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Model testing and numerical investigation of interference effect of closely spaced ring and circular footings on reinforced sand

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A B S T R A C T

Due to heavy loads and the non-availability of suitable construction sites, engineers are often required to place footings at close spacing. These footings influence each other, including effects on load-settlement and bearing capacity behavior. In this research the bearing capacity of closely located ring and circular footings on reinforced sand has been investigated numerically and experimentally. The goal of this study is to evaluate the interference effect on the bearing capacity of adjacent circular and ring footings. Footings on reinforced and unreinforced sand have been investigated. In this research, interference effect of footings, shape effects, effect of spacing between footings and also the effect of reinforcement layer on the bearing capacity are studied. To achieve these objectives laboratory circular and ring footing models and also numerical models were used. Finite element computer code PLAXIS 3D Foundation was used for numerical modeling. Experimental and numerical analysis results show that the ultimate bearing capacity of two closely spaced circular and ring footings is greatest when they stand exactly beside each other and decreases with increase in the spacing to footing diameter ratio (Δ/D). It is found that for $\Delta/D > 4$, the bearing capacity of each adjacent footing is almost the same as that for single footing. This means that for a center-to-center spacing greater than 4D, no significant interference effect was observed and each footing acted more or less independently, similar to a single footing.

1. Introduction

Bearing capacity of soil plays an important role in design of foundations. Considering the amount of bearing capacity more than the actual amount can cause severe damage to the structures or even destroy them. On the other hand, underestimating the bearing capacity makes the foundation non-economic. Ring foundation is exceedingly used in practice. This foundation is used for communication towers, storage tanks, silos, chimneys. For most of the axisymmetric structures it is both suitable and economical. Fisher (1957) was the first to study ring foundation behavior. Egorov (1965), Ohri et al. (1997), Hataf and Razavi (2003) and Boushehrian and Hataf (2003) have also investigated the behavior of ring footings.

On the other hand, the use of soil reinforcement, which began with the work of Vidal (1966) is considered as one of the most promising techniques of improving bearing capacity of foundation soils. The introduction of geosynthetics broadened the concept of soil reinforcement even further. Initial studies on the effect of soil reinforcement for the improvement of bearing capacity of footings was carried out by Binquet and Lee (1975a, b) and subsequently pursued by many others.

Due to heavy loads and shortage of suitable sites for construction of structures, engineers are often required to place foundations at close spacing. Therefore, the foundations in the field generally interfere with each other to some extent and are rarely isolated. Interference of foundations can cause changes in bearing capacity and load-settlement behavior, when compared to isolated foundations. Khing et al. (1992), Al-Ashou et al. (1994), Kumar and Saran (2003), Ghosh and Kumar (2009), and more recently Ghazavi and Lavasan (2012) and Srinivasana and Ghosh (2013) have investigated the interference effect of square, strip and circular footings. Closely located ring footings on reinforced and unreinforced soils have not received attention in this regard. The goal of this study is to evaluate the interference effect on the bearing capacity of adjacent circular and ring footings. To investigate this, in this paper, the results of a series of laboratory tests on closely spaced model ring and circular footings on reinforced sand together with the results of numerical investigation of such footings are presented. The experimental and numerical results are then compared and conclusions are made.

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2. Behavior of ring and circular footings on sand

Fisher (1957) proposed a solution to calculate the settlement of ring footings on a semi-infinite elastic media. Egorov (1965) later proposed some relations to calculate the bearing pressure under the ring footing and its settlement. Ohri et al. (1997) performed a series of laboratory tests on model ring footings using dune sand and found that for a ratio of internal to external diameter of the ring equal to 0.375 the unit bearing capacity reaches its maximum. Hataf and Razavi (2003) found that bearing capacity of ring footing with ratio of internal to external diameter in the range of 0.2–0.4 is around 25% more than the bearing capacity of circular footing with the same external diameter. Also, Boushehrian and Hataf (2003) investigated the bearing capacity of ring and circular footing on reinforced sand with laboratory models and numerical analysis. In their research the effects of depth of the first layer of reinforcement, number of layers and vertical distance between layers on bearing capacity were studied. Basudhar et al. (2007) experimentally studied the circular footings resting on semi-infinite layer of sand reinforced with geotextiles. They also conducted analytical and numerical analyses to predict the load-settlement behavior and to compare with experimental observations. Laman and Yildiz (2007) numerically studied the ultimate bearing capacity of ring foundations supported by a sand bed with and without geogrid reinforcement. Their analyses were carried out using the finite element package Plaxis, and the effect of depth of first reinforcement layer, the number of reinforcement layers and reinforcement layer size on the ultimate bearing capacity of ring foundations were investigated. El Sawwaf and Nazir (2012) experimentally studied the behavior of an eccentrically loaded model ring footing resting on a compacted replaced layer of sand that overlies an extended layer of loose sand, and more recently Chakraborty and Kumar (2013) studied the ultimate bearing capacity of a circular footing placed over granular and cohesive-frictional soils reinforced with a horizontal circular sheet of reinforcement, using analytical analyses.

3. Behavior of closely spaced footings on reinforced sand

Das and Labri-Cherif (1983) studied the ultimate bearing capacity of two closely spaced strip foundations on sand using laboratory model tests. Selvadurai and Rabbaa (1983) experimentally studied the contact stress distribution beneath two interfering rigid strip foundations resting in frictionless contact with a layer of dense sand underlain by a smooth rigid base. Khing et al. (1992) performed experimental tests on closely spaced strip footings on sand reinforced with a geogrid layer. Al-Ashou et al. (1994) studied the effect of number of reinforcement layers on bearing capacity of closely spaced strip and square footings on reinforced sand experimentally. They reported that the interference effect was less pronounced in square footings, but had a significant effect on bearing capacity of closely spaced strip footings on reinforced sand. Kumar and Saran (2003) studied the pressure, settlement and tilting of adjacent strip and square footings on reinforced sand using model tests and reported similar results. Kumar and Ghosh (2007) investigated the ultimate bearing capacity of two interfering strip footings using the method of stress characteristics. Ghazavi and Lavasan (2008) carried out numerical studies on the bearing capacity of closely spaced rough square footings on geogrid reinforced sand and compared the results with experimental results of other researchers. They examined the effects of distance between reinforcing layers and footings, and the width and depth of reinforcing layers on the bearing capacity using finite difference program FLAC3D. They also studied the distribution of shear strain and displacement in the soil for both reinforced and unreinforced footings. Kumar and Bhoi (2009) investigated the interference effect on the ultimate bearing capacity of two closely spaced strip footings, placed on the surface of dry sand, using small scale model tests. No tilt of the footing was allowed in the tests and the effect of clear spacing (s) between two footings was explicitly studied. They reported that the interference of footings leads to a significant increase in their bearing capacity and the interference effect becomes even more substantial with an increase in the relative density of sand. Lee and Eun (2009) investigated the effects of multiple-footing configurations in sand on bearing capacity, using field plate load tests and finite element analyses. They observed that the load responses of multiple footings are similar to those of the single footing at distances greater than three times the footing width. Ghosh and Kumar (2009) conducted a number of model tests to study the interference effect of two nearby strip footings on dry, reinforced sand. A single layer of uniaxial geogrid was used for reinforcing the sand foundation bed. Kouzer and Kumar (2010) investigated the ultimate bearing capacity of a new strip footing placed on a cohesionless soil medium in the presence of an existing strip footing using an upper bound finite element limit analysis. Ghosh and Sharma (2010) using the theory of elasticity investigated the settlement behavior of two strip footings placed in close spacing on layered soil deposit consisting of a strong top layer underlying a weak bottom layer. Mabrouki et al. (2010) numerically studied the bearing capacity of two interfering strip footings, subjected to centered vertical loads with smooth and rough interfaces, using the finite difference program FLAC. Ghosh and Kumar (2011) studied the interference effect of two near-by strip footings on dry, cohesionless, layered soil, using a number of model tests. The effect of center to center spacing (S) between two footings on their bearing capacity and settlement at failure is the main focus in their research. Nainegali and Basudhar (2011) studied the finite element modeling of two closely spaced footings resting on linearly elastic foundation soil whose modulus of elasticity is either constant or linearly varying with depth. Ghazavi and Lavasan (2012) experimentally investigated the bearing capacity, settlement and the tilt of closely spaced square and circular footings on unreinforced and reinforced sand. Srinivasana and Ghosh (2013) experimentally investigated the interaction between two nearby surface circular footings, by conducting a number of laboratory scaled model tests on a dry, cohesionless Ennore sand bed. The experimental study indicates that the ultimate bearing capacity of interfering footing increases with the decrease in the spacing between the footings. Nainegali et al. (2013) using finite element method (FEM) studied the effect of interference on the settlement of two closely spaced rough rigid strip footings resting on the surface of linearly elastic finite and infinite non-homogeneous soil bed with the modulus of elasticity linearly varying with depth.

4. Material tested

Considering the model footing size and the scale factor effect, relatively fine-grained sand was used for tests. Grain size distribution and other characteristics of the sand are provided in Table 1.

Table 1
Characteristics of the test sand.

Parameter	Value
Effective grain size, D_{10} (mm)	0.55
D_{30} (mm)	1.97
D_{60} (mm)	4.75
D_{85} (mm)	7.79
Coefficient of uniformity (Cu)	8.64
Coefficient of concavity (Cc)	1.48
Dry unit weight (kN/m^3)	16
Angle of internal friction (Ø)	43

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Fig. 1. Placement of beam, gauges and footings in the tests.

This sand was classified as SW in the Unified Soil Classification System. The friction angle was determined using the results of direct shear tests on the soil at a relative density of 50%, which was close to the relative density of the soil in loading tests.

A commercially available biaxial geogrid made of stretched, monolithic polypropylene (PP) flat bars with welded junctions was used as reinforcement element. The geogrid was Secugrid 30/30 Q1, which was produced by NAUE. The elastic normal stiffness of the geogrid (EA) measured in the laboratory was 30 kN/m and had 32 mm × 32 mm size openings. The size of geogrid was 0.95 m × 0.95 m and was placed at a depth of 60 mm. Although the dimension of the geogrid used is realistic compared to the dimension of the model footing, this does not affect the results since only the number of openings which are placed under the foundation varies for the model and large scale footings.

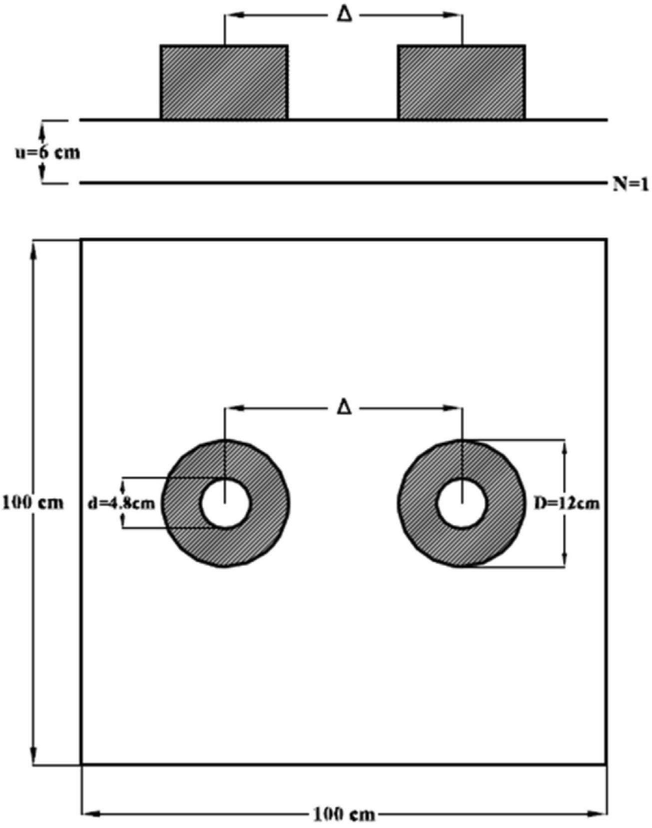

Fig. 2. Schematic figure of footings and dimensions.

5. The apparatus

A cubic box with 1.0 m length, 1.0 m width and 1.0 m height was built to accommodate the sand. The box was prepared using steel in order for it to be stiff enough not to deform under the range of loading applied on the model footing. In order to determine the size of model footings, it was considered that, the outer diameter should be such that the effect of box sides on footing behavior is negligible. For this purpose the preliminary numerical modeling of footings was conducted and the external diameter of footings were considered so that the stress increase in the soil due to footing loading near the boundary would be almost zero. Preliminary results of Plaxis 3D Foundation analysis indicated that an external diameter of 120 mm would be suitable for both ring and circular footings. Other investigators such as Hataf and Razavi (2003), also using experimental results, found that a ratio of inner diameter to outer diameter between 0.2 and 0.4 leads to maximum bearing capacity of model ring footings. Further, Boushehrian and Hataf (2003) using finite element program Plaxis 2D achieved the ratio of inner diameter to outer diameter equal to 0.4 for maximum bearing capacity of ring footings. Hence the external and internal diameter of model ring footings was taken equal to 120 and 48 mm, respectively, giving the value of this ratio as equal to 0.4. The diameter of the model circular footing was taken equal to 120 mm. These dimensions were kept constant for both laboratory experiments and numerical analysis. Model footings had 70 mm thickness and were constructed of rigid plastic. Sand paper was fixed to the base of the footings to achieve a rough base. The loading system was a hydraulic jack and the load was transformed equally to both of the footings using a rigid steel beam.

6. Test procedure

For each loading test, the tank was initially filled by pouring 100 mm thick layers of sand using a raining method. In this method, the layers of soil were poured from a constant height of 200 mm using a funnel. To obtain uniform compaction, each layer was tamped using a steel plate dropping from a 200 mm height 10 times

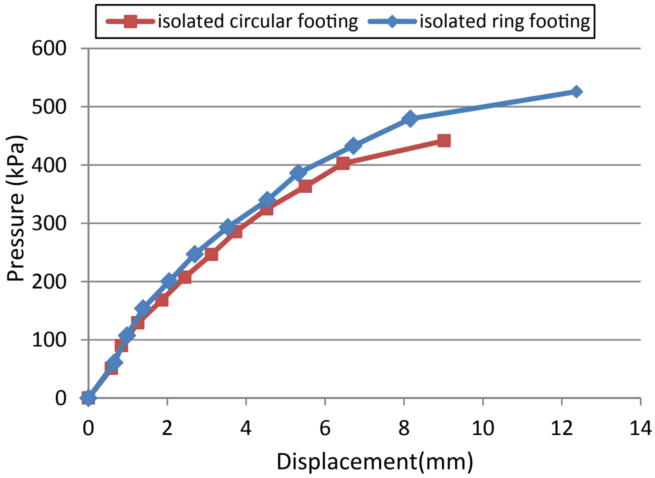

Fig. 3. Laboratory test results for isolated circular and ring footing on unreinforced sand.

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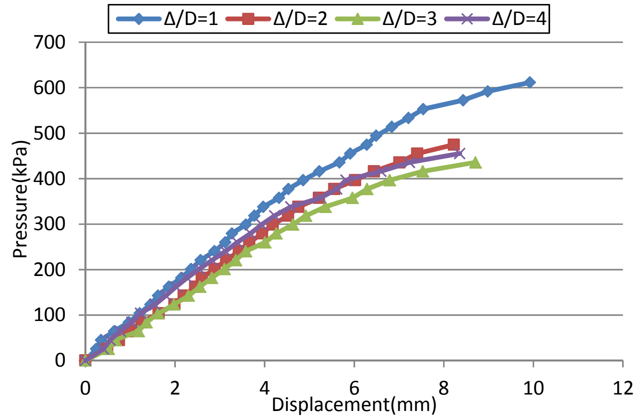

Fig. 4. Laboratory test results for interfering circular footings on unreinforced sand.

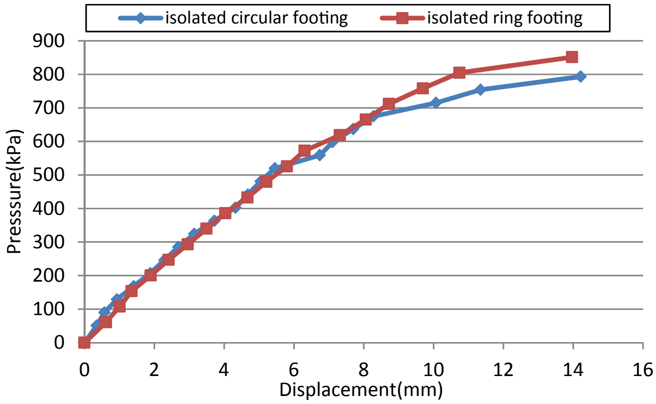

Fig. 6. Laboratory test results for isolated circular and ring footing on reinforced sand.

before the next layer was poured. This compaction procedure resulted in a relative density of about 50 ± 5%. The steel plate used to tamp the sand had dimensions of 0.45 m × 0.45 m length and 5 mm thick.

Displacement of footings was measured using two dial gauges placed at equal distance from the edges of the hydraulic jack. Thus, according to symmetry of system the equal load applied to footings and the average displacement measured by gauges was equal to average settlement of footings. Placement of the beam, gauges and footings in the tests is shown in Fig. 1. It should be noted that, because the beam was placed directly on the footings, they did not permit tilting. While in reality and in the numerical modeling footings can tilt, but as shown in the following, tilting of footings is small and does not have an important influence on the bearing capacity.

The footings were loaded statically using the hydraulic jack and the displacement was measured for each load. The load increased up to soil failure and the load–displacement curve was finally plotted. The bearing capacity was obtained by the tangent method. The tangent method has often been used by the corps of engineers to determine the load that corresponds to the point at which the settlement curve has a significant change in slope. In this method the load corresponding to a distinctive marked change in settlement is chosen. This method is also referred to as the Tangent Intersection Method (Trautmann and Kulhawy, 1988).

A total of 20 tests were carried out using the footings. Initially, isolated circular and ring footings were tested, then closely spaced circular and ring footings were tested. Testing of closely spaced footings included 4 tests for ring footings and 4 tests for circular footings. The center to center distance between footings (Δ) changed in each test and spacings of 12 cm, 24 cm, 36 cm and 48 cm were examined. After these 10 tests on unreinforced sand were conducted, 10 similar tests were carried out on sand reinforced with one layer of geogrid. A schematic figure of footings and dimensions is shown in Fig. 2.

7. Results and discussion

Model footings were loaded with a stress control method in laboratory tests. Displacement controlled loading may provide a better indication of failure and load controlled loading may cause instability as ultimate load occurs, but the tests have been conducted using load controlled method and the loading steps were chosen as small as possible in order to not pass the ultimate load. Pressure–displacement diagrams were drawn and the bearing capacities determined using the tangent method. The horizontal axis of pressure–displacement diagrams of interfering footings show average settlement of two adjacent footings. In order to evaluate the soil reinforcement effects and also the interference effect, two non-dimensional parameters were used. The aforementioned studies have shown that when the sand is reinforced, the bearing capacity of the footing increases. This increase is normally defined in a non-dimensional form, BCR, called the bearing capacity ratio and defined by

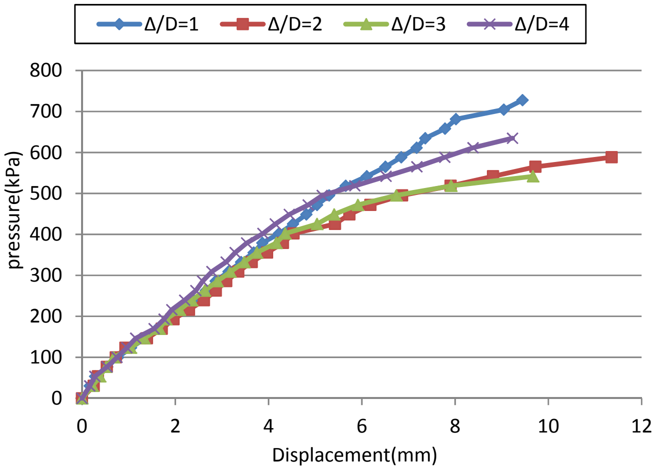

Fig. 5. Laboratory test results for interfering ring footings on unreinforced sand.

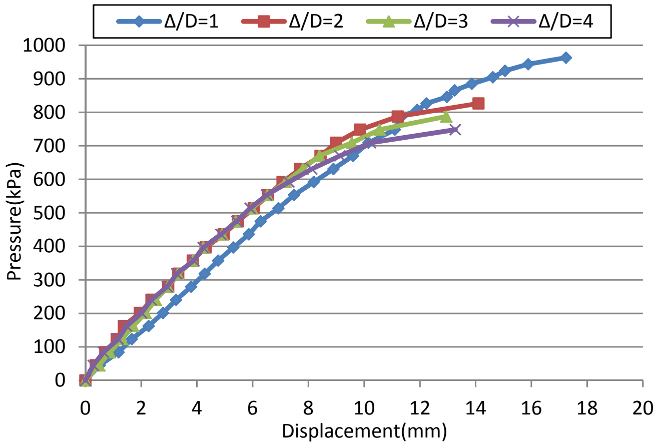

Fig. 7. Laboratory test results for interfering circular footings on reinforced sand.

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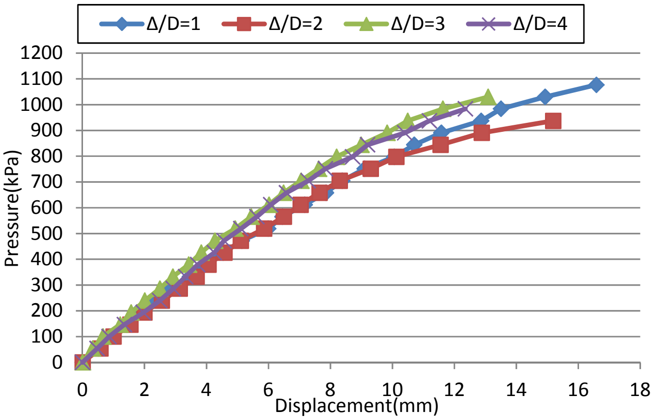

Fig. 8. Laboratory test results for interfering ring footings on reinforced sand.

$$BCR = \frac{qu(\text{reinforced})}{qu(\text{unreinforced})} \tag{1}$$

Where $qu(\text{reinforced})$ is the bearing capacity of an isolated or interfering footing on the reinforced soil with one layer of geogrid, and $qu(\text{unreinforced})$ is the bearing capacity of the same footing on unreinforced soil.

To evaluate the bearing capacity of an interfering footing on reinforced soil, the interference factor, *If* and *I′f*, may be defined as

$$If = \frac{qu(\text{int.}) - N}{qu(\text{single})} \tag{2}$$

$$I'f = \frac{qu(\text{int.}) - N}{qu(\text{single}) - N} \tag{3}$$

Where qu(int.) − N is the bearing capacity of one of two closely spaced footings on reinforced soil with N layer of reinforcement; and $qu(\text{single})$ and $qu(\text{single})\text{-}N$ represent the bearing capacity of the same isolated footing on unreinforced and reinforced soil with N layer of geogrid, respectively.

8. Unreinforced sand

The pressure-settlement diagrams of isolated circular and ring model footings on unreinforced sand are shown in Fig. 3. This figure shows that results of laboratory testing indicates that the bearing capacity of isolated ring footing on unreinforced sand is about 21% greater in comparison to the bearing capacity of isolated circular footing on the same soil; which is similar to results reported by Hataf and Razavi (2003).

Table 2
Summary of laboratory test results.

		Circular footings			Ring footings		
		Bearing capacity (kPa)	$I'f$	BCR	Bearing capacity (kPa)	$I'f$	BCR
Unreinforced	Isolated	357	–	–	432	–	–
	$\Delta = 12$ cm	454	1.272	–	504	1.165	–
	$\Delta = 24$ cm	431	1.207	–	478	1.106	–
	$\Delta = 36$ cm	386	1.082	–	455	1.052	–
	$\Delta = 48$ cm	366	1.025	–	443	1.026	–
Reinforced	Isolated	656	–	1.837	728	–	1.684
	$\Delta = 12$ cm	859	1.309	1.892	820	1.127	1.628
	$\Delta = 24$ cm	725	1.105	1.682	797	1.095	1.667
	$\Delta = 36$ cm	691	1.053	1.788	776	1.067	1.707
	$\Delta = 48$ cm	658	1.003	1.797	743	1.02	1.674

In Fig. 4 the pressure–settlement diagram for interfering circular footings with different spacing on unreinforced sand is shown. For $\Delta/D = 1$ where two footings are placed next to each other, compared to isolated circular footing on the same soil, there is a significant increase in bearing capacity. This increase in bearing capacity is due to the interference effect which adjacent footings apply to each other. With increasing distance between footings, the bearing capacity reduces and for $\Delta/D = 4$ the bearing capacity approaches that of an isolated circular footing on unreinforced sand.

In Fig. 5, behavior of interfering ring footings on unreinforced sand is shown. The behavior of ring footings was similar to circular footings but the bearing capacity of ring footings for each spacing was greater than that for the bearing capacity of circular footings with the same spacing. When two adjacent footings were placed next to each other ($\Delta/D = 1$), the maximum bearing capacity was achieved and with increasing distance between footings, the bearing capacity reduced. For center to center distance between footings equal to 48 cm ($\Delta/D = 4$), it can be seen that footings had very little influence on each other and behave almost like isolated ring footings.

9. Reinforced sand

The results of tests on isolated circular and ring model footings on a single layer of reinforcement are shown in Fig. 6. The bearing capacity of isolated circular footing on reinforced sand had about an 84% increase compared to the bearing capacity of the same isolated footing on unreinforced sand. Also, the bearing capacity of isolated ring footing had about 68% increase in comparison with the bearing capacity of the same isolated ring footing on unreinforced sand. It can be seen that using one layer of geogrid in the foundation soil beneath footings for both ring and circular footings leads to an increase in the bearing capacity, but the effect of reinforcing the

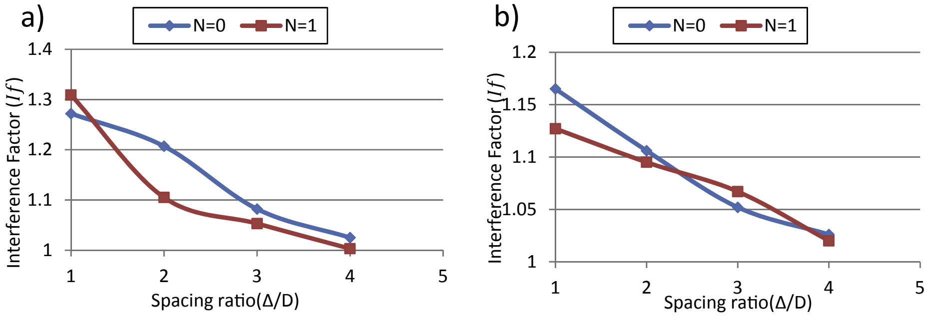

Fig. 9. Variation of *I′f* versus spacing ratio for interfering circular and ring footings on reinforced and unreinforced sand: a) Interfering circular footings b) Interfering ring footings.

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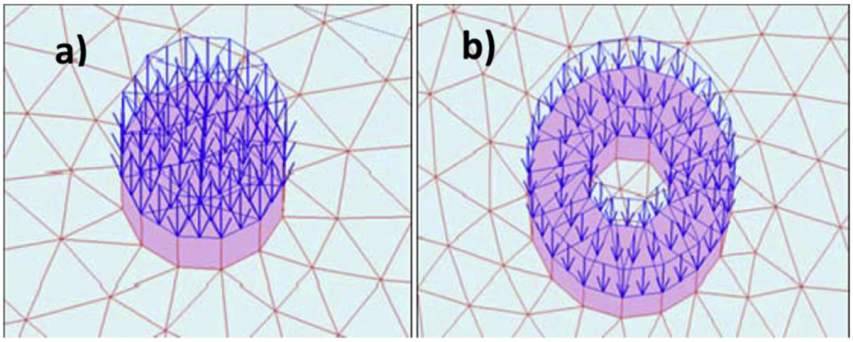

Fig. 10. Diagram showing configuration of footings and loadings in Plaxis program: a) circular footing b)ring footing.

Table 3
Parameters used in the numerical analysis.

Material	Parameter	Value
Sand	Unit weight (kN/m^3)	16
	Cohesion (kPa)	1
	Friction angle (Ø)	43
	Poisson's ratio	0.3
	Modulus of elasticity (kPa)	18,000
Floor (assumed as reinforcement)	Thickness (mm)	1
	Unit weight (kN/m^3)	17
	Modulus of elasticity in X direction (MPa)	1500
	Modulus of elasticity in Y direction (MPa)	15
	Modulus of elasticity in Z direction (MPa)	1500
	Poisson's ratio	0.3
Footing	Unit weight (kN/m^3)	25
	Poisson's ratio	0.3
	Modulus of elasticity (kPa)	2E8

foundation sand on the bearing capacity of the circular footing is more pronounced than for the ring footing.

The results of tests on interfering circular and ring footings on reinforced sand are shown in Figs. 7 and 8, respectively. It can be seen that similar to unreinforced footings, the bearing capacity of reinforced footings also decreases with the increase in spacing ratio from $\Delta/D = 1$ to $\Delta/D = 4$ and for $\Delta/D = 4$, footings had a negligible effect on each other and behave like isolated footings. In all tests of footings that bear on foundations soils that are reinforced, there is an increase in bearing capacity. Moreover, in comparing the settlements of reinforced and unreinforced footings, it can be seen that in any specific stress value, the settlement of footings on reinforced sand is less than the settlement of the same footings on unreinforced sand. Further, regarding the failure stress, the settlement values for the footings on reinforced sand are greater than settlement for footings on unreinforced sand.

10. Interference effect

According to interference effect (I_f) defined in equation (3), the interference effect versus spacing ratio (Δ/D) diagrams can be drawn. In Fig. 9 the interference effect diagram for closely spaced ring and circular footings on reinforced and unreinforced sand are shown. As it can be seen, for footings on reinforced and unreinforced foundations, the maximum interference effect occurs when $\Delta/D = 1$ and with increasing spacing, the interference effect reduces and, finally, in the $\Delta/D = 4$ the value of If is about 1; which means that for this condition, the footings had little influence on each other and almost behave like individual footings. The interference effect equal to 1 means that footings are far enough that they do not effect each other.

This behavior of interference effect may be attributed to the so-called "blocking effect". Because of this effect, the soil between two footings forms an inverted arch and the combined system moves down upon loading as a single unit. The area of this single unit is greater than that of the sum of two footings and results in greater bearing capacity. A summary of laboratory test results can be seen in Table 2.

11. Numerical analysis

The finite element program Plaxis 3D Foundation (version 1.1) was used to model the tests of circular and ring footings on reinforced sand previously described. Plaxis is intended for the analysis of deformation and stability in geotechnical engineering projects.

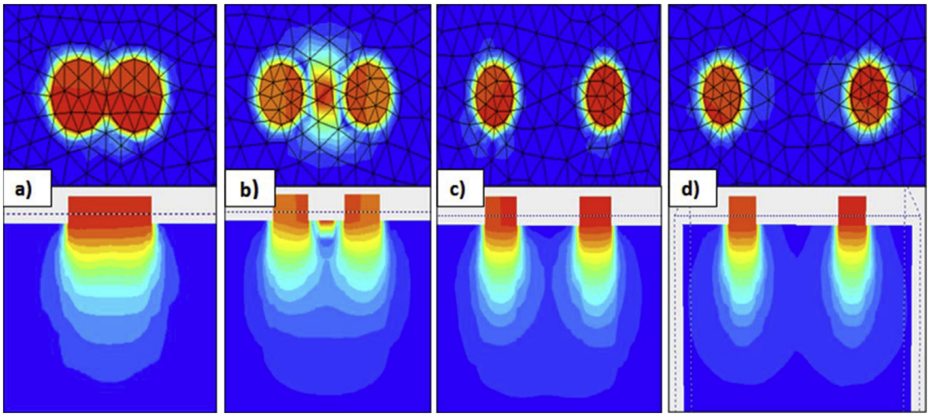

Fig. 11. Total displacement of interfering circular footings in plan and section view for different spacings: a) $\Delta/D = 1$, b) $\Delta/D = 2$, c) $\Delta/D = 3$, d) $\Delta/D = 4$.

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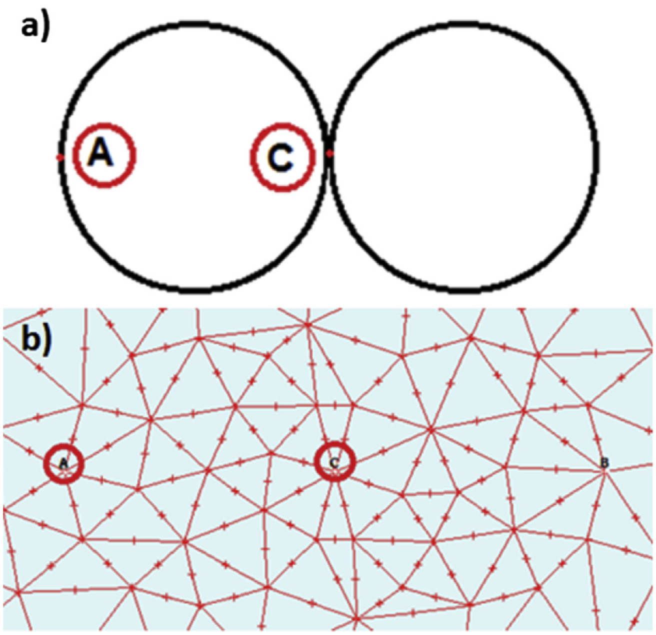

Fig. 12. Location of points A and C a) schematic figure of points location b) points location in Plaxis mesh.

The Mohr–Coulomb model was used for soil and 15-noded triangular elements were used for the analysis. It was found that the mesh density should be more refined around the footings than in the corners of the tank because of the stress concentration on these parts. However, a very coarse mesh can be used around the footings and a coarse mesh can be used in other parts of the tank. Because of the existence of two ring or circular footings in the modeling, the axi-symmetric condition and 2D modeling cannot be utilized. Due to limitations of Plaxis 3D Foundation program, two innovative methods were used in the modeling. Firstly, because circular clusters cannot be drawn in the Plaxis program, footings were defined as rigid piles which were placed on the ground surface. Footing materials were defined so that they were rigid using the Linear-elastic material model in the software. The distributed loads were applied on the surface of these footings. A diagram showing the configuration of the footing and loadings for Plaxis analysis are shown in Fig. 10.

Secondly because Plaxis 3D Foundation program does not allow for modeling the geogrid directly, the reinforcement layer was modeled using the “Floor” option in the program. Reinforcements are slender objects with a normal stiffness but with no bending stiffness. In order to model this behavior, the floor layer was defined with a small thickness and elastic modulus of floor material was assumed very small in depth direction (Y). Parameters used in the numerical analysis are summarized in Table 3.

Total displacement of interfering circular footings on unreinforced foundation, in the plan view and view of sections passing from the centers of both adjacent footings are shown in Fig. 11. In the section view, stresses under footings and the interference effect on them can be seen well. For $\Delta/D = 1$, closely spaced footings behave like one footing with larger width, and stress contours under the footings are quite continual. When spacing between footings increased, stress contours become distant from each other and separate gently, which indicates that effect of footings on each other reduces with increasing distance. For $\Delta/D = 4$, it can be seen that stress contours are not totally separated but have a slight influence on each other, and this confirms the results that the bearing capacities and interference effects had shown.

12. Verifying the footings tilting

In the tests, because the beam was placed directly on the footings, they did not allow for tilting; while in the numerical modeling the footings can tilt due to applied loading. In this section, using Plaxis results the tilting angle has been evaluated. In this case, the tilting of two interfering ring footings with $\Delta/D = 1$, which was one of the conditions that had maximum interference effect, were assessed. At first, two points (A & C) on the line passing through the centers of both interfering footings and on the sides of one of the footings were considered as it is shown in Fig. 12, and then the pressure-displacement diagram for each point was drawn. Using the distance between two points and maximum settlement

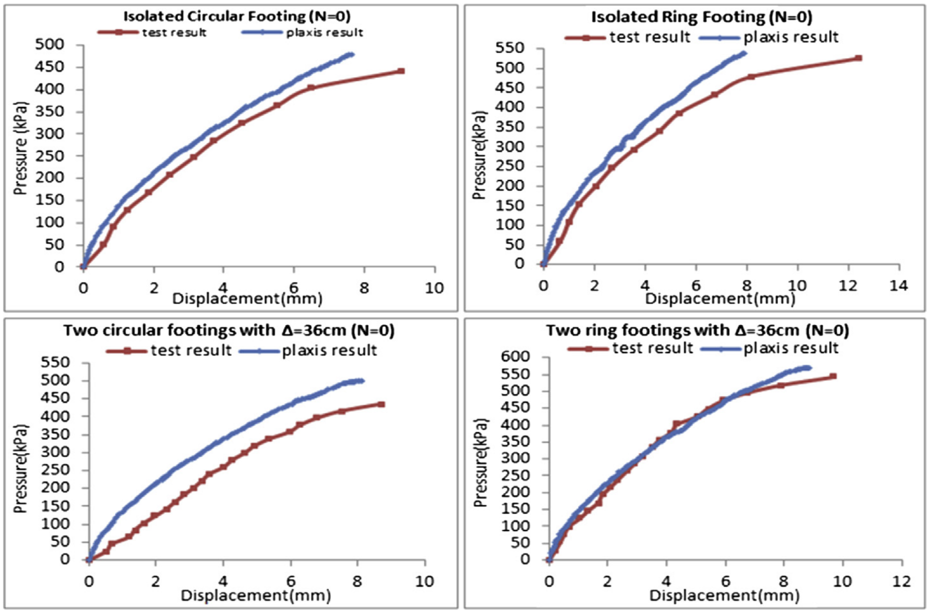

Fig. 13. Comparing Plaxis and laboratory test results for isolated and $\Delta/D = 3$ state on unreinforced soil.

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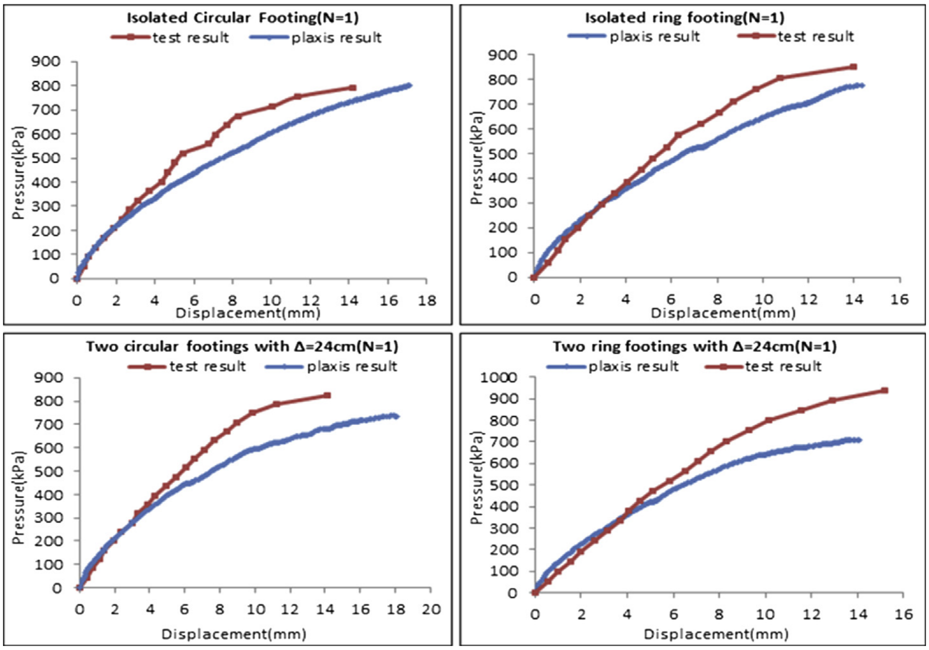

Fig. 14. Comparing Plaxis and laboratory test results for isolated and $\Delta/D = 2$ state on reinforced soil.

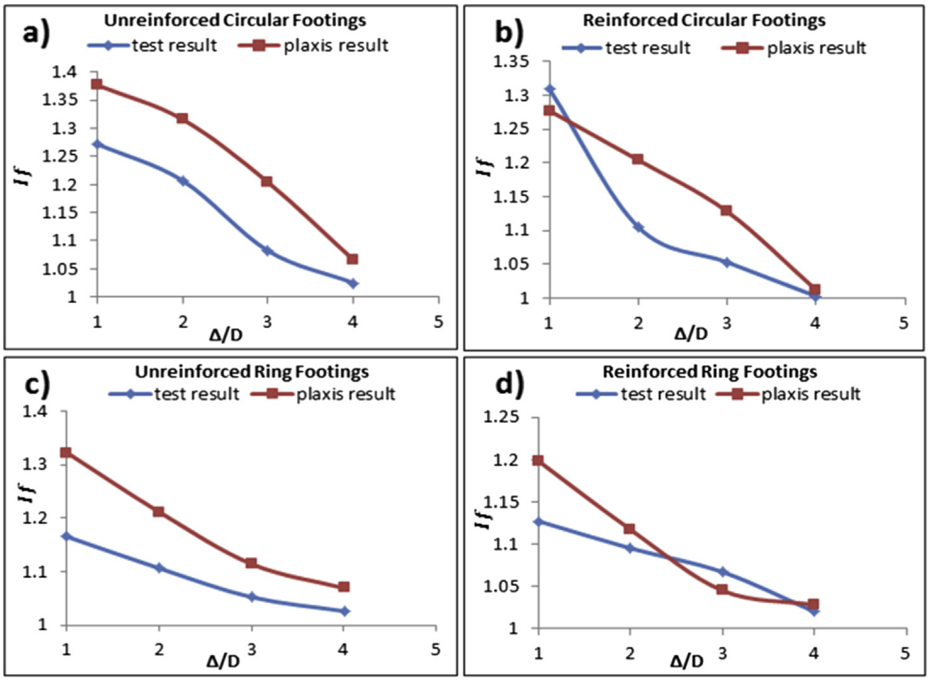

Fig. 15. Comparing Plaxis and laboratory test results of interference factor ($\acute{I}f$) for closely spaced footings: a) Circular footings on unreinforced sand b) Circular footings on reinforced sand c) Ring footings on unreinforced sand d) Ring footings on reinforced sand.

Table 4
Comparison of bearing capacity ratio for isolated footings.

Footing shape	Reference	BCR	Description
Circular	Boushehrian and Hataf (2003)	1.65	Test, $\varnothing = 38^{\circ}$, $\Upsilon = 17$ KN/m^3, EA (geonet) = 28 kN/m, $D = 15$ cm, SW
Ring		1.32	
Circular	Present study	1.84	Test, $\varnothing = 43^{\circ}$, $\Upsilon = 16$ KN/m^3, EA (geogrid) = 30 kN/m, $D = 12$ cm, SW
ring		1.68	

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Table 5
Comparison of interference factor *If* for closely spaced footings.

Footing shape	Reference	N	Δ/D or Δ/B						Description
			1	1.5	2	2.5	3	4	
Strip	Das and Labri-cherif (1983)	0	1.8	2	1.7	1.6	–	–	Test, $Ø = 38°$, $\gamma = 15.88$ KN/m^3, Dr = 54%
	Kumar and Ghosh (2007)	0	2	1.7	1.4	1.2	–	–	Theoretical, mechanism I, $Ø = 35°$
			2	2.5	1.9	1.4			Theoretical, mechanism II, $Ø = 35°$
	Kumar and Saran (2003)	0	2	1.8	1.3	1.2	–	–	Test, $Ø = 37°$, $\gamma = 17$ KN/m^3, Dr = 60%, EA (geogrid) = 20 kN/m, $B = 10$ cm, SP
		1	1.4[a]	2.6[a]	2.3[a]	2.1[a]			
Square	Ghazavi and Lavasan (2008)	0	1.5	1.7	1.9	1.6	–	–	Numerical, $Ø = 35°$
		1	2.1	2.4	2.7	2.2			
	Ghazavi and Lavasan (2012)	0	1.3	1.6	1.2	–	–	–	Test, $Ø = 34°$, $\gamma = 15.1$ KN/m^3, Dr = 40%, $B = 40$ cm, SP
		1	1.6	1.8	1.4				
	Kumar and Saran (2003)	0	1.4	1.9	1.4	1.2	–	–	Test, $Ø = 37°$, $\gamma = 17$ KN/m^3, Dr = 60%, EA (geogrid) = 20 kN/m, $B = 10$ cm, SP
		1	1.1[a]	1.2[a]	1.1[a]	1.1[a]			
Circular	Ghazavi and Lavasan (2012)	0	1.6	1.3	1.2	1.2	–	–	Test, $Ø = 34°$, $\gamma = 15.1$ KN/m^3, Dr = 40%, $B = 40$ cm, SP
		1	1.9	–	1.4	–			
	Lee and Eun (2009)	0	1.8	1.7	1.3	1.2	–	–	Test, $Ø = 35°$, SP-SM
	Present study	0	1.27	–	1.21	–	1.08	1.02	Test, $Ø = 43°$, $\gamma = 16$ KN/m^3, EA (geogrid) = 30 kN/m, $D = 12$ cm, SW
		1	2.41		2.03		1.94	1.84	
Ring	Present study	0	1.17	–	1.11	–	1.05	1.03	Test, $Ø = 43°$, $\gamma = 16$ KN/m^3, EA (geogrid) = 30 kN/m, $D = 12$ cm, SW
		1	1.90		1.84		1.80	1.72	

[a] Values correspond to $I'f$ (Eq. (3)).

difference between two diagrams and according to rigidity of footings, the maximum tilting degree of footings can be attained as shown below:

$$\text{Point A}: x = 120\text{mm}$$

$$\text{Point C}: x = 0\text{mm}$$

$$\text{Maximum settlement difference} = 0.334\text{mm}$$

$$\text{Tilting angle (degree)} = \tan^{-1}\left(\frac{0.334}{120}\right) = 0.159°$$

So it can be seen that even in the maximum interference effect state, the tilting angle of footings was very small and assuming it was unobserved did not have significant influence on the results of bearing capacity. As the available facilities in the laboratory were limited and the footings size was small, if the footings had the possibility to tilt, the eccentric loading was probable and could cause significant errors in the results.

13. Comparison between the experimental and numerical results

The load-displacement curves for some cases obtained experimentally and numerically are compared in Figs. 13 and 14 for footings on unreinforced and reinforced sand, respectively. As can be seen in Fig. 13 numerical and experimental load-displacement curves for some cases of circular and ring footings on unreinforced sand are in relatively good agreement. For the unreinforced soils, the results indicate that load-displacement curves from Plaxis are slightly above the curves obtained from laboratory tests; which means that for the unreinforced sand at a given stress load, numerical modeling predicts settlements that are less than the laboratory test results. In Fig. 14 similar curves for some cases with reinforced foundation soil are shown. As can be seen, the results are similar to an acceptable content. For the reinforced soils it is observed that load-displacement curves from tests are above the curves from the numerical modeling. This may be due to the soil parameters such as modulus of elasticity and methods used in the analysis and also the values of displacement measured in the laboratory.

Comparison between experimental and numerical results for interference factor ($I'f$) of ring and circular footings can be seen in Fig. 15. Plaxis results also indicate that for both circular and ring footings, the maximum interference effect takes place when footings are next to each other. In the numerical results like the laboratory test results, it can be seen that with increase in spacing between footings, interference effect decreases and in spacing ratio equal to 4 ($\Delta/D = 4$) the interference effect is almost eliminated.

The bearing capacity ratio (BCR) of ring and circular footings on reinforced sand from Boushehrian and Hataf's (2003) research is compared with the results of this article and presented in Table 4. In both researches the bearing capacity ratio of the circular footing is greater than that of the ring footing. The difference in the values may refer to reinforcement tensile strength and sand properties.

The values of interference factor *If*, for two closely spaced footings, are presented in Table 5. As observed in the table, the interference factor of the strip footings is almost greater than that of the other shaped footings. This may be due to the plain strain condition of such foundations. Almost all of the research works show that the bearing capacities of square and strip footings are maximized at the critical spacing. This is in contrast to that which occurs for round-shaped closely spaced foundations where the maximum bearing capacity was observed when two footings were placed exactly beside each other.

14. Conclusions

In this study experimental and numerical investigation on the bearing capacity of closely spaced circular and ring footings on reinforced and unreinforced sand was performed.

Based on the results of this study the following conclusions are obtained:

- The bearing capacity of closely spaced small-scale model footings on unreinforced and reinforced sand increases due to interference effects.
- The bearing capacity of closely spaced footings increases with the provision of a reinforcement layer in the foundation soil beneath the footing, and the effect of reinforcing in the case of circular footings is more, when compared to ring footings ($BCR_{\text{circ.}} > BCR_{\text{ring}}$.

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- The ultimate bearing capacity of two closely spaced circular and ring footings is maximum when they are next to each other and decreases with increasing in spacing ratio (Δ/D).
- For around $\Delta/D > 4$, no significant interference effect was observed and each footing more or less acted independently, similar to a single isolated footing.
- The bearing capacity of ring footings was observed to be greater than for circular footings for the same test conditions.

It should be noted that the results presented in this study are related to model footings on a sand foundation and are limited to these conditions and the effect of some other parameters such as scale effect, density of soil, type of reinforcement, etc. have not been investigated.

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