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BETA random waypoint mobility model for wireless network simulation

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ABSTRACT

It is widely known that the instantaneous average node speed for the random waypoint (RWP) mobility model may not reach a steady state regime due to velocity gradual decaying which can cause inaccurate results in simulations and communication protocol validations for wireless networks. This paper presents a modification to the RWP model, in which we propose to choose node speeds from a BETA(α, β) distribution, demonstrating analytically and by simulations that depending on the values of α and β parameters the instantaneous average node speed and consequently other important network metrics, like control overhead and number of dropped data packets may reach (or not) a steady state regime. Therefore, by allowing α and β to vary, a multitude of probability distributions for speed choice is obtained and the resulting limiting state behavior for the mobility model can straightforwardly be determined, offering to the research community a generalized BETA random waypoint mobility model. Accordingly, the generic analytical closed form for the instantaneous average node velocity $\overline{V}$ as $V_{min} \to 0$ is obtained as a function of α and β to be given by $\lim_{V_{min}\to 0} \overline{V} = V_{max}\frac{\alpha-1}{\beta+\alpha-1}$ in which V_{min} and V_{max} are the minimum and maximum velocities, respectively, that a node can select.

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1. Introduction

Wireless ad hoc networks require no base station and all the control and access tasks are distributed among nodes acting as peers [1]. That is, there is no network infrastructure, while nodes can be static or mobile. This makes such networks attractive in situations such as battle fields, catastrophe-relief efforts or environmental monitoring. Accordingly, communication protocols for ad hoc networks must be decentralized and utilize few resources, like information processing and energy. Such protocols must be tested under conditions reflecting various possible practical scenarios that a user may confront. In this context, the mobility effect on ad hoc networks has been investigated by many authors [2–8]. These studies showed that the performance of mobile ad hoc networks is highly dependent on the mobility employed in simulations and its characteristics.

On the other hand, in order to be considered valid, the results from any network simulation must be obtained under steady state behavior, i.e., the convergence time shall be smaller than the total simulation interval. It implies that the initial transient is discarded for performance analysis. Therefore, mobility models that never attain a stationary regime must be avoided.

Among many mobility models used in the literature and in network simulators for ad hoc networks and wireless networks in general, the random waypoint (RWP) model is one of the most employed as observed in [9] and for example used in [2,3,6,8,10–37]. Its main features are the random choice of position and speed for the nodes, as well as the use of pause time between direction changes [2,10]. In [6], Yoon et al. showed that the RWP model does not attain steady state regime under certain conditions. More specifically, they proved that the instantaneous average node speed consistently decreases over time for a given set of parameters which interferes with the network performance and therefore should not be directly used for simulations. Note that authors unaware of this problem have analyzed communication protocols under such conditions [3,8,10–16,21–24,26–28]. Therefore, it is of uttermost importance to discuss and disseminate the correct way of using the RWP mobility model as we do in this paper.

Previous works have proposed new mobility models with stable average node speed [38–41]; however, they did not present the impacts over networking performance metrics. In addition, the

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main features of the resultant models deviated from the principal characteristics of the RWP model, e.g., the average velocities of the used distributions for choosing the node speeds differ from the average value $\frac{V_{max}+V_{min}}{2}$ of the uniform distribution employed in RWP model, where V_{min} and V_{max} are the minimum and maximum velocities, respectively, that a node can select. In [42], Boudec and Vojnovic proposed a general mobility model called *random trip* which contains as special case the random waypoint. Accordingly, such work deals with a broader class of mobility models. On the other hand, our work focus on the RWP model aiming to provide the RWP with a stable average node speed, but preserving its original features.

In [43], it was proposed an alteration in the way the node speeds are chosen such that the RWP model always attains a steady state. More specifically, it was proposed a modification to the RWP model, in which node speeds are chosen from a BETA(2,2) distribution [44], demonstrating analytically and by simulations that it stabilizes the instantaneous average node speed and consequently other important network metrics, like control overhead and number of dropped data packets. The proposal of alteration not only eliminates the decaying problem of the average node speed but also provides average values closer to the commonly supposed average velocity $\frac{V_{max}+V_{min}}{2}$ than those of the original RWP model.

This paper extends the work in [43], generalizing the distribution employed to be a BETA(α, β) [44] in which we perform a study of the mobility showing analytically and by simulations how the use of a BETA distribution for node speed can stabilize (or not) this model for utilization in performance analysis of mobile networks. The reason for choosing the BETA distribution is because it does not change the RWP main features, like node spatial concentration in the centre of the network area and average number of neighbors [43]. In addition, this BETA distribution can be readily incorporated into network simulators since it is available in common programming languages (see GSL [45] or SSJ [46] libraries, for example). Furthermore, the BETA(α, β) distribution can reduce to the previous cases depending on the α and β parameters; for example, for $\alpha = 1$ and $\beta = 1$, BETA(1,1) is the uniform distribution for the speed choice of the original RWP model [2,10]. On the other hand, for $\alpha = 2$ and $\beta = 2$, BETA(2,2) is the distribution investigated in [43]. In addition, for $\alpha = 2$ and $\beta = 1$, BETA(2,1) is the linear distribution proposed in [39] and [7]. Therefore, by allowing the α and β parameters to vary, a multitude of distributions can be obtained and the resulting limiting state behavior for the mobility model can readily be scrutinized, offering to the research community a generalized BETA random waypoint mobility model that can be used to explore numerous speed test scenarios. Also, we find that by setting the parameter $\alpha \geq 2$ provides the BETA RWP mobility model with stabilized average steady state speed for the nodes.

The rest of this paper is organized as follows. Section 2 proposes the BETA RWP mobility model and presents its basic definitions together with a detailed steady state analysis leading to a final closed formula for the average steady state speed as a function of the BETA distribution parameters (α, β). Accordingly, it emphasizes the cases in which a stabilized RWP mobility model is attained by appropriately choosing the BETA parameters. Section 3 contains network performance results obtained in the ns-2 simulator [47] comparing cases in which the α and β parameters imply mobility models stabilized or not. Finally, Section 4 concludes the paper.

2. BETA random waypoint mobility model

A mobility model governs node movements in a network. The RWP model introduces pause time between changes in direction of the nodes and provides random choice of position and speed for the nodes [2,10]. It has been widely used to validate wireless communication protocols for mobile networks. Furthermore, the RWP mobility model is most employed due to its simplicity of implementation in network simulators.

In our proposal, we expand the random speed choice by allowing its probability density function to be a BETA(α, β) in which many distinct distribution shapes are possible according to the selection of the α and β parameters. Beyond the BETA(1,1) resulting in the uniform distribution, the BETA(2,1) producing the linear distribution case, and the BETA(2,2) generating the parabolic distribution behavior, by varying α and β, the model provides speed distributions to fit distinct simulation scenarios. For example, if α is greater than β the upper speeds are reinforced compared to lower speeds which may be the case for scenarios where the majority of the nodes move with high velocities. Moreover, the reinforcement, i.e., the rate at which the upper speeds are chosen over the lower speeds can be appropriately adjusted by the skewness of the distribution. The opposite behavior is obtained for $\beta > \alpha > 1$. On the other hand, if α is equal to β ($\alpha = \beta > 1$) the distribution is symmetric with relation to its mean. Thus, upper and lower speeds are evenly chosen on average, however, the greater the value of α and β ($\alpha = \beta$), a more impulsive shaped distribution centered at the mean is obtained which can be used to test scenarios that avoid the velocities near the extreme values V_{min} and V_{max}, for example.

Here we introduce the BETA RWP model considering a rectangular network area with dimensions $X_{max} \times Y_{max}$. The BETA(α, β) probability density function used by a node to randomly choose a speed v is given by

$$f_V(v) = \frac{(v - V_{min})^{\alpha-1}(V_{max} - v)^{\beta-1}}{B(\alpha, \beta)(V_{max} - V_{min})^{\alpha+\beta-1}}, \quad V_{min} \leq v \leq V_{max} \tag{1}$$

for $V_{min} \leq v \leq V_{max}$, where $0 < V_{min} < V_{max}$ and $B(\alpha, \beta) = \int_0^1 x^{\alpha-1}(1-x)^{\beta-1}dx = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$, in which $\Gamma(n) = (n-1)!$ for n integer. The parameters α and β are positive integers.

In order to simplify the upcoming analysis we expand the exponential terms in (1) using the Newton's Binomial Theorem $(x+y)^n = \sum_{a=0}^{n}\binom{n}{a}x^{n-a}y^a$ to obtain

$$(v - V_{min})^{\alpha-1} = \sum_{a=0}^{\alpha-1}\binom{\alpha-1}{a}(-1)^a V_{min}^a v^{\alpha-a-1} \tag{2}$$

and

$$(V_{max} - v)^{\beta-1} = \sum_{b=0}^{\beta-1}\binom{\beta-1}{b}(-1)^{\beta-b-1} V_{max}^b v^{\beta-b-1}. \tag{3}$$

Replacing (2) and (3) into (1), the BETA(α, β) probability density function can be rewritten as

$$f_v(v) = \sum_{a=0}^{\alpha-1}\sum_{b=0}^{\beta-1}\binom{\alpha-1}{a}\binom{\beta-1}{b}\frac{(-1)^{\beta+a-b-1}V_{min}^a V_{max}^b v^{\alpha+\beta-a-b-2}}{B(\alpha, \beta)(V_{max} - V_{min})^{\alpha+\beta-1}}. \tag{4}$$

Assumption 1. BETA RWP model: (i) Each node randomly chooses an initial position (x, y) in the network, where x and y are both uniformly distributed over [0, X_{max}] and [0, Y_{max}], respectively. (ii) Then, every node selects a destination (x', y') uniformly distributed in the network area and a speed v according to the BETA(α, β) distribution given in (1). (iii) A node will then start travelling toward the (x', y') destination on a straight line using the chosen velocity v. (iv) Upon reaching the selected destination, the node remains there for a pause time, either constant or randomly chosen from a given distribution. Upon expiration of the pause time, the next destination and speed are chosen in the same way as in (ii) and the process repeats until the end of the simulation.

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Note that the above model reduces to the original RWP model [2,10], if $\alpha = \beta = 1$.

2.1. Steady state analysis

The results from mobile wireless network simulations are meaningful as long as they are obtained under steady state regime. Yoon *et al.* [6] showed that the stationary regime is directly related to the instantaneous average node speed of a mobility model. For a mobility model, the instantaneous average node speed is defined as [6]

$$\overline{v}(t) = \frac{\sum_{i=1}^{N} v_i(t)}{N} \tag{5}$$

where N is the total number of nodes and $v_i(t)$ is the speed of node i at time t.

In [6], it was shown that a steady state for the original RWP mobility model cannot be attained in the cases where $V_{min} \to 0$, i.e., $\overline{v}(t)$ decay to zero over time such that $\lim_{t\to\infty} \overline{v}(t) = 0$. In practical terms, it means that if the range of $(0, V_{max}]$ is used for speed choices then the network simulation will never reach a stationary regime in terms of average node speed which may lead to inconsistent results when this model is employed to validate communication protocols in mobile networks. An intuitive explanation for this fact is to observe that the original RWP model selects destination and velocity for each node in a random and independent fashion where each node will maintain the chosen speed until it reaches the selected destination and then the process is repeated. During this procedure, the nodes that choose low speeds and long distances will remain trapped for a long time to these trips and depending on the total simulation period they may never reach their destinations. The nodes that select higher speeds and shorter distances will rapidly reach their destinations and soon they can choose new courses and velocities. As they repeat the procedure, these other nodes can choose low speeds and far destinations and they will also remain confined to slow journeys which dominate the average node speed, gradually taking the network to stagnation.

Note that it is possible to have the RWP stable as long as $V_{min} > > 0$ in simulations in order to quickly attain a stationary regime, because the smaller V_{min}, the longer the decay period until the steady state is achieved [6,43]. On the other hand, a mobility model must be flexible and robust regardless the values of its parameters. Also, the solution of setting $V_{min} > > 0$ to avoid the consistent speed decay restricts the use of the RWP model, e.g., preventing its utilization in testing scenarios where node velocity can be very low. Consequently, the following analysis will investigate the RWP behavior for $V_{min} \to 0$.

It was observed in [6] that pause times lead to fluctuations in the beginning of simulations; however, such effect is gradually reduced and the average node speed decaying is not consequence of pause time. Thus, we are going to assume zero pause time in our analysis. Accordingly, we propose the following BETA RWP model to perform the steady state investigation.

The destination of a node is uniformly chosen from a circle of radius R_{max} centered at the current location of the node. Therefore, the cumulative density function of the travel distance R for this node can be obtained by

$$P(R \le r) = \int_0^{2\pi}\int_0^{r} \frac{1}{\pi R_{max}^2} r' dr' d\theta = \frac{r^2}{R_{max}^2}, \quad 0 \le r \le R_{max}. \tag{6}$$

The probability density function of the random variable R is

$$f_R(r) = \frac{\partial P(R \le r)}{\partial r} = \frac{2r}{R_{max}^2}. \tag{7}$$

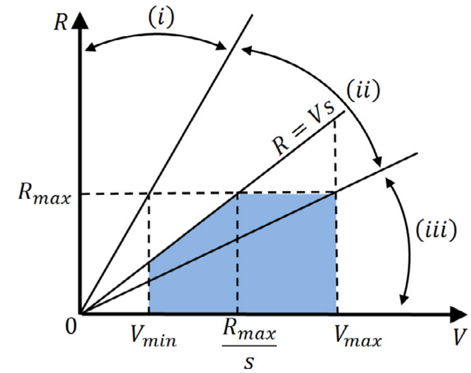

Fig. 1. Distance-speed graph. (*i*), (*ii*) and (*iii*) are the three regions (cases) of interest.

Consequently, the expected value of R is given by

$$E[R] = \int_0^{R_{max}} r\frac{2r}{R_{max}^2} dr = \frac{2}{3} R_{max}. \tag{8}$$

Assuming that the ensemble average equals time average as $t \to \infty$, it can be shown that the time average of the speed ($\overline{V}$) for a given node can be obtained from $v(t)$ in the following way [6]

$$\overline{V} = \lim_{T\to\infty} \frac{1}{T}\int_0^T v(t)dt = \frac{E[R]}{E[S]} \tag{9}$$

in which S is the random variable representing the travel time from one waypoint to the following waypoint and $E[S]$ is its average. Consequently, $\overline{V}$ can be taken as the steady state expected node speed.

Since we know $E[R]$ from (8), in order to derive a general expression for $\overline{V}$, we need to obtain $E[S]$. Therefore, the following development focus on the description of the travel time random variable S. Starting from its cumulative density function $P(S \le s)$ and by noting that travel time, speed and distance are related by $S = \frac{R}{V}$, we can use Fig. 1 considering three possible cases $s \ge \frac{R_{max}}{V_{min}}$, $\frac{R_{max}}{V_{max}} \le s \le \frac{R_{max}}{V_{min}}$, and $0 \le s \le \frac{R_{max}}{V_{max}}$. Also, from Assumption 1, R and V are independent, thus, $f_{R,V}(r,v) = f_R(r) \cdot f_V(v)$. We now consider each possible case in detail.

(*i*) For the case $s \ge \frac{R_{max}}{V_{min}}$ (i.e., $R_{max} \le V_{min}s$), we have

$$P(S \le s) = \int_{V_{min}}^{V_{max}}\int_0^{R_{max}} f_{R,V}(r,v)drdv = 1. \tag{10}$$

(*ii*) For the case $\frac{R_{max}}{V_{max}} \le s \le \frac{R_{max}}{V_{min}}$ (i.e., $V_{min}s \le R_{max} \le V_{max}s$), we obtain

$$\begin{aligned} P(S \le s) &= \int_{V_{min}}^{\frac{R_{max}}{s}}\int_0^{vs} f_{R,V}(r,v)drdv + \int_{\frac{R_{max}}{s}}^{V_{max}}\int_0^{R_{max}} f_{R,V}(r,v)drdv \\ &= \int_{V_{min}}^{\frac{R_{max}}{s}}\int_0^{vs} f_V(v)f_R(r)drdv + \int_{\frac{R_{max}}{s}}^{V_{max}}\int_0^{R_{max}} f_V(v)f_R(r)drdv \\ &= \int_{V_{min}}^{\frac{R_{max}}{s}} f_V(v)dv\int_0^{vs} f_R(r)dr + \int_{\frac{R_{max}}{s}}^{V_{max}} f_V(v)dv\int_0^{R_{max}} f_R(r)dr. \end{aligned} \tag{11}$$

By noting that

$$\int_0^{vs} f_R(r)dr = \int_0^{vs} \frac{2r}{R_{max}^2}dr = \frac{v^2}{R_{max}^2}s^2 \tag{12}$$

and

$$\int_0^{R_{max}} f_R(r)dr = \int_0^{R_{max}} \frac{2r}{R_{max}^2}dr = 1 \tag{13}$$

we rewrite (11) as

$$P(S \le s) = \underbrace{\frac{s^2}{R_{max}^2}\int_{V_{min}}^{\frac{R_{max}}{s}} v^2 f_V(v)dv}_{H_1} + \underbrace{\int_{\frac{R_{max}}{s}}^{V_{max}} f_V(v)dv}_{H_2} \tag{14}$$

where

$$H_1 = \frac{s^2}{R_{max}^2}\int_{V_{min}}^{\frac{R_{max}}{s}} v^2 f_V(v)dv \tag{15}$$

and

$$H_2 = \int_{\frac{R_{max}}{s}}^{V_{max}} f_V(v)dv. \tag{16}$$

Substituting (4) into H_1, it follows that

$$\begin{aligned} H_1 &= s^2 \sum_{a=0}^{\alpha-1}\sum_{b=0}^{\beta-1}\binom{\alpha-1}{a}\binom{\beta-1}{b}\frac{(-1)^{\beta+a-b-1}V_{min}^a V_{max}^b \int_{V_{min}}^{\frac{R_{max}}{s}} v^{\alpha+\beta-a-b}dv}{R_{max}^2 B(\alpha,\beta)\,(V_{max}-V_{min})^{\alpha+\beta-1}} \\ &= \sum_{a=0}^{\alpha-1}\sum_{b=0}^{\beta-1}\binom{\alpha-1}{a}\binom{\beta-1}{b}\frac{(-1)^{\beta+a-b-1}V_{min}^a V_{max}^b}{R_{max}^2 B(\alpha,\beta)\,(V_{max}-V_{min})^{\alpha+\beta-1}} \\ &\quad\times\left(\frac{R_{max}^{\alpha+\beta-a-b+1}\,s^{1-\alpha-\beta+a+b} - V_{min}^{\alpha+\beta-a-b+1}\,s^2}{\alpha+\beta-a-b+1}\right). \end{aligned} \tag{17}$$

Analogously, for H_2 we obtain

$$\begin{aligned} H_2 &= \sum_{a=0}^{\alpha-1}\sum_{b=0}^{\beta-1}\binom{\alpha-1}{a}\binom{\beta-1}{b}\frac{(-1)^{\beta+a-b-1}V_{min}^a V_{max}^b}{B(\alpha,\beta)\,(V_{max}-V_{min})^{\alpha+\beta-1}} \\ &\quad\times\left(\frac{V_{max}^{\alpha+\beta-a-b-1} - R_{max}^{\alpha+\beta-a-b-1}\,s^{1-\alpha-\beta+a+b}}{\alpha+\beta-a-b-1}\right). \end{aligned} \tag{18}$$

Finally, substituting the expressions for H_1 and H_2 into (14) and rearranging terms we arrive at

$$\begin{aligned} P(S \le s) &= \sum_{a=0}^{\alpha-1}\sum_{b=0}^{\beta-1}\binom{\alpha-1}{a}\binom{\beta-1}{b}\frac{(-1)^{\beta+a-b-1}V_{min}^a V_{max}^b}{B(\alpha,\beta)\,(V_{max}-V_{min})^{\alpha+\beta-1}} \\ &\quad\times\left[\left(\frac{s}{R_{max}}\right)^2\left(\frac{\left(\frac{R_{max}}{s}\right)^{\alpha+\beta-a-b+1} - V_{min}^{\alpha+\beta-a-b+1}}{\alpha+\beta-a-b+1}\right)\right. \\ &\quad\left.+\left(\frac{V_{max}^{\alpha+\beta-a-b-1} - R_{max}^{\alpha+\beta-a-b-1}\,s^{1-\alpha-\beta+a+b}}{\alpha+\beta-a-b-1}\right)\right] \\ &= \sum_{a=0}^{\alpha-1}\sum_{b=0}^{\beta-1}\binom{\alpha-1}{a}\binom{\beta-1}{b}\frac{(-1)^{\beta+a-b-1}V_{min}^a V_{max}^b}{B(\alpha,\beta)\,(V_{max}-V_{min})^{\alpha+\beta-1}}\left[\frac{V_{max}^{\alpha+\beta-a-b-1}}{\alpha+\beta-a-b-1}\right. \\ &\quad\left.+\left(\frac{R_{max}^{\alpha+\beta-a-b}}{\alpha+\beta-a-b+1} - \frac{R_{max}^{\alpha+\beta-a-b-1}}{\alpha+\beta-a-b-1}\right)s^{1-\alpha-\beta+a+b} - \left(\frac{V_{min}^{\alpha+\beta-a-b+1}}{R_{max}^2(\alpha+\beta-a-b+1)}s^2\right)\right]. \end{aligned} \tag{19}$$

(*iii*) For the case $0 \le s \le \frac{R_{max}}{V_{max}}$, (i.e., $0 \le sV_{max} \le R_{max}$), it follows that

$$\begin{aligned} P(S \le s) &= \frac{s^2 \int_{V_{min}}^{V_{max}} \sum_{a=0}^{\alpha-1}\sum_{b=0}^{\beta-1}\binom{\alpha-1}{a}\binom{\beta-1}{b}(-1)^{\beta+a-b-1}V_{min}^a V_{max}^b\, v^{\alpha+\beta-a-b}dv}{R_{max}^2 B(\alpha,\beta)\,(V_{max}-V_{min})^{\alpha+\beta-1}} \\ &= \frac{\sum_{a=0}^{\alpha-1}\sum_{b=0}^{\beta-1}\binom{\alpha-1}{a}\binom{\beta-1}{b}(-1)^{\beta+a-b-1}V_{min}^a V_{max}^b\left(\frac{V_{max}^{\alpha+\beta-a-b+1} - V_{min}^{\alpha+\beta-a-b+1}}{\alpha+\beta-a-b+1}\right)s^2}{R_{max}^2 B(\alpha,\beta)\,(V_{max}-V_{min})^{\alpha+\beta-1}}. \end{aligned} \tag{20}$$

From (10), (19) and (20) and using that the probability density function can be obtained from the cumulative density function, *i.e.*, $f_S(s) = \frac{\partial P(S\le s)}{\partial s}$, we arrive at the following result:

$$f_S(s) = \begin{cases} \dfrac{\sum_{a=0}^{\alpha-1}\sum_{b=0}^{\beta-1}\binom{\alpha-1}{a}\binom{\beta-1}{b}(-1)^{\beta+a-b-1}V_{min}^a V_{max}^b\left(\frac{V_{max}^{\alpha+\beta-a-b+1} - V_{min}^{\alpha+\beta-a-b+1}}{\alpha+\beta-a-b+1}\right)2s}{R_{max}^2 B(\alpha,\beta)\,(V_{max}-V_{min})^{\alpha+\beta-1}}, \\ \text{for } 0 \le s \le \frac{R_{max}}{V_{max}}; \\ \dfrac{\sum_{a=0}^{\alpha-1}\sum_{b=0}^{\beta-1}\binom{\alpha-1}{a}\binom{\beta-1}{b}(-1)^{\beta+a-b-1}V_{min}^a V_{max}^b}{B(\alpha,\beta)\,(V_{max}-V_{min})^{\alpha+\beta-1}}\left[\left(\frac{2R_{max}^{\alpha+\beta-a-b-1}}{\alpha+\beta-a-b+1}\right)s^{-\alpha-\beta+a+b}\right. \\ \quad\left.-\left(\frac{2sV_{min}^{\alpha+\beta-a-b+1}}{R_{max}^2(\alpha+\beta-a-b+1)}\right)\right], \quad \text{for } \frac{R_{max}}{V_{max}} \le s \le \frac{R_{max}}{V_{min}}; \\ 0, \quad \text{for } s \ge \frac{R_{max}}{V_{max}}. \end{cases} \tag{21}$$

From the probability density function in (21), the expected travel time is given by

$$\begin{aligned} E[S] &= \int_0^{\infty} s\, f_S(s)ds \\ &= \sum_{a=0}^{\alpha-1}\sum_{b=0}^{\beta-1}\binom{\alpha-1}{a}\binom{\beta-1}{b}\frac{(-1)^{\beta+a-b-1}V_{min}^a V_{max}^b}{B(\alpha,\beta)\,(V_{max}-V_{min})^{\alpha+\beta-1}} \\ &\quad\times\int_{\frac{R_{max}}{V_{max}}}^{\frac{R_{max}}{V_{min}}}\left[\left(\frac{2R_{max}^{\alpha+\beta-a-b-1}}{\alpha+\beta-a-b+1}\right)s^{-\alpha-\beta+a+b+1}\left(\frac{2s^2V_{min}^{\alpha+\beta-a-b+1}}{R_{max}^2(\alpha+\beta-a-b+1)}\right)\right]ds \\ &\quad+\sum_{a=0}^{\alpha-1}\sum_{b=0}^{\beta-1}\binom{\alpha-1}{a}\binom{\beta-1}{b}\frac{2(-1)^{\beta+a-b-1}V_{min}^a V_{max}^b}{R_{max}^2 B(\alpha,\beta)(V_{max}-V_{min})^{\alpha+\beta-1}} \\ &\quad\times\int_0^{\frac{R_{max}}{V_{max}}}\left(\frac{V_{max}^{\alpha+\beta-a-b+1} - V_{min}^{\alpha+\beta-a-b+1}}{\alpha+\beta-a-b+1}\right)s^2 ds. \end{aligned} \tag{22}$$

Developing the previous equation we arrive at

$$E[S] = T_1 + T_2 + T_3 + T_4 \tag{23}$$

in which the terms T_1, T_2, T_3 and T_4 are given, respectively, by

$$T_1 = \frac{\sum_{a=0}^{\alpha-1}\sum_{b=0}^{\beta-1}\binom{\alpha-1}{a}\binom{\beta-1}{b}2R_{max}(-1)^{\beta+a-b-1}V_{min}^a V_{max}^b\left(\frac{V_{max}^{\alpha+\beta-a-b+1} - V_{min}^{\alpha+\beta-a-b+1}}{\alpha+\beta-a-b+1}\right)}{3V_{max}^3 B(\alpha,\beta)(V_{max}-V_{min})^{\alpha+\beta-1}} \tag{24}$$

$$\begin{aligned} T_2 &= \sum_{\substack{a=0 \\ \alpha+\beta-a-b\neq 2}}^{\alpha-1}\sum_{b=0}^{\beta-1}\binom{\alpha-1}{a}\binom{\beta-1}{b}\frac{(-1)^{\beta+a-b-1}V_{min}^a V_{max}^b}{B(\alpha,\beta)\,(V_{max}-V_{min})^{\alpha+\beta-1}} \\ &\quad\times\frac{2R_{max}}{(\alpha+\beta-a-b+1)(2-\alpha-\beta+a+b)}\left(\frac{1}{V_{min}^{2-\alpha-\beta+a+b}} - \frac{1}{V_{max}^{2-\alpha-\beta+a+b}}\right) \end{aligned} \tag{25}$$

$$T_3 = \frac{2\,(-1)^{\alpha-1}R_{max}V_{min}^{\alpha-1}V_{max}^{\beta-1}}{3B(\alpha,\beta)\,(V_{max}-V_{min})^{\alpha+\beta-1}}\ln\left(\frac{V_{max}}{V_{min}}\right) \tag{26}$$

$$T_4 = \frac{\sum_{a=0}^{\alpha-1}\sum_{b=0}^{\beta-1}\binom{\alpha-1}{a}\binom{\beta-1}{b}2(-1)^{\beta+a-b-1}V_{max}^b R_{max}\left(\frac{V_{min}^{\alpha+\beta-b+1}}{V_{max}^3} - V_{min}^{\alpha+\beta-b-2}\right)}{3B(\alpha,\beta)\,(V_{max}-V_{min})^{\alpha+\beta-1}(\alpha+\beta-a-b+1)}. \tag{27}$$

The special case $\alpha = 1$ and $\beta = 1$ reduces to the well-known formula for the RWP [6, Eq. (5)]

$$E[S] = \frac{2\,R_{max}}{3\,(V_{max}-V_{min})}\ln\left(\frac{V_{max}}{V_{min}}\right).$$

If we now consider the condition $V_{min} \to 0$ in T_1, T_2, T_3 and T_4 yielding

$$\lim_{V_{min}\to 0} T_1 = \frac{2R_{max}}{3V_{max}B(\alpha,\beta)}\sum_{b=0}^{\beta-1}\binom{\beta-1}{b}\frac{(-1)^{\beta-b-1}}{\alpha+\beta-b+1} \tag{28}$$

$$\lim_{V_{min}\to 0} T_2 = \frac{2R_{max}}{V_{max}B(\alpha,\beta)}\sum_{b=0}^{\beta-1}\binom{\beta-1}{b}\frac{(-1)^{\beta-b-1}}{(\alpha+\beta-b+1)(\alpha+\beta-b-2)} \tag{29}$$

$$\lim_{V_{min}\to 0} T_3 = \begin{cases} 0, & \text{if } \alpha \geq 2 \\ \infty, & \text{if } \alpha = 1 \end{cases} \tag{30}$$

$$\lim_{V_{min}\to 0} T_4 = \begin{cases} 0, & \text{if } \alpha \geq 2 \\ \frac{-2Rmax}{9B(1,\beta)V_{max}}, & \text{if } \alpha = 1. \end{cases} \tag{31}$$

This shows that choosing the node velocity from a BETA(1, β) distribution, $E[S] \to \infty$, as $V_{min} \to 0$, which implies that

$$\lim_{V_{min}\to 0} \overline{V} = \frac{E[R]}{E[S]} = 0. \tag{32}$$

In this case, the BETA RWP will never achieve steady state, and this result agrees with the particular case for BETA(1, 1) distribution, that is, the original RWP.

Hereafter, we consider $\alpha \geq 2$ and $\beta \geq 1$. It follows from (23) and (28) and (29) that

$$\begin{aligned} \lim_{V_{min}\to 0} E[S] &= \lim_{V_{min}\to 0} T_1 + \lim_{V_{min}\to 0} T_2 \\ &= \frac{2R_{max}}{V_{max}B(\alpha,\beta)} \sum_{b=0}^{\beta-1} \binom{\beta-1}{b} \frac{(-1)^{\beta-b-1}}{\alpha+\beta-b+1} \\ &\quad \times \left(\frac{1}{3} + \frac{1}{\alpha+\beta-b-2}\right) \\ &= \frac{2R_{max}}{3V_{max}B(\alpha,\beta)} \sum_{b=0}^{\beta-1} \binom{\beta-1}{b} \frac{(-1)^{\beta-b-1}}{\alpha+\beta-b-2}. \end{aligned} \tag{33}$$

In order to develop (33) we need the following lemma.

Lemma 1. *Let $\beta \geq 1$ and $\alpha \geq 2$ be integers. Then*

$$\sum_{b=0}^{\beta-1} \binom{\beta-1}{b} \frac{(-1)^{\beta-1-b}}{(\beta-1)+(\alpha-1)-b} = \frac{\Gamma(\alpha-1)\Gamma(\beta)}{\Gamma(\alpha+\beta-1)} = B(\alpha-1,\beta). \tag{34}$$

Proof: See Appendix A.

Using Lemma 1 in (33), we get

$$\lim_{V_{min}\to 0} E[S] = \frac{2R_{max}}{3V_{max}} \frac{B(\alpha-1,\beta)}{B(\alpha,\beta)} \tag{35}$$

$$= \frac{2R_{max}}{3V_{max}} \frac{(\alpha-2)!\,\Gamma(\beta)}{(\alpha+\beta-2)!} \frac{(\alpha+\beta-1)!}{(\alpha-1)!\,\Gamma(\beta)} \tag{36}$$

$$= \frac{2R_{max}}{3V_{max}} \frac{(\alpha+\beta-1)}{(\alpha-1)} \tag{37}$$

which implies from (8), (9) and (37) that the average node speed as $V_{min} \to 0$ is given by

$$\overline{V} = V_{max} \frac{\alpha-1}{\beta+\alpha-1}. \tag{38}$$

Thus, the average node speed for the BETA RWP mobility model which employs the BETA(α, β) probability density function for velocity choice (with $\alpha \geq 2$) is a function of the parameters V_{max}, α and β only, as $V_{min} \to 0$. It is clear that choosing node velocity from a BETA($n, n-1$) distribution, for $n \geq 2$, the BETA RWP will always achieve steady state with $\overline{V} = \frac{V_{max}}{2}$.

Table 1 shows the steady state average speed for several values of α and β. Accordingly, if α is set to be one, irrespective of the value of β, the final average node speed is zero and agrees with the original RWP model, confirming analytically the limitation detected by Yoon et al. in [6]. Therefore, in order to avoid the instantaneous average node speed decaying problem one has to set $\alpha \geq 2$. Also, the result obtained of $\overline{V} = \frac{V_{max}}{3}$ for BETA(2,2) agrees with [43]. It is possible to observe that for several chosen values of α and β, $\overline{V}$ assumes values that demonstrate the stability of the BETA RWP model when $V_{min} \to 0$, that is, $\overline{V} \neq 0$.

Table 1
$\overline{V}$ as $V_{min} \to 0$ for several values of α and β.

α	β	$\lim_{V_{min}\to 0} \overline{V} = V_{max} \frac{\alpha-1}{\beta+\alpha-1}$
1	$n \geq 1$	0
$n \geq 2$	$n-1$	$\frac{V_{max}}{2}$
2	2	$\frac{V_{max}}{3}$
2	3	$\frac{V_{max}}{4}$
2	4	$\frac{V_{max}}{5}$
3	3	$\frac{V_{max}}{\frac{5}{2}}$
3	4	$\frac{V_{max}}{3}$
3	1	$\frac{V_{max}}{\frac{3}{2}}$
4	2	$\frac{V_{max}}{\frac{5}{3}}$
4	4	$\frac{V_{max}}{\frac{7}{3}}$

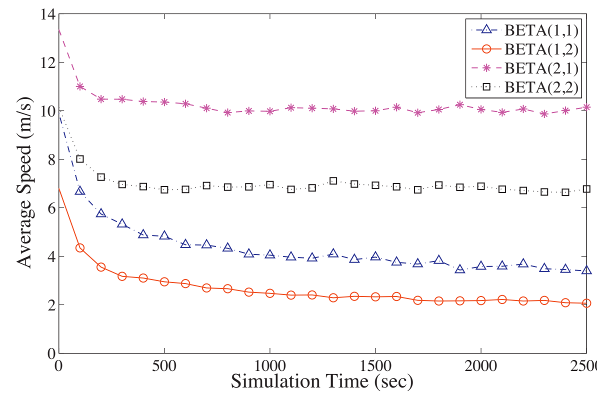

Fig. 2. Instantaneous average node speed for the BETA RWP mobility model employing speed range of (0,20] m/s.

Fig. 2 illustrates the behavior of the instantaneous average node speed for the BETA RWP model using the range of (0,20] meters per second (m/s) with zero pause time for a rectangular area with dimensions 1500 m × 500 m containing 50 mobile nodes. The presented curves are averaged over 30 distinct scenarios. It is clear that for the cases of BETA(1,1) and BETA(1,2) the average speed gradually decays with time, whereas the cases BETA(2,1) and BETA(2,2) the average speed stabilizes around 10 m/s and 6.67 m/s, respectively, as expected from (38).

3. Network performance evaluation

To verify the impact of the BETA RWP mobility model, we simulated several distinct scenarios, each using different BETA parameters for node velocity. The distributions employed are the BETA(1, 1), BETA(1, 2), BETA(2, 1) and BETA(2, 2). The ns-2 network simulator [47] is used for our simulations. The simulation environment consists of a rectangle measuring 1500 m × 500 m, containing 50 nodes employing speed range of (0,20] m/s. Simulation runs are 2000 s long, and results are averaged over 30 runs, and calculated at the end of each interval of 100 s, for each scenario. The data traffic pattern employed is CBR (constant bit rate), with rates of four packets per second, and packet lengths of 64 bytes. For routing, ad hoc on-demand distance vector routing protocol (AODV) is employed [17]. Our purpose is to verify network performance stabilization over several parameters of the BETA distribution. We investigate two important network metrics: dropped data packets and routing overhead packets.

- Dropped data packets: this performance measure quantifies discarded data packets by routers along the path to destinations due to errors. Fig. 3 illustrates the behaviour of dropped data

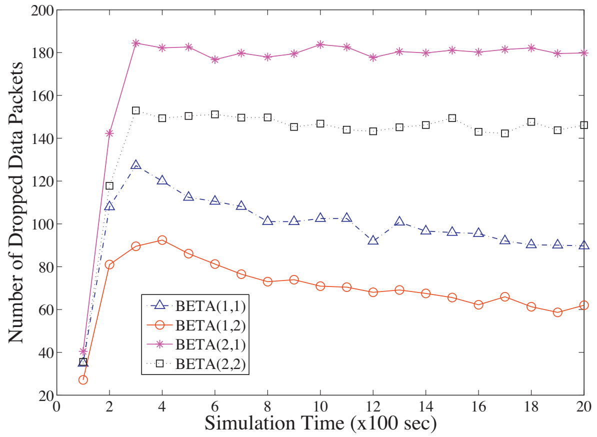

Fig. 3. Number of dropped data packets as a function of simulation time.

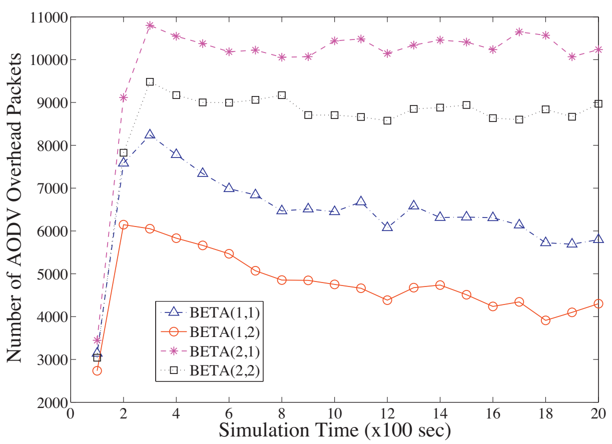

Fig. 4. Number of routing overhead packets as a function of simulation time.

packets. It is clear the influence of the instantaneous average speed decaying over the curves for the case of the original RWP BETA(1,1) and the BETA(1,2), i.e., the unstable cases which present dropped data packets decaying as simulation time increases. However, in the stable cases of BETA(2,1) and BETA(2,2), the number of dropped data packets stabilizes as simulation time increases as consequence of instantaneous average speed reaching steady state.

- Number of routing overhead packets: this performance metric quantifies the amount of control routing packets used by the AODV protocol to create and update routes in the network. Fig. 4 illustrates the behaviour of overhead packets as a function of simulation time. It is also clear the influence from instantaneous average speed decaying over the overhead packets for the cases of the BETA(1,1) and the BETA(1,2), while the metric attains stability for the cases of BETA(2,1) and BETA(2,2).

In all simulated cases, it is clear the influence of the behavior of the instantaneous average speed. Therefore, it is very important to employ a mobility model that attains steady state regime as simulation time evolves. Our investigation indicates that the BETA(α, β) RWP mobility model can be used for simulation of wireless mobile networks with any speed range (including $V_{min} = 0$), as long as $\alpha \geq 2$.

Another important observation from all figures in this paper and also from the figures in [6] and [43] is that the investigated network metrics keeps their relatives values as a function of the average node speed.

4. Conclusion

This paper proposed and analyzed a modification that stabilizes the random waypoint mobility model used to evaluate performance of mobile wireless networks. We showed that the use of a BETA(α, β) distribution for choosing node speed, results a steady state expected node speed equals to $V_{max}\frac{\alpha-1}{\beta+\alpha-1}$ as $V_{min} \to 0$. Therefore, by choosing $\alpha \geq 2$, it avoids the gradual decaying with time of the instantaneous average node speed when the minimum velocity in the choice range is set to zero. On the other hand, if $\alpha = 1$, irrespective of the value of β, the instantaneous average speed decays to zero. Analytical and simulation results were presented which showed the impact of instantaneous average node speed in network performance metrics. Beyond providing a RWP mobility model spanning a multitude of speed distributions, our investigation corroborates the importance of using a stable mobility model when communication protocols are under evaluation in wireless networks.

Future work can extend our analysis to consider changing the BETA Random Waypoint model to address the node concentration issue, as well as to investigate its behavior in other network topologies like circle, torus and sphere.

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Appendix A. Proof of Lemma 1

The proof of Lemma 1 is by induction on β. For $\beta = 1$, then $B(\alpha-1, 1) = 1/(\alpha-1)$, and (34) holds. We suppose next that the claim is satisfied for some $\beta \geq 1$. Using the relation

$$\binom{\beta}{b} = \binom{\beta-1}{b} + \binom{\beta-1}{b-1}$$

we write the left-hand side of (34) for $\beta+1$ as

$$\sum_{b=0}^{\beta}\binom{\beta}{b}\frac{(-1)^{\beta-b}}{\beta+(\alpha-1)-b}$$

$$= \sum_{b=0}^{\beta-1}\binom{\beta-1}{b}\frac{(-1)^{\beta-b}}{\beta+(\alpha-1)-b} + \sum_{b=1}^{\beta}\binom{\beta-1}{b-1}\frac{(-1)^{\beta-b}}{\beta+(\alpha-1)-b}$$

$$= -\sum_{b=0}^{\beta-1}\binom{\beta-1}{b}\frac{(-1)^{\beta-1-b}}{(\beta-1)+\alpha-b} + \sum_{b=1}^{\beta}\binom{\beta-1}{b-1}\frac{(-1)^{\beta-1-(b-1)}}{(\beta-1)+(\alpha-1)-(b-1)}$$

$$= -\sum_{b=0}^{\beta-1}\binom{\beta-1}{b}\frac{(-1)^{\beta-1-b}}{(\beta-1)+\alpha-b} + \sum_{b=0}^{\beta-1}\binom{\beta-1}{b}\frac{(-1)^{\beta-1-b}}{(\beta-1)+(\alpha-1)-b}.$$

Using the inductive hypothesis

$$\sum_{b=0}^{\beta}\binom{\beta}{b}\frac{(-1)^{\beta-b}}{\beta+(\alpha-1)-b} = -B(\alpha,\beta) + B(\alpha-1,\beta)$$

$$= -\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)} + \frac{\Gamma(\alpha-1)\Gamma(\beta)}{\Gamma(\alpha+\beta-1)}.$$

Since $\Gamma(x+1) = x\Gamma(x)$, we get

$$\sum_{b=0}^{\beta} \binom{\beta}{b} \frac{(-1)^{\beta-b}}{\beta + (\alpha-1) - b} = -\frac{(\alpha-1)\Gamma(\alpha-1)\Gamma(\beta)}{\Gamma(\alpha+\beta)} + \frac{(\alpha+\beta-1)\Gamma(\alpha-1)\Gamma(\beta)}{\Gamma(\alpha+\beta)} = \frac{\Gamma(\alpha-1)\beta\Gamma(\beta)}{\Gamma(\alpha+\beta)} = \frac{\Gamma(\alpha-1)\Gamma(\beta+1)}{\Gamma(\alpha+\beta)}.$$

Thus, Eq. (34) is satisfied for $\beta + 1$.

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