Anti-lock Brake System Design Based on an Adaptive Second Order Sliding Mode Controller

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***Abstract*—The aim of this paper is to propose a second order sliding mode controller for a brake system. The main objective of the controller is to induce anti-lock feature by means of tracking the slip rate of the wheel, ensuring a shorter distance in the braking process and improving the vehicle safety. The closed-loop system is robust in presence of external disturbances and parameter variations. To show the performance of the proposed design, a simulation study is carried on, where results show good performance of the antilock-brake system.**

I. INTRODUCTION

The ABS control problem consists of imposing a desired vehicle motion and as a consequence, provides adequate vehicle stability. The main difficulties arising in the brake and active suspension control design are high non-linearities, uncertainties caused by external perturbations and parameter variations which are unknown. Therefore, the ABS has become an attractive research area in nonlinear systems control framework.

On the other hand, sliding mode approaches have been widely used for the problems of dynamic systems control and observation due to their characteristics of finite time convergence, robustness to uncertainties and insensitivity to external bounded disturbances [1], [2]. Then, sliding mode control emerges as an very interesting alternative for ABS design. Several researchers have dealt with the issue of designing sliding-mode controllers for the ABS application are [3], [4], [5], [6], including the problem of extremum seeking [7].

In this work, our purpose is to discuss an ABS based on sliding mode using a simple model, regarding external disturbances and parameter variations. Similar methods has been treated previously in the above works. Subsequently, a relative degree one sliding surface is proposed; and, the for induce sliding mode dynamics, the use of a recent variation of the Super-Twisting Algorithm [8], a Lyapunov design of adaptive Super-Twisting Algorithm (ASTW) [9], is proposed, with adaptation rule based on the Lyapunov approaches presented in [10] and [11]. This control law provides finite time convergence to a bounded second order sliding set with reduction of chattering effect.

Note that the sliding modes techniques are based on the idea of the sliding manifold, that is an integral manifold with finite reaching time [12]. This manifold can be implemented by different methods including use of discontinuous function or continuous with discontinuous derivatives (so called higher order sliding modes). Let us note, that this issue of implementation, as demonstrated clearly by Utkin in [13] and earlier works is computational and depends on the system behavior in the boundary layer of the sliding manifold. Thus, the main difficulty and innovations in continuous-time sliding mode research is in the choice of the manifold rather than in the reaching phase that belongs more to numerical issue.

Indeed, once the sliding manifold $\sigma(x) = 0$ is chosen, the derivatives $\sigma^{(k)}$ of the function σ along the system trajectories can be expressed as function of control that has exactly same dimension as σ. Practically in all cases the sliding control is implemented via digital computers, so, discrete-time sliding mode is used, which is a version of a deadbeat control that makes σ to converge to zero in finite time. Let us note, that this algorithm can be dynamic, i.e. include past values of $\sigma(t_k)$ and in continuous-time will look as integrals of a function of σ.

In the following, in Section II a mathematical model of the brake system is presented. In Section III an ABS controller design based over the ASTW method is proposed. An example of the proposed controller is presented in Section IV. Finally, the Section V presents the conclusions of the current propose.

II. MODEL

In this section, the dynamic model of a vehicle is shown. Here we use a quarter of vehicle model, this model considers the pneumatic brake system, the wheel motion and the vehicle motion. We study the task of controlling the wheels rotation, such that, the longitudinal force due to the contact of the wheel with the road, is near from the maximum value in the period of time valid for the model. This effect is reached as a result of the ABS valve throttling. In this work we only consider the braking stage.

A. Pneumatic Brake System Equations

The specific configuration of this system considers brake disks, which hold the wheels, as a result of the increment of the air pressure in the brake cylinder, Fig. 1. The entrance of the air trough the pipes from the central reservoir and the expulsion from the brake cylinder to the atmosphere is regulated by a common valve. This valve allows only one pipe to be open, when 1 is open 2 is closed and vice versa.

The time response of the valve is considered small, compared with the time constant of the pneumatic system.

Let us consider Fig. 1, we suppose the brake torque T_b is proportional to the pressure P_b in the brake cylinder

$$T_b = k_b P_b \tag{1}$$

with $k_b > 0$. For the brake system we use an approximated model of pressure changes in the brake cylinder due to the opening of the valve with a first order relation [14], this relation can be represented as

$$\tau \dot{P}_b + P_b = P_c u \tag{2}$$

where τ is the time constant of the pipelines, P_c is the pressure inside the central reservoir, u is the valve input signal. Besides, the atmospheric pressure P_a is considered equal to zero.

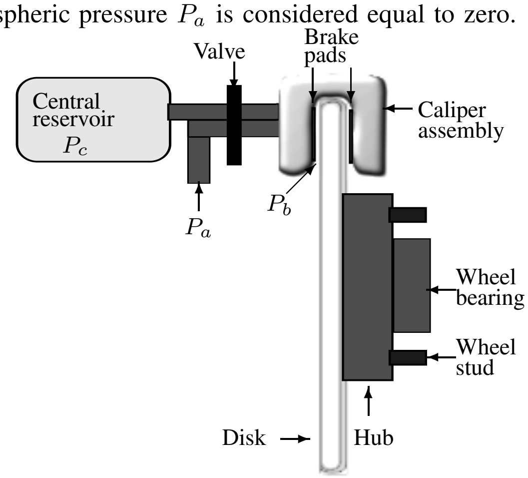

Fig. 1. Pneumatic brake scheme

B. Wheel Motion Equations

To describe the wheels motion we will use a partial mathematical model of the dynamic system as is done in [15], [16], [17] and [18].

Consider Fig. 2, the dynamics of the angular momentum change relative to the rotation axis are given by

$$J\dot{\omega} = rf(s) - B_b\omega - T_b \tag{3}$$

where ω is the wheel angular velocity, J is the wheel inertia moment, r is the wheel radius, B_b is a viscous friction coefficient due to wheel bearings and $f(s)$ is the contact force of the wheel.

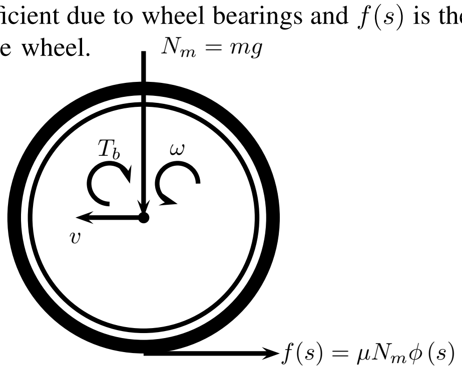

Fig. 2. Wheel forces and torques

The expression for longitudinal component of the contact force in the motion plane is

$$f(s) = \mu f_m \phi(s) \tag{4}$$

where μ is the nominal friction coefficient between the wheel and the road, f_m is the normal reaction force in the wheel

$$f_m = mg$$

with m equal to the mass supported by the wheel and g is the gravity acceleration. The function $\phi(s)$ represents a friction/slip characteristic relation between the tyre and road surface. Here, we use the Pacejka model [19], defined as follows

$$\phi(s) = D\sin(C\arctan(Bs - E(Bs - \arctan(Bs)))) \tag{5}$$

in general, this model produces a good approximation of the tyre/road friction interface. With the following parameters $B = 10$, $C = 1.9$, $D = 1$ and $E = 0.97$ that function represents the friction relation under a dry surface condition. A plot of this function is shown in Fig. 3

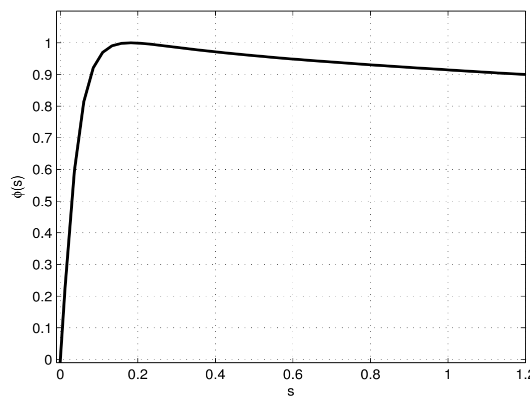

Fig. 3. Characteristic function $\phi(s)$

The slip rate s is defined as

$$s = \frac{v - r\omega}{v} \tag{6}$$

where v is the longitudinal velocity of the wheel mass centre. The equations (1)-(6) characterize the wheel motion.

C. The Vehicle Motion Equation

The vehicle longitudinal dynamics without lateral motion considered are represented as

$$M\dot{v} = -F(s) - F_a \tag{7}$$

where M is the vehicle mass; F_a is the aerodynamic drag force, which is proportional to the vehicle velocity and is defined as

$$F_a = \frac{1}{2}\rho C_d A_f (v + v_w)^2$$

where ρ is the air density, C_d is the aerodynamic coefficient, A_f is the frontal area of vehicle, v_w is the wind velocity; the contact force of the vehicle $F(s)$ is modelled of the form

$$F(s) = \mu\phi(s) f_M$$

where f_M is the normal reaction force of the vehicle, $f_M = Mg$ with M equal to the vehicle mass.

The dynamic equations of the whole system (1)-(7) can be rewritten using the state variables

$$\mathbf{x} = [x_1, x_2, x_3]^T = [\omega, P_b, v]^T$$

with initial conditions $x_0 = x(0)$ results the following form:

$$\begin{aligned} \dot{x}_1 &= -a_0 x_1 + a_1 f(s) - a_2 x_2 \\ \dot{x}_2 &= -a_3 x_2 + bu \\ \dot{x}_3 &= -a_4 F(s) - f_w(x_3) \end{aligned} \tag{8}$$

with output

$$y = s = h(x) = 1 - r\frac{x_1}{x_3}$$

where $a_0 = B_b/J$, $a_1 = r/J$, $a_2 = k_b/J$, $a_3 = 1/\tau$, $a_4 = 1/M$, $b = P_c/\tau$ and $f_w(x_3) = \frac{1}{2M}(\rho C_d A_f)(x_3 + v_w)^2$.

III. SLIDING MODE CONTROLLER FOR ABS

Given s^* as the desired value of the relative slip s, which must maximize the function $\phi(s)$, the considered problem is to design a controller that obtains reference tracking in despite of the perturbations in the system. As a solution, we propose a sliding mode controller based on the ASTW for system (8).

Throughout the development of the controller, we will assume that all the state variables are available for measurement.

A. Control Design

Let s^* be the desired value for the slip s, taking into account the direct action of the pressure P_b in the brake cylinder over the wheels motion, we define the output tracking error as

$$e \triangleq x_1 - \frac{1-s^*}{r}x_3, \tag{9}$$

and the manifold

$$\sigma \triangleq \dot{e} + \lambda e, \tag{10}$$

where $\lambda > 0$.

Hence, from (8) and (10) the derivative of σ is

$$\dot{\sigma} = \frac{\partial\sigma}{\partial t} + \frac{\partial\sigma}{\partial\mathbf{x}}\dot{\mathbf{x}} + \frac{\partial\sigma}{\partial\mathbf{x}}\bar{b}u \tag{11}$$

where $\bar{b} = [0 \quad b \quad 0]^T$.

Defining, $\varphi(t,\mathbf{x}) = \frac{\partial\sigma}{\partial t} + \frac{\partial\sigma}{\partial\mathbf{x}}\dot{\mathbf{x}}$, $\psi(t,\mathbf{x}) = \frac{\partial\sigma}{\partial\mathbf{x}}\bar{b}$ and under the assumptions:

A1 The function $\psi(t,\mathbf{x}) \in \mathbb{R}$ is presented as

$$\psi(t,\mathbf{x}) = \psi_0(t,\mathbf{x}) + \Delta\psi(t,\mathbf{x}) \tag{12}$$

where the nominal part $\psi_0(t,\mathbf{x})$ is a known function and $\Delta\psi(t,\mathbf{x})$ is a bounded disturbance so that

$$\frac{|\Delta\psi(t,\mathbf{x})|}{\psi_0(t,\mathbf{x})} = \gamma(t,\mathbf{x}) \le \hat{\gamma} < 1$$

for all $\mathbf{x}$ and $t > 0$, with $\hat{\gamma}$ unknown.

A2 The function $\varphi(t,\mathbf{x}) \in \mathbb{R}$ is presented as

$$\varphi(t,\mathbf{x}) = \varphi_1(t,\mathbf{x}) + \varphi_2(t,\mathbf{x}) \tag{13}$$

with bounded terms

$$|\varphi_1(t,\mathbf{x})| \le \delta_1|\sigma|^{\frac{1}{2}} \text{ and } |\dot{\varphi}_2(t,\mathbf{x})| \le \delta_2$$

where $0 < \delta_1, \delta_2 < \infty$ are unknown.

The following control law is proposed:

$$u = u_1 + u_2 \tag{14}$$

where $u_1 = -\alpha|\sigma|^{\frac{1}{2}}\text{sign}(\sigma)$, $\dot{u}_2 = -\frac{\beta}{2}\text{sign}(\sigma)$ and the adaptation rule for the gains $\alpha(\sigma,\dot{\sigma},t)$, $\beta(\sigma,\dot{\sigma},t)$ is [9]:

$$\dot{\alpha} = \begin{cases} \omega_1\sqrt{\frac{\gamma_1}{2}}\text{sign}(|\sigma| - \mu), & \text{if } \alpha > 0 \\ 0, & \text{if } \alpha = 0 \end{cases} \tag{15}$$

$$\beta = 2\epsilon\alpha \tag{16}$$

where ϵ, γ_1, ω_1 and μ are positive constants.

Remark 1. *We only consider the braking stage, then the value for α tends to zero when the car comes to stop. For this reason we have to put a suitable initial condition different of zero for α in the simulation case. However, for real time implementation when we use the adaptive gains we have an extra advantage because we can manage the control effort.*

B. Stability Analysis

The stability of (8) closed loop by (14) is outlined in a step by step procedure:

Step A) Reaching phase of the projection motion (11) closed loop by (14);

Step B) Sliding mode stability of (9); and

Step C) Stability of the zero dynamics x_3.

Step A): From the equations (11) and (13), follows that

$$\dot{\sigma} = \varphi(t,\mathbf{x}) + g(t,\mathbf{x})\xi \tag{17}$$

where $g(t,\mathbf{x}) = 1 + \frac{\Delta b(t,\mathbf{x})}{\psi_0(t,\mathbf{x})}$ and $\xi = \psi_0(t,\mathbf{x})u$. The closed loop system can be written as:

$$\begin{aligned} \dot{\sigma} &= -\alpha g(t,\mathbf{x})|\sigma|^{\frac{1}{2}}\text{sign}(\sigma) + \bar{\xi} + \varphi_1(t,\mathbf{x}) \\ \dot{\bar{\xi}} &= -\frac{\beta g(t,\mathbf{x})}{2}\text{sign}(\sigma) + \dot{\varphi}_2(t,\mathbf{x}) + \dot{g}(t,\mathbf{x})u_2 \end{aligned} \tag{18}$$

with $\bar{\xi} = \varphi_2 + g(t,\mathbf{x})u_2$ and $\bar{\xi}(0) = 0$.

Under the assumptions **A1** and **A2**, for the system (18) a *real 2-sliding mode* [8] is established in finite time [9], i.e. $|\sigma| < \eta_1$ and $|\dot{\sigma}| < \eta_2$, where $\eta_1 \ge \mu$, $\eta_2 > 0$.

Step B): The sliding mode dynamics is given by:

$$\dot{e} = -\lambda e + \bar{\delta} \tag{19}$$

where $\bar{\delta}$ is a bounded term, $|\bar{\delta}| < \delta$ with $\delta \in \mathbb{R}$, due to the error the sliding surface σ, which is bounded by η_1.

Finally, it is clear that the error e converges exponentially to the vicinity $|e| < \frac{\delta}{\lambda}$.

Step C) Note that during braking process $x_3 > 0$ and with $|e| < \frac{\delta}{\lambda}$ due to the control action, the zero dynamics becomes

$$\dot{x}_3 = -a_4 F(s^*) - f_w(x_3). \tag{20}$$

From the mechanics of the vehicle, the term $f_w(x_3)$ can be assumed bounded $|f_w(x_3)| \leq \varsigma$. In addition, $a_4 F(s^*) \gg \varsigma$. Therefore, let $\rho = a_4 F(s^*) - \varsigma$, $\rho > 0$. Let $V = \frac{1}{2}x_3^2$, then $\dot{V} = -x_3 [a_4 F(s^*) + f_w(x_3)] < -\rho x_3$.

Hence, $\dot{V} < -\rho_0 \sqrt{V}$, where $\rho_0 = \rho\sqrt{2}$. Therefore, $x_3 = 0$ in finite time. Also, from (9), $x_1 = 0$ in finite time.

IV. SIMULATION CASE

To show the effectiveness of the proposed control law, simulations have been carried out on the wheel model design example, the system parameters used are listed in Table 1.

TABLE 1
Values of Parameters (MKS Units)

Parameter	Value	Parameter	Value
A_f	6.6	v_w	-6
P_c	8	v	0.5
M	1800	B	10
J	18.9	C	1.9
r	0.3	D	1
m	450	E	0.97
ρ	1.225	g	9.81
C_d	0.65	B_b	0.08

In order to maximize the friction force, we suppose that slip tracks a constant signal during the simulations

$$s^* = 0.203$$

which produces a value close to the maximum of the function $\phi(s)$.

The parameters used in the control law are $\epsilon = 0.3$, $\gamma_1 = 3$, $\omega_1 = 0.5$, $\mu = 30$ and $\lambda = 100$.

On the other hand, to show robustness property of the control algorithm in presence of parametric variations we introduce a change of the friction coefficient ν which produces different contact forces, namely F and $\hat{F}$. Then, $\nu = 0.5$ for $t < 1$ s, $\nu = 0.52$ for $t \in [1, 2.5)$ s, and $\nu = 0.5$ for $t \geq 2.5$ s. Notice that just the nominal values were considered in the control design.

The error variable e is shown in Fig. 4. Here, it can be noted the very quick response of the ABS controller and its robustness in presence of the perturbation given by the variation of ν

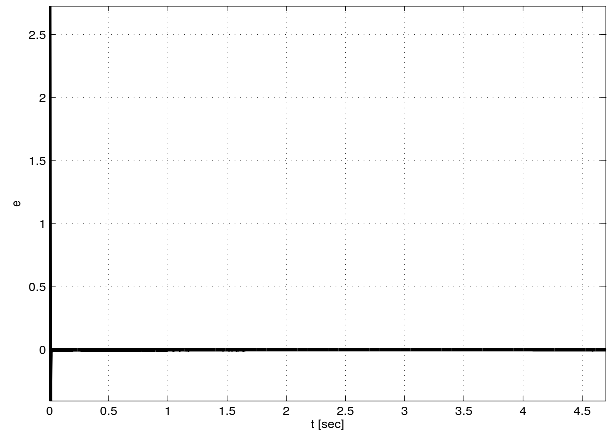

Fig. 4. Error variable e

In Fig. 5 *(a)* the slip performance trough the simulation is shown, *(b)* shows a zoom of the transient

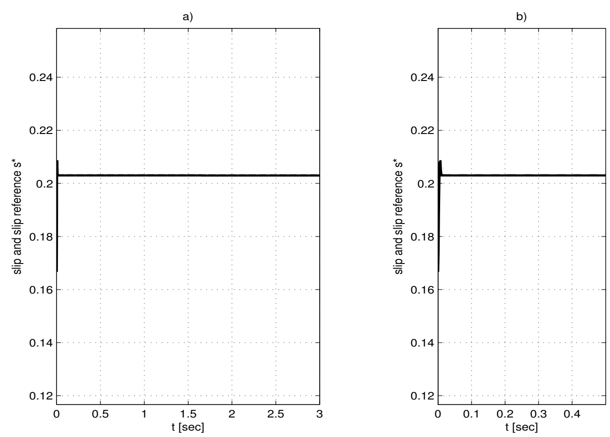

Fig. 5. Slip performance s

The finite time convergence to zero of the wheel velocity x_1 (dashed) and the vehicle velocity x_3 (solid) is shown in Fig. 6

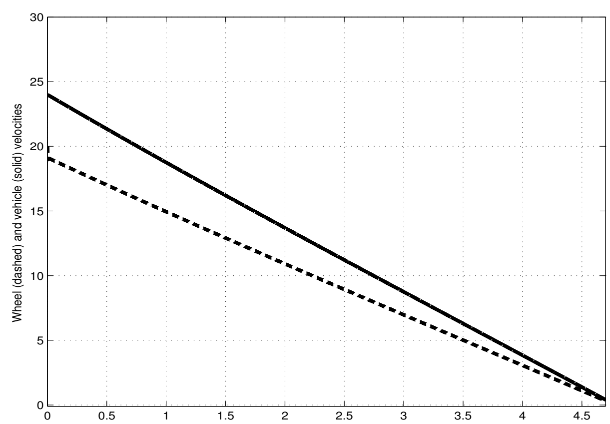

Fig. 6. Wheel velocity x_1 (dashed) and vehicle velocity x_3 (solid)

Figure 7 shows the control signal

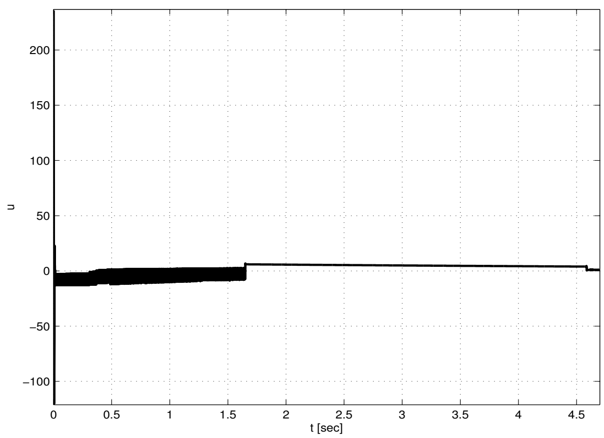

Fig. 7. Control signal u

Finally, Fig. 8 shows the parameter α. The decreasing trend of the adaptive gain is due to control effort needed as the vehicle is stopping

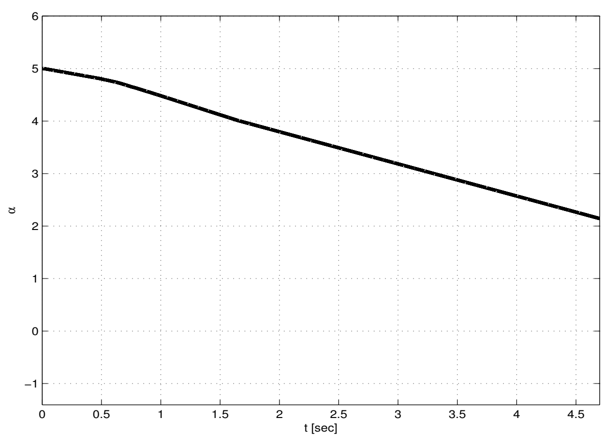

Fig. 8. Parameter α

V. CONCLUSIONS

In this work an adaptive second order sliding mode control for ABS has been proposed. The simulation results show good performance and robustness of the closed-loop system in presence of both the matched and unmatched perturbations, namely, parametric variations and neglected dynamics. We only consider the braking stage, then the value for α tends to zero when the car comes to stop. The use of adaptive gains provides an extra advantage because it can manage the control effort.

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