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Short communication

A note on "A new method for solving fully fuzzy linear programming problems"

H. Saberi Najafi [a], S.A. Edalatpanah [a,b,*]

[a] Department of Applied Mathematics, Lahijan Branch, Islamic Azad University, Lahijan, Iran
[b] Young Researchers Club, Lahijan Branch, Islamic Azad University, Lahijan, Iran

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ABSTRACT

This note shows that solving fully fuzzy linear programming (FFLP) model presented by Kumar et al. [A. Kumar, J. Kaur, P. Singh, A new method for solving fully fuzzy linear programming problems, Appl. Math. Model. 35 (2011) 817–823] needs some corrections to make the model well in general. A new version is provided in this note. A simple example is also presented to demonstrate the new form.

1. Introduction

The fully fuzzy linear programming (FFLP) in which all the parameters as well as the variables are represented by fuzzy numbers is an attractive topic for researchers, (see [1–3] and references therein). Dehghan et al. [1] proposed some practical methods to solve fully fuzzy linear system (FFLS) that are comparable to the well known methods and extended a new method employing Linear Programming (LP) for solving square and non-square fuzzy systems. Lotfi et al. [2] applied the concept of the symmetric triangular fuzzy number and obtained a new method for solving FFLP by convert a FFLP into two corresponding LPs. Kumar et al. [3], pointed out the shortcomings of the above methods and to overcome these shortcomings, proposed a new method for finding the fuzzy optimal solution of FFLP problems with equality constraints.

In this paper we study Kumar et al. model [3] and present a revised version for it. This paper is organized as follows. In Section 2, we present some notation, definitions and preliminary arithmetic. In Section 3, we point out the problem of Kumar et al. model and then we provide the correct version for it.

2. Preliminaries

We begin with some basic notation and preliminary results which we refer to later. For details, we refer to [1–3].

Definition 2.1. Let X denote a universal set. Then a fuzzy subset $\tilde{A}$ of X is defined by its membership function $\mu_{\tilde{A}} : X \to [0,1]$; which assignsm a real number $\mu_{\tilde{A}}(x)$ in the interval [0,1], to each element $x \in X$, where the value of $\mu_{\tilde{A}}(x)$ at x shows the grade of membership of x in $\tilde{A}$.

* Corresponding author. Tel./fax: +98 1925672445.
E-mail addresses: hnajafi@guilan.ac.ir (H. Saberi Najafi), saedalatpanah@gmail.com (S.A. Edalatpanah).

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A fuzzy subset $\tilde{A}$ can be characterized as a set of ordered pairs of element x and grade $\mu_{\tilde{A}}(x)$ and is often written $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)); x \in X\}$. The class of fuzzy sets on X is denoted with $\Gamma(X)$.

Definition 2.2. A fuzzy number $e\,\tilde{A} = (a, b, c)$ is said to be a triangular fuzzy number if its membership function is given

$$\mu_{\tilde{A}}(x) = \begin{cases} (x-a)/(b-a), & a \leqslant x \leqslant b, \\ (x-c)/(b-c), & b \leqslant x \leqslant c, \\ 0, & \text{else}. \end{cases}$$

Definition 2.3. A triangular fuzzy number $\tilde{A} = (a, b, c)$ is said to be nonnegative fuzzy number if and only if $a \geqslant 0$.

Definition 2.4. Let $\tilde{A} = (a, b, c)$, $\tilde{B} = (d, e, f)$ be two triangular fuzzy numbers then;

(i) $\tilde{A} + \tilde{B} = (a, b, c) + (d, e, f) = (a + d, b + e, c + f)$,
(ii) $\tilde{A} - \tilde{B} = (a, b, c) - (d, e, f) = (a - f, b - e, c - d)$,
(iii) if $\tilde{B} = (d, e, f)$ be a nonnegative triangular fuzzy number then;

$$\tilde{A} \times \tilde{B} = \begin{cases} (ad, be, cf), & a \geqslant 0, \\ (af, be, cf), & a < 0, c \geqslant 0, \\ (af, by, cd), & c < 0. \end{cases}$$

Furthermore, the ranking function for triangular fuzzy number $\tilde{A} = (a, b, c)$ is;

$$R(\tilde{A}) = 0.25(a + 2b + c).$$

3. Main results

Consider the following fully fuzzy linear programming(FFLP) with m constraints and n variables;

$$\begin{aligned} &Min \quad \tilde{z} = \tilde{C} \times \tilde{\mathbf{x}} \\ &s.t: \quad \tilde{A} \times \tilde{\mathbf{x}} = \tilde{\mathbf{b}}, \\ &\qquad \tilde{\mathbf{x}} \textit{ is nonnegative fuzzy number}. \end{aligned} \tag{3.1}$$

where $\tilde{C}^T = [\tilde{c}_j]_{1\times n}$, $rank(\tilde{A}, \tilde{b}) = rank(\tilde{A}) = m$ and all the parameters $\tilde{c}_j, \tilde{x}_j, \tilde{a}_{i,j}$ and $\tilde{b}_i$ are represented by triangular fuzzy numbers (p_j, q_j, r_j), (x_j, y_j, z_j), $(a_{i,j}, b_{i,j}, c_{i,j})$, and (b_i, g_i, h_i), respectively. To solve it, Kumar et al. [3] proposed the following Method;

Let, $(m_{i,j}, n_{i,j}, o_{i,j}) = (a_{i,j}, b_{i,j}, c_{i,j}) \times (x_j, y_j, z_j)$. First, by using preliminaries section, convert the fuzzy linear programming problem, into the following CLP problem;

$$\begin{aligned} &Min \quad R(\tilde{z}) = R\left(\sum_{j=1}^{n} (p_j, q_j, r_j) \times (x_j, y_j, z_j)\right) \\ &s.t: \quad \sum_{j=1}^{n} m_{i,j} = b_i, \quad \forall i = 1, 2, \ldots, m, \\ &\qquad \sum_{j=1}^{n} n_{i,j} = g_i, \quad \forall i = 1, 2, \ldots, m, \\ &\qquad \sum_{j=1}^{n} o_{i,j} = h_i, \quad \forall i = 1, 2, \ldots, m, \\ &\qquad y_j - x_j \geqslant 0, \quad z_j - y_j \geqslant 0. \end{aligned} \tag{3.2}$$

Then, find the fuzzy optimal solution by putting the optimal values of x_j, y_j, z_j in $\tilde{x}_j = (x_j, y_j, z_j)$ and finally, find the fuzzy optimal value by putting $\tilde{x}_j$ in $\sum_{j=1}^{n} \tilde{c}_j \times \tilde{\mathbf{x}}_j$.

This method is an interesting algorithm for FFLS. However, Because in (3.2), the conditions $y_j - x_j \geqslant 0$ and $z_j - y_j \geqslant 0$ don't guarantee that $\tilde{x}$ is nonnegative fuzzy number, we cannot obtain a nonnegative optimal solution $\tilde{x}$. The following example is given to show that our demonstration.

Example. Consider the following FFLP

$$\begin{aligned} &Min \quad \tilde{z} = (1, 3, 9) \times \tilde{\mathbf{x}}_1 + (1, 2, 8) \times \tilde{\mathbf{x}}_2 \\ &s.t: \quad (1, 3, 5) \times \tilde{\mathbf{x}}_1 + (2, 3, 4) \times \tilde{\mathbf{x}}_2 = (1, 9, 22), \\ &\qquad (1, 2, 3) \times \tilde{\mathbf{x}}_1 + (2, 3, 4) \times \tilde{\mathbf{x}}_2 = (1, 8, 18), \\ &\qquad \tilde{\mathbf{x}}_j \textit{ is nonnegative fuzzy number}. \end{aligned}$$

By the Kumar et al. [3] method we have,

$$\begin{aligned} &Min \quad R(\tilde{z}) = \tfrac{1}{4}(x_1 + x_2 + 6y_1 + 4y_2 + 9z_1 + 8z_2) \\ &s.t: \quad x_1 + 2x_2 = 1;\ 3y_1 + 3y_2 = 9;\ 2y_1 + 3y_2 = 8; \\ &\quad 5z_1 + 4z_2 = 22;\ 3z_1 + 4z_2 = 18; \\ &\quad y_1 - x_1 \geqslant 0,\ z_1 - y_1 \geqslant 0,\ y_2 - x_2 \geqslant 0,\ z_2 - y_2 \geqslant 0. \end{aligned}$$

The optimal solution of the above CLP is $x_1 = -1$, $y_1 = 1$, $z_1 = 2$, $x_2 = 1$, $y_2 = 2$, $z_2 = 3$ and $R(\tilde{z}) = 14$. Using (3.2), the fuzzy optimal solution is given by $\tilde{x}_1 = (-1, 1, 2), \tilde{x}_2 = (1, 2, 3)$. However, $\tilde{x}_1$ is not nonnegative fuzzy number. Therefore, to correct solution we need add following conditions to (3.2);

$$\text{For} \quad \tilde{x}_j = (x_j, y_j, z_j); \quad x_j \geqslant 0, \quad \forall j = 1, 2, \ldots, n. \tag{3.3}$$

Then, for above example we solve the following CLP;

$$\begin{aligned} &Min \quad R(\tilde{z}) = \tfrac{1}{4}(x_1 + x_2 + 6y_1 + 4y_2 + 9z_1 + 8z_2) \\ &s.t: \quad x_1 + 2x_2 = 1;\ 3y_1 + 3y_2 = 9;\ 2y_1 + 3y_2 = 8; \\ &\quad 5z_1 + 4z_2 = 22;\ 3z_1 + 4z_2 = 18; \\ &\quad y_1 - x_1 \geqslant 0,\ z_1 - y_1 \geqslant 0,\ y_2 - x_2 \geqslant 0,\ z_2 - y_2 \geqslant 0. \\ &\quad x_1 \geqslant 0, x_2 \geqslant 0. \end{aligned} \tag{3.4}$$

The optimal solution of the above CLP is $x_1 = 0$, $y_1 = 1$, $z_1 = 2$, $x_2 = 0.5$, $y_2 = 2$, $z_2 = 3$ and $R(\tilde{z}) = 14.125$. Using (3.2), the fuzzy optimal solution is given by $\tilde{x}_1 = (0, 1, 2), \tilde{x}_2 = (0.5, 2, 3)$. Hence, the fuzzy optimal value of the given FFLP problem is $(1, 7, 42)$.

4. Conclusions

In this paper we studied Kumar et al. model [3], for solving fully fuzzy linear programming problems. We have shown that this model is not correct, generally. Furthermore, a new version is provided in this note.

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