Sliding mode Controller for Wheel-slip Control of Anti-lock Braking System

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***Abstract*—The anti-lock braking system is a technology for passengers' safety in recently developed sophisticated vehicles. It is a part of the braking system of a vehicle which exhibits controlled braking torque to prevent the wheels from locking whenever a hard brake is applied. The highly non-linear dynamics of longitudinal tire force and the uncertain parameters of the vehicle and friction coefficient of the road motivate to use a non-linear robust controller. An efficient sliding mode controller has been proposed here to maintain the slip ratio to an optimum value so that the vehicle stopping without skidding is ensured within minimum possible distance. The non-linear Dugoff's friction model for the wheel has been taken for modeling the quarter car. The chattering phenomena caused by sliding mode have been minimized with an efficient technique. Slip-tracking performance and the robust performance of the proposed controller have been compared with the recent existing non-linear controller.**

Keywords-quarter car, anti-lock braking system, road-friction model, Dugoff's friction model, sliding mode control, chattering.

I. INTRODUCTION

The development of anti-skid braking controller boosted the development of Anti-lock Braking System. The first anti-skid controller was developed n 1908 for a train. Thereafter in 1940's electro-hydraulic ABS was designed for aircraft. In 1969 Ford introduced this system for motor car [11]. The implementation of electronic ABS came frequently in the market for the ground vehicle improving the maneuverability of the vehicle under severe braking condition. Today ABS comes in every sophisticated car in order to improve the passengers' safety. Whenever driver presses a hard brake in an emergency situation in order to stop the vehicle as early as possible the wheel of the vehicle becomes locked and the vehicle starts sliding. During this, it is very difficult to control the vehicle in a desired manner. ABS exhibits directional control of the vehicle under such condition. The basic function of ABS controller is to provide a firm braking torque so that the slip ratio between tire and road surface is maintained properly.

However the braking torque provided by the controller strongly depends on the longitudinal force of the tire. The nature of the tire force depends on the tire model [3, 8, 9, 12, 13, 15, and 18]. Therefore choice of the friction model is an important task for this system. The quarter car model [1, 2, 5, 12 and 16] has been used in ABS for simplicity and the non-linear Dugoff's dynamic friction model [2, 21] has been considered for modeling the system. The parameters such as vehicle mass, position of the center of gravity of the vehicle and the road tire friction coefficient are uncertain in nature. Therefore the controller should deal with the non-linear system and should be insensitive to the parametric variation and disturbances.

The sliding mode controller [1, 4, 17, and 20] is very popular in this context as it has the property of finite convergence for a highly non-linear system and exhibits good robustness against the variation of the plant parameters and the external disturbances. This control algorithm for ABS appears frequently in the ABS literature. C. Űnsal[17] designed an ABS with sliding mode measurement feedback. Schinkle et al. [11] proposed a sliding mode approach to control the slip ratio of an ABS. The braking torque was taken as control input and wheel-slip ratio control algorithm was developed by the first-order sliding mode controller. In 2008 K Park et al. [5] suggested the sliding mode controller for the ABS using feedback linearization. The chattering phenomenon [20] is a major shortcoming of the sliding mode control. Several chattering suppression method is proposed in the literature [6]. However the second- order sliding mode controller is generally not acceptable for ABS because of the chattering having high frequency. Predictive non-linear control with integral feedback [2] has been employed to improve the tracking performance of the controller. This method has been suggested as an alternative option to the sliding mode controller. The proposed sliding mode control law has been developed with appropriate sliding surface for the quarter car with dugoff's friction model. As high speed switching of SMC limits the performance and produces unwanted pulsation in the brake pedal, the control law is further modified to reduce the chattering in steady state. A simplified block diagram of the system is shown in Fig. 1. The performance of the vehicle with sliding mode controller has been checked and the controller performances in terms of slip tracking, robustness etc. have been compared with the non-linear predictive controller [2] with the same simulation parameters.

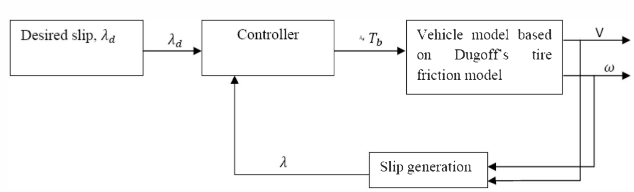

Fig 1. Simplified block diagram of the system with controller

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II. System Dynamics

The dynamics of the whole system comprises of the car model, tire friction model and the slip dynamics.

A. Quarter Car model

A simplified quarter car model [2] which holds necessary characteristics of the whole vehicle has been taken here to model the system. The free body diagram of the single-wheel model is shown in the Fig. 2.

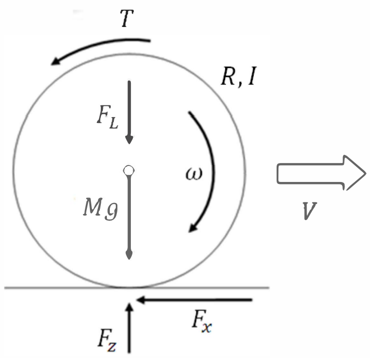

Fig 2. Free body Diagram of the quarter car model

The equations of motion of the vehicle during braking can be written as

$$\dot{V} = \frac{-F_x}{M} \tag{1}$$

$$\dot{\omega} = \frac{1}{I}(RF_x - T) \tag{2}$$

Where V denotes the longitudinal velocity of the vehicle, ω denotes angular velocity of the wheel, M is the total mass of the quarter car, F_x is the longitudinal tire force, I is the total moment of inertia of the wheel and R is radius of the wheel.

The total mass of the quarter car can be represented as

$$M = \frac{1}{4}m_{vs} + m_w \tag{3}$$

where, m_{vs} is sprung mass of the vehicle and m_w is the mass of the single wheel. The normal force of the tire has two components, one is due to the load of the vehicle and the other one is due to the load transfer during braking. So the normal load acting on the wheel is defined as

$$F_z = Mg - F_L \tag{4}$$

where g denotes the gravitational acceleration and F_L is the dynamic load of the wheel which can be written as

$$F_L = \frac{m_{vs}h_{cg}}{2l}\ddot{x} \tag{5}$$

Where h_{cg} denoted the height of the center of the gravity of vehicle is sprung mass and $\ddot{x}$ is longitudinal acceleration of the vehicle. $l$denotes the height of the CG from the wheel axle.

B. Wheel-slipdynamics

The dynamic equations of the car have been described in the previous section. As our main aim is to design a wheel-slip controller for vehicle, so the primary task is to measure wheel longitudinal slip. A positive torque should be applied to the wheel to accelerate the vehicle. During braking the applied torque on the wheel should be negative and the longitudinal speed of the vehicle decreases as speed of the wheel decreases. When a driver presses the brake pedal in order to reduce the speed, with the increase of brake torque the angular velocity of the wheel is decreased (2) which results in a change in longitudinal speed of the vehicle and the wheel. The differential speed of the vehicle and the wheel generates the slip at the contact surface of the tire and road. The longitudinal slip, λ of the wheel is defined as the relative difference between the driven wheel angular velocity and the vehicle absolute velocity.

Slip ratio is expressed as

$$\lambda = \frac{V - R\omega}{V} \tag{6}$$

Differentiating (6) with respect to time and substituting in (1) and (2) gives

$$\dot{\lambda} = -\frac{1}{V}\left[\frac{F_x}{M}(1-\lambda) + \frac{R^2 F_x}{I}\right] + \left(\frac{R}{VI}\right)T \tag{7}$$

The friction force F_x strongly depends on the road-tire friction model which comes from the contact area of the wheel and road surface. The friction model establishes the relationship between the slip ratio λ and friction coefficient μ.

C. Dugoff's tire model

The tire-friction model is a critical factor during emergency maneuvering. Linear tire model is not always sufficient for accurate maneuvering. To account for the nonlinearity due to saturation property of the tire force, the non-linear Dugoff's tire model [2, 21] has been taken. It calculates pure longitudinal and lateral tire forces and side slip of the wheel. The longitudinal force is given by

$$F_x = \frac{C_i}{1-\lambda} f(s) \tag{8}$$

Where,

$$f(s) = \begin{cases} s(2-s), if\ s < 1 \\ 1, if\ s > 1 \end{cases} \tag{9}$$

and
$$S = \frac{\mu F_z\left(1 - \varepsilon_r V\sqrt{\lambda^2 + \tan^2\alpha}\right)(1-\lambda)}{2\sqrt{C_i^2\lambda^2 + C_\alpha^2\tan^2\alpha}} \tag{10}$$

Where, C_i is the tire longitudinal stiffness C_α is tire cornering stiffness, μ denotes road friction coefficient, α is side slip angle and ε_r denotes road adhesion reduction factor. The non-linear longitudinal force characteristics curve is shown in the Fig. 3 for dry asphalt taking the value of the parameters as given in table 1.

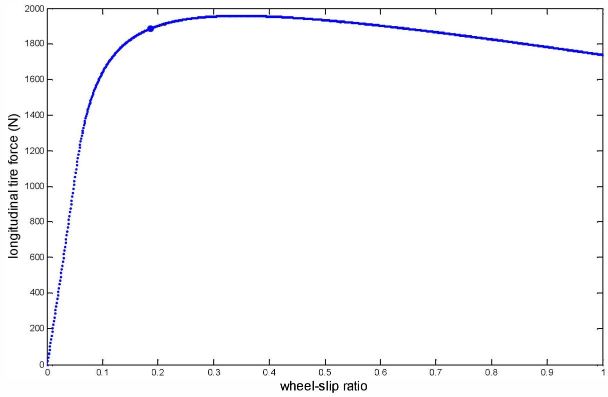

Fig 3.Typical longitudinal force characteristic of the wheel.

III. Controller Design

The state space model of the whole system [2] can be represented as

$$\dot{x}_1 = \frac{-F_x}{M} \tag{11}$$

$$\dot{x}_2 = -\frac{1}{V}\left[\frac{F_x}{M}(1-\lambda) + \frac{R^2 F_x}{I}\right] + \frac{R}{VI}T \tag{12}$$

$$y_1 = x_2 \tag{13}$$

Where the state vector $X = \begin{bmatrix} V \\ \lambda \end{bmatrix}$. The output y_1 is longitudinal slip ratio of the wheel and braking torque T is considered as control input.

A. Desired slip

According to the dugoff's friction model it has been seen from the Fig. 3 that the peak of the tire force curve lies between the slip value of 0.1 to 0.2. So the maximum tire force is achieved nearly at λ =0.15.

To prevent large tracking error, the following model [2] to get desired slip dynamic is introduced which includes good transient response.

$$\lambda_d(t) = \lambda_{opt} - \lambda_{opt} e^{-at} \tag{14}$$

Where$\lambda_{opt} = 0.15$, is the optimum wheel slip and a=20 is the time constant. The transient response of the desired slip response is shown in Fig. 4.

B. Sliding mode control law

As mentioned before, due to the high nonlinearity and the parametric uncertainty present in the system the sliding mode control approach has been considered. In general, the SMCexhibits high speed switching law to drive the nonlinear system onto a specified surface which is called sliding surface. The first order sliding mode controller [19] has been implemented for the above mentioned system. The switchingsurface $s_m(\lambda, t)$ is defined by equating the sliding variable $s$which is defined below, to zero

$$s_m(\lambda, t) \equiv \left(\frac{d}{dt} + \gamma\right)^{r-1} (\lambda - \lambda_d) \tag{15}$$

Where γ is strictly positive constant and r =1 for the first order sliding mode. The sliding variables are chosen as the error between current slip λ and the desired slip ratio λ_d.

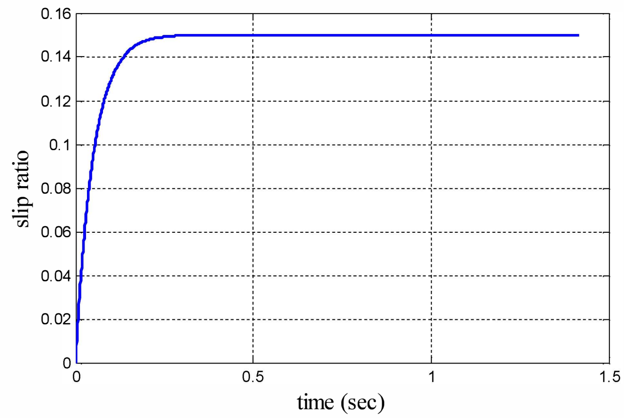

Fig 4. The desired slip response.

So the sliding manifolds are taken as

$$s_m = \lambda_e = \lambda - \lambda_d = 0 \tag{16}$$

Now the main objective is to design a continuous control law for the braking torque T, which is capable of enforcing sliding mode on the sliding manifolds. Once the sliding mode occurs, within finite time the current slip ratio tracks the desired slip ratio since the sliding manifold $\lambda_e = 0$.

The system dynamics which is shown in (12) can be rewritten as follows:

$$\dot{\lambda} = f + hT \tag{17}$$

Where,

$$f = -\frac{1}{V}\left[\frac{F_x}{M}(1-\lambda) + \frac{R^2 F_x}{I}\right] \tag{18}$$

And $h = \frac{R}{VI}$

The first derivative of the sliding variable s is given by

$$\dot{s}_m = \dot{\lambda}_e = \dot{\lambda} - \dot{\lambda}_d \tag{19}$$

Substituting (17) into (19) we get

$$\dot{s}_m = f + hT - \dot{\lambda}_d \tag{20}$$

$$\text{Or,}T = \frac{1}{h}[\dot{s}_m + \dot{\lambda}_d - f] \tag{21}$$

Applying first order sliding mode control law

$$T = \frac{1}{h}[-Lsign(s_m) + \dot{\lambda}_d - f] \tag{22}$$

Where, the controller's gain is $L > 0$.

The value of L is chosen such that the tracking error of slip ratio asymptotically approaches to zero by satisfying the following condition

$$s_m(\lambda, t)\dot{s}_m(\lambda, t) \tag{23}$$

So finally the control law is derived as

$$T = \frac{1}{\frac{R}{VI}}\left[-Lsign(s_m) + \dot{\lambda}_d + \frac{1}{V}\left[\frac{F_x}{M}(1-\lambda) + \frac{R^2 F_x}{I}\right]\right] \tag{24}$$

C. Proposed control law

In order to have good slip-tracking performance of the controller, the conventional sliding mode controller mentioned in (26) is modified here. As the controller is a tracking controller type, the PID type sliding surface has been chosen. Hence the switching surface is redefined as

$$s_0 = K_I \int_0^t e(\tau)d\tau + K_P e(t) + K_D \dot{e}(t) \tag{25}$$

Where K_P, K_I and K_D are proportional gain, integral gain and derivative gain respectively.

Then the control input T for ABS can then be defined as

$$T = \frac{1}{h}[-Lsign(K_I \int_0^t e(\tau)d\tau + K_P e(t) + K_D \dot{e}(t)) + \dot{\lambda}_d - f] \tag{26}$$

Where the switching gain$L > 0$

It has been seen that one of the shortcomings of the sliding mode control scheme is chattering at high controller gain, which limits its tracking performance in the steady state.

Several chattering reduction techniques are there in the literature [6, 19, and 20] to have a smooth control law. Here, an equivalent control law [6] is employed to have the chattering free controller. As the magnitude of the chattering is proportional to the switching gain L, to reduce chattering the gain should be adjusted by choosing small value at the steady state. The switching gain L may be a function of equivalent controlU_{eq}. Since U_{eq} decreases at the steady state, the sliding mode occurs along with the discontinuous surface $\sigma = 0$ then the switching gain also decreases. The control input is taken as

$$u = -\left(K_0 \left|\frac{1}{\tau s+1} sign(s_0)\right| + \Delta\right) sign(s_0) \tag{27}$$

Where K_0 and Δ are positive constants and $\left|\frac{1}{\tau s+1} sign(s_0)\right|$ denotes the average of $sign(s_0)$. The low pass filter is also helpful to eliminate the disturbance. Derivative of the sliding variable σ is given as

$$\dot{\sigma} = f(x,t) - (K_0|k| + \Delta)sign(s_0) = 0 \tag{28}$$

It is clear from the above equations that the switching gain $L = K_0|k| + \Delta$ decreases as the average of $sign(s_0)$ decreases. The switching surface σ is defined as mentioned in (25).

So the final control law for ABS is defined as

$$T = \frac{1}{h}[\Big(K_0 \left|\frac{1}{\tau s+1} sign\Big(K_I \int_0^t e(\tau)d\tau + K_P e(t) + \right.$$
$KDet+\delta sign(KI0te\tau d\tau+KPet+ \quad KDe(t))+\lambda d-f]$
(29)

IV. Simulation Result

Simulations have been carried out for the ABS with quarter car model based on Dugoff's tire model. The performances of the controller have been studied here. The table 1 shows the values which are used for simulation. In this maneuver, the vehicle travels along straight path on a flat dry road (μ=0.8). Initial longitudinal velocity of the vehicle is assumed to be 20m/s. Before applying brake, the wheel speed and the vehicle longitudinal speed was same but after applying a braking torque the wheel speed gradually decreases and closely follows the vehicle speed without locking. Finally they coincide when $\omega = 0$. Therefore vehicle stopping without skidding is ensured. The desired slip ratio is considered as 0.15, which is perfectly tracked by controller and converge the slip error to zero. The stopping distance is achieved by integrating the vehicle velocity over the period of braking time. The magnified view of the slip error curve is shown in Fig. 6. The parameter $K_p = 1$, $K_I = 500$ and $K_D = 0.05$ are taken for the sliding surface. The smooth controlled braking torque profile is shown in Fig. 7. Fig. 8 and Fig. 9 are the comparative study of the tracking performance of the proposed controller with the conventional sliding mode controller and the existing predictive controller [2]. The 'sign' function in (29) has been approximated with the comparatively smooth saturation function.

To evaluate the robustness of the controller in the presence of parametric uncertainty the same car model has been taken with their nominal value according to table 1. It is assumed that the uncertainties have raised out of 15% uncertainty in total mass of the vehicle, 10% uncertainty in friction coefficient and 5% uncertainty in the position of the vehicle's C.G. The performance indices of the proposed controller have been compared in the table 2.

TABLE I. SIMULATION PARAMETERS

Wheel radius, R	0.326 m
Wheel base, l	2.5 m
Center of gravity height, h_{cg}	0.5 m
Wheel mass, m_w	40 kg
¼ of vehicle sprung mass, $\frac{1}{4}m_{vs}$	415 kg
Total moment of inertia of the wheel, I	1.7 kg m^2
Tire longitudinal stiffness, C_i	17349.8 N
Tire cornering stiffness, C_α	2720.55422 (N/rad)
K_0	537
τ	5
Δ	0.1
L	678

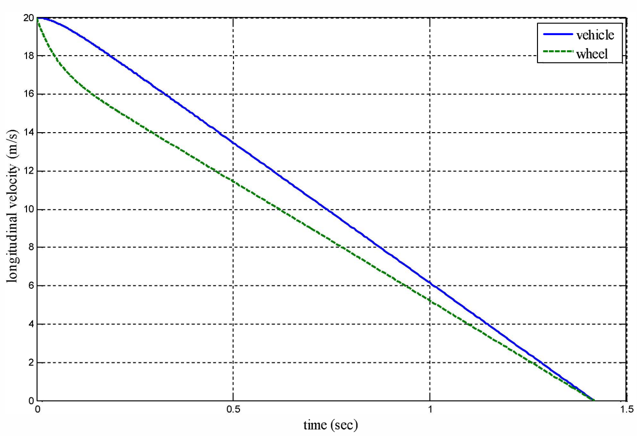

Fig 5. Longitudinal velocity profiles of the wheel and vehicle

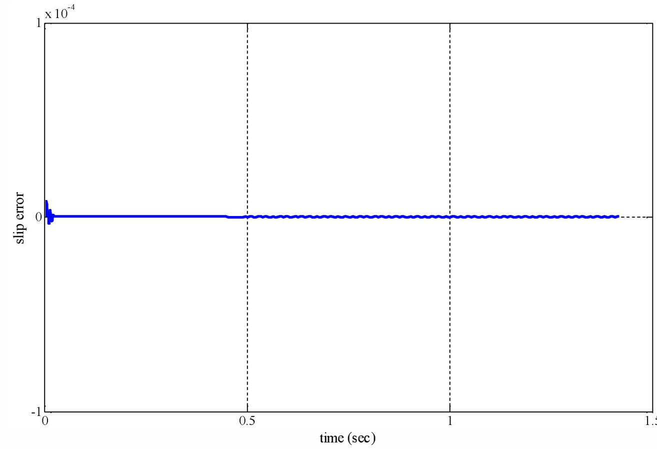

Fig 6. Slip error

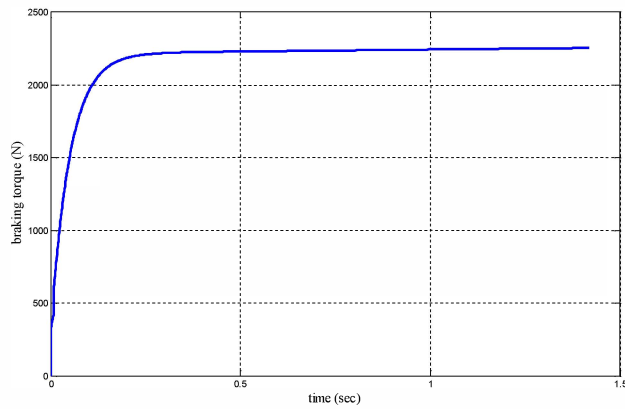

Fig 7 Braking torque profiles

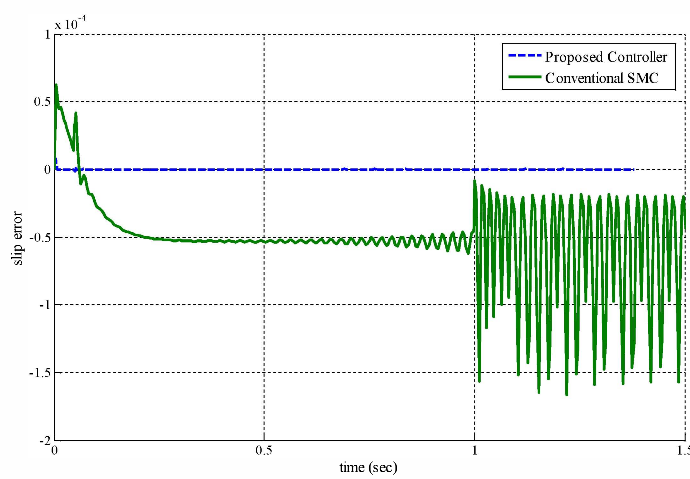

Fig 8. Slip error comparison with the conventional sliding mode controller

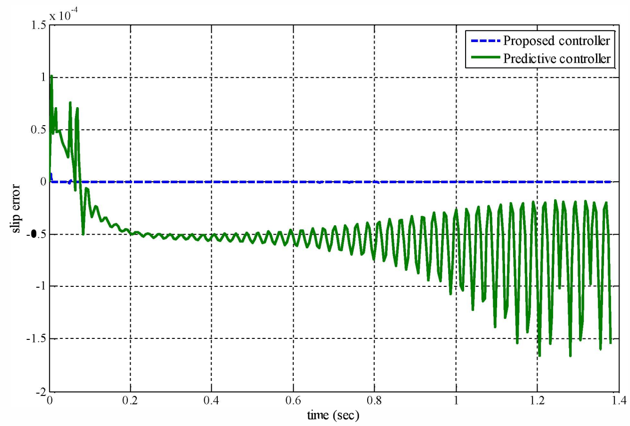

Fig 9. Slip error comparisons with the predictive controller

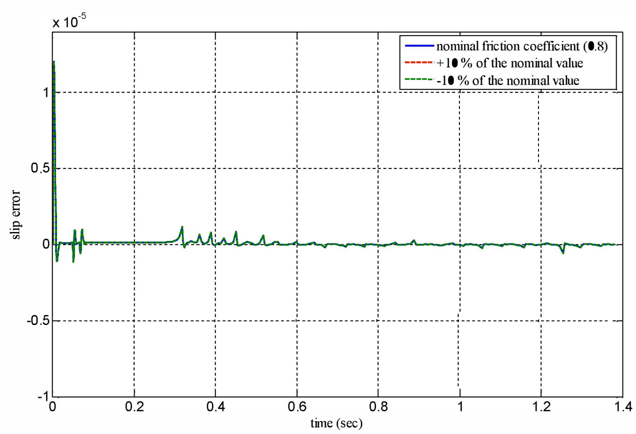

Figure 10. Slip errors in the presence of uncertainty in the friction coefficient.

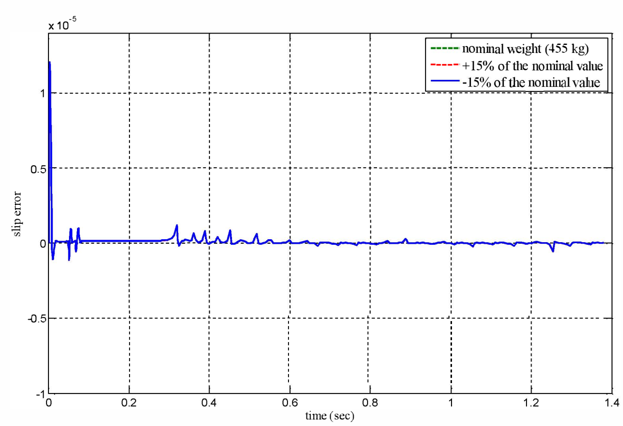

Fig 11. Slip errors in the presence of uncertainty in mass of the vehicle.

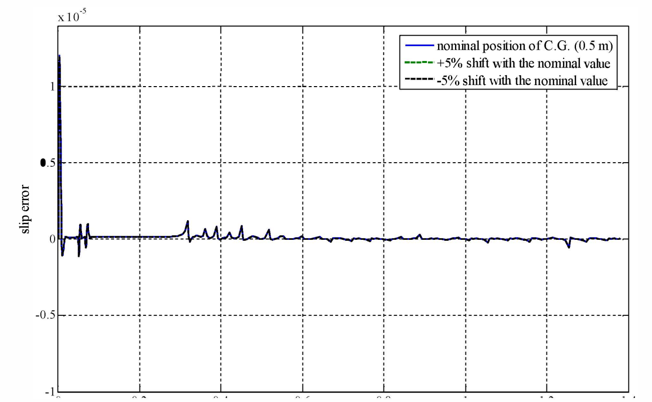

Fig 12. Slip errors in the presence of uncertainty in the position of C.G. of the vehicle.

TABLE II. PERFORMANCE INDEX

Performance Index	Predictive controller	Conventional SMC	Proposed SMC
$\int_0^t T_b^2 dt \times 10^{-6} ((N.m)^2 \text{sec})$	3.104	4.2	4.1
$\int_0^t (\lambda - \lambda_d)^2 dt \times 10^{-7}$ (sec)	0.4398	0.393	7.245 $\times 10^{-5}$
$\int_0^t V dt \ (m)$	21.890	14.39	14.1

V. CONCLUSION

The proposed sliding mode controller exhibits good slip tracking performance. Comparatively shorter stopping distance is achieved by the proposed controller. It shows good robustness in the presence of uncertainties and disturbance. This controller can be applied for the front wheel and rare wheel separately. Simulation can be performed considering banking angle and side slip angle.

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