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Harmonic Interaction between the Power Inverters and the Distribution Network in Terms of Stochastic Dependence: a Case Study in Tehran's Network

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Abstract :
This paper introduces modeling of the probabilistic dependence between different structures of power system devices, employing the "*Copulas*" analytical tool for correlating among multivariate outcomes. Copulas have become a popular analytical tool in multivariate modeling, where recently has been applied in many fields. Here, the basic properties and theorems of Copulas along with their contributions to Monte Carlo method are described. A case study has been performed on a distribution substation in Tehran, in which enormous information was gathered using an installed data logger. Then, investigation is carried out based on the measured data. The recorded data paves the way for pursuing further analysis that is associated with simulating statistical correlation between uncharacteristic harmonics and realistic unbalanced conditions for a voltage-source inverter at the point of common coupling (PCC).

1. Introduction

In power systems analysis, it is preferred to use completely analytical and deterministic methods anywhere possible; that is, the equations describing system models were solved either explicitly or numerically. In order to finding these solutions we often had to make simplifying assumptions and approximations, alluding at the same time to the possibility of obtaining more accurate results with the help of simulation methods. In fact, simulation methods can often be the only means of obtaining the solution to the system model, especially when the systems studied is large and complex or when the effects of certain sequences of events are of a particular interest or when the probability distributions, rather than only the means and variances, are required.

Monte Carlo simulation as a numerical simulation is a process of selecting a set of values of system parameters and obtaining a solution of the system model for a selected set. By repeating the simulation process for different sets of system parameters, different sample solutions are obtained. The key activity in the Monte Carlo simulation process is the selection of system parameters to obtain sample solutions; which is applied to problems involving random variables with known, modeled or assumed probability distributions. A sample from a Monte Carlo simulation is similar to a sample of experimental observations. Therefore, the results of these studies, as modeled samples, can then be used to study mathematical models of real-world systems, or for statistical studies.

Systematic generation of the appropriate values of the random variables in accordance with the respective prescribed probability distribution is accomplished by first generating a uniformly distributed random number between 0 and 1 and then, through appropriate transformations, obtaining the corresponding random number with the specified probability distribution. However, one of the main difficulties associated with the application of the analytical methods in probabilistic power system studies is that the random variables are often not independent and that the joint probability distribution functions must be used. This introduces an additional difficulty in the already complex problems, and therefore the majority of analytical approaches assume independence of the random variables or somehow inaccurate dependencies through the correlations only.

One of the main applications of copulas is in the Monte Carlo studies where a multivariate dependency structure exists [1]. Using of copulas fits the stochastic modeling of dependant chaotic variables and time series in power systems well. They can efficiently used to produce non-conventional multivariate distributions for Monte Carlo studies. On the other hand, the use of copulas for modeling purposes includes two straightforward steps. The first step is modeling the marginal distributions along with their correlation matrix and the second consists in fitting the proper copula. It should be mentioned that it is an obscure task to find a multivariate distribution and fit it to our data. The use of copulas is practical as some good software packages have already provided its complete implementation (such as [2], [3]).

The concepts of stochastic analysis in power systems from the viewpoints of designing, planning and operation are rather modern phenomena. Generally speaking, there are four main circumstances that necessitate using of such methods:

1) Generation, demand and configuration uncertainty mainly due to the prevalence of the renewable and distributed sources (e.g. [4], [5]);
2) measurement inaccuracy (e.g. [6]);
3) modeling and forecasting uncertainty (e.g. [7], [8]);
4) uncharacteristic and parametric aggregate uncertainty (e.g. [9], [10]).

The accurate deterministic modeling of such circumstances is not trivial [11] and a quantitative or a qualitative uncertainty modeling is required. Dealing with these uncertainties in power systems refer to a vast amount of research work, many related to reliability studies farther ahead (e.g. see the bibliographies in [12]). Recently, more attention is focused on the uncertainties due to the prevalence of the renewable and distributed resources in the deregulated environment. With regard to uncertainty studies addressing parametric and aggregate aftermath or complex dependence structures in power systems, there are fewer papers, and these mainly consider uncertainties using case study experiments [13] or assume simplifying presumptions [14].

Relevant methods to the stochastic uncertainty analysis combine deterministic simulation techniques with stochastic analyses. These do not consider uncertainties in parameter values, and neglect the modeling of dependence structures between parameters of an integral system. While these studies are important in their own right, they lack adequate accuracy addressing the parametric uncharacteristic uncertainties related to the operation of power electronic devices in interaction with each other and the distribution network.

In this context, the use of an integrated deterministic and probabilistic simulation algorithm may be the forthcoming solution if the stochastic dependence structures are modeled in addition to the deterministic dependence in a system with its interacting devices [22]. Therefore, the key point of such an analysis is the modeling of stochastic dependence that can be suitably done using the copulas. The basics for such an analysis are introduced in this paper.

In the following sections, the main theorems and procedures for using copulas are presented. The discussions are followed by a case study which includes the use of copulas in a Monte Carlo simulation for estimating the correlation between uncharacteristic current THD produced by a voltage-source inverter (VSI) and the levels of exchanged reactive power when a realistic unbalance exists at the point of common coupling (PCC).

2. Principles of Copulas and Dependence

2.1. Basic Definitions

According to [1], copulas are "functions that join or couple multivariate distribution functions to their one-dimensional marginal distribution functions" or equivalently in terms of mathematical representation [15], a copula is a function C of n variables on the unit *n-cube* $[0,1]^n$ with the following properties:

1) The range of C is the unit interval [0,1];
2) $C(\mathbf{u})$ is zero for all $\mathbf{u}=(u_1,\ldots,u_n)$ in $[0,1]^n$ for which at least one coordinate equals zero;
3) $C(\mathbf{u})=u_k$ if all coordinates of $\mathbf{u}$ are 1 except the k-th one;
4) C is n-increasing in the sense that for every $\mathbf{a} \le \mathbf{b}$ in $[0,1]^n$ the measure ΔC_a^b assigned by C to the n-box $[\mathbf{a},\mathbf{b}]=[a_1,b_1]\times\cdots\times[a_n,b_n]$ is nonnegative, i.e.

$$\Delta C_a^b := \sum_{(\varepsilon_1,\ldots,\varepsilon_n)\in\{0,1\}^n} (-1)^{\sum_{i=1}^{n}\varepsilon_i} C\big(\varepsilon_1 a_1 + (1-\varepsilon_1)b_1,\ldots,\varepsilon_n a_n + (1-\varepsilon_n)b_n\big) \ge 0 \quad (1)$$

where, n is the number of dependent outcomes that should be modeled and all marginal distributions of the random vector $\mathbf{u}=(u_1,\ldots,u_n)$ are uniform. It can be illustrated from the definition that copulas have many useful properties, such as uniform continuity and existence of all partial derivatives.

To complete the construction of copula, a set of arbitrary marginal distribution functions can be assumed and therefore, the C defines a multivariate distribution function evaluated at $x_1, x_2, \ldots, x_n$ as:

$$C\big[F_1(x_1),F_2(x_2),\ldots,F_n(x_n)\big] = F(x_1,x_2,\ldots,x_n). \quad (2)$$

Sklar [16] showed that any multivariate distribution function F can be written in the form of (2) that is using copula representation. He also showed that if the marginal distributions are continuous, then there is a unique copula representation. The aforementioned statements are the key theorem of copulas referred to as *Sklar's theorem* and clarify the relations of dependence and the copula of a distribution. It should be mentioned that constructing multivariate distributions without the concept of copula has some drawbacks such as (1) necessarily a different family is needed for each marginal distribution, (2) extension to more than just the bivariate case are not clear, and (3) measures of association often appear in the marginal distributions. The use of copulas, however, does not suffer from these drawbacks.

2.2. Correlation Measures

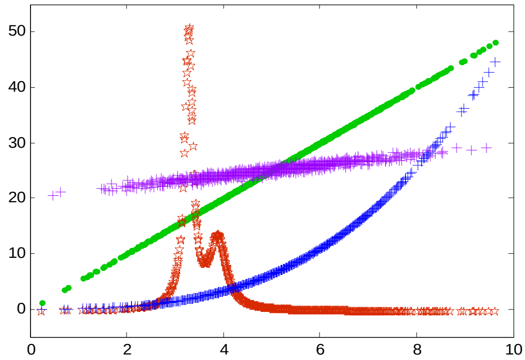
To continue with the concepts of applying copula fits to the simulation of dependency, a brief reminding of *correlation* and its measures seems to be necessary. The familiar form of correlation is the Pearson's pairwise linear coefficient and defined as

$$\rho(X,Y)=\frac{\mathrm{cov}[X,Y]}{\sqrt{\sigma^2[X]\sigma^2[Y]}}. \qquad (3)$$

This correlation has some shortcomings to measure the strength of dependence other than the linear case: (1) it is not ideal for a dependence measure of heavy-tailed marginal distributions; (2) zero correlation for a joint distribution with a non-linear relationship may not reflect its dependence other than for the multivariate normal case; and (3) it is not invariant under non-linear scale transformations. Instead, a rank correlation coefficient, such as Kendall's τ or Spearman's ρ, is more appropriate to fulfill the desirable characteristics of a measure of dependence. Spearman's rank correlation has been used in the succeeding analysis which is given by

$$\rho_S(X,Y)=\rho\left(F_X(X),F_Y(Y)\right) \qquad (4)$$

where, $F_X(X)$ and $F_Y(Y)$ are the distribution functions of the random variables X and Y respectively. It should be mentioned that for the jointly normal distribution, Spearman's rank correlation is almost identical to the linear correlation; however, this is not true when transformations apply. The performance of the Pearson's linear correlation versus the Spearman's rank correlation is illustrated in Fig. 1, where their values are shown with different deterministic transformations. It is obvious that the rank correlation provides a more desirable measure of dependence than the linear correlation. Note that there is a deterministic relationship and a good measure should equal one in any case (Pearson's ρ completely fails in the 3rd extremely non-linear case).



Sign	Relationship between axis	Pearson's ρ	Spearman's ρ
•	$5x$	1.000	1.000
+	$0.05 \times x^3$	0.9308	1.000
*	Humps curve	-0.4355	0.9806
†	Pre-specified ρ of 0.95	0.9522	0.9486

Fig. 1. Comparison of different dependence measures.

2.3. Modeling of Stochastic Dependence

There are various situations in the applications of power system analysis where we might wish to simulate dependent random vectors and arrangements (as evidenced by Monte Carlo algorithms). Examples of such applications are in the noise modeling, reliability studies, materials and natural phenomena uncertainty, risk assessment, complex modeling and etc. Randomly behaving variables of such circumstances may be assumed completely dependent, linearly correlated, superposed, or completely independent; the most appropriate choice is influenced by several factors such as the characteristics of the system and the required accuracy. In the power system problems, anyhow, many cases involve high levels of dependency. Therefore, it is very tempting to approach the problem in the following way:

1) Estimate matrix of pairwise rank correlations,
2) estimate marginal distributions,
3) combine this information using a copula.

To perform a simulation, therefore, the following information should be specified from the measured or calculated data:

1) the copula family and any required shape parameters,
2) the rank correlations among variables, and
3) the marginal distributions for each variable.

It should be mentioned that the copula is

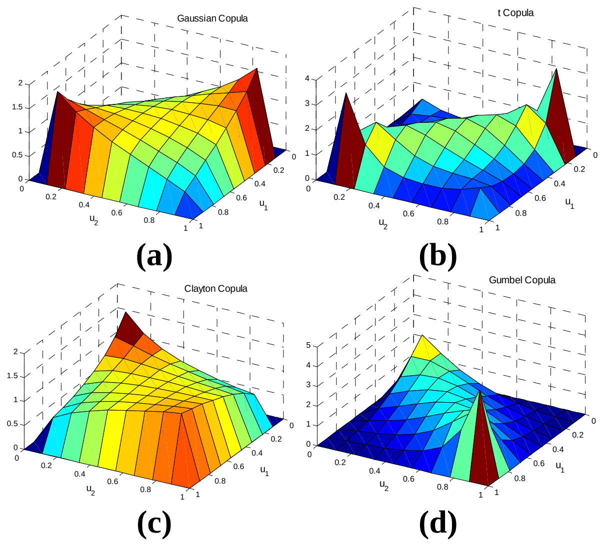


Fig. 2. Samples of different bivariate copula functions: (a) Gayssian copula; (b) t copula; (c) Clayton copula; and (d) Gumbel copula.

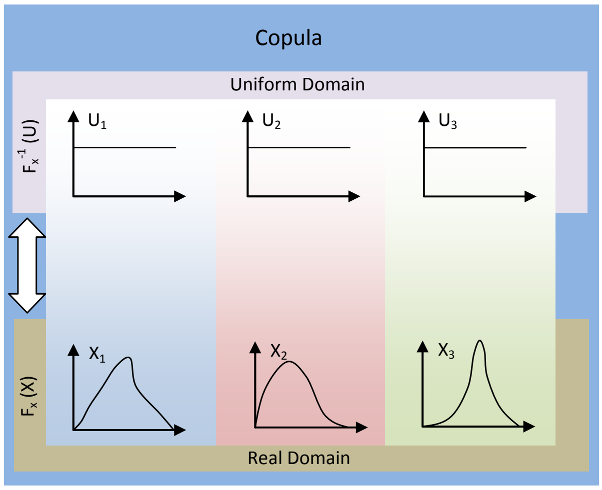


Fig. 3. Copula modeling structure.

assumed to be chosen by the modeler based on his experience, ease of use and analytical tractability. The most commonly used copulas are the Gaussian copula for linear correlation, Gumbel copula for extreme distributions, and the Archimedean copula and the t-copula for dependence in tail [1], [17]. Some popular copulas are shown in Fig. 2.

Furthermore, a realistic correlation matrix must be positive semi-definite, real-valued, and symmetric. Proper modeling of such a matrix based on the realistic data is an important stage of the algorithm; specifically when there is some data misplacement or the values are noisy, unavailable, or inapt. There are practical methods to deal with the problems associated with the non-proper correlation matrices [18]. Anyhow, these methods generally apply to abnormal data recordings and become conspicuous in Gaussian copula construction algorithms.

Another key point in a reliable dependency modeling is building the marginal distributions. One could fit a parametric model separately to each dataset, and use those estimates as the marginal distributions; however, a parametric model may not be sufficiently flexible. Instead, a nonparametric model to transform to the marginal distributions seems to be appropriate. Meanwhile, using empirical cumulative distributions results in a discrete representation which may not desirable for a continuous distribution. Therefore, it is advisable to apply a smoothing technique such as kernel smoothing or interpolate between the midpoints of the steps with a piecewise linear function. The general modeling structure based on copulas is illustrated in Fig. 3.

2.4. Simulation Algorithm

For the simulation, it is a good idea to experiment with different copulas and correlations. The two main simulation strategies are the Archimedean and compounding methods [19]. Both methods can be easily implemented for more than two dimensions (multivariate case). Nonetheless, the compounding algorithms are computationally more straightforward than the conditional distribution approach used in Archimedean methods. Meanwhile, it requires the generation of an additional variable which can be computationally expensive in applications. Regarding the fact that the power system problems typically require extensive calculations, addition of extra variables may not be acceptable. Therefore, as popularly used in most software [3], the Archimedean construction is used in this paper. One method is briefly as follows [19]:

1) Generate independent uniform random numbers $U_1, U_2, \ldots, U_n$.
2) Set $X_1 = F_1^{-1}(U_1)$ and $c_0 = 0$.
3) For $k = 2, \ldots, n$, recursively calculate X_k by

$$U_k = F_k\left(X_k \middle| x_1,\ldots,x_{k-1}\right) = \frac{\Phi^{-1(k-1)}\left\{c_{k-1} + \Phi\left[F_k\left(x_k\right)\right]\right\}}{\Phi^{-1(k-1)}\left(c_{k-1}\right)}. \quad (5)$$

This algorithm is to generate $X_1, X_2, \ldots, X_n$ having modeled distribution function of (1), the copula is

$$C\left(u_1,u_2,\ldots,u_n\right) = \Phi^{-1}\left[\Phi\left(u_1\right)+\cdots+,\Phi\left(u_n\right)\right], \quad (6)$$

and $c_k = \Phi\left[F_1\left(x_1\right)\right]+\cdots+\Phi\left[F_k\left(x_k\right)\right]$.

Equation (6) defines a class of copulas known as Archimedean. The Archimedean representation allows us to reduce the study of a multivariate copula to a single univariate function. The function Φ is a generator of the copula and uniquely determines it.

Given a dataset, choosing a copula to fit the data is an important but difficult problem. Since the real data generation mechanism is unknown, it is possible that several candidate copulas fit the data reasonably well or that none of the candidates fit the data well. When maximum likelihood method is used, the general practice is to fit the data with all the candidate copulas and choose the ones with the highest likelihood. Considering the maximum likelihood, the Frank copula is chosen in this paper because it fits the studied data well.

The technique for random vectors can be applied for time series as well [20]. A moving window with a certain number of vectors is taken as a sample vector for a stationary time series. The marginal distributions and the copula are then estimated with this sample according to the above algorithm.

In the following we demonstrate a novel application. First, the recorded three-phase active and reactive powers at a distribution substation in Tehran, Iran and their stochastic dependence are realistically modeled using a copula based on the algorithm proposed by [22]. Second, a Monte Carlo simulation is applied to a VSI for estimating the correlation between uncharacteristic harmonic distortion, levels of unbalance, and the exchanged reactive power at the PCC.

3. Stochastic Dependency between Uncharacteristic Harmonics of a VSC and its Reactive Power Exchange

In practice, any VSC-based applications such as static compensators, act as a source of producing harmonics for power systems. It could also interact with possible harmonic distortions and unbalances of the power network. These interactions would be complex, making the deterministic analysis of steady-state harmonic levels and full assessment of their dynamic behavior difficult tasks [9]. Therefore, it is required to pursue evaluation of these harmonic interactions through a suitable combination of measurements and statistical simulation studies. Here, we present a novel methodology for analyzing such interactions. To demonstrate a practical application, a VSC with Selective Harmonic Elimination (SHE) is considered to be used as a reactive power compensator.

Selective harmonic elimination techniques use pre-calculated switching angles based on assuming ideal conditions (e.g. fixed dc bus voltage). This method presents several advantages in comparison to the conventional carrier-based sinusoidal PWM schemes [21]. On the other hand, load-terminal harmonics and unbalance of the distribution system impose distortion on both dc and ac sides introducing additional uncharacteristic harmonics generated by the VSC. Considering the SHE, the pre-calculated chopping angles will not then be optimal under these conditions. Hence, the amount of uncharacteristic harmonics that is injected to

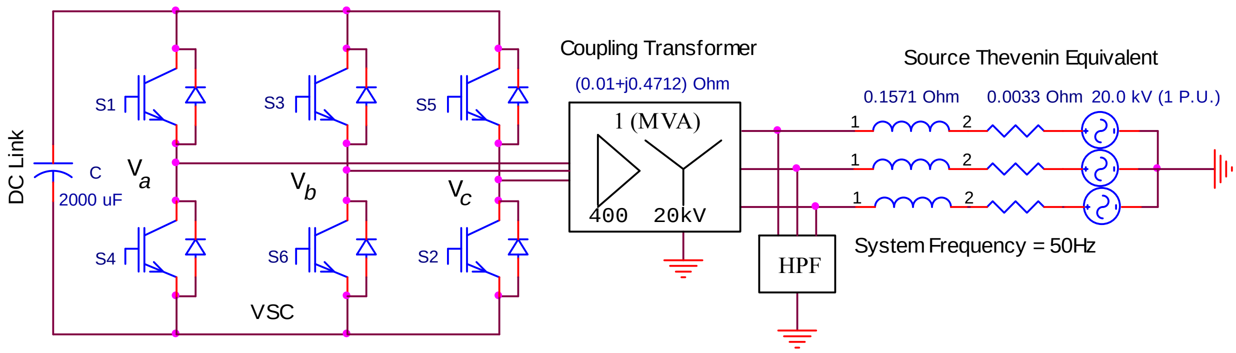


Fig. 4. Recorded three-phase active and reactive powers by data logger at 400 V substation of Alestom.

the distribution system stochastically depends on several factors such as operating conditions of the VSC [9].

The evaluation of this stochastic dependency should embrace all of the realistic uncertainties in the network. It should also reckon with the harmonics sensitivity to the non-optimal chopping angles. In this study, a Monte Carlo simulation based on the described copula approach is proposed to this problem. The emphasis is on the estimation of the degrees of dependence between a realistic voltage unbalance of the network, the produced uncharacteristic harmonics, and the reactive power operating point.

The harmonic-domain model and topology of [9] is used here in order to provide the required calculation efficiency and accuracy. A schematic diagram of the network is shown in Fig. 4. The ac system on the high voltage side of the transformer is rated at 20 kV. Five chopping angles are used to control in steady state, thereby permitting the regulation of the fundamental component while eliminating the 5th, 7th, 11th, and 13th harmonics.

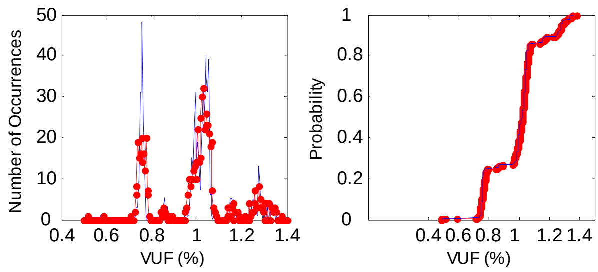


Fig. 5. Recorded vs. simulated VUF% considering stochastic dependency in the system load.

First, the three-phase active and reactive powers data, as simulated in [22], are used to estimate the statistical behavior of the voltage unbalance by a three-phase load flow calculation. The estimation result is shown in Fig. 5 which demonstrates a good conformity

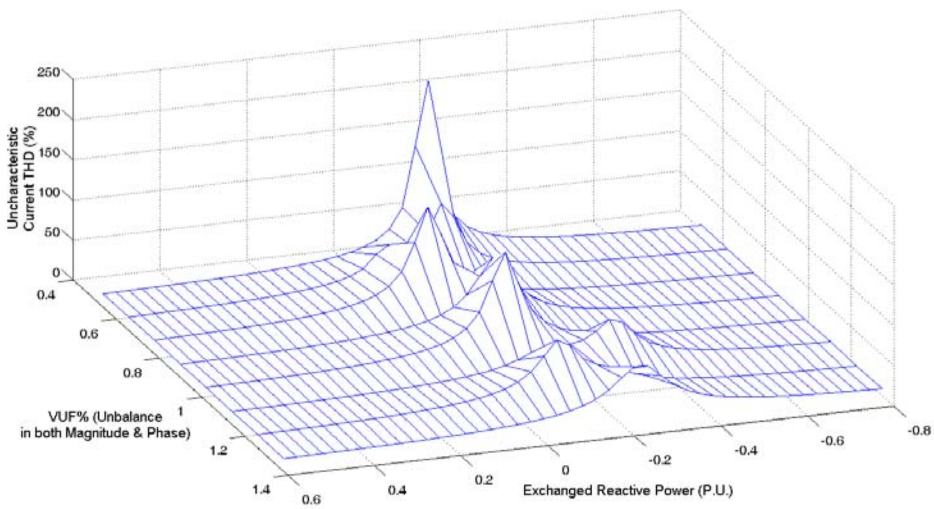


Fig. 6. Simulation of the penetrated uncharacteristic current THD based on the measured unbalanced powers by a surface plot.

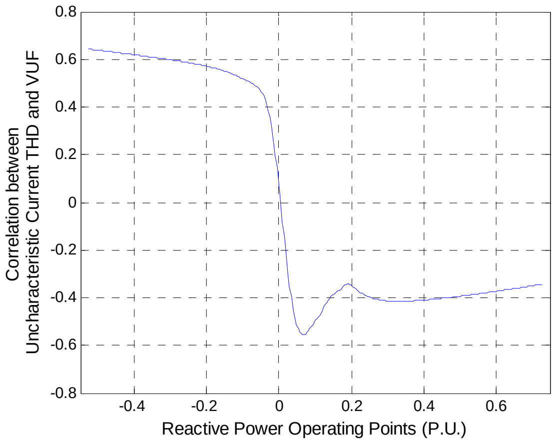


Fig. 7. Variation pattern of the correlation between uncharacteristic current THD and the exchanged reactive power of the VSC.

with the measured data. The obtained accuracy is owing to the fact that the dependence structure of six powers is taken into account by a suitable copula.

Then, The VSC-based compensated network of Fig. 4 is considered along with the simulated voltage unbalance. Furthermore, background harmonics are represented by voltage harmonic source obtained from field measurements. If the capacitance of the DC-link capacitor is assumed 2 mF, the resultant current THD% is then calculated for all data as shown in Fig. 6. This is a surface plot in which the THD variation over various reactive power loadings is depicted for each observed VUF. Note that the diversity of various probable VUF's, causing different THD's, are also realistically modeled by the proposed procedure.

It can be seen from Fig. 6 that a realistic voltage unbalance would not dramatically modifies the uncharacteristic THD of VSC ac current except for situations that the compensator absorbs relatively small amounts of reactive power; nonetheless, there are rather few observed higher VUF's for which the uncharacteristic THD became very risky.

To further clarify the estimated stochastic dependency, a pattern of variation for the correlation coefficient between the uncharacteristic current THD and the VUF is shown in Fig. 7. This correlation is around zero for very small reactive power loadings; conversely, it arises for higher amounts of reactive power absorption or generation. It is can be inferred from Fig. 7 the pattern of Fig. 6 implicitly.

It should also be noted that the THD is affected by changing the DC-link capacitance. Also, the power system equivalent impedance influences the THD under variation of the voltage unbalance percent. This analysis can be easily extended to include other realistic conditions. In this manner, a purely deterministic analysis will become more complex and the proposed stochastic simulation would be more useful.

4. Conclusion

This paper suggests a new type of analysis related to the deterministic-stochastic dependencies in power system devices and signals with an emphasis on power electronic switching interaction with the network. In order to model stochastic dependencies in a multivariate Monte Carlo simulation, the copula theory has been proposed and briefly introduced. A case study is arranged based on the recorded data from a 20 kV/400 V distribution substation located in Tehran for a one week period. Firstly, the complete dependence structure of the three-phase active and reactive powers is modeled using a copula. As verified with the measured data, it demonstrated useful characteristics with a sufficient accuracy [22].

Secondly, a Monte Carlo simulation is implemented based on the described copula approach to estimate the degrees of dependence between a realistic voltage unbalance of the network, the produced uncharacteristic harmonics, and the reactive power loadings. The presented results are used to evaluate harmonic performance of a VSC-based compensator. Future work will focus on other possible applications and will be extended to include other realistic conditions.

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