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Fuzzy models for analysis of rock mass displacements due to underground mining in mountainous areas

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Abstract

The displacement and deformations of rock mass due to underground mining has often resulted in major disasters throughout the world, frequently inflicting heavy losses of life and damage to property. And these disasters have motivated the development of rock mass mechanics. The prediction of displacement of rock mass and their surface effects is an important problem of the rock mass mechanics in the excavation activities especially the coal and metal mining in mountainous areas. Based on results of the statistical analysis of a large amount of measured data in mining engineering, the fundamental fuzzy model of displacements and deformations of rock mass is established by using the theory of fuzzy probability measures. The theories of both two- and three-dimensional problems are developed and applied to the analysis of engineering problems in excavation and underground mining in mountainous areas. The agreement of the theoretical results with the field measurements shows that our model is satisfactory and the formulae obtained are valid and thus can be effectively used for predicting the displacements and deformations and the safety evaluation of the buildings on the ground.

Keywords: Underground mining; Rock mass displacement; Mountainous areas; Fuzzy probability measure

1. Introduction

In both the underground and surface mining, if a void is excavated in a rock continuum, the load formerly applied on the rock in the opening will be transferred either to the rock surrounding the opening or to the supports (pillars) within the opening or both, and finally to the ground surface, hence resulting in a macroscopically nonuniform deformation of the surface in the horizontal or vertical direction. If the uneven deformation or subsidence (i.e. differential subsidence on the top of the mined-out area) cannot be effectively controlled then it will cause damage and even a disaster, such as deformation or even cracking of buildings, particularly tall buildings. This means that the failure of a building is to a great extent controlled by the presence of differential subsidence rather than the absolute magnitude of subsidence [1–13].

It is difficult to calculate the accurate displacement of every point in a body of rock because of the complexity of the problem. Instead, various approximate methods have been used for this calculation. In recent years, in mining engineering in particular, theory of fuzzy mathematics has been applied to analyze the problems of displacement and deformations of rock mass due to underground mining [2–8]. In fact, the movement of each point at a level of overburden can be regarded as a fuzzy event. In other words, this displacement will take place at a fuzzy probability, and so the theory of fuzzy probability measures can be applied in describing the ground subsidence and deformation of rock mass [8].

In this paper, the application of the fuzzy probability measures to the analysis of the rock mass displacements due to underground mining in mountainous areas is described.

2. Fuzzy mathematical models

We first briefly give several definitions of fuzzy probability [14,15].

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Definition 1. Suppose a triplet (R^n, B, M) is a probability (non-fuzzy) space, where R^n is a sample space and B is the fuzzy σ-field of Borel sets in R^n and M is a probability measure over R^n.

Let fuzzy events A_1 and A_2 be two fuzzy sets in R^n, and f_{A_1} and $f_{A_2}(f_{A_1}, f_{A_2} : R^n \to [0,1])$ be two membership functions which are Borel measurable.

Definition 2. If A_1 and A_2 are two fuzzy sets in R^n, then we can define fuzzy probability measures of A_1 and A_2 as follows:

$$M(A_1) = \int_{R_1} f_{A_1}(x)\,\mathrm{d}p(x), \tag{1}$$

$$M(A_2) = \int_{R_2} f_{A_2}(y)\,\mathrm{d}p(y). \tag{2}$$

Here formulae (1) and (2) are Lebesgue–Stieltjes integrals.

Because f_{A_1} and f_{A_2} are Borel measurable, there exist the Lebesgue–Stieltjes.

Definition 3. Let A_1 and A_2 be two fuzzy sets in the probability space (R^n, B, M). A_1 and A_2 are said to be independent if

$$M(A_1 A_2) = M(A_1) \cdot M(A_2) \tag{3}$$

with the above definitions and theories as a basis, now we shall describe practical engineering problems.

We shall restrict our discussion to the ground subsidence due to ore mining, although a more general subsidence due to excavation may be discussed in a similar manner.

When the subsidence of overburden takes place due to the underground mining, some points in the associated surface will deviate from their equilibrium positions, and so displacement will occur. The positions of these points can be denoted by a fixed rectangular coordinate system, as shown in Fig. 1.

Because a ground subsidence is controlled by many factors such as geologic conditions, the presence of ground water, the properties of rock mass and mining conditions, it is difficult to predict the ground subsidence accurately. Because the theory of fuzzy mathematics is generally taken to embrace the whole field of imprecisely described systems, the above theory of fuzzy probability can be used for this prediction. For this purpose, we first describe the assignment of the membership function for the plane problem.

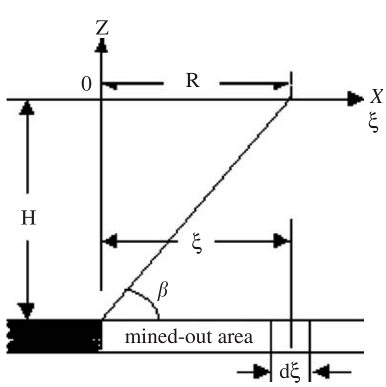

Fig. 1. A rectangular coordinate system of underground mining.

Consider an *xoz* cross-section of overburden, for the points x in which plane the expression for the fuzzy probability of subsidence can be established. The set (denoted by A_1) is called "associated set", whose elements are the points of the x-axis involved in the subsidence. Clearly, A_1 is a fuzzy subset in the set R^n.

Now the membership function for engineering problems can be assigned by the membership function of the subsidence surface to the fuzzy subset A_1 (as mentioned above, A_1 is the set of the points in the subsidence surface). As shown in Fig. 1, we choose a rectangular coordinate system such that the x-axis is directed to the mined-out area (we restrict our attention to the points on the top of the mined-out area), and assign unity to the maximum degree of membership of x to A_1. We assume that following expression holds:

$$f_{A_1}(x) = f(x), \tag{4}$$

where $f(x)$ is a relation function to be determined by the field measured data.

The statistical analysis is made by using a large amount of measured data, and the results obtained show that

$$f(x) = f(R, x) \tag{5}$$

is a transcendental function, where R is a parameter, x are the measured points in the subsidence surface ($x = x_i$; $i = 1, 2, \ldots, n$). From formula (4) we have the following expression of the membership function:

$$f_{A_1}(x) = \exp\left\{-\left[\frac{\sqrt{\pi}(x-\xi)}{\sqrt{2}R}\right]^2\right\}, \tag{6}$$

where R is called "the primary influence radius". It is a parameter determined by such a factor as the mining conditions of a particular mining district and the occurrence of ore deposits:

$$R = \frac{H}{\tan\beta}, \tag{7}$$

where $\tan\beta$ is a fuzzy parameter to be determined by the field measured data, H is a constant.

The above density distribution function (i.e. the distribution function for the surface subsidence) can be established by using the statistical theory of probability. In the case of the mining of ore, the subsidence of ground surface depends on many factors. We shall consider the theory of the plane problems as particular cases of rock mass movements. In the *xoz* plane of the rectangular coordinates system, let $\mathrm{d}p(x)$ be the density function of the ground subsidence at point x in the xoz cross section, then we have

$$\mathrm{d}p(x) = \frac{1}{R}\exp\left[-\frac{\pi}{2R^2}(x-\xi)^2\right], \tag{8}$$

where R *is a* fuzzy parameter.

From Definition 3, we obtain the expression of fuzzy probability for the ground subsidence in the xoz cross-section:

$$M(A_1) = \int_{R_1} f_{A_1}(x)\,\mathrm{d}p(x). \tag{9}$$

The fuzzy probability of slope,

$$M_t(A_1) = \frac{\partial}{\partial x} M(A_1). \tag{10}$$

The value calculated from formula (9) is the fuzzy probability for the ground subsidence. However, the mining thickness and subsidence factor must be taken into account for calculating the practical surface subsidence W:

$$W(x,z) = k_1 k_2 M(A_1), \tag{11}$$

where $W(x, z)$ is the practical ground subsidence; k_1 and k_2 are parameters depending on the mining method and the rock properties, and can be determined from the measured data of mining districts. For example, given the fuzzy probability of surface subsidence in a mining district $M(A_1) = 0.55$, $k_1 = 0.7555$, $k_2 = 1800\,\mathrm{mm}$, then $W(x,z) = 747.9450\,\mathrm{mm}$.

When a subsidence takes place in the underground rock mass, there must exist one subsidence point $P(x, y, z)$ on the corresponding ground surface, and similarly the fuzzy probability $M(A_1A_2)$of the subsidence can be obtained in the three-dimensional case.

In the space rectangular coordinates, let $M(A_1)$ be a subsidence fuzzy probability of the fuzzy event A_1 on the xoz plane, and $M(A_2)$ a subsidence fuzzy probability of the fuzzy event A_2 on the yoz plane. From Definition 3, we can find the subsidence fuzzy probability $M(A_1A_2)$at the point $P(x, y, z)$ to be $M(A_1A_2) = M(B)$, and

$$M(A_1A_2) = M(A_1)M(A_2). \tag{12}$$

In the xoz plane, we have

$$\begin{aligned} M(A_1) &= \int_{D_1} \frac{1}{R}\exp\left\{-\left[\frac{\sqrt{\pi}(x-\xi)}{\sqrt{2}R}\right]^2\right\} \\ &\quad \times \exp\left[-\frac{\pi}{2R^2}(x-\xi)^2\right]\mathrm{d}\xi \\ &= \int_{D_1} \frac{1}{R}\exp\left[-\frac{\pi}{R^2}(x-\xi)^2\right]\mathrm{d}\xi, \\ &\quad D_1 \in (-\infty,+\infty). \end{aligned} \tag{13}$$

In the yoz plane, we have

$$M(A_2) = \int_{D_2} \frac{1}{R}\exp\left[-\frac{\pi}{R^2}(y-\eta)^2\right]\mathrm{d}\eta,$$
$$D_2 \in (-\infty,+\infty) \tag{14}$$

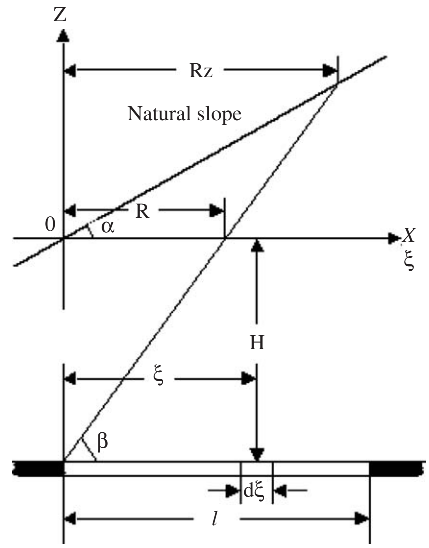

Fig. 2. A rectangular coordinate system of underground mining in mountainous areas (xoz).

In the case of the underground mining in mountainous areas (as shown in Fig. 2), the fuzzy measure formula of ground subsidence in the xoz cross section

$$M(A_1) = \int_{D_1} \frac{1}{R_z}\exp\left[-\frac{\pi}{R_z^2}(x-\xi)^2\right]\mathrm{d}\xi, \tag{15}$$

where R_z is the primary influence radius in mountainous area:

$$R_z = \frac{H + \xi\tan\alpha}{\tan\beta}, \tag{16}$$

where H is the mining depth (as shown in Fig. 2), α is the angle of natural slope in mountainous areas, β is the primary influence angle correlate with rock properties.

The fuzzy measure formula of ground subsidence in the yoz cross–section

$$M(A_2) = \int_{D_2} \frac{1}{R_z}\exp\left[-\frac{\pi}{R_z^2}(y-\eta)^2\right]\mathrm{d}\eta. \tag{17}$$

From formula (16) we have the following expression of the fuzzy measure of ground subsidence due to underground mining in mountainous areas in the xoz cross section:

$$M(A_1) = \int_{D_1} \frac{\tan\beta}{H + \xi\tan\alpha} f_1(x,\xi)\mathrm{d}\xi, \tag{18}$$

$$f_1(x,\xi) = \exp\left[-\frac{\pi\tan^2\beta}{(H+\xi\tan\alpha)^2}(x-\xi)^2\right]. \tag{19}$$

The fuzzy measure of ground subsidence in the yoz cross-section

$$M(A_2) = \int_{D_2} \frac{\tan\beta}{H + \eta\tan\alpha} f_2(x,\eta)\mathrm{d}\eta, \tag{20}$$

$$f_2(y,\eta) = \exp\left[-\frac{\pi \tan^2\beta}{(H+\eta\ \tan\ \alpha)^2}(y-\eta)^2\right]. \tag{21}$$

In the two-dimensional case, the ground subsidence W (x, z) can be determined by the following formulae:

$$W(x,z) = k_1 k_2 M(A_1), \tag{22}$$

$$W(x,z) = k_1 k_2 \int_{D_1} \frac{\tan\ \beta}{H+\xi\ \tan\ \alpha} f_1(x,\xi)\mathrm{d}\xi. \tag{23}$$

In the three-dimensional case, the ground subsidence $W(x, y, z)$ can be determined by the following formula:

$$W(x,y,z) = k_1 k_2 M(B), \tag{24}$$

where k_1 and k_2 are parameters depending on the mining method and the rock properties.

The formula of tilt (x-direction),

$$\begin{aligned} T_x(x,y,z) &= \frac{\partial}{\partial x} W(x,y,z) \\ &= \int_0^l \frac{2\pi \tan^3\beta k_1 k_2 (x-\xi)}{(H-\xi\ \tan\ \alpha)^3} \\ &\quad \times \exp\left[\frac{-\pi\tan^2\beta}{(H+\xi\ \tan\ \alpha)^2}(x-\xi)^2\right]\mathrm{d}\xi, \end{aligned} \tag{25}$$

where l is the mining length.

In the y-direction,

$$\begin{aligned} T_y(x,y,z) &= \frac{\partial}{\partial y} W(x,y,z) \\ &= \int_0^l \frac{2\pi \tan^3\beta k_1 k_2 (y-\eta)}{(H-\eta\ \tan\ \alpha)^3} \\ &\quad \times \exp\left[\frac{-\pi\tan^2\beta}{(H+\eta\ \tan\ \alpha)^2}(y-\eta)^2\right]\mathrm{d}\eta, \end{aligned} \tag{26}$$

where l is the mining length.

The formula of directional slope

$$T_\delta(x,y,z) = T_x \cos\delta + T_y \sin\delta, \tag{27}$$

where δ is the directional angle.

The formula of curvature in the x-direction is given by

$$C_{Ux}(x,y,z) = \frac{\partial}{\partial x} T_x(x,y,z), \tag{28}$$

and in the y-direction by

$$C_{Uy}(x,y,z) = \frac{\partial}{\partial y} T_y(x,y,z). \tag{29}$$

The formula of directional curvature

$$C_{U\delta} = C_{Ux}\cos\delta + 2C_{Uxy}\sin\delta\cos\delta + C_{Uy}\sin\delta, \tag{30}$$

where δ is the directional angle, and C_{Uxy} is second partial derivative of W with respect to x and y:

$$C_{Uxy}(x,y,z) = \frac{\partial^2}{\partial x\,\partial y} W(x,y,z). \tag{31}$$

The formula of horizontal displacement in the x-direction is given by

$$U_x(x,y,z) = B_{S1}\frac{\partial}{\partial x} W(x,y,z), \tag{32}$$

in the y-direction by

$$U_y(x,y,z) = B_{S2}\frac{\partial}{\partial y} W(x,y,z), \tag{33}$$

where B_{S1} and B_{S2} are engineering parameters ($0.1 \leqslant B_{S1} \leqslant 0.4$, $0.1 \leqslant B_{S2} \leqslant 0.4$).

The formula of horizontal strain in the x-direction is given by

$$E_x(x,y,z) = \frac{\partial}{\partial x} U_x(x,y,z), \tag{34}$$

in the y-direction by

$$E_y(x,y,z) = \frac{\partial}{\partial y} U_y(x,y,z). \tag{35}$$

The formula of vertical strain is

$$V_S(x,y,z) = \frac{\partial}{\partial z} W(x,y,z). \tag{36}$$

The similar discussion can be made for the three-dimensional problems of the multiseam mining.

3. Application of fuzzy models to mining engineering

Example 1. Mou-Mine, Shan-dong Province

The topography in the district is complex, with the maximum relative difference of height up to 30 m. The mining thickness of the ore is 50 m, average dip 40°, mining depth 400 m.

3.1. The process of estimating the displacements

In order to demonstrate the application of the formula for the fuzzy probability measures of rock mass displacements some examples are given of the practical application of the above theoretical results.

Fig. 3 is a full flow chart of steps in the method of estimating displacement values.

3.2. The method of determining engineering parameters

The engineering parameters can be determined by the artificial neural networks (ANNs) method.

3.2.1. The artificial neural networks (ANN)

Recently, ANNs, has begun to be used in rock and soil mechanics and geotechnical engineering [16–29]. An ANN is a highly simplified model of the biological structures found in a human brain. Their layered structure is composed of a large number of interconnected elementary processing elements to mimic the biological neurons. The characteristics of a neural network come from the activation function and connection weights. Since the neural network stores data as patterns in a set of processing elements by adjusting the connection weights, it is possible to realize complex mapping through its characteristics of distributed representations. The neural network can automatically find the closest match through its content addressable property, even if the data are incomplete or vague (e.g. rock mass displacement parameters k_1, k_2, B_{S1}, B_{S2}, $\tan\beta$ in the case examples). Most of the researches to date show that an ANNs can be applied successfully to engineering problems without any restriction. It has also been seen that the capability of an ANN is suitable for inherent uncertainties and imperfections found in geotechnical engineering problems.

Neural networks are nonlinear dynamic systems that have important features, such as self-learning, adaptive recognition, nonlinear dynamic processing and associative memory. They have the ability to learn knowledge from historical data (e.g. A–E in Fig. 4) and bring forth new knowledge and generalization. Therefore, neural networks have been successfully used in geotechnical engineering areas, for e.g., to predict reliability [27], to evaluate the permeability of compacted clay liners [28] and to predict the parameters of rock mass movement [29]. These applications show that neural network models are superior at solving problems in which many parameters influence the process and results, when the process and results are not fully understood and when there is historical or experimental data available. The problem involving prediction of rock mass displacement parameters is of this type.

Neural network models are set up by learning or training. If a network model is trained with a large number of input–output pairs, it can produce an appropriate output for untrained inputs. More than 50 neural network models have been devised so far, and it is found that the back-propagation learning algorithm based on the generalized delta rule by Rumelhart et al. [30] is the most popular

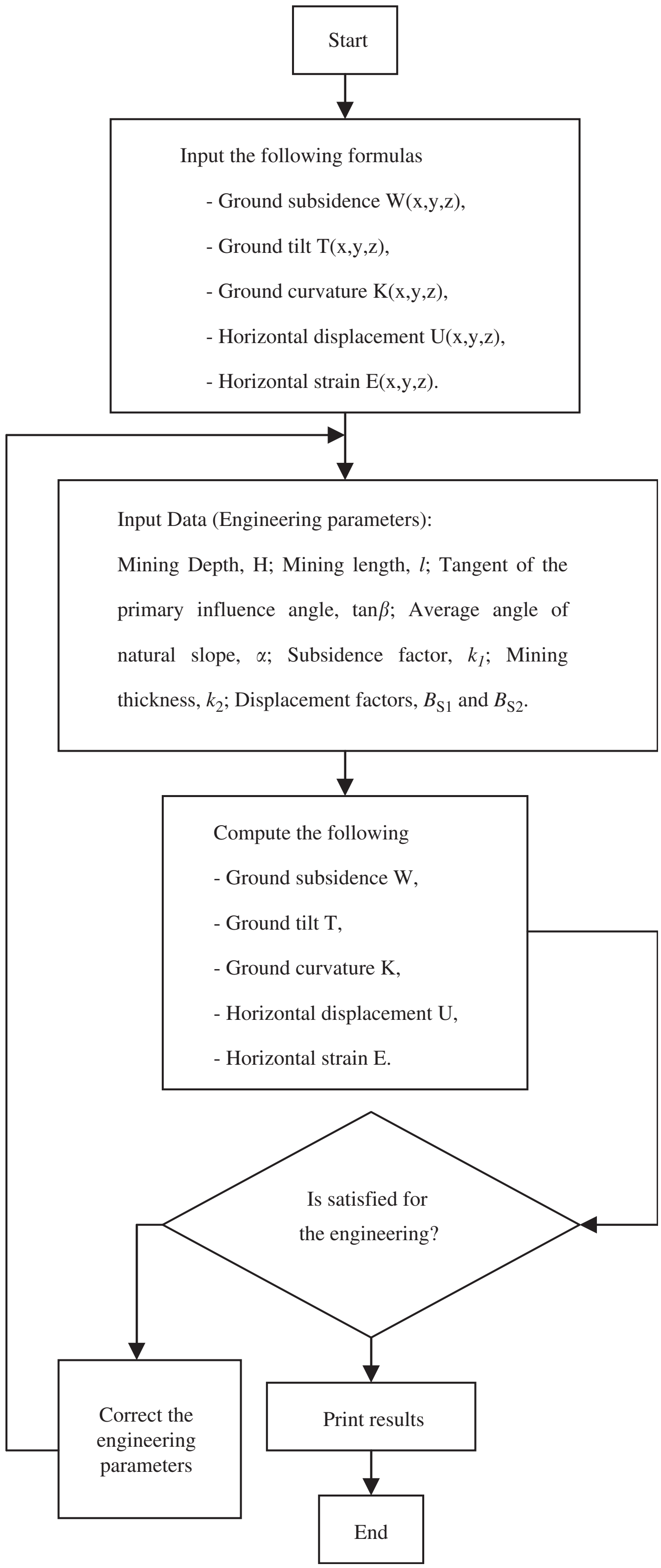

Fig. 3. The flow chart for estimating the displacement values.

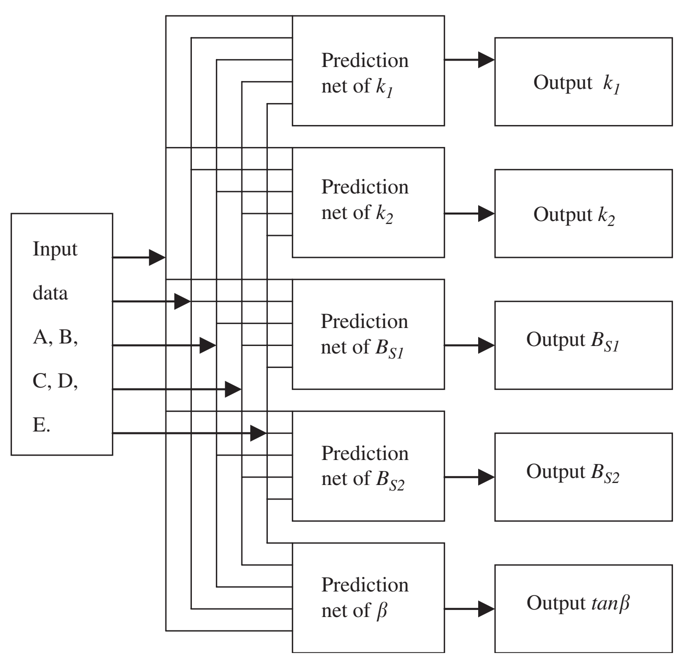

Fig. 4. Topological structure of back-propagation neural network.

and efficient learning procedure for multi-layer neural networks.

3.2.2. Topological structure of the back-propagation neural network (BPNN)

The BPNN generally consists of many sets of nodes arranged in layers (e.g. input, hidden and output layers). The output signals from one layer are transmitted to the subsequent layer through links that amplify or attenuate or inhibit the signals using weighting factors. Except for the nodes in the input layer, the net input to each node is the sum of the weighted outputs of the nodes in the previous layer. An activation function such as the sigmoid logistic function is used to calculate the output of the nodes in the hidden and output layers. We can divide the BPNN into five subsidiary nets for the sake of predicting five parameters (k_1, k_2, B_{S1}, B_{S2}, $\tan\beta$). Fig. 4 is the topological structure of BPNN [29].

In topological structure of BPNN in Fig. 4, different symbols have different meaning. ((A) historical data, the measured ground subsidence; (B) historical data, the measured horizontal displacement; (C) historical data, the hardness factor of rock mass in mining area; (D) historical data, the dip of mining seam; (E) historical data, the ratio of mining depth to the mining thickness).

3.2.3. The parameters of rock mass displacements

The parameters of rock mass displacements are obtained using a BPNN method (Table 1).

3.3. The results of estimating displacements

From these data in Table 1, we obtain the predicted results of ground surface subsidence due to underground mining (Fig. 5).

Example 2. Wangjiazhai Coal Mine, Shui-cheng Bureau of Mining, Guizhou Province

The topography in the district is complex, with the maximum relative difference of height up to 125 m, maximum slope 61°, average slope 31°. The overlying strata consist of argillaceous limestone and purple sandy shale intercalated with siltstone and fine- and medium-grained sandstone between them. The mining thickness of the coal seam is 1.80 m, strike NE 40°, inclination SE 130°, average dip 13°.

The stope face has a strike length 330 m, inclination length 130 m, and a mining depth 50–120 m. The seam was mined by longwall retreat mining method. The engineering parameters can be determined by using BPNN (Table 2).

Table 1
Parameters of rock mass displacements

α (°)	k_2 (mm)	β (°)	H (m)
30.00	2000	67.00	400.00
$\tan\beta$	k_1	B_{S1}	B_{S2}
2.3559	0.1601	0.20	0.21

Table 2
Parameters of rock mass displacements

α (°)	k_2 (mm)	β (°)	H (m)
31.00	1800	64.00	92.00
$\tan\beta$	k_1	B_{S1}	B_{S2}
2.0503	0.8950	0.23	0.21

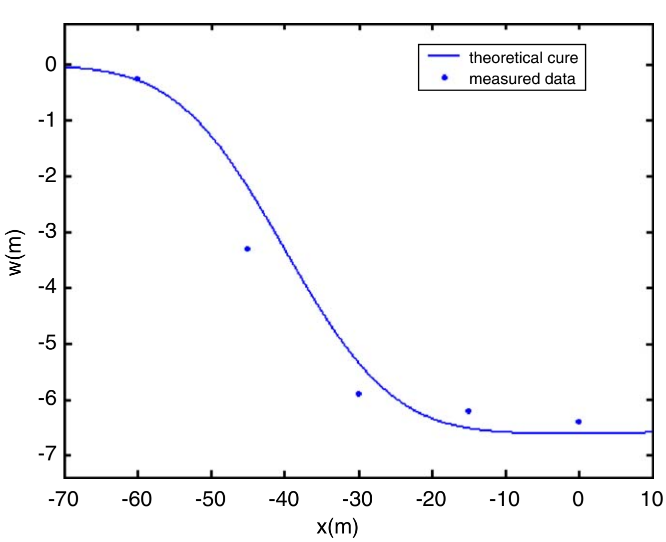

Fig. 5. Comparison between the data points and the theoretical curve of the ground subsidence.

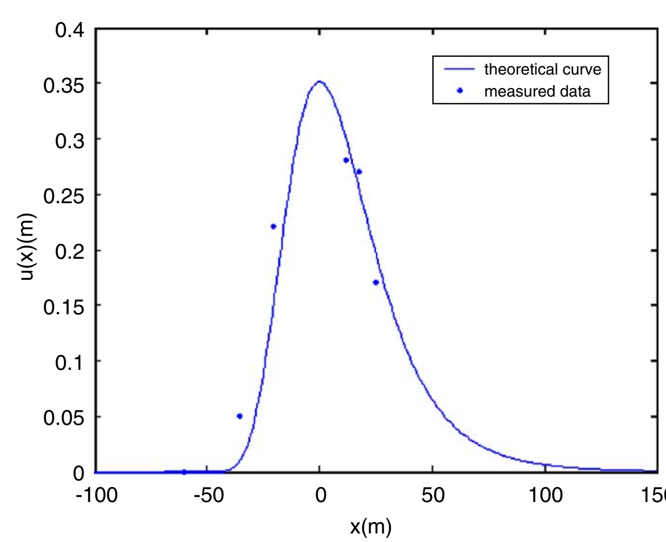

Fig. 6. Comparison between the data points and the theoretical curve of the horizontal displacement.

From these data in Table 2, we obtain the formulae of the ground subsidence due to underground mining in mountainous areas:

$$W(x,z) = 0.8950k_2 \int_{D_1} \frac{\tan 64^\circ}{92 + \xi \tan 31^\circ} f_1(x,\xi)\mathrm{d}\xi,$$

$$f_1(x,\xi) = \exp\left[-\frac{\pi \tan^2 64^\circ}{(92 + \xi \tan 31^\circ)^2}(x-\xi)^2\right].$$

From these data in Table 2, we obtain the formulae of the horizontal displacement due to underground mining in mountainous areas:

$$U_x(x,z) = 0.23k_1k_2 \int_{D_1} \frac{\tan 64^\circ}{92 + \xi \tan 31^\circ} f_1(x,\xi)\mathrm{d}\xi,$$

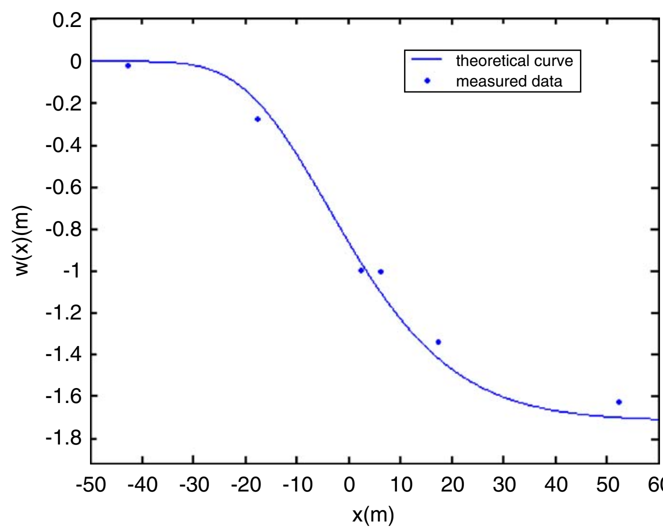

Fig. 7. Comparison between the data points and the theoretical curve of the ground subsidence.

$$f_1(x,\xi) = \exp\left[-\frac{\pi \tan^2 64^\circ}{(92 + \xi \tan 31^\circ)^2}(x-\xi)^2\right],$$

where $k_1 = 0.8950$, $k_2 = 1.80$ m.

The theoretical horizontal displacement curve was plotted using the above formulas and was compared with the observed data (Fig. 6).

The theoretical subsidence curve was plotted using the above formulae and was compared with the observed data (Fig. 7).

Example 3. Hong-lingwan No.4 Coal Mine

The mining thickness of the coal seam is 1.80 m, strike NE 40°, inclination SE 130°, average dip 13°.

The topography in the district is complex, with the maximum relative difference of height up to 100 m, maximum slope 32°.

The stope face has a strike length 140 m, a mining depth 100 m. The engineering parameters can be determined by BPNN (Table 3).

From these data in Table 3, we obtain the formulae of the ground subsidence due to underground mining in mountainous areas:

$$W(x,z) = k_1k_2 \int_{D_1} \frac{\tan 53^\circ}{101 + \xi \tan 32^\circ} f_1(x,\xi)\mathrm{d}\xi,$$

Table 3
Parameters of rock mass displacements

α (°)	k_2 (mm)	β (°)	H (m)
32.00	1800	53.00	101.00
$\tan\beta$ 1.3271	k_1 0.9010	B_{S1} 0.26	B_{S2} 0.25

Table 4
Numerical results of the subsidence due to underground mining in mountainous area

x (m)	W (m)	x (m)	W (m)	$-x$ (m)	W (m)	$-x$ (m)	W (m)
0	0.89985692	65	1.50639443	2	0.85688603	70	0.02389424
2	0.94178230	70	1.48741001	4	0.81401701	75	0.01556352
4	0.98343325	75	1.46224364	6	0.77112701	80	0.00985950
6	1.02430182	80	1.43130112	8	0.72840821	85	0.00613986
8	1.06422137	85	1.39496551	10	0.68604910	90	0.00317777
10	1.10303141	90	1.35334920	15	0.58292163	95	0.00219628
15	1.19418002	95	1.30716664	20	0.48579800	100	0.00126624
20	1.27547003	100	1.25628014	25	0.39681036	110	0.00039073
25	1.34507006	110	1.14229051	30	0.31747903	120	0.00010922
30	1.40363010	120	1.01512301	35	0.24865867	130	0.00000000
35	1.44962461	150	0.60699309	40	0.19055708	150	0.00000132
40	1.48381001	180	0.27416241	45	0.14281991	180	0.00000000
45	1.50689321	200	0.13542401	50	0.10464481	200	0.00000000
50	1.51973022	230	0.03544140	55	0.07492890	230	0.00000000
55	1.52332315	250	0.01189062	60	0.05241710	250	0.00000000
60	1.51859000	300	0.00037891	65	0.03584103	300	0.00000000

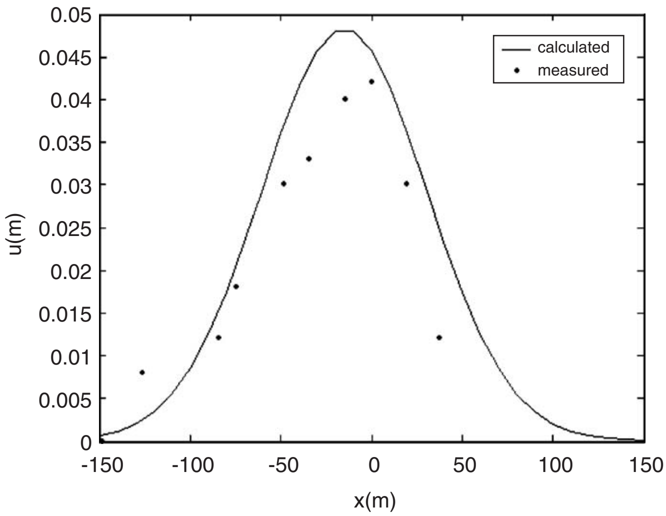

Fig. 8. Comparison between the data points and the theoretical curve of the horizontal displacement.

$$f_1(x,\xi) = \exp\left[-\frac{\pi \tan^2 53^\circ}{(101+\xi\tan 32^\circ)^2}(x-\xi)^2\right],$$

where $k_1 = 0.9010$, $k_2 = 1.80\,\mathrm{m}$.

The theoretical results are given in Table 4 (subsidence) and a theoretical horizontal displacement curve was plotted using the above formulae and was compared with the observed data (Fig. 8).

The agreement of the theoretical results with the field measurements shows that the model is satisfactory and the formulae obtained are valid, and thus can be effectively used for predicting the displacements and deformations due to underground mining in mountainous areas.

4. Conclusions

In this paper, by applying the concepts of fuzzy probability measures to actual cases of excavation, mining, ground surface movement and subsidence have been analyzed and the corresponding membership function is established. The approximate subsidence and horizontal displacement have been calculated and compared with the recorded data obtained from monitoring stations. The comparison shows that the theoretical prediction is in agreement with the observations.

The formulae derived in this paper have been confirmed by a large amount of measured data. The fuzzy model, therefore, is valid for solving the problems of the rock mass displacement and ground subsidence due to underground mining, especially the mining of coal and metal in mountainous areas.

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