Information Sciences 268 (2014) 185–201

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Combining possibilistic linear programming and fuzzy AHP for solving the multi-objective capacitated multi-facility location problem

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ARTICLE INFO

Article history:
Received 12 May 2009
Received in revised form 26 December 2013
Accepted 15 January 2014
Available online 24 January 2014

Keywords:
Possibilistic linear programming
Fuzzy AHP
Supply chain network design
Capacitated multi-facility location problem
Multi-objective programming

ABSTRACT

The capacitated multi-facility location problem is a complex and imprecise decision-making problem which contains both quantitative and qualitative factors. In the literature, many objectives for optimizing many types of logistics networks are described: (i) minimization objectives such as cost, inventory, transportation time, environmental impact, financial risk and (ii) maximization objectives such as profit, customer satisfaction, and flexibility and robustness. However, only a few papers have considered quantitative and qualitative factors together with imprecise methodologies. Unlike traditional cost-based optimization techniques, the approach proposed here evaluates these factors together while considering various viewpoints. Decision-makers must deal both factors together to model complex structure of real-world applications. In this paper, a two-phase possibilistic linear programming approach and a fuzzy analytical hierarchical process approach have been combined to optimize two objective functions ("minimum cost" and "maximum qualitative factors benefit") in a four-stage (suppliers, plants, distribution centers, customers) supply chain network in the presence of vagueness. The results and findings of this method are illustrated with a numerical example, and the advantages of this methodology are discussed in the conclusion.

1. Introduction

Today's global market competition and high customer expectations have forced enterprises to consider their supply chains (SC) more carefully. Supply chain decisions are important strategic decisions which affect every member of the chain because the various functions performed by these members are integrated with each other. Among these functions are marketing, distribution, planning, manufacturing, and purchasing.

The capacitated multi-facility location and SC network design problem is one of the most comprehensive strategic decision problems that need to be optimized for long-term efficient operation of the whole supply chain. This problem determines the number, location, capacity, and type of the plants, warehouses, and distribution centers to be used. It also establishes distribution channels and the quantities of materials and items to consume, produce, and ship from suppliers to customers [1].

Location-allocation decisions involve substantial capital investment and result in long-term constraints on the production and distribution of goods. These problems are complex and, like most real-world problems, depend on a number of tangible

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http://dx.doi.org/10.1016/j.ins.2014.01.024
0020-0255/

and intangible factors which are unique to each problem. The complexity of these systems arises from a multitude of quantitative and qualitative factors which influence location choices as well as from the intrinsic difficulty of making numerous tradeoffs among those factors [5]. Over and above this complexity, global SC management is difficult because multiple sources of uncertainty and complex interrelationships at various levels between diverse entities exist in the SC, and therefore it is very difficult to determine simultaneously the supply chain configuration and the SC total cost. Fast-changing transportation and facilities costs, facility capacities, and customer demands are some of the SC parameters which are difficult to predict accurately because of imprecision in the environment.

Supply chain network (SCN) design problems reviewed in the literature have been examined for situations ranging from a single product type to complex multi-product systems; the models developed range from linear deterministic models to complex nonlinear stochastic ones. The number of objective functions also depends on the degree of complexity of the problem. Generally, these problems involve multiple and conflicting objectives such as cost, service level, and resource utilization. To deal with multiple objectives and to enable the decision-maker to evaluate a greater number of alternative solutions, various numbers of supply chain levels or stages and various solution approaches and methodologies have been used. Supply chain network design levels are determined according to the components of the supply chain network problem being considered. In this research, papers in the literature have been categorized based on the number of SCN levels. The criteria considered in the objective functions and the solution methods and methodologies used in the literature are also reviewed.

Vercellis [40] presented a capacitated master production planning and capacity allocation problem for a multi-plant manufacturing system with two serial stages in each plant. The objective of the problem is to minimize the sum of the various cost factors, namely the production cost in stages 1 and 2, inventory, lost demand, transportation, and overtime. The resulting mixed $\{0,1\}$ linear programming model is solved by means of LP-based heuristic algorithms.

Zhou and Liu [47] proposed a mathematical model and an efficient solution procedure for a bi-criteria allocation problem involving multiple warehouses with different capacities. They also considered two conflicting objectives, transit time and shipping cost, with respect to the warehouse allocation problem. Their proposed solution procedure used a genetic algorithm that is designed to find Pareto optimal solutions for this problem in a short period of time. Romeijn et al. [32] considered a traditional deterministic single-DC multi-retailer (SDMR) model. They tried to minimize the location and transportation costs and the two-level inventory costs. An additional cost term that represents costs related to safety stocks or capacity issues was also proposed. They formulated the problem as a set covering model.

Cakravastia et al. [9] aimed to develop an analytical model for the supplier selection process when designing a supply chain network. The assumed objective of the supply chain is to minimize the level of customer dissatisfaction, which is evaluated by two performance criteria: (i) price and (ii) delivery lead time. The overall model operates at two levels of decision-making: the operational level and the chain level. An optimal solution in terms of the models for the two levels can be obtained using a mixed-integer programming technique. Syam [36] extended traditional facility location models by introducing several logistical cost components such as holding, ordering, and transportation costs in a multi-commodity, multi-location framework. Their paper provided an integrated model and sought to minimize total physical distribution costs by simultaneously determining optimal locations, flows, shipment compositions, and shipment cycle times. Two sophisticated heuristic methodologies, based on Lagrangean relaxation and simulated annealing respectively, were provided and compared in an extensive computational experiment. Yan et al. [43] proposed a strategic production–distribution model for supply chain design with consideration of bills of materials (BOM). Logical constraints were used to represent BOM and the associated relationships among the main entities of a supply chain such as suppliers, producers, and distribution centers. Moreover, these relationships were formulated as logical constraints in a mixed integer programming (MIP) model, thus capturing the role of BOM in supplier selection in the strategic design of a supply chain. The total cost of the supply chain included purchasing cost, production cost, transportation and distribution cost, and fixed costs such as the fixed ordering cost, the fixed cost to open and operate a producer, and the fixed cost to open and operate a DC. Chen and Lee [11] proposed a multi-product, multi-stage, and multi-period scheduling model to deal with multiple incommensurable goals for a multi-echelon supply chain network with uncertain market demands and product prices. The supply chain scheduling model is constructed as a mixed integer nonlinear programming problem to satisfy several conflicting objectives, including fair profit distribution among all participants, safe inventory levels, maximum customer service levels, and robustness of decisions to uncertain product demands. For the solution, a two-phase fuzzy decision-making method was presented.

Amiri [3] developed a mixed integer programming model and presented a Lagrangean-based solution procedure for the problem. The model minimizes total costs, including the costs to serve the demands of customers from the warehouses, the costs of shipments from the plants to the warehouses, and the costs associated with opening and operating the warehouses and the plants. Yilmaz and Çatay [44] addressed a strategic planning problem for a three-stage production–distribution network. The problem consisted of a single-item, multi-supplier, multi-producer, and multi-distributor production–distribution network with deterministic demand. The objective was to minimize the costs associated with production, transportation, and inventory as well as capacity expansion costs over a given time horizon. The problem was formulated as a 0–1 mixed integer programming model. Efficient relaxation-based heuristics were considered to obtain a good feasible solution. Tsiakis and Papageorgiou [39] proposed a mixed integer linear programming (MILP) model to assist senior operations management in making decisions about production allocation, production capacity per site, purchase of raw materials, and network configuration, while taking into account financial aspects (exchange rates, duties, etc.) and costs. The objective function included fixed infrastructure costs, production costs, material-handling costs at distribution centers, transportation costs, and duties.

Pirkul and Jayaraman [30] presented a Lagrangian relaxation of this model and developed a heuristic solution procedure which uses the information provided by this relaxation to generate good feasible solutions. Their model minimized the sum of the costs to distribute products from open warehouses to customers, the costs for transporting units of different commodities from plants to warehouses, and the fixed costs associated with locating and operating manufacturing plants and warehouses.

Jayaraman and Pirkul [18] studied an integrated logistics model for locating production and distribution facilities in a multi-echelon environment. The objective function minimized the total cost of the supply chain, including the fixed costs of operating and opening plants and warehouses, the variable costs of production and distribution, and the costs of transportation of raw materials from vendors to plants and of transportation of the finished products from plants to customer outlets through warehouses. A mixed integer programming approach was formulated, and a Lagrangean relaxation scheme was applied to the resulting model.

Syarif et al. [37] considered a logistic chain network problem formulated by a 0–1 mixed integer linear programming model. This problem involved the choice of facilities to be opened and the design of the distribution network to satisfy the demand at minimum cost. To solve this problem, a spanning-tree-based genetic algorithm using a Prüfer number representation was used. Results were compared with those from a traditional matrix-based genetic algorithm and from the LINDO professional software package.

Braun et al. [8] first described a six-node network and a model predictive control (MPC)-based management policy. The objective function for an MPC controller was constructed using three terms: penalized predicted setpoint tracking error, excess movement of the manipulated variable, and deviation of the manipulated variable from a target value. The optimization problem can be readily solved using standard quadratic programming (QP) algorithms. Melo et al. [27] proposed a mathematical modeling framework that captures many practical aspects of supply chains, such as dynamic planning horizon, generic supply chain network structure, external supply of materials, inventory opportunities for goods, distribution of commodities, facility configuration, availability of capital for investments, and storage limitations. A mixed integer linear programming model (MILP) for the dynamic relocation problem was formulated. Altiparmak et al. [1] considered three objectives: (1) minimization of total cost, including fixed costs of plants and distribution centers and inbound and outbound distribution costs, (2) maximization of customer services that can be rendered to customers in terms of acceptable delivery time, and (3) maximization of capacity utilization balance for distribution centers. They used a new solution procedure based on genetic algorithms to find the set of Pareto-optimal solutions for the multi-objective SCN design problem.

In recent years, many people have brought fuzzy theory into facility location and design to deal with the supply-chain network problem. Zhou et al. [48] proposed three types of fuzzy programming models to model capacitated location-allocation problem with fuzzy demands. Kahraman et al. [19] used fuzzy theory for the select a facility location among alternative locations. Bilgen [7] presented a fuzzy model consisting of multiple manufacturers, multiple production lines and multiple distribution centers for application in the consumer goods industry. Liang [26] presented an interactive fuzzy multi-objective linear programming (f-MOLP) model for solving integrated production and transportation problem with multiple fuzzy goals in fuzzy environments. Roghanian et al. [31] considered a "probabilistic bi-level linear multi-objective programming problem" and its application to an enterprise-wide supply-chain planning problem. Sakawa et al. [34] are introduced fuzzy goals into the formulated fuzzy random noncooperative bilevel linear program by taking into account the vagueness of decision makers' judgements. Selim and Ozkarahan [35] developed an interactive fuzzy goal program for supply-chain distribution network design. Chen and Chang [13] developed an approach for deriving the membership function of the fuzzy minimum total cost in a multi-product, multi-echelon, multi-period supply chain with fuzzy parameters. Ghatee and Hashemi [15] in their work dealt with fuzzy quantities and relations in multi-objective minimum-cost flow problem in a supply-chain network. Torabi and Hassini [38] proposed a new multi-objective possibilistic mixed integer linear programming model (MOPMILP) for integrating procurement, production and distribution planning considering various conflicting objectives simultaneously as well as the imprecise nature of certain critical parameters such as market demands, cost/time coefficients and capacity levels.

According to the literature described above, few researchers have considered the inclusion of qualitative factors in multi-objective problems. Although several effective techniques and models have been used to design the best supply chain network and to optimize various objectives, little work has been done on incorporating vagueness and imprecision of information into the capacitated multi-facility location problem. In this paper, an integrated approach using possibilistic linear programming and fuzzy AHP is developed to consider both quantitative and qualitative factors. The major objective of this study is to model the uncertainty problems faced by decision-makers and supply chain managers. A multi-objective linear programming technique is first used to solve the problem. Fuzzy theory is used to deal with transportation costs between the stages of the supply chain, fixed costs of facilities, and expert opinions in AHP. Consequently, possibilistic linear programming (PLP) is proposed for solving the problem because it appears a convenient approach for incorporating the imprecise nature of the real world [28]. Alternatively, bi-level programming can be used to describe model for decision-making situations where a hierarchy exists. The bi-level programming model consists of two sub-models, which is defined as an upper-level problem and the other as a lower-level problem. The choice of dominant level limits or strongly affects the choice of strategy on the lower level.

The rest of the paper is organized as follows: in Section 2, the theoretical background of the possibilistic linear programming and fuzzy AHP methodologies is given. In Section 3, the problem assumptions and the mathematical model are defined. In Section 4, the model is developed as a crisp model with the aid of the discussions in Section 2. The proposed method is illustrated with an example in Section 5. The results and findings are also discussed in this section. Finally, Section 6 concludes the study.

2. Possibilistic linear programming and fuzzy AHP approaches

2.1. Two-phase possibilistic linear programming

In this study, a two-phase multi-objective possibilistic linear programming (MOPLP) methodology is used. The theoretical background of the two phases is explained below.

2.1.1. Phase 1

The first phase of the two-phase approach involves defining the possibilistic coefficients and structuring their triangular distribution functions. An ambiguous datum is represented by a possibility distribution π_{ij} in possibilistic programming. A possibility distribution π_{ij} is defined by a fuzzy set, with A_{ij} representing a linguistic expression such as "approximately a_{ij}" as $\pi_{ij} = \mu_{Aij}$, where μ_{Aij} is a membership function of A_{ij}. A variable α_{ij} restricted by a possibility distribution π_{ij} is called a possibilistic variable [17]. Zadeh [45] proposed the concept of generalized constraints which classifies uncertainty situations as possibilistic, probabilistic, and veristic constraints. In the possibilistic programming approach, a vague aspiration is represented by a fuzzy goal G_i. A possibilistic linear programming (PLP) problem with imprecise fuzzy coefficients can be stated as:

$$\begin{aligned} &\max \sum_{i=1}^{n} \tilde{c}_i x_i \\ &\text{s.t. } x \in X = \{x | Ax \leqslant \tilde{b} \text{ and } x \geqslant 0\}, \end{aligned} \tag{1}$$

where $\tilde{c}_i = (c_i^p, c_i^m, c_i^o)$, for all i, are imprecise fuzzy coefficients and have triangular possibility distributions. Although various distributions can be chosen, the triangular and trapezoidal distributions are the most commonly used in solving possibilistic mathematical programming problems. In this study, only triangular fuzzy numbers will be used because it is simpler to do so. Because real-world problems usually involve uncertain data, decision-makers should address this imprecise or fuzzy environment. Hence, the possibility distributions estimated by the decision makers can be described more simply by triangular fuzzy numbers. The most possible value is c_i^m (possibility = 1, if normalized); c_i^p (the most pessimistic value) and c_i^o (the most optimistic value) are the least possible values. The possibility distributions (π_i) can be expressed as the degree of occurrence of an event [22].

The imprecise objective function with a triangular possibility distribution can be written as:

$$\max \left((c^m)^T x, \left(c^P\right)^T x, (c^o)^T x \right). \tag{2}$$

The auxiliary MOLP problem for solving Eq. (2) can be formulated as follows [25]:

$$\begin{aligned} &\min z_1 = (c^m - c^p)^T x, \\ &\max z_2 = c^{m^T} x, \\ &\max z_3 = (c^o - c^m)^T x, \\ &\quad \text{s.t.} \quad x \in X. \end{aligned} \tag{3}$$

The multi-objective programming problem can be solved using the concept of fuzzy set developed by Zimmermann [49]:

$$\begin{aligned} &\max Z = [c_1 x, c_2 x, \ldots, c_l x]^T \\ &\min W = [c_1 x, c_2 x, \ldots, c_r x]^T \\ &\quad \text{s.t.} \quad Ax \leqslant b, \\ &\qquad x \geqslant 0. \end{aligned} \tag{4}$$

The membership functions for the objective are defined as

$$\begin{aligned} &\mu_k(Z_k) = \frac{Z_k(x) - Z_k^{NIS}}{Z_k^{PIS} - Z_k^{NIS}}, \quad k = 1, 2, \ldots, l, \\ &\mu_k(W_s) = \frac{W_s^{NIS} - W_s(x)}{W_s^{NIS} - W_s^{PIS}}, \quad s = 1, 2, \ldots, r, \end{aligned} \tag{5}$$

where Z_k^{PIS}, W_s^{PIS} and Z_k^{NIS}, W_s^{NIS} are the positive and negative ideal solutions. With the "max–min" operator and degree of satisfaction $\lambda^{(1)}$, the MOLP problem can be solved as a single-objective problem:

$$\begin{aligned} & \max \lambda^{(1)} \\ & \text{s.t. } \lambda^{(1)} \leqslant \left(Z_k(x) - Z_k^{NIS}\right) / \left(Z_k^{PIS} - Z_k^{NIS}\right), \quad k = 1,2,\ldots,l, \\ & \lambda^{(1)} \leqslant \left(W_s^{NIS} - W_s(x)\right) / \left(W_s^{NIS} - W_s^{PIS}\right), \quad s = 1,2,\ldots,r, \\ & x \in X \\ & \lambda \in [0,1] \end{aligned} \tag{6}$$

The biggest disadvantage of Eq. (6) is that the results obtained by the "max–min" operator represent the worst situation and cannot be compensated for by other members which have better outcomes. Obviously, it is much more desirable for a compensatory operator to be used to obtain a compromise solution [24].

2.1.2. Phase 2 – two-phase approach

The two-phase method uses the max–min operator in its first phase. In the second phase, the solution is forced to improve from that obtained by the max–min operator by adding the Phase 1 satisfaction degree, $\lambda^{(1)}$, to Phase 2 as a constraint. The arithmetic average operator $\bar{\lambda}_{k,s}^{(2)}$ is used to split $\lambda^{(1)}$ to obtain new degrees of satisfaction that represent the degrees of satisfaction of the MOLP objectives. The problem can be reformulated according to Eq. (6) as follows:

$$\begin{aligned} & \max \bar{\lambda}_{k,s}^{(2)} = \frac{1}{l+r}\sum_{i=1}^{l+r}\lambda_i \\ & \text{s.t. } \lambda^{(1)} \leqslant \lambda_k^{(2)} \leqslant \left(Z_k(x) - Z_k^{NIS}\right) / \left(Z_k^{PIS} - Z_k^{NIS}\right), \quad k = 1,2,\ldots,l, \\ & \lambda^{(1)} \leqslant \lambda_s^{(2)} \leqslant \left(W_s^{NIS} - W_s(x)\right) / \left(W_s^{NIS} - W_s^{PIS}\right), \quad s = 1,2,\ldots,r, \\ & x \in X \\ & \lambda \in [0,1]. \end{aligned} \tag{7}$$

The optimal solution obtained in Phase 1 may not be an efficient solution; on the other hand, if $\bar{\lambda}_{k,s}^{(2)}$ is improperly given in Phase 2, it will make the interaction process more complicated. The steps for defining $\bar{\lambda}_{k,s}^{(2)}$ properly are given by Li and Li [25] as:

(1) Take the negative ideal solution as the initial solution of the max–min operator (6), i.e., $O = Z^{NIS}$; solve model (6) to get an optimal solution; then calculate the relative membership $\lambda^{(1)}$ of each objective value's degree of satisfaction.
(2) Set $\bar{\lambda}_{k,s}^{(2)} = \lambda^{(1)}$ and solve model (7) to obtain an optimal solution of the two-phase approach for solving the MOLP.

2.2. Fuzzy AHP

The analytic hierarchy process (AHP), first suggested by Saaty [33], is one of the most extensively used multiple-criteria decision-making methods. AHP is a straightforward method with the ability to handle effectively both qualitative and quantitative data. When using AHP, a hierarchical decision model is constructed by decomposing the decision problem into its decision criteria. The importance ratings or preference degrees of the decision criteria are compared using pairwise comparisons the criterion preceding each in the hierarchy [14,20,21]. In the traditional AHP formulation, human judgments are represented as crisp values. However, in many practical cases, the human preference model is uncertain, and decision-makers may be reluctant or unable to assign crisp values to the comparison judgments [10,14] because human assessment of qualitative attributes is always subjective and therefore imprecise. Therefore, conventional AHP seems inadequate to capture decision-makers' requirements explicitly. To model this kind of uncertainty in human preference, fuzzy sets can be used in the AHP approach to generate an extended version of AHP [4]. Pedrycz [29] considered a granular generalization of AHP approach which helps endow existing modeling paradigms and practice with new conceptual and algorithmic features, making the resulting models more reflective of complexities of real-world phenomena.

This research uses the four-step fuzzy AHP procedure as given by Cheng and Mon [12] and Ayağ and Özdemir [4]. The procedure presented in this paper contains a "calculation of geometric mean of weighted decision-makers" [42] step in addition. For details of the steps used, please see the papers mentioned above.

3. Problem formulation

3.1. Problem description and assumptions

Decision-making in such a complex supply-chain network requires consideration of conflicting objectives as well as different constraints imposed by suppliers, manufacturers and distributors [37]. In this research a four-stage (suppliers, plants, distribution centers, customers) supply-chain network is considered. A number of potential facilities with a certain capacity, such as service centers, plants, DCs are given and the problem is to assign facilities to locations in such way that the objective functions ("minimum cost" and "maximum qualitative factors benefit") are optimized. Moreover, in practical situations, the environmental coefficients and related parameters are uncertain in a medium time horizon. Therefore, the forecast demand and related operating costs are generally imprecise. Here possibility theory was used to model this imprecision. This theory uses possibility distributions to handle inherently ambiguous phenomena in the problem parameters [28,37,40].

The main assumptions used in the problem are:

- The supply chain network has four stages; suppliers, plants, distribution centers, and customers.
- The numbers of suppliers and customers are known beforehand; there are three suppliers and four customers in the network. The numbers of plants and DCs to be opened will be determined from among a maximum of five alternatives in each phase.
- There is only one product in the network.
- Suppliers, capacities of alternative plants and DCs, and customer demands are known to follow imprecise triangular distributions.
- Triangular distributions are used in the objective functions because of their simplicity and their widespread use in the literature.
- The entire demand of customers is met. There is no backorder from DCs.
- In the first-phase objective function, total cost is the sum of transportation costs and fixed costs of alternative facilities.
- The qualitative factors used in the second-phase objective function can be changed depending on the business sector.

3.2. Notation

The following notation is used here:

Indices

I	number of suppliers ($i = 1,2,\ldots,I$)
J	number of plants ($j = 1,2,\ldots,J$)
K	number of distribution centers ($k = 1,2,\ldots,K$)
L	number of customers ($l = 1,2,\ldots,L$)
M	number of qualitative factors for plant locations ($m = 1,2,\ldots,M$)
N	number of qualitative factors for DC locations ($n = 1,2,\ldots,N$)

Parameters

$\tilde{a}_i$	capacity of supplier i
$\tilde{b}_j$	capacity of plant j
$\tilde{c}_k$	capacity of DC k
$\tilde{d}_l$	demand of customer l
$\tilde{s}_{ij}$	unit cost of production in plant j using material from supplier i
$\tilde{t}_{jk}$	unit cost of transportation from plant j to DC k
$\tilde{u}_{kl}$	unit cost of transportation from DC k to customer l
$\tilde{f}_j$	total of fixed costs for operating plant j
$\tilde{g}_k$	total of fixed costs for operating DC k
V	upper limit on the total number of plants that can be opened
R	upper limit on the total number of DCs that can be opened
$\widetilde{W}_m\_FAHP\_p_j$	qualitative factor weight of m for plant j calculated using the fuzzy AHP
$\widetilde{W}_n\_FAHP\_dc_k$	qualitative factor weight of n for DC k calculated using the fuzzy AHP

Variables

x_{ij}	quantity produced at plant j using raw material from supplier i
y_{jk}	amount shipped from plant j to DC k
z_{kl}	amount shipped from DC k to customer l

$$v_j = \begin{cases} 1 & \text{if plant } j \text{ is opened,} \\ 0 & \text{otherwise} \end{cases}$$

$$r_k = \begin{cases} 1 & \text{if DC } k \text{ is opened,} \\ 0 & \text{otherwise} \end{cases}$$

3.3. Possibilistic multiobjective programming model

3.3.1. Objective functions

Objective 1: *Minimum Total Transportation and Facilities Costs*

Most theoretical and practical decisions made to solve capacitated multi-facility location problems (CMFLP) usually consider total supply-chain costs. To minimize total costs, the objective function used by Pirkul and Jayaraman [30], Syarif et al. [37], and Altiparmak et al. [1] was used here. The total costs are the sum of the transportation costs between the members of the SC network and the total fixed costs for all facilities. The objective function coefficients are imprecise because of fast-changing market conditions. The modified version of the objective function is:

$$\min \sum_i \sum_j \tilde{s}_{ij} x_{ij} + \sum_j \sum_k \tilde{t}_{jk} y_{jk} + \sum_k \sum_l \tilde{u}_{kl} z_{kl} + \sum_j \tilde{f}_j v_j + \sum_k \tilde{g}_k r_k, \quad (8)$$

where $\tilde{s}_{ij}, \tilde{t}_{jk}, \tilde{u}_{kl}, \tilde{f}_j, \tilde{g}_k$ are imprecise coefficients with triangular possibility distributions. The transportation cost is made up of three parts. $\sum_i \sum_j \tilde{s}_{ij} x_{ij}$ is the transportation costs between suppliers and plants, $\sum_j \sum_k \tilde{t}_{jk} y_{jk}$ is the transportation costs between plants and distribution centers, and $\sum_k \sum_l \tilde{u}_{kl} z_{kl}$ is the transportation costs between distribution centers and customers. $\sum_k \tilde{f}_j v_j$ is the total fixed cost for a candidate plant if it will be opened, and a similar formulation can be written for distribution centers as $\sum_k \tilde{g}_k r_k$.

Objective 2: *Maximum Total Qualitative Factor Benefits For Facility Location*

The facility location decision is a more complex problem because of the uncertainty and volatility of distribution environments. The location decision process involves qualitative as well as quantitative factors. Decision-makers can no longer ignore the influence of sensitive factors such as the worker status of a candidate region, transportation conditions, market environment, and location properties. Moreover, the process can become highly judgmental if a wide variety of qualitative factors are present. In such cases, the selection process may lack consistency and flexibility. The fuzzy analytic hierarchy process (FAHP) methodology has been successfully used to provide consistent evaluation (weighting and ranking) of location alternatives [23]. The qualitative factors used in the present paper are shown in Table 1. The list of these factors could be expanded or changed depending on business field, location properties, or country. The maximization of FAHP weights has been added to the multiobjective problem as follows:

$$\max \sum_j \widetilde{W}\_FAHP\_p_j y_{jk} + \sum_k \widetilde{W}\_FAHP_d c_k z_{kl}, \quad \forall j, \forall k, \forall l \quad (9)$$

where $\widetilde{W}\_FAHP\_p_j$ and $\widetilde{W}\_FAHP\_dc_k$ are the fuzzy AHP weights for plants and distribution centers respectively.

3.3.2. Constraints

$$\sum_j x_{ij} \leqslant \tilde{a}_i, \quad \forall i, \quad (10)$$

$$\sum_k y_{jk} \leqslant \tilde{b}_j v_j, \quad \forall j, \quad (11)$$

$$\sum_j v_j \leqslant P \quad (12)$$

Table 1
Main qualitative factors for facility location.

1	Proximity to markets
2	Transportation opportunities
3	Labor skills
4	Education and field schools
5	Energy alternatives
6	Water availability
7	Infrastructure availability (roads, sewer system, municipality services)
8	Security
9	Expand capability
10	Housing and residence availability for workers
11	Closeness health services facilities
12	Disaster risks
13	Climate

$$\sum_l z_{kl} \leqslant \tilde{c}_k z_k, \quad \forall k, \tag{13}$$

$$\sum_k r_k \leqslant V \tag{14}$$

$$\sum_k z_{kl} \geqslant \tilde{d}_l, \quad \forall l, \tag{15}$$

$$v_j, z_k = \{0,1\} \quad \forall j, \forall k, \tag{16}$$

$$x_{ij}, y_{jk}, z_{kl} \geqslant 0, \quad \forall i, \forall j, \forall k, \forall l \tag{17}$$

where Eqs. (10), (11), and (13) denote capacity constraints for suppliers, plants, and distribution centers respectively. The "∼" signs over a_i, b_j, and c_k mean that these variables are imprecise. Plant and distribution locations are alternative locations, and binary variables v_j and z_k will determine the open/close decisions. $\sum v_j$ and $\sum z_k$ cannot exceed the values of P and V given in Eqs. (12) and (14). These values can be the maximum number of potential plant and distribution centers or a value which is determined by the decision-maker.

4. Model development

Traditional mathematical programming techniques have used deterministic variables and coefficients for modeling. In fact, most real systems are not deterministic. Traditional modeling techniques may sometimes disagree with real situations. Zadeh [46] introduced fuzzy set theory to deal with the uncertainty and imprecision associated with information about various parameters. In possibilistic linear programming, possibilistic distributions are generally represented by triangular possibility distributions because of their simplicity and ease of use. Triangular possibilistic distributions have been used by Ozgen et al. [28], Wang and Liang [41], and Torabi and Hassani [38] to represent imprecise coefficients for solving different types of supply chain problems. The triangular possibility distribution of an imprecise number can be represented as $\tilde{a} = (a^p, a^m, a^o)$, where a^p denotes the smallest possible value (lower bound), a^m a midrange value, and a^o the largest possible value (upper bound) that describes a fuzzy event. The imprecise data for the first-phase objective function and constraints can be modeled using triangular possibility distributions as follows (the optimistic values are the smallest possible values and the pessimistic values are the largest values because the coefficients represent cost components):

$$\tilde{s}_{ij} = (s_{ij}^o, s_{ij}^m, s_{ij}^p) \quad \forall i, \forall j,$$

$$\tilde{t}_{jk} = (t_{jk}^o, t_{jk}^m, t_{jk}^p) \quad \forall j, \forall k,$$

$$\tilde{u}_{kl} = (u_{kl}^o, u_{kl}^m, u_{kl}^p) \quad \forall k, \forall l,$$

$$\tilde{f}_j = (f_j^o, f_j^m, f_j^p) \quad \forall j,$$

$$\tilde{g}_k = (g_k^o, g_k^m, g_k^p) \quad \forall k,$$

$$\tilde{a}_i = (a_i^p, a_i^m, a_i^o) \quad \forall i,$$

$$\tilde{b}_j = (b_j^p, b_j^m, b_j^o) \quad \forall j,$$

$$\tilde{c}_k = (c_k^p, c_k^m, c_k^o) \quad \forall k,$$

$$\tilde{d}_l = (d_l^p, d_l^m, d_l^o) \quad \forall l,$$

4.1. Converting imprecise objective-function and constraint coefficients into crisp numbers

The coefficients of the imprecise first-phase objective function of the multiobjective model have a triangular possibility distribution. Geometrically, this imprecise objective function is fully defined by three prominent points: (z^o, 0), (z^m, 1), and (z^p, 0). Consequently, minimizing the imprecise objective function z requires minimizing z^o, z^m, and z^p simultaneously. However, there may exist a conflict in the simultaneous minimization of these crisp objectives. Therefore, using Lai and Hwang's [22] approach which has also been adopted by other researchers [22,28,40], one can minimize z^m, maximize $(z^m - z^o)$, and minimize $(z^p - z^m)$ instead of minimizing z^o, z^m, and z^o simultaneously. In this manner, the imprecise total cost of the supply chain in the first-phase objective function can be converted into crisp objectives as follows:

$$\text{Min } z_1 = z_1^m = \sum_i \sum_j s_{ij}^m x_{ij} + \sum_j \sum_k t_{jk}^m y_{jk} + \sum_k \sum_l u_{kl}^m z_{kl} + \sum_j f_j^m v_j + \sum_k g_k^m r_k \tag{18}$$

$$\begin{aligned}\text{Max } z_2 &= z_1^m - z_1^o \\ &= \sum_i \sum_j \left(s_{ij}^m - s_{ij}^o\right) x_{ij} + \sum_j \sum_k \left(t_{jk}^m - t_{jk}^o\right) y_{jk} + \sum_k \sum_l (u_{kl}^m - u_{kl}^o) z_{kl} + \sum_j \left(f_j^m - f_j^o\right) v_j + \sum_k (g_k^m - g_k^o) r_k\end{aligned} \tag{19}$$

$$\begin{aligned}\text{Min } z_3 &= z_1^p - z_1^m \\ &= \sum_i \sum_j \left(s_{ij}^p - s_{ij}^m\right) x_{ij} + \sum_j \sum_k \left(t_{jk}^p - t_{jk}^m\right) y_{jk} + \sum_k \sum_l (u_{kl}^p - u_{kl}^m) z_{kl} + \sum_j \left(f_j^p - f_j^m\right) v_j + \sum_k (g_k^p - g_k^m) r_k.\end{aligned} \tag{20}$$

To solve the imprecise left sides of the imprecise constraint coefficients, the most likely values method as proposed by Lai and Hwang [22] was used. If the minimum acceptable degree of feasibility, β, is given, then the equivalent auxiliary crisp constraints (Eqs. (10), (11), (13), and (15)) can be represented as follows:

$$\sum_j x_{ij} \leqslant w_1 a_{i,\beta}^p + w_2 a_{i,\beta}^m + w_3 a_{i,\beta}^o \quad \forall i, \tag{21}$$

$$\sum_k y_{jk} \leqslant \left(w_1 b_{j,\beta}^p + w_2 b_{j,\beta}^m + w_3 b_{j,\beta}^o\right) v_j, \quad \forall j, \tag{22}$$

$$\sum_l z_{kl} \leqslant (w_1 c_{k,\beta}^p + w_2 c_{k,\beta}^m + w_3 c_{k,\beta}^o) r_k, \quad \forall k, \tag{23}$$

$$\sum_k z_{kl} \geqslant w_1 d_{l,\beta}^p + w_2 d_{l,\beta}^m + w_3 d_{l,\beta}^o, \quad \forall l, \tag{24}$$

and the most likely values are assumed to be $w_1 = 1/6$, $w_2 = 4/6$, $w_3 = 1/6$, and $\beta = 0.5$ [22]. The midrange weight w_2 is the most possible value, and it indicates why more weight should be assigned to it.

4.2. Solving the auxiliary of the first-phase objective function using a two-phase PLP approach

To solve Eqs. (18)–(20), any MOLP technique, such as utility theory, goal programming, or fuzzy programming, may be used. Zimmermann developed the first fuzzy approach for solving a MOLP, called the max–min approach [49], but the max–min operator may not be unique. To remove this deficiency, a two-phase approach has been used by Lee and Li [24], Guu and Wu [16], Lee and Li [24], Amid et al. [2], and Ozgen et al. [28]. In this paper, a two-phase PLP approach is used to evaluate the degree of satisfaction of the objective functions. To solve the auxiliary objective functions and the AHP objective function together, it is necessary to combine the second-phase objective function with three auxiliary objective functions. The crisp AHP solutions can be obtained using fuzzy AHP steps as described earlier. Consequently, before converting these four objectives into a single-goal problem, it is necessary to determine the positive ideal solution (PIS) and the negative ideal solution (NIS) for each objective function by solving the corresponding MILP model as follows:

$$z_1^{PIS} = \text{Min } z_1^m, \quad z_1^{NIS} = \text{Max } z_1^m \tag{25a}$$

$$z_2^{PIS} = \text{Max } (z_1^m - z_1^o), \quad z_2^{NIS} = \text{Min } (z_1^m - z_1^o) \tag{25b}$$

$$z_3^{PIS} = \text{Min } (z_1^p - z_1^m), \quad z_3^{NIS} = \text{Max } (z_1^p - z_1^m) \tag{25c}$$

$$z_4^{PIS} = \text{Max } z_2^m, \quad z_4^{NIS} = , \text{Min } z_2^m \tag{25d}$$

The linear membership function for each objective function can then be specified as follows:

$$\mu z_1 = \begin{cases} 1, & z_1 < z_1^{PIS}, \\ \frac{z_1^{NIS} - z_1}{z_1^{NIS} - z_1^{PIS}}, & z_1^{PIS} \leqslant z_1 \leqslant z_1^{NIS}, \\ 0, & z_1 > z_1^{NIS}, \end{cases} \tag{26}$$

$$\mu z_2 = \begin{cases} 1, & z_2 < z_2^{PIS}, \\ \frac{z_2 - z_2^{NIS}}{z_2^{PIS} - z_2^{NIS}}, & z_2^{NIS} \leqslant z_2 \leqslant z_2^{PIS}, \\ 0, & z_2 > z_2^{NIS}, \end{cases} \tag{27}$$

where μz_3 is similar to μz_1 and μz_4 is similar to μz_2.

Using the fuzzy decision-making approach of Bellman and Zadeh [6] and Zimmermann's [49] fuzzy programming method, the complete equivalent single-goal LP model (*Phase 1*) for solving the capacitated plant location problem [46] becomes:

$$
\begin{aligned}
&\text{Max } \lambda^{(1)}\\
&\quad \text{s.t.}\\
&\qquad \lambda^{(1)} \leqslant \mu z_k \quad k = 1,..,4\\
&\qquad \sum_j x_{ij} \leqslant w_1 a^p_{i,\beta} + w_2 a^m_{i,\beta} + w_3 a^o_{i,\beta} \quad \forall i,\\
&\qquad \sum_k y_{jk} \leqslant \left(w_1 b^p_{j,\beta} + w_2 b^m_{j,\beta} + w_3 b^o_{j,\beta}\right) v_j, \quad \forall j,\\
&\qquad \sum_j v_j \leqslant P\\
&\qquad \sum_l z_{kl} \leqslant \left(w_1 c^p_{k,\beta} + w_2 c^m_{k,\beta} + w_3 c^o_{k,\beta}\right) r_k, \quad \forall k,\\
&\qquad \sum_k r_k \leqslant V\\
&\qquad \sum_k z_{kl} \geqslant w_1 d^p_{l,\beta} + w_2 d^m_{l,\beta} + w_3 d^o_{l,\beta}, \quad \forall l,\\
&\qquad v_j, z_k = \{0,1\} \quad \forall j, \forall k,\\
&\qquad x_{ij}, y_{jk}, z_{kl} \geqslant 0, \quad \forall i, \forall j, \forall k, \forall l\\
&\qquad \lambda^{(1)} \in [0,1].
\end{aligned}
\tag{28}
$$

In the second phase (*Phase 2*), the degree of satisfaction of each objective function can be represented separately. After obtaining an optimal solution from *Phase 1* (Eq. (28)), this solution will be used in constraints as follows:

$$
\begin{aligned}
&\text{Max } \bar{\lambda}^{(2)}_{k,s}\\
&\quad \text{s.t.}\\
&\qquad \lambda^{(1)} \leqslant \lambda^{(2)}_s \leqslant \mu z_s, \quad s = 1,3\\
&\qquad \lambda^{(1)} \leqslant \lambda^{(2)}_k \leqslant \mu z_k, \quad k = 2,4\\
&\qquad \sum_j x_{ij} \leqslant w_1 a^p_{i,\beta} + w_2 a^m_{i,\beta} + w_3 a^o_{i,\beta} \quad \forall i,\\
&\qquad \sum_k y_{jk} \leqslant \left(w_1 b^p_{j,\beta} + w_2 b^m_{j,\beta} + w_3 b^o_{j,\beta}\right) v_j, \quad \forall j,\\
&\qquad \sum_j v_j \leqslant P\\
&\qquad \sum_l z_{kl} \leqslant \left(w_1 c^p_{k,\beta} + w_2 c^m_{k,\beta} + w_3 c^o_{k,\beta}\right) z_k, \quad \forall k,\\
&\qquad \sum_k z_k \leqslant V\\
&\qquad \sum_k z_{kl} \geqslant w_1 d^p_{l,\beta} + w_2 d^m_{l,\beta} + w_3 d^o_{l,\beta}, \quad \forall l,\\
&\qquad v_j, z_k = \{0,1\} \quad \forall j, \forall k,\\
&\qquad x_{ij}, y_{jk}, z_{kl} \geqslant 0, \quad \forall i, \forall j, \forall k, \forall l\\
&\qquad \lambda^{(1)}, \lambda^{(2)}_{k,s} \in [0,1].
\end{aligned}
\tag{29}
$$

After solving the problem in Phase 2, decision-makers can clearly see the tradeoffs among multiple objectives. An interactive solution set will help them to make appropriate decisions in response to fast-changing market conditions.

5. Numerical example

5.1. Objective 1 data

A simple but comprehensive model is used here to illustrate the effectiveness of the possibilistic linear programming procedure developed in this paper. For purposes of comparison, the numerical example data set presented by Syarif et al. [37] is used. Their work used deterministic costs in the model. On the other hand, the present model is focused on fuzzifying the parameters of decision variables by transforming them into triangular possibility distributions. The fuzzy sets are constructed with 10–20% right- and left-side tolerance bounds for midrange values. Table 2 shows the possibilistic capacities

Table 2
Imprecise capacities, demands and fixed costs.

Supplier		Plant			Distribution center			Customer	
i	Capacity ($\tilde{a}_i$)	j	Capacity ($\tilde{b}_j$)	Fixed cost ($\tilde{f}_j$)	k	Capacity ($\tilde{c}_k$)	Fixed cost ($\tilde{g}_k$)	l	Demand ($\tilde{d}_l$)
1	(400, 500, 550)	1	(350, 400, 475)	(1500, 1800, 2000)	1	(460, 530, 600)	(800, 1000, 1250)	1	(350, 460, 550)
2	(550, 650, 800)	2	(425, 550, 625)	(800, 900, 1200)	2	(500, 590, 650)	(750, 900, 1100)	2	(250, 330, 420)
3	(320, 390, 450)	3	(440, 490, 570)	(1700, 2100, 2300)	3	(350, 400, 460)	(1400, 1600, 1800)	3	(375, 450, 500)
		4	(275, 300, 345)	(850, 1100, 1300)	4	(325, 370, 425)	(1300, 1500, 1650)	4	(250, 300, 345)
		5	(425, 500, 530)	(750, 900, 1100)	5	(500, 580, 650)	(1100, 1400, 1600)		

Table 3
Imprecise transportation costs between each stage of SC.

Supplier	Plant				
$\tilde{s}_{ij}$	1	2	3	4	5
1	(4, 5, 7)	(4, 6, 8)	(3, 4, 5)	(5,7, 9)	(4, 5, 7)
2	(4, 6, 8)	(4, 5, 6)	(4, 6, 8)	(4, 6, 8)	(5, 8, 10)
3	(5, 7, 9)	(5, 6, 8)	(2, 3, 4)	(7, 9, 12)	(5, 6, 8)
Plant	**Distribution center**				
$\tilde{t}_{jk}$	1	2	3	4	5
1	(4, 5, 6)	(6, 8, 10)	(4, 5, 7)	(6, 8, 9)	(4, 5, 7)
2	(6, 8, 10)	(5, 7, 8)	(6, 8, 10)	(5,6, 8)	(6, 8, 11)
3	(3, 4, 5)	(5, 7, 9)	(3, 4, 5)	(4, 5, 7)	(3, 4, 5)
4	(2, 3, 4)	(4, 5, 7)	(2, 3, 4)	(4, 5, 7)	(2, 3, 4)
5	(3, 5, 6)	(4, 6, 7)	(4, 6, 8)	(7, 8, 9)	(2, 3, 5)

Distribution center	Customer			
$\tilde{u}_{kl}$	1	2	3	4
1	(6, 7, 9)	(3, 4, 5)	(4, 5, 7)	(5, 6, 8)
2	(4, 5, 7)	(3, 4, 6)	(4, 6, 7)	(5, 7, 9)
3	(5, 7, 8)	(4, 5, 6)	(2, 3, 5)	(5, 6, 8)
4	(2, 3, 4)	(4, 5, 6)	(5, 6, 8)	(3, 4, 6)
5	(3, 4, 6)	(5, 6, 8)	(4, 5, 7)	(6, 7, 9)

of suppliers, plants, and distribution centers, the possibilistic fixed costs of alternative plants and distribution centers, and the imprecise customer demands. Table 3 shows the imprecise transportation costs between the nodes of the SC network.

5.2. Objective 2 data

To facilitate the use of FAHP, the qualitative factors for plant and distribution centers can be summarized into a hierarchy which shows the overall goal of the decision process, each decision criterion to be used, and the decision alternatives to be considered as candidates for location. The most commonly used six criteria to be used in deciding on plant and distribution center locations are shown in Tables 4 and 5 respectively. Information needed for application of the FAHP steps is given in Table 6.

Before making pairwise comparisons, it is required to gather expert opinions using a FAHP scale. For this purpose, it is advised to prepare a questionnaire which evaluates each comparison. First, plant and DC quantitative factors must be compared, and then the location alternatives related to every factor for both plant and distribution center location alternatives. The fuzzy comparison matrix of pairwise comparisons (from five experts) for plant qualitative factors using fuzzy numbers is given in Table 7. The fuzzy comparison matrix of plant location (PL) alternatives with respect to the qualitative factor "proximity to markets" (PLQF1) is shown in Table 8.

5.3. Solution steps

The solution steps for the possibilistic capacitated multi-commodity facility location problem can be described as follows:

Step 1: Formulate the multiobjective model for the facility location problem using Eqs. (8) and (9).
Step 2: Model the imprecise data using triangular possibility distributions. Tables 2 and 3 list the triangular possibility distributions of the imprecise coefficients for objective function 1 and the right-hand sides of the constraints.
Step 3: Transform the imprecise objective functions into new crisp objective functions using Eqs. (18)–(20).
Step 4: Transform the imprecise constraints into new crisp constraints using Eqs. (21) and (24) with $\beta = 0.5$.
Step 5: Determine qualitative factors and subfactors (if available) for objective 2 from Table 1.

Table 4
Qualitative factors for plant location (PLQF).

PLQF1 – Proximity to market
PLQF2 – Labor skills
PLQF3 – Education and field schools
PLQF4 – Transportation alternatives
PLQF5 – Infrastructure availability (roads, sewer system, municipality services)
PLQF6 – Housing and residence availability for workers

Table 5
Qualitative factors for distribution center location (DCLQF).

DCLQF1 – Proximity to market
DCLQF2 – Transportation alternatives
DCLQF3 – City planning
DCLQF4 – Security
DCLQF5 – Natural disaster
DCLQF6 – Climate

Table 6
AHP data entry.

Goal	Best plant location
Number of alternatives (m)	5
Names of alternatives	PL1(m_1), PL2(m_2), PL3(m_3), PL4(m_4), PL5(m_5)
Index of optimism (μ)	0.5 (default value: 0.5, $0 < \mu < 1$)
Confidence level (α)	0.5 (default value: 0.5, $0 < \alpha < 1$)
	Matrix of paired comparisons for the attributes using triangular fuzzy numbers ($n \times n = 6 \times 6$)
	Matrices of paired comparisons results for the alternatives (m_1, m_2, m_3, m_4, m_5) with respect to each attribute using triangular fuzzy numbers, respectively

Table 7
Fuzzy comparison matrix of the plants qualitative factors (with 5 experts opinions).

	PLQF1	PLQF 2	PLQF 3	PLQF 4	PLQF 5	PLQF 6
PLQF 1	1	$\tilde{5},\tilde{5},\tilde{7},\tilde{3},\tilde{9}$	$\tilde{3},\tilde{5},\tilde{7},\tilde{3},\tilde{1}$	$\tilde{1},\tilde{3},\tilde{1},\tilde{5},\tilde{1}$	$\tilde{5},\tilde{3},\tilde{5},\tilde{7},\tilde{5}$	$\tilde{7},\tilde{5},\tilde{5},\tilde{7},\tilde{5}$
PLQF 2	$\tilde{5}^{-1},\tilde{5}^{-1},\tilde{7}^{-1},\tilde{3}^{-1},\tilde{9}^{-1}$	1	$\tilde{1},\tilde{1}^{-1},\tilde{1},\tilde{5},\tilde{1}$	$\tilde{3}^{-1},\tilde{7}^{-1},\tilde{1}^{-1},\tilde{3}^{-1},\tilde{5}^{-1}$	$\tilde{3}^{-1},\tilde{9}^{-1},\tilde{1}^{-1},\tilde{3}^{-1},\tilde{5}^{-1}$	$\tilde{5},\tilde{7},\tilde{5},\tilde{5},\tilde{1}$
PLQF 3	$\tilde{3}^{-1},\tilde{5}^{-1},\tilde{7}^{-1},\tilde{3}^{-1},\tilde{1}^{-1}$	$\tilde{1}^{-1},\tilde{1},\tilde{1}^{-1},\tilde{5}^{-1},\tilde{1}^{-1}$	1	$\tilde{3}^{-1},\tilde{5}^{-1},\tilde{1}^{-1},\tilde{3}^{-1},\tilde{7}^{-1}$	$\tilde{5}^{-1},\tilde{5}^{-1},\tilde{3}^{-1},\tilde{5}^{-1},\tilde{9}^{-1}$	$\tilde{1},\tilde{3},\tilde{1},\tilde{3},\tilde{1}$
PLQF 4	$\tilde{1}^{-1},\tilde{3}^{-1},\tilde{1}^{-1},\tilde{5}^{-1},\tilde{1}^{-1}$	$\tilde{3},\tilde{7},\tilde{1},\tilde{3},\tilde{5}$	$\tilde{3},\tilde{5},\tilde{1},\tilde{3},\tilde{7}$	1	$\tilde{3},\tilde{5},\tilde{3},\tilde{5},\tilde{7}$	$\tilde{7},\tilde{9},\tilde{5},\tilde{7},\tilde{9}$
PLQF 5	$\tilde{5}^{-1},\tilde{3}^{-1},\tilde{5}^{-1},\tilde{7}^{-1},\tilde{5}^{-1}$	$\tilde{3},\tilde{9},\tilde{1},\tilde{3},\tilde{5}$	$\tilde{5},\tilde{5},\tilde{3},\tilde{5},\tilde{9}$	$\tilde{3}^{-1},\tilde{5}^{-1},\tilde{3}^{-1},\tilde{5}^{-1},\tilde{7}^{-1}$	1	$\tilde{5},\tilde{3},\tilde{5},\tilde{5},\tilde{5}$
PLQF 6	$\tilde{7}^{-1},\tilde{5}^{-1},\tilde{5}^{-1},\tilde{7}^{-1},\tilde{5}^{-1}$	$\tilde{5}^{-1},\tilde{7}^{-1},\tilde{5}^{-1},\tilde{5}^{-1},\tilde{1}^{-1}$	$\tilde{1}^{-1},\tilde{3}^{-1},\tilde{1}^{-1},\tilde{3}^{-1},\tilde{1}^{-1}$	$\tilde{7}^{-1},\tilde{9}^{-1},\tilde{5}^{-1},\tilde{7}^{-1},\tilde{9}^{-1}$	$\tilde{5}^{-1},\tilde{3}^{-1},\tilde{5}^{-1},\tilde{5}^{-1},\tilde{5}^{-1}$	1

Table 8
Fuzzy comparison matrix for plant location alternatives with respect to the first attribute; PLQF1 (proximity to markets).

PLQF 1	PL1	PL2	PL 3	PL 4	PL 5
PL1	1	$\tilde{5},\tilde{3},\tilde{5},\tilde{7},\tilde{5}$	$\tilde{3}^{-1},\tilde{3}^{-1},\tilde{5}^{-1},\tilde{1}^{-1},\tilde{5}^{-1}$	$\tilde{3},\tilde{3}^{-1},\tilde{3},\tilde{3},\tilde{1}^{-1}$	$\tilde{7},\tilde{7},\tilde{5},\tilde{7},\tilde{5}$
PL2	$\tilde{5}^{-1},\tilde{3}^{-1},\tilde{5}^{-1},\tilde{7}^{-1},\tilde{5}^{-1}$	1	$\tilde{5}^{-1},\tilde{3}^{-1},\tilde{5}^{-1},\tilde{3}^{-1},\tilde{7}^{-1}$	$\tilde{3},\tilde{3},\tilde{5},\tilde{1},\tilde{3}$	$\tilde{5},\tilde{7},\tilde{5},\tilde{3},\tilde{5}$
PL3	$\tilde{3},\tilde{3}\tilde{5},\tilde{1}\tilde{5}$	$\tilde{5},\tilde{3},\tilde{5},\tilde{3},\tilde{7}$	1	$\tilde{7},\tilde{5},\tilde{7},\tilde{7},\tilde{5}$	$\tilde{9},\tilde{7},\tilde{9},\tilde{9},\tilde{5}$
PL4	$\tilde{3}^{-1},\tilde{1},\tilde{3}^{-1},\tilde{3}^{-1},\tilde{1}$	$\tilde{3}^{-1},\tilde{3}^{-1},\tilde{5}^{-1},\tilde{1}^{-1},\tilde{3}^{-1}$	$\tilde{7}^{-1},\tilde{5}^{-1},\tilde{7}^{-1},\tilde{7}^{-1},\tilde{5}^{-1}$	1	$\tilde{3},\tilde{5},\tilde{5},\tilde{5},\tilde{1}$
PL5	$\tilde{7}^{-1},\tilde{7}^{-1},\tilde{5}^{-1},\tilde{7}^{-1},\tilde{5}^{-1}$	$\tilde{5}^{-1},\tilde{7}^{-1},\tilde{5}^{-1},\tilde{3}^{-1},\tilde{5}^{-1}$	$\tilde{9}^{-1},\tilde{7}^{-1},\tilde{9}^{-1},\tilde{9}^{-1},\tilde{5}^{-1}$	$\tilde{3}^{-1},\tilde{5}^{-1},\tilde{5}^{-1},\tilde{5}^{-1},\tilde{1}^{-1}$	1

Step 6: Enter the values found in Table 6 into the FAHP steps. Create a similar table for distribution centers.
Step 7: Collect expert opinions for evaluating plant location qualitative factors using the FAHP scale (Table 6).
Step 8: Collect expert opinions for evaluating the plant location alternatives related to every qualitative factor (Table 7).
Step 9: Repeat Steps 7 and 8 for the pairwise matrix for distribution centers.
Step 10: Calculate the $\widetilde{W}\_FAHP\_p_j$ and $\widetilde{W}\_FAHP\_dc_k$ weights according to the steps described in papers given in Section 2.2 (the overall e-vector calculations are shown in Tables 9 and 10).

Table 9
The FAHP final ranking of plant location alternatives.

Qualitative factors for plants		Alternative plant locations					
		PL1	PL2	PL3	PL4	PL5	CR < 0.10
PLQF1	0.382	0.279	0.135	0.458	0.091	0.037	0.084
PLQF2	0.096	0.186	0.148	0.520	0.101	0.044	0.051
PLQF3	0.069	0.143	0.172	0.532	0.108	0.045	0.027
PLQF4	0.254	0.201	0.080	0.514	0.167	0.038	0.100
PLQF5	0.165	0.206	0.119	0.529	0.093	0.053	0.038
PLQF6	0.034	0.170	0.140	0.523	0.106	0.062	0.029
Overall *e*-vector		0.225	0.123	0.497	0.113	0.042	

Table 10
The FAHP final ranking of distribution centers alternatives.

Qualitative factors for DC's		Alternative DC locations					
		DCL1	DCL2	DCL3	DCL4	DCL5	CR < 0.10
DCLQF1	0.339	0.077	0.046	0.506	0.227	0.144	0.056
DCLQF2	0.271	0.101	0.052	0.484	0.261	0.102	0.034
DCLQF3	0.209	0.409	0.257	0.051	0.095	0.188	0.068
DCLQF4	0.095	0.217	0.151	0.343	0.165	0.125	0.094
DCLQF5	0.050	0.516	0.233	0.058	0.094	0.100	0.030
DCLQF6	0.036	0.109	0.193	0.362	0.236	0.100	0.026
Overall *e*-vector		0.189	0.116	0.362	0.196	0.136	

Table 11
Positive and negative ideal solutions of auxiliary objective functions.

	PIS	NIS
z_1	Min $z_1^m = 29.323$	Max $z_1^m = 43.414$
z_2	Max $(z_1^m - z_1^o) = 10.974$	Min $(z_1^m - z_1^o) = 6.535$
z_3	Min $(z_1^p - z_1^m) = 7.069$	Max $(z_1^p - z_1^m) = 11.825$
z_4	Max $z_4 = 763$	Min $z_4 = 381$

Step 11: Calculate positive (PIS) and negative (NIS) ideal solutions of the auxiliary objective functions using Eqs (25a)–(25d) (Table 11).
Step 12: Construct the single-objective model with the aid of the membership functions (Eqs. (26) and (27)) and calculate the optimum degree of satisfaction (λ^1) for Phase 1 (Eq. (28)).
Step 13: Take the optimum result of Step 12, find the satisfaction levels of the (λ^1) results for every objective, and add in the two-phase approach model as a constraint. Solve Phase 2 (Eq. (29)) and calculate the optimum degree of satisfaction $(\lambda^{(2)})$ for Phase 2.
Step 14: If a satisfactory result cannot be obtained, perform a tradeoff between $\lambda_{k,s}^{(2)}$ satisfaction levels using a trial-and-error method. First make a concession from the pessimistic and/or optimistic auxiliary objective functions and then try to increase $\lambda_{k,s}^{(2)}$ for the midrange objective functions (by assigning them more weight than the auxiliary objective functions).

5.4. Results and findings

After implementing the steps described above, in Step 12, the Phase 1 degree of satisfaction was calculated as $(\lambda^1) = 0.65$. In this solution, the values of the objective functions are: $z_1 = 9440$, $z_2 = 34{,}195$, $z_3 = 8664$, and $z_4 = 631$. However, this solution, which is found by the Zimmermann max–min method, is always inefficient and can be dominated by the Phase 2 approach and the trial-and-error method. Moreover, this level of satisfaction may not be enough to satisfy decision-makers (DM) in terms of the net cost objective function. It is realistic in some cases that a poor performance on one criterion can provide better results on other criteria.

In Phase 2, four degrees of satisfaction for each objective functions can be easily observed, $(\lambda_1^{(2)}, \lambda_2^{(2)}, \lambda_3^{(2)}, \lambda_4^{(2)})$. In accordance with DM preferences, each objective function has equal weight. To start trading off among these degrees of satisfaction to achieve better results, the $\lambda_2^{(2)}$, $\lambda_3^{(2)}$, $\lambda_4^{(2)}$ satisfaction levels were increased to 0.7, 0.8, and 0.9 respectively by accepting poor performance on $\lambda_1^{(2)}$. The results are given in Table 12.

Table 12
Trading off between satisfaction degrees of auxiliary objective functions with trial-and-error method in Phase 2.

Max $0.25\,\lambda_1^{(2)} + 0.25\lambda_2^{(2)} + 0.25\lambda_3^{(2)} + 0.25\lambda_4^{(2)}$ (all objective functions have equal weights)	$\lambda_1^{(2)}$	$\lambda_2^{(2)}$	$\lambda_3^{(2)}$	$\lambda_4^{(2)}$	$z_2 - z_1$	z_2	$z_2 + z_3$	z_4
Solution 1 $\lambda_2^{(2)}, \lambda_3^{(2)}, \lambda_4^{(2)} \geqslant$ **0.7**	**0.595** **Preferable**	0.7 Good	0.7338 Good	0.70 Good	**24,378**	**33,555**	**41,890**	**647**
Solution 2 $\lambda_2^{(2)}, \lambda_3^{(2)}, \lambda_4^{(2)} \geqslant$ **0.8**	**0.333** **Poor**	0.80 Very good	0.80 Very good	0.801 Very good	**24,131**	**32,144**	**40,164**	**681**
Solution 3 $\lambda_1^{(2)} \geqslant 0.01$ $\lambda_2^{(2)}, \lambda_3^{(2)}, \lambda_4^{(2)} \geqslant$ **0.9**	No feasible solution found							
Solution 4 $\lambda_1^{(2)} \geqslant 0.01$ $\lambda_2^{(2)}, \lambda_3^{(2)}, \lambda_4^{(2)} \geqslant 0.833$ max	**0.219** **Poor**	0.833 Very good	0.833 Very good	0.833 Very good	**24,167**	**31,678**	**39,542**	**693**
Solution 5 $\lambda_1^{(2)} \geqslant 0.01$ $\lambda_3^{(2)} \geqslant 0.75$ $\lambda_2^{(2)}, \lambda_4^{(2)} \geqslant 0.85$	0.07 Very poor	0.85 Very good	**0.75** **Good**	0.99 Perfect	**24,568**	**31,439**	**39,697**	**758**
Solution 6 $\lambda_1^{(2)}, \lambda_3^{(2)} \geqslant 0.01$ $\lambda_2^{(2)}, \lambda_4^{(2)} \geqslant$ **0.09**	No feasible solution found							
Solution 7 $\lambda_1^{(2)}, \lambda_3^{(2)} \geqslant 0.01$, $\lambda_2^{(2)} = 0.893$, $\lambda_4^{(2)} = 0.90$	0.16 Very poor	**0.893** **Perfect**	**0.573** **Preferable**	0.90 Perfect	**23,576**	**30,832**	**39,929**	**727**

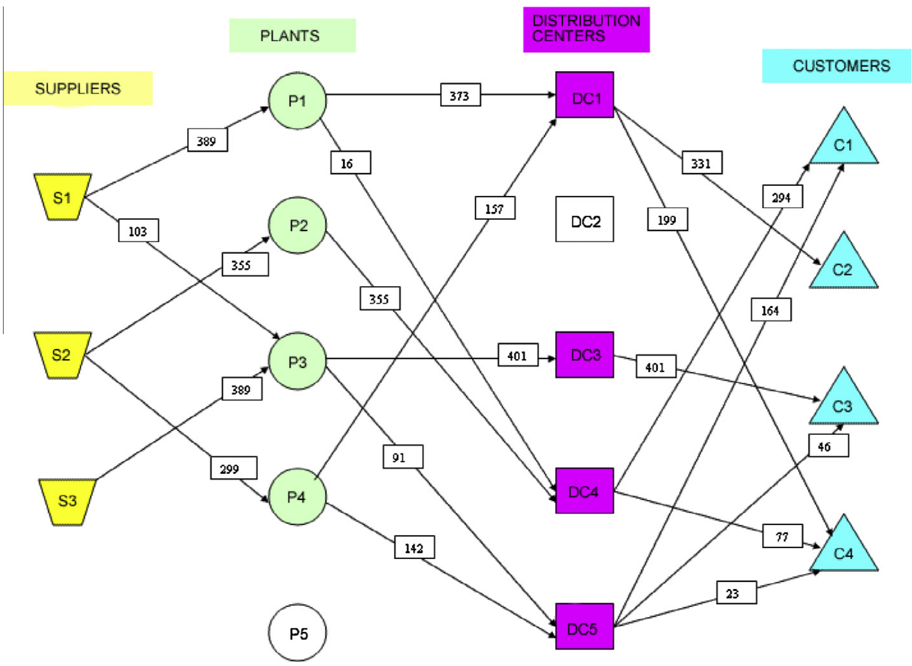

Fig. 1. Solution 5 SC network.

In Solution 1, presented in Table 12, the three degrees of satisfaction are all greater than 0.7; however, $\lambda_1^{(2)}$ can be at maximum 0.595. In Solution 2, to obtain $\lambda_2^{(2)}, \lambda_3^{(2)}, \lambda_4^{(2)} \geqslant 0.8$, the DM must accept $\lambda_1^{(2)} = 0.33$. In Solution 3, no feasible solution was found for $\lambda_2^{(2)}, \lambda_3^{(2)}, \lambda_4^{(2)} \geqslant 0.9$ even when the satisfaction level of $\lambda_1^{(2)} \cong 0$. In Solution 4, the maximum possible satisfaction

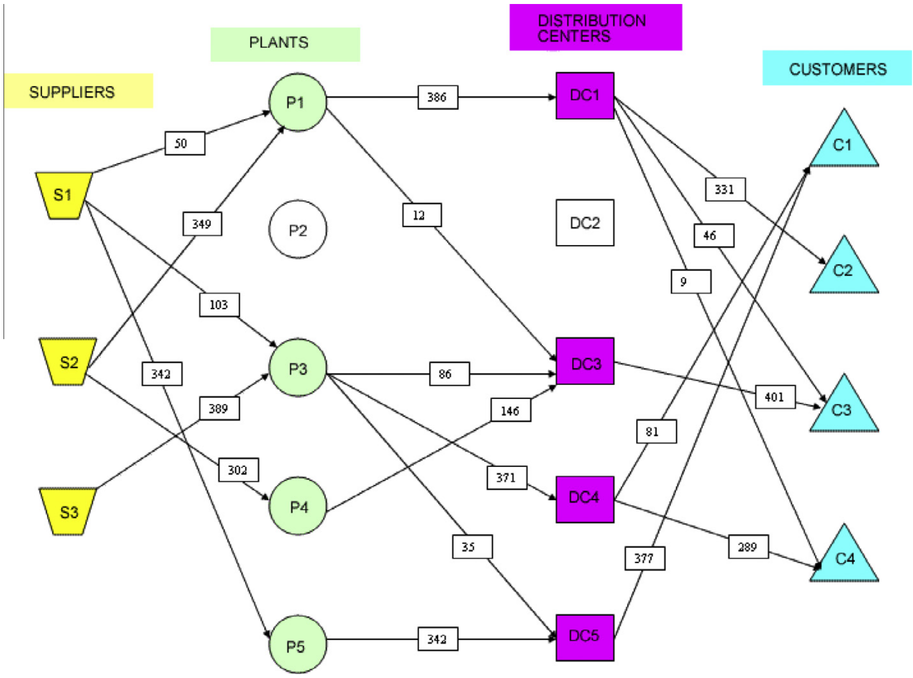

Fig. 2. Solution 7 SC network.

Table 13
Comparison of PLP and crisp LP.

Item	LP1	LP2	LP3	LP4	PLP (PIS, NIS)
Objective function	Min z_1	Max z_2	Min z_3	Max z_4	
z_1	29,323	–	–	–	(29,323, 43,414)
z_2	–	10,974	–	–	(10,974, 6535)
z_3	–	–	7069	–	(7069, 11,825)
z_4	–	–	–	763	(763, 381)

level with a feasible solution was calculated. The result was $\lambda_2^{(2)}, \lambda_3^{(2)}, \lambda_4^{(2)} \geqslant 0.83$. In the next step, $\lambda_3^{(2)}$ was also compensated, because it is also an auxiliary objective function degree of satisfaction value. After finding the value of 0.83 in Solution 4, an attempt was made to increase $\lambda_2^{(2)}, \lambda_4^{(2)} \geqslant 0.85$ in Solution 5; with the other two satisfaction degrees $\lambda_1^{(2)} \cong 0$ and $\lambda_3^{(2)} \geqslant 0.75$, this approach gives a feasible solution. In Solution 6, the final experiment was to see whether $\lambda_2^{(2)}, \lambda_4^{(2)}$ could be made greater than 0.90, and this was found to be possible with $\lambda_1^{(2)}, \lambda_3^{(2)} \geqslant 0.01$, but no feasible solution was found. The final solution (Solution 7) examined the maximum possible levels for $\lambda_2^{(2)}, \lambda_4^{(2)}$, and the results were $\lambda_2^{(2)} = 0.89, \lambda_4^{(2)} = 0.9$ with the other satisfaction-degree constraints $\lambda_1^{(2)}, \lambda_3^{(2)} \geqslant 0.01$.

From Table 12, it is obvious that Solutions 5 and 7 provide the best results. Solution 5 satisfies the maximum qualitative factors benefit objective function near its maximum level: 0.99 with 31,439 unit cost. However, many different solutions can be reached at different degrees of satisfaction, corresponding to DM preferences. The DM can choose different results according to his needs in different time periods. For example, the DM can think that spending less money will be better than satisfying $\lambda_4^{(2)}$ at a 0.99 level. In such a situation, Solution 7 is better than Solution 5. Because the level of satisfaction for $\lambda_4^{(2)}$ was 0.90 with a higher value of $\lambda_2^{(2)}$ which is 0.89 (≅0.90), two important objective functions have been perfectly satisfied. Figs. 1 and 2 show the results for Solutions 5 and 7 for the SC network. Routes, quantity of units shipped over the routes, and facilities opened or closed can be seen in these figures.

The proposed PLP approach outputs more wide-ranging decision information than other models. Notably, the optimal goal values using the proposed approach should be imprecise because the related transportation costs and capacity are always imprecise in nature. In practice, dealing with crisp LP models could be much easier because of the advantage of acquiring a solution interval. The model proposed here can generate better decisions and also warns decision-makers and managers about the possibility of pessimistic and optimistic solutions. The crisp LP and PLP approach solutions are summarized in Table 13.

6. Conclusions

The multi-facility location problem is one of the most important strategic decision problems which can affect the future of companies. This importance is increased even more when the supply chain is considered globally. Multi-facility location problems are multi-criteria decision-making problems which contain both imprecise quantitative and qualitative factors.

Estimating the customer requirements and expert opinions for facility location problems is not easy due to the scarcity and volatility of data. To cope with ambiguity and vaguness problems, fuzzy set theory has been used in this research. In this paper, interactive integrated ''two-phase PLP'' and ''fuzzy AHP'' approaches were used for solving the multi-objective multi-facility location problem. Combining two approaches can effectively handle the imprecision of input data. The auxiliary multiple-objective linear programming model attempts to minimize total SC transportation and facilities costs and maximize the qualitative-factor benefits. The proposed model tries to minimize as much as possible the imprecise total cost, to maximize the possibility of obtaining lower total cost, to minimize the risk of obtaining higher total cost, and to maximize qualitative-factor benefits.

It must be noted that this study also used a two-phase approach to MOPLP. The two-phase approach provides some advantages to DMs. First, the degree of satisfaction can be improved with the use of MOPLP. Moreover, various types of interactive solutions achieved by use of this approach could help decision-makers to formulate decisions under variable conditions.

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