and outputs for 16 projects in the IT/IS investment problem.

The reliability values assigned to projects criteria identified by experts in three category of likely, usually and sure are presented in Table 2. The reliability values are selected randomly for the three categories.

4.2. Selecting efficient projects with Z-number DEA model

DMUs, inputs and outputs are modeled into Z-numbers and the proposed DEA model to select the efficient projects for portfolio evaluation. The efficiency value of each project is calculated by running the model several times (number of projects). The indices, parameters and variables entered into the model are presented by Eq. (5).

Indices	
i	Indices of DMUs
j	Indices of inputs
r	Indices of outputs
n	Number of DMUs
m	Number of inputs
s	Number of outputs
Parameters	
$\widetilde{Zx}_{ji}$	The Z-number amount of input j related to DMU (project) i
$\widetilde{Ax}_{ji}$	Fuzzy value of input j related to DMU (project) i
$\widetilde{Bx}_{ji}$	Fuzzy reliability value of input j related to DMU (project) i
$\widetilde{Zy}_{ri}$	The Z-number amount of output r related to DMU (project) i
$\widetilde{Ay}_{ri}$	Fuzzy value of output r related to DMU (project) i
$\widetilde{By}_{ri}$	Fuzzy reliability value of output r related to DMU (project) i
$\widetilde{U}$	The interaction matrix of projects for input criterion
z_t	The particular selection of projects in portfolio t (if project i participates in portfolio t, then $z_{it} = 1$)
$\tilde{u}_{kt}$	The interaction between project k and project t for input criterion
$\widetilde{V}^r$	The interaction matrix of projects for output r
$\tilde{v}^r_{kt}$	The interaction between project k and project t for output r
α	Parameter between [0, 1] for α-cut programming
$\tilde{p}_{ij}$	The possibility of success of project i when the project j is participating in the portfolio that belongs to project i
Variables	
Eff_i	Value of efficiency for DMU (project or portfolio) i calculated by proposed model
λ_i	Weight variables in the proposed model for obtaining the efficiencies of DMUs
$\widetilde{ZInput}_t$	Z-number amount of input related to portfolio t
$\widetilde{AInput}_t$	Fuzzy value of input related to portfolio t
$\widetilde{BInput}_t$	Fuzzy reliability value of input related to portfolio t
$\widetilde{ZOutput}_{rt}$	Z-number amount of output r related to portfolio t
$\widetilde{AOutput}_{rt}$	Fuzzy value of output r related to portfolio t
$\widetilde{BOutput}_{rt}$	Fuzzy reliability value of output r related to portfolio t

Projects with low efficiency values are removed from portfolio generation step.

4.3. Generating portfolios for selected projects

Portfolios generation begins for decision making process after selecting the efficient projects. The branching procedure to generate candidate maximal portfolios is used in this study Eilat et al. [49]. The maximal portfolio is the one in which adding any new project to it violates the resource restrictions. Suppose that the numbers of selected projects are 4. To do this, we start with an empty portfolio with no projects. Through next step, we branch 4 nodes by adding the single project to the portfolio in each node according to projects indices. In next steps, remaining projects are added to portfolio by considering the input limitations, projects indices and maximal portfolios. This process is repeated for remaining nodes to achieve maximal portfolios.

After generating the maximal portfolios, the input and output values of each portfolio will be calculated. The accumulation

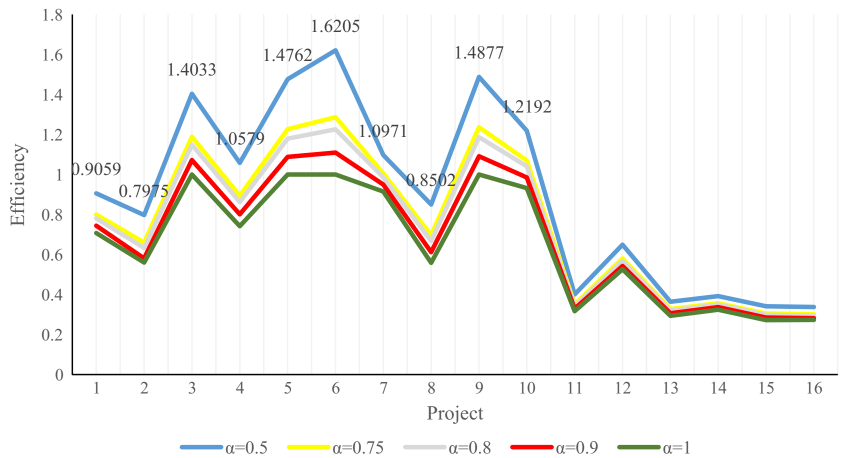

Fig. 4. The results of project selection phase for different satisfaction degrees.

Table 4
Sorted projects with their efficiency values.

Project number	Efficiency
6	1.6205
9	1.4877
5	1.4762
3	1.4033
10	1.2192
7	1.0971
4	1.0579
1	0.9059
8	0.8502
2	0.7975
12	0.6493
11	0.4009
14	0.3923
13	0.3641
15	0.3415
16	0.3373

function approach proposed by Ghapanchi et al. [53] is used to calculate the input and output values of the portfolios, because projects of one influenced each other and then the input and output values of portfolio can be changed. Then, the input and output values of portfolios that are considered as DMUs can be computed by adding the project values with specific structure that is called accumulation functions. The accumulation function considers input, output and possibility interactions. Eqs. (20) and (21) show the accumulation functions. In Eq. (20), the input value of each portfolio is equal to sum of the input values of projects corporate on it ($\sum_{j=1}^{n}\widetilde{Ax}_{j1}z_{jt}$) and sum of the interaction values of its projects (Uz_t). Eq. (21) is similar to Eq. (20) with difference that in calculating the output values of each portfolio, the success of participating projects in portfolio ($\tilde{p}_{ji}$) is important. For more information please refer to Ghapanchi et al. [53]

$$\widetilde{AInput}_t = \sum_{j=1}^{n}\widetilde{Ax}_{j1}z_{jt} + Uz_t; \quad \forall t \tag{20}$$

$$\widetilde{AOutput}_{rt} = \sum_{j=1}^{n} z_{jt}\left(\sum_{i=1}^{n}\tilde{p}_{ji}z_{it}\right)\left[\widetilde{Ay}_{rj} + \sum_{i=1}^{j-1}\tilde{v}_{ji}^{r}\left(\sum_{k=1}^{n}\tilde{p}_{ik}z_{kt}\right)z_{it}\right]; \quad \forall r,t \tag{21}$$

Reliability values of inputs and outputs of portfolios are assumed with calculation of mean of fuzzy values, because the linguistic variables of reliabilities in the range of the [0, 1], then the mean of reliabilities is in the same range and is a certainty for portfolio values. Then, mean fuzzy reliability values of projects that are presented in each portfolio are computed. Eqs. (22) and (23) show the stated procedure.

$$\widetilde{BInput}_t = \frac{\left(\sum_{i=1}^{n}\widetilde{Bx}_{1i}z_{it}\right)}{n_t}; \quad \forall t \tag{22}$$

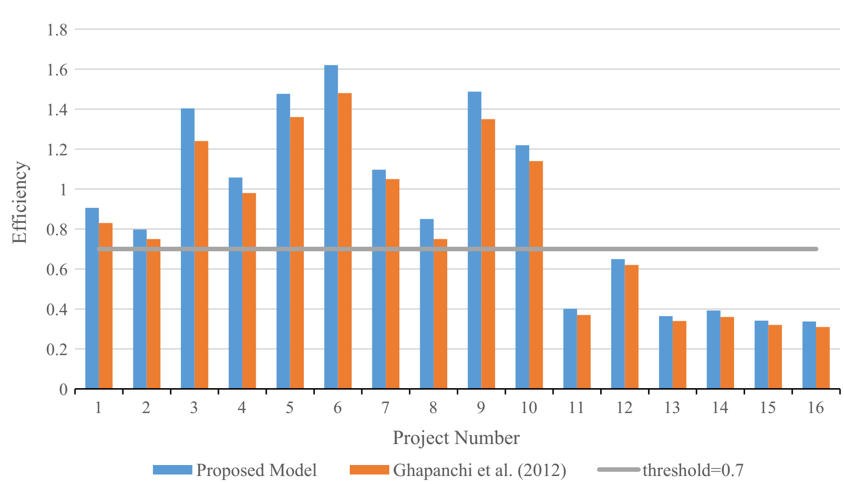

Fig. 5. Comparison of the proposed model with Ghapanchi et al. [53] in project selection.

Table 5
The interaction values between projects.

Project (i, j)	U1	V1	V2	V3	P(i, j)
(1, 3)			(0.3, 0.44, 0.58)		
(1, 10)	(−32, −28.4, −24.8)	(3.75, 4.25, 4.5)			
(2, 7)	(−14.14, −12.5, −10.6)				(0.108, 0.111, 0.114)
(3, 4)		(1, 1, 1)			
(4, 5)	(−24, −21, −18)			(7.1, 8.7, 10.3)	
(4, 6)				(9.7, 10.6, 11.5)	
(5, 6)		(1, 2, 3)		(12.5, 13.7, 14.9)	
(5, 9)				(4.8, 13.7, 14.9)	
(6, 7)	(−19, −16.8, −14.6)				
(6, 8)	(−5, −4.2, −3.6)				
(7, 6)					(0.148, 0.167, 0.186)
(7, 8)	(−14, −13.3, −12.6)	(2, 2, 2)	(0.05, 0.15, 0.3)	(7.8, 8.9, 10)	(0.187, 0.194, 0.201)
(7, 10)			(0.07, 0.15, 0.27)		
(8, 6)					(0.0064, 0.0701, 0.078)
(9, 10)	(−19, −17.4, −15.8)				

Table 6
Fuzzy values of portfolios criteria.

Portfolio		Cost of the portfolio ($ million)	Number of potential subsequent investments	Contribution to the workflow improvement	Percentage of contribution to electronic readiness
Number	Label	Input	Output 1	Output 2	Output 3
1	P1000000010	(563, 589, 615)	(88, 100, 111)	(0.51, 0.69, 0.94)	(33, 39, 46)
2	P1000100000	(587, 624, 661)	(93, 101, 109)	(0.5, 0.69, 0.85)	(56, 63, 70)
3	P1100000000	(586, 613, 640)	(104, 115, 126)	(0.34, 0.5, 0.67)	(22, 27, 33)
4	P1000010000	(496, 528, 560)	(76, 83, 91)	(0.32, 0.44, 0.56)	(33, 37, 41)
5	P0100110010	(584, 614, 644)	(151, 167, 184)	(0.48, 0.8, 1.23)	(91, 103, 116)
6	P0110000010	(550, 574, 598)	(103, 116, 130)	(0.66, 0.92, 1.37)	(42, 52, 64)
7	P0100000011	(571, 596, 620)	(113, 128, 142)	(0.62, 0.88, 1.28)	(40, 49, 58)
8	P0110100000	(574, 609, 644)	(108, 117, 128)	(0.65, 0.92, 1.27)	(65, 76, 88)
9	P0110010000	(483, 513, 543)	(91, 100, 109)	(0.47, 0.67, 0.98)	(42, 51, 60)
10	P0100011000	(574, 612, 650)	(126, 145, 166)	(0.41, 0.62, 0.86)	(35, 43, 53)
11	P0100010001	(523, 552, 581)	(101, 111, 122)	(0.43, 0.63, 0.9)	(40, 47, 54)
12	P0101010000	(566, 594, 622)	(108, 121, 135)	(0.28, 0.49, 0.75)	(64, 74, 84)
13	P0100000110	(570, 603, 636)	(96, 109, 122)	(0.32, 0.54, 0.88)	(27, 35, 43)
14	P0100100100	(594, 638, 682)	(100, 110, 120)	(0.31, 0.55, 0.79)	(50, 59, 67)
15	P0100010100	(498, 538, 577)	(86, 95, 105)	(0.15, 0.33, 0.55)	(29, 36, 42)
16	P0010100010	(551, 585, 619)	(92, 102, 113)	(0.83, 1.11, 1.54)	(81, 93, 107)
17	P0010010010	(460, 489, 518)	(75, 85, 95)	(0.65, 0.86, 1.26)	(53, 62, 73)
18	P0010110000	(484, 524, 564)	(80, 88, 96)	(0.64, 0.86, 1.16)	(89, 100, 112)
19	P0010010001	(574, 616, 658)	(66, 73, 81)	(0.8, 1, 1.33)	(57, 67, 78)
20	P0011000000	(533, 565, 597)	(51, 59, 68)	(0.63, 0.8, 1.06)	(56, 67, 78)
21	P0010010100	(549, 602, 654)	(51, 57, 64)	(0.53, 0.7, 0.98)	(47, 56, 66)
22	P0010001000	(574, 612, 650)	(43, 52, 63)	(0.61, 0.75, 0.96)	(30, 39, 48)
23	P0001110000	(543, 584, 625)	(97, 109, 121)	(0.44, 0.68, 0.92)	(118, 132, 146)
24	P0001010010	(543, 570, 597)	(92, 106, 120)	(0.46, 0.68, 1.02)	(75, 86, 97)
25	P0001000001	(573, 604, 635)	(60, 69, 79)	(0.59, 0.76, 0.98)	(54, 63, 72)
26	P0001000100	(553, 594, 635)	(43, 51, 59)	(0.29, 0.42, 0.58)	(42, 49, 57)
27	P0000100011	(572, 607, 641)	(102, 113, 125)	(0.78, 1.07, 1.46)	(79, 90, 101)
28	P0000110001	(524, 563, 602)	(90, 99, 108)	(0.59, 0.82, 1.08)	(87, 96, 106)
29	P0000111000	(589, 635, 681)	(108, 125, 143)	(0.57, 0.79, 1)	(81, 92, 103)
30	P0000100110	(571, 614, 657)	(85, 95, 105)	(0.49, 0.73, 1.06)	(66, 76, 86)
31	P0000110100	(499, 549, 598)	(75, 83, 91)	(0.32, 0.52, 0.72)	(76, 85, 94)
32	P0000010011	(481, 511, 540)	(85, 96, 107)	(0.61, 0.82, 1.17)	(51, 59, 67)
33	P0000011010	(565, 600, 635)	(103, 122, 142)	(0.58, 0.79, 1.1)	(45, 54, 64)
34	P0000010110	(475, 514, 552)	(70, 80, 91)	(0.33, 0.52, 0.82)	(40, 47, 55)
35	P0000010101	(589, 641, 692)	(61, 69, 76)	(0.48, 0.66, 0.89)	(45, 52, 60)
36	P0000001100	(580, 628, 675)	(60, 72, 85)	(0.5, 0.72, 0.97)	(31, 38, 47)

$$\widetilde{BOutput}_{rt} = \frac{\left(\sum_{i=1}^{n} \widetilde{By}_{ri} z_{it}\right)}{n_t}; \quad \forall r, t \tag{23}$$

n_t is the number of projects in portfolio t.

4.4. Selecting efficient portfolio with Z-number DEA model

Portfolios are selected and modeled by resource inputs and specific outputs. Moreover, Z-number DEA model is applied for portfolios to specify efficiency scores for maximal portfolios. The model is run for different satisfaction degrees to show the sensitivity of results for decision makers.

5. Experimental results

In this section the case study has been solved with proposed model and experiments are described. The linguistic reliability values assigned to projects are represented in Table 2. According to Azadeh et al. [62,63] for these three categories we assigned triangular fuzzy sets that are represented in Fig. 3. Table 3 shows the membership functions of reliability sets. Moreover, the proposed Z-number DEA model is applied for projects criteria to select the efficient projects for portfolio evaluation. We used the linear programming toolbox in MatLab software for coding the proposed model.

Fig. 4 presents the results of project selection phase. As seen some projects have greater efficiencies than others. For value of $\alpha = 0.5$ the results of project selection is presented in Table 4. Decision maker has a minimal threshold equal to 0.7 for efficiency index as stated in [53]. We also choose this threshold to compare the results of new method with the stated study. However, in real problems decision makers should choose proper threshold with respect their objectives and constraints. Moreover, according to Table 4 projects are selected for portfolio evaluation. Fig. 5 shows the comparison of projects selection results between our method and Ghapanchi et al. [53] for $\alpha = 0.5$. The only differences are between projects 9 and 5 and projects 8 and 2 due to the reliability values in our method. According to these results we conclude that the new model is very important in real problems because it considers the reliability values in decision making process.

Now selected projects are entered in portfolio generation phase for generating maximal portfolios. Branching procedure for generating maximal feasible portfolios is used. The generated portfolios are labeled and presented in Table 6. For example, "P1000000010" refers to the fact that in this portfolio, projects 1 and 10 are participated. After constructing the maximal portfolios, experts determine project interactions in brainstorming meeting in each candidate maximal portfolios for the actual case study. They proposed fuzzy interaction values between projects according to Table 5. The values of Table 5 have been obtained from [25].

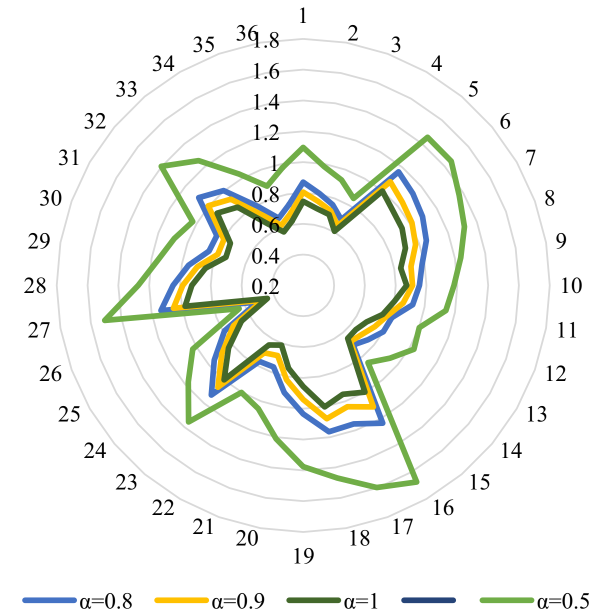

Fig. 6. Efficiencies of candidate maximal portfolios with different satisfaction degrees.

Table 7
Fuzzy reliabilities of portfolios criteria.

Portfolio Number	Portfolio Label	Cost of the portfolio ($ million) Input reliability	Number of potential subsequent investments Output 1 reliability	Contribution to the workflow improvement Output 2 reliability	Percentage of contribution to electronic readiness Output 3 reliability
1	P1000000010	(0.575, 0.675, 0.775)	(0.5, 0.6, 0.7)	(0.575, 0.675, 0.775)	(0.5, 0.6, 0.7)
2	P1000100000	(0.65, 0.8, 0.85)	(0.65, 0.8, 0.85)	(0.575, 0.675, 0.775)	(0.575, 0.675, 0.775)
3	P1100000000	(0.5, 0.6, 0.7)	(0.65, 0.8, 0.85)	(0.65, 0.8, 0.85)	(0.5, 0.6, 0.7)
4	P1000010000	(0.5, 0.6, 0.7)	(0.65, 0.8, 0.85)	(0.65, 0.8, 0.85)	(0.575, 0.675, 0.775)
5	P0100110010	(0.6125, 0.7375, 0.8125)	(0.725, 0.9, 0.9625)	(0.725, 0.875, 0.925)	(0.575, 0.675, 0.775)
6	P0110000010	(0.6, 0.7, 0.8)	(0.7, 0.87, 0.9)	(0.65, 0.783, 0.85)	(0.55, 0.65, 0.75)
7	P0100000011	(0.65, 0.79, 0.85)	(0.7, 0.87, 0.9)	(0.7, 0.84, 0.9)	(0.55, 0.65, 0.75)
8	P0110100000	(0.65, 0.79, 0.85)	(0.8, 1, 1)	(0.65, 0.79, 0.85)	(0.6, 0.7, 0.8)
9	P0110010000	(0.55, 0.65, 0.75)	(0.8, 1, 1)	(0.7, 0.87, 0.9)	(0.6, 0.7, 0.8)
10	P0100011000	(0.6, 0.74, 0.8)	(0.75, 0.92, 0.95)	(0.8, 1, 1)	(0.55, 0.65, 0.75)
11	P0100010001	(0.6, 0.74, 0.8)	(0.8, 1, 1)	(0.75, 0.92, 0.95)	(0.6, 0.7, 0.8)
12	P0101010000	(0.6, 0.74, 0.8)	(0.8, 1, 1)	(0.75, 0.92, 0.95)	(0.65, 0.79, 0.85)
13	P0100000110	(0.65, 0.79, 0.85)	(0.65, 0.79, 0.85)	(0.65, 0.79, 0.85)	(0.6, 0.74, 0.8)
14	P0100100100	(0.7, 0.87, 0.9)	(0.75, 0.92, 0.95)	(0.65, 0.79, 0.85)	(0.65, 0.79, 0.85)
15	P0100010100	(0.6, 0.74, 0.8)	(0.75, 0.92, 0.95)	(0.7, 0.87, 0.9)	(0.65, 0.79, 0.85)
16	P0010100010	(0.7, 0.84, 0.9)	(0.7, 0.87, 0.9)	(0.6, 0.7, 0.8)	(0.6, 0.7, 0.8)
17	P0010010010	(0.6, 0.7, 0.8)	(0.7, 0.87, 0.9)	(0.65, 0.79, 0.85)	(0.6, 0.7, 0.8)
18	P0010110000	(0.65, 0.79, 0.85)	(0.8, 1, 1)	(0.65, 0.79, 0.85)	(0.65, 0.75, 0.85)
19	P0010010001	(0.65, 0.79, 0.85)	(0.8, 1, 1)	(0.65, 0.79, 0.85)	(0.65, 0.75, 0.85)
20	P0011000000	(0.725, 0.875, 0.925)	(0.8, 1, 1)	(0.575, 0.675, 0.775)	(0.725, 0.875, 0.925)
21	P0010010100	(0.65, 0.79, 0.85)	(0.75, 0.92, 0.95)	(0.6, 0.74, 0.8)	(0.7, 0.84, 0.9)
22	P0010001000	(0.725, 0.875, 0.925)	(0.725, 0.875, 0.925)	(0.65, 0.8, 0.85)	(0.575, 0.675, , 0.775)
23	P0001110000	(0.7, 0.87, 0.9)	(0.8, 1, 1)	(0.7, 0.84, 0.9)	(0.7, 0.84, 0.9)
24	P0001010010	(0.65, 0.79, 0.85)	(0.7, 0.87, 0.9)	(0.7, 0.84, 0.9)	(0.65, 0.79, 0.85)
25	P0001000001	(0.8, 1, 1)	(0.8, 1, 1)	(0.65, 0.75, 0.85)	(0.725, 0.875, 0.925)
26	P0001000100	(0.8, 1, 1)	(0.725, 0.875, 0.925)	(0.575, 0.675, 0.775)	(0.8, 1, 1)
27	P0000100011	(0.75, 0.92, 0.95)	(0.7, 0.87, 0.9)	(0.65, 0.75, 0.85)	(0.6, 0.7, 0.8)
28	P0000110001	(0.7, 0.87, 0.9)	(0.8, 1, 1)	(0.7, 0.84, 0.9)	(0.65, 0.75, 0.85)
29	P0000111000	(0.7, 0.87, 0.9)	(0.75, 0.92, 0.95)	(0.75, 0.92, 0.95)	(0.6, 0.7, 0.8)
30	P0000100110	(0.75, 0.92, 0.95)	(0.65, 0.79, 0.85)	(0.6, 0.7, 0.8)	(0.65, 0.79, 0.85)
31	P0000110100	(0.7, 0.87, 0.9)	(0.75, 0.92, 0.95)	(0.65, 0.79, 0.85)	(0.7, 0.84, 0.9)
32	P0000010011	(0.65, 0.79, 0.85)	(0.7, 0.87, 0.9)	(0.7, 0.84, 0.9)	(0.6, 0.7, 0.8)
33	P0000011010	(0.65, 0.79, 0.85)	(0.65, 0.79, 0.85)	(0.75, 0.92, 0.95)	(0.55, 0.65, 0.75)
34	P0000010110	(0.65, 0.79, 0.85)	(0.65, 0.79, 0.85)	(0.65, 0.79, 0.85)	(0.65, 0.79, 0.85)
35	P0000010101	(0.7, 0.87, 0.9)	(0.75, 0.92, 0.95)	(0.65, 0.79, 0.85)	(0.7, 0.84, 0.9)
36	P0000001100	(0.8, 1, 1)	(0.65, 0.75, 0.85)	(0.65, 0.8, 0.85)	(0.65, 0.8, 0.85)

Eqs. (20) and (21) with project values in Table 1 and interaction matrixes in Table 5 are used for calculating input and outputs values of portfolios. The first part of Z-number values of Portfolio criteria is presented in Table 6. Value of criteria for portfolio number 1 is obtained by adding the criteria values of projects 1 and 9 because there are no interactions between projects. The fuzzy reliabilities of portfolios criteria are obtained by means of the reliabilities of participated projects in each portfolio. These values are shown in Table 7.

The data in Tables 6 and 7 are entered into the proposed Z-number DEA model to generate the efficiencies of each portfolio. Fig. 6 shows the results of portfolio efficiencies. Furthermore, it shows that portfolio number 16 has more efficiency than others with different satisfaction degrees. It has an efficiency value of 1.6. Moreover, projects 3, 5 and 9 take part in this portfolio. It is claimed that projects 3 and 5 have more influence on efficiencies of portfolios (by investigating other portfolios with higher efficiency values). In addition, these projects must be invested in IS/IT master plan of organization. Fig. 7 compares the results of proposed model with Ghapanchi et al. [53]. It is concluded that in our method, the efficiency value of portfolio number 5 is less than efficiencies values of portfolio numbers 27 and 18 due to reliability values of projects criteria, then our model considered the reliability values in its results. In the most engineering problems in the R&D departments, mangers must be decided between different portfolios to invest on them and in the R&D projects the

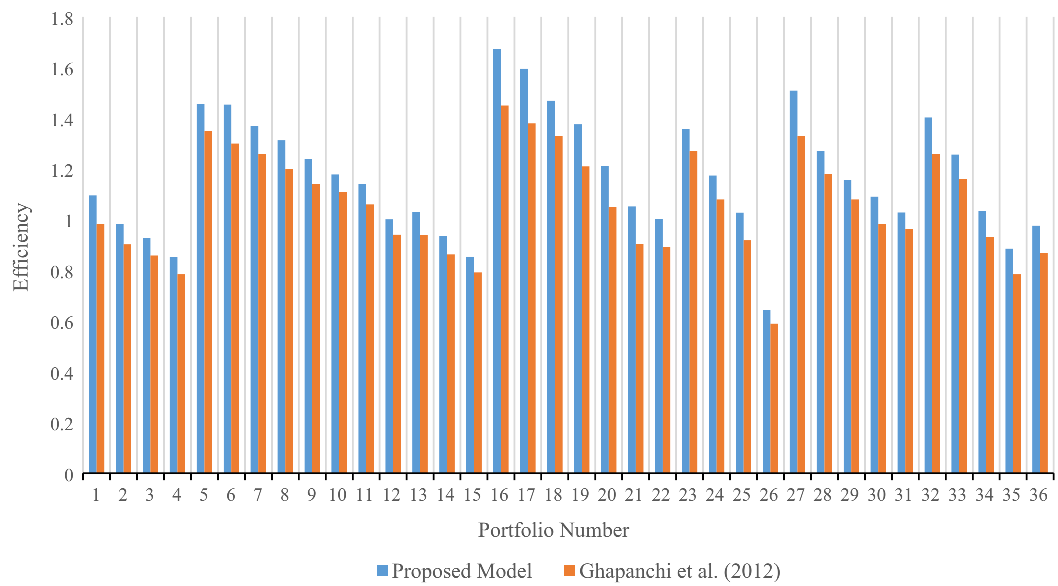

Fig. 7. Results of the proposed model versus Ghapanchi et al. [53].

Table 8
Features of proposed model versus other studies.

	DEA	FDEA	Z-DEA	Deterministic	Reliability	Z-numbers	Interactions	Restrictions
Proposed model	✓	✓	✓	✓	✓	✓	✓	✓
CCR	✓			✓				
BCC	✓							
Sueyoshi [12]	✓			✓				
Lertworasirikul et al. [4,32,33]	✓	✓						
Saati et al. [20]	✓	✓		✓				
Danila [36]				✓				✓
Cooper et al. [37]				✓				✓
Schmidt [46]				✓			✓	✓
Bardhan et al. [48]				✓			✓	✓
Eilat et al. [49]	✓			✓			✓	✓
Huang et al. [50]				✓			✓	✓
Chen and Cheng [52]				✓			✓	✓
Ghapanchi et al. [53]	✓	✓		✓			✓	✓

outcome is often unpredictable and decision makers reliability numbers are important in decision making. The project selection is complicated by many factors, such as vision and preferences of decision makers, allocating the right human resources, interrelationships between projects, and changes over time and success factors that are difficult to measure, thus our model is very strong method in development of engineering problems.

6. Conclusions

Presenting data with uncertainty is one of the most important features in most real problems. As stated by Zadeh [6] in his paper: (entitled as "A note on Z-numbers") reliability is the inseparable part of uncertainty data in real problems and experts usually represent the data with linguistic variables that is called reliability.

In this paper the Z-number version of CCR and BCC DEA models are suggested for vague and incomplete data especially for future analysis of decision making units. We used the concept of Z-numbers for adding the reliability into the fuzziness. We proposed the method for converting these models to possibilistic models and then used the α-cut approach for obtaining equivalent crisp linear programming models. The suggested ranking approach in this paper is an application of fuzzy theory and Z-numbers in DEA. This model is also capable to rank the DMUs that consume Z-number inputs to produce Z-number outputs.

We used actual portfolio selection case problem in IS/IT environments to show the applicability of proposed model. We considered the uncertainty, reliability and interactions in the stated case to show how the proposed model can handle such important issues. Table 8 presents the features of the proposed model versus other studies.

It is stated that the proposed model can solve complex decision making issues for engineering problems. In addition, it is stated that the portfolio selection is the most important tool in R&D environments and any engineering development department needs this tool to manage its resources.

For the future work one may consider the trapezoid membership functions for inputs and outputs that are more general to triangular membership function. Also, the triple interactions between projects in the actual case of this study can be considered.

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