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Structural damage diagnosis using modal data

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Abstract This paper presents a global algorithm for damage assessment of structures, based on a parameter estimation method, using the finite element and measured modal response of the structure. Damage is considered as a localized reduction in structural stiffness. Unmeasured parts of the mode shapes of a structure are characterized as a function of the structural parameter and measured parts of the mode shape. Elemental damage equations, which relate the partially measured mode shapes of a damaged structure to a change in structural parameters, are developed using incomplete measured mode shapes. These equations are solved to find the changes in structural parameters, utilizing an optimization method. Noise polluted data are used through Monte Carlo simulation to investigate the sensitivity of the proposed method to errors present in the measured modal data. The algorithm is verified in a numerical simulation environment using a planer truss and frame. Results show the good ability of this method to detect any damage of structures in the presence of errors in the acquired data.

1. Introduction

Many existing structures, which were constructed several decades ago, are still in service, and many have deteriorated. For example, it has been pointed out that nearly 1/3–1/2 of America's infrastructure, such as bridges, railways and school buildings, is structurally deficient and needs to be repaired [1]. Therefore, the early detection, monitoring and analysis of a damaged structure is vital for its safe performance. While there are many techniques and approaches presented in the nondestructive evaluation (NDE) of structural systems, existing damage identification methods can be categorized into dynamic and static identification methods, using corresponding data. The purpose is to adjust the parameters of numerical or analytical models to match analytical and measured data.

Structural damage is considered to be changes in structural parameters that adversely affect performance. Damage may also be defined as any deviation in the original geometrics of structures or material properties that may cause undesirable stresses, displacements or vibrations in the structure. These weaknesses and deviations may be due to cracks, loose bolts, broken welds, corrosion, fatigue, etc.

During the past few decades, dynamic identification techniques have been developed more maturely compared with the static approach, and the corresponding literature is quite extensive. The basic premise of a vibration-based damage detection method is that changes in physical properties will cause changes in the measured dynamic response of the structure or modal parameters. These changes provide a feature to evaluate the structural state. Detailed literature reviews have been provided by Doebling et al. [2], Stubbs et al. [3] and Mottershead and Friswell [4].

The modal-based model updating technique relies on modal characteristics data obtained from an experimental modal analysis extracted from the measured FRF data indirectly. Modal based methods attempt to correlate the changes in natural frequencies, mode shapes, mode shape curvature or frequency response functions with the occurrence of structural damage. Thus, these methods are developed essentially as applications of traditional experimental modal analysis procedures [2,3,5]. Natural frequencies are the global properties of the structure and, thus, they can be measured at a few locations or even at one point [6]. Furthermore, a number of methods, based on mode

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shapes, for the damaged structure are proposed, such as sensitivity (perturbation) methods [7,8], modal force error methods [3,9,10], modal residual methods [11,12] and techniques using genetic algorithms and neural networks [13,14].

A main category of model updating methods use structural responses in the frequency domain to tune structural models. In this class of model updating methods, FE models are updated in view of the fully damped response along a frequency axis. The main advantage of FRF methods is that the amount of available test data is not limited to a few identified eigenvalues and eigenvectors, and FEM updating can be performed using many more data points [15,16]. It must be noted that although FRF methods provide model updating techniques with more data, modal data represent more information about structural conditions using less data points.

The main difficulties lie in uncertainties in FE modeling and errors related to modal testing [17]. Uncertainties in the FE model exist due to inaccurate physical parameters, non-ideal boundary conditions and structural non-linear properties. With respect to modal testing, measurement noise is inevitable and the maximum number of measurement locations is limited. Moreover, it has not been possible until now to measure some degrees of freedom, such as rotational and internal. Therefore, the number of equations in model updating is usually smaller than that of the unknown parameters of the model. Hence, it is an under-determined problem and a small error may cause a large deviation in the results [18].

Model reduction or data expansion is a challenge to overcome incomplete measurements. Lim [19] proposed a systematic method that provides precise identification of damage locations and extent, when exact measured modes at every finite element DOF are used. Also, a procedure was presented to perform damage detection with inaccurate and incomplete measured modes. Sanayei et al. [20] use natural frequencies and associated mode shapes measured at a selected subset of Degrees Of Freedom (DOF) for stiffness and mass parameter estimation, through a condensation algorithm. A mode-based damage identification method is proposed by Ren and De Roeck [21,22] to predict the location and severity of damage, based on the work done by Araújo dos Santos et al. [23] (1998). In this work, it was demonstrated that multiplying damaged eigenvalue equations with damaged or undamaged modes provides more equations than the strain energy-based method to guarantee damage localization.

Also, to achieve a linear sensitivity equation of modal parameter sensitivity, Chen and Bicanic [7] expressed the change of any mode shape as a linear combination of the original eigenvectors of the intact structure. Mode shape participation factors are a function of measured natural frequencies of a damaged structure, the stiffness matrix perturbation and the changes of mode shape itself. The number of unknowns is equal to the number of structural parameters, plus the number of measured natural frequencies multiplied by the number of mode shapes, which increases the number of unknowns in the optimization problem.

As a drawback of FEM-update techniques, the requirement of reducing FEM degrees of freedom or extending measured modal parameters may result in the loss of physical interpretability and errors, due to the stiffness diffusion that smears the damage-induced localized changes in the stiffness matrix into the entire stiffness matrix. Using incomplete acquired data, Pothisiri and Hjelmstad [24] introduced a method by rearranging the degrees of freedom. Bakhtiari-Nejad et al. [25] proposed a damage detection method using incomplete measured mode shapes, by assuming one of the modal displacements to be equal to one and, then, deriving a set of equations that related the modal displacement of the damaged structure to changes in structural parameters. Furthermore, they extended their previous work and presented a diagnostic algorithm, based on the damage equation method of Ren and De Roeck [21]. The authors applied the damage equations, using the incomplete mode shape data [26].

Furthermore, most modal based model updating techniques are forced to use a number of equations less than the number of unknown parameters. Therefore, proposing various types of sensitivity equation, using the same input data, will improve the robustness of model updating algorithms against measurements errors.

In the present work, the unmeasured part of the modal displacement (eigenvector) of a structure is expressed as a function of the measured part of the mode shape, frequency (eigenvalue), structural parameters and mass matrix of the structure. An element level damage equation was characterized using the mode shapes of an intact structure and the partially measured mode shapes of a damaged structure. An optimization criterion is used to solve equations for estimating the structural parameters. Noise in the measurement is simulated by adding a proportional random error to the exact data obtained from the finite element model of the damaged structure and the issue of selection of measurement locations is investigated.

2. Theory

2.1. Mode shape condensation

The modal characteristics of an intact structure are described by the eigenvalue equation:

$$(K - \omega_i^2 M)\phi_i = 0, \tag{1}$$

where $K(n \times n)$ and $M(n \times n)$ are stiffness and mass matrices of the structure, respectively; ω_i and ϕ_i are the ith eigenvalue and mode shape of the structure in the same order and n is the number of degrees of freedom. Degrees of freedom of a structure can be portioned into two categories; measured and unmeasured. Therefore, the stiffness and mass matrices of the structure can be rewritten as:

$$K = \begin{bmatrix} K_{aa} & K_{ab} \\ K_{ba} & K_{bb} \end{bmatrix}, \qquad M = \begin{bmatrix} M_{aa} & M_{ab} \\ M_{ba} & M_{bb} \end{bmatrix}, \tag{2}$$

where subscripts a and b indicate the degrees of freedom associated with the measured and unmeasured location of the structure, respectively. Substituting Eq. (2) into Eq. (1) and expanding it yields:

$$(K_{aa} - \omega_i^2 M_{aa})\phi_{ia} + (K_{ab} - \omega_i^2 M_{ab})\phi_{ib} = 0, \tag{3a}$$

$$(K_{ba} - \omega_i^2 M_{ba})\phi_{ia} + (K_{bb} - \omega_i^2 M_{bb})\phi_{ib} = 0. \tag{3b}$$

Using Eq. (3b), the unmeasured part of mode shapes can be calculated as:

$$\phi_{ib} = (K_{bb} - \omega_i^2 M_{bb})^{-1}(K_{ba} - \omega_i^2 M_{ba})\phi_{ia}. \tag{4}$$

Eq. (4) expresses the unmeasured part of the mode shapes of structures as a function of the stiffness matrix (structural parameters), mass matrix and natural frequencies.

2.2. Element damage equations

The eigenvalue problem of the damaged structure of the lth mode shape can be written as:

$$[(K + \delta K) - \omega_{ld}^2 M]\phi_{ld} = 0. \tag{5}$$

Pre-multiplying Eq. (5) by ϕ_i^T, transposing and rearranging it yields:

$$\phi_{ld}^T(K+\delta K)\phi_i = \omega_{ld}^2\phi_{ld}^T M\phi_i. \tag{6}$$

Using the eigenvalue problem of the intact structure, as given by Eq. (1), and substituting $M\phi_i$ by $\frac{1}{\omega_i^2}K\phi_i$ in the right hand side of Eq. (6) results in:

$$\phi_{ld}^T(K+\delta K)\phi_i = \frac{\omega_{ld}^2}{\omega_i^2}\phi_{ld}^T K\phi_i. \tag{7}$$

expanding the left hand side of Eq. (7) and rearranging it yields:

$$\phi_{ld}^T\delta K\phi_i = \left(\frac{\omega_{ld}^2}{\omega_i^2}-1\right)\phi_{ld}^T K\phi_i. \tag{8}$$

Eq. (8) expresses the relation between the measured modal parameter of the damaged structure and the change in the stiffness matrix of the structure. This equation requires a complete measured mode shape of a structure, which is time consuming and expensive for most structures. Additionally, in structures which have translational and rotational degrees of freedom, the measuring of rotational degrees of freedom requires expensive equipment. Using Eq. (3), Eq. (7) can be rewritten as:

$$\begin{aligned}&\begin{bmatrix}\phi_{lda}^T & \phi_{ldb}^T\end{bmatrix}\begin{bmatrix}\delta K_{aa} & \delta K_{ab}\\ \delta K_{ba} & \delta K_{bb}\end{bmatrix}\begin{bmatrix}\phi_{ia}\\ \phi_{ib}\end{bmatrix}\\ &= \left(\frac{\omega_{ld}^2}{\omega_i^2}-1\right)\begin{bmatrix}\phi_{lda}^T & \phi_{ldb}^T\end{bmatrix}\begin{bmatrix}K_{aa} & K_{ab}\\ K_{ba} & K_{bb}\end{bmatrix}\begin{bmatrix}\phi_{ia}\\ \phi_{ib}\end{bmatrix}.\end{aligned} \tag{9}$$

The transposition of Eq. (9) can be expanded and rearranged as:

$$\begin{aligned}&\phi_{ia}^T\delta K_{aa}\phi_{lda} + \phi_{ib}^T\delta K_{ba}\phi_{lda} + \phi_{ia}^T\delta K_{ab}\phi_{ldb} + \phi_{ib}^T\delta K_{bb}\phi_{ldb}\\ &\quad - \left(\frac{\omega_{ld}^2}{\omega_i^2}-1\right)(\phi_{ia}^T K_{ab}\phi_{ldb} + \phi_{ib}^T K_{bb}\phi_{ldb})\\ &= \left(\frac{\omega_{ld}^2}{\omega_i^2}-1\right)(\phi_{ia}^T K_{aa}\phi_{lda} + \phi_{ib}^T K_{ba}\phi_{lda}) = R_{li},\\ &\quad l = 1,\ldots,nm,\ i = 1,\ldots,nu,\end{aligned} \tag{10}$$

where R_{li} is the vector of the residual. Using Eq. (4), the unmeasured portion of the mode shapes of the damaged structure can be computed based on the measured part. Therefore, Eq. (10) can be rewritten as:

$$\begin{aligned}&\phi_{ia}^T\delta K_{aa}\phi_{lda} + \phi_{ib}^T\delta K_{ba}\phi_{lda} - \Bigg[\phi_{ia}^T\delta K_{ab} + \phi_{ib}^T\delta K_{bb}\\ &\quad - \left(\frac{\omega_{ld}^2}{\omega_i^2}-1\right)(\phi_{ia}^T K_{ab} + \phi_{ib}^T K_{bb})\Bigg] \times (K_{bb} + \delta K_{bb}\\ &\quad - \omega_{ld}^2 M_{bb})^{-1}(K_{ba} + \delta K_{ba} - \omega_{ld}^2 M_{ba})\phi_{lda} = R_{li}.\end{aligned} \tag{11}$$

Using nu mode shapes of intact structures and nm mode shapes of damaged structures, the $nm \times nu$ equation will be derived. The solution of element damage equations for the unknowns allows locating and quantifying damage. These two types of damage equation, expressed by Eqs. (4) and (11), can be used either independently or combined. The advantage of combining two equations is that more equations are available for damage detection. To investigate the efficiency of the proposed method to handle the element damage equation, in the present work, only this type of equation has been used in the damage detection process. Since the obtained set of equations is under-determined, it cannot result in a unique solution. To increase the confidence of the solution, these equations have been solved by the optimization criterion as follows.

2.3. Optimization function

The eigenvalue problem of the damaged structures is described by:

$$[K+\delta K-(\omega_i^2+\delta\omega_i^2)M]\phi_{id} = 0. \tag{12}$$

It is assumed that a good approximation of the eigenvector of a damaged structure is a corresponding eigenvector of an undamaged structure. Then, the eigenvalue problem of the damaged structure can be stated as:

$$E_i = (K+\delta K)\phi_i - (\omega_i^2+\delta\omega_i^2)M\phi_i, \tag{13}$$

where E_i is a vector of residuals. Simplification of Eq. (13) provides an expression for E_i as:

$$E_i = \delta K\phi_i - \delta\omega_i^2 M\phi_i. \tag{14}$$

To stay true, with the assumption that the damaged eigenvectors remain close to the undamaged ones, the square of the magnitude of the residuals is minimized. The square of the magnitude of R_i is expressed by:

$$\begin{aligned}\|E_i\|^2 &= E_i^T E_i = \phi_i^T(\delta K)^2\phi_i - 2\delta\omega_i^2\phi_i^T\delta KM\phi_i\\ &\quad + \delta\omega_i^2\phi_i^T M^2\phi_i.\end{aligned} \tag{15}$$

Summation over the m measured modes provides the error produced by all equations as:

$$g = \sum_{i=1}^{m}\|E_i\|^2 = \sum_{i=1}^{m}\phi_i^T(\delta K)^2\phi_i - 2\sum_{i=1}^{m}\delta\omega_i^2\phi_i^T\delta KM\phi_i. \tag{16}$$

Since a constant term does not influence a minimization procedure, $\delta\omega_i^2\phi_i^T M^2\phi_i$ has been dropped. Since change in the structural stiffness parameter is always negative, an inequality constraint is introduced as:

$$\delta P < 0. \tag{17}$$

Minimization of the cost function, given by Eq. (16), subjected to equality and inequality constraints, expressed by Eqs. (11) and (17), results in a change of evaluated structural parameters.

3. Parameterized stiffness matrix

The stiffness matrix of the structure can be described as follows:

$$[K] = [A][P][A]^T. \tag{18}$$

Matrix $A_{n_p\times n_p}$, defined as the stiffness connectivity matrix, and the $(n_p \times n_p)$ diagonal matrix of $[P]$, have the elemental stiffness parameters of the $(n_p \times 1)$ vector $\{P\}$ as its diagonal entries, mathematically defined as:

$$\operatorname{diag}[P] = \{P\}. \tag{19}$$

The global stiffness matrix is a linear function of the elemental stiffness parameters and $[A]$ is independent of $[P]$. Therefore, Eq. (18) can be perturbed to get:

$$[K+\delta K] = [A][P+\delta P][A]^T. \tag{20}$$

Expanding Eq. (20) and subtracting Eq. (18) from it yields a parameterized form of the perturbed global stiffness matrix, $[\delta K]$, as:

$$[\delta K] = [A][\delta P][A]^T. \tag{21}$$

With the above definition, a computer program is developed, and verification of the program is undertaken as follows.

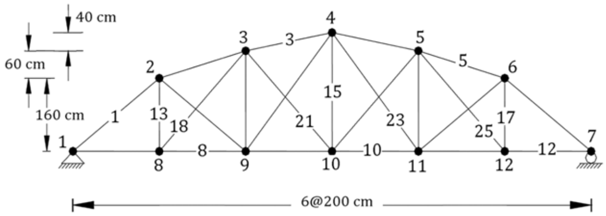

Figure 1: Geometry of bowstring truss.

Table 1: Cross-sectional area of truss members.

Member	Area (cm^2)
1–6	18
7–12	15
13–17	10
18–25	12

Minimization of the objective function of Eq. (16), subject to the nonlinear equality constraints given in Eq. (11) and inequality constraints of Eq. (17), can be done using the MATLAB optimization toolbox and the FMINCON routine. This routine implements Sequential Quadratic Programming (SQP) to minimize the nonlinear cost function, subject to linear and nonlinear equality and inequality constraints. SQP converts a nonlinear minimization to a linear minimization, using a Hessian matrix of cost function and a gradient of nonlinear constraints. The presented damage detection algorithm is programmed as a nonlinear constrained optimization problem. Hence, this problem must be solved iteratively, and like any iterative algorithm, the estimators need initial values for unknown parameters to start the iteration. The choice of initial values controls the convergence of the algorithm and dictates, to some extent, the computational effort required to achieve a solution. In this paper, origin ($\delta P = 0$) is considered an initial trial for the optimization problem. This assumption may increase the required number of iterations, but it does not influence the uniqueness of the results. Since, therefore, the inequality constraints of Eq. (17) bound the search domain of the optimization criteria, the results are unique. Examination of other random initial trials indicates that the initial trials do not influence the results of this study.

4. Numerical verification

4.1. Noise free data

4.1.1. Truss

A two-dimensional truss, as shown in Figure 1, is used to investigate the ability of the present damage detection method. The unknown parameters are; the axial stiffness of elements, *EA*, where *A* is the cross-sectional area of the truss element and *E* is Young's modulus. Cross-sectional areas of truss members are given in Table 1.

Four damage cases are considered to investigate the influence of the location, severity and number of damaged elements on the results. In the first damage case, the stiffness of elements 20 and 15 were reduced by 40%. In damage case number two, the stiffness of elements 11 and 25 were decreased by 30% and 40%, respectively. In damage case number three, the stiffness of elements 10, 16 and 24 were reduced by 20%, 30% and 30%, respectively. In damage case number four, the stiffness of elements 11, 14 and 25 were decreased by 20%, 30% and 40%, respectively.

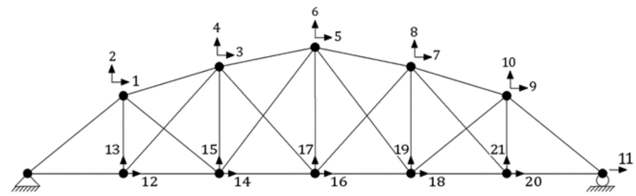

Figure 2: Degrees of freedom of bowstring truss.

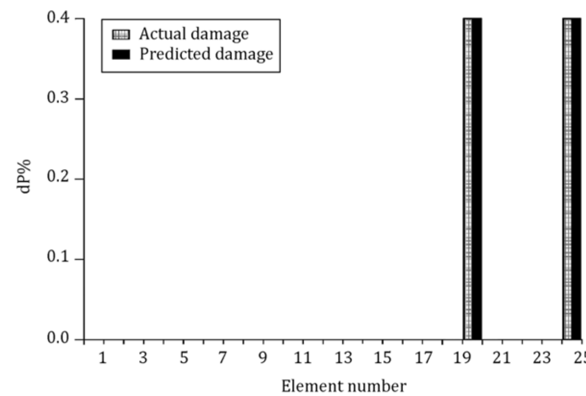

Figure 3: Predicted damage of truss, Scenario 1 (noise free data).

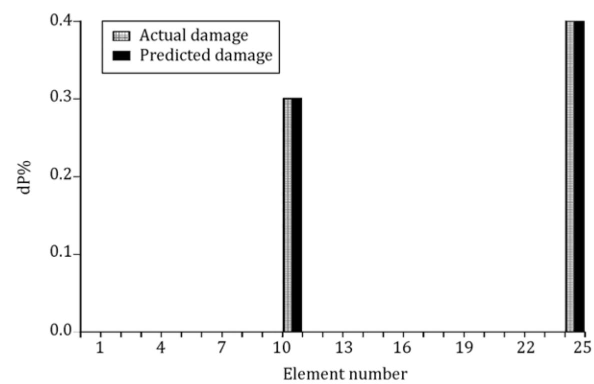

Figure 4: Predicted damage of truss, Scenario 2 (noise free data).

The first two, partially measured mode shapes of damaged structures, and the first five mode shapes of intact structures were considered in the process of damage detection, and measurement locations were selected by practical experience and engineering judgment. Degrees of freedom of the investigated truss are shown in Figure 2, and degrees of freedom 5, 6, 11, 17, and 18 have been considered as measurement locations. The predicted damage in simulated damage cases is illustrated in Figures 3–6.

Results confirm that the severity and location of damaged elements can be detected exactly using noise free data, by employing few measurement efforts.

4.1.2. Frame

A one-story, one-bay frame, as shown in Figure 7, is considered to verify the damage identification method described in this paper. The FEM analysis is carried out to simulate the experimental data, using two-node beam elements. The number of nodes and elements are 16 and 15, respectively. The unknown parameters are the flexural rigidity of elements, EI, where *I* is the moment of inertia of the cross-sectional beam elements.

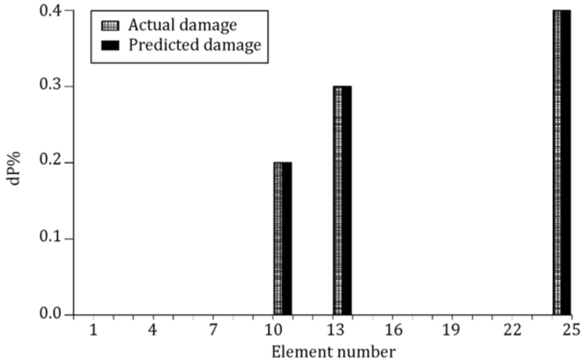

Figure 5: Predicted damage of truss, Scenario 3 (noise free data).

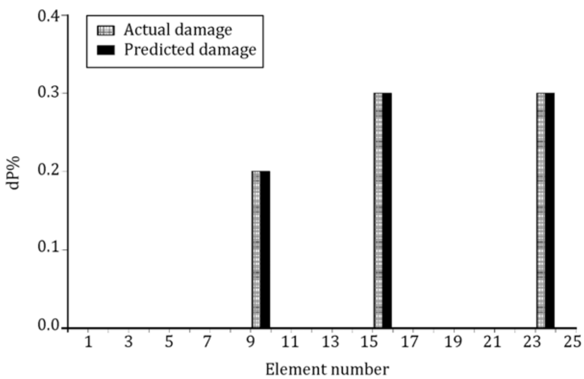

Figure 6: Predicted damage of truss, Scenario 4 (noise free data).

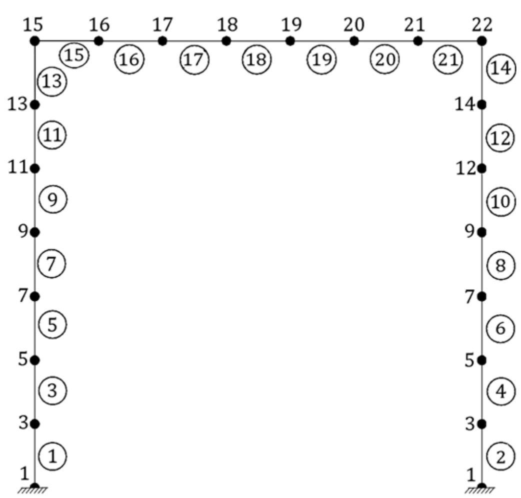

Figure 7: Planer frame structure.

Here, four damage cases are assumed to investigate the capabilities of the present method in detection of the occurred damage of a flexural structure. In the first damage case, the stiffness of element 10 was decreased by 20%. In damage case number two, the stiffness of elements 12 and 20 were decreased by 20% and 40%, respectively. In damage case number three, the stiffness of elements 3, 9 and 18 were decreased by 30%, 20% and 40%, respectively. In damage case number four, the stiffness of elements 7, 12 and 20 were decreased by 20%, 30% and 20%, respectively.

First, the partially measured mode shape of damaged structures and fifteen first mode shapes of intact structures

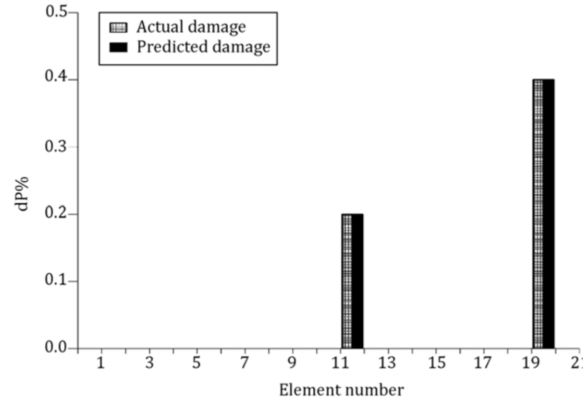

Figure 8: Predicted damage of frame, Scenario 1 (noise free data).

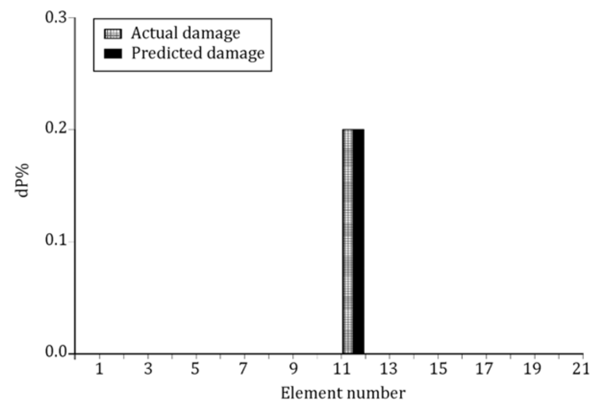

Figure 9: Predicted damage of frame, Scenario 2 (noise free data).

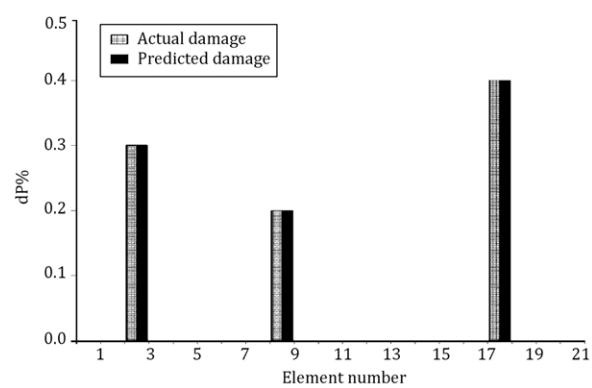

Figure 10: Predicted damage of frame, Scenario 3 (noise free data).

have been considered in the damage detection process, and it is assumed that only translational degrees of freedom are measurable. Node numbers 6, 10, 15, 11 and 19 are selected as measurement locations to measure the translational displacements. Predicted damages of the frame are shown in Figures 8–11.

As a truss example, the severity and location of the damage are detected exactly using noise free data.

4.2. *Noisy measurements*

For experimental modal testing, some deviation of results, due to measurement noise, is expected. In the numerical examples, noise is simulated by adding a series of pseudorandom numbers to the theoretically calculated frequencies and mode

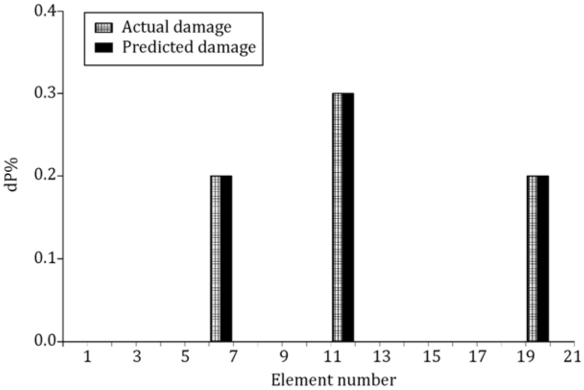

Figure 11: Predicted damage of frame, Scenario 4 (noise free data).

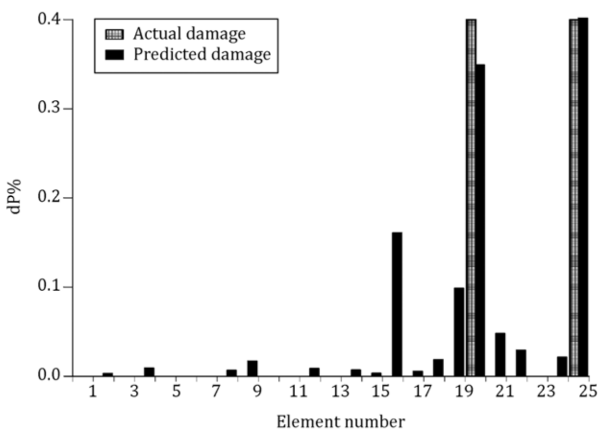

Figure 12: Predicted damage of truss, Scenario 1 (noisy data).

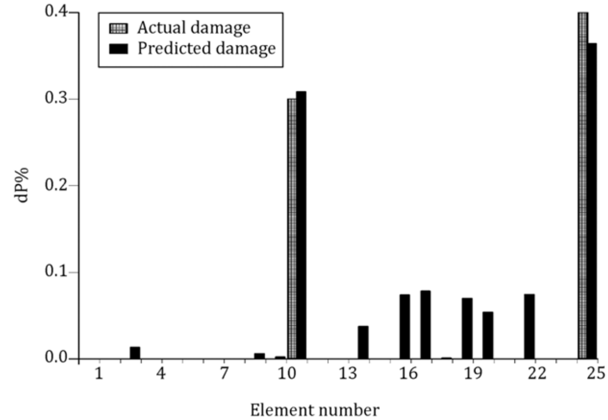

Figure 13: Predicted damage of truss, Scenario 2 (noisy data).

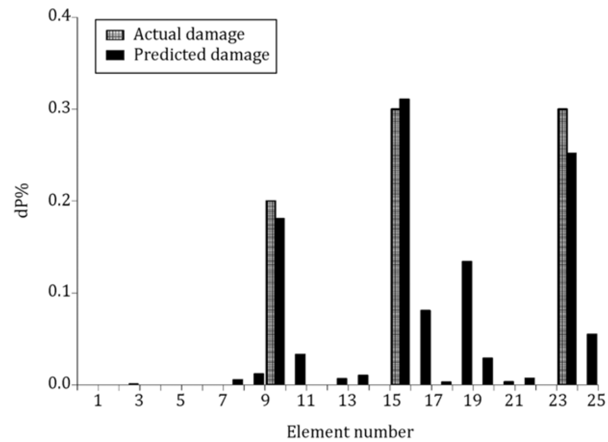

Figure 14: Predicted damage of truss, Scenario 3 (noisy data).

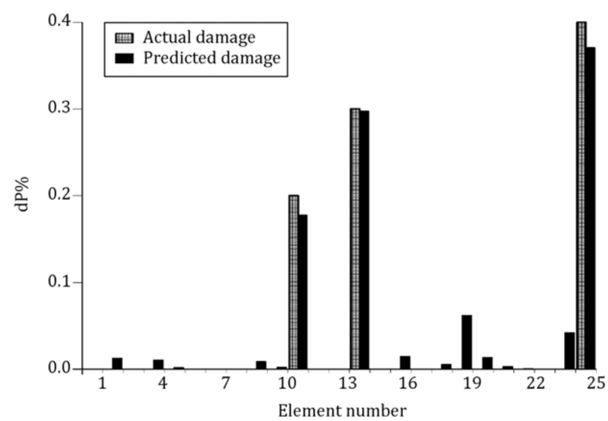

Figure 15: Predicted damage of truss, Scenario 4 (noisy data).

shapes. There are many type of error that can be introduced into the mathematical model to simulate noisy measurements. Due to the complexity of the measurement process, any single type of random error may be experienced in the field. Therefore, two types of simple random error were used to model measurement noise. Uniform error, which represents an equal probability at any one time, and normal distribution, which represents a higher probability of noise level closer to a mean value and a lower probability of larger noise. In the present study, 1% proportional uniform noise has been applied to the model displacement, and natural frequencies have been considered noise free. Next, in order to investigate the effect of measurement error on parameter estimates, the kth component of the noisy measured eigenvector, ϕ_{lk}^{m}, can be computed from the lth simulated noise free eigenvector, ϕ_{lk}^{0}, as:

$$\phi_{lk}^{m} = \phi_{lk}^{0}(1 + \zeta_{l}^{k}), \tag{22}$$

where ζ_{l}^{k} is a normally distributed random number with a mean value of zero and a standard deviation of 1, indicating the noise level in the numerical simulation. Structural model updating will be repeated by a set of error polluted data created by Eq. (22), and an average of all predictions will be reported as the model updating outcome. Standard deviation of the predicted results can be calculated, in order to investigate method robustness against measurement errors. The results of model updating, using simulated noise polluted data analysis for truss and frame models, are given in Figures 12–19.

As results show, this method is capable of detecting the magnitude and location of damaged elements with noisy data. The damaged elements are identified with acceptable accuracy, but an additional slight damage appears on the intact element, due to errors present in the mode shape measurements. Maximum errors in parameter identification of the truss example are less than 10% at damaged elements and less than 20% at intact elements. Also, maximum errors in parameter identification of the frame example are less than 5% at damaged elements and less than 10% at intact elements.

Averages of the estimated parameters do not reflect the robustness and confidence of the parameters estimation process. To investigate the robustness of a method, it is necessary

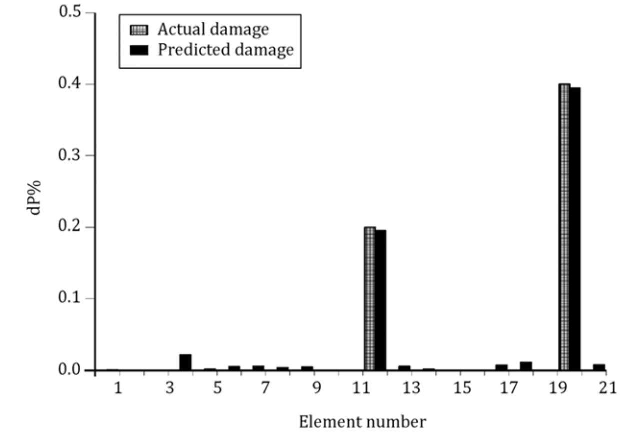

Figure 16: Predicted damage of frame, Scenario 1 (noisy data).

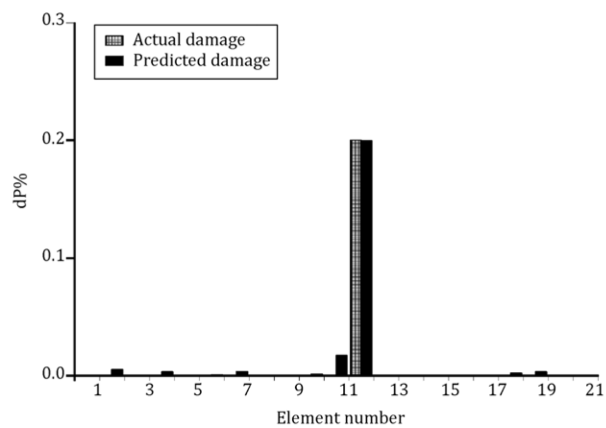

Figure 17: Predicted damage of frame, Scenario 2 (noisy data).

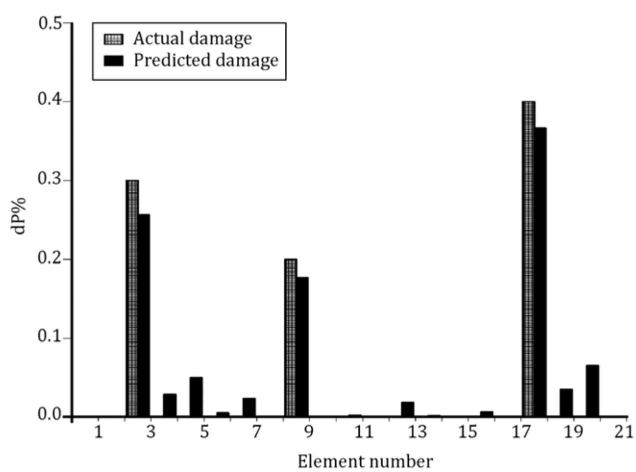

Figure 18: Predicted damage of frame, Scenario 3 (noisy data).

to evaluate the standard deviation and/or Coefficient Of Variation (COV) of the predicted unknown parameters in Monte Carlo simulations. Low standard deviation and COV indicate less scatter in the predicted parameters. For illustration purposes, and as two templates, standard deviation of the estimated parameters for truss and frame identification cases are plotted in Figures 20 and 21. Because this paper deals with the percentage of changes and not absolute values, presented standard deviations are unit-less.

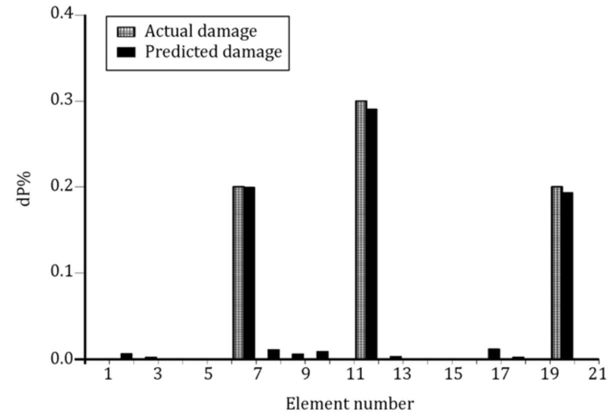

Figure 19: Predicted damage of frame, Scenario 4 (noisy free data).

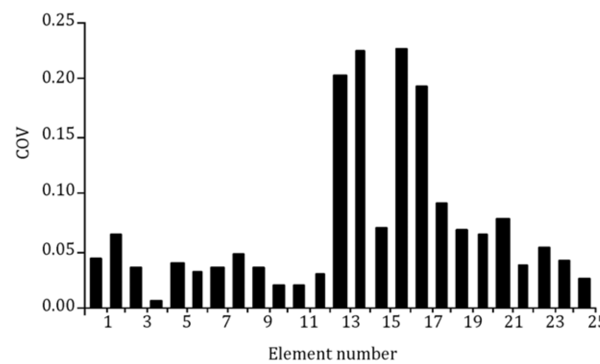

Figure 20: Example of the standard deviation of predicted parameters of a truss identification case.

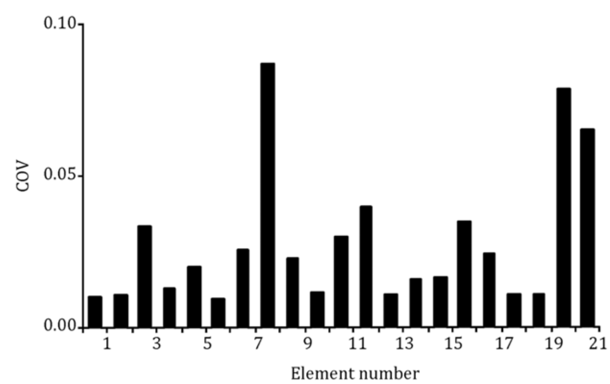

Figure 21: Example of the standard deviation of predicted parameters of a frame identification case.

Low values of the standard deviation of the predicted results prove a robust model updating algorithm. Also, these figures indicate more accurate identification of frame structures, in comparison with truss structures.

Without using element damage equations to obtain the same results, at least three first mode shapes of truss examples and two first mode shapes of frame examples are required [26]. Since the amplitude of the mode shape decreases at higher mode shape, measurements of higher mode shape are more noise contaminated, which adversely affects the results of damage detection. The efficiency of the proposed method and the necessity of developing more sensitive equations can become more significant when increasing the number of unknowns in large structures.

5. Conclusion

This paper presents an approach to damage detection in structures utilizing incomplete measured mode shapes and natural frequencies. The unmeasured part of the mode shapes of a structure is characterized as a function of structural stiffness parameters and measured modal displacements. More equations have been obtained, using element damage equations, which need complete mode shapes. This drawback is solved by presenting mode shape equations and dividing structural degrees of freedom into measured and unmeasured parts. An optimal criterion is used to solve these sets of equations to obtain changes in structural parameters. Results of bowstring truss and planer frames represent the ability of this method to evaluate the severity and location of damage, using exact and noise polluted data. Additionally, results demonstrate that this method is capable of detecting structural damage using less modal data and measurement effort.

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