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Economic production quantity model with repair failure and limited capacity

Ata Allah Taleizadeh^a, Hui-Ming Wee^{b,*}, Seyed Gholamreza Jalali-Naini^a^a Department of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran^b Department of Industrial & Systems Engineering, Chung Yuan Christian University, Chungli, Taiwan

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ABSTRACT

We develop an economic production quantity (EPQ) model with random defective items and failure in repair. The existence of only one machine results with limited production capacity and shortages. The aim of this research is to derive the optimal cycle length, the optimal production quantity and the optimal back ordered quantity for each product so as to minimize the total expected cost (holding, shortage, production, setup, defective items and repair costs). The convexity of the model is derived and the objective function is proved convex. Two numerical examples illustrate the practical usage of the proposed method.

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1. Introduction

One of the critical factors in any production process is material. The management of material concerns the regulation of the flow of materials to, within, and from the organization. The efficiency of the material flow can substantially influence costs as well as revenue generation capabilities [1]. The management of material involves a balance between the shortages and excesses of stock in an uncertain environment. With the globalization of business in recent years, firms are sourcing and distributing raw materials, components, and finished goods across the globe. Customers want to receive their quality products quickly. As a result, efficient inventory management, production planning and scheduling to achieve flexibility and quick response has become a core competitive advantage. To achieve operation strategies goals, the company must be able to effectively utilize resources and minimize costs. In manufacturing companies, when items are internally produced instead of being obtained from an outside supplier, the economic production quantity (EPQ) model is often employed to determine the optimal production lot size that minimizes the overall production/inventory costs. The classic EPQ model assumes that during a production run a manufacturing facility functions perfectly. However, due to process deterioration or some other factors, imperfect quality items are inevitable. Some examples of the rework processes are: printed circuit board assembly in the PCBA manufacturing, metal components, and plastic injection molding. A considerable amount of research has been carried out by Cheng [2], Chiu et al. [3], Chung [4], Lee and Rosenblatt [5], and Rosenblatt and Lee [6] to address the imperfect quality EPQ problem. They assumed that at some random time, the process might shift from an in-control to an out-of-control state. Hayek and Salameh [7] derived an optimal operating policy for finite production (EPQ) model with rework and imperfect quality items. They assumed that all defective items were repairable and that backorders were allowed. Numerous studies have been carried out to address the problems of imperfect quality EPQ model with rework (see, for

* Corresponding author.

E-mail addresses: ata.taleizadeh@gmail.com, Taleizadeh@iust.ac.ir (A.A. Taleizadeh), weehm@cycu.edu.tw (H.-M. Wee), sgjalali@iust.ac.ir (S.G. Jalali-Naini).

example, [7–12]). Chan et al. [13] presented a new EPQ model with increasing lower pricing, rework and reject situations. Teng et al. [14] studied optimal ordering decisions with returns and excess inventory. Islam and Roy [15] formulated an EPQ model considering flexible and reliable production process and with fuzzy demand-dependent-unit production cost. Bayindir et al. [16] considered the EPQ model with general inventory cost rate function and piecewise linear concave production costs, and proposed an effective solution procedure for deriving the economic order quantity. Hou [17] studied an EPQ model with setup cost and process quality as a function of capital expenditure and developed an efficient procedure to derive the optimal production run time, setup cost, and process quality. Chiu et al. [10] investigated an EPQ model with scrap, rework, and stochastic machine breakdowns to determine the optimal run time and production quantity. Chiu [18] later showed that the same problem can be derived without derivatives. Li et al. [19] developed an EPQ-based model with planned backorders to evaluate the impact of the postponement strategy on a manufacturer in a supply chain. Pentico et al. [20] extended the EPQ model with partial back ordering where the decision variables were production quantity and period length. Teng and Chung [21] considered the EPQ model under two levels of trade credit policy to optimize the production quantity and period length. Chiu et al. [22] considered the effects of random defective rate and imperfect rework process on economic production quantity model. Wee et al. [23] developed an inventory model for items with imperfect quality and shortage backordering. Taleizadeh et al. [24] developed an EPQ model under limited production capacity and scraped items production. Taleizadeh et al. [25] developed an EPQ model with stochastic scraped production rate, partial back ordering and service level constraint. From our literature search, none of the above has so far developed an economic production quantity (EPQ) model with random defective items and failure in repair with capacity constraint. In the case of multi product-single machine systems, Haji et al. [26] studied an imperfect manufacturing process with rework where several products are manufactured on a unique machine. Recently, Widyadana and Wee [27] studied the optimal deteriorating items production inventory models with random machine breakdown and stochastic repair time.

2. Modeling and formulation

The imperfect quality EPQ model by Chiu et al. [14] considered a manufacturing process with a constant production rate P larger than the demand rate D . This process randomly generates x percent of defective items at a rate λ . All items produced are screened and the inspection cost per item is included in the unit production cost C^p . All defective items produced can be reworked at a rate of P^1 , and rework starts when the regular production process ends. A random portion θ of the reworked items is assumed to be scrap. Let λ denote production rate of defective items during regular manufacturing process, and λ can be expressed as the product of production rate P and the defective percentage x . Therefore, $\lambda = Px$. Let λ^1 denote production rate of scrap items during rework, and λ^1 can be expressed as the product of reworking rate times the percentage of scrap items produced during rework process. Hence, $\lambda^1 = P^1\theta$. A real constant production capacity limitation on a single machine on which all products are produced and that the setup cost is considered nonzero. Since all products are manufactured on a single machine with a limited capacity, the cycle length for all of them are equal ($T_1 = T_2 = \dots = T_n = T$). From Table 1, the main differences between this research and others are as follows: Firstly, our model investigates multi-product single-machine. Secondly, we consider capacity limitation. Moreover, during the regular production time, defective items may be produced randomly. The random fraction of defective items is reworked during the rework process and complete backordering is allowed.

Since the problem at hand is of multiproduct with products $i = 1, 2, \dots, n$, the following notations are used in this research:

- Q_i : production lot size of i th product for each cycle;
- B_i : allowable backorder level of i th product, in units for each cycle;
- A_i : setup cost for each production run of i th product;
- C_i^R : repair cost for each imperfect quality item reworked of i th product, \$/item;
- C_i^S : disposal cost per scrap item produced of i th product during the rework process, \$/scrap item;
- C_i^H : holding cost of i th product per item per unit time, \$/item/unit time;
- C_i^{H1} : holding cost for each imperfect quality items of i th product being reworked per unit time;
- C_i^B : shortage cost of i th product per item per unit time, \$/item/unit time;
- I_i : maximum level of on-hand inventory of i th product when regular production process stops;
- I_i^{Max} : maximum level of on-hand inventory of i th product in units, when the reworking ends;
- S_i : setup time of machine to produce the i th product;
- N : number of cycles per year;
- $TC(Q, B)$: total inventory costs per year;
- $E(.)$: denotes the expected value.

2.1. Formulation

Initially the problem is modeled as a single product case and then it is modified as a multi product case. The basic assumption of EPQ model with imperfect quality items produced is that P_i must always be greater than or equal to the sum of demand rate D_i and the production rate of defective items is λ_i . One has:

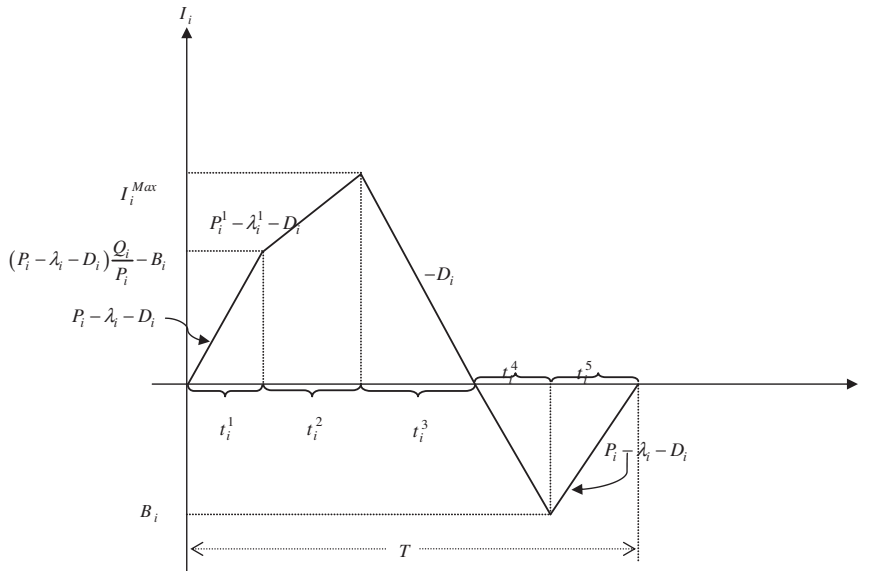


Fig. 1. On hand inventory of perfect quality items.

$$I_i = (P_i - \lambda_i - D_i) \frac{Q_i}{P_i} - B_i, \tag{8}$$

$$I_i^{Max} = I_i + (P_i - \lambda_i - D_i)t_i^2 = Q_i \left(1 - \frac{D_i}{P_i} - \frac{\lambda_i^1 \lambda_i}{P_i^1 P_i} - \frac{\lambda_i D_i}{P_i^1 P_i} \right) - B_i. \tag{9}$$

During rework process, the production rate of scrap items can be written as in Eqs. (10) and (11):

$$\lambda_i^1 = P_i^1 E[\theta_i], \quad \text{where } 0 \leq \theta_i \leq 1, \tag{10}$$

$$\lambda_i^1 t_i^2 = (P_i^1 E[\theta_i]) \left(\frac{\lambda_i Q_i}{P_i^1 P_i} \right) = E[\theta_i] \left(\frac{P_i E[X_i] Q_i}{P_i} \right) = E[\theta_i] E[X_i] Q_i. \tag{11}$$

Hence, the cycle length for a single product state is:

$$T = \frac{Q_i E[1 - \theta_i X_i]}{D_i}, \quad \text{where } 0 \leq \theta_i \leq 1 \tag{12}$$

or

$$Q_i = \frac{D_i T}{E[1 - \theta_i X_i]}, \quad \text{where } 0 \leq \theta_i \leq 1. \tag{13}$$

During the imperfect rework process, the random defective rate has a range of [0,1], and the scrap rate has a range of [0, $Q_i E[\theta_i] E[X_i]$]. The total inventory cost per year $TC(Q, B)$ is:

$$\begin{aligned}
 C(Q, B) = & \underbrace{NC_i^P Q_i}_{\text{Production Cost}} + \underbrace{NC_i^R E[X_i] Q_i}_{\text{Rework Cost}} + \underbrace{NC_i^S E[X_i] Q_i E[\theta_i]}_{\text{Disposal Cost}} + \underbrace{NC_i^h \left[\frac{I_i}{2} (t_i^1) + \frac{I_i + I_i^{Max}}{2} (t_i^2) + \frac{I_i^{Max}}{2} (t_i^3) + \frac{\lambda_i (t_i^1 + t_i^5)}{2} (t_i^1 + t_i^5) \right]}_{\text{Holding Cost of Perfect Quality Items}} \\
 & + \underbrace{NC_i^{h1} \left[\frac{P_i^1 t_i^2}{2} (t_i^2) \right]}_{\text{Holding Cost of perfect Quality Items}} + \underbrace{NC_i^B \frac{B_i}{2} (t_i^4 + t_i^5)}_{\text{Shortage Cost}} + \underbrace{NA_i}_{\text{Setup Cost}}. \tag{14}
 \end{aligned}$$

The joint production policy (Multi-Product Single-Machine) from Eq. (14) becomes:

$$\begin{aligned}
 TC(Q, B) = & \sum_{i=1}^n \frac{C_i^P Q_i}{T} + \sum_{i=1}^n \frac{C_i^R E[X_i] Q_i}{T} + \sum_{i=1}^n \frac{C_i^S E[X_i] Q_i E[\theta_i]}{T} + \sum_{i=1}^n \frac{C_i^h}{T} \left[\frac{I_i}{2} (t_i^1) + \frac{I_i + I_i^{Max}}{2} (t_i^2) + \frac{I_i^{Max}}{2} (t_i^3) + \frac{\lambda_i (t_i^1 + t_i^5)}{2} (t_i^1 + t_i^5) \right] \\
 & + \sum_{i=1}^n \frac{C_i^{h1}}{T} \left[\frac{P_i^1 t_i^2}{2} (t_i^2) \right] + \sum_{i=1}^n C_i^B \frac{B_i}{2T} (t_i^4 + t_i^5) + \sum_{i=1}^n \frac{A_i}{T}. \tag{15}
 \end{aligned}$$

2.2. The constraint

Since $t_i^1 + t_i^2 + t_i^5$ are the production and rework times and S_i is the setup time for i th product, the summation of the total production, rework and setup time (for all products) will be $\sum_{i=1}^n (t_i^1 + t_i^2 + t_i^5) + \sum_{i=1}^n S_i$, and it should be smaller or equal to the period length (T). So the constraint of the model is:

$$\sum_{i=1}^n (t_i^1 + t_i^2 + t_i^5) + \sum_{i=1}^n S_i \leq T. \tag{16}$$

Then, based on the Eqs. (3)–(5) and (8), we have:

$$\sum_{i=1}^n \frac{D_i(P_i^1 + \lambda_i)}{P_i P_i^1 E[1 - \theta_i X_i]} T + \sum_{i=1}^n S_i \leq T. \tag{17}$$

2.3. Final model

From Eqs. (3)–(9) and Eq. (13), $TC(Q, B)$ in Eq. (14) and constraint in Eq. (16), one can formulate the problem as:

$$\text{Min : } TC(Q, B) = \sum_{i=1}^n C_i^1 \frac{(B_i)^2}{T} - \sum_{i=1}^n C_i^2 B_i + \sum_{i=1}^n C_i^3 T + \sum_{i=1}^n C_i^4 + \sum_{i=1}^n \frac{A_i}{T}. \tag{18}$$

$$\text{S.t. : } T \geq \frac{\sum_{i=1}^n S_i}{\left[1 - \sum_{i=1}^n \frac{D_i(P_i^1 + \lambda_i)}{P_i P_i^1 E[1 - \theta_i X_i]} \right]} \quad T, B_i \geq 0 \quad \forall i, i = 1, 2, \dots, n, \tag{19}$$

where,

$$C_i^1 = \frac{C_i^B (P_i - \lambda_i)}{D_i (P_i - \lambda_i - D_i)} + \frac{C_i^h}{2(P_i - \lambda_i - D_i)} - \frac{C_i^h}{2D_i} > 0, \tag{20}$$

$$C_i^2 = \frac{C_i^h D_i}{P_i E[1 - \theta_i X_i]} + \frac{C_i^h \lambda_i D_i}{P_i^1 P_i E[1 - \theta_i X_i]} + \frac{C_i^h}{E[1 - \theta_i X_i]} \left(1 - \frac{D_i}{P_i} - \frac{\lambda_i^1 \lambda_i}{P_i^1 P_i} - \frac{\lambda_i D_i}{P_i^1 P_i} \right) > 0. \tag{21}$$

$$C_i^3 = \frac{C_i^{h1} \lambda_i^2 D_i^2}{2P_i^1 (P_i)^2 (E[1 - \theta_i X_i])^2} + \frac{C_i^h (P_i - \lambda_i - D_i) D_i^2}{2(P_i)^2 (E[1 - \theta_i X_i])^2} + \frac{C_i^h (P_i - \lambda_i - D_i) \lambda_i D_i^2}{2P_i^1 (P_i)^2 (E[1 - \theta_i X_i])^2} + C_i^h \left[\frac{D_i}{2E[1 - \theta_i X_i]} + \frac{\lambda_i D_i^2}{2P_i P_i^1 (E[1 - \theta_i X_i])^2} \right] \left(1 - \frac{D_i}{P_i} - \frac{\lambda_i^1 \lambda_i}{P_i P_i^1} - \frac{\lambda_i D_i}{P_i P_i^1} \right) > 0, \tag{22}$$

$$C_i^4 = \frac{[C_i^p + C_i^R E[X_i] + C_i^S E[X_i] E[\theta_i]] D_i}{E[1 - \theta_i X_i]} + \frac{\lambda_i D_i}{2P_i E[1 - \theta_i X_i]} > 0. \tag{23}$$

3. Solution method

In order to derive the optimal solution of the final model, a proof of the convexity of the objective function is provided. A classical optimization technique using partial derivatives is performed to derive the optimal solutions [24,25].

Theorem 1. The objective function $TC(Q, B)$ in (18) is convex.

Proof. To proof the convexity of $TC(Q, B) = Z$, the following Hessian matrix is developed:

$$[T, B_1, B_2, \dots, B_n] \times \begin{bmatrix} \frac{\partial^2 Z}{\partial T^2} & \frac{\partial^2 Z}{\partial T \partial B_1} & \frac{\partial^2 Z}{\partial T \partial B_2} & \dots & \frac{\partial^2 Z}{\partial T \partial B_n} \\ \frac{\partial^2 Z}{\partial B_1 \partial T} & \frac{\partial^2 Z}{\partial B_1^2} & \frac{\partial^2 Z}{\partial B_1 \partial B_2} & \dots & \frac{\partial^2 Z}{\partial B_1 \partial B_n} \\ \frac{\partial^2 Z}{\partial B_2 \partial T} & \frac{\partial^2 Z}{\partial B_2 \partial B_1} & \frac{\partial^2 Z}{\partial B_2^2} & \dots & \frac{\partial^2 Z}{\partial B_2 \partial B_n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial^2 Z}{\partial B_n \partial T} & \frac{\partial^2 Z}{\partial B_n \partial B_1} & \frac{\partial^2 Z}{\partial B_n \partial B_2} & \dots & \frac{\partial^2 Z}{\partial B_n^2} \end{bmatrix} \begin{bmatrix} T \\ B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix} = \frac{2 \sum_{i=1}^n A_i}{T} \geq 0. \tag{24}$$

From Appendix 1, the objective function for all nonzero T and B_j is shown to be strictly convex. T and B_j are solved by letting the partial derivatives equal to zero [24,25]. One has:

$$\frac{\partial Z}{\partial T} = 0 \rightarrow T = \sqrt{\frac{\sum_{i=1}^n A_i}{\sum_{i=1}^n C_i^3 - \sum_{i=1}^n \frac{(C_i^2)^2}{4C_i^1}}}, \tag{25}$$

$$\frac{\partial Z}{\partial B_i} = 0 \rightarrow B_i = \frac{C_i^2}{2C_i^1} T. \tag{26}$$

Then

$$Q_i = \frac{D_i}{E[1 - \theta_i X_i]} T. \tag{27}$$

The constraint below must be satisfied, otherwise the minimum value of T will be considered as the optimal point.

$$T_{Min} = \frac{\sum_{i=1}^n S_i}{\left[1 - \sum_{i=1}^n \frac{D_i(P_i^1 + \lambda_i)}{P_i P_i^1 E[1 - \theta_i X_i]} \right]}. \tag{28}$$

To ensure feasibility, the following solution procedure must be performed. □

4. Solution procedure

Step 1: Check for feasibility.

If $\sum_{i=1}^n \frac{D_i(P_i^1 + \lambda_i)}{P_i P_i^1 E[1 - \theta_i X_i]} < 1$, go to step 2, else the problem will be infeasible.

Step 2: Calculate T by Eq. (25).

Step 3: Calculate T_{Min} by Eq. (28).

Step 4: If $T \geq T_{Min}$ then $T^* = T$ else $T^* = T_{Min}$.

Step 5: Calculate B_i^* by Eq. (26).

Step 6: Calculate Q_i^* by Eq. (27).

Step 7: Terminate procedure.

5. Numerical example

Consider a multi-product inventory control problem with five products where the general and specific data are given in Tables 2–4, respectively. We consider two numerical examples with uniform and normal probability distributions for X_i and θ_i . Tables 5 and 6 show the optimal results for the two numerical examples.

Table 2
General data for the examples.

P	D_i	P_i	P_i^1	S_i	A_i	C_i^p	C_i^h	C_i^{h1}	C_i^s	C_i^B	C_i^R
1	300	3000	2000	0.003	500	15	5	2	3	10	1
2	400	3500	2500	0.004	450	12	4	2	3	8	2
3	500	4000	3000	0.005	400	10	3	2	3	6	3
4	600	4500	3500	0.006	350	8	2	2	3	4	4
5	700	5000	4000	0.007	300	6	1	2	3	2	5

Table 3
Specific data for example 1.

Items	$X_i \sim U[a_i, b_i]$				$\theta_i \sim U[a_i, b_i]$			
	a_i	b_i	$E[X_i]$	$\lambda_i = P_i E[X_i]$	a_i	b_i	$E[\theta_i]$	$\lambda_i^1 = P_i^1 E[\theta_i]$
1	0	0.05	0.025	75	0	0.15	0.075	150
2	0	0.1	0.05	175	0	0.2	0.1	250
3	0	0.15	0.075	300	0	0.25	0.125	375
4	0	0.2	0.1	450	0	0.3	0.15	525
5	0	0.25	0.125	625	0	0.35	0.175	700

Table 4
Specific data for example 2.

Items	$X_i \sim N[\mu_i, \sigma_i^2]$		$\lambda_i = P_i E[X_i]$	$\theta_i \sim N[\mu_i, \sigma_i^2]$		
	$\mu_i = E[X_i]$	σ_i^2		$\mu_i = E[\theta_i]$	σ_i^2	$\lambda_i^1 = P_i^1 E[\theta_i]$
1	0.1	0.01	150	0.15	0.01	300
2	0.15	0.02	350	0.18	0.02	450
3	0.2	0.03	600	0.21	0.03	630
4	0.25	0.04	900	0.24	0.04	840
5	0.3	0.05	1250	0.27	0.05	1080

Table 5
The best results for example 1 (uniform distribution).

Items	Uniform					
	T_{Min}	T	T^*	B_i	Q_i	Z
1	0.7909	0.9183	0.9183	77.72	276	32,129
2				101.31	369.16	
3				124.49	463.49	
4				147.34	559.36	
5				169.91	657.17	

Table 6
The best results for example 2 (normal distribution).

Items	Normal					
	T_{Min}	T	T^*	B_i	Q_i	Z
1	1.0822	0.9060	1.0822	91.85 8	327.11	34,656
2				119.92	440.81	
3				147.36	558.69	
4				174.02	682.05	
5				199.78	812.37	

Table 7
Effects of parameter changes for the uniform distribution case.

% Changes in parameters and their values		% Changes in			
		T_{Min}	T	T^*	Z
P_i	+50	-39.52	+0.69	+0.69	+0.79
	+20	-24.63	+0.31	+0.31	+0.39
	-20	+96.14	-0.32	+68.93	+8.16
	-50	Infeasible	-	-	-
A_i	+50	0	+22.47	+22.47	+5.38
	+20	0	+9.54	+9.54	+2.29
	-20	0	-10.56	-10.56	-2.53
	-50	0	-29.29	-13.87	-5.22
$E[\theta_i]$	+50	1.32	-0.04	-0.04	+0.62
	+20	+0.52	-0.02	-0.02	+0.25
	-20	-0.51	+0.01	+0.01	-0.25
	-50	-1.26	+0.03	+0.03	-0.62
$E[X_i]$	+50	+13.03	-0.60	-0.60	+1.96
	+20	+4.8	-0.24	-0.24	+0.78
	-20	-4.36	+0.23	+0.23	-0.78
	-50	-10.20	+0.57	+0.57	-1.94
S_j	+50	+50	0	+29.19	+3.48
	+20	+20	0	+3.35	+0.36
	-20	-20	0	0	0
	-50	-50	0	0	0

Table 8
Effects of parameter changes for the normal distribution case.

% Changes in parameters and their values		% Changes in			
		T_{Min}	T	T^*	Z
P_i	+50	-47.87	+0.61	-15.77	-1.27
	+20	-31.47	+0.26	-16.06	-1.69
	-20	+221.22	-0.24	+221.22	+36.76
	-50	Infeasible	-	-	-
A_i	+50	0	+22.47	+2.53	+2.96
	+20	0	+9.54	0	+1.07
	-20	0	-10.56	0	-1.07
	-50	0	-29.29	0	-2.66
$E[\theta_i]$	+50	+57.24	-1.48	+57.24	+14.25
	+20	+16.74	-0.57	+16.74	+4.43
	-20	-12.33	+0.54	-12.33	-3.50
	-50	-25.73	+1.31	-15.19	-6.58
$E[X_i]$	+50	+7.52	-0.16	+7.52	+3.23
	+20	+2.83	-0.06	+2.83	+1.25
	-20	-2.62	+0.06	-2.62	-1.19
	-50	-6.21	+0.14	-6.21	-2.88
S_j	+50	+50	0	+50	+7.85
	+20	+20	0	+20	+2.96
	-20	-20	0	-16.28	-2.1
	-50	-50	0	-16.28	-2.1

6. Sensitivity analysis

To study the effects of parameter changes on the optimal result derived by the proposed method, this investigation performs a sensitivity analysis by increasing or decreasing the parameters, one at a time, by 20% and 50%. Section 4 gives two numerical examples, and section 5 gives the sensitivity analyses. Tables 7 and 8 show the results of the sensitivity analysis for uniform and normal distribution cases, respectively.

The following conclusions can be drawn from Table 7:

- T and Z are slightly sensitive to the changes in the values of parameter P_i . T_{min} and T^* are highly sensitive to the changes in the values of parameter P_i . By increasing P_i by 50%, the problem becomes infeasible.
- T_{min} and Z are insensitive and slightly sensitive to the changes of parameter A_i respectively. T and T^* are moderately sensitive to the changes of parameter A_i .
- T , T_{min} , T^* and Z are very lightly sensitive to the changes of parameter $E[\theta_i]$.
- T and T^* are very lightly sensitive to the changes of parameter $E[X_i]$. T_{min} and Z are moderately and slightly sensitive to the changes of parameter $E[X_i]$ respectively.
- T_{min} and T are highly sensitive and insensitive to the changes of parameter S_j respectively. T^* and Z are highly and slightly sensitive in the increasing changes of parameter S_j respectively, and both of them are insensitive to the decreasing changes of parameter S_j .

The following conclusions can be drawn from Table 8:

- T_{Min} , T^* and Z are highly sensitive to the changes of parameter P_i and T is very lightly sensitive to the changes of parameter P_i . By increasing P_i by 50%, the problem becomes infeasible.
- T_{min} and Z are insensitive and slightly sensitive to the changes of parameter A_i respectively. T and T^* are highly and slightly sensitive to the changes of parameter A_i .
- T_{min} and T^* are highly sensitive, T is slightly sensitive and Z is moderately sensitive to the changes of parameter $E[\theta_i]$.
- T and T^* are very slightly sensitive to the changes of parameter $E[X_i]$. T_{min} and Z are moderately and slightly sensitive to the changes of parameter $E[X_i]$ respectively.
- T_{min} and T^* are highly sensitive, T is insensitive and Z is slightly sensitive to the changes of parameter S_j .

7. Conclusion

This study develops an EPQ model with production capacity limitation and random defective production rate and failure during repair. Our objective is to determine the optimal period lengths, backordered quantities, and order quantities. The objective function of the proposed numerical model is proved to be convex. Two numerical examples and sensitivity analysis using uniform and normal distribution functions for X_j and θ_i are used to illustrate the practical applications of the proposed

methodology. The study provides managerial insights for practitioners in designing an EPQ model with random defective items and failure in repair. Future research should focus on multi-product multi-constraint problems in an uncertain environment.

Appendix 1

$$\frac{\partial Z}{\partial T} = -\frac{\sum_{i=1}^n C_i^1 (B_i)^2}{T^2} + \sum_{i=1}^n C_i^3 - \frac{\sum_{i=1}^n A_i}{T}; \quad \frac{\partial^2 Z}{\partial T^2} = \frac{2\sum_{i=1}^n C_i^1 B_i}{T^3} + \frac{2\sum_{i=1}^n A_i}{T^2}; \quad \frac{\partial^2 Z}{\partial T \partial B_i} = \frac{2C_i^1 B_i}{T^2}$$

$$\frac{\partial Z}{\partial B_i} = \frac{2C_i^1 B_i}{T} - C_i^2; \quad \frac{\partial^2 Z}{\partial^2 B_i} = \frac{2C_i^1}{T}; \quad \frac{\partial^2 Z}{\partial B_i \partial T} = \frac{2C_i^1 B_i}{T^2}$$

$$[T, B_1, B_2, \dots, B_n] \times \begin{bmatrix} \frac{2\sum_{i=1}^n C_i^1 B_i}{T^3} + \frac{2\sum_{i=1}^n A_i}{T^2} & -\frac{2C_1^1 B_1}{T^2} & -\frac{2C_1^1 B_2}{T^2} & \dots & -\frac{2C_1^1 B_n}{T^2} \\ -\frac{2C_1^1 B_1}{T^2} & \frac{2C_1^1}{T} & 0 & \dots & 0 \\ -\frac{2C_2^1 B_2}{T^2} & 0 & \frac{2C_2^1}{T} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\frac{2C_n^1 B_n}{T^2} & 0 & 0 & \dots & \frac{2C_n^1}{T} \end{bmatrix} \times \begin{bmatrix} T \\ B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix} = \left[\frac{2\sum_{i=1}^n A_i}{T^2}, 0, 0, \dots, 0 \right] \times \begin{bmatrix} T \\ B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix}$$

$$= \frac{2\sum_{i=1}^n A_i}{T} \geq 0$$

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