Bioeconomics of even- vs. uneven-aged forestry: the case of Norway spruce

Olli Tahvonen, Timo Pukkala, Olavi Laiho, Erkki Lähde and Sami Niinimäki

Abstract

An economic optimization model is specified to analyze forest management without any restrictions on the forest management system. The data on forest growth comes from unique field experiments and is used to estimate a nonlinear transition matrix or size structured model for Norway spruce. The model includes nonlinear density-dependent submodels for ingrowth, transition between the size classes, and natural mortality. The economic model includes detailed harvesting cost specifications and the optimization problem is solved in its most general dynamic form. In optimal uneven-aged management, the level of stand density is shown to be a cause of the density effects on natural regeneration. If the goal is volume maximization, even-aged management with artificial regeneration (and thinnings from above) is superior to uneven-aged management. After including regeneration and harvesting costs, the interest rate, and the price differential between saw timber and pulpwood, uneven-aged management becomes superior to even-aged management. In a forest management model with unrestricted thinning and natural regeneration the optimal rotation period increases with increases in the interest rate and cost of artificial regeneration and with decreases in timber price. When these parameter changes are high enough, the even-aged management solution always transforms into uneven-aged management.

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1 Introduction

Forest economic analysis rests strongly on the classical optimal rotation framework (Faustmann 1849). This approach has proven to be economically valid and open to numerous extensions. However, economists seldom ask whether the underlying forest management specification, i.e. producing trees in even-aged cohorts, is economically superior to all possible alternatives[1]. This study estimates a size-structured stand growth model and applies the model in analyzing optimal timber production without any preconditions on the forest management system. The model yields new understanding on the economic profitability of forest management alternatives and several surprising results for Norway spruce, a tree species that in Finland and Sweden has been managed—almost without exception—as even-aged stands.

The seminal paper on the economics of uneven-aged forestry is by Adams and Ek (1974). They utilize a transition matrix model and compute a maximum sustainable yield type of steady state and a transitional solution that is specified to reach the equilibrium within a given time period. As surveyed by Getz and Haight (1989) and Pukkala et al. (2009b), similar simplifications for solving such a complex numerical optimization problem have been utilized in several later studies. In Haight (1987) and Getz and Haight (1989), the uneven-aged model is computed as a general infinite horizon problem. In Haight and Monserud (1990), the procedure is to take some initial state and then compute the unrestricted optimal harvesting solution with the option to switch (any time) to even-aged plantation forestry. They found that in the case of mixed-conifer stands[2] in the Northern Rocky Mountains, both management systems are capable of producing approximately equal timber volume. However, when the objective is to maximize present value stumpage revenues net of planting cost, the uneven-aged solution without clearcuts becomes superior.

During the last 60 years or so conifer forests in Finland and Sweden have been managed under a planting/thinning-from-below/clearcut management system (Siiskonen 2007). This can be criticized by referring to several economic studies, which almost without exception, have shown that it is optimal to thin coniferous stands from above (Haight and Monserud 1990; Valsta 1992; Pukkala and Miina 1998; Hyytiäinen et al. 2005, 2004). Although this result is a rather serious anomaly to the orthodox thinning-from-below/clearcut management system, Nordic economic studies open to a thinning-from-above/no clearcuts alternative have been almost nonexistent.

A set of Nordic studies has performed field experiments where a number of similar sites have been managed as even- or uneven-aged systems over several decades. A representative study of this type is Andreassen and Øyen (2002). It compares clearcutting, tree selection, and group selection in six initially mature Norway spruce stands. Selection cutting is specified to remove 30-40% of stand volume immediately and about 30% in 20-year cutting cycles. The economic outcome from this strategy is compared with even-aged management specified as an immediate clearcut and the value of the bare land. Clearcutting is found to yield slightly higher net present value compared to uneven-aged management but all three forest management methods were found to be economically reasonable. Lundqvist et al. (2007) performed somewhat similar field experiments

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1   Even-aged management is typically not even mentioned among the background assumptions of the Faustmann framework. This list includes assumptions like perfect capital markets, constant timber price and regeneration cost, no environmental preferences, no uncertainty in forest growth or in economic parameters, competitive markets and no connections between different forest stands.

2   The stands include western white pine, grand fir, western hemlock, western red cedar and Douglas-fir.

for Norway spruce and found volume increments to be considerably higher in plots thinned from above than from plots thinned from below. The overall level of natural regeneration was found to be sufficient for replacing the trees harvested.

Tahvonen (2007, 2009) applies a nonlinear transition matrix model for Norway spruce taken from pioneering studies by Kolström (1993) and Pukkala and Kolström (1988). His result is that an unrestricted optimal infinite horizon solution represents uneven-aged management with a cyclical steady state. Comparing this solution with an optimal even-aged solution yields the result that the uneven-aged solution appears to be economically superior. The element with the highest uncertainty in this comparison was the density free regeneration specification in Kolstöm (1993). However, it is possible to decrease the regeneration level below one-tenth of the original (i.e. to 16 seedlings per year) without changing the superiority of uneven-aged management over even-aged management. With respect to Andreassen and Øyen (2002), the major difference was that the even-aged solution was compared with the economically optimal uneven-aged solution and not with a harvesting strategy that is *ad hoc*.

Pukkala et al. (2009a) used a large data set and estimated individual tree diameter increment, height, survival, and ingrowth models for major Finnish tree species. They did not perform economic analysis but found that sustainable harvesting may yield $5-7m^3ha^{-1}a^{-1}$—output that compares well with the output under even-aged management. A somewhat similar study for Norway has been carried out by Bollandsås et al. (2008) who estimated a transition matrix model for mixed species stands in Norway but did not apply their model in analyzing forest harvesting alternatives.

Pukkala et al. (2009b) used their estimated model in Pukkala et al. (2009a) and computed the investment-efficient steady states following the ideas in Chang (1981). Investment-efficient steady states were compared with the maximized forest values under even-aged management as given in Pukkala (2005). Excluding some more fertile and rare southern Finland site types and low rates of interest (1%), uneven-aged forestry was found superior to even-aged forestry both for Scots pine and Norway spruce.

This study performs comparisons between forest management alternatives similar to those in Getz and Haight (1990). However, instead of using only bare land values from the estimated model at hand, this study uses bare land values from three earlier even-aged studies. A priori this may be crucial because there is no guarantee that the same stand growth model is capable of describing the even-aged solutions accurately enough. In addition, this study presents a couple of new and intuitive ways to compare the economic superiority of the forest management alternatives. Furthermore, it will be shown that in a generalized even-aged model with unrestricted thinnings, an increase in interest rates or replanting costs or a decrease in timber prices lead to longer and finally infinitely long rotation, i.e. to uneven-aged forestry.

Compared with Pukkala et al. (2009b) this study analyzes uneven-aged management as a dynamic problem. This approach yields an optimal steady state as a side product but, in addition, enables the comparison of management systems given any initial tree size class distribution. The cost of this generality in optimization is the transition matrix model that can be understood as a simplified version of the single tree model in Pukkala et al. (2009a). However, based on detailed comparisons, Getz and Haight (1989) found that the differences in projections between these models may remain slight and recommended that the transition matrix model be used in dynamic economic optimization (see also Liang 2005). Compared to Tahvonen (2007, 2009) the transition matrix model of this study is more detailed and the ingrowth model includes the appropriate density dependence features. In addition, the economic analysis includes empirically estimated harvesting cost specifications.

The next section introduces the bioeconomic optimization framework. Section three presents the estimated transition matrix model and section four the main results of the study. The first results show that given pure volume maximization, even-aged management (with thinning from above) dominates uneven-aged management if the number of artificially planted seedlings exceeds 1450 per hectare. However, under saw logs volume maximization, the same result requires 1950 seedlings per hectare. The next section adds economic parameters and presents solutions that maximize the present value of timber revenues net of harvesting costs. Excluding a case where the initial stand is ready for immediate clearcut and unfavorable for natural regeneration, thinning the stand from above without clearcuts dominates the even-aged artificial planting/thinnings-from-above/clearcut alternative. The final section of this paper discusses some elements of greatest uncertainty and directions for future studies.

2 The size-structured optimization model

This study applies the size-structured transition matrix (Usher 1966, 1969) model for describing the stand growth. Let x_{st}, $s = 1,...,n$, $t = 0,1,...$ denote the number of trees in size class s at period t. Let $\alpha_s(\mathbf{x}_t)$, $s = 1,...,n$, $t = 0,1,...$ denote the fraction of trees that reaches the next size class for period $t+1$. This fraction is size class specific and depends on the stand density. Denote the fraction of trees that die by $\mu_s(\mathbf{x}_t)$, $s = 1,...,n$, $t = 0,1,...$. The remaining fraction, i.e. $1 - \alpha_s(\mathbf{x}_t) - \mu_s(\mathbf{x}_t)(\leq 1)$ will stay in their present size class. Ingrowth, or the number of trees that enters the smallest size class, is given by $\phi(\mathbf{x}_t)$ and the number of trees cut from each size class by h_{st}, $s = 1,...,n$, $t = 0,1,...$. The stand development can now be specified as the set of the following difference equations:

$$x_{1,t+1} = \phi(\mathbf{x}_t) + [1 - \alpha_1(\mathbf{x}_t) - \mu_1(\mathbf{x}_t)]x_{1t} - h_{1t}, \tag{1}$$

$$x_{s+1,t+1} = \alpha_s(\mathbf{x}_t)x_{st} + [1 - \alpha_{s+1}(\mathbf{x}_t) - \mu_{s+1}(\mathbf{x}_t)]x_{s+1,t} - h_{s+1,t}, \tag{2}$$

$$x_{n,t+1} = \alpha_{n-1}(\mathbf{x}_t)x_{n-1,t} + [1 - \mu_n(\mathbf{x}_t)]x_{nt} - h_{nt}. \tag{3}$$

Note that the state of the forest, i.e. the number of trees, is given at the beginning of each period and the number of trees harvested at the end of each period. According to Equation (3) trees that have reached the largest size class n remain in this size class until they die or are harvested. Let the gross revenues for a harvest be denoted as R_t. The revenue is given as:

$$R_t = \sum_{s=1}^{n} h_{st}(\omega_{s1}p_1 + \omega_{s2}p_2) \tag{4}$$

where ω_{si}, $s = 1,...,n$, $i = 1,2$ denote the saw logs and pulpwood volume of a trunk in m^3 and $p_i, i = 1,2$ denote the corresponding roadside prices (in €). Logging and hauling costs are denoted by C_t and depend on the volume of timber harvested and on the dimensions of harvested trees:

$$C_t = C(\mathbf{h}_t, \mathbf{d}) + C_f, \tag{5}$$

where C_f denotes the fixed cost caused by harvesting and vector $\mathbf{d} = d_1...,d_n$ denotes the tree diameter at breast height in cm. Given that $b=1/(1+r)$ denotes the discount factor and r the interest rate, the problem of maximizing net revenues from a given stand can be given as:

$$\max_{\{h_{st}, s=1,...,n, t=0,1,...\}} V_1(\mathbf{x}_0) = \sum_{t=0}^{\infty} (R_t - C_t) b^t, \tag{6}$$

subject to (1)-(5) and subject to the following nonnegativity constraints and stand initial size distribution:

$$h_{st} \geq 0, \ x_{st} \geq 0, \ s = 1,...,n, \ t = 0,1,..., \tag{7a,b}$$
$$x_{s0}, \ s = 1,...,n \ given.$$

The problem is complicated by the presence of fixed harvesting costs because it may not be optimal to cut the forest every (three-year) time period. A simple, albeit somewhat unsatisfactory, possibility that takes this into account is to allow cutting every $k \geq 1$ periods only. By varying k and observing the value of the objective function it is then possible to find the optimal constant cutting period. Formally this feature can be specified by adding a restriction

$$h_{st} = 0 \ when \ t \neq k, 2k, 3k,..., \tag{8}$$

where the value of k is a positive integer.

3   Model estimation and optimization method

Data from two long-term experiments (Vessari and Honka) of uneven-aged forest management were used to develop models for the probability that a tree moves to the next diameter class (upgrowth), and the probability that the tree dies during the next three-year period. Another model was fitted for ingrowth, expressed as the number of trees that will pass the 5 cm diameter limit during the following three-year period. Three years was selected as the time step because the plots of the experiments had been measured at three-year intervals. As in Pukkala et al. (2009), 5 cm was taken as the lower limit of the smallest diameter class, and 4 cm was used as the class width. The midpoints of diameter classes were therefore 7, 11, 15,…,43 cm.

The two experiments have 92 sample plots in total. The plots have been measured seven times, starting in 1990. This results in six three-year measurement intervals. The plots are located in Central Finland, and they represent *Oxalis-Myrtillus* (OMT) and *Myrtillus* (MT) forest site types, which are typical sites for Norway spruce (see Cajander 1949 for forest site types). The annual temperature sum in the study sites is approximately 1200 $(°Cd)$. For ingrowth estimation, the total number of observations was 459 (plots with two measurements at three-year intervals). The models for the probability of upgrowth were based on 19,811 observations, of which 17758 were trees that remained in the same diameter class and 2058 were trees that moved up one class. For mortality modeling, the observations were the same 19811 trees that survived for the three-year period, plus 971 additional trees that died between the two measurements.

Ingrowth for the 5–9 cm diameter class is given by

$$x_{1,t+1} = e^{2.1368 + 0.104\sqrt{N_t} - 0.107 y_t} - 1, \tag{9}$$

where N_t is the total number of trees and y_t is the total basal area (in m^2), i.e.

$$y_t = \sum_{s=1}^{n} x_{st} \pi (d_s / 2)^2 10^{-4}. \tag{10}$$

Thus, ingrowth increases with the number of trees and decreases with the total basal area. The degree of explained variance (R^2) of the ingrowth model is 0.391. Let variable y_{st}, $s = 1,...,n$, $t = 0,1,...$ denote the sum of basal areas of trees in size classes larger than size class s including half of the basal area of trees in size class s. Thus

$$y_{st} = \frac{x_{st}(d_s / 2)^2 10^{-4}}{2} + \sum_{k=s+1}^{n} x_{st}(d_s / 2)^2 10^{-4}, \ s = 1,...,n. \tag{11}$$

The fraction of trees that move to the next size class during the next three years equals

$$\alpha_s = \left\{ + e^{\left[3.752 - 2.560\sqrt{d_s} + 0.296 d_s + 0.849 \ln(y_t) + 0.035 y_{st} \right]} \right\}^{1}. \tag{12}$$

Thus, this fraction decreases with the total basal area and the basal area of larger diameter trees. In addition, it increases with low diameter levels but decreases when the diameter reaches a level of 18 cm. The percentage of correct predictions of the upgrowth model was 89.6%, and the Nagelkerke R^2 was 0.173.

A fraction of trees that die is given by

$$\mu_s = \left\{ 1 + e^{[3.606 - 0.075 y_{st} + 0.997 \ln(d_s)]} \right\}^{-1}. \tag{13}$$

The percentage of correct predictions was 95.3, and the Nagelkerke R^2 was 0.208. To obtain the volumes of saw logs and pulpwood per trunk, the height of a tree of diameter d_s, $s = 1,...,n$ is estimated from the data as follows

$$h_s = \frac{39691 d_s^2}{1000 d^2 + 25683 d + 37785} \tag{14}$$

Using the computation procedure described in Heinonen (1994), the sawn timber and pulpwood volumes per size class are given in Table 1. It is assumed that the maximum and minimum lengths of saw logs are 5.5 and 4.3 meters and that the minimum diameter for saw logs and pulp logs are 15 cm and 6 cm respectively. The roadside price for saw logs equals 51.7€m⁻³ and pulp logs 25€ m⁻³. For harvesting cost we apply the empirically estimated models by Kuitto et al. (1994) (see Appendix). These detailed models present cutting and hauling costs for thinnings and clearcuts under even-aged management and can be used to form a cost model for uneven-aged management as well. Costs depend on total harvested volume of saw logs and pulpwood and on the average volume of logs. The cost of artificial regeneration (including pre-commercial thinning) equals 1500€ per hectare[3].

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3 T. Saksa, personal communication.

Table 1. Sawn timber and pulpwood volumes m^3 per size classes.

Diameter, cm	7	11	15	19	23	27	31	35	39	43
Sawn timber	0	0	0	0.14136	0.29572	0.45456	0.66913	0.88761	1.12891	1.39180
Pulp wood	0.01189	0.05138	0.12136	0.08262	0.06083	0.06703	0.04773	0.04596	0.04672	0.04119

Applying the estimation results given in (9)-(14), the problem of maximizing (6) subject to restrictions (1)-(5), (7a and b) and (8) constitutes a nonlinear programming problem. Because of nonlinearities, the problem is potentially nonconvex and may yield multiple locally optimal solutions. The numerical analysis utilizes Knitro optimization software (Byrd et al. 1999, 2006; Wächter and Biegler 2006) that applies gradient-based, state-of-the-art, interior point solution methods. About 200 initial guesses are chosen randomly to find the globally optimal solution. The infinite horizon solutions are approximated by applying finite time horizons. These are increased (up to 800 periods) until the solutions represent transitions toward some steady states or stationary cycles that do not vary with further lengthening of the time horizon.

4 Results

4.1 Maximization of cubic meter yield

It is illustrative first to study the model properties in the simplest possible form, i.e. assuming that the aim is the maximization of physical yield. The highest physical yield is reached by applying the shortest possible cutting period, i.e. three years. Based on this, Figure 1 shows the development of an optimal solution in basal area/cuttings space. The solutions approach a steady state with constant yield and basal area over time.[4] These solutions are robust in the sense that varying the properties of the model, for example, increasing the effect of larger trees on the fraction of trees that move to the next size class, does not change the qualitative outcome (cf. equation 12).

As shown in Figure 1 and Table 2, second row (baseline solution), the optimal steady state yield equals 17.5m^3 per three years; the maximized annual yield per hectare is about 6 m^3. The corresponding density measured in basal area before cuttings is about 12.8 m^2 and the basal area after cuttings about 10.9 m^2 (not shown). It should be noted that the basal area includes only trees with diameter at least 5 cm. According to the data, the basal area for trees smaller than 5 cm was typically about 1–2m^2 implying that the total basal area per hectare varies between 12 – 15m^2.

The development of the forest size class structure or the number of trees per size class before cutting is shown in Figure 2. At the steady state it is optimal to cut trees when they reach a diameter class of 31 cm. The long-run size class structure deviates from the classical inverted J-form (Usher 1966) since natural mortality for diameter classes 21-33 is almost zero. The ingrowth per three years is 27 trees and the number of trees harvested is 25, implying that about 7% of trees die before reaching the diameter class of 31 cm.

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4   This is far from self-evident because some studies have shown that the long-run equilibrium may be a
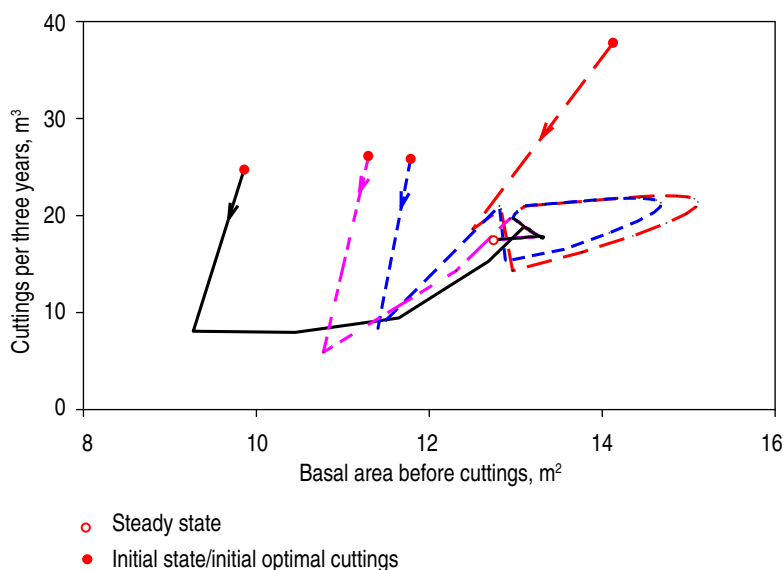   stationary cycle as well (Tahvonen 2007, 2008).

Figure 1. Optimal development of basal area and cuttings toward the MSY steady state.

Table 2. The baseline solution and effects of stand ingrowth variation on density and cuttings

	Ingrowth per three years/ ha	Yield m3 /ha per three years	Basal area before cutting m^2 /ha	Number of trees before cutting/ha	Cuttings per size class in number of trees/ha
Baseline solution	27	17.5	12.76	512	h_7 25
Exogenous ingrowth: 10% increase to the baseline	30	18.58	15.80	661	h_7 25.0 h_8 1
Increased ingrowth: $x_{1,t+1} = 1.1(e^{2.1368+0.104\sqrt{N_t}-0.107y_t}-1),$	41.7	20.15	12.70	665	h_6 39

It would be interesting to understand the explanation for the somewhat low steady state stand density $12-15 \mathrm{m}^3$. It may be a consequence of either the negative effects of stand density (measured by basal area variables) on tree growth and survival or a consequence of low ingrowth. If low density is a consequence of the density effects on transition and mortality, a variation where ingrowth is given exogenously and is increased from its steady state level should shift cuttings to smaller trees and should not cause any major increase in basal area. The third row of Table 2 shows the optimal steady state levels of cuttings and other variables if ingrowth is given exogenously and increased by 10% from the baseline level. This variation increases the basal area and number of trees and switches cuttings toward larger size classes. Thus, low stand density must be a cause of low ingrowth instead of the density effects in transition and mortality. The fourth row in Table 2 shows the outcome of increasing the endogenous ingrowth by 10%, i.e. multiplying the right hand side of equation (9) by a factor of 1.1. This variation causes a slight decrease in steady state basal area and a more than 10% increase in ingrowth and steady state cuttings. In addition, it becomes optimal to cut trees when they reach the diameter of 27 cm. Thus, in this case it is not optimal to let the basal area and stand density increase. Together this shows that in the optimal MSY solution stand density is kept at a low level because of its negative effect on ingrowth (Equation 9).
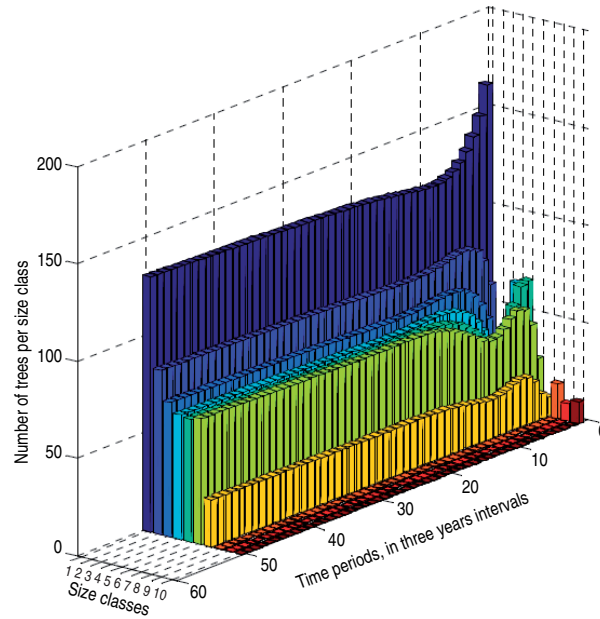
Figure 2. Development of the size class distribution over time. Number of trees before cuttings.

4.2 Comparing forest management systems

Figure 3 compares the maximized (steady state) cubic meter yield from uneven-aged management and natural regeneration with the maximized yield from even-aged management based on artificial regeneration. The latter is computed by choosing the rotation period and harvesting strategy in order to maximize the average annual yield, i.e. by solving the problem,

$$\max_{\{T,h_{st},s=1,\ldots,n,t=0,1,\ldots\}} V_2(\mathbf{x}_0) = \frac{\sum\limits_{t=0}^{T}\sum\limits_{s=1}^{n} h_{st}(\omega_{s1}+\omega_{s2})}{T}$$

subject to restrictions (1)-(3) and (7a and b) where natural regeneration has been eliminated. In even-aged management the maximum yield depends on the number of seedlings. In what follows, the initial number of seedlings is varied between 1300 and 2300 (per ha). After 26 years of clearcutting and 24 years of planting, 70% of trees are assumed to reach the smallest size class and 30% the next size class. For maximizing physical yield under even-aged management, it is optimal to apply thinnings from above. In this case the physical output from even-aged management exceeds the output from uneven-aged management given that the number of seedlings is higher than 1450 (see Figure 3a). Given thinnings from below, i.e. if thinnings are restricted to the four smallest size classes only, even-aged management becomes inferior to uneven-aged management when the number of seedlings remains below 1900/ha.

Another possibility for such a comparison is to take the figures for maximized timber production from existing studies that apply models specified for even-aged management. In Hyytiäinen et al. (2006) the maximized yield for approximately similar forest type varies between 5 and 7.2m^2 when the number of seedlings varies between 1300 and 2300 (per ha). This compares well with the figures obtained applying the model developed here.

Figure 3b shows a similar comparison, if the aim is to maximize saw timber yield. In this case, even-aged management with unrestricted thinnings produces highest yield if the initial number of trees is above 1950/ha. If thinnings are restricted to the four smallest size classes, even-aged management yields lower output given any initial number of trees between 1300-2300 (per ha). Along the uneven-aged steady state about 7% of the trees die. The percentage of dying trees under even-aged management increases with the initial number of trees and varies between 10% and 13%. Uneven-aged steady state cutting of 25 trees produces $17.5m^3$, i.e. yield per harvested tree equals about $0.7m^3$. The yield per harvested tree under even-aged management is 0.45, 0.38 or $0.33\,m^3$ given that the initial number of trees is 1300, 1800 or 2300 (per ha) respectively. Uneven-aged management yields higher yield per harvested tree because trees are harvested from the diameter class of 31 cm while under even-aged management it is optimal to harvest trees mainly from diameter class 27 cm. In addition, at the clearcut trees are harvested from all size classes. Such a harvest is not selective and thus decreases the cubic meter yield compared to the outcome of continuously accurate tree selection in uneven-aged management.
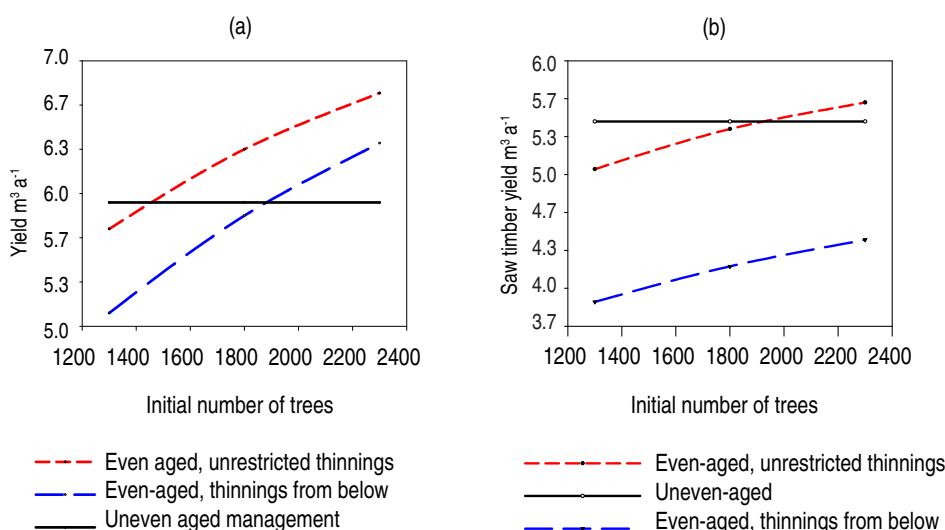

Figure 3a, b. Comparison of maximum yields in even- and uneven-aged management, a) Maximization of the saw and pulp log volume. b) Maximization of saw log volume. Initial state: 24 years after regeneration 70% of trees enter size class 1 and 30% size class 2. Thinnings from below: thinnings allowed to four smallest size classes.

4.3 Optimal solutions under discounting and other economic factors

Figures 4 and 5 show the optimal solutions over time when the model includes harvesting costs, separate timber prices for saw logs and pulpwood and an interest rate equal to zero (Figure 4) or 3% (Figure 5). Given no discounting and the fixed harvesting cost of 300€/ha it is optimal to apply a 15-year harvesting period, while the optimal cutting period with a 3% rate of interest is 12 years. Given 0% interest, the steady state average annual yield is $5.1m^3$ of which $4.8m^3$ is saw logs. Compared to the solutions that maximize volume output, the reduction in volume output is due to the 15-year harvesting interval. Figures 6a and b show the steady state size distribution of trees just before each cutting and optimal tree selection. Given no discounting each harvest removes all trees equal to or larger than 27 cm. When the interest rate equals 3% it is, in addition, optimal to cut about half from size class 24 cm. With the 3% interest rate, the annual output of saw timber and pulpwood are 4.6 and $4.1m^3$.
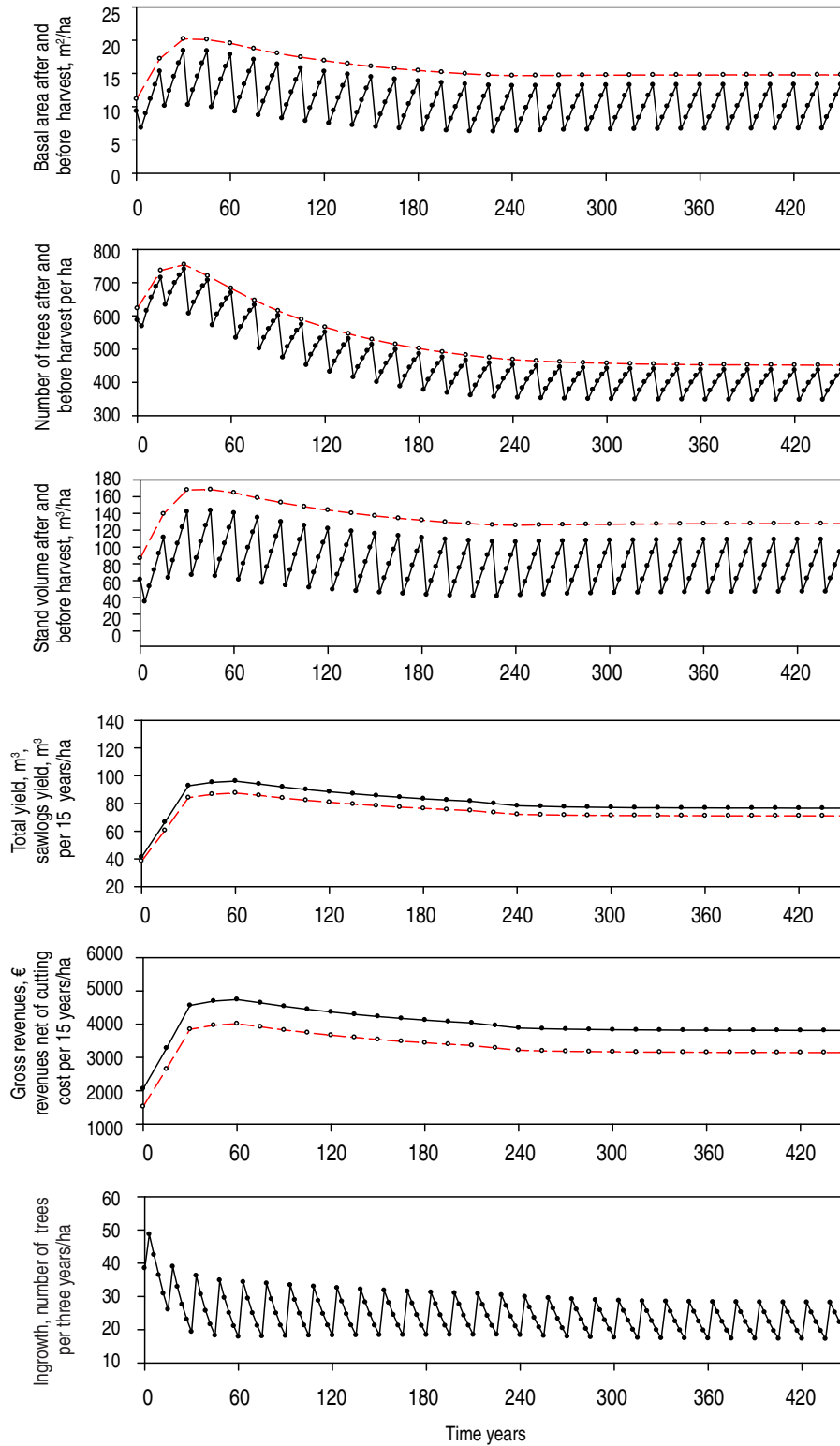
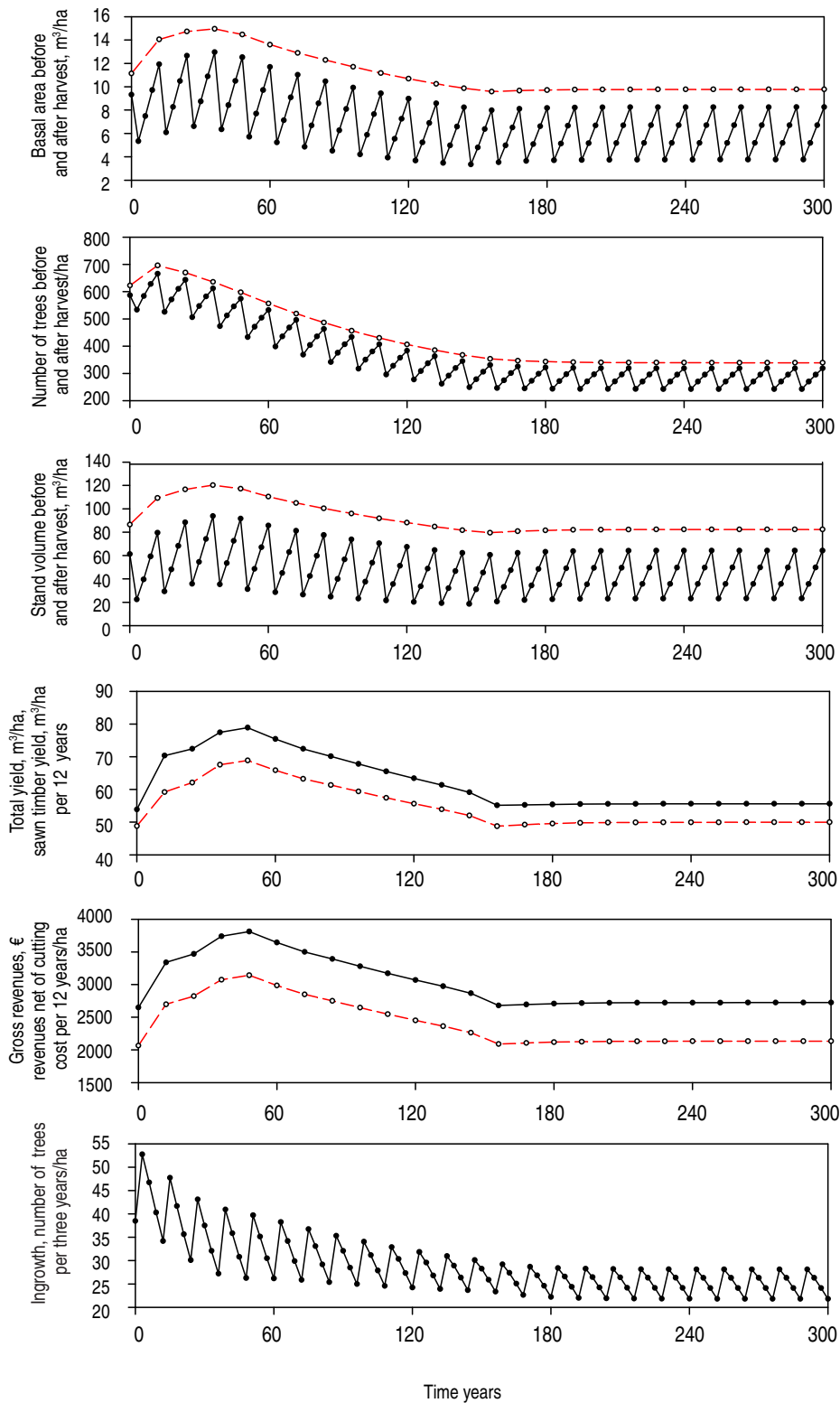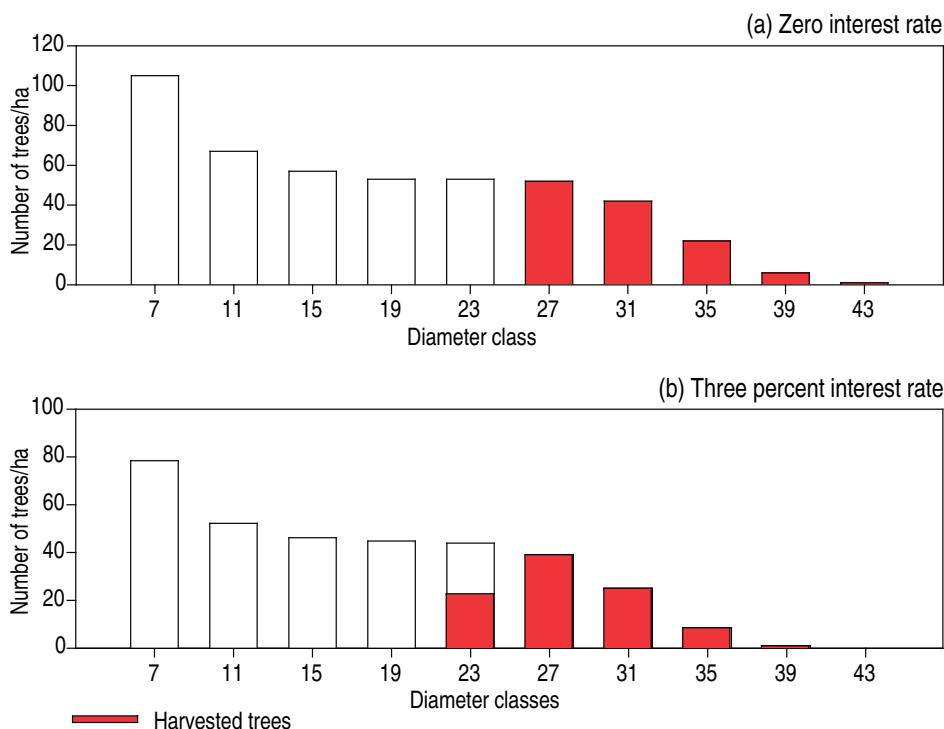**Figure 4.** Optimal uneven-aged solution with no discounting and a 15-year cutting cycle.

Figure 5. Optimal steady state size distribution and selection of harvested trees Interest rate is 3%.

Figures 6a and b. Optimal steady state size distribution and selection of harvested trees. Cutting periods 15 years in Figure 6a and 12 years in Figure 6b.

Given 0% interest, the total steady state revenue per cutting equals 3808€ and the revenue net of cutting cost 3147€. This yields annual revenue per hectare equal to 210€. Cutting and hauling costs per cubic meter become 8.6€. The corresponding annual net income with a 3% interest rate is 178€ and cutting and hauling costs per cubic meter are about 11€. With a 3% interest rate, the economic value of the forest per hectare just before and after cutting are 7143 and 5010€. Because the stand density is lower with a 3% interest rate, the ingrowth level is somewhat higher compared with the optimal solution with 0% interest.

4.4 Comparing the economic superiority of silvicultural systems

The setup of this study enables various comparisons between the economic outcomes of even- and uneven-aged management. A basic set of comparisons is obtained by assuming some initial size class distribution and then computing whether it is optimal to choose even-aged management immediately or to switch to even-aged management after some optimized transition period or to continue with uneven-aged management forever. Denote the value of bare land by V_{BL}. Thus the problem is to subject to conditions (1)-(8). If the optimal T in the solution of problem (15) is zero, it is optimal to apply even-aged management immediately. If the optimal T is above zero but finite, it pays to switch to even-aged management after some transitional period. If the optimal T is infinite, it is better to apply uneven-aged management forever and solving problem (15) yields the same present value for net revenues as solving the uneven-aged problem (1)-(8). This setup can next be applied to computing a breakeven bare land value that implies equality between the even- and uneven-aged solutions. If this breakeven bare land value is higher than the actual bare land value, uneven-aged management is superior to even-aged management and *vice versa*.

$$\max_{\{h_{st},s=1,...,n,t=0,...,T,T\}} V_T(\mathbf{x}_0) = \sum_{t=0}^{T-1} b^t (R_t - C_t) + b^T (R_T - C_T^{cc} + V_{BL}) \tag{15}$$

The remaining question is how to obtain estimates for the value of bare land. One possibility is to compute the maximized bare land value by applying the model and data of this study. For this purpose, the objective function is specified as where w is regeneration cost, $t_i, i=1,...,k$ the dates for thinnings and T_F the Faustmann rotation period. Maximization of (16) is subject to the conditions (1)-(5), (7) and (8) assuming, however, that trees are only regenerated artificially.

$$\max_{\{T_F,h_{st},s=1,...,n,t_i,i=0,1,...,k\}} V_{BL}(\mathbf{x}_0) = \frac{\sum_{i=0}^{k} -w + b^{t_i} (R_{t_i} - C_{t_i}^{thin}) + b^{T_F} (R_{T_F} - C_{T_F}^{cc})}{1 - b^{T_F}}, \tag{16}$$

Instead of applying (16), another possibility is to use bare land values from existing studies. Hyytiäinen et al. (2006) perform detailed computations for optimal even-aged management for the Finnish Forest Extension Service Tapio to be used in developing silvicultural guidelines. The computations for Norway spruce were based on the detained single-tree model and optimization procedure developed in Valsta (1992) and Cao et al. (2006). The maximized bare land values of this study are given in Table 3, column (2) for interest rates between 1-5% (excluding the proportional profit tax of 29%). Pukkala (2005) applies a similar single-tree model and stumpage prices and presents the bare land values given in column (3). Hyytiäinen and Tahvonen (2003) used a simpler extended univariate model and presented bare land values (excluding proportional profit tax) shown in Table 3, column (4). The fifth column in Table 3 shows the bare land values obtained using the model at hand and the solution to (16). It is assumed that 26 years after the clearcut (24 years after the regeneration), size class 1 includes 1750 trees and that the cost of artificially planting one hectare equals 1500€. Separate harvesting costs for thinnings and clearcutting are given in the appendix. Very similar harvesting cost functions are used in Hyytiäinen et al. (2006) and Hyytiäinen and Tahvonen (2003).

Table 3. Bare land values for various interest rates according to different studies.

	Study			
Interest rate	Hyytiäinen et al. 2006[1]	Pukkala 2005[2]	Hyytiäinen and Tahvonen 2003[3]	This study[4]
0.5%	-		-	36979€
1%	16197€	12030€	19202€	15645€
2%	4437€	3393€	5769€	4867€
3%	1396€	1207€	2105€	1721€
4%	116€	514€	652€	324€
5%	-290€	-	-	-300€

1. Single tree model; $p_1 = 51.5€$, $p_2 = 32.6€$, $w = 1000€$, number of seedlings 1800/ha.
2. Single tree model, stumpage prices, $p_1 = 40€$, $p_2 = 25€$, $w = 935€$, number of seedlings 1600/ha.
3. Extended univariate model; $p_1 = 47€$, $p_2 = 32€$, $w = 500 - 1000€$, number of seedlings 1800/ha.
4. $w = 1500$, initial state: $x = 1750, 0,...,0$, (8 periods after the clearcut).

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Figures 7-9 compare these bare land values with the breakeven bare land value defined above. The figures are based on three different initial size class distributions. In Figure 7 the initial state represents a steady state from the optimal uneven-aged solution (in the beginning of the period where cutting occurs) based on a 3% interest rate. The superiority of the even-aged management requires the bare land value to be above the solid black line. As shown only one bare land value estimate fulfills this requirement, i.e. the bare land value for the 1% interest rate from Hyytiäinen and Tahvonen (2003). All other bare land value estimates suggest that uneven-aged management is superior to even-aged management, i.e., the optimal T in (15) is infinite.
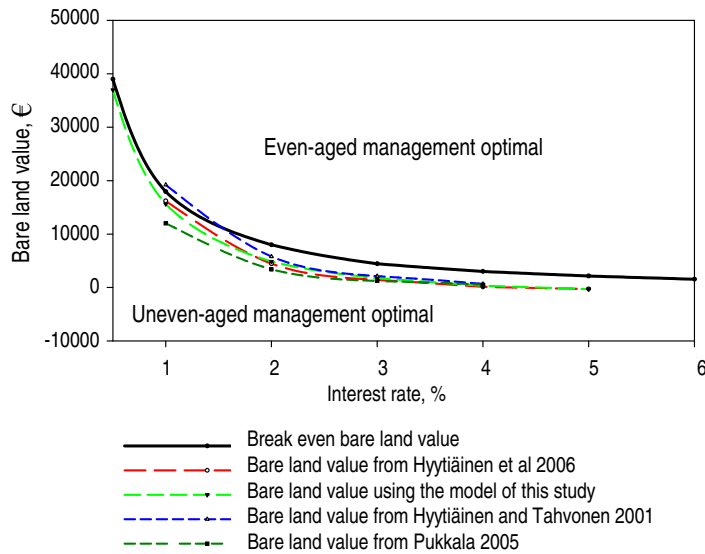

Figure 7. Comparing the economic superiority of forest management forms. If bare land value is above the solid black line even-aged management yields higher present value of net revenues than uneven-aged management. Initial state: $\mathbf{x}_0 = 76, 53, 47, 45, 43, 35, 17, 3, 0, 0$

Figure 8. Comparing the economic superiority of forest management forms. If bare land value is above the solid black line even-aged management yields higher present value of net revenues than uneven-aged management. Initial state: $\mathbf{x}_0 = 450, 600, 450, 0, 0, 0, 0, 0, 0, 0$
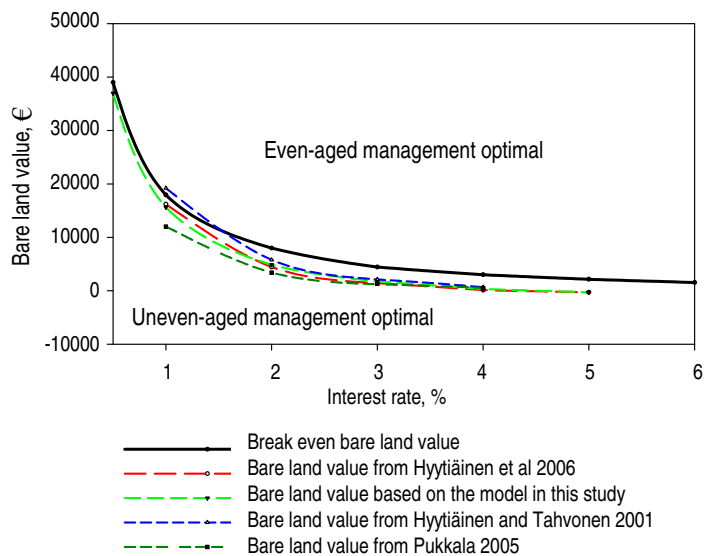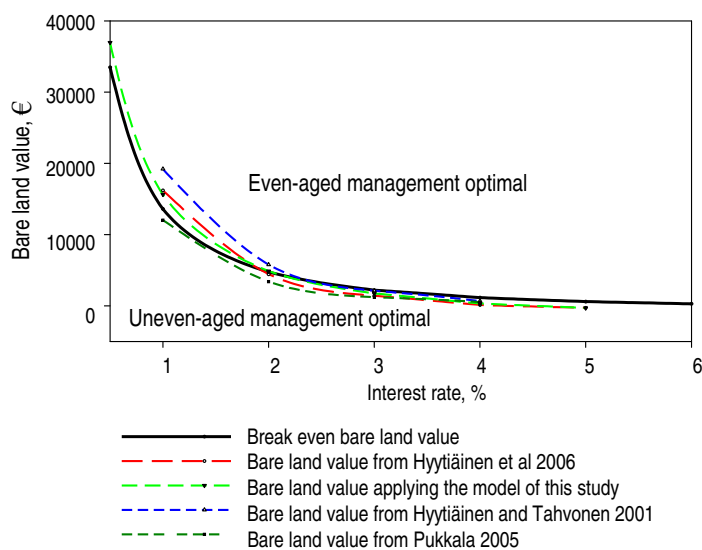
18

Figure 9. Comparing the economic superiority of forest management forms. If bare land value is above the solid black line even-aged management yields higher present value of net revenues than uneven-aged management. Initial state: $\mathbf{x}_0 = 0,0,0,0,20,80,140,135,0,0$

In Figure 8, the initial size class distribution is $\mathbf{x}_0 = 450,600,450,0,...,0$. This state may represent an even-aged stand about 35-45 years after regeneration. In this case, all bare land value estimates fall short of the breakeven curve implying that from this initial state it is optimal to apply uneven-aged management and rely on natural regeneration.

A third case represents an initial size class structure that may exist under even-aged management close to clearcut. In Figure 9 most of the bare land value estimates exceed the breakeven bare land values given that the interest rate is below 2%. The reason why clearcutting and planting yield a higher present value of net revenues follows from the fact that this initial state is unfavorable for ingrowth and natural regeneration. Thus, given low interest rates, the costly regeneration is superior to (free but scanty) natural regeneration. However, the remaining question is whether, after this clearcut, it is optimal to continue even-aged management forever. The answer appears to be negative since about 30 years after the regeneration the stand will reach a state like the one in Figure 8. From this and similar states it is optimal to continue with uneven-aged management.

Given the same initial state as in Figure 9, the optimal transition to uneven-aged management is shown in Figures 10 and 11. Figure 10 shows the development of total cuttings and ingrowth. As can be seen under uneven-aged management it is also optimal to apply rather heavy initial cutting. However, as shown in Figure 11 the initial cutting is not a clearcut and it is optimal to maintain a fraction of large trees in the stand. The ingrowth in Figure 10 is initially very low but it increases as the stand density is decreased by initial cuttings.
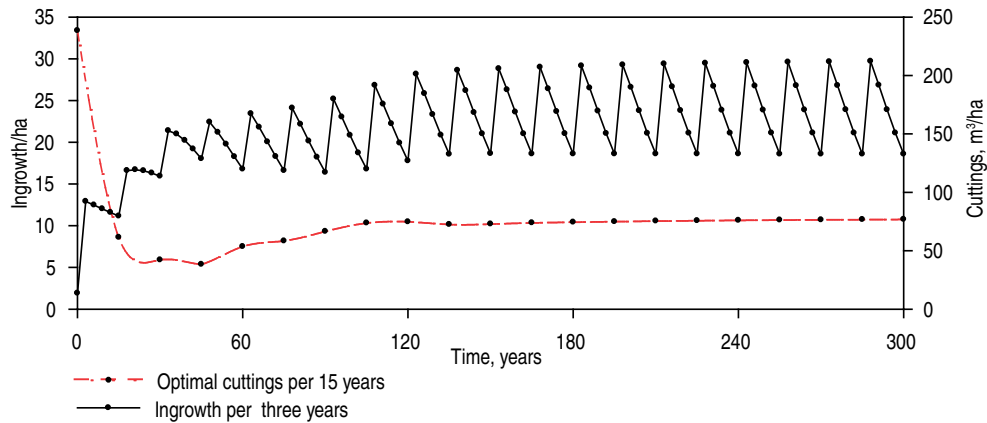
Figure 10. Cuttings and ingrowth along the optimal solution.
Initial state: $\mathbf{x}_0 = 0,0,0,0,20,80,140,135,0,0$; interest rate is 1%
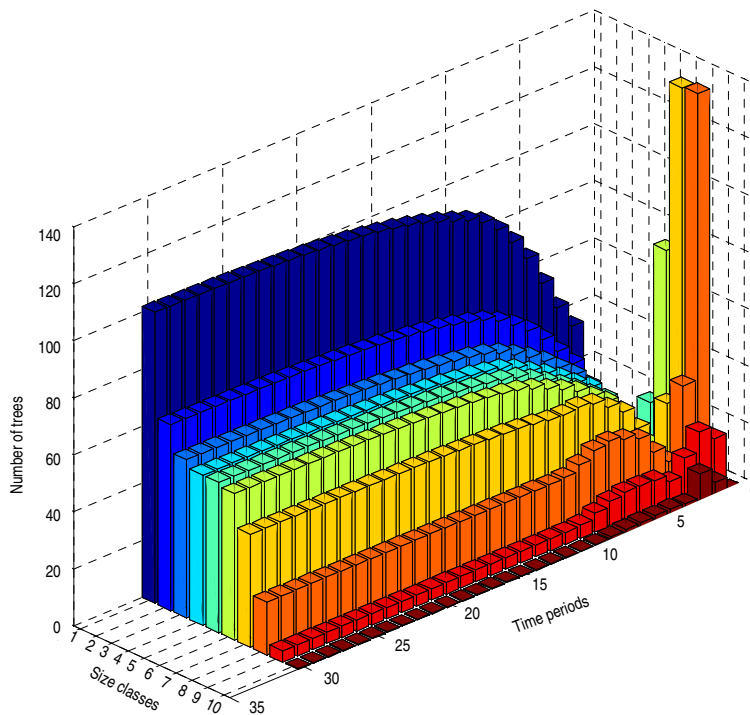
Figure 11. Optimal development of tree size classes.
Initial state: $\mathbf{x}_0 = 0,0,0,0,20,80,140,135,0,0$, r=1%

4.5 Effects of economic parameters on the optimal choice between forest management systems

According to an established result, in optimal rotation models without optimized thinnings the rotation period decreases with the rate of interest. Typically, regeneration cost is constant, implying that with a high enough rate of interest the bare land value becomes negative. This result does not change if the model includes thinnings from below. However, allowing unrestricted thinnings

has somewhat surprising consequences. When the rate of interest is increased and the bare land value becomes negative it becomes non-optimal to cut the last tree because that will only lead to the realization of the negative bare land value. Thus the optimal rotation period becomes infinite. Obviously such an outcome would not occur if regeneration effort and cost could be chosen optimally. However, there are very few (if any) models that could specify empirically estimated connection between the level of regeneration activities and stand growth. In spite of this problem, it is possible to apply the model from this study and analyze the effects of economic parameters on the superiority of the management alternatives.

Figure 12a (the lowest dotted line) shows optimal rotation when the interest rate is increased from 0% to 4% and thinnings are restricted to the six smallest size classes. Optimal rotation decreases with the interest rate confirming the established economic result. The dashed and solid curves show optimal rotation when thinnings are unrestricted and natural regeneration is included in the model. These changes increase the rotation length as expected. If the interest rate is zero, a planting cost equal to 1500€ is not too high and it is optimal to make such an investment instead of relaying only on natural regeneration. However, with the interest rate greater than or equal to 2.5%, the optimal rotation period becomes infinite, i.e. it becomes optimal to apply uneven-aged forestry instead of even-aged forestry. With a lower regeneration cost (800€), the rotation period is shorter and uneven-aged forestry becomes optimal when the interest rate exceeds 3.5%. The rotation period increases with the rate of interest because with higher interest it becomes optimal to apply costly regeneration investment less frequently. Figure 12b shows the effect on increasing the regeneration cost and Figure 12c the effect of timber prices. Increasing the regeneration cost or decreasing the timber price causes a switch from even-aged to uneven-aged management. These outcomes are in sharp contrast with typical even-aged economic models that do not include the trade off between artificial and natural regeneration or the possibility to continue thinnings from above without clearcuts.
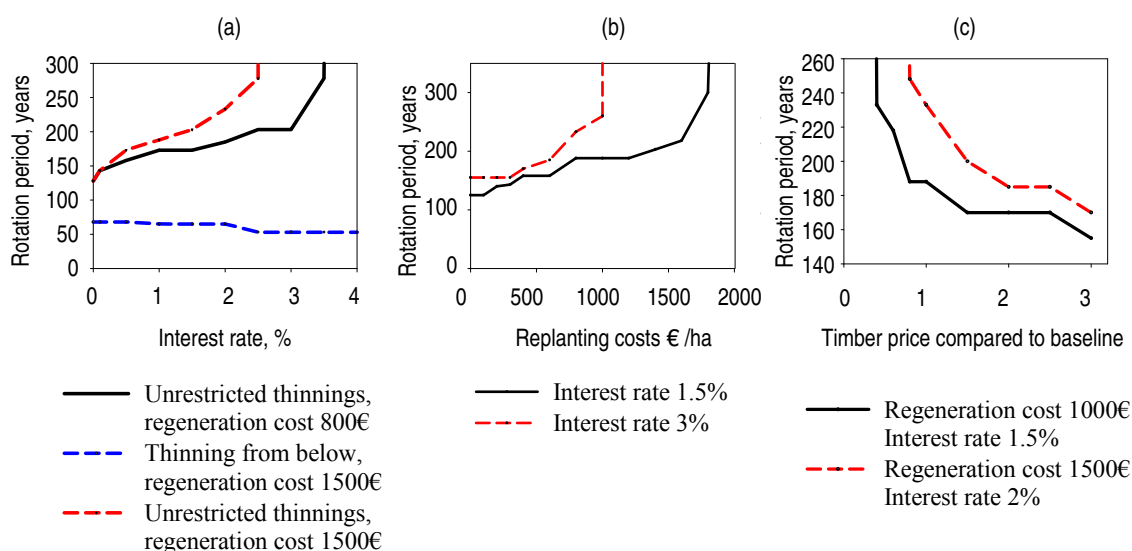

Figure 12a,b,c. The effect of economic parameters on optimal rotation and choice between forest management systems. In Figure 12b: interest rate 1.5%. In Figure 12c: interest rate 1.5% and regeneration cost 1,000€. In all cases $\mathbf{x}_0 = 300, 700, 600, 0, ..., 0$ and the delay between regeneration and initial state is 21 years.

5   Conclusions

Haight and Monserud (1990) found that in contrast to widely-held views, both even-aged management (with thinnings from above) and uneven-aged management are capable of yielding almost identical volume output. This study finds that in the case of Finnish Norway spruce, physical volume is maximized under even-aged management if the number of seedlings is above 1450. This result requires that even-aged management be applied with thinnings from above. However, the picture changes in favor of uneven-aged management if the goal is to maximize saw log volume. Uneven-aged management remains optimal given that the number of seedlings remains less than 1950. The interpretation behind this result is the finding that under uneven-aged management the average volume of harvested trees is almost twice as high as in even-aged management. A similar finding was reported by Andreassen and Øyen (2002).

In Getz and Haight (1990), uneven-aged management becomes clearly superior when the aim is the maximization of present value stumpage prices net of planting cost. They assumed that stumpage prices are independent of the forest management system. This study obtains a similar result but applies roadside prices and detailed harvesting cost specifications that take into account the cost differences between clearcutting and thinning operations. In addition, this study finds that such a result for Norway spruce was independent of whether bare land value was computed using the same underlying model or whether the value was taken from existing studies designed for the specific purposes of even-aged management.

The economic superiority of uneven-aged management was found to be sensitive to the initial state of the stand; an initially mature stand with unfavorable conditions for natural regeneration may be optimal to clearcut once. However, after this it is optimal to manage the stand toward uneven-aged structure and apply thinnings from above without clearcuts. This result is in line with the findings in Andreassen and Øyen (2002) who suggested that the conversion to uneven-aged management should be started earlier than in their experiments.

Given both volume maximization and economically optimal solutions the stand density remains at quite a low level (12-$15m^2$). This was shown to be a consequence of the negative effects of density on ingrowth. Lundqvist et al. (2007) found an average ingrowth to the 5 cm class equal to 21 stems per year per hectare. This is considerably more than the ingrowth in the optimization runs of this study, where the annual ingrowth to the 7 cm class typically varies between 6 to 10 stems. Even small increases in ingrowth imply considerable additions to yield and net revenues from uneven-aged management.

According to an established result in forestry economics, the optimal rotation period decreases with increases in interest rates. This study shows (numerically) that including thinnings and natural regeneration turns this result upside down: the rotation period increases (without bound) with increases in the interest rate. This follows because it becomes optimal to make the costly regeneration investment less frequently. Similarly, increases in regeneration costs or decreases in timber prices do not only lengthen the rotation period but cause the optimal solution to switch from even-aged management to uneven-aged management. Similar results have not been shown previously in forest economic studies.

This study supports the earlier findings in Tahvonen (2007, 2009) and Pukkala et al. (2009b) that challenge the general belief of unquestionable economic superiority of even-aged management for Norway spruce. Economic analysis makes it apparent that there is a trade off between costless

natural regeneration and costly but perhaps more abundant artificial regeneration. Another difference between the management alternatives follows from the fact that continuous thinning from above leads to more accurate tree selection compared to the rather crude clearcutting operation. The relative superiority of these systems is an outcome of a complex optimization problem that includes a great number of biological and economic details. Sharpening the understanding of these questions calls for inclusion of logging damage, a more reliable and detailed ingrowth model, non-timber values, and nonconstant cutting intervals into the models, to mention just some examples.

Appendix

Following Kuitto et al. (1994) and assuming logging costs per hour equal to 82.5€ and hauling cost per hour equal to 59.5€, the costs for thinning and clearcutting in the case of even-aged management can be given as

$$
\begin{aligned}
C_t^{th} = {} & 21.906306 + 3.3457762 H_t^{sawvol} + 25.5831144 + 3.77754938 H_t^{pulpvol} + \\
& 0.1374 e^{-0.005469 N_t} + 0.2104 e^{-0.001147 N_t} + \\
& \sum_{s=1}^{n} \left\{ 0.50001 + 0.59 vol_s - \frac{22.386}{vol_s 1000 + 85.621} \frac{h_{st}}{N_t} \right\} 2.1001366 N_t + 300,
\end{aligned}
$$

$$
\begin{aligned}
C_t^{cc} = {} & 26.350495 + 2.82183045 H_t^{sawvol} + 25.701440 + 3.33144 H_t^{pulpvol} + \\
& 0.1594 e^{-0.003509 N_t} + 0.09911 e^{-0.001162 N_t} + \\
& \sum_{s=1}^{n} \left\{ 0.44472 + 0.94 vol_s - \frac{146.17}{vol_s 1000 + 862.05} \frac{h_{st}}{N_t} \right\} 2.1001366 N_t + 300,
\end{aligned}
$$

where H_t^{sawvol} and $H_t^{pulpvol}$ are the total volumes of sawlogs and pulpwood yields per cut vol_s is the total (commercial) volume of a stem from size class s.

The linear parts in both cost functions denote the hauling costs and the two nonlinear components the logging cost. In the case of uneven-aged management the cost function in (5) is formed by taking the hauling cost components from the thinning cost function and the logging costs using the logging cost component from the clearcut cost function multiplied by a factor equal to 1.15. This specification follows the suggestions in Surakka and Siren (2004). Fixed harvesting cost equals 300€.

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