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Estimation of the tensile elastic modulus using Brazilian disc by applying diametrically opposed concentrated loads

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ABSTRACT

The tensile elastic modulus E_t of a rock is different from the compressive elastic modulus E_c , due to inhomogeneity and microcracks. There is no convenient method to obtain E_t except using direct tension tests. However, the direct tension test for rock materials is difficult to perform, because of stress concentrations, and the difficulty of preparing specimens. We have developed a new method to determine E_t of rock materials easily and conveniently. Two strain gauges are pasted at the center part of a Brazilian disc's two side faces along the direction perpendicular to the line load to record tensile strain, and a force sensor is used to record the force applied; then the stress–strain curve can be obtained; finally the E_t can be calculated according to those related formulas which are derived on the basis of elasticity theory. Our experimental results for marble, sandstone, limestone and granite indicate that E_t is less than E_c , and their ratio is generally between 0.6 and 0.9.

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1. Introduction

In general, rocks are inhomogeneous, and contain numerous microcracks. Consequently, rocks show different behavior under tensile and compressive conditions. Accordingly, there are two kinds of elastic modulus: the compressive elastic modulus E_c and the tensile elastic modulus E_t . Parameters such as Young's modulus and Poisson's ratio are expected to be different under compressive or tensile stress [1,2]. The data reported by Krech et al. [3] and Liao et al. [4] indicate such differences for Young's modulus and Poisson's ratio in some rock types (granite, quartzite, sandstone, limestone, argillite).

The modulus E_c and the compressive strength σ_c are easy to measure in the laboratory by uniaxial compression tests. But the parameter E_t and the tensile strength σ_t are difficult to obtain by direct tension tests, because it is very difficult and complicated to prepare test specimens, and it is easy to generate stress concentrations at the ends of the specimen. In order to solve this problem, the International Society for Rock Mechanics (ISRM) officially proposed the Brazilian test as a suggested method for determining the tensile strength σ_t of rock materials [5]; however, there is no indirect test proposed by the ISRM to determine the tensile elastic modulus E_t . In this study, a convenient and

maneuverable method for determining E_t of rock materials with the Brazilian disc approximately is to be developed.

In the Brazilian test, a disc specimen is compressed with diametrically opposite and symmetric line loads [6,7]. The theoretical basis for the Brazilian test is the analytical solutions that have been obtained by many researches for isotropic or transverse isotropic materials under concentrated loads, loads that are distributed over a small arc of the disc's circumference [8–11]. Fairhurst [12] discussed the validity of the Brazilian test, and concluded that failure is expected to initiate at the center of disc, but actually the failure sometimes initiates at the loading points. Hudson [13] verified this conclusion with experiments. Guo et al. [14] developed a simple method to measure the opening mode (mode-I) fracture toughness K_{Ic} with the Brazilian disc. Wang et al. [15] later made some improvement to Guo's method for determining K_{Ic} .

Much attention has been placed on the elastic modulus by researchers. Hondros [8] developed an approach to measure the elastic modulus E and Poisson's ratio ν with the Brazilian disc. He also gave a complete stress analytic solution for the case of a radial load distributed over a finite circular arc of the disc. However, this kind of loading is very difficult to obtain in the laboratory. Consequently, there will be some differences between the actual stress field and the ideal analytical solution. And there is another problem for the method: the theory of the method is based on the strain of the center of the disc, but the strain measured by the strain gauge is the contribution of a line segment

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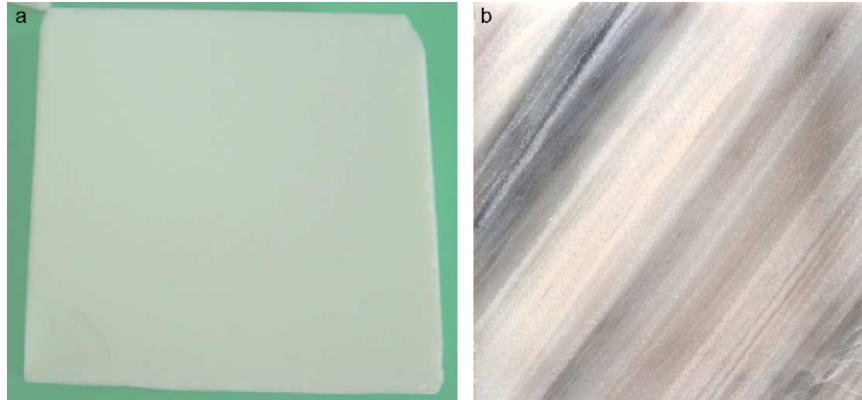


Fig. 1. (a) is white marble which seem to be homogenous and isotropic and (b) banded marble shows obvious transverse isotropy.

near the center of the disc. Yu et al. [16] invented a method for determining E with the Brazilian test proposed by ISRM. They recorded the force applied and the displacement of loading point in experiment, and then a force–displacement curve could be obtained. The slope of the line section of force–displacement curve was defined as E_d . The elastic modulus E could be determined by E_d multiplying a correction coefficient k . According to finite-element analysis (ANSYS) and experiment, using Three Gorges granite, they concluded that k is about 19.2. This method was an improvement, but it was just based on linear elastic, isotropic finite-element analysis and regressing, fitting test data. Its credibility and reliability may be low for other rock types. Wang et al. [17–19] developed a similar method for determining E with flattened Brazilian disc. They gave out an approximate formula to calculate the relative displacement between the two ends according to Cauwellaert's result for a uniformly and parallelly distributed load applied on a section of a circular arc [20]. However, You and Su [21] disputed Wang's method. Because the loading and geometry between flattened Brazilian disc and Cauwellaert's complete disc were completely different, they concluded that it was incorrect to use Cauwellaert's results.

The elastic modulus E mentioned above is the compressive elastic modulus. Some attention has also been placed on the tensile elastic modulus. Li and Yin [22] used pure bending beam to measure the E_c and E_t . Two strain gauges were pasted on the upper surface (compressive zone) and lower surface (tensile zone), to record the compressive and tensile strain, respectively. The strongpoint of this method is that it can obtain E_c and E_t at the same time. Zhang et al. [23] devised a method to calculate the tensile elastic modulus E_t of rock with a cracked Brazilian disc. A small vertical, straight and through notch was required at the center of disc. However, the theory of this method is immature at present, and it is difficult to make the notch required in Brazilian disc.

As we know, some rock types show non-homogeneity to some extent, and contain many microcracks. These two factors lead to some anisotropy for rock materials. Furthermore, the level of microcracking and the orientation of microcracks also have great effect on the mechanical properties of rock materials. However, it is very difficult to study the effect quantitatively at present. Therefore, for the sake of simplicity, and also adopting the same model proposed by ISRM for the Brazilian test who treat the rock material as an equivalent isotropic continuum medium, we also consider the disc to be an equivalent isotropic continuum medium. Correspondingly, in fact the elastic modulus and Poisson's ratio are the equivalent ones. Of cause, the model adopted here is not suitable for those rock types that show

anisotropy or transverse isotropy. For example, Fig. 1a is a white marble, which seems to be homogenous and isotropic. Fig. 1b is a banded marble, which shows transverse isotropy. Certainly, the white marble is suitable for the model proposed here, whereas the banded marble is not suitable.

In this study, a simple and convenient test method is proposed for determining E_t for isotropic rock materials with the Brazilian disc. The configuration of the test is shown in Fig. 2. The core idea of the proposed method that two strain gauges are pasted, respectively, at the center of disc on the both side faces along the direction perpendicular to the line load P (Fig. 3) to record the tensile deformation of the center part. Then, according to the stress obtained through elasticity theory and the recorded strain, the equivalent tensile elastic modulus E_t can be calculated. Obviously, the loading manner of the test method proposed here is completely different from that proposed by ISRM. The ISRM suggests that two concave loading plates can be used to apply load in order to distribute the load along an arc of the disc. The reasons that the test configuration shown in Fig. 2 is adopted in this study are as following. On the one hand, the stress analytic solution obtained by Hondros [8] for a pair of distributed loads applied over an arc of the disc oppositely and diametrically (Fig. 4) based on isotropy is

$$\begin{aligned}\sigma_r &= -\frac{2p}{\pi} \left\{ \alpha + \sum_{n=1}^{n=\infty} \left[1 - \left(1 - \frac{1}{n} \right) \left(\frac{r}{R} \right)^2 \right] \left(\frac{r}{R} \right)^{2n-2} \sin 2n\alpha \cos 2n\theta \right\} \\ \sigma_\theta &= -\frac{2p}{\pi} \left\{ \alpha - \sum_{n=1}^{n=\infty} \left[1 - \left(1 + \frac{1}{n} \right) \left(\frac{r}{R} \right)^2 \right] \left(\frac{r}{R} \right)^{2n-2} \sin 2n\alpha \cos 2n\theta \right\} \\ \tau_{r\theta} &= -\frac{2p}{\pi} \left\{ \sum_{n=1}^{n=\infty} \left[1 - \left(\frac{r}{R} \right)^2 \right] \left(\frac{r}{R} \right)^{2n-2} \sin 2n\alpha \cos 2n\theta \right\} \quad (1)\end{aligned}$$

where p is the applied pressure, R is the radius of disc, r and θ are the polar coordinates of a point in disc, and α is the half central angle related to the distributed load applied. From the equation above, we know that the stress field of the disc subjected to a pair of distributed load is the function of α . Obviously, the magnitude of α affects the stress distribution in the disc directly. According to ISRM's suggestion [5], if the standard concave loading plates are used in the Brazilian test, the 2α is about 10° at failure. As far as we know, the disc would show some plastic properties, rather than complete elasticity when approaching failure. However, the parameter of tensile elastic modulus E_t is a physical quantity of the elastic stage in tension of rock materials. Therefore, 2α is certainly not 10° , and there is no way to exactly know the specific value of 2α when the disc is in the elastic stage. Furthermore, 2α is a variable in the processing of loading. Additionally, the value of

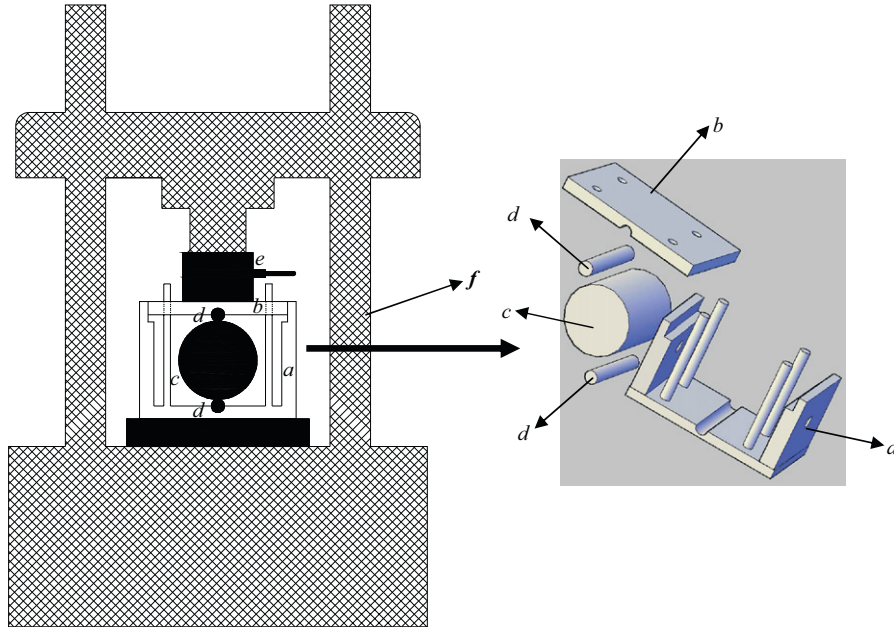


Fig. 2. The configuration of the test adopted in this study: (a) the experiment box, (b) the loading plate, (c) the Brazilian disc, (d) steel bar, (e) force sensor and (f) the press machine.

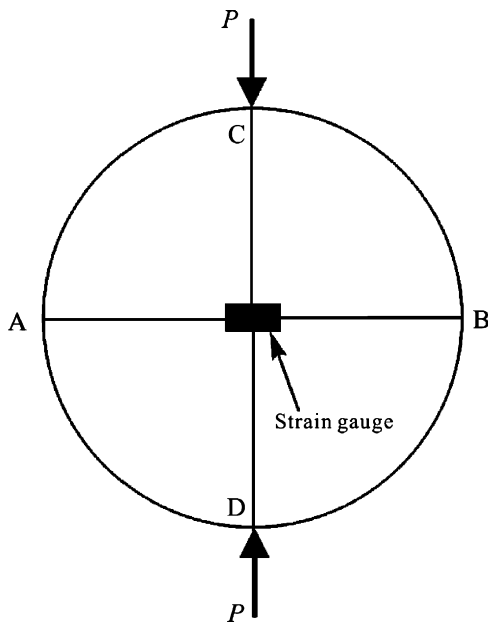


Fig. 3. Two extra strain gauges are pasted, respectively, at the center part of Brazilian disc on both side faces.

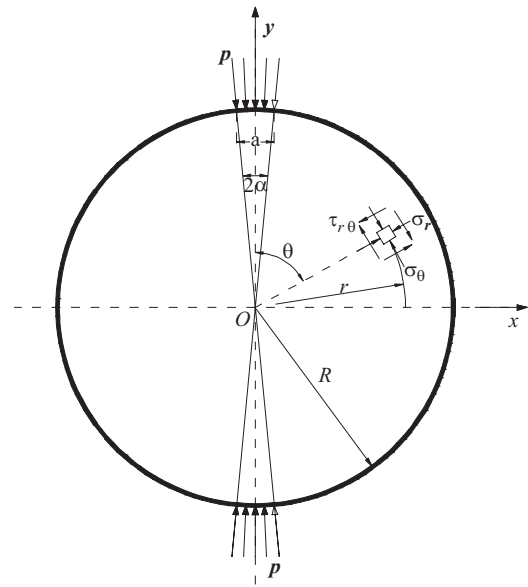


Fig. 4. A pair of distributed load applies over an arc of the disc oppositely and diametrically on the disc.

2α would vary despite using the same concave loading plates and under the same loading, due to the type of rock, component and size of mineral, brittleness and stiffness. In short, these factors make it very difficult to know and calculate the value of 2α when the disc is in its elastic stage. Accordingly, it is also very difficult for us to calculate exactly the stress field in the disc subjected to distributed loads over an arc when the disc is in elastic stage. This brings difficulty to determine the tensile elastic modulus E_t through the strain measurement due to the uncertainty of the stress field in disc of elastic stage. On the other hand, in the test configuration proposed here showed in Fig. 2, two steel bars are used to provide loads. Because the diameter of the steel bars is

much smaller than the diameter of the disc, the area of contact between the steel bars and the disc is very small. Correspondingly, the loads applied by the two steel bars can be considered as two concentrated loads applied to the disc oppositely and diametrically along the diameter. This type of loading can effectively avoid the problem of the uncertainty of the stress field in disc subjected to distributed load over an arc due to the difficulty in knowing 2α .

The analytical solution for a pair of diametrically opposite, symmetric and compressive line loads applying on a disc of isotropic rock materials (Fig. 5) has been given by Muskhelishvili [9]. Therefore, the distribution of the stress at the center part of disc is completely known. As long as the tensile strain is recorded by strain gauges in the experiment, the tensile elastic

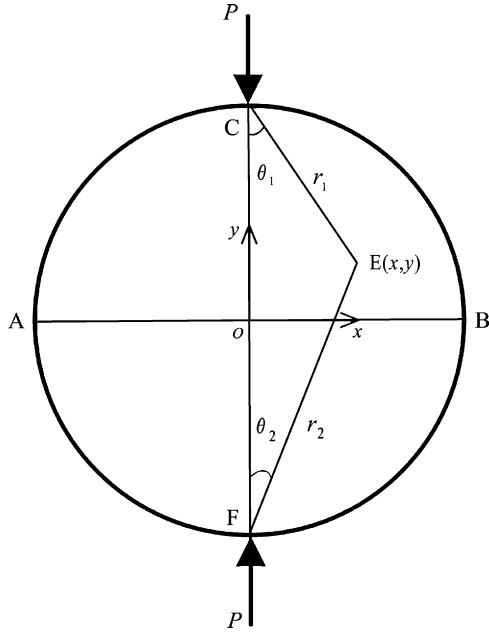


Fig. 5. A Brazilian disc subject to line load P .

modulus E_t of isotropic rock materials can be calculated from the slope of the line section of stress–strain curve, the Poisson ratio ν , the diameter of disc D and the half length of strain gauge L (see formula (12)).

Some experiments have been performed on four rock types to support this method and related theory, including the uniaxial compression tests and the Brazilian tests. The experimental results indicate that the ratio between E_t and E_c for the same rock type is generally about 0.6–0.9, which is consistent with the data reported in Ref. [22]. It proves to some extent that the method proposed is flexible and reliable.

2. Analytic solution for a pair of diametrically opposed, symmetric and compressive line loads applied on an isotropic Brazilian disc (Fig. 5)

The solution for an isotropic Brazilian disc subjected to concentrated loads is [9]

$$\begin{aligned}\sigma_x &= \frac{2p}{\pi l} \left(\frac{\cos \theta_1 \sin^2 \theta_1}{r_1} + \frac{\cos \theta_2 \sin^2 \theta_2}{r_2} \right) - \frac{2P}{\pi D l} \\ \sigma_y &= \frac{2p}{\pi l} \left(\frac{\cos^3 \theta_1}{r_1} + \frac{\cos^3 \theta_2}{r_2} \right) - \frac{2P}{\pi D l} \\ \tau_{xy} &= \frac{2p}{\pi l} \left(\frac{\cos^2 \theta_1 \sin \theta_1}{r_1} + \frac{\cos^2 \theta_2 \sin \theta_2}{r_2} \right)\end{aligned}\quad (2)$$

where P is the line load applied, whose units are (N/m), l is the thickness of the disc, D is the diameter of the disc. θ_1, θ_2 are positive when the point E is at the right of load P , and they are negative when E is at the left of load P . r_1 and r_2 are the distances from the point E to the loading points C and F (Fig. 5). There are following relations in the triangle CEF :

$$\begin{aligned}r_2^2 &= r_1^2 + D^2 - 2r_1 D \cos \theta_1 \\ \cos \theta_2 &= \frac{D^2 + r_2^2 - r_1^2}{2r_2 D} = \frac{D - r_1 \cos \theta_1}{r_2} \\ \sin \theta_2 &= \sqrt{1 - \cos^2 \theta_2} = \frac{r_1 \sin \theta_1}{r_2}\end{aligned}\quad (3)$$

Using the relation formula above, expression (2) can be written as

$$\begin{aligned}\sigma_x &= \frac{2P}{\pi l} \left[\frac{\cos \theta_1 \sin^2 \theta_1}{r_1} + \frac{(D - r_1 \cos \theta_1) r_1^2 \sin^2 \theta_1}{(r_1^2 + D^2 - 2Dr_1 \cos \theta_1)^2} - \frac{1}{D} \right] \\ \sigma_y &= \frac{2P}{\pi l} \left[\frac{\cos^3 \theta_1}{r_1} + \frac{(D - r_1 \cos \theta_1)^3}{(r_1^2 + D^2 - 2Dr_1 \cos \theta_1)^2} - \frac{1}{D} \right] \\ \sigma_{xy} &= \frac{2P}{\pi l} \left[\frac{\sin \theta_1 \cos^2 \theta_1}{r_1} - \frac{(D - r_1 \cos \theta_1)^2 r_1 \sin \theta_1}{(r_1^2 + D^2 - 2Dr_1 \cos \theta_1)^2} \right]\end{aligned}\quad (4)$$

Creating a rectangular coordinate system $O-x-y$ at the center of disc by taking the point O for the origin (Fig. 5), the relationship between the rectangular coordinates (x, y) and the polar coordinates (r_1, θ_1) of point E is

$$\begin{cases} x = r_1 \sin \theta_1 \\ y = \frac{D}{2} - r_1 \cos \theta_1 \end{cases} \rightarrow \begin{cases} r_1 = \sqrt{\left(\frac{D}{2} - y\right)^2 + x^2} \\ \cos \theta_1 = \frac{((D/2) - y)}{r_1}, \sin \theta_1 = \frac{x}{r_1} \end{cases}\quad (5)$$

Substituting (5) into Eq. (4), we get

$$\begin{aligned}\sigma_x &= \frac{2P}{\pi l} \left\{ \frac{((D/2) - y)x^2}{(((D/2) - y)^2 + x^2)^2} + \frac{((D/2) + y)x^2}{(((D/2) + y)^2 + x^2)^2} - \frac{1}{D} \right\} \\ \sigma_y &= \frac{2P}{\pi l} \left\{ \frac{((D/2) - y)^3}{(((D/2) - y)^2 + x^2)^2} + \frac{((D/2) + y)^3}{(((D/2) + y)^2 + x^2)^2} - \frac{1}{D} \right\} \\ \tau_{xy} &= \frac{2P}{\pi l} \left\{ \frac{((D/2) - y)^2 x}{(((D/2) - y)^2 + x^2)^2} + \frac{((D/2) + y)^2 x}{(((D/2) + y)^2 + x^2)^2} - \frac{1}{D} \right\}\end{aligned}\quad (6)$$

Fig. 6 shows the distributions of $\sigma_x, \sigma_y, \tau_{xy}$ in the disc.

From formula (6) and Fig. 6, it is obvious that the points C and F are singular points in the stress field, and there is stress concentration phenomenon at the region near C and F . However, the phenomenon disappear at the region far away the force acting point. The sections on which the strain gauges are pasted are the center part of the disc, which is far away from the points C and F . Therefore, the validity of the method presented in this paper is certainly not affected.

3. Estimation of tensile elastic modulus of rock

3.1. The method and theory

The stress field of the Brazilian disc based on elasticity mechanics for isotropic rock materials has been known completely according to formula (6). It is the theoretical foundation for measuring the tensile elastic modulus of rock through the strain measurement.

The constitutive equation expresses the relation between the stress and the strain through Young's modulus and Poisson's ratio. Accordingly, as long as the strain and the force applied are recorded in experiment by strain gauge and force sensor, the tensile elastic modulus can be determined with the slope of the line section of stress–strain curve according to the constitutive equation. Therefore, the most principal problem needed to be solved is to record the stress–strain curve of elastic stage when rock is in tensile condition. The method adopted in this paper is that two strain gauges are pasted, respectively, at the center on the both side faces of disc along the direction perpendicular to the line load P to record tensile deformation of the center part, and a force sensor is used to record the line load P applied. The stress and strain data are picked by stress–strain acquisition system in experiment. At last, the tensile elastic modulus can be determined by using the computer processing system in which the strain used

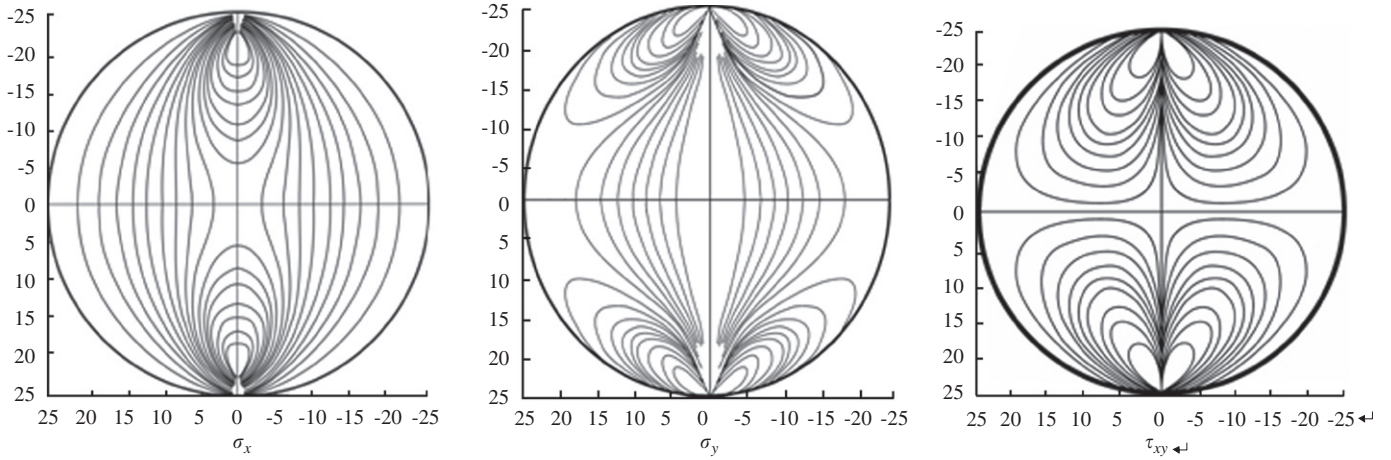


Fig. 6. The distributions of the stress analytic solution in Brazilian disc ($D = 50$ mm).

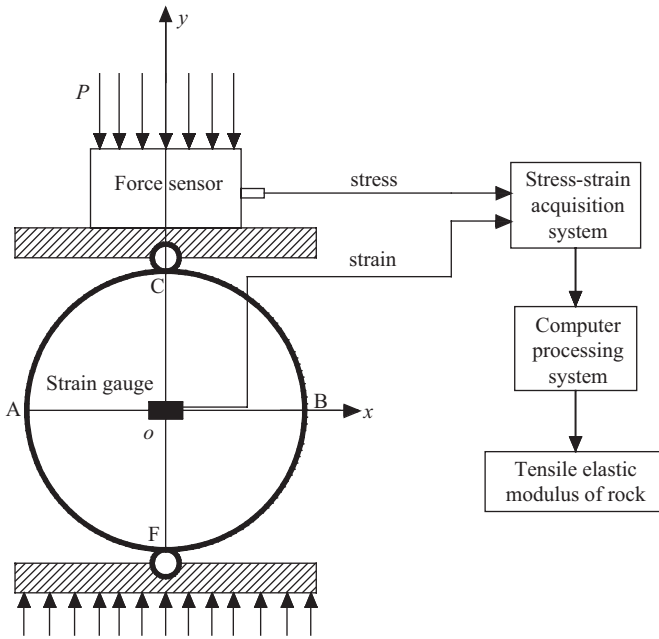


Fig. 7. The method of measuring tensile elastic modulus with Brazilian disc.

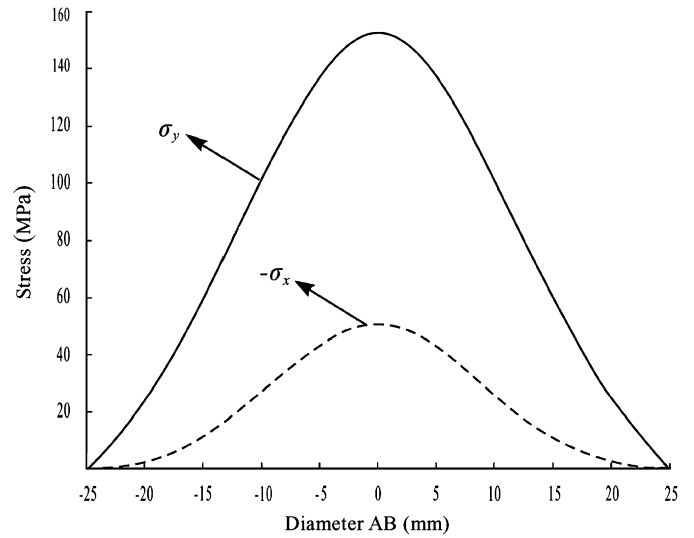


Fig. 8. The distribution of the stress on diameter AB ($D = 50$ mm).

in calculating and plotting should be the average value of the two strain gauges (Fig. 7).

The measuring theory is illustrated in detail in the following. Assuming $y = 0$ in formula (6), we obtain the following expression for the stress state on the diameter AB :

$$\begin{cases} \sigma_x = \frac{2P}{\pi D l} \left\{ \frac{16D^2 x^2}{(4x^2 + D^2)^2} - 1 \right\} \\ \sigma_y = \frac{2P}{\pi D l} \left\{ \frac{4D^2}{(4x^2 + D^2)^2} - 1 \right\} \\ \tau_{xy} = 0 \end{cases} \quad (7)$$

The distribution of the stress on the diameter AB is showed in Fig. 8. From Fig. 8 we can know that the compressive stress σ_y is two times than the absolute value of the tensile stress σ_x at the

center part of Brazilian disc. Consequently, the tensile strain generated by compressive stress σ_y due to Poisson's effect cannot be ignored in estimation of tensile elastic modulus.

The tensile strain at the center part of disc along the diameter AB in the range the strain gauges are pasted can be calculated with the following expression:

$$\varepsilon_t = \frac{1}{2L} \int_{-L}^L \frac{1}{E_t} (-\sigma_x + \nu \sigma_y) dx = \frac{1}{E_t L} \int_0^L (-\sigma_x + \nu \sigma_y) dx \quad (8)$$

where L is the half-length of the strain gauges pasted at the center part of disc, E_t is the tensile elastic modulus of rock, and ν is Poisson's ratio. The tensile stress is intended to be negative in this paper, so an extra minus sign is appended to σ_x in formula (8). Substituting formula (7) into (8), we obtain

$$\varepsilon_t = -\frac{2PD}{\pi l E_t L} \int_0^L \frac{16y^2}{(4y^2 + D^2)^2} dy + \frac{2\nu P D^3}{\pi l E_t L} \int_0^L \frac{4}{(4y^2 + D^2)^2} dy + \frac{2P}{\pi D l E_t} (1 - \nu) \quad (9)$$

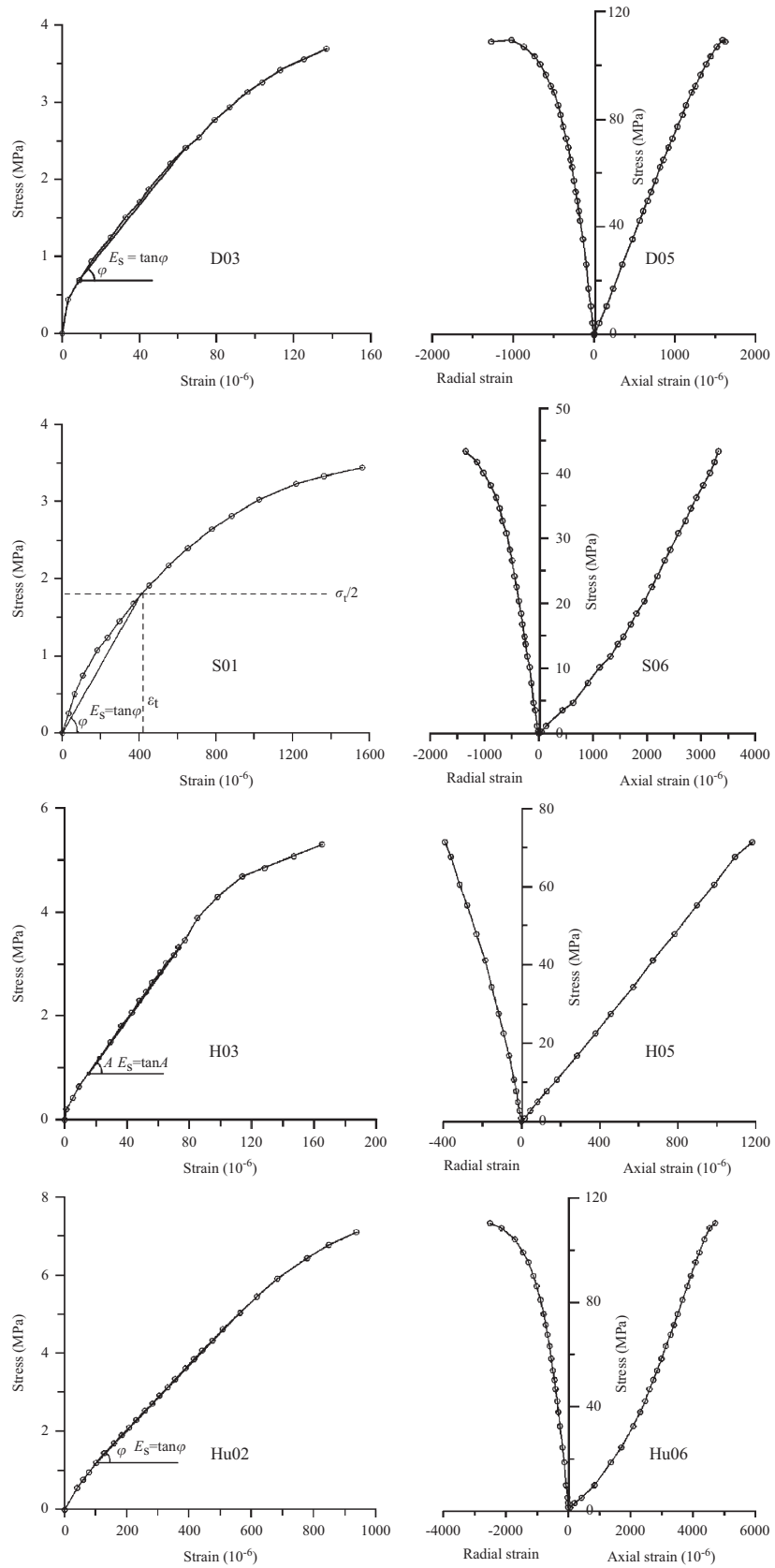


Fig. 9. The typical stress–strain curves recorded in the Brazilian test and the uniaxial compressive test.

The two definite integrals in above expression (9) can be worked out analytically as follows:

$$\int_0^L \frac{16x^2}{(4x^2 + D^2)^2} dx = \frac{1}{D} \arctan \frac{2L}{D} - \frac{2L}{4L^2 + D^2}$$

$$\int_0^L \frac{4}{(4x^2 + D^2)} dx = \frac{1}{D^3} \arctan \frac{2L}{D} + \frac{2L}{D^2(4L^2 + D^2)} \quad (10)$$

If the formula (10) is substituted into (9), the following expression is obtained:

$$\varepsilon_t = \frac{2P}{\pi D I E_t} \left\{ \left(1 - \frac{D}{L} \arctan \frac{2L}{D} \right) (1 - \nu) + \frac{2D^2(1 + \nu)}{4L^2 + D^2} \right\} \quad (11)$$

Exchanging the ε_t and E_t in formula (11), and taking $E_s = 2P/\pi D I \varepsilon_t$, we obtain

$$E_t = E_s \left\{ \left(1 - \frac{D}{L} \arctan \frac{2L}{D} \right) (1 - \nu) + \frac{2D^2(1 + \nu)}{4L^2 + D^2} \right\} = A^* E_s \quad (12)$$

$$A = \left(1 - \frac{D}{L} \arctan \frac{2L}{D} \right) (1 - \nu) + \frac{2D^2(1 + \nu)}{4L^2 + D^2} \quad (13)$$

where E_t is the tensile elastic modulus of rock, E_s is defined as splitting elastic modulus which can be determined from the stress–strain curve recorded in the Brazilian test, ν is Poisson's ratio. A is a correction coefficient which is related to D , L and ν . From formula (12), we see that after the diameter of Brazilian disc D and the half-length of strain gauge L are known; Poisson's ratio ν has been determined with the uniaxial compression test; and the splitting elastic modulus E_s is obtained which is equal to the slope of line section of the stress–strain curve recorded in the Brazilian test. Then the tensile elastic modulus E_t can be determined easily and conveniently.

In fact, the stress–strain curve recorded in test sometime shows the properties of non-linearity. There is no obvious linear section in the stress–strain curve in the elastic stage. Accordingly, if there is no obvious linear section in the stress–strain curve due to a high degree of non-linearity, the parameter of splitting elastic modulus E_s would be determined by the following formula:

$$E_s = \frac{(1/2)\sigma_t}{\varepsilon_t} \quad (14)$$

where σ_t is the maximum stress in test, namely tensile strength. $\sigma_t/2$ is the stress in stress–strain curve which is half of the tensile strength. ε_t is the strain related to $\sigma_t/2$ in the stress–strain curve (Fig. 9—S01). Actually, this suggestion of determining elastic modulus is also proposed by some specification in China.

3.2. The experimental results for the measuring method and theory

Four kinds of rock materials are used in the experiment: marble, limestone, sandstone and granite. These specimens are comparatively homogenous and fine grained. Therefore, they can be treated as approximately isotropic. Six specimens are prepared for each kind of rock and they are all circular cylinders. Three specimens of the six are used for uniaxial compression test; their height and diameter are about 100 and 50 mm, respectively. The other three are used for the Brazilian tests; their height and diameter are about 25 and 50 mm, respectively. The compressive strength σ_c , Poisson's ratio ν , and compressive elastic modulus E_c can be determined by the uniaxial compression test. The tensile strength σ_t and splitting elastic modulus E_s can be determined by the Brazilian test. The half-length L of strain gauge used in experiment is 5 mm. The loading rate for the Brazilian test and the uniaxial compressive test is 200 and 2000 N/s, respectively. Those

Table 1
The experimental results of the uniaxial compression test

Rock materials	Serial number	σ_c (MPa)	ν	E_c (GPa)
Marble	D04	70.32	0.353	77.54
	D05	109.34	0.320	75.31
	D06	113.44	0.317	77.72
	Average value	97.70	0.330	76.86
Sandstone	S04	28.84	0.199	7.39
	S05	40.1	0.112	11.86
	S06	43.4	0.197	10.8
	Average value	37.45	0.169	10.02
Limestone	H04	68.67	0.222	54.17
	H05	71.43	0.268	60.25
	H06	72.66	0.202	58.77
	Average value	70.92	0.231	57.73
Granite	Hu04	125.26	0.226	21.85
	Hu05	104.1	0.166	19.45
	Hu06	110.42	0.181	19.56
	Average value	113.26	0.191	20.29

Table 2
The experimental results of the Brazilian test ($L = 5$ mm)

Rock materials	Serial number	ν	D (mm)	A	E_s (GPa)	E_t (GPa)
Marble	D01	0.330	50.0	1.905	33.4	63.6
	D02		50.0		32.6	62.1
	D03		50.0		39.2	74.7
	Average value		50.0		35.1	66.8
Sandstone	S01	0.169	50.0	1.439	4.22	6.1
	S02		50.0		4.99	7.2
	S03		50.0		5.49	7.9
	Average value		50.0		4.9	7.1
Limestone	H01	0.271	55.5	1.749	22.57	39.5
	H02		55.5		24.34	42.6
	H03		55.5		27.25	47.7
	Average value		55.5		24.72	43.3
Granite	Hu01	0.191	50.0	1.502	9.31	14.0
	Hu02		50.0		9.25	13.9
	Hu03		50.0		9.40	14.1
	Average value		50.0		9.32	14.0

stress–strain curves have been recorded by applying the measuring method stated formerly. Fig. 9 shows the typical stress–strain curves of the Brazilian test and the uniaxial compressive test in the laboratory. The experimental results are shown in Tables 1 and 2.

We can see from Tables 1 and 2 that the tensile elastic modulus E_t of the four kinds of rock materials are all less than their compressive elastic modulus E_c . The average value of the E_t , E_c and their ratio of the four kinds of rock materials are listed in Table 3.

The experimental results show that the ratio between average tensile elastic modulus E_t and average compressive elastic modulus E_c of the four kinds of rock materials are 87%, 71%, 75%, and 69%, respectively. The scope of these ratios is about 60–90%. This result is roughly consistent with the conclusion of Ref. [22].

Table 3The average value of E_t , E_c and their ratio of the four kind of rock materials

Rock	Marble	Sandstone	Limestone	Granite
E_t	66.8	7.1	43.3	14.0
E_c	76.86	10.02	57.73	20.29
Ratio (%)	86.9	70.9	75.0	69.0

Table 4The value of A corresponding to different Poisson's ratio ν ($D = 50$ mm, $L = 5$ mm)

ν	0.05	0.1	0.15	0.2	0.25	0.3	0.35
A	1.094	1.239	1.384	1.528	1.673	1.818	1.963

4. Discussion

From formula (6), the compressive stress σ_y is about two times than the absolute value of the tensile stress σ_x at the center part of the Brazilian disc. Therefore, the tensile strain generated by compressive stress σ_y due to Poisson's effect must be considered, unless the measuring result is inaccurate. However, the tensile strain contributed by σ_y usually accounts for a small proportion of the total tensile strain, because Poisson's ratio ν usually is a small value of 0.1–0.3. That is to say, the tensile stress σ_x contributes the majority in the total tensile strain.

From formula (12), we know that the determination of the tensile elastic modulus E_t is not only affected by the diameter of disc D and the half-length of strain gauge L , but also by Poisson's ratio ν . Furthermore, Poisson's ratio ν is the most important effective factor whose accuracy directly determines the reliability of the E_t . Table 4 lists the different values of A when Poisson's ratio is 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, and 0.35 ($D = 50$ mm, $L = 5$ mm). Table 4 indicates that the A is very sensitive to Poisson's ratio. However, frequently, there is uncertainty when measuring Poisson's ratio ν in the laboratory with uniaxial compressive tests. Therefore, in order to obtain reliable value of E_t , the uniaxial compressive test must be performed carefully and at least three specimens are used for each type of rock.

As stated above, it is difficult for us to perform the direct tension test in laboratory. Consequently, the experimental data of the direct tension test for the four kinds of rock have not been acquired. Therefore, there is no way to compare the results obtained by applying the measuring method proposed in this paper with the results obtained from direct tension test. It is a deficiency, and also a research direction later.

Lots of rock types show anisotropy or transverse isotropy. How to measure or estimate their tensile elastic modulus at all directions with the Brazilian disc is also worthy of research further later.

5. Conclusions

The stress analytic solution of rectangular coordinates form for the Brazilian disc (formula (6)) are given based on the results obtained by Muskhelishvili for isotropic rock materials. The origin of coordinates is the center of Brazilian disc. It is the theoretical foundation for determining the tensile elastic modulus with Brazilian disc.

The tensile elastic modulus E_t is an important parameter, which characterizes the tensile property of rock materials. But it is difficult to obtain attributing to the fact that the direct tensile test

is difficult to perform in laboratory. A new measuring method with the Brazilian disc is proposed in this paper that can determine the E_t easily and conveniently (Fig. 7). The method is that two strain gauges are pasted, respectively, at the center on the both side faces of disc along the direction perpendicular to the line load P (Fig. 3) which are used to record tensile strain of the center part, and a force sensor is used to record the force applied; then the splitting elastic modulus E_s can be obtained from the slope of the line section of the stress–strain curve recorded or according to formula (14); the ν are determined by uniaxial compression test; finally, the E_t can be determined according to formula (12). Additionally, a new experimental set-up is invented which is used to provide the line concentration loads (Fig. 2).

Our experimental results show that the E_t of rock materials are all less than their E_c , and the ratio between E_t and E_c of rock materials, including marble, sandstone, limestone, and granite is about 60–90%. These results are basically consistent with the conclusion of Ref. [22]. Consequently, the flexibility, maneuverability, and the reliability of the measuring method proposed here are shown to be good.

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