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Dynamic Reactive Power Control of Islanded Microgrids

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***Abstract*—Reactive power control is a fundamental issue in microgrids, especially during islanded mode operation with no support from the main grid. Lack of infinite bus, tightly coupled generation and consumption, and existence of nondispatchable intermittent renewable power sources reinforce the need for a new VVC scheme. This paper presents a new model predictive control (MPC)-based dynamic voltage and var control (VVC) scheme, which includes the dynamics of the microgrid in the VVC formulation. The MPC-based controller uses a simplified voltage prediction model to predict the voltage behavior of the system for a time horizon ahead. The advantage of this method is that it can avoid unstable voltage conditions in microgrids by prediction of the instability ahead of time. This method can also avoid voltage drops or swells in any of the phases of the system since the model can predict the voltage of each phase separately. Also, the presented method can be implemented online so it can efficiently use the time-variant reactive capabilities of the distributed generators to compensate for reactive power needs of the system. This controller is tested for different operating conditions of the microgrid and the simulation results confirm that the MPC controller successfully keeps the system stable and achieves a smooth voltage profile.**

***Index Terms*—Loss minimization, microgrid, power system dynamics, voltage and var control (VCC), voltage and var optimization, wind energy.**

I. Introduction

REACTIVE power and voltage control in a distribution system is very important for ensuring the security of the network. The main objective of voltage and var control (VVC) is to advise a control policy for reactive power sources in order to minimize the peak hour demand, reduce losses, while all of the bus voltages are kept within the permissible range. Reactive power control of microgrids integrating wind and photovoltaic sources has many technical challenges [1].

To begin with, significant penetration of renewable distributed generators (DG) in microgrids can affect the frequency, angle, and voltage stability of the system [2]. Wind and PV energy sources that are abundant in microgrids are intermittent and not dispatchable and are usually operated at the maximum power point tracking to reach the highest active power generation efficiency [3]. Thus, changes in the weather have significant influence on the amount and quality of generated power by these sources. Abrupt changes in the wind energy resource can lead to sudden loss of production, causing voltage changes and frequency excursions which may result in dynamically unstable situations if these frequency fluctuations cause the frequency relay to trip [2], [4]. Photovoltaic sources cause similar problems when they start operating in the morning time or through transients during cloudy weather conditions [5].

Lack of an infinite bus in the microgrid when operating in the islanded mode makes VVC even more challenging. Islanded operation mode occurs intentionally or due to a fault in the system. During islanded operation, the main grid does not provide voltage and frequency references anymore. Therefore, either each DG can seamlessly balance the power of the islanded microgrid using a droop controller [3] or at least one suitably sized DG unit with fast dynamic response has to act as master generator and regulate the voltage and frequency [6]. Obviously, the inertia of this master generator is less than the inertia of the generators of grid; therefore, microgrids are weak distribution systems in islanded mode. Thus, islanded microgrid are vulnerable to load and source changes which means fast and accurate control actions should be taken to keep the voltages smooth and stable in this operation mode [7].

With proper control and coordination, the power electronics interfaced renewable DG units and the synchronous generators are capable of injecting reactive power at their connection point to the system [8]–[11]. Thus, DG units have the ability to enhance the power quality as well as the reliability of the system [3]. Using DGs as reactive compensators reduces the need for adding extra voltage and reactive compensators in the microgrid. However, reactive support capacity of DGs is time-variant, which means a fast online coordination of these new reactive sources in the microgrid is necessary if they are used for reactive support.

The relatively new problem of Volt/Var optimization of microgrids using global optimization techniques has been studied in [5], [12], [13]. Ghadimi and Rastegar [12] tried to improve VVC of microgrids by using a multilayer reactive power control method. They formulated reactive power control problem to minimize the total loss of the system subject to static load flow equations and inequality constraints of the system. Madureira and Lopes [13] used particle swarm intelligence to solve reactive power control of a system including multiple microgrids, a diesel generator, a wind generator, a hydro unit, and a combined heat and power unit. The same authors further proceeded the work by adding the concept of "micro-generation shedding" in [5]. They simply added the amount of micro-generation shedding to the objective function of [13]. The solution methodology in this paper is similar to that in [13]. In an interesting case study, they showed that, in a sunny day, where the PV sources generate

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more power than the total load, micro-generation shedding results in lower power loss in the system. All of these methods are inspired by reactive power control of conventional distribution systems [2]. However, since islanded power systems are more vulnerable to changes in voltage and the changes can be sharp, classic reactive control methods do not reflect a realistic impression of where and when overvoltage and under voltage occurs in the system. Therefore, some control actions taken by classic methods may result in unacceptable transient response in the islanded power system. In addition, conventional VVC methods are offline and offline control methods are not suitable for microgrids due to fast changes in the system.

A dynamic model of microgrid predicts the voltage response of the system to certain control actions more accurately than static models. This means that an online controller designed to use dynamic voltage prediction of a microgrid takes more accurate and efficient control actions to keep the bus voltage profile smooth and stable than the conventional VVC methods.

This paper presents a new online dynamic reactive power control method that considers the dynamics of the system to control reactive power in unbalanced microgrids. First, the system model is simplified for voltage control. The model of the system is then linearized and descretized. This simplified discrete model, which only predicts the voltage behavior of the system in future, is used as system model in model predictive control (MPC). The MPC control approach used in this paper uses a transform called mixed logical dynamics (MLD), which transforms the linearized dynamic and static equations to equality and inequalities to be able to solve the problem in real-time. Efficient solvers such as CPLEX can solve the resulting linear optimization problem sufficiently fast to be implemented for online control. The optimization problem is solved online using CPLEX, and the control signals are sent to reactive compensators and DGs to dynamically coordinate the reactive power generation and consumption. The remainder of this paper is organized as follows. Section II presents the problem formulation of dynamic reactive power control of unbalanced microgrids. Section III briefly discusses MPC and mixed logical dynamics. Simulation results are presented in Section IV. Finally, concluding remarks appear in Section V.

II. Problem Formulation

Dynamics reactive control using the prediction of the voltage behavior of the system has been used for voltage and reactive power control of transmission systems in the past decade [14]–[17]. However, in distribution systems, the large size of the network and lack of an accurate network model make dynamic modeling a difficult task. Nevertheless, dynamic control can be successfully adopted for small-scale distribution systems, specifically isolated power systems such as microgrids because the size of the system is small and the operator has more information of the topology and measurements of the bus voltages and line currents in real time.

Diesel, wind, and solar generators are the major sources of active and reactive power in most microgrids during islanded operation. Depending on the size and response time of the generators in the system, one generator should be chosen as the master generator in islanded operation mode. The master generator dictates the voltage and frequency level of the islanded system. The MPC controller can use the voltage setpoint of this generator to achieve the Volt/Var control objectives in this system. Additional control inputs are the setpoints of dynamic compensators, DGs, and capacitor banks.

The dynamic reactive power control problem can be divided into two layers. The first layer is a local control layer, which regulates the reactive power output of each DG unit to the desired setpoint. The second layer, which is the global layer, can determine the optimal value of the control signals including the reactive power setpoint of the DGs, dynamic reactive compensators and capacitor banks.

The second-layer optimization problem, which is the focus of this paper, is generally formulated for the isolated power system as follows:

$$\min_{\boldsymbol{v}}(\mathbf{0},\ldots\boldsymbol{N}-\mathbf{1}),\boldsymbol{\sigma}(\mathbf{0},\ldots,\boldsymbol{N}-\mathbf{1})J(x,\boldsymbol{u},k) \tag{1}$$

$$\boldsymbol{x}(k+1)=f(\boldsymbol{x},\boldsymbol{u},k) \tag{2}$$

$$g(\boldsymbol{x},\boldsymbol{u},k)=0 \tag{3}$$

$$h(\boldsymbol{x},\boldsymbol{u},k)\leq 0 \tag{4}$$

where $\boldsymbol{x}(k)$, $\boldsymbol{u}(k)$, $\boldsymbol{v}(k)$, and $\boldsymbol{\sigma}(k)$ are the vectors of the state space variables, control inputs, continuous control inputs, and discrete control inputs respectively. In distribution systems, losses increase operation cost significantly. Therefore, decreasing losses is often selected as the objective function of VVC. However, islanded microgrids can easily go unstable which means stability of voltage is more concerning. Thus, the objective function (1) is chosen to minimize the sum of voltage deviations of load buses in the system, over a prediction horizon on N. The prediction horizon and step size is chosen by the operator to cover the slowest dynamic of the studied microgrid. Assuming the current time instant as t_0, the voltages of the critical buses of the system should be predicted over the horizon of N to calculate the cost function. The objective function should also penalize the change in control action since too much control action causes actuator aging and damage in the system. Thus, the objective function is chosen as follows.

$$\begin{aligned}J(\boldsymbol{x},\boldsymbol{v},\boldsymbol{\sigma},k)=\sum_{k=0}^{N-1}\Bigg\{\sum_{i\in N_t}\Bigg\{&\|V_i^A(k+t_0|t_0)-V_i^{Nom}\|_2\\&+\|V_i^B(k+t_0|t_0)-V_i^{Nom}\|_2\\&+\|V_i^C(k+t_0|t_0)-V_i^{Nom}\|_2\Bigg\}\\&+\|\boldsymbol{W}_1\Delta\boldsymbol{v}(k)\|_2+\boldsymbol{W}_2\Delta\boldsymbol{\sigma}(k)\|_2\Bigg\}.\end{aligned} \tag{5}$$

In (5), N_t is the set of the buses which are more important for voltage control, i.e., load buses, N is the horizon of the optimization, $\boldsymbol{W}_1$ and $\boldsymbol{W}_2$ are user definable input weight matrices, $V_i(k+t_0|t_0)$ is the phasor of estimated voltage of bus i at $k+t_0$ based on the bus voltage measurement at time t_0, and $\Delta\boldsymbol{v}(k)$, $\Delta\boldsymbol{\sigma}(k)$ are the changes in continuous and discrete control inputs from previous step respectively. The inequalities defined by (4) include the bus voltage limits, compensator reactive production limits, and cable thermal limits given by following equations:

$$V_{b_{\min}} \leq |V_b(k)| \leq V_{b\ Max} \quad b = 1, \ldots, N_{Bus} \tag{6}$$

$$Q_{c\min}(k) \leq Q_c(k) \leq Q_{c\ Max}(k), \qquad c = 1, \ldots, N_{\text{comp}} \tag{7}$$

$$|I_{cl}(k)| \leq I_{cl\ Max}, \qquad cl = 1, \ldots, N_{cl} \tag{8}$$

where N_{Bus} is the number of buses, N_{comp} is the number of dynamic compensators, N_{cl} is the number of cables, $|V_b(k)|$ is the voltage magnitude of bus b at time k, and $Q_c(k)$ is the reactive power generation of compensator c at time k. Notice that the limits in (7) are time-varying to account for the reactive production limit variations of DGs, which can be calculated using the estimated active power production at time k [18].

In (1), $\boldsymbol{x}(k)$ is the vector of the state space variables as follows:

$$\boldsymbol{x}(k) = [x_1(k)x_2(k)\ldots x_i(k)\ldots x_{L_q j}(k)\ldots]^T$$
$$i = 2, \ldots, N_{\text{DGs}}, \quad j = 1, \ldots, N_{Load} \tag{9}$$

where x_1 is the state of the master generator, x_i's are states of the other sources, and $x_{L_q j}$ are the states of the loads. These state variables correspond to DG and load dynamic (12), (13), and (16).

The continuous control variables in the $\boldsymbol{v}(k)$ vector in (1) include voltage reference of the master generator, reactive power setpoint of DGs, and reactive power setpoint of dynamic reactive compensators defined as follows:

$$\boldsymbol{v}(k) = [V_{\text{Ref,DG1}}(k)\ Q_{\text{Ref,DG n}}(k)\ldots Q_{c_m}(k)\ldots]^T$$
$$n = 1, \ldots, N_{\text{DG}}, \quad m = 1, \ldots, N_{\text{comp}} \tag{10}$$

where V_{ref} is the reference voltage of the master generator, Q_{DG_n} is the reactive power setpoint of the nth DG, and Q_{c_m} is the reactive power reference of the mth reactive compensator if dynamic compensators are present in the system.

In (1), $\boldsymbol{\sigma}(k)$ is the vector of discrete inputs to the system defined as follows:

$$\boldsymbol{\sigma}(k) = [\sigma_1(k)\ldots\sigma_l(k)\ldots\sigma_{N_{sw}}(k)]^T \tag{11}$$

where $\sigma_l(k)$ is the status of capacitor bank l at time k and N_{sw} is the number of capacitor banks. The control vector $\boldsymbol{u}(k)$ is defined as the augmented vector of discrete control and continuous control inputs, i.e., $\boldsymbol{u}(k) = [\boldsymbol{v}(k)^T\ \boldsymbol{\sigma}(k)^T]^T$.

The differential constraints of the system expressed in (2) include the dynamics of the generators and the loads in the power system. These equations are usually expressed in continuous time domain and then discretized and linearized.

A. Sources

Sources should be modeled with an appropriate amount of detail for the study [19], [20]. As mentioned earlier, using the full detailed dynamic model of the sources as the prediction model makes the computational time of the optimization too long. Thus the full model may not be used for prediction and simplifications should be applied. A system identification method is used to achieve a simple input–output model of the component that can be used in the prediction model in this paper. Since mainly the output voltage of the master generator is important in volt/var studies, the model should describe the change in output voltage as a result of a change in the input voltage reference. Hence, an input–output model with the reference voltage as the input is enough for the prediction model. In this paper, a first-order model which is derived based on the step response of the reference voltage is used as the prediction model for the master generator:

$$\frac{V_{\text{DG 1}}(s)}{V_{\text{Ref,DG1}}(s)} = \frac{1}{T_V s + 1} \tag{12}$$

where T_V is the time constant which is identified with system identification, $V_{\text{Ref,DG1}}(s)$ is the reference voltage of the master generator, and V_{DG1} is the output voltage of the master generator. The step response of the master generator and the 63% rule is used to determine the time constant T_V [21].

The equations for wind and PV sources are not presented in this paper due to page limitation but they may be found in [11], [22]. The same system identification approach was used for these sources to limit the number of dynamic equations for the prediction model. However, since the PV and wind sources are controlled in PQ mode, the dynamic response of the reactive power outputs of the sources to changes in reactive power setpoints is required for the prediction model. The identified model for these sources is shown as

$$\frac{Q_{\text{DG}\ i}(s)}{Q_{\text{Ref,DG i}}(s)} = \frac{1}{T_{Q,i}\, s + 1} \tag{13}$$

where $T_{Q,i}$ is the time constant which is identified with system identification, $Q_{\text{Ref,DG i}}$ is the reactive power reference of ith DG, and $Q_{\text{DG i}}$ is the reactive power output of ith DG.

B. Loads

The power consumed by most of the loads in a power system is voltage dependent. Hence, the load admittance varies with the voltage, which means that when the voltage decreases the consumed active and reactive power of most of the loads decrease. However, after a disturbance, internal controllers of most loads, like thermostats of electrical heating and power electronics converters which regulate the rotational speed of machines, restore the power demand of the load. The new power demand usually settles below or equal to the pre-disturbance level. This self-restoring behavior of loads can be described using second order differential equations as described in [23]. The model for the reactive power of load is defined as follows:

$$\frac{dx_{L_q}(t)}{dt} = -\frac{x_{L_q}(t)}{T_q} + Q_{L0\ q}\left(V_n^{b_s}(t) - V_n^{b_t}(t)\right) \tag{14}$$

$$Q_{L\ q}(t) = \left(\frac{x_{L_q}(t)}{T_q} + Q_{L0\ q}\left(V_n^{b_t}(t)\right)\right) \tag{15}$$

where x_{Lq} is an internal state variable which models the load recovery dynamic with the time constant T_q, $Q_{L0\,q}$ is the nominal reactive power of load q, and $Q_{L\,q}$ is the actual reactive power of the load. The instantaneous voltage dependency is expressed by $V_n^{b_t}$ and the steady-state voltage dependency is given by $V_n^{b_s}$, V_n is the bus voltage of bus n connected to dynamic load q, b_t is the transient reactive load-voltage dependence, and b_s is the

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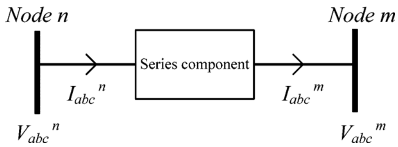

Fig. 1. Two consecutive nodes in a radial power system.

steady-state reactive load-voltage dependence. Linearizing (14) around an operating point of the system yields

$$\frac{dx_{L_q}(t)}{dt} = -\frac{x_{L_q}(t)}{T_q} + Q_{L0\,q}\left[b_s(V_n^*)^{b_s-1} - b_t({V_n}^*)^{b_t-1}\right] V_n(t) \tag{16}$$

where V_n^* is the operating voltage of the bus connected to the load.

Mathematical manipulation of (15) and (16) yields the following equation in the frequency domain:

$$Q_{L\,q}\,(s) = Q_{L0\,q}\left(\frac{C + T_q D s}{T_q s + 1}\right) V(s) \tag{17}$$

where $C = b_s({V_n}^*/V_{n0})^{b_s-1}$, $D = b_t\,(V_n^*/V_{n_0})^{b_t-1}$, and V_{n_0} is the nominal voltage of bus n. It should be noted that (15) and (16) are in continuous time domain but they can be discretized easily using [24]

$$\frac{dx_{L_q}(t)}{dt} \approx \frac{x\,((k+1)\,\Delta t) - x\,(k\Delta t)}{\Delta t}. \tag{18}$$

C. Network Equations

The algebraic constraints presented in (3) include the network equations of the system. For an unbalanced distribution system, the network equations can be solved iteratively with each iteration described as follows:

$$\boldsymbol{V^n_{ABC}}(k) = \boldsymbol{a}\,\boldsymbol{V^m_{abc}}(k) + \boldsymbol{b}\,\boldsymbol{I^m_{abc}}(k) \tag{19}$$
$$\boldsymbol{I^n_{abc}}(k) = \boldsymbol{c}\,\boldsymbol{V^m_{abc}}(k) + \boldsymbol{d}\,\boldsymbol{I^m_{abc}}(k) \tag{20}$$

where currents and voltages are shown in Fig. 1, and the matrices $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}$ are derived from characteristics of the series component [25].

In this formulation the loads and PQ generators should be converted to current source as follows:

$$\boldsymbol{I_i} = \left(\frac{S_i(k)}{V_i(k)}\right)^*. \tag{21}$$

The power flow equations should be solved many times at each stage of the optimization and the iterative method takes too much time to be solved. Thus the power flow equations can be solved offline with each of the parameters sweeping their whole operation range. Then a piecewise linearization algorithm such as the one presented in [26] can be used to identify linear equations in different operating regions of the system based on the data points. The linear equations are presented in the following general form:

$$[\delta_1\,(k) = 1] \leftrightarrow \begin{cases} \boldsymbol{x}\,(k+1) = \boldsymbol{A_1 x}\,(k) + \boldsymbol{B_1 u}\,(k) \\ \boldsymbol{V}\,(k) = \boldsymbol{C_2 x}\,(k) + \boldsymbol{D_2 u}\,(k) \end{cases}$$

$$[\delta_2\,(k) = 1] \leftrightarrow \begin{cases} \boldsymbol{x}\,(k+1) = \boldsymbol{A_2 x}\,(k) + \boldsymbol{B_2 u}\,(k) \\ \boldsymbol{V}\,(k) = \boldsymbol{C_2 x}\,(k) + \boldsymbol{D_2 u}\,(k) \end{cases}$$
$$\vdots$$
$$[\delta_n\,(k) = 1] \leftrightarrow \begin{cases} \boldsymbol{x}\,(k+1) = \boldsymbol{A_n x}\,(k) + \boldsymbol{B_n u}\,(k) \\ \boldsymbol{V}\,(k) = \boldsymbol{C_n x}\,(k) + \boldsymbol{D_n u}\,(k) \end{cases} \tag{22}$$

where, $\boldsymbol{A_n}$, $\boldsymbol{B_n}$, $\boldsymbol{C_n}$, $\boldsymbol{D_n}$ are time invariant matrices, $\boldsymbol{u}(k)$ is the vector of all the inputs and $\boldsymbol{V}(k)$ is the vector of bus phase voltages of the system and $\delta_i(k)$ are modes of operation, which are auxiliary binary variables correspondent to hyperplanes introduced during linearization. Each hyperplane is limited by inequalities of the following general form:

$$\boldsymbol{Hx}(k) \leq \boldsymbol{K} \tag{23}$$

where $\boldsymbol{H}$ and $\boldsymbol{K}$ are rectangular matrices and $\boldsymbol{x}$ is the vector of the states of the system. More details of the linearization technique can be found in [26]. Increasing the number of sets makes the prediction of voltage more accurate; however, the optimization algorithm takes more time to converge to the global minimum. Equations (22) and (23) describe the system in piecewise affine (PWA) form. Since the optimization software does not understand if-then rules, the PWA model should be transformed to a unified format. Thus, the PWA model of the system is transformed to MLD platform in this study which will be discussed next.

III. Model Predictive Control

Most of the control system theory has been developed for smooth dynamic systems. However, reactive power control of a power system includes logic variables such as switches and capacitor banks and the model of the system presented in (22) is piecewise linear and not smooth. Therefore, the relatively new concept of hybrid systems and hybrid control that has been introduced in the literature for this class of systems is used in this paper to solve the VVC [27]. Different platforms have been introduced in the hybrid control literature to model hybrid systems. In this paper, the model of the microgrid is transformed to the MLD platform.

A. Mixed Logical Dynamical Systems

One design procedure for hybrid systems is to transform PWA described by (22) and (23) into linear inequalities involving integer and continuous variables. This leads to a model of the system named MLD which describes the system by linear dynamic equations subject to linear mixed-integer inequalities. The PWA model of the microgrid described in (22) and (23) can easily be converted to the MLD form as follows [27]:

$$\begin{aligned} \boldsymbol{x}\,(k+1) &= \boldsymbol{Ax}\,(k) + \boldsymbol{B_1 u}\,(k) \\ &\quad + \boldsymbol{B_2 \delta}\,(k) + \boldsymbol{B_3 z}(k) \end{aligned} \tag{24}$$
$$\begin{aligned} \boldsymbol{y}\,(k) &= \boldsymbol{Cx}\,(k) + \boldsymbol{D_1 u}\,(k) \\ &\quad + \boldsymbol{D_2 \delta}\,(k) + \boldsymbol{D_3 z}\,(k) \end{aligned} \tag{25}$$
$$\boldsymbol{E_2 \delta}\,(k) + \boldsymbol{E_3 z}\,(k) \leq \boldsymbol{E_4 x}\,(k) + \boldsymbol{E_1 u}\,(k) + \boldsymbol{E_5} \tag{26}$$

where $k \in \mathbb{N}$ is the discrete time instant, $\boldsymbol{x} \in \boldsymbol{X}$ is the continuous states of the system, $\boldsymbol{u} \in \boldsymbol{U}$ are the inputs, and $\boldsymbol{y} \in \boldsymbol{Y}$ are the outputs. Also, $\boldsymbol{\delta} \in \{0,1\}^{n_\delta}$ and $\boldsymbol{z} \in \mathbb{R}^{n_z}$ are binary and auxiliary continuous variables which are introduced when translating logic or PWA functions into linear inequalities. All

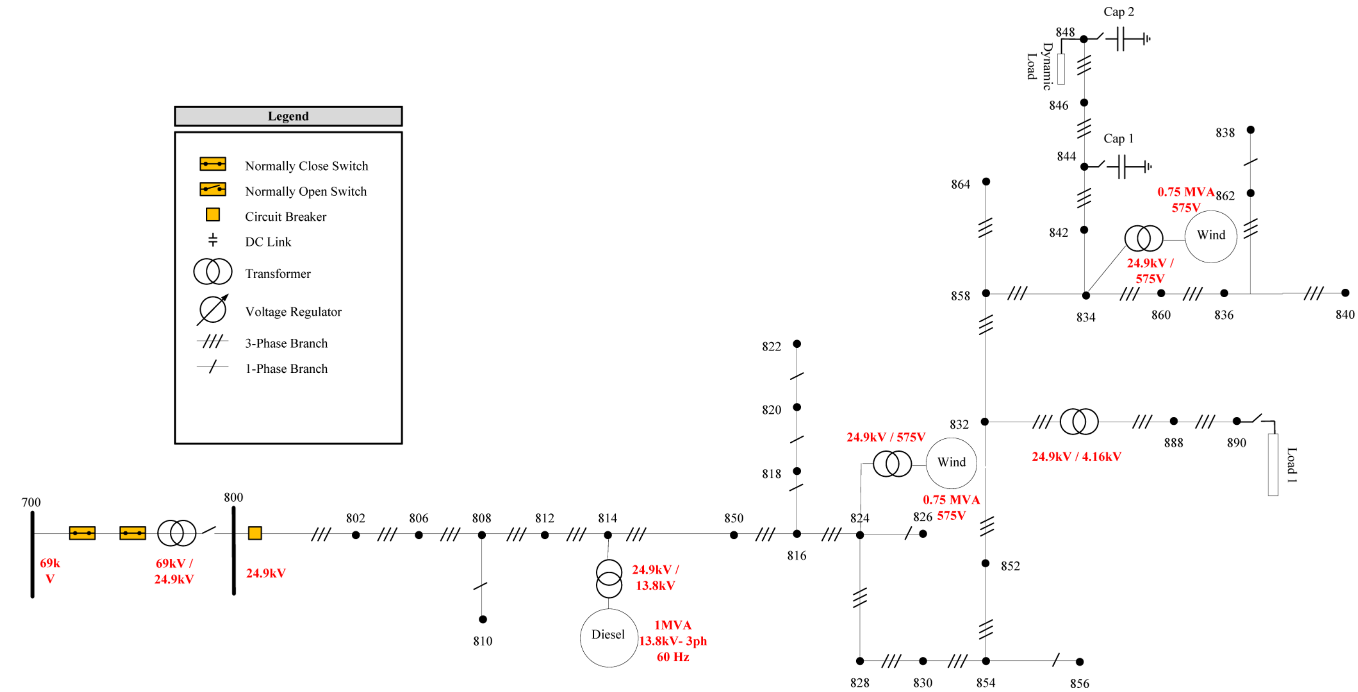

Fig. 3. Schematic diagram of the studied microgrid.

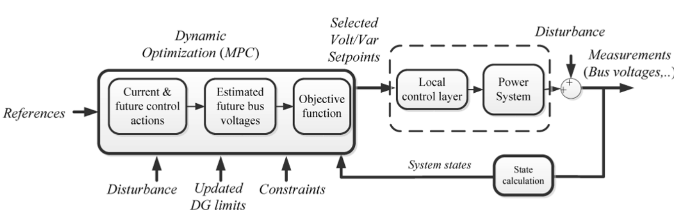

Fig. 2. Block diagram of MPC of the microgrid.

constraints on states, inputs, outputs, and auxiliary variables are summarized in the mixed-integer linear inequality constraint (26). It should be noted that the equality (24), (25) are linear and the nonlinearity is hidden in the integrality constraints on the binary variables included in (26) [27], [28].

A major advantage of MLD platform is that a generalized stabilizing MPC controller can be derived for MLD systems. For example, a model predictive control scheme that can stabilize the MLD systems on desired reference trajectories is discussed in [27]. Further, a problem described in MLD platform can include thresholds on states, inputs and internal variables [28], [29] which makes it suitable for the voltage control problem of power systems.

B. MPC of Hybrid Systems

MPC works based on a receding horizon policy which means a sequence of future control actions is chosen based on the prediction of the future behavior of the system and these control actions are applied to the system until future measurements are available. When new measurements are available, a new control sequence is calculated over the shifted horizon and replaces the previous one. This means that the receding horizon combines the constrained optimal control, which is an open loop procedure with a receding horizon policy, which provides the feedback to the controller and closes the control loop. More details about MPC can be found in [30]–[32].

The optimization problem solved by the MPC can be a linear programming, quadratic programming (QP), mixed integer quadratic programming (MIQP) or mixed integer linear programming (MILP) depending on the selected objective function and presence of integer variables in the system. A block diagram of the proposed model predictive controller based on MLD model of the microgrid is presented in Fig. 2.

IV. Simulation Results

The microgrid studied in this paper depicted in Fig. 3, was designed based on the IEEE 34 node test feeder [33]. The system was composed of three DGs, and one dynamic load. The DGs added to the system were consisted of one diesel generator and two wind generators. The wind generators were doubly fed induction generators (DFIG) connected to the system through back-to-back converters and isolation transformers. The grid-side converter is designed to be able to inject reactive power back to the grid. The diesel generator acts as the master generator and regulates the system frequency in islanded mode. Two switched capacitor banks were connected to buses 844 and 848 in the system for reactive compensation. The total load of the system was 2.5 MVA, and the total generation capacity was 3 MVA. Parameters of the sources used in test system simulation are presented in Table I. The cables were kept with the same ratings as the IEEE 34 node [33].

The control inputs to the system are the reference voltage of synchronous diesel master generator (V_{ref}), reactive power setpoints of the DGs and the status of switched capacitor banks (σ_1, σ_2). Thus, the continuous and discrete control input vectors are as follows:

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TABLE I
SOURCE PARAMETERS

Diesel	Quantity	Wind	Quantity
$V_{gl\text{-}l}$	575V	Xq''	0.26
P_g	1.5MW	Tqo'	N.A
p_{fg}	0.8	Tqo''	0.061s
f	60 Hz	$V_{gl\text{-}l}$	575V
R_a	0.199Ω	P_g	0.75 MW
X_p	0.18	Vs	400 V
Xd	1.25	Vr	1975 V
Xd'	0.24	F	60 Hz
Xd''	0.17	Rs	0.023 (p.u.)
Tdo'	4.11s	Ls	0.18 (p.u.)
Tdo''	0.023s	Rr	0.016 (p.u.)
Xq	0.62	Lr	0.16 (p.u.)
Xq'	N.A	$V_{gl\text{-}l}$	575V

$$\boldsymbol{v}(k) = [V_{ref}(k)Q_{W1}(k)Q_{W2}(k)]^T \tag{27}$$

$$\boldsymbol{\sigma}(k) = [\sigma_1(k)\,\sigma_2(k)]^T. \tag{28}$$

Since there are three distributed generators and one dynamic load in the system, the number of the states of the system was four. The input weight matrix was defined as $\boldsymbol{W}_1 = \mathrm{diag}([1e5, 0.01, 0.01])$, $\boldsymbol{W}_2 = \mathrm{diag}([15, 15])$ which was designed to penalize the reactive power generation by DGs less than the capacitor banks. In addition, time constants were $T_V = 0.18$ s, and $T_{Q,1} = T_{Q,2} = 0.03$ s for DGs, and $b_s = 1$, $b_t = 2$ for the dynamic load.

As mentioned earlier, direct modeling of hybrid systems in MLD form is time consuming. In this paper, HYbrid Systems DEscription Language (HYSDEL) [34] was used to derive the MLD form of the microgrid. The MLD model of the system was derived with the approach discussed in the paper for a horizon of 30 time steps ahead and each time step was 0.2 s. The MLD model of the system was then used as the prediction model in the MPC scheme.

The 34-bus microgrid was implemented in SIMULINK using the DG data from Table I and load and cable data from [33]. This system was used as the actual power system and voltage and current measurements of this system were sent to the MPC controller. The MPC control code written in MATLAB updated the measurements and called the optimization algorithm.

Solving the multiparametric problem leads to solving MILP or MIQP problem and efficient solvers exist for these problems. CPLEX 12.2 engine was used to solve the MIQP in this paper. CPLEX solves a MIQP using a branch and bound algorithm. The MPC used the MLD model achieved from HYSDEL compiler as the prediction model and performed the optimization over the horizon N using the feedback of the system as the initial state. The optimal control sequence achieved from solving the optimization problem was sent to the sources and the compensators of the microgrid.

A. Case-Study 1: Load Change

Microgrids may experience sharp load changes during their operation. This case study demonstrates the performance of the MPC controller during sharp load changes in the system. The load change was selected to be relatively large and abrupt to show the performance of the controller. All loads of the microgrid were energized at $t = 0$. Load 2 connected to node 844, which is 513 kVA, was disconnected from the system at $t = 2.5$ s. Further, load 1 connected to node 890, which is 503 kVA, was disconnected at $t = 3.5$ s and both of these loads were energized again at $t = 5.5$ s. The generalized MPC controller received the state feedback from the system and tried to send optimal inputs to reduce the effect of the load change in the system.

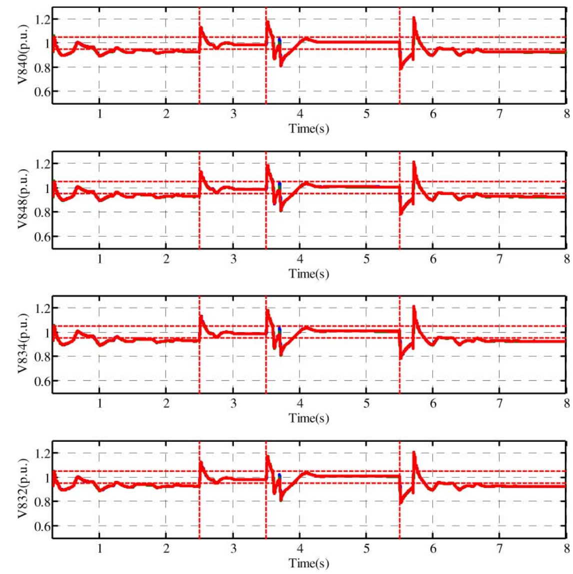

Fig. 4. Microgrid's bus voltages (rms)—Case study 1.

The optimization problem led to solving MIQP problem since the second norm was used in the objective function. The assumption in this case-study was that the MPC controller did not have a prediction of the exact time of load change. Thus, it took 0.2 s for the controller to send the updated control inputs to the system after the load change. As can be seen in Fig. 4, the MPC controller was able to keep the voltages within the limits by adjusting the control inputs. Fig. 5 shows the control inputs to the system generated by the MPC. A local voltage and reactive controller, which did not use global optimization, was also implemented for the same case-study and the system voltages jumped too high and started to oscillate after the second load was disconnected at $t = 3.5$ s. This means that the local controller was not capable of stabilizing the system in this case. The results for the local control are not presented in this paper due to the page limit.

As can be seen in Fig. 4, the islanded system experiences higher voltage variations compared to grid connected system. However, these voltage variations are acceptable for islanded systems. For example, the present requirements in IEEE Standard 1547 [6] for under-voltage tripping time are 0.16 s for voltage less than 50% and 2 s for voltages less than 88%.

B. Case Study 2: Wind Change

Variations in wind speed results in variation of the output power of the wind generator. These variations can affect the stability of the system as well as the voltage level of the load buses. All loads of the microgrid were energized at $t = 0$ s. In

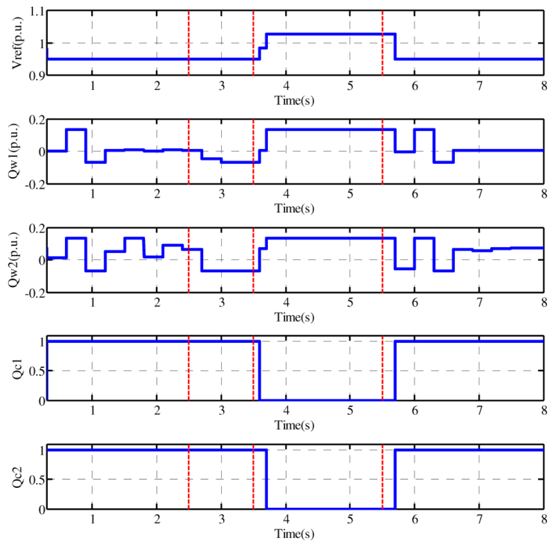

Fig. 5. Control inputs to the microgrid—Case study 1.

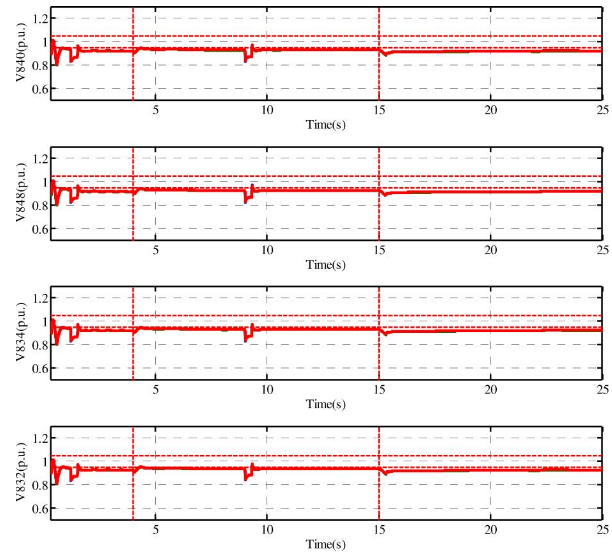

Fig. 6. Microgrid's bus voltages (rms)—Case study 2.

this case-study, the wind speed dropped to 60% of rated wind speed between $t = 5$ to 15 s. The model predictive control had an estimate of the reactive capacity of wind generators for a few seconds ahead based on an estimate of active power production. The controller had to determine what percentage of the output power could be assigned to reactive power generation based on the total generated power.

During this sharp wind transient, the wind generators were not capable of injecting enough reactive power to the grid. To reduce the effect of the reduction in reactive power production of wind generators, the MPC controller increased the voltage level of system by increasing the excitation of the synchronous generator.

The bus voltages of load buses that typically experience more voltage drop in the system are shown in Fig. 6. As can be seen in Fig. 6, the wind profile resulted in a glitch in one of the wind turbines at $t = 8.6$ s, which affected the bus voltages in the system. Fig. 7 demonstrates how the MPC controller utilized the reactive injection capability of the DGs to overcome the voltage drop as a result of the wind speed change.

C. Case Study 3: Load Change With Varying Wind

All loads of the microgrid were energized at $t = 0$ s. Load 2 connected to node 844 reduced to half at $t = 2.5$ s. Further, load 1 connected to node 890 reduced to half at $t = 3.5$ s. Both of the loads got back to the normal value at $t = 5.5$ s.

In addition, the wind speed was changing moderately during this case-study. This case study was intentionally chosen in an operating condition where both capacitor banks were connected to the system. Thus, this case study clearly shows the coordination of reactive injection of the DGs for voltage control. The assumption in this case study was that the MPC controller did not have a prediction of the exact time of load change. Therefore, after the load changed, it took 0.2 s for the controller to send the updated control inputs to the system since the step time of the control algorithm was set to 0.2 s. As can be seen in Fig. 10, the MPC controller was able to keep the voltages within the limits by adjusting the control inputs. In this case-study, to reduce the losses in the system, the reference voltage setpoint was

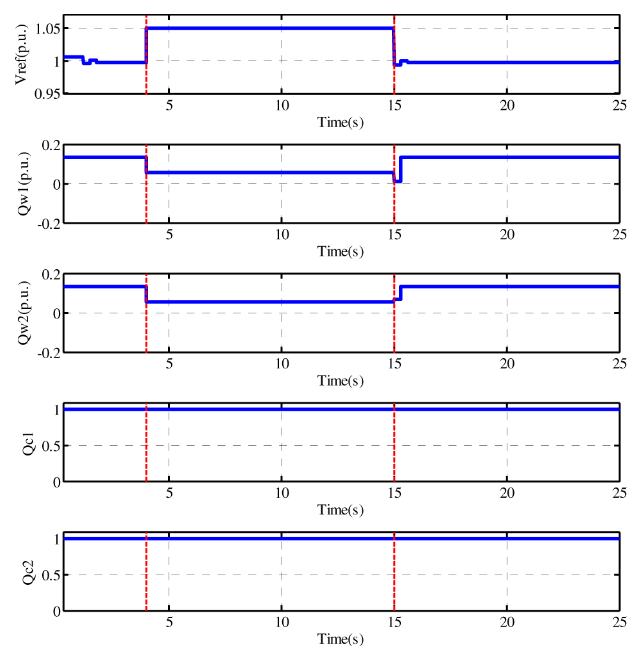

Fig. 7. Control inputs to the microgrid—Case study 2.

chosen as 0.96 p.u. which means the system was operating close to lower bus voltage limits. Fig. 9 shows the wind profile for case study 3. The rated wind speed is 15 m/s.

In case study 1 (Fig. 5), the system experienced a significant decrease in total load. Therefore, the reactive controller disconnected the capacitor banks to avoid overvoltage and instability. However, in case study 2, only the wind started varying while the capacitors were both connected to the system. During the wind transient, the system required extra reactive support while both capacitor banks were already connected to system. Thus, the controller tried to find other reactive sources to keep bus voltages stable. In this case, since no other reactive sources were available, the controller increased the voltage setpoint of the master generator. (Fig. 7) In case study 3, loads 1 and 2 were only reduced to half. In addition, wind was not at rated speed

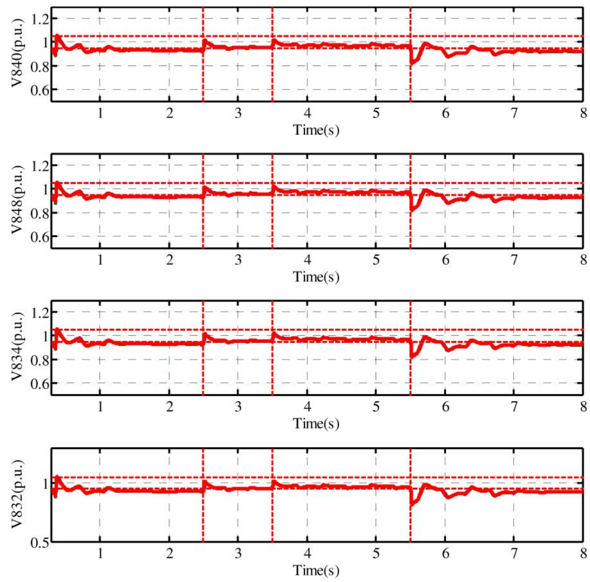

Fig. 8. Microgrid's bus voltages (rms)—Case study 3.

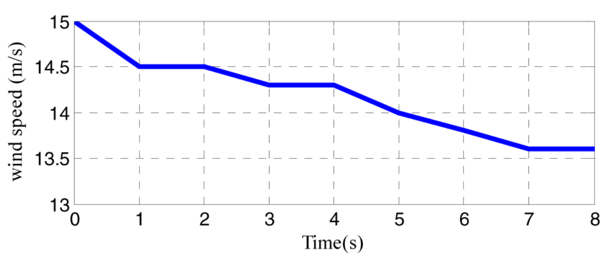

Fig. 9. Wind profile—Case study 3.

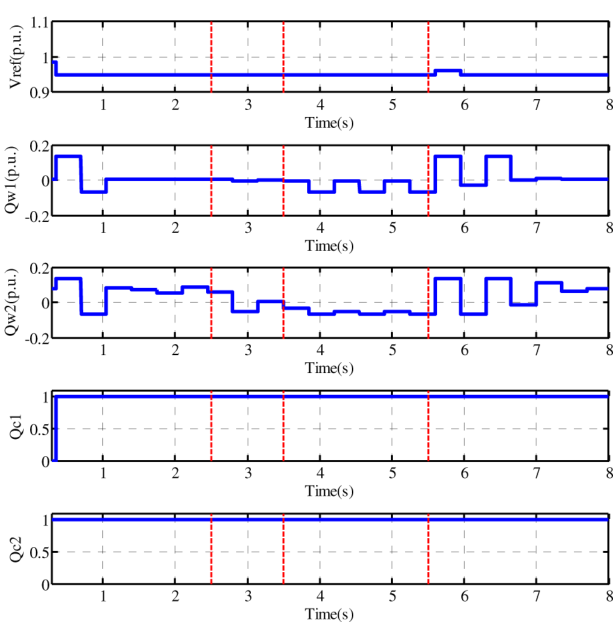

Fig. 10. Control inputs to the microgrid—Case study 3.

and the system still needed reactive support. Therefore, the capacitor banks remained connected to the grid (Fig. 9). To give an illustration for the concept, it should be noted that capacitor banks are quantized inputs. If a reactive controller needs to add 73 kVar reactive compensation at a specific point due to a load or condition change in the system, the controller cannot achieve this by only connecting one 100-kVar capacitor bank. One way to reach this is to connect the capacitor banks and reduce the reactive power generation of a nearby DG. Another way to reach this amount is to only increase the reactive power production of a nearby DG. It should also be noted that, most controllers are typically designed to reduce the number of switching of capacitor banks for many technical reasons.

The proposed MPC-based method can be applied to small and medium sized microgrids. The method can be applied to large and very large microgrids if a simplified model can be derived by lumping buses and loads. If this method is applied to a very large microgrid and simplifications are not performed on the model, the method may suffer from state explosion which should be addressed according to software limits [35].

V. Conclusion

This paper presented a new dynamic reactive power control method for the microgrid which is tailored for small scale unbalanced distribution systems. The discussed method dynamically coordinates between voltage/reactive power generation and consumption to keep the bus voltages close to their nominal value. It uses an optimization with prediction of the voltages in the system to achieve desired voltage profile by constantly adjusting the voltage and reactive power setpoints of the microgrid. The method is able to achieve a smooth voltage profile in various operating conditions of the microgrid in all the phases of the microgrid. It is also able to efficiently use the time-varying reactive power generation capability of the DGs in the system to regulate the voltage in the system. This method detects and prevents voltage swells, drops and voltage collapse ahead of time. The method results in more control action comparing to other VVC methods, however, it is faster and more robust. This method is suitable for small and medium sized microgrids and is not recommended for very large microgrids unless simplifications can be made to the system model.

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